# An Angular Correlation Test of Time Reversal Invariance* <br> Richard A Kuebbing ${ }^{\dagger}$ and Karl J. Casper Department of Physics, Western Reserve University, Cleveland, Ohio 

The discovery of the violation of CP-invariance in the $K_{2}^{0}$ decay has led to the suggestion that $T$-invariance may be violated in electromagnetic interactions. ${ }^{1}$ Several tests have been devised by Henley and Jacobson ${ }^{2}$ utilizing ordinary gamma-ray angular correlation and beta-gamma-gamma triple correlation experiments. The purpose of this Letter is to point out an additional gamma-ray angular correlation experiment which is sensitive to the T-violating amplitude. Although it is necessary to measure three separate correlations to determine this amplitude, the experiment is relatively simple in that these are the ordinary two-gammaray correlations.

The experiment proposed requires a three gamma-ray cascade as shown in Fig. 1. The second transition must be mixed, although, for this experiment, the mixing ratio may have almost any value. Experimentally, it is only necessary to measure the second Legendre polynomial coefficient for the $1-2,2-3$, and 1-3 gamma ray angular correlations.
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If the third radiation is unmixed, then the coefficient $A_{8}(2-3)$ for the angular correlation between the second and third gamma rays can be written ${ }^{3}$

$$
\begin{align*}
& A_{2}(2-3)=\left[F_{2}\left(L_{2} L_{8} j_{2} j_{3}\right)+2 \delta_{2} F_{3}\left(L_{R} I_{8}^{\prime} j_{2} j_{3}\right)\right. \\
& \left.\quad+\delta_{2}^{2} F_{2}\left(L_{2}^{\prime} L_{2}^{\prime} j_{2} j_{3}\right)\right] F_{2}\left(L_{B} L_{3} j_{4} j_{3}\right) /\left(1+\delta_{2}^{a}\right) \tag{1}
\end{align*}
$$

In a similar fashion, the coefficient $A_{z}(1-2)$ is given by

$$
\begin{aligned}
& A_{2}(1-2)=\left[F_{2}\left(L_{1} I_{1} J_{1} J_{2}\right)+2 \delta_{1} F_{2}\left(L_{1} L_{2}^{\prime} J_{2} J_{3}\right)\right.
\end{aligned}
$$

$$
\begin{align*}
& \left.+8_{2}^{2} F_{8}\left(L_{8}^{\prime} L_{8}^{1} j_{3} J_{2}\right)\right] /\left(1+\delta_{i}^{2}\right)\left(1+\delta_{2}^{B}\right) \tag{2}
\end{align*}
$$

The phases have been chosen so that the term linear in 8 , the mixing ratio, is positive if the transition is the first one in the double gamma-ray cascade whose correlation is being measured. As a number of authors have pointed out ${ }^{4}$, this term is then opposite in sign when the transition is the second one in the cascade. The sign change has been incorporated into equations (1) and (2).

The coefficient $A_{0}(1-3)$ is given by

$$
\begin{gather*}
A_{2}(1-3)=\left\{\left[F_{2}\left(L_{2} L_{2} j_{1} j_{3}\right)+2 \delta_{1} F_{2}\left(L_{1} L_{1}^{\prime} j_{1} j_{2}\right)+\delta_{1}^{2} F_{2}\left(L_{1}^{1} L_{1}^{\prime} j_{1} j_{3}\right)\right]\right. \\
\left(2 j_{2}+1\right)^{\frac{1}{2}}\left(2 j_{3}+1\right)^{\frac{1}{2}}\left[W\left(j_{2} j_{2} j_{3} j_{3} ; 2 L_{2}\right)-\delta_{2}^{3} W\left(j_{2} j_{2} j_{3} j_{3} ; 2 L_{2}^{\prime}\right)\right] \\
(-1)^{\left.L_{2}-j_{2}-j_{3} F_{2}\left(L_{3} L_{3} j_{4} j_{3}\right)\right\} /\left(1+\delta_{1}^{2}\right)\left(1+\delta_{2}^{2}\right)} \tag{3}
\end{gather*}
$$

Both equations (2) and (3) involve characteristics of the first transition, but this can be eliminated by forming the ratio ${ }^{5}$

$$
\begin{align*}
& R= A_{2}(1-2) / A_{8}(1-3)=\left[F_{2}\left(L_{8} L_{2} j_{3} j_{2}\right)-2 \delta_{2} F_{3}\left(L_{8} L_{8}^{\prime} j_{3} j_{3}\right)\right. \\
&\left.+\delta_{2}^{a} F_{2}\left(L_{8}^{f} L_{8}^{\prime} j_{3} j_{2}\right)\right] /\left\{(-1) L_{2}-j_{2} j_{3}\left[\left(2 j_{2}+1\right)\left(2 j_{3}+1\right)\right]^{\frac{1}{6}}\right. \\
& {\left.\left[W\left(j_{2} J_{3} J_{3} j_{3} ; 2 L_{8}\right)-\delta_{3}^{2} W\left(j_{2} J_{2} j_{3} j_{3} ; 2 L_{6}^{\prime}\right)\right] F_{2}\left(L_{8} I_{8} j_{4} j_{3}\right)\right\} } \tag{4}
\end{align*}
$$

If time reversal invariance is not valid, then the mixing ratio will be complex, that is $\delta=|\delta| e^{i \eta}$

$$
\text { where } \eta=\eta\left(L_{8}\right)-\eta\left(L_{6}^{\prime}\right)
$$

and is different from $0^{\circ}$ or $180^{\circ}$.
Equations (1) and (4) can be combined in two ways:

$$
A_{8}\left(1+\delta_{2}^{2}\right)+\operatorname{RGF}_{2}\left(L_{8} I_{B} J_{4} J_{3}\right)=2\left[F_{3}\left(L_{8} I_{8} J_{3} J_{2}\right)\right.
$$

$$
\begin{equation*}
\left.+8_{2}^{8} F_{2}\left(L_{6}^{\prime} L_{8}^{\prime} J_{3} J_{2}\right)\right] / F_{2}\left(L_{8} L_{3} J_{4} J_{3}\right) \tag{5}
\end{equation*}
$$

$A_{2}\left(1+\delta_{2}^{2}\right)-R F_{3}\left(L_{3} L_{B} j_{4} j_{3}\right)=4 F_{2}\left(L_{8} L_{2}^{1} j_{3} j_{2}\right)\left|\delta_{2}\right| \cos \eta / F_{3}\left(L_{B} L_{3} j_{4} j_{3}\right)$
where
$G=(-1)^{L_{8}-j_{2}-j_{3}}\left[\left(2 j_{2}+1\right)\left(2 j_{3}+1\right)\right]^{\frac{1}{2}}\left[W\left(j_{2} j_{2} j_{3} j_{3} ; 2 L_{8}\right)-88_{2}^{2} W\left(j_{a} j_{2} j_{3} j_{3} ; 2 L<d\right]\right.$

The unknown quantities $A_{0}(2-3)$ and $R$ are measured experimentally.
Equation (5) can be solved for $\left|\delta_{2}\right|$ if the spins of the nuclear levels are known, and this value.then used in equation (6) to determine $\eta$.

The disadvantage of this method over others proposed is that $\eta^{3}$, rather than $\eta$, is determined. Therefore, $T$-noninvariant amplitudes less than a few per cent are difficult to detect. On the other hand, only a two gamma-ray angular correlation is being measured, and $A_{2}$ coefficients can be measured very accurately. In addition, this method is rather insensitive to the magnitude of the mixing ratio. Since a precision measurement of the coefficient is necessary, considerable care must be taken in
evaluating the finite solid angle correction. It has been recently ${ }^{6}$ shown that the usual calculation using the method of Rose ${ }^{7}$ can be inaccurate by several per cent.

A survey of the literature uncovered many cases where two of the three necessary correlations had been measured, but not all three. Some of the more favorable cases are shown in Table 1 , with the appropriate transitions indicated. In general, most of the cases were $2(M 1, E 2) 2(E 2) 0$ transitions, but the only real requirement is that the second transition in the cascade be mixed. In looking for other cases, it is probably best to avoid transitions from a second $2+$ collective state to a first $2+$ excited state since many of these transitions are pure electric quadrupole and have a low intensity. Finally, it is not necessary that the first transition in this cascade be a gamma ray, since all of the characteristics of this transition are canceled out, nor that it be a single gamma ray since the coupling of several gamma rays as the first transition in the angular correlation expressions is linear.


Table 1

| Parent Nucleus | Cascade | Spin Sequence | Correlations Measured |
| :---: | :---: | :---: | :---: |
| $\mathrm{Fe}^{57}(\mathrm{n}, \gamma)$ | 8.345-0.81-0.805 | 0,1-2-2-0 | 2-3 |
| Ga ${ }^{74}$ | 1.72-0.60-0.596 | J-2-2-0 | 2-3 |
|  | $\beta-0.60-0.596$ | 3-2-2-0 | 2-3 |
|  | 1.00-0.60-0.596 | J-2-2-0 | 2-3 |
| $\mathrm{Br}^{82}$ | 0.6-10-0.698-0.777 | 3-2-2-0 | 1-2 |
|  |  |  | 2-3 |
| $\mathrm{Rh}^{100}$ | 1.58-0.820-0 538 | J-2-2-0 | 2-3 |
|  | 1.11-0.820-0.538 | J-2-2-0 | 2-3 |
|  | 1.23-0.613-0.513 | - $\mathrm{T}-2-2-0$ | 2-3 |
|  | 1.14-0.613-0.513 | J-2-2-0 | 2-3 |
|  | 0.82-0.613-0.513 | J-2-2-0 | 2-3 |
|  | 1.20-1.050-0.513 | J-2-2-0 | 2-3 |
|  | 0.78-1.050-0.513 | J-2-2-0 | 2-3 |
|  | 0.74-1.050-0.513 | J-2-2-0 | 2-3 |
| $\mathrm{Tb}^{156}$ | 0.5352-1.2253-0.1992 | 4-4-4-2 | 1-2 |

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