FINAL REPORT

on

NULL GRAVITY SIMULATOR

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by

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As was pointed out in the proposal, a condition of zero gravity produced by mechanical means on the surface of the earth is impossible. The question arises as to whether or not one could approximate the condition of zero gravity by the utilization of some simple mechanism.

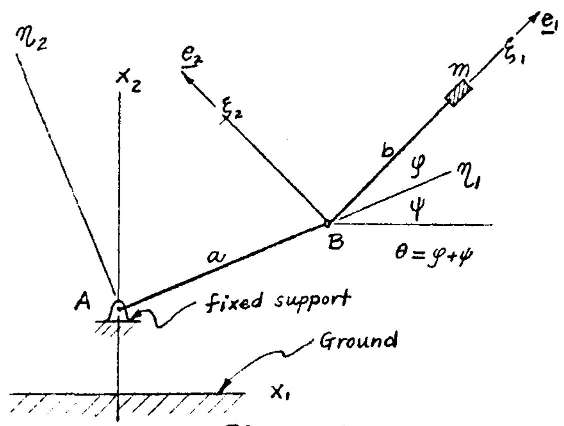


Figure 1

Figure 1 shows a coplanar two-bar mechanism with fixed cam at A and movable cam at B. From such a mechanism one hopes that the state of zero gravity can be approximated for mass m by appropriate prescriptions of the motions of the two bars. The kinematic consideration of m leads to the expressions for the velocity

$$\bar{v}_m = a\dot{\psi}\sin\varphi\,\bar{e}_1 + (a\dot{\psi}\cos\varphi + b\dot{\theta})\,\bar{e}_2 \qquad (1)$$

and the acceleration

$$\bar{a}_m = (a\ddot{\psi}\sin\varphi - a\dot{\psi}^2\cos\varphi - b\dot{\theta}^2)\,\bar{e}_1 + (a\ddot{\psi}\cos\varphi + b\ddot{\theta} + a\dot{\psi}^2\sin\varphi)\,\bar{e}_2 \qquad (2)$$

with Figure 2

$$a_{m_1} = a\ddot{\psi}\sin\varphi - a\dot{\psi}^2\cos\varphi - b\dot{\theta}^2 \quad \text{and} \quad a_{m_2} = a\ddot{\psi}\cos\varphi + b\ddot{\theta} + a\dot{\psi}^2\sin\varphi \qquad (3)$$

The dynamic equation of motion of m gives

$$R - mg\sin\theta = m\,a_{m_1}$$
$$T - mg\cos\theta = m\,a_{m_2} \qquad (4)$$

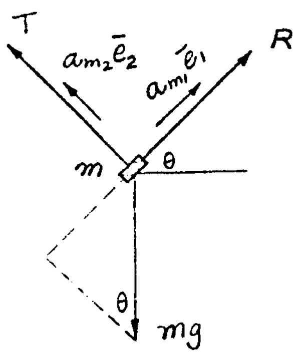


Figure 2

The weightless state for m requires that no surface force, i.e., R and T acts on it and that the resultant inertial force acting on it balances that of the gravitational one.

Hence for total weightlessness equation (4) reduces to

$$a_{m_1} + g\sin\theta = 0$$
$$a_{m_2} + g\cos\theta = 0 \qquad (5)$$

or, after substituting a_{m1} and a_{m2} from equation (3)

$$a\ddot{\psi}\sin\psi - a\dot{\psi}^2\cos\psi - b\dot{\theta}^2 + g\sin\theta = 0$$
$$a\ddot{\psi}\cos\psi + a\dot{\psi}^2\sin\psi + b\ddot{\theta} + g\cos\theta = 0 \qquad (6)$$

which has also been confirmed by Hamilton's Principle.

It can be shown from equation (6) that

$$\left(\ddot{\psi} + \frac{g}{a}\cos\psi\right)^2 + \left(\dot{\psi}^2 - \frac{g}{a}\sin\psi\right)^2 = \left(\ddot{\theta}^2 + \dot{\theta}^4\right)\frac{b^2}{a^2} \qquad (7)$$

letting

$$\dot{\psi} = \sqrt{2q} \qquad (8)$$

one has

$$\ddot{\psi} = q' \qquad (9)$$

where $\varrho' = \frac{d\varrho}{d\psi}$

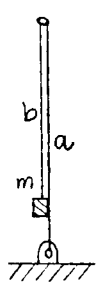
Equation (7) can then be transformed to

$$\varrho'^2 + 2\frac{g}{a}(\cos\psi)\varrho' + 4\varrho^2 - 4\frac{g}{a}\varrho\sin\psi + \frac{g^2}{a^2} = (\ddot{\theta}^2 + \dot{\theta}^4)\frac{b^2}{a^2} \qquad (10)$$

As no general solution for equations (6), (7), and (10) has been found, it is common to simplify equation (10) for certain particular cases of constant angular velocities.

Adopting the new initial configuration, $t = 0$, $\psi_o = \theta_o = \varphi_o = 0$ as shown in Figure 3, equation (10) becomes

$$\varrho'^2 - 2\frac{g}{a}(\sin\psi)\varrho' + 4\varrho^2 - 4\varrho/a\cos\psi + \frac{g^2}{a^2} = (\ddot{\theta}^2 + \dot{\theta}^4)\frac{b^2}{a^2} \qquad (11)$$



The initial condition requires that

$$\dot{\theta} = \dot{\psi} = \omega_0$$

and $$(a-b)\omega_0^2 = g \qquad (12)$$

For $a = 2b$ equation (12) leads to

Figure 3

$$g^2 = b^2\omega_0^4 \qquad (13)$$

Case A $\dot{\theta} = \omega_o$ = const.

Equation (11) is then reduced to

$$\varrho'^2 - 2\frac{g}{a}(\sin\psi)\varrho' + 4\varrho^2 - 4\frac{g}{a}\varrho\cos\psi = 0 \qquad (14)$$

from which

$$\varrho' = K_0\sin\psi\left[1 \pm \sqrt{1 - \frac{4\varrho(\varrho - K_0\cos\psi)}{K_0^2\sin^2\psi}}\right] \qquad (15)$$

where $K_0 = g/a$

One manages, first of all, to linearize it by assuming

$$\left| \frac{4q(q-K_o\cos\psi}{K_o^2 \sin^2\psi} \right| \leq 1 \tag{16}$$

whose validity will be verified with the solution one gets therefrom. Equation (15) becomes, after linearization

$$q' = K_o \sin\psi \left\{ 1 \mp \left[1 - \frac{2q(q-K_o\cos\psi)}{K_o^2 \sin^2\psi} \right] \right\} \tag{17}$$

The solution for the first branch has been obtained as

$$q = \frac{K_o}{[\cos\psi - \sin^2\psi \log \tan\frac{\psi}{2} - 2\gamma_0 \sin^2\psi]} \tag{18}$$

where γ_o is an arbitrary constant and evidently $q = K_o = g/a$ for $t=0$, $\psi=0$.

A computer program has been set up to evaluate q for $\kappa_o = 1$ and arbitrarily assigned γ_o's under the condition of equation (16). It does not provide any output, because of the breakdown of the assumption. The second branch of equation (17) is also an equation of the Riccati type whose solution depends on the solution of

$$y'' - \frac{1}{4}(18 + 3\cot^2\psi)y = 0 \tag{19}$$

The solution of this equation was not investigated because of the reason mentioned above.

Case B $\dot{\psi} = \omega o = \text{const}$. Equation (7) becomes

$$\ddot{\theta}^2 + \dot{\theta}^4 = 5\omega_o^4 - 4\omega_o^4 \cos\omega_o t \tag{20}$$

No solution to equation (20) has been found.

Case C Small angle changes. One tends to use Taylor's expansion to approximate equation (6) and (7). It does not, however, appear to ease the non-linear situation of the equations. For example, linearization of equation (20) gives

$$\ddot{\theta} + \dot{\theta}^4 = \omega_0^4 - 2\omega_0^6 t^2 \tag{21}$$

It is still non-linear and the solution of this equation is not expected to yield very practical information.

At this point one might feel that the reacting forces R and T could be minimized instead of requiring them to vanish. Such is not the case, in the classical sense, since the minimizing principles require R and T to vanish identically (this is not surprising).

Since the above approach fails to arrive at any fruitful result, a further investigation into the equation (14) is therefore necessary.

The initial conditions at t = 0 ($\psi_0 = \theta_0 = \varphi = 0$) require, from equations (12), (13), and (14), that

$$\begin{aligned} q_0 &= \frac{\dot{\psi}_0^2}{2} = \frac{\omega_0^2}{2} = K_0 \ , \\ \ddot{\psi}_0 &= q_0' = 0 \end{aligned} \tag{22}$$

and $q_0'' =$ indefinite.

Solving q in terms of q' and assuming ψ being close to its initial value, the first two terms of equation (14) of second order of magnitude, can be neglected. Hence as a result of the approximation equation (14) becomes

$$q^2 - K_0 q \cos\psi = 0$$

or

$$q = K_0 \cos\psi \tag{23}$$

and

$$q' = -K_o \sin\psi \tag{24}$$

substituting equation (24) into equation (14), one has, after linearization, the second iteration for q

$$q = \frac{K_o}{2}[\cos\psi + \cos 2\psi] \tag{25}$$

The third iteration can be obtained following the same procedure as

$$q = \frac{K_o}{2}[1 + \cos\psi - \frac{1}{2}(\frac{9}{4}\sin^2\psi + 3\sin\psi\sin 2\psi + \sin^2 2\psi)] \tag{26}$$

Substitution of equations (23) and (24) into the left hand of equation (14) leaves $3\kappa_o^2 \sin^2\psi$ which vanishes only at the initial position.

Though equation (23) does not satisfy equation (14) exactly, it does satisfy the condition of equation (16). Equations (25) and (26), however, are not as such, for equation (25) makes

$$\lim_{\psi \to 0} \left| \frac{4q(q - K_o\cos\psi)}{K_o^2 \sin^2\psi} \right| = 3$$

Hence equation (23) should be the best approximation among all the solutions obtained by the above mentioned iteration procedure. To ascertain such a claim, numerical evaluations of R and T according to equations (23), (25), and (26) respectively are necessary.

The radial and tangential components of the surface force acting on the point mass are given respectively by

$$\begin{aligned} \frac{R}{m} &= -a\ddot{\psi}\sin\varphi + a\dot{\psi}^2\cos\varphi - b\dot{\theta}^2 - g\cos\theta \\ \frac{T}{m} &= -a\ddot{\psi}\cos\varphi - a\dot{\psi}^2\sin\varphi + b\ddot{\theta} + g\sin\theta \end{aligned} \tag{4'}$$

For $\dot{\theta} = \omega_o = \text{const.}$, $\theta = \omega_o t$ and

$$\frac{R}{m} = -a\ddot{\psi}\sin\varphi + a\dot{\psi}^2\cos\varphi - b\omega_o^2 - g\cos\omega_o t$$
$$\frac{T}{m} = -a\ddot{\psi}\cos\varphi - a\dot{\psi}^2\sin\varphi + g\sin\omega_o t \qquad (5)'$$

or

$$RA = \frac{R}{mg} = -\frac{2}{\omega_o^2}\ddot{\psi}\sin(\omega_o t-\psi) + \frac{2}{\omega_o^2}\dot{\psi}^2\cos(\omega_o t-\psi) - (1+\cos\omega_o t)$$
$$TA = \frac{T}{mg} = -\frac{2}{\omega_o^2}\ddot{\psi}\cos(\omega_o t-\psi) - \frac{2}{\omega_o^2}\dot{\psi}^2\sin(\omega_o t-\psi) + \sin\omega_o t \qquad (6)'$$

where the relations $b = g/\omega_o^2$, $a = \frac{2g}{\omega_o^2}$ and $K_o = \frac{1}{2}\omega_o^2$ due to equations (12) and (13) has been applied.

Computer programs are then devised to evaluate the $\frac{R}{mg}$ and $\frac{T}{mg}$'s from equation (6)'.

For the first iteration

$$\frac{\dot{\psi}^2}{2} = g = K_o\cos\psi \qquad (7)'$$

or

$$\frac{d\psi}{\sqrt{2K_o\cos\psi}} = dt \;, \qquad f(\psi)d\psi = dt \qquad (8)'$$

where

$$f(\psi) = \frac{1}{\sqrt{2K_o\cos\psi}}$$

or

$$t = \int_0^{\psi} f(\psi)\,d\psi = F(\psi) \qquad (9)'$$

then

$$\psi = G(t) \qquad (10)'$$

also

$$\ddot{\psi} = -K_o\sin\psi$$
$$\dot{\psi} = \sqrt{2K_o\cos\psi}$$

The values of $\frac{R}{mg}$ and $\frac{T}{mg}$ are computed for a total range of 45° for the unit interval of 1°.

The computations for the second and third iterations are carried out in a similar way using IBM 7040 computer.

Their results follow from Table I:

TABLE I

ψ(°)	R/mg			T/mg		
	First	Second	Third	First	Second	Third
0	0	0	0	0	0	0
1	-.000154	-.000615	-.001884	.035063	.061354	.133637
2	-.000614	-.002456	-.007535	.070111	.122646	.266934
3	-.001381	-.005520	-.016953	.105131	.183819	.399544
4	-.002453	-.009796	-.030134	.140109	.244816	.531118
5	-.003827	-.015274	-.047070	.175032	.305579	.661313
6	-.005502	-.021939	-.067749	.209886	.366052	.789784
7	-.007476	-.029772	-.092155	.244658	.426179	.916193
8	-.009744	-.038754	-.120268	.279336	.485904	1.040203
9	-.012303	-.048858	-.152062	.313905	.545173	1.161483
10	-.015149	-.060059	-.187506	.348352	.603932	1.279707
11	-.018278	-.072326	-.226563	.382665	.662130	1.394553
12	-.021683	-.085626	-.269188	.416830	.719715	1.505707
13	-.025361	-.099922	-.315329	.450835	.776638	1.612863
14	-.029303	-.115175	-.364924	.484667	.832848	1.715719
15	-.033506	-.131343	-.417905	.518312	.888301	1.813986
16	-.037960	-.148380	-.474192	.551759	.942949	1.907381
17	-.042659	-.166239	-.533694	.584995	.996750	1.995632
18	-.047595	-.184868	-.596312	.618007	1.049661	2.078476
19	-.052759	-.204214	-.661931	.650784	1.101642	2.155662
20	-.058142	-.224219	-.730427	.683314	1.152655	2.226951
21	-.063736	-.244825	-.801662	.715583	1.202664	2.292117
22	-.069531	-.265968	-.875483	.747582	1.251633	2.350944
23	-.075516	-.287584	-.951726	.779297	1.299532	2.403233
24	-.081680	-.309603	-1.030213	.810718	1.346329	2.448797
25	-.088012	-.331955	-1.110749	.841833	1.391997	2.487466
26	-.094501	-.354564	-1.193131	.872632	1.436510	2.519081
27	-.101134	-.377355	-1.277138	.903102	1.479844	2.543503
28	-.107899	-.400245	-1.362540	.933234	1.521978	2.560605
29	-.114783	-.423152	-1.449094	.963017	1.562893	2.570278
30	-.121772	-.445986	-1.536549	.992439	1.602572	2.572425
31	-.128851	-.468658	-1.624648	1.021492	1.640999	2.566964
32	-.136007	-.491071	-1.713133	1.050164	1.678163	2.553823
33	-.143223	-.513125	-1.801752	1.078446	1.714052	2.532934
34	-.150486	-.534717	-1.890276	1.106328	1.748657	2.504228
35	-.157777	-.555735	-1.978523	1.133800	1.781972	2.476607
36	-.165081	-.576064	-2.066441	1.160853	1.813990	2.422903
37	-.172379	-.595581	-2.154395	1.187476	1.844707	2.369692
38	-.179656	-.614155		1.213662	1.874119	
39	-.186890	-.631647		1.239400	1.902225	
40	-.194065	-.647907		1.264682	1.929021	
41	-.201160	-.662773		1.289499	1.954504	
42	-.208154	-.676066		1.313841	1.978671	
43	-.215027	-.687593		1.337701	2.001513	
44	-.221757	-.697136		1.361068	2.023021	
45	-.228322	-.704452		1.383934	2.043179	

It is obvious then that the first iteration gives the best fit for weightless state for m as predicted.

Conclusion

1. Exact solution of the null gravity state for m has not been found except at its initial configuration shown in Figure 3. It is unlikely that a prolonged exact solution for null gravity exists for the simple mechanism shown in Figure 1 or, possible, any mechanism one can construct on the earth's surface. It is possible only when m happens either to be a satellite of the earth or to be subjected to a free falling condition.
2. No minimization of either R or T of the surface force is possible.
3. An approximate solution for the motion of the mechanism close to its initial configuration (Figure 3) has been found in the form of equation (10)'. The angular displacement as a function of time t can be found, numerically, using a digital computer. The deviations of this equation from the ideal null gravity state are listed in Table I. It can be seen that the deviation will reach about one-g after the motion carries on for 0.5 seconds from its initial configuration or for a corresponding angular displacement $\psi \doteq 30°$.
4. No better approximate methods have been found.