EML RESEARCH REPORT NO. 1

COMPENSATION OF THE EFFECTS OF THE SLOT IN A RECTANGULAR WAVEGUIDE

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This research was supported by
the National Aeronautics and Space Administration
and partially funded under NGR-34-002-047

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Raleigh, North Carolina

October 1966

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Summary - A method of compensating the effects of a slot in a rectangular waveguide on the propagation properties of the TE_{10} wave mode has been developed. The compensation consists of an extension of the side walls of the slot into the interior of the waveguide. The extended walls form ridges which deform the field configuration in the waveguide in such a manner that the impedance and propagation properties of the waveguide approach those of the non-slotted guide.

The cross-sectional field configuration of the compensated-slot structure is evaluated by conformal mapping using a TEM-wave approach and the transformation function derived. Compensation criteria are derived which lead to relationships between the dimensions of the compensation ridges and the slot width of the rectangular guide for specific cross-sectional ratios a/b. As an example, the compensation of X-band waveguides is treated numerically.

INTRODUCTION

The propagation properties of a rectangular wave guide excited in the TE_{10} wave mode are affected by a longitudinal slot in the center of the broad wall of the guide in two ways. The presence of the slot causes on one hand a change of the propagation constant and, on the other hand, a change of the characteristic impedance or matching properties of the guide. If such a slotted wave guide is applied in components of microwave circuitry or as a slotted section of a standing-wave or immittance meter, reflections and errors will be introduced by the slot. A compensation of the effects of the slots is necessary if reflections by the various components of the circuitry and if the error of the standing-wave meter have to be kept low.

The compensation of the slot is particularly important in the latter case of a standing-wave meter with a probe moving in the slot along the slotted section. Such an instrument can be used as an absolute impedance or matching standard for a particular wave guide cross-section if sufficiently accurate compensation of the effects of the slot can be accomplished. Its characteristic impedance and the properties as a reference guide are then uniquely defined by the geometric dimensions of the rectangular guide disregarding the slot.

The objective of this paper is the presentation of results of studying the possibilities of compensating the slot effects with emphasis on one specific approach. After introductory considerations, the paper deals in particular with:

(1) A description of the method of compensation.

(2) With the derivation of relationships for describing the field configuration in the compensated structure by conformal mapping and the derivation of the essential equations.

(3) Presentation of a method of approximation for obtaining the dimensions of the compensating structure for given waveguide and slot dimensions.

(4) Presentation of numerical results for an X-band waveguide as an example.

The effects of a slot in a rectangular waveguide were considered in the past and relationships shown for the change of the guide wavelength [1, 2] and for an equivalent admittance representing the slot at cutoff wavelength of the guide [2]. No attempts were made, however, to introduce a compensation. The probable reason for this was the fact that the changes of the propagation properties caused by the slot were considered negligeably small at the time. The increased sophication and complexity of present-day microwave circuitry and particularly the high accuracies required in measurements, make, however, compensation necessary today, particularly in the upper frequency region of microwaves.

In a slotted section of a waveguide, the guide wavelengths λ_g' in the slotted section differs from that of the non-slotted guide λ_g by a relative amount [1]

$$\frac{\Delta \lambda_g}{\lambda_g} = \frac{1}{8\pi} \left(\frac{d}{a}\right)^2 \frac{\lambda_g^2}{d\,b}, \tag{1}$$

where d is the width of the slot and a, b are the width and height of the rectangular guide respectively. The slotted wall of the guide is assumed to be very thick. The effect of the slot can also be expressed by a hypothetical equivalent change of the relative guide width $\Delta a/a$ given by [1]

$$\frac{\Delta a}{a} = \frac{1}{2\pi}\left(\frac{d}{a}\right)^2 \frac{a}{b} \; . \tag{2}$$

The characteristic impedance of the guide is hence increased by a relative amount slightly higher than that of the relative wavelength as indicated in [1].

For the compensation of the effects of the slot, there exist two simple ways, namely, either by reducing slightly the height or increasing slightly the width of the guide in accordance with the above equations. The equations indicate, however, that these compensations are frequency dependent due to the frequency dependence of λ_g. Hence, other methods of compensation have to be considered.

Method of Compensation

A practically frequency independent slot compensation can be achieved by extending the walls of the slot into the interior region of the waveguide as indicated in Figure 1b. The two extended walls deform the field configuration inside the waveguide in such a manner that the impedance and propagation characteristics of the compensated waveguide approach those of the non-slotted guide. Figure 1 shows the approximate field configurations for the noncompensated and compensated slotted guides for comparison. The field configurations indicate that the electric flux of the slotted guide is reduced in the slotted region which leads to a reduced capacitance per unit length of the guide. This decrease is compensated in the structure of Figure 1b by the increase of the capacitance caused by the two ridges on both sides of the slot. The same rules applied to the magnetic field indicate an increase of the inductance per unit length in the slot region which also is reduced in the field structure of Figure 1b by the two ridges. The conformal mapping of these field configurations into those between parallel conducting walls will later confirm these findings.

Conformal Mapping of the Compensated-Guide Contour

The method of conformal mapping is applied here to yield relationships which can be applied for the determination and description of the field configuration in the compensated slotted guide and for finding relationships for the dimensions of the compensating ridges as a function of slot width. The transformation is carried out in two steps as indicated in Figure 2. The figure shows on the left the contour of one-half of the symmetric, slotted and compensated section in the complex z-plane ($z = x + jy$). This contour is first transformed by use of the Schwartz-Christoffel Theorem into the real axis of the complex t-plane ($t = r + js$). An additional mapping operation transforms the field in the upper half of the t-plane, which corresponds to the interior region of the wave guide between the contours in the z-plane, into the field between parallel boundaries in the complex w-plane (w = u + jv).

In the present derivation, the assumption is made first that the width of the wave guide (in the direction of the y-coordinate in the z-plane) is infinite, so that the field distribution is identical to that in a parallel-strips line excited in the TEM wave mode with a centrally located, compensated, longitudinal slot. This also means that the field distribution is the same as in the case of static fields. With this assumption the compensated-slot structure can be transformed conformally directly into an unperturbed parallel-strip line. The approximation is justified since the slot and the compensated ridges are located in the center of the rectangular wave guide in the region where the sinusoidal distributions of the transverse compenents of the electric and magnetic field intensities have maxima and vary only slightly. In the rectangular waveguide, the magnetic field intensity has also a longitudinal compenent, but its magnitude is zero in the center and relatively small in the central region where the compensated slot is located, so that this component of the field is practically not affected by the slot and the compensation. This fact also justifies the TEM-wave and static-field approach. In the later course of the derivation, sidewalls will be assumed at a distance a/2 from the center and the computations carried out accordingly.

The characteristic points along the contour of the cross section are indicated in Figure 2 by A B C D E. The same letters are used for the corresponding points in the other complex mapping planes. The electric field inside this contour is caused by a potential difference between the walls, i. e. the sections of the contour A to B and B C D E. The line A to B is contained in the symmetry plane of the wave guide and follows an electric field line between these two points. In the w-plane, the electric field lines are straight lines in the u-direction between the contours A E and B E. The magnetic field lines are perpendicular and point in the b direction. They coincide with the lines representing constant electric potentials. In the t-plane the corresponding lines are hyberbolas and ellipses. The transformation between the z- and t-planes gives then the field lines in the actual wave guide cross-section indicated by its contour in the z-plane. We observe that the x-axis is the axis of symmetry of the actual total guide cross-section with a width a and height b. The width of the slot is d and the height of one of the compensating ridges is Δh. The ridge represented by the section of the contour from C to D is indicated in Figure 2 by two lines but it is assumed that it has practically zero thickness.

Essential Relationships

The Schwartz-Christoffel transformation [3] maps the region inside the contour A B C D E onto the upper half of the t-plane. The mapping gradient between the z- and t-plane is found to be

$$\frac{dz}{dt} = K_1 \frac{t - r_1}{t\sqrt{(t-r_1)(t-r_2)}}. \qquad (3)$$

According to this theorem, complex values of z within the contour are related to those on the upper half of the t-plane by the following integral

$$z = K_1 \int (t-t_1)^{-\delta_1/\pi} (t-t_2)^{-\delta_2/\pi} (t-t_3)^{-\delta_3/\pi} \cdots dt + K_2 ,$$

where the values t_ν represent the values of t in the characteristic points and where δ_ν represents the angular difference of the contour directions in the characteristic point ν in the z-plane. Insertion into the integral and evaluation yields

$$z = K_1 \left(\cosh^{-1} \xi + \frac{r_1}{\sqrt{r_2}} \cos^{-1} \eta \right) + K_2 , \qquad (4)$$

where

$$\xi = \frac{2t + (1 - r_2)}{1 + r_2} , \qquad (5)$$

and

$$\eta = \frac{t(1-r_2) - 2r_2}{t(1+r_2)} . \qquad (6)$$

In a similar manner, the mapping gradient between the w- and the t-plane can be found. It is

$$\frac{dw}{dt} = K_3 \frac{1}{\sqrt{t(t+1)}} . \qquad (7)$$

Integration yields

$$w = K_3 \cosh^{-1}(2t+1) + K_4 . \qquad (8)$$

The next step consists in the determination of the various constants. The constants K_1 and K_3 can be found by evaluating the functions z and w in the point F. Herewith one proceeds in the t-plane along the real axis toward infinity where, for an infinite value of $|t|$, the path follows a circle with infinite radius in the upper half plane to $t = -\infty$ and continues then along the real axis toward A. Going along the half circle corresponds to moving in the z-plane from the line D E to E A covering a distance -b.

The mathematical description of the procedure is given by the following equations. First

$$t = R e^{j\phi} \tag{9}$$

has been introduced and R has been made to approach infinity. Substituted into Eqs. (3) and (7), the mapping gradients become

$$\frac{dz}{dt} = K_1 \frac{1}{t} \quad \text{and} \quad \frac{dw}{dt} = K_3 \frac{1}{t} . \tag{10a}$$

Differentiation of Eq. (9) and substitution into (10a) yields

$$dz = K_1 j \, d\phi ,$$

so that

$$\int_{\text{at E on DE}}^{\text{to E on EA}} dz = -b = j \begin{Bmatrix} K_1 \\ K_3 \end{Bmatrix} \int_0^{\pi} d\phi , \tag{10b}$$

and hence

$$K_1 , K_3 = j \frac{b}{\pi} . \tag{10c}$$

An analogous evaluation of the complex function z in point B where t approaches zero and proceeding along a half-circle [redacted] around B in the t-plane yields a relationship for the factor $r_1/\sqrt{r_2}$ in front of the second term of Eq. (4), namely

$$\frac{r_1}{\sqrt{r_2}} = \frac{d}{2b} . \tag{10d}$$

Considering the functions z and w in point B yields the integration constants $K_3 = K_4 = b$. With these constants, the equations for z and w are

$$z = j \frac{b}{\pi} \left(\cosh^{-1} \xi + \frac{d}{2b} \cos^{-1} \eta \right) + b , \tag{11a}$$

and

$$w = j \frac{b}{\pi} \cosh^{-1} (2t + 1) + b . \tag{11b}$$

Equation (11b) can be inverted to give an expression for t in terms of w,

$$2t = \cosh\left[-j\pi\left(\frac{w}{b} - 1\right)\right]. \tag{12}$$

Insertion into Eqs. (5) and (6) and substitution into (11a) permits writing the complex function z in the form

$$z = x + jy = F(w) = F(u + jv). \tag{13}$$

This function then describes the electric and magnetic field lines in the slotted compensated parallel-strip line in the z-plane. The complete equation is

$$z = j\frac{b}{\pi}\left\{\cosh^{-1}\left(\frac{1-r_2}{1+r_2} + \frac{\cosh\left[-j\pi\left(\frac{w}{b}-1\right)\right]}{1+r_2}\right) + \right. \tag{14}$$
$$\left. + \frac{r_1}{\sqrt{r_2}}\cos^{-1}\left[\frac{1-r_2}{1+r_2} - \frac{4r_2}{1+r_2}\left(\cosh\left[-j\pi\left(\frac{w}{b}-1\right)\right]\right)^{-1}\right]\right\} + b.$$

The equation actually represents the mapping function between the complex z- and w-planes in Figure 2. The electric field lines in the slotted section are obtained by keeping v constant and varying u between the limits 0 and b. The lines of constant electrical potential which coincide with the magnetic field lines are obtained by keeping u constant and varying v. For large values of v, the mapping function becomes a constant factor.

Criteria for the Compensation

Before we can consider a compensation of the slot and derive equations which permit the computation of the proper height of the ridges Δh, it is necessary to discuss briefly the effects of the slot from the view point of the derived relationships. It is obvious that the derived equation (14) is also applicable to the description of the field distribution in the presence of a slot only. Herewith the assumption is made that $\Delta h = 0$ and $r_1 = r_2$. We thus can map the slotted strip line in the z-plane (where now C and D coincide) into a non-slotted line in the w-plane as indicated at the right in Figure 2. If we do this, a slotted line of width a will be transformed into a non-slotted line of width a - Δa, which is slightly narrower due to the effects of the slot. Correspondingly, a slotted wave guide of width a will have the same propagation properties as a non-slotted guide of width a - Δa where Δa is given by Eq. (2).

If we now introduce the compensation ridges and increase Δh from 0 to finite values, the effects of the slot will decrease with increasing Δh until full compensation is achieved. In this process of compensation, the value of Δa which indicates the reduction of the width of the transformed stripline will become smaller until it reaches zero for full compensation. If the height of the ridges is additionally increased, an over-compensation occurs which will increase the equivalent width of the non-slotted waveguide to a + Δa.

The physics concept of the compensation can also be considered qualitatively by evaluating the field configurations in Figure 1. The sizes of the equivalent squares between the field lines of the left-hand configuration give a clear indication of the reduced width a - Δa (increased size of the squares). In the case of the compensation in the right-hand figure, the squares, particularly those in the lower row, have approximately the original size typical for the non-slotted configuration. This indicates that the width of the compensated slotted guide is identical to that of the non-slotted guide.

The preceding considerations indicate that the height of the ridges formed by the extended slot walls, Δh, should be such that the width of the contour enclosing the field configuration indicated in Figure 2 in the z-plane should be identical to that of the transformed configuration in the w-plane (in the jv direction). This condition will henceforth serve as a criterion for the compensation.

The formulation of these conditions is shown in Figure 3. It shows at the right the contour A B F G which indicates one half of the unslotted rectangular waveguide cross-section. This contour represents in the w-plane the transform of the left hand contour A B C D F G in the z-plane which represents one half of the cross-section of the compensated slotted structure. The transformation is carried out by Eq. (14).

EQUATIONS FOR THE COMPENSATED STRUCTURE

The mathematical formulation of the criteria for the compensated structure is indicated in the following equations:

$$w = j\frac{a}{2} = j\frac{b}{\pi}\cosh^{-1}(2t_a + 1), \tag{15}$$

and

$$z = j\frac{a}{2} = j\frac{b}{\pi}\left[\cosh^{-1}(-\xi_a) + \frac{r_1}{\sqrt{r_2}}\cos^{-1}\eta_a\right],$$

$$\xi_a = \frac{1-r_2}{1+r_2} + \frac{2t_a}{1+r_2}\ ;\ \eta_a = \frac{1-r_2}{1+r_2} - \frac{2r_2}{t_a(1+r_2)}. \tag{16}$$

The equations are evaluated by the following procedure. For a given ratio a/b of the rectangular guide, Eq. (15) yields a specific value of t_a. This value is substituted into Eq. (16) which in turn yields a relationship between r_1 and r_2. In combination with Eq. (10d), $r_1/\sqrt{r_2} = d/(2b)$, it permits determination of r_1 and r_2. Since Eq. (16) is transcendental, it is convenient to evaluate the equations in diagram form for a succession of values of r_2 determining ξ and η and from those

$$\frac{r_1}{\sqrt{r_2}} = \frac{\cosh^{-1}(2t_a + 1) - \cosh^{-1}(-\xi_a)}{\cos^{-1}\eta_a} . \tag{17}$$

For each value of r_2 the value of r_1 and d/2b can then be found. This means that r_1 and r_2 are known as a function d/2b which is one half of the relative width of the slot for a given ratio a/b.

The next step is the determination of the relative height of the compensation ridges $\Delta h/b$. For doing this, the transformation function is needed for the contour between B and D (see Figure 2). Evaluation of the transformation gradient in this region yields

$$z = \frac{b}{\pi}\left[-\cos^{-1}\xi + \frac{r_1}{\sqrt{r_2}}\cosh^{-1}(-\eta)\right] + b + j\frac{d}{2} . \tag{18}$$

Use of this equation in the point C and D of the z-plane (see Figure 2) for which the values of t in the t-plane are r_1 and r_2, gives

$$\Delta h = z_D - z_C = -\frac{b}{\pi}\left[-\cos^{-1}\xi_c + \frac{r_1}{\sqrt{r_2}}\cosh^{-1}(-\eta_c)\right] \tag{19}$$

where

$$\xi_C = \frac{1-r_2}{1+r_2} + \frac{2r_1}{1+r_2} \;;\; \eta_C = \frac{1-r_2}{1+r_2} - \frac{2r_2}{r_1(1+r_2)} ,$$

Evaluation of Eq. (19) for the originally assumed value of r_2 and the found value of r_1 gives finally $\Delta h/b$ for each value of r_2 and the corresponding value of d/2b. The results permit plotting $\Delta h/b$ as a function of d/2b in diagram form.

NUMERICAL EXAMPLES

Numerical examples computed by using the above equations are shown in Table I and Figure 4. The computed data are valid for the X-band waveguide RG 52/U, with outer dimensions 1 x 1½ inches and with inner dimensions .9 x .4 inches. The ratio a/b is 2.25 and the corresponding value of t in the t-plane is $t_a = -9.125$. The table shows the values of the quantities essential for computing the data for this cross-sectional ratio. Figure 4 shows in diagram form the relationship between the relative height of the ridges $\Delta h/b$ and one half of the relative slot width d/2b for full compensation.

Conclusion

Relationships were derived for finding the dimensions of compensation ridges in a slotted rectangular wave guide section. The ridges are formed by extending the slot walls into the interior of the wave guide. The field deformation caused by the ridges compensates the effects of the slot on the guide wave length and impedance. Such a compensation is necessary in precise microwave components and in slotted lines for standing-wave and immittance measurements particularly at the higher frequencies of the microwave region and at millimeter waves where the width of the slot is in the magnitude of the width of the guide and where the effect of the slot becomes very pronounced.

The derivations are based on a TEM-wave approach and the equations and results represent hence approximations. Since the slots in waveguides usually are kept narrow and since the effects of the slot are of second order magnitude, first order approximations of the relationships for the compensation seem to give satisfactory accuracy. If higher accuracy is desired, this can be achieved by taking the averages of the transformed values of z in the points F and G (see Figure 3) setting them equal to a/2 instead of using the value of z in G directly and equating it to a/2. The results indicate that the relationship between the height of the compensated ridges and the width of the slot is linear in the region of small slot width.

REFERENCES

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2 "Waveguide Handbook," edited by N. Marcuvitz. Vol. X of MIT Radiation Lab. Series. McGraw Hill Book Co., Inc., 1951.

3 S. A. Schelkunoff: "Applied Mathematics for Engineers and Scientists," pp. 282-300. D. Van Nostrand Co., Inc., 1948.

TABLE I.

$2t_a = -18.25$					
b	ξ	η	r_1	$r_1/\sqrt{r_2}$	$\Delta h/b$
0.02	-16.93	0.9651	0.0099	0.0705	0.0239
0.05	-16.47	0.9152	0.0248	0.1108	0.0376
0.10	-15.77	0.8381	0.0492	0.1555	0.0529
0.15	-15.13	0.7677	0.0732	0.1889	0.0643
0.20	-14.54	0.7032	0.0968	0.2164	0.0737
0.30	-13.49	0.5891	0.1430	0.2611	0.0892

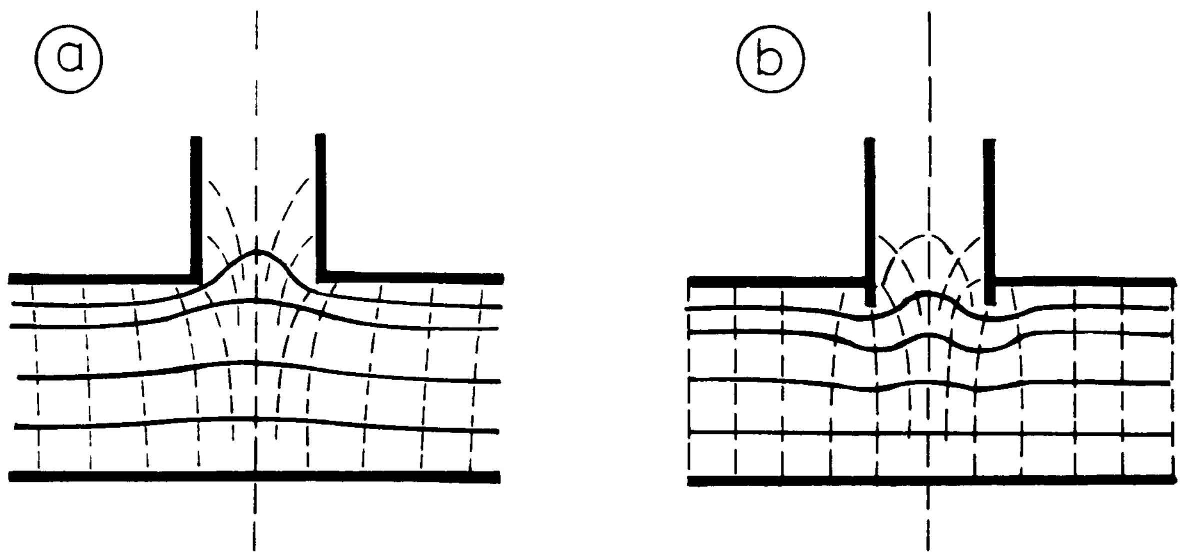

Figure 1 - Approximate field configurations in slotted strip lines.
(a) Slot only.
(b) with compensation.

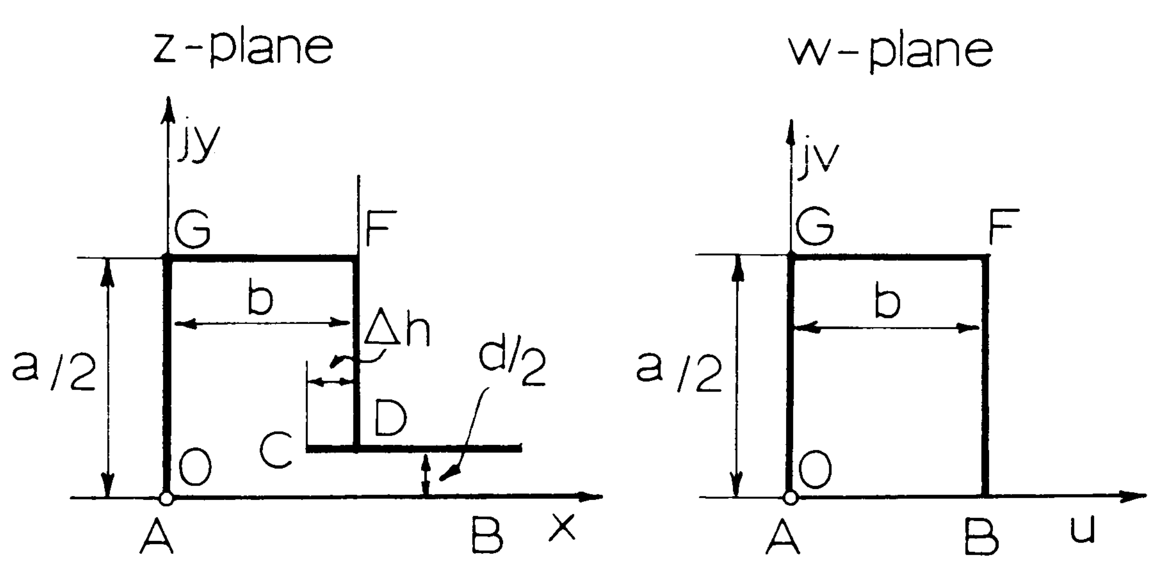

Figure 3 - Conformally mapped contours of partial waveguide cross-sections.

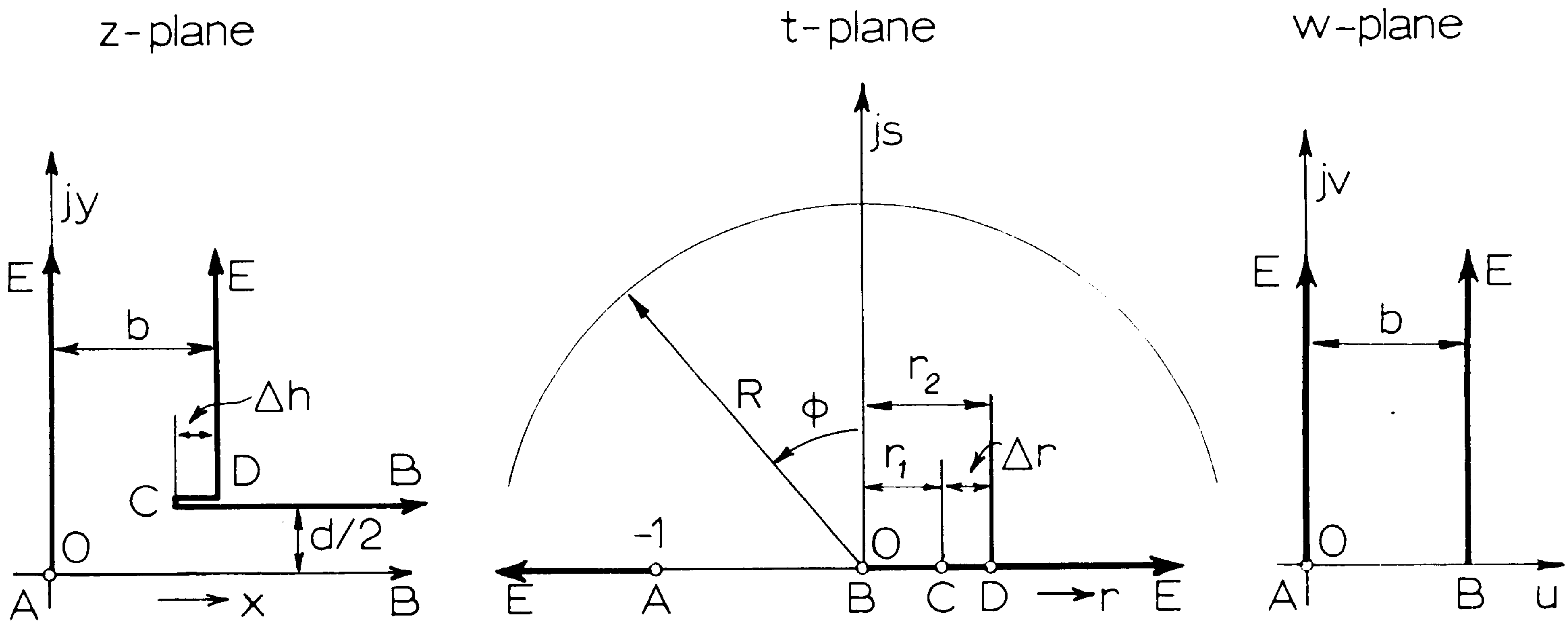

Figure 2 - Conformal contour configurations in the complex z-, t-, and w-planes.

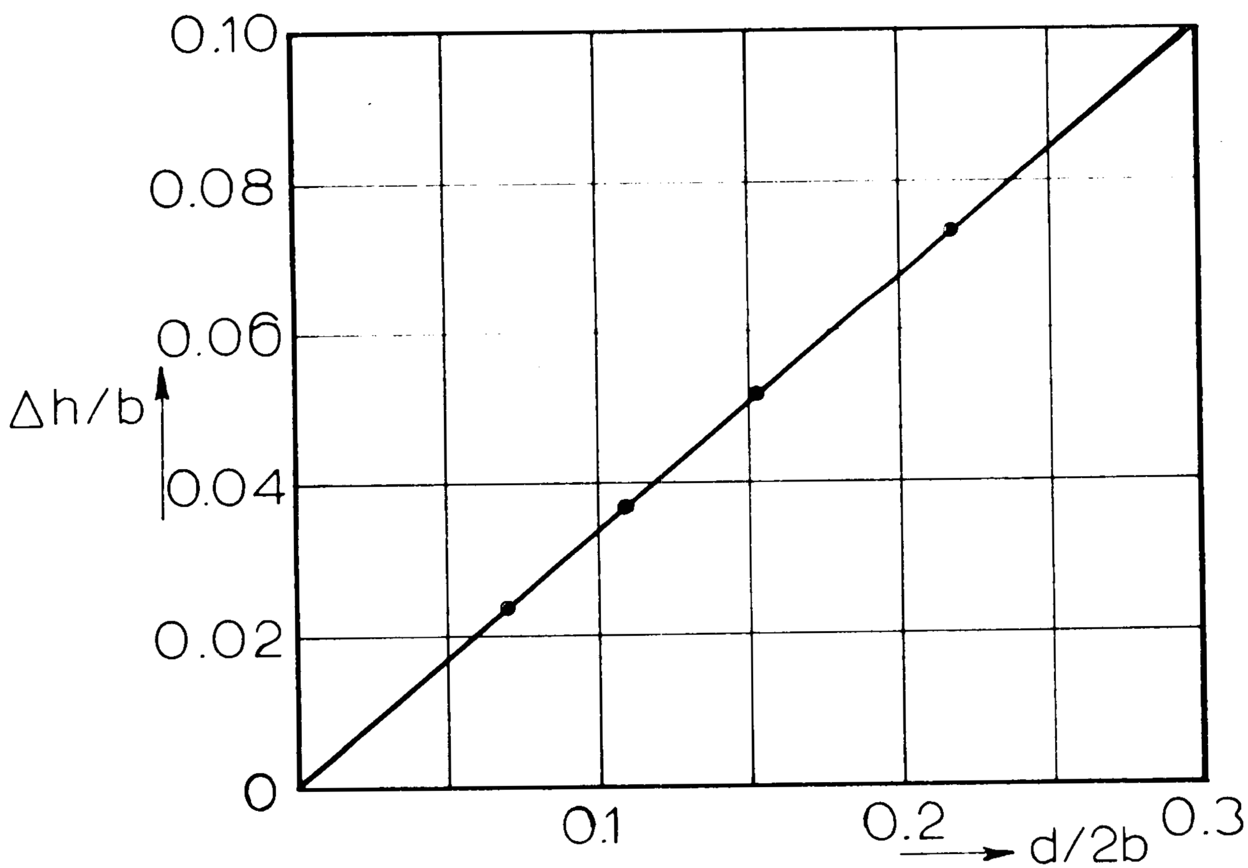

Figure 4 - Relationship between relative height of the compensation ridges and relative slot width.