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NASA Contract

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GENERAL ELECTRIC

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CALCULATIONS OF OPTIMUM SEED CONCENTRATIONS IN CESIUM AND POTASSIUM NON-EQUILIBRIUM PLASMAS*

By

J. Marlin Smith
F. H. Shair

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MISSILE AND SPACE DIVISION

GENERAL ELECTRIC

Abstract

The electrical conductivity of a gas can be enhanced by the use of a mixture of a parent gas of small electron elastic collision cross section (typically a noble gas) seeded with a gas of low ionization potential (typically an alkali metal). Further enhancement for a given gas temperature is possible through the creation of a non-equilibrium condition where the electron temperature is higher than the gas temperature. In this paper an extension for the optimum seed concentration in a non-equilibrium plasma is obtained for conditions applicable to MHD power generators or positive columns of discharges. This result is applied to Cesium and Potassium seeds and it is shown that the concentration of seed at optimum conductivity is one to two orders of magnitude lower than it is in the equilibrium plasma.

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I. INTRODUCTION

The electrical conductivity of a gas can be greatly increased by the addition of a small amount of an easily ionized "seed" material and also by the creation of a non-equilibrium condition of the electrons. The creation of such non-equilibrium conditions (where the average electron temperature is higher than the average neutral particle temperature) is being widely studied in near atmospheric pressure plasmas where it has direct application to the field of closed cycle MHD power generation.

The optimum in the seed concentration occurs as a result of the fact that the electron elastic collision cross-section is much greater for the seed (typically an alkali metal) than for the parent gas (typically a noble gas) so that the increase in conductivity resulting from the increase in electron density due to seed addition is eventually offset by the increase in electron collision frequency. Rosa[1] has briefly discussed the optimum seed concentration in an equilibrium plasma. Russians Zimin and Popov[2,3] have greatly extended the analysis of the optimum composition of a gas mixture, also at equilibrium. Recently Ellington[4] observed an order of magnitude reduction from the theoretical optimum in his experiments on low-voltage diffuse discharges in noble gases seeded with alkali metals. This reduction was attributed to non-equilibrium effects. The theoretical analysis presented below tends to substantiate Ellington's conclusion.

II. ANALYSIS

The scalar electrical conductivity is expressed in its usual form as

$$\sigma = \frac{n_e e^2}{m\nu} \qquad (1)$$

where n_e is the free electron concentration, e is the unit electric charge, m is the mass of an electron, and ν is the total collision frequency given by the sum of contributions from electron seed atom collisions, electron-parent gas atom collisions, and electron-ion collisions.

$$\nu = bQ_{es}(n_s - n_e)T_e^{1/2} + bQ_{en}n_n T_e^{1/2} + cn_e T_e^{-3/2} \ln\left(d\frac{T_e^3}{n_e}\right) \qquad (2)$$

where b, c, and d are constants which will be discussed later, Q_{es} = electron-seed atom cross section, Q_{en} = electron-parent gas atom cross section, n_s = seed atom density, n_n = parent gas atom density, T_e = average electron temperature. cgs units are used throughout and for simplicity of calculation the cross sections are taken to be constant.

The electron density is assumed to be appropriately represented by Saha's equation evaluated at the electron temperature

$$\frac{n_e^2}{n_s - n_e} = \varphi(T_e) \qquad (3)$$

where for an alkali metal

$$\varphi(T_e) = 2.4147 \times 10^{15}\, T_e^{3/2}\, e^{-T_i/T_e} \qquad (4)$$

and T_i is the ionization potential in $^{\circ}$K.

The electron temperature is given by the electron heating equation which is obtained by equating the joule heating of the electrons to their energy loss due to collisions. For an MHD generator with segmented electrodes this equation takes the form

$$\frac{T_e}{T} = 1 + \frac{f}{\nu^2} \qquad (5)$$

where T is the gas temperature and f is a constant. This is only approximately true due to the mass disparity of the seed and parent gas. However, for the cases considered the reduction in seed concentration appears sufficient to make the approximation a valid one at least for low degrees of ionization. The above expression is also valid in the positive column of a gaseous discharge.

Differentiating equations (1), (2), (3), (4) and (5) with respect to n_s we obtain

$$\frac{d\sigma}{dn_s} = \frac{e^2}{m\nu}\frac{dn_e}{dn_s} - \frac{n_e e^2}{m\nu^2}\frac{d\nu}{dn_s} \tag{6}$$

$$\frac{d\nu}{dn_s} = bQ_{es}\left(1 - \frac{dn_e}{dn_s}\right)T_e^{1/2} + \frac{1}{2}\nu\frac{1}{T_e}\frac{dT_e}{dn_s} + cT_e^{-3/2}\,\ell n\left(d\frac{T_e^3}{n_e}\right)\frac{dn_e}{dn_s}$$

$$- 2n_eT_e^{-3/2}\,\ell n\left(d\frac{T_e^3}{n_e}\right)\frac{1}{T_e}\frac{dT_e}{dn_s} + cn_eT_e^{-3/2}\left(3\frac{1}{T_e}\frac{dT_e}{dn_s} - \frac{1}{n_e}\frac{dn_e}{dn_s}\right) \tag{7}$$

$$\frac{dn_e}{dn_s} = \frac{\varphi}{\sqrt{\varphi^2 + 4n_s\varphi}} - \frac{1}{2}\left[1 - \frac{\varphi + 2n_s}{\sqrt{\varphi^2 + 4n_s\varphi}}\right]\frac{d\varphi}{dT_e}\frac{dT_e}{dn_s} \tag{8}$$

$$\frac{d\varphi}{dT_e} = \left(\frac{3}{2} + \frac{T_i}{T_e}\right)\frac{1}{T_e}\varphi \tag{9}$$

$$\frac{dT_e}{dn_s} = -2T\frac{f}{\nu^3}\frac{d\nu}{dn_s} \tag{10}$$

The condition for optimum seed concentration is obtained from equation (6) by setting $\frac{d\sigma}{dn_s} = 0$. Equations (7) - (10) are then substituted into this condition to obtain, after considerable algebraic manipulation

$$\frac{n_n}{n_s} = \frac{1}{\alpha} = \frac{Q_{es}}{Q_{en}} \left\{ \frac{-1 + \frac{2}{\beta\alpha}\left(-1 + \sqrt{1+\beta\alpha}\right)\sqrt{1+\beta\alpha} + 2\left(1 - \frac{T}{T_e}\right)\left(1 + \frac{T_i}{T_e}\right)\left[1 - \frac{2}{\beta\alpha}\left(-1 + \sqrt{1+\beta\alpha}\right)\right]}{2 - \frac{T}{T_e}} \right\}$$

$$+ \frac{c}{bQ_{en}T_e^2}\left[3\left(1 - \frac{T}{T_e}\right)\ln\left(d\,\frac{T_e^3}{n_e}\right) - 6\left(\frac{7}{6} - \frac{T}{T_e}\right)\right]\frac{\frac{2}{\beta\alpha}\left(-1 + \sqrt{1+\beta\alpha}\right)}{2 - \frac{T}{T_e}} \qquad (11)$$

where

$$\beta = \frac{4n_n}{\varphi}$$

III. DISCUSSION OF RESULTS

In the equilibrium case $T_e = T$ and for slight degrees of ionization, i.e., $\beta\alpha >> 1$, we obtain

$$\frac{n_n}{n_s} = \frac{1}{\alpha} = \frac{Q_{es}}{Q_{en}} \tag{12}$$

which is the common expression given for the seed concentration. In the non-equilibrium case, i.e., $T_e > T$, and for low degrees of ionization, $\beta\alpha >> 1$, we obtain

$$\frac{n_n}{n_s} = \frac{1}{\alpha} = \frac{Q_{es}}{Q_{en}} \left[\frac{1+2\left(1 - \frac{T}{T_e}\right)\left(1 + \frac{T_i}{T_e}\right)}{2 - \frac{T}{T_e}} \right]$$

or

$$\alpha = \frac{Q_{en}}{Q_{es}} \frac{2 - \frac{T}{T_e}}{1+2\left(1 - \frac{T}{T_e}\right)\left(1 + \frac{T_i}{T_e}\right)} \tag{13}$$

which for $T_i >> T_e >> T$ yields

$$\alpha = \frac{Q_{en}}{Q_{es}} \frac{T_e}{T_i}$$

and since $T_i/T_e \cong 10$ we obtain the order of magnitude decrease in α for the non-equilibrium case as observed by Ellington.

In the more general case of arbitrary $\beta\alpha$ and electron temperature equation (11) has been solved numerically for various gas temperatures and pressures. In Figures 1 - 8 the ratio of the non-equilibrium optimum seed concentration as given by equation (11) to the equilibrium optimum seed concentration as given by equation (12) is plotted as a function of electron temperature. It is noted from equations 11 and 12 that this ratio is independent of the parent gas cross section but not of the seed cross section.

This is unfortunate since this cross section is not well known, e.g., see Figure I of reference 6. Therefore the results have been plotted for two values of the alkali metal cross section, 1×10^{-14} cm^2 and 6×10^{-14} cm^2. Furthermore at low electron densities the gas will tend to behave like a Lorentz gas while at higher electron densities the electrons will tend to Maxwellianize about their own temperature. Therefore the results have been computed for these two models. The constants b, c, and d being 7.32×10^5, .53, and 1.53×10^8 respectively for the Lorentz gas and 6.21×10^5, 1.79, and 1.53×10^8 respectively for the Maxwellianized gas, i.e., collision frequencies are computed by averaging over a Maxwellian distribution.

In Figure I the ratio is plotted for Potassium seed in the Lorentz gas approximation as a function of electron temperature for a gas temperature of 1000°K and gas pressures of .15, 1, 15 atmospheres. It is noted that for the larger cross section the ratio is of order .1 and is relatively independent of the gas pressure and electron temperatures. However, for the smaller cross section the ratio is found to be sensitive to both the gas pressure and electron temperature and to decrease with increasing electron temperature and decreasing gas pressure. This decrease is due to electron-ion collision effects which are represented by the second term of equation (11). It can be seen that at equilibrium, i.e., $T_e = T$, this term is negative and therefore would result in an increase in the optimum seed concentration. However, for the non-equilibrium case and the regime considered here $\ln\left(d \frac{T_e^3}{n_e}\right) > 2$ and hence the term makes a positive contribution thereby reducing the optimum concentration. Also since this term varies inversely with the seed cross section it is more pronounced the smaller the cross section is. This effect is also evident in the other figures.

In Figure 2 the ratio is plotted for Potassium seed in the Lorentz gas approximation for a gas pressure of 1 atmosphere and gas temperatures of 700, 1000, 1300, and 1600 $^\circ$K. The general character of Figure I with regard to the cross section is evident and as one might expect for $T_e >> T$ the result for a given cross section becomes independent of the gas temperatures.

In Figures 3 and 4 the gas conditions represented in Figures 1 and 2 respectively are presented for the Maxwellianized gas approximation. The general character of the previous curves is evident, however, the previously described effect of electron-ion collisions is more pronounced. This is due to the fact that $\left.\frac{c}{b}\right|_{\text{Lorentz}} < \left.\frac{c}{b}\right|_{\text{Maxwellian}}$.

Figures 6 - 8 represent the conditions of Figures 1 - 4 respectively for Cesium seed. A comparison shows that there is very little change in the overall results.

IV. CONCLUSIONS

From the above analysis it is concluded that the effect of operation in the non-equilibrium mode is to reduce the concentration of seed material necessary to obtain optimum electrical conductivity. Calculations show this reduction to be one to two orders of magnitude; to be sensitive to electron-seed atom cross section and gas pressure; and to be nearly independent of gas temperature; electron-parent gas atom cross section, and seed material. The reduction in seed concentration is a relatively important result in that it shows that a MHD generator operating in the non-equilibrium mode will operate optimally at a lower seed concentration than the equilibrium generator which should greatly reduce the corrosion problem inherent with alkali metal atmospheres.

V. ACKNOWLEDGEMENTS

The authors wish to express their appreciation to Miss M. Singer for carrying out the programming of numerical calculations for the IBM 1620 computer.

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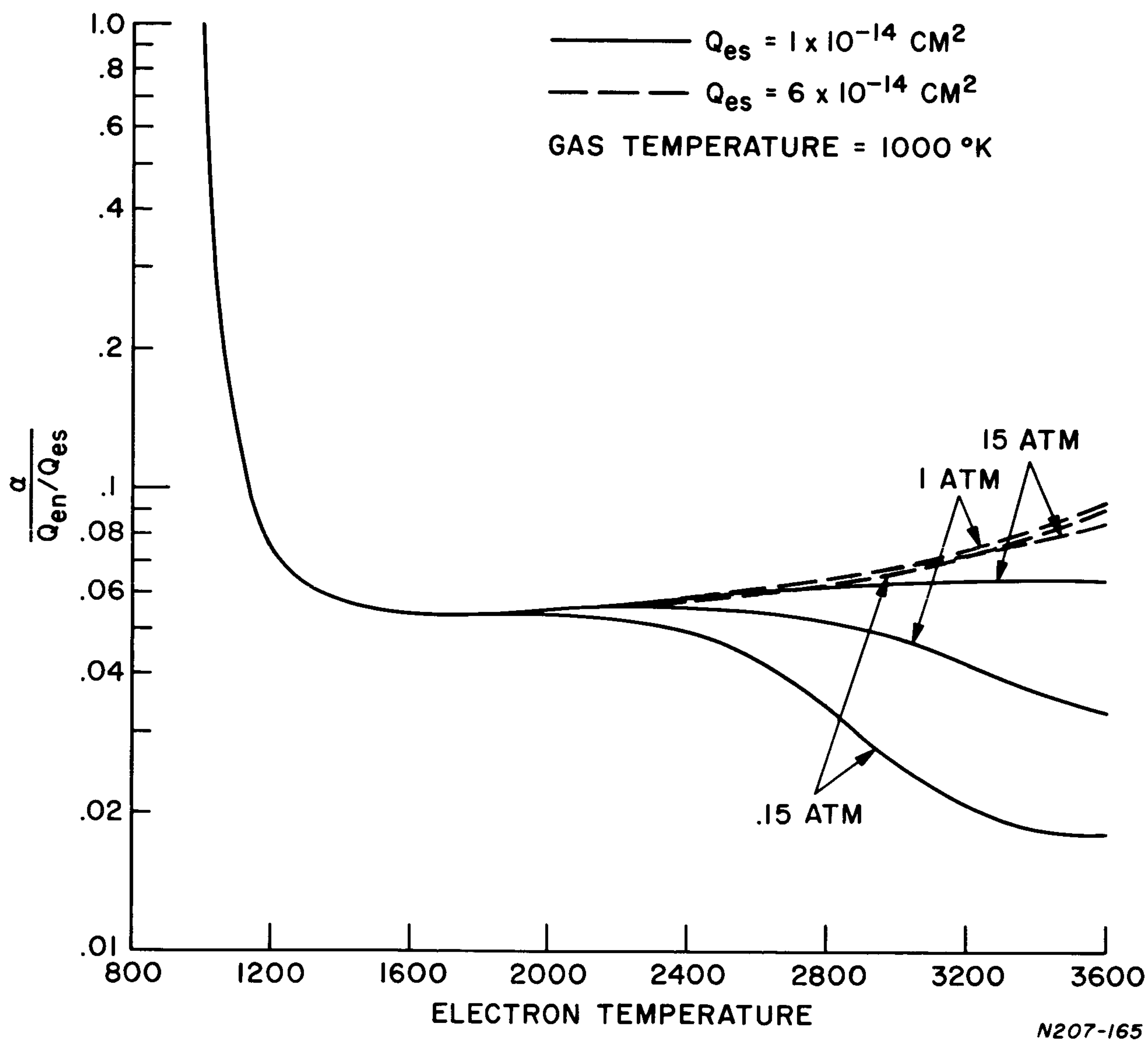

Figure 1. Ratio of Non-equilibrium to Equilibrium Seed Concentration Vs. Electron Temperature as Function of Gas Pressure - Potassium Seed, Lorentz Gas

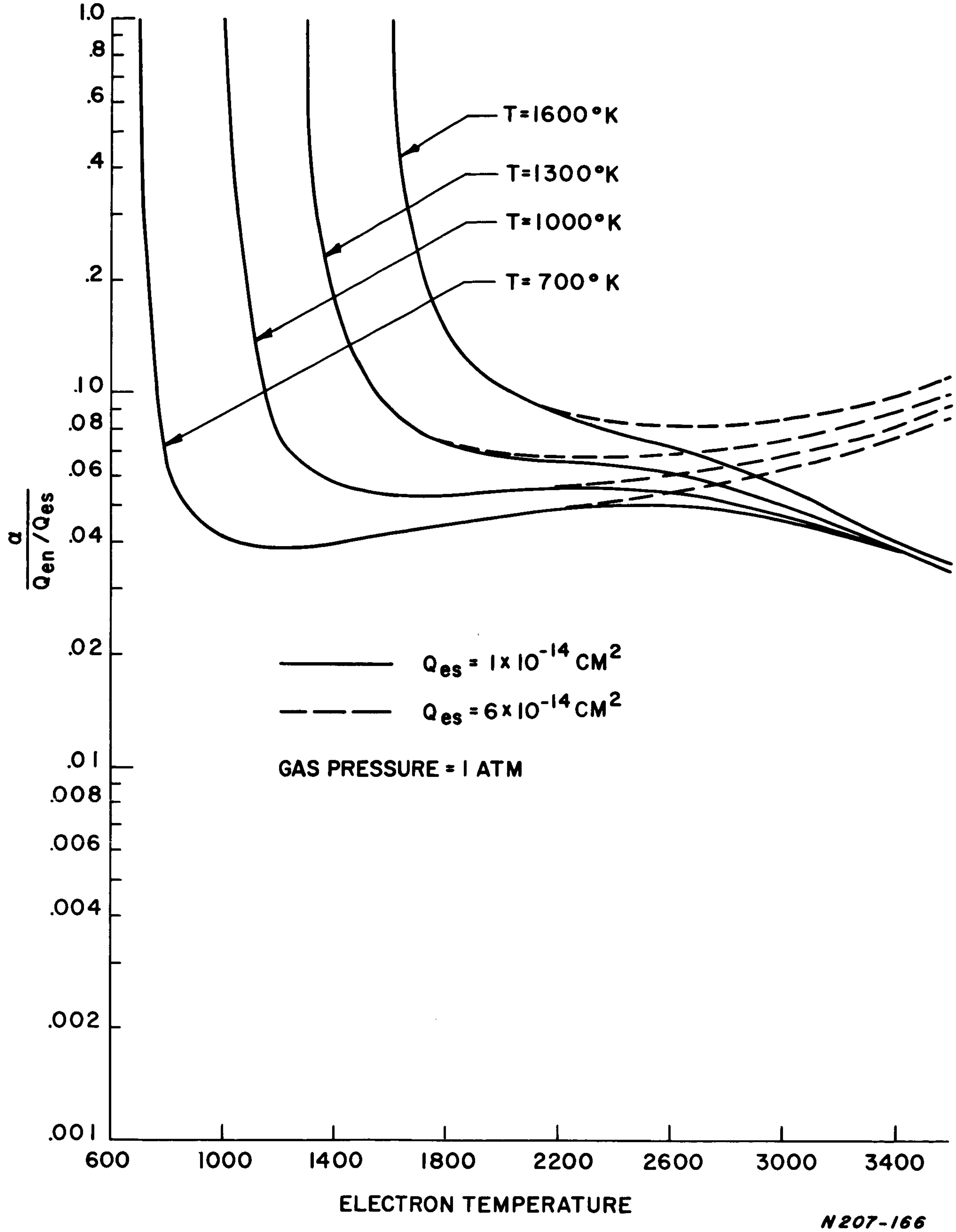

Figure 2. Ratio of Non-equilibrium to Equilibrium Seed Concentration Vs. Electron Temperature as Function of Gas Temperature - Potassium Seed, Lorentz Gas

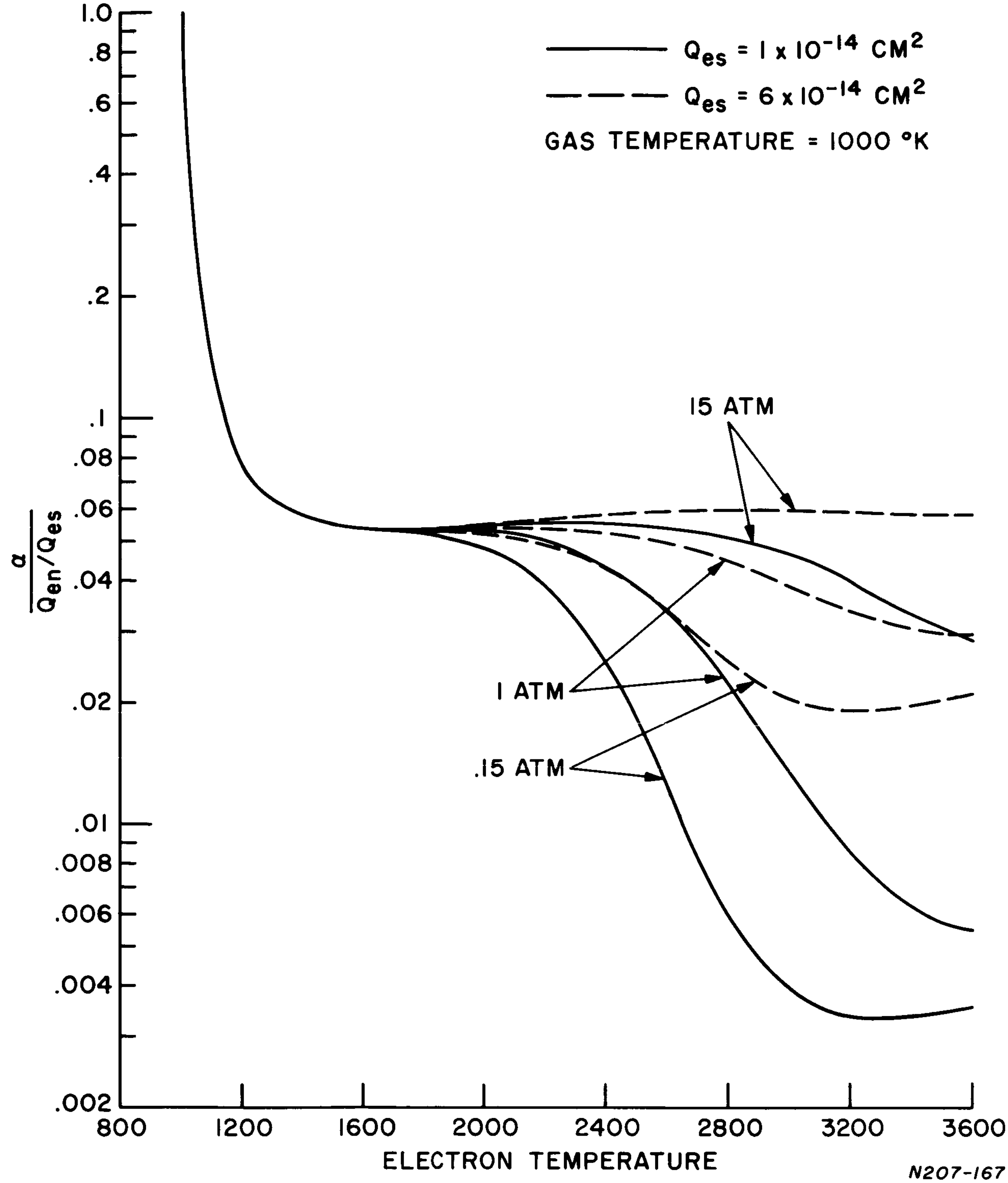

Figure 3. Ratio of Non-equilibrium to Equilibrium Seed Concentration Vs. Electron Temperature as Function of Gas Pressure - Potassium Seed, Maxwellianized Gas

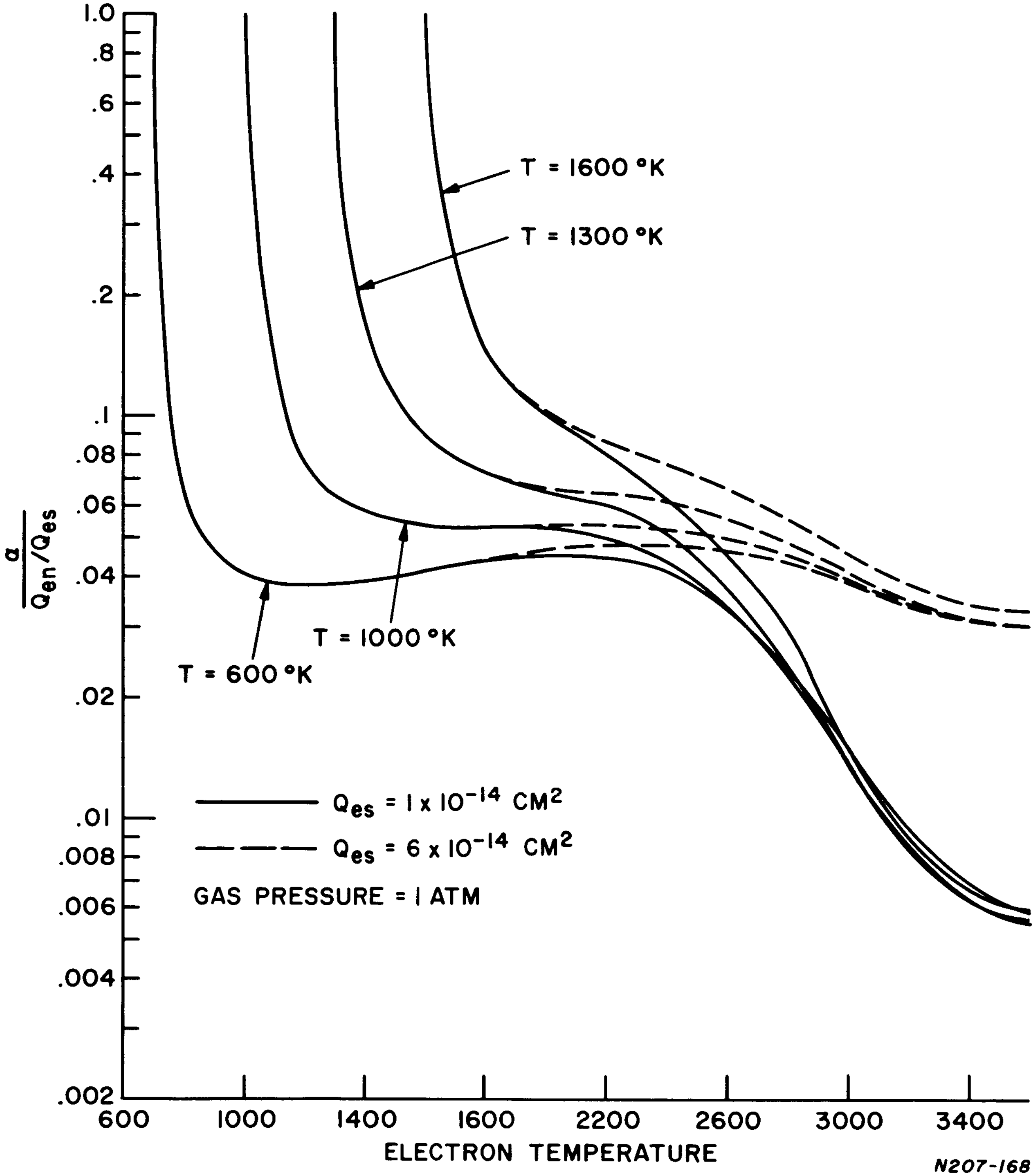

Figure 4. Ratio of Non-equilibrium to Equilibrium Seed Concentration Vs. Electron Temperature as Function of Gas Temperature - Potassium Seed, Maxwellianized Gas

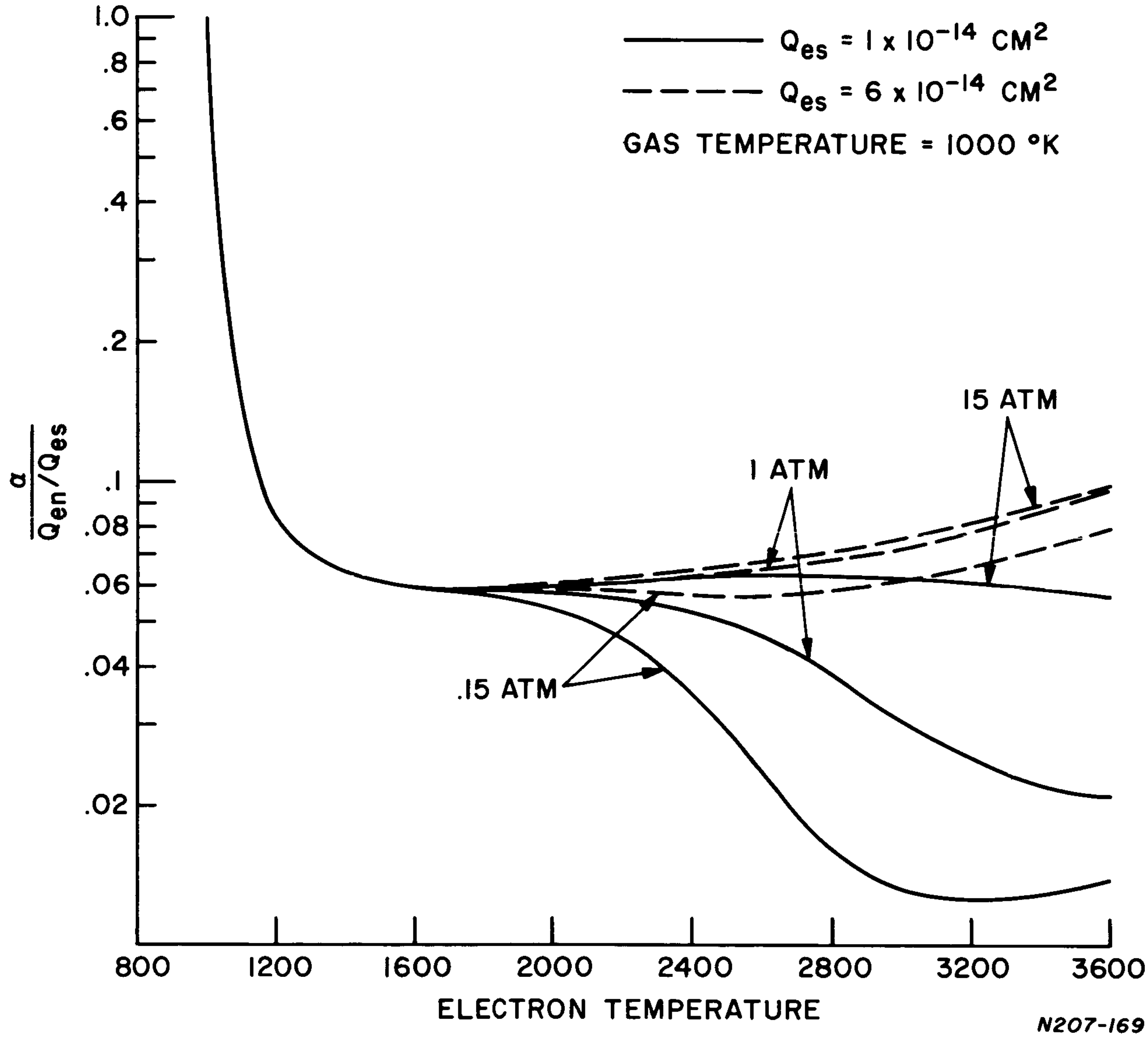

Figure 5. Ratio of Non-equilibrium to Equilibrium Seed Concentration Vs. Electron Temperature as Function of Gas Pressure - Cesium Seed, Lorentz Gas

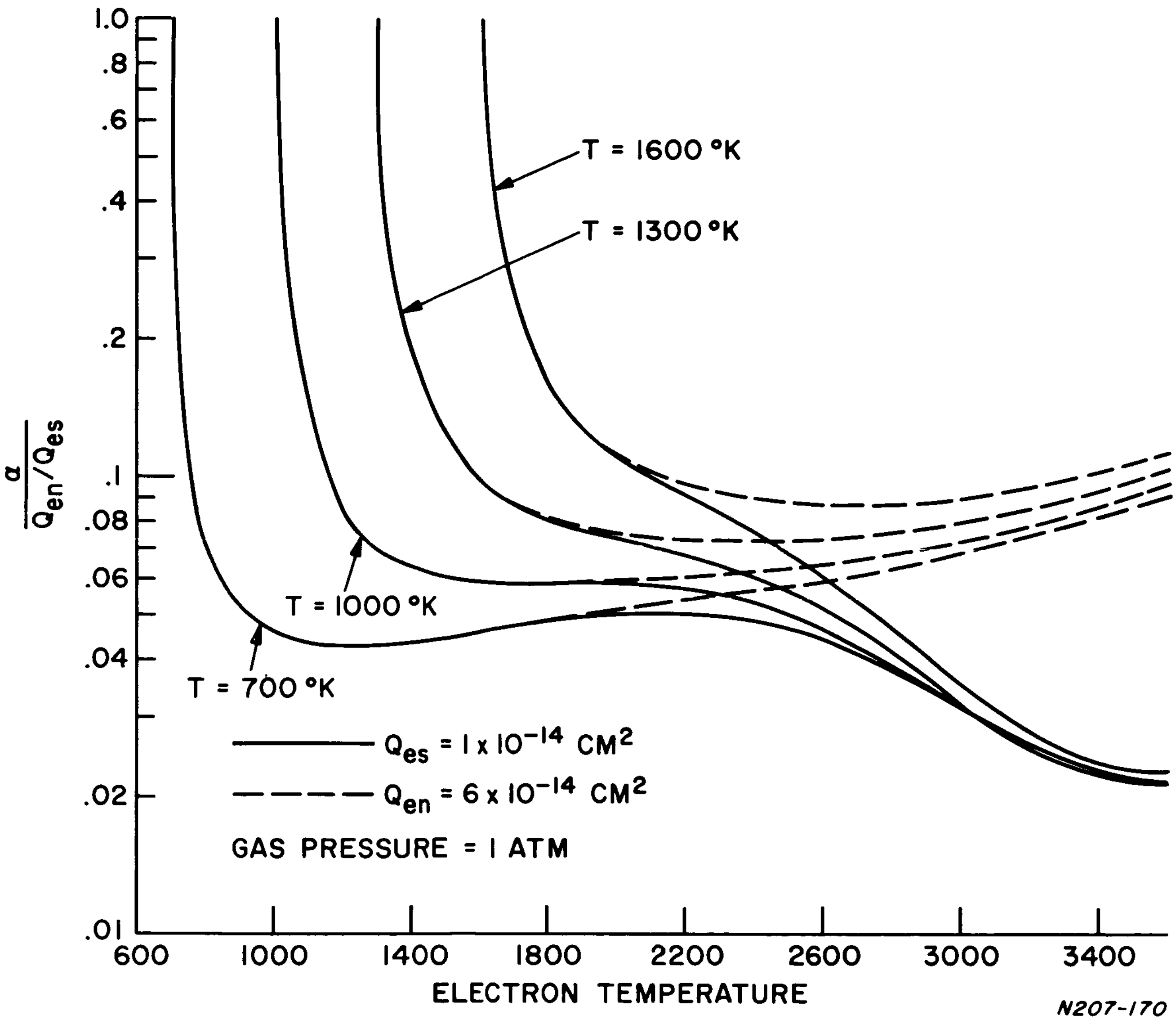

Figure 6. Ratio of Non-equilibrium to Equilibrium Seed Concentration Vs. Electron Temperature as Function of Gas Temperature - Cesium Seed, Lorentz Gas

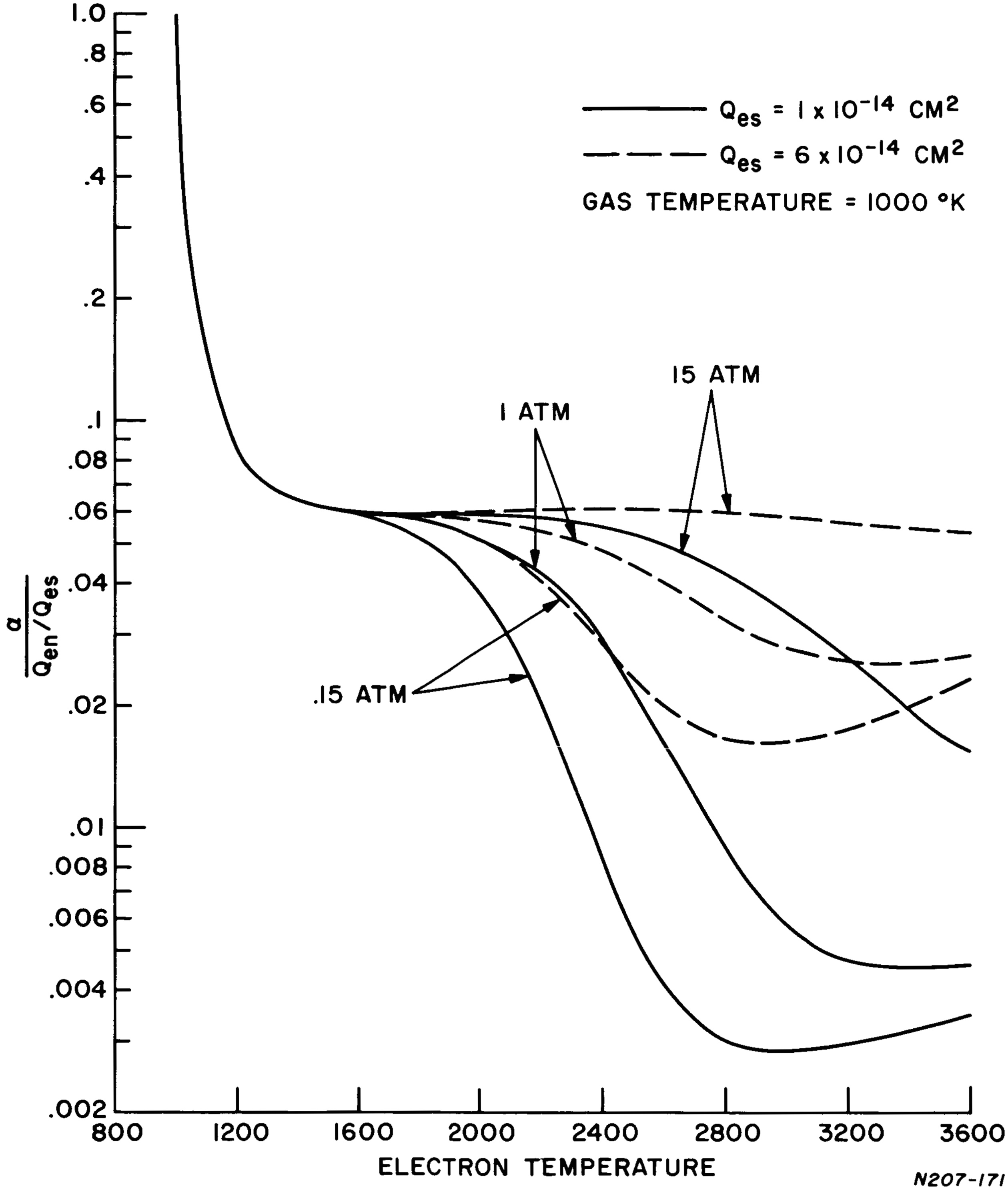

Figure 7. Ratio of Non-equilibrium to Equilibrium Seed Concentration Vs. Electron Temperature as Function of Gas Pressure - Cesium Seed, Maxwellianized Gas

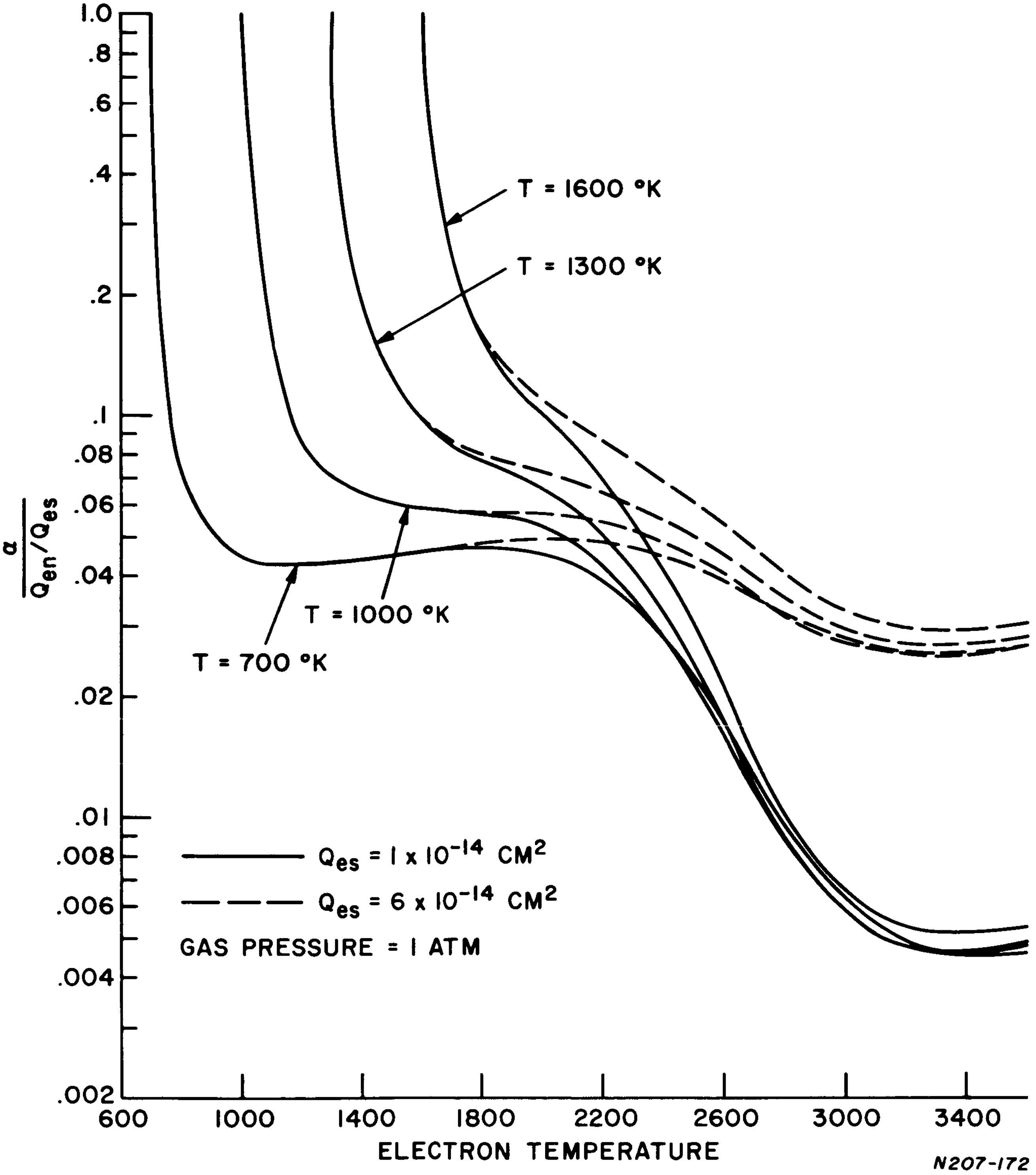

Figure 8. Ratio of Non-equilibrium to Equilibrium Seed Concentration Vs. Electron Temperature as Function of Gas Temperature - Cesium Seed, Maxwellianized Gas

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SUMMARY The electrical conductivity of a gas can be enhanced by the use of a mixture of a parent gas of small electron elastic collision cross section (typically a noble gas) seeded with a gas of low ionization potential (typically an alkali metal). Further enhancement for a given gas temperature is possible through the creation of a non-equilibrium condition where the electron temperature is higher than the gas temperature. In this paper an extension for the optimum seed concentration in a non-equilibrium plasma is obtained for conditions applicable to MHD power generators or positive columns of discharges. This result is applied to Cesium and Potassium seeds and it is shown that the concentration of seed at optimum conductivity is one to two orders of magnitude lower than it is in the equilibrium plasma.

KEY WORDS

Magnetohydrodynamics, Gas conductivity, Non-equilibrium plasma, Seed concentration

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