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This report presents a development of the Lagrangian equations of motion and a FORTRAN computer program for the motion of two rigid bodies in space, separating as a result of any one, or a combination of, the following force mechanisms: springs with ball ends, springs with one end guided, pyrotechnics, rockets, cold-gas jets, air pistons, and coulomb drag. Two constraints, treated by the method of Lagrange multipliers, are included as options. These constraints are the *pin-puller delay* and the *hard-mounted spring constraint*. The pin-puller delay represents the situation when one discrete separation device (e.g., a pin puller) actuates later than the others, providing a ball and socket type of joint prior to final separation-device firing. The hard-mounted spring constraint takes into account the fact that sliding, at the tip of a guided spring, is usually prohibited either by design or by inherent friction. The forces arising from the various mechanisms are represented by the customary mathematical models. No approximations are used in the derivation of the classical equations of motion; they are numerically integrated by the Adams-Moulton method.

Author

I. INTRODUCTION

The separation of two bodies in space is distinguished from the many other current applications of dynamics to space technology because it is associated with every space mission. Thus, the problem is of unique, practical importance, and the need for an analytical tool to solve this dynamic problem is clear. The purpose of the work presented in this report has been to develop such a tool that is appropriate to a variety of separation schemes.

The basic approach to the problem is a straightforward application of the Lagrange equations augmented by undetermined multipliers. The mathematical complexity of the resultant equations is primarily due to the large number of degrees of freedom (12) and the coordinate transformations involved. Many of the more complex and tedious derivations have been relegated to the Appendixes.

The computer program, which consists of three chain links, is written in the FORTRAN/FAP system, and is compatible with the JPL/IBM 7094 installation.

The *input link* reads the input data and performs initial conversions. The locations and directions of all force-producing devices are converted to the appropriate body-fixed coordinate system (CS), and initial conditions for the coordinates are generated.

The *integration link* applies the JPL integration subroutine FMARK to the differential equations. Separate subroutines for each force type compute the contributions to the generalized forces. Small matrix operation subroutines are used throughout this link.

The *plotting link* accepts the data to be plotted, generates scale factors, and prepares the other necessary plotting instructions.

II. DESCRIPTION OF THE PHYSICAL PROBLEM AND THE CORRESPONDING MATHEMATICAL MODEL

A. Physical Problem

All space exploration missions require, at some point in their lifetime, a separation of two bodies. The separation of an aerodynamic shroud from a launch vehicle, the separation of a spacecraft from a launch vehicle, and the separation of an exploratory module from its parent spacecraft are a few of the more common requirements.

In connection with the common types of separations that will be encountered, the following force-producing devices are considered to be significant: rockets, pyrotechnics, cold-gas jets, pneumatic pistons, coulomb drag, and springs.

Two constraint conditions are considered to be important aspects of the physical problem: (1) there is a dispersion among latching device release times, and (2) guided spring tips will not slip on their bearing surfaces.

B. Mathematical Model

The mathematical model assumes two rigid bodies subjected to the action of forces from the following types of idealized devices:

1. Linear springs hard mounted on one body and bearing on the other (either with frictionless slippage or no slippage)
2. Linear springs between the two bodies with universal joints at both ends
3. Pyrotechnic devices supplying impulsive forces
4. Cold-gas jets—adiabatic processes of an ideal gas
5. Pneumatic pistons—adiabatic processes of an ideal gas
6. Coulomb drag between the bodies
7. Rockets (constant force), including linear changes in inertia properties

The primary assumptions concerning the forces are implicitly contained in the descriptions of these idealized devices. It has been assumed that the bodies do not have separation distances that are significant relative to the distance from the common center of mass (CM) to the center of any external force-field. The equations of motion are written in a reference frame that has fixed directions in inertial space, but that moves with the pre-separation trajectory of the two bodies; this frame is taken to be an inertial one. The results are then correct (when used in combination with a trajectory solution for the common CM) if there are no forces acting other than the ones considered in the analysis, or if there is an external force-field exerting approximately the same vector force per unit mass on each element of the system.

III. SYMBOLS AND CONVENTIONS

A. Symbols

To make the analysis more useful for reference when writing input for the computer program, a descriptive listing is made at the beginning of each section detailing the notation for that particular section.

For clarity, the symbols that are used repeatedly throughout the text are listed below.

1. Scalars

a_{ij}	element of the $\mathbf{A}_{\mathrm{I}}$ matrix (i^{th} row, j^{th} column)
b_{ij}	element of the $\mathbf{A}_{\mathrm{II}}$ matrix
c_{ij}	generalized element of either $\mathbf{A}_{\mathrm{I}}$ or $\mathbf{A}_{\mathrm{II}}$
k	spring rate
m	mass
$\mathcal{Q}$	generalized force
q	generalized coordinate
T	kinetic energy
$\mathbf{T}$	matrix product $\mathbf{A}_{\mathrm{I}}\mathbf{A}_{\mathrm{II}}^{-1}$
t	time
Δt	length of integration interval
x, y, z	inertial coordinates of the CM
x', y', z'	body–fixed coordinates
x'', y'', z''	drawing board coordinates
θ, ϕ, ψ	Euler angles (see Appendix A)
$\lambda_x, \lambda_y, \lambda_z$	direction cosines with body I-fixed coordinate axes
μ_x, μ_y, μ_z	direction cosines with drawing board axes when referring to body I
μ'_x, μ'_y, μ'_z	direction cosines with drawing board axes when referring to body II
ρ_x, ρ_y, ρ_z	direction cosines with body II-fixed coordinate axes
$\omega_{x''}, \omega_{y''}, \omega_{z''}$	components of the initial angular velocity of body II in the drawing board CS
$\omega_{\mathrm{I}x'}, \omega_{\mathrm{I}y'}, \omega_{\mathrm{I}z'}$	components of the initial angular velocity of body I about the body I-fixed CS
$\omega_{\mathrm{II}x'}, \omega_{\mathrm{II}y'}, \omega_{\mathrm{II}z'}$	components of the initial angular velocity of body II about the body II-fixed CS

2. Vectors and Matrixes

$\mathbf{A}_{\mathrm{I}}$	(3×3) orthogonal transformation matrix that takes vectors in the inertial CS into vectors in the body I-fixed CS (see Appendix A)
$\mathbf{A}_{\mathrm{II}}$	(3×3) orthogonal transformation matrix that takes vectors in the inertial CS into vectors in the body II-fixed CS (see Appendix A)
$\mathbf{e}$	unit vector or column matrix in direction indicated by subscript
$\mathbf{I}$	inertia matrix
$\mathbf{r}$	$\mathbf{r}_{\mathrm{II}} - \mathbf{r}_{\mathrm{I}}$
$\mathbf{r}'_{\mathrm{I}i}$	location vector of i^{th} device in the body I-fixed CS
$\mathbf{r}'_{\mathrm{II}i}$	location vector of i^{th} device in the body II-fixed CS
$\mathbf{r}_{\mathrm{I}}$	$\begin{bmatrix} x_{\mathrm{I}} \\ y_{\mathrm{I}} \\ z_{\mathrm{I}} \end{bmatrix}$
$\mathbf{r}_{\mathrm{II}}$	$\begin{bmatrix} x_{\mathrm{II}} \\ y_{\mathrm{II}} \\ z_{\mathrm{II}} \end{bmatrix}$
$\mathbf{v}$	velocity vector or column matrix
$\boldsymbol{\rho}$	$\begin{bmatrix} \rho_x \\ \rho_y \\ \rho_z \end{bmatrix}$
$\boldsymbol{\lambda}$	$\begin{bmatrix} \lambda_x \\ \lambda_y \\ \lambda_z \end{bmatrix}$
$\boldsymbol{\omega}$	angular velocity vector or column matrix

3. Subscripts

i, j, r, s	nonspecific indexes
k, l	specific indexes
I	refers to body I
II	refers to body II
0	initial value

B. Notation Conventions

1. Vector-Matrix

In the interests of economy and clarity, the same notation is used for vectors and their associated (3 × 1) column matrixes. The symbolism **v** is used where it is to be considered a vector, as in $(\mathbf{v} \times \boldsymbol{\omega})$ or $(\mathbf{f} \cdot \mathbf{r})$, and where it is to be considered a column matrix, as in $\mathbf{v}^T \mathbf{I}\, \mathbf{v}$.

2. Trigonometric Function

Since the sine and cosine functions of the Euler angles occur quite frequently throughout the text, cos θ is replaced by $(c\theta)$ and sin θ is replaced by $(s\theta)$. The other Euler angles are treated similarly. Such expressions, or groups of expressions, are always enclosed in parentheses [e.g., $(-c\theta\, s\phi\, s\psi + c\phi\, c\psi)$].

3. Miscellaneous

The miscellaneous notation conventions used in this report are as follows:

Bold face lower case letters	vector, (3 × 1) matrix
Bold face capital letters	square matrix
$\mathbf{a}_{(i)}$	(3 × 1) matrix formed from i^{th} row of corresponding (3 × 3) matrix
$\mathbf{a}^{(i)}$	(3 × 1) matrix formed from i^{th} column of corresponding (3 × 3) matrix
Dot (•) between vectors	scalar product (e.g., $\mathbf{f} \cdot \mathbf{r}$)
Multiplication sign (×) between vectors	cross product (e.g., $\mathbf{f} \times \mathbf{r}$)
Dot (˙) over symbol	derivative with respect to time
Two dots (¨) over symbol	second derivative with respect to time
$(\ )^T$	transpose
$(\ )^{-1}$	inverse
$\lvert\mathbf{v}\rvert$	magnitude of **v**
δ	variational operator
δ_{ij}	Kronecker delta

IV. COORDINATE SYSTEMS

The four coordinate systems used in this analysis are: (1) drawing board CS, (2) inertial CS, (3) body I-fixed CS, and (4) body II-fixed CS.

A. Symbols

c (subscript)	refers to common CM of bodies I and II
$\mathbf{r}$	location vector
$\boldsymbol{\varepsilon}$	location vector of CM indicated by subscript
$\lambda_{ix}, \lambda_{iy}, \lambda_{iz}$	cosines of angles between rocket directions, spring directions, etc., and drawing-board axes when applicable to body I
$\mu_{ix}, \mu_{iy}, \mu_{iz}$	cosines of angles between rocket directions, spring directions, etc., and drawing-board axes when applicable to body II
$\mu'_{ix}, \mu'_{iy}, \mu'_{iz}$	cosines of angles between rocket directions, spring directions, etc., and body I-fixed axes
$\rho_{ix}, \rho_{iy}, \rho_{iz}$	cosines of angles between rocket directions, spring directions, etc., and body II-fixed axes
ω	angular velocity about axis indicated by subscript

B. Generalized Coordinates

To apply the Lagrange equations, it is necessary to choose a set of generalized coordinates. By definition, the generalized coordinates must be independent and must fully describe the position and orientation of the system. This analysis employs the three inertial coordinates of each CM and the three Euler angles for each body (coordinates are defined in Subsection D). These coordinates form an independent and fully descriptive set unless certain special values of either θ-coordinate are reached. The methods used to circumvent this difficulty are described in Subsections F and G.

C. Drawing Board Coordinates

The 3-dimensional orthogonal drawing board coordinate system may be selected by the user, subject to the following restrictions:

1. If the hard-mounted spring constraint is to be present, the z'' axis must be perpendicular to the separation plane (this situation is assumed in Appendix B, Eq. B-1).
2. Since the solutions for θ, ϕ, ψ, pitch and yaw rates, etc., depend on the definition of the body-fixed CS, which is in turn controlled by the definition of the drawing board CS, it will usually be convenient to make the drawing board CS parallel to any symmetry or geometric axes that the bodies may have.

D. Inertial Coordinates

The point at which the common CM would be located if there were no separation is used as the origin of the inertial CS. The directions of these axes are taken to be fixed in the inertial CS attached to the fixed stars. The inertial CS is initially parallel to the drawing board CS. Because of its motion in the gravitational fields of the Sun, the planets, the galaxy, etc, this CS is not a true inertial system; however, as long as the bodies are sufficiently close to the origin to be in essentially the same gravitational field, they will move as though this CS is exactly an inertial system. The results of this analysis are then deviations from the pre-separation trajectory. To provide initial conditions for new trajectory calculations, it is recommended that the translation results be considered in conjunction with the initial trajectory.

Since the orientation and location of the two rigid bodies relative to the stars and planets is not considered, certain force effects must necessarily be neglected. For example, solar pressure effects are not included in the analysis. The pre-separation trajectory, however, includes the translational effects of solar pressure, based on the orientation relative to the Sun before separation (solar pressure is a function of paint patterns, etc., so that it is usually a function of orientation). Then, in translation, it is only the deviation from pre-separation solar pressure effects that is neglected. Solar pressure torques are neglected completely.

E. CM Body-Fixed Coordinates and Eulerian Angles

A body-fixed CS is associated with each body. The respective CM's are the origins of these systems, and they are the references used for locating force-producing devices.

The orientation of each system is specified by a set of Eulerian angles that are generated in the following manner (Fig. 1):

1. Roll through angle ϕ
2. Pitch through an angle θ
3. Roll through an angle ψ

A more complete description of the Eulerian angles is given in Appendix A.

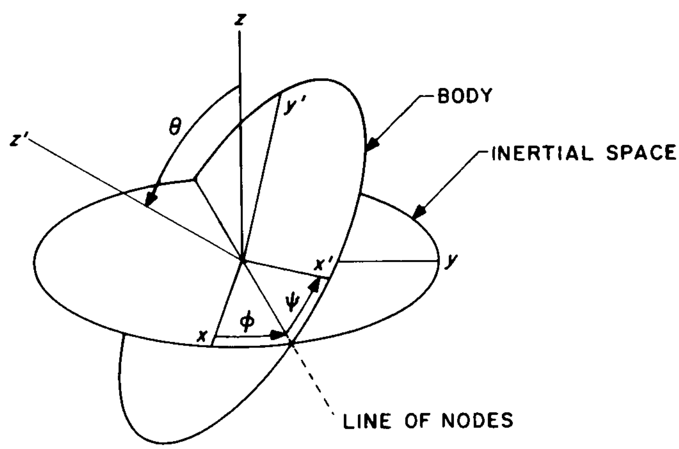

Fig. 1. Eulerian angles

F. Coordinate Conversions From Drawing Board CS to Body-Fixed CS

Although it would be convenient to have the body-fixed CS initially parallel to the drawing board or inertial CS, it is not feasible. In both Section VI and Appendix C, *sin* θ appears in some denominators; this corresponds to dependence among the coordinates when either sin θ_{I} or sin θ_{II} vanishes. To avoid this, the body-fixed CS are defined so that ordinary initial conditions do not have either θ equal to zero. Also, provision must be made to rotate (redefine) the body-fixed CS relative to the body if sin θ approaches zero.

The CM body-fixed CS (Fig. 2) is defined so that the initial value of θ is 90 deg. The conversions between

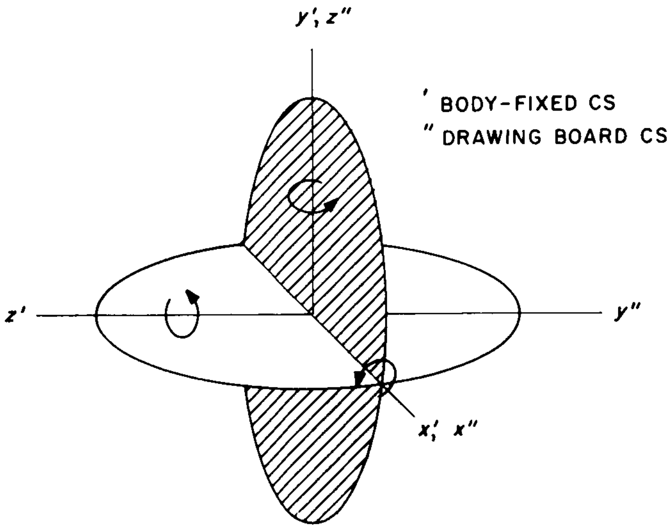

Fig. 2. Relation between drawing board coordinates and body-fixed coordinates

drawing board coordinates and body-fixed coordinates are given in Eqs. (1) through (3).

$$x'_{\mathrm{I}i} = x''_i - \varepsilon''_{\mathrm{I}x} \quad (1)$$
$$x'_{\mathrm{II}i} = x''_i - \varepsilon''_{\mathrm{II}x}$$

$$y'_{\mathrm{I}i} = z''_i - \varepsilon''_{\mathrm{I}z} \quad (2)$$
$$y'_{\mathrm{II}i} = z''_i - \varepsilon''_{\mathrm{II}z}$$

$$z'_{\mathrm{I}i} = -(y''_i - \varepsilon''_{\mathrm{I}y}) \quad (3)$$
$$z'_{\mathrm{II}i} = -(y''_i - \varepsilon''_{\mathrm{II}y})$$

In addition to coordinate conversions, there are also direction cosine and inertia matrix conversions that must be made (Eqs. 4 through 12).

$$\lambda_{ix} = \mu_{ix} \quad (4)$$
$$\rho_{ix} = \mu'_{ix}$$

$$\lambda_{iy} = \mu_{iz} \quad (5)$$
$$\rho_{iy} = \mu'_{iz}$$

$$\lambda_{iz} = -\mu_{iy} \quad (6)$$
$$\rho_{iz} = -\mu'_{iy}$$

$$\mathrm{I}_{x'x'} = \mathrm{I}_{x''x''} \quad (7)$$

$$\mathrm{I}_{x'y'} = \mathrm{I}_{x''z''} \quad (8)$$

$$\mathrm{I}_{x'z'} = \mathrm{I}_{x''y''} \quad (9)$$

$$\mathrm{I}_{y'y'} = \mathrm{I}_{z''z''} \quad (10)$$

$$\mathrm{I}_{y'z'} = -\mathrm{I}_{y''z''} \quad (11)$$

$$\mathrm{I}_{z'z'} = \mathrm{I}_{y''y''} \quad (12)$$

For simplicity in notation, the primes on the subscripts of the I's are dropped for the remainder of this analysis.

G. Body-Fixed CS Redefinition

If $\theta \leqq 0.1$ and $\dot{\theta} < 0$, or if $\theta \geqq 3.04$ and $\dot{\theta} > 0$, all body-fixed coordinates, direction cosines, and inertia matrix elements relating to the body concerned are converted according to Eqs. (13) through (24); this is sufficient to assure that sin θ will not approach zero.

$$x'_{i\,\mathrm{new}} = x'_{i\,\mathrm{old}} \quad (13)$$

$$y'_{i\,\mathrm{new}} = z'_{i\,\mathrm{old}} \quad (14)$$

$$z'_{i\,\mathrm{new}} = -y'_{i\,\mathrm{old}} \quad (15)$$

$$\lambda_{ix\,\mathrm{new}} = \lambda_{ix\,\mathrm{old}} \quad (16)$$
$$\rho_{ix\,\mathrm{new}} = \rho_{ix\,\mathrm{old}}$$

$$\lambda_{iy\,\mathrm{new}} = \lambda_{iz\,\mathrm{old}} \quad (17)$$
$$\rho_{iy\,\mathrm{new}} = \rho_{iz\,\mathrm{old}}$$

$$\lambda_{iz\,\mathrm{new}} = -\lambda_{iy\,\mathrm{old}} \quad (18)$$
$$\rho_{iz\,\mathrm{new}} = -\rho_{iy\,\mathrm{old}}$$

$$\mathrm{I}_{xx\,\mathrm{new}} = \mathrm{I}_{xx\,\mathrm{old}} \quad (19)$$

$$\mathrm{I}_{xy\,\mathrm{new}} = \mathrm{I}_{xz\,\mathrm{old}} \quad (20)$$

$$\mathrm{I}_{xz\,\mathrm{new}} = -\mathrm{I}_{xy\,\mathrm{old}} \quad (21)$$

$$\mathrm{I}_{yy\,\mathrm{new}} = \mathrm{I}_{zz\,\mathrm{old}} \quad (22)$$

$$\mathrm{I}_{yz\,\mathrm{new}} = -\mathrm{I}_{yz\,\mathrm{old}} \quad (23)$$

$$\mathrm{I}_{zz\,\mathrm{new}} = \mathrm{I}_{yy\,\mathrm{old}} \quad (24)$$

The foregoing redefinition is accomplished by rotating the body-fixed CS about the x' axis until y'_{new} coincides with z'_{old}, and z'_{new} coincides with $-y'_{\mathrm{old}}$. Since the Eulerian angles are defined relative to the inertial CS, they now assume new values (see Fig. 3).

Straightforward application of the spherical trigonometric laws of cosines and sines to the spherical triangle containing the old and new Eulerian angles (Fig. 4) gives Eqs. (25) through (27).

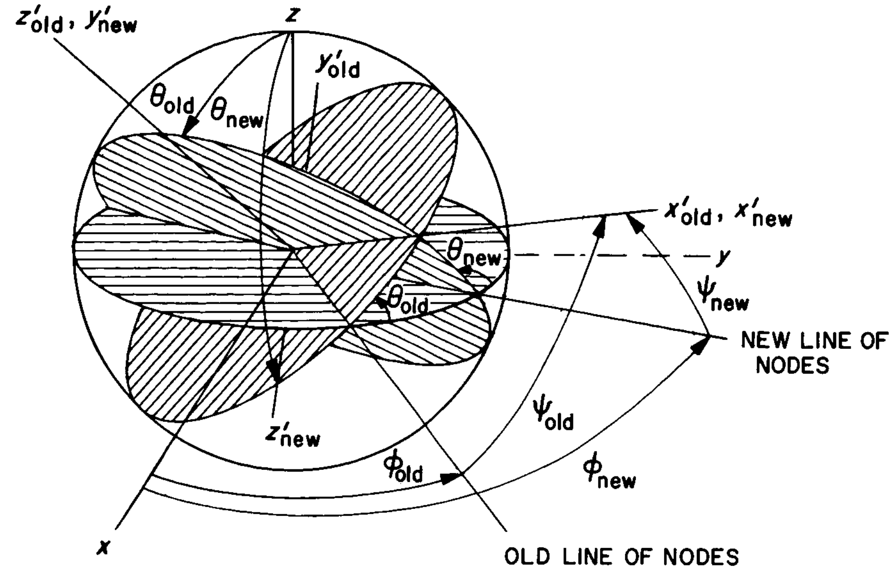

Fig. 3. Eulerian angle change due to CS redefinition

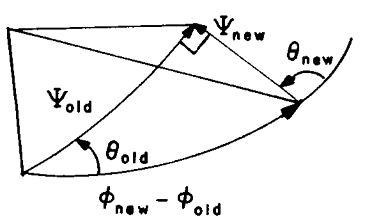

Fig. 4. Spherical triangles containing new and old Eulerian angles

$$\cos\theta_{new} = -\sin\theta_{old}\cos\psi_{old} \tag{25}$$

$$\sin(\phi_{new} - \phi_{old}) = \frac{\sin\psi_{old}}{(1 - \sin^2\theta_{old}\cos^2\psi_{old})^{1/2}} \tag{26}$$

$$\sin\psi_{new} = \frac{\sin\psi_{old}\sin\theta_{old}}{(1 - \sin^2\theta_{old}\cos^2\psi_{old})^{1/2}} \tag{27}$$

Since the computer does not have subroutines for the arc sine or arc cosine, it is necessary to restate the new Eulerian angles in terms of arc tangents (Eqs. 28 through 30).

$$\theta_{new} = \tan^{-1}\frac{(1 - \sin^2\theta_{old}\cos^2\psi_{old})^{1/2}}{\sin\theta_{old}\cos\psi_{old}} \tag{28}$$

$$\phi_{new} = \phi_{old} + \tan^{-1}\frac{\tan\psi_{old}}{\cos\theta_{old}} \tag{29}$$

$$\psi_{new} = \tan^{-1}(\tan\theta_{old}\sin\psi_{old}) \tag{30}$$

The quadrant of θ_{new} may be obtained from Eq. (25). For the three ranges of cos ψ_{old}, θ_{new} has a value within the indicated range:

$$\cos\psi_{old} < 0, \qquad 0 < \theta_{new} < \frac{\pi}{2}$$

$$\cos\psi_{old} = 0, \qquad \theta_{new} = \frac{\pi}{2}$$

$$\cos\psi_{old} > 0, \qquad \frac{\pi}{2} < \theta_{new} < \pi$$

The quadrant of $\phi_{new} - \phi_{old}$ may be obtained from Eq. (29) and Fig. 3. For the two ranges of cos θ_{old}, the quadrant of $\phi_{new} - \phi_{old}$ is as follows:

$\cos\theta_{old} > 0$, $\phi_{new} - \phi_{old}$ is in the same quadrant as ψ_{old}

$\cos\theta_{old} < 0$, $\phi_{new} - \phi_{old}$ is in the same quadrant as $2\pi - \psi_{old}$

The quadrant of ψ_{new} may be obtained from Eq. (27) and Fig. 3. For the two ranges of sin ψ_{old}, the quadrant of ψ_{new} is as follows:

$\sin\psi_{old} \geqq 0$, ψ_{new} is in the same quadrant as θ_{old}

$\sin\psi_{old} < 0$, ψ_{new} is in the same quadrant as $2\pi - \theta_{old}$

These redefinitions are carried out such that the angles have the ranges of $0 < \theta < \pi$, $0 \leqq \phi < 2\pi$, and $0 \leqq \psi < 2\pi$.

H. Initial Values

If the locations of the CM's of bodies I and II are $\boldsymbol{\varepsilon}_{\rm I}''$ and $\boldsymbol{\varepsilon}_{\rm II}''$ in the drawing board CS, then

$$\varepsilon_{cx}'' = \frac{m_{\rm I}\,\varepsilon_{{\rm I}x}'' + m_{\rm II}\,\varepsilon_{{\rm II}x}''}{m_{\rm I} + m_{\rm II}} \tag{31}$$

$$\varepsilon_{cy}'' = \frac{m_{\rm I}\,\varepsilon_{{\rm I}y}'' + m_{\rm II}\,\varepsilon_{{\rm II}y}''}{m_{\rm I} + m_{\rm II}} \tag{32}$$

$$\varepsilon_{cz}'' = \frac{m_{\rm I}\,\varepsilon_{{\rm I}z}'' + m_{\rm II}\,\varepsilon_{{\rm II}z}''}{m_{\rm I} + m_{\rm II}} \tag{33}$$

and the initial values of the generalized coordinates are as given in Eqs. (34) through (37).

$$\begin{aligned} x_{{\rm I}0} &= \varepsilon_{{\rm I}x}'' - \varepsilon_{cx}'' \\ x_{{\rm II}0} &= \varepsilon_{{\rm II}x}'' - \varepsilon_{cx}'' \end{aligned} \tag{34}$$

$$\begin{aligned} y_{{\rm I}0} &= \varepsilon_{{\rm I}y}'' - \varepsilon_{cy}'' \\ y_{{\rm II}0} &= \varepsilon_{{\rm II}y}'' - \varepsilon_{cy}'' \end{aligned} \tag{35}$$

$$\begin{aligned} z_{{\rm I}0} &= \varepsilon_{{\rm I}z}'' - \varepsilon_{cz}'' \\ z_{{\rm II}0} &= \varepsilon_{{\rm II}z}'' - \varepsilon_{cz}'' \end{aligned} \tag{36}$$

$$\begin{aligned} &\theta_{{\rm I}0} = \theta_{{\rm II}0} = \frac{\pi}{2}, \qquad \phi_{{\rm I}0} = \phi_{{\rm II}0} = 0 \\ &\psi_{{\rm I}0} = \psi_{{\rm II}0} = 0 \end{aligned} \tag{37}$$

The initial values of the components of the translational velocities are

$$\begin{aligned} \dot{x}_{{\rm I}0} &= \omega_{{\rm I}y'}\, z_{{\rm I}0} - \omega_{{\rm I}z'}\, y_{{\rm I}0} \\ \dot{y}_{{\rm I}0} &= \omega_{{\rm I}z'}\, x_{{\rm I}0} - \omega_{{\rm I}x'}\, z_{{\rm I}0} \\ \dot{z}_{{\rm I}0} &= \omega_{{\rm I}x'}\, y_{{\rm I}0} - \omega_{{\rm I}y'}\, z_{{\rm I}0} \end{aligned}$$

and

$$\begin{aligned} \dot{x}_{{\rm II}0} &= \omega_{{\rm II}y'}\, z_{{\rm II}0} - \omega_{{\rm II}z'}\, y_{{\rm II}0} \\ \dot{y}_{{\rm II}0} &= \omega_{{\rm II}z'}\, x_{{\rm II}0} - \omega_{{\rm II}x'}\, z_{{\rm II}0} \\ \dot{z}_{{\rm II}0} &= \omega_{{\rm II}x'}\, y_{{\rm II}0} - \omega_{{\rm II}y'}\, z_{{\rm II}0} \end{aligned} \tag{38}$$

The components of the initial angular velocities are determined in Appendix C, Eqs. (C-10) through (C-12). Note that initially

$$\omega_{{\rm I}x'} = \omega_{{\rm II}x'} = \omega_{x''} \tag{39}$$

$$\begin{aligned} \omega_{{\rm I}y'} &= \omega_{{\rm I}y'0} + \omega_{z''} \\ \omega_{{\rm II}y'} &= \omega_{z''} \end{aligned} \tag{40}$$

$$\omega_{{\rm I}z'} = \omega_{{\rm II}z'} = -\omega_{y''} \tag{41}$$

Where $\omega_{x''}$, $\omega_{y''}$, $\omega_{z''}$ are the initial angular velocities of body II in the drawing board CS; the bodies are assumed to be connected at $t = 0$, but a relative roll rate about z'', namely $\omega_{{\rm I}y'0}$, is permitted.

V. SOLUTION TECHNIQUE

Because of the ease with which the constraint conditions can be included in the Lagrangian formulation, this method was chosen for the derivation of the equations of motion.

A. Symbols

- $\mathbf{f}_i$ force due to i^{th} device
- $\mathcal{Q}$ generalized force
- q generalized coordinate
- T kinetic energy
- t time
- λ_l Lagrange multipliers

B. Lagrange Equations

The Lagrange equations, in the form most useful for this problem, are given by

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_s}\right) - \frac{\partial T}{\partial q_s} = \mathcal{Q}_s \tag{42}$$

The generalized forces for the chosen coordinates are given by Eqs. (43) through (48) (see Appendix D).

$$\mathcal{Q}_x = \Sigma\, \mathbf{f}_i \cdot \mathbf{e}_{ix} \tag{43}$$

$$\mathcal{Q}_y = \Sigma\, \mathbf{f}_i \cdot \mathbf{e}_{iy} \tag{44}$$

$$\mathcal{Q}_z = \Sigma\, \mathbf{f}_i \cdot \mathbf{e}_{iz} \tag{45}$$

$$\mathcal{Q}_\theta = \Sigma \text{ (all torques about } \theta \text{ axis)} \tag{46}$$

$$\mathcal{Q}_\phi = \Sigma \text{ (all torques about } \phi \text{ axis)} \tag{47}$$

$$\mathcal{Q}_\psi = \Sigma \text{ (all torques about } \psi \text{ axis)} \tag{48}$$

The θ axis is defined as a line passing through the CM perpendicular to the plane in which θ is measured; the ϕ axis and the ψ axis are defined similarly.

C. Constraints

Since the coordinates are no longer independent when constraints are present, Eq. (42) cannot be directly applied.

There are several methods of incorporating constraints; the method used here is the Lagrange multiplier technique. This technique uses nearly all of the mathematical machinery available in the unconstrained case, and is readily adaptable to numerical solution.

It can be shown[1] that if the constraints can be put in the form

$$\sum_s e_{ls}\, dq_s + e_{lt}\, dt = 0 \tag{49}$$

where l is used to indicate the different constraint equations, the equations of motion may be written in the form

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_s}\right) - \frac{\partial T}{\partial q_s} = \mathcal{Q}_s + \sum_l \lambda_l e_{ls}$$

$$s = 1, 2, \cdots, 12 \tag{50}$$

together with

$$\sum_s e_{ls}\, \dot{q}_s + e_{lt} = 0$$

$$l = 1, 2, \cdots, l_m, \quad l_m = \text{number of constraints} \tag{51}$$

where λ_l are the undetermined Lagrange multipliers. These make $12 + l_m$ equations in the $12 + l_m$ unknowns

$$[q_s\,(s = 1, \cdots, 12), \qquad \text{and} \qquad \lambda_l\,(l = 1, \cdots, l_m)]$$

that must be solved.

Although more equations must now be solved, most of the terms are calculated in the same manner as when no constraints are acting.

[1]Goldstein, H., *Classical Mechanics,* Addison–Wesley Publishing Company, Inc., Reading, Massachusetts, 1959, pp. 40–43.

Since the constraints to be dealt with are both scleronomic and holonomic, they are of the form

$$f_l\,(q_1, \cdots, q_{12}) = 0 \tag{52}$$

from which we obtain

$$\sum_s \frac{\partial f_l}{\partial q_s} dq_s = 0, \qquad \sum_s \frac{\partial f_l}{\partial q_s}\, \dot{q}_s = 0 \tag{53}$$

and

$$e_{ls} = \frac{\partial f_l}{\partial q_s}, \qquad e_{lt} = 0 \tag{54}$$

To make the constraint Eqs. (51) compatible (for numerical solution) with the Lagrange equations, they must be differentiated once more, with respect to time, giving

$$\sum_s (\dot{e}_{ls}\, \dot{q}_s + e_{ls}\, \ddot{q}_s) = 0 \tag{55}$$

or

$$\sum_s e_{ls}\, \ddot{q}_s = r_l \tag{56}$$

where

$$r_l = -\sum_s \dot{e}_{ls}\, \dot{q}_s \tag{57}$$

Equations (50) and (56) are then the complete set for the $12 + l_m$ unknowns.

D. Units

Since rocket thrusts, spring forces, etc., will normally be reported in pounds (force), and masses will normally be reported as earth-surface weights (pounds mass), and dimensional locations will normally be reported in inches, it will be convenient to employ the following system:

Quantity	Unit
Force	lbf
Mass	lbm
Time	sec
Length	in.

This is not a consistent system of units; that is, the equations of motion will not be dimensionally correct if the values reported in this system are entered directly into the equations. Dimensional consistency is obtained by multiplying the generalized forces by the constant $k = 385.7$ in./sec^2.

VI. FORCES

The forces must be expressed explicitly so that they may be entered into the equations of motion. The forces are divided into groups (depending on how they are produced) and are then added appropriately. For any particular problem, however, most of the forces will be absent. The program (Sections IX through XIII) has been written so that the sets of force calculations may be avoided to save computation time.

A. Components of $\mathcal{Q}$

$\mathcal{Q}$ generalized force

$\mathcal{D}$ component of $\mathcal{Q}$ due to coulomb drag forces

$\mathcal{E}$ component of $\mathcal{Q}$ due to explosive forces (pyrotechnics)

$\mathcal{J}$ component of $\mathcal{Q}$ due to cold-gas jet forces

$\mathcal{N}$ component of $\mathcal{Q}$ due to pneumatic forces

$\mathcal{R}$ component of $\mathcal{Q}$ due to hot rocket forces

$\mathcal{S}$ component of $\mathcal{Q}$ due to body II hard-mounted springs

$\mathcal{U}$ component of $\mathcal{Q}$ due to universally-jointed springs

The summation equation for the components of $\mathcal{Q}$ is

$$\mathcal{Q}_s = \mathcal{D}_s + \mathcal{E}_s + \mathcal{J}_s + \mathcal{N}_s + \mathcal{R}_s + \mathcal{S}_s + \mathcal{U}_s \tag{58}$$

where

$$s = \mathrm{I}_x, \cdots, \mathrm{II}_\psi$$

B. Coulomb Drag

A coulomb drag force is assumed to act between two points, one on each body, which are initially coincident. The drag force ceases when the separation distance exceeds some selected value. This model represents the usual idealization of electrical disconnects and similar devices. The forces are constant during their action time and take the direction of $\mathbf{v}_i$ (defined below).

1. Symbols

$\mathbf{D}_i$ estimated potential drag force at point i

D_i magnitude of i^{th} drag force

$\mathbf{d}_i$ separation distance vector at point i

d_i separation distance at point i (see Eqs. 59 and 60)

d_{if} separation distance beyond which i^{th} drag force ceases

$\mathbf{v}_i$ separation velocity at point i (see Appendix E)

$\mathbf{e}_\theta, \mathbf{e}_\phi, \mathbf{e}_\psi$ unit vectors in the $\dot{\theta}$, $\dot{\phi}$, and $\dot{\psi}$ directions

2. Distance $\mathbf{d}_i$, Between a Point on Body I and a Point on Body II

Clearly

$$\mathbf{d}_i = \mathbf{r}_{\mathrm{I}i} - \mathbf{r}_{\mathrm{II}i} = \mathbf{r} + \mathbf{A}_{\mathrm{I}}^{\mathrm{T}}\mathbf{r}'_{\mathrm{I}i} - \mathbf{A}_{\mathrm{II}}^{\mathrm{T}}\mathbf{r}'_{\mathrm{II}i} \tag{59}$$

where

$$\mathbf{d}_i = \begin{bmatrix} d_{ix} \\ d_{iy} \\ d_{iz} \end{bmatrix}, \quad \mathbf{r}'_{\mathrm{I}i} = \begin{bmatrix} x'_{\mathrm{I}i} \\ y'_{\mathrm{I}i} \\ z'_{\mathrm{I}i} \end{bmatrix}$$

$$\mathbf{r}'_{\mathrm{II}i} = \begin{bmatrix} x'_{\mathrm{II}i} \\ y'_{\mathrm{II}i} \\ z'_{\mathrm{II}i} \end{bmatrix}, \quad \mathbf{r} = \mathbf{r}_{\mathrm{I}} - \mathbf{r}_{\mathrm{II}} \tag{60}$$

3. Generalized Forces

The generalized forces are given in Eqs. (61) through (66).

$$\mathcal{D}_{\mathrm{I}x} = \sum_i \left(-D_i \frac{v_{ix}}{|\mathbf{v}_i|} \right) = -\mathcal{D}_{\mathrm{II}x} \tag{61}$$

$$\mathcal{D}_{\mathrm{I}y} = \sum_i \left(-D_i \frac{v_{iy}}{|\mathbf{v}_i|} \right) = -\mathcal{D}_{\mathrm{II}y} \tag{62}$$

$$\mathcal{D}_{Iz} = \sum_i \left(- D_i \frac{v_{iz}}{|\mathbf{v}_i|} \right) = -\mathcal{D}_{IIz} \tag{63}$$

$$\mathcal{D}_{I\theta} = \left\{ \sum_i \left(- \frac{D_i}{|\mathbf{v}_i|} \right) [(\mathbf{A}_I^T \mathbf{r}_{Ii}) \times \mathbf{v}_i] \right\} \cdot \mathbf{e}_{I\theta} \tag{64}$$

$$\mathcal{D}_{I\phi} = \left\{ \sum_i \left(- \frac{D_i}{|\mathbf{v}_i|} \right) [(\mathbf{A}_I^T \mathbf{r}_{Ii}) \times \mathbf{v}_i] \right\} \cdot \mathbf{e}_{I\phi} \tag{65}$$

$$\mathcal{D}_{I\psi} = \left\{ \sum_i \left(- \frac{D_i}{|\mathbf{v}_i|} \right) [(\mathbf{A}_I^T \mathbf{r}_{Ii}) \times \mathbf{v}_i] \right\} \cdot \mathbf{e}_{I\psi} \tag{66}$$

$$\begin{aligned} D_i &= \mathrm{D}_i \quad \text{if} \quad d_i \leqq d_{if} \\ D_i &= 0 \quad \text{if} \quad di > d_{if} \end{aligned} \tag{67}$$

$\mathcal{D}_{II\theta}$, $\mathcal{D}_{II\phi}$, and $\mathcal{D}_{II\psi}$ are found by replacing I by II and reversing sign in Eqs. (64), (65), and (66). The unit vectors are

$$\begin{aligned} \mathbf{e}_{I\theta} &= \begin{bmatrix} (c\phi_I) \\ (s\phi_I) \\ 0 \end{bmatrix}, \quad \mathbf{e}_{I\phi} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{e}_{I\psi} = \begin{bmatrix} (s\theta_I \, s\phi_I) \\ (-s\theta_I \, c\phi_I) \\ (c\theta_I) \end{bmatrix} \\ \mathbf{e}_{II\theta} &= \begin{bmatrix} (c\phi_{II}) \\ (s\phi_{II}) \\ 0 \end{bmatrix}, \quad \mathbf{e}_{II\phi} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{e}_{II\psi} = \begin{bmatrix} (s\theta_{II} \, s\phi_{II}) \\ (-s\theta_{II} \, c\phi_{II}) \\ (c\theta_{II}) \end{bmatrix} \end{aligned} \tag{68}$$

C. Pyrotechnics

Forces due to pyrotechnic devices are considered to be impulsive in nature, and are taken to act between the two bodies, as in the case of bolt cutters, explosive bolts, and linear separation charges. The devices are assumed to have fixed locations and directions relative to body II.

1. Symbols

E_i force due to i^{th} pyrotechnic device

I_i total impulse of i^{th} device

t_i firing time of i^{th} device

Δt integration interval

$\boldsymbol{\rho}_i$ direction cosine vector of the i^{th} device in the body II CS.

2. Forces

The impulses are idealized as constant forces acting over one integration interval. Thus

$$\begin{aligned} E_i &= 0 \quad \text{for} \quad t < t_i - \frac{\Delta t}{2} \quad \text{or} \quad t > t_i + \frac{\Delta t}{2} \\ E_i &= \frac{I_i}{\Delta t} \quad \text{for} \quad t_i - \frac{\Delta t}{2} \leqq t \leqq t_i + \frac{\Delta t}{2} \end{aligned} \tag{69}$$

The generalized forces are given in Eqs. (70) through (78).

$$\mathcal{E}_{Ix} = \sum_i E_i \mathbf{b}^{(1)T} \cdot \boldsymbol{\rho}_i \frac{(x_I - x_{II})}{|x_I - x_{II}|} = -\mathcal{E}_{IIx} \tag{70}$$

$$\mathcal{E}_{Iy} = \sum_i E_i \mathbf{b}^{(2)T} \cdot \boldsymbol{\rho}_i \frac{(x_I - x_{II})}{|x_I - x_{II}|} = -\mathcal{E}_{IIy} \tag{71}$$

$$\mathcal{E}_{Iz} = \sum_i E_i \mathbf{b}^{(3)T} \cdot \boldsymbol{\rho}_i \frac{(x_I - x_{II})}{|x_I - x_{II}|} = -\mathcal{E}_{IIz} \tag{72}$$

$$\mathcal{E}_{I\theta} = \left\{ \sum_i E_i [(\mathbf{A}_{II}^T \mathbf{r}'_{IIi} + \mathbf{r}) \times (\mathbf{A}_{II}^T \boldsymbol{\rho}_i)] \right\} \cdot \mathbf{e}_{I\theta} \tag{73}$$

$$\mathcal{E}_{I\phi} = \left\{ \sum_i E_i [(\mathbf{A}_{II}^T \mathbf{r}'_{IIi} + \mathbf{r}) \times (\mathbf{A}_{II}^T \boldsymbol{\rho}_i)] \right\} \cdot \mathbf{e}_{I\phi} \tag{74}$$

$$\mathcal{E}_{I\psi} = \left\{ \sum_i E_i [(\mathbf{A}_{II}^T \mathbf{r}'_{IIi} + \mathbf{r}) \times (\mathbf{A}_{II}^T \boldsymbol{\rho}_i)] \right\} \cdot \mathbf{e}_{I\psi} \tag{75}$$

$$\mathcal{E}_{II\theta} = \left\{ \mathbf{A}_{II}^T \left[\sum_i E_i (\mathbf{r}'_{IIi} \times \boldsymbol{\rho}_i) \right] \right\} \cdot \mathbf{e}_{II\theta} \tag{76}$$

$$\mathcal{E}_{II\phi} = \left\{ \mathbf{A}_{II}^T \left[\sum_i E_i (\mathbf{r}'_{IIi} \times \boldsymbol{\rho}_i) \right] \right\} \cdot \mathbf{e}_{II\phi} \tag{77}$$

$$\mathcal{E}_{II\psi} = \left\{ \mathbf{A}_{II}^T \left[\sum_i E_i (\mathbf{r}'_{IIi} \times \boldsymbol{\rho}_i) \right] \right\} \cdot \mathbf{e}_{II\psi} \tag{78}$$

D. Cold-Gas Jets

Cold-gas jet reaction systems are assumed to be of the constant volume, unregulated type. The *adiabatic process of an ideal gas approximation* is made in this section.

1. Symbols

- A_{ie} area at exit of nozzle jet i
- A_{ig} area at throat of nozzle jet i
- C_{i1}, C_{i2}, C_{i3} constants for jet i depending on nozzle geometry and properties of working fluid
- f_{i1}, f_{i2}, f_3 constants for jet i depending on initial conditions, nozzle geometry, and properties of working fluid
- $\mathbf{J}_i$ vector force of jet i
- J_i magnitude of the vector force of jet i
- J_{st} value of jet force below which J_i may be considered zero
- M_{ie} mach number at exit of jet i
- p pressure
- p_{it0} initial pressure in bottle for jet i
- R gas constant for working fluid as in $pv = RT$
- T absolute temperature
- T_{it0} initial temperature in bottle for jet i
- t time

t_{ie} time at which i^{th} jet fires

t_{if} time at which i^{th} jet cuts off

t_i $t - t_{ie}$

V_{i0} volume of bottle for jet i

v specific volume

α_i i^{th} nozzle divergence half angle

γ ratio of specific heats, c_p/c_v, for working fluid

λ_i nozzle divergence factor for jet i

$\lambda_{ix}, \lambda_{iy}, \lambda_{iz}$ cosines of angles between thrust vector for jet i and body I axes

$\rho_{ix}, \rho_{iy}, \rho_{iz}$ cosines of angles between thrust vector for jet i and body II axes

It is assumed (in Appendix F) that the total mass expelled by the cold-gas jets is negligible compared to the total mass of the body.

2. Forces

The vector force is given in Eq. (79), where either

$$\mathbf{J}_i = J_i (\lambda_{ix} \mathbf{e}_{\mathrm{I}x'} + \lambda_{iy} \mathbf{e}_{\mathrm{I}y'} + \lambda_{iz} \mathbf{e}_{\mathrm{I}z'})$$

or

$$\mathbf{J}_i = J_i (\rho_{ix} \mathbf{e}_{\mathrm{II}x'} + \rho_{iy} \mathbf{e}_{\mathrm{II}y'} + \rho_{iz} \mathbf{e}_{\mathrm{II}z'}) \tag{79}$$

This subsection contains all of the definitions necessary to use the force equations. Appendix F details the derivation of the following equations.

$$t_{ie} \leqq t \leqq t_{if}, \qquad J_i = \frac{\mathrm{f}_{i1}}{(1 + \mathrm{f}_{i2} t_i)^{\mathrm{f}_3}} \tag{80}$$

$$t < t_{ie} \quad \text{or} \quad t > t_{if} \quad \text{or} \quad J_i < J_{st}, \qquad J_i = 0 \tag{81}$$

Where

$$\mathrm{f}_{i1} = (-\lambda_i C_{i1} C_{i2} + A_{ie} C_{i3})\, p_{ito} \tag{82}$$

$$\mathrm{f}_{i2} = \left(\frac{\gamma - 1}{2}\right)\left(\frac{2}{\gamma + 1}\right)^{\gamma + 1/[2(\gamma - 1)]} (\gamma R\, \mathrm{T}_{ito})^{1/2} \frac{A_{ig}}{V_{i0}} \tag{83}$$

$$\mathrm{f}_3 = \frac{2\gamma}{\gamma - 1} \tag{84}$$

And where

$$\lambda_i = \frac{1}{2}(1 + \cos \alpha_i) \tag{85}$$

$$C_{i1} = -A_{ig}\left(\frac{2}{\gamma + 1}\right)^{\gamma + 1/[2(\gamma - 1)]} \left(\frac{\gamma}{R}\right)^{1/2} \tag{86}$$

$$C_{i2} = \frac{M_{ie}(\gamma R)^{1/2}}{\left(1 + \dfrac{\gamma - 1}{2} M_{ie}^2\right)^{1/2}} \tag{87}$$

$$C_{i3} = \left(1 + \frac{\gamma - 1}{2} M_{ie}^2\right)^{-\gamma/(\gamma-1)} \tag{88}$$

M_{ie} or the nozzle area ratio is to be determined from Eq. (89).

$$\frac{A_{ig}}{A_{ie}} = \left(\frac{\gamma + 1}{2}\right)^{\gamma+1/[2(\gamma-1)]} M_{ie} \left(1 + \frac{\gamma - 1}{2} M_{ie}^2\right)^{-\gamma+1/[2(\gamma-1)]} \tag{89}$$

The generalized forces are given in Eqs. (90) through (102).

$$\mathcal{F}_{Ix} = \mathbf{a}^{(1)T} \left[\sum_i J_i \boldsymbol{\lambda}_i\right] \tag{90}$$

$$\mathcal{F}_{Iy} = \mathbf{a}^{(2)T} \left[\sum_i J_i \boldsymbol{\lambda}_i\right] \tag{91}$$

$$\mathcal{F}_{Iz} = \mathbf{a}^{(3)T} \left[\sum_i J_i \boldsymbol{\lambda}_i\right] \tag{92}$$

$$\mathcal{F}_{I\theta} = \left[\sum_i J_i (\mathbf{r}'_{Ii} \times \boldsymbol{\lambda}_i)\right] \cdot \mathbf{e}'_{I\theta} \tag{93}$$

$$\mathcal{F}_{I\phi} = \left[\sum_i J_i (\mathbf{r}'_{Ii} \times \boldsymbol{\lambda}_i)\right] \cdot \mathbf{e}'_{I\phi} \tag{94}$$

$$\mathcal{F}_{I\psi} = \left[\sum_i J_i (\mathbf{r}'_{Ii} \times \boldsymbol{\lambda}_i)\right] \cdot \mathbf{e}'_{I\psi} \tag{95}$$

$$\mathcal{F}_{IIx} = \mathbf{b}^{(1)T} \left[\sum_i J_i \boldsymbol{\rho}_i\right] \tag{96}$$

$$\mathcal{F}_{IIy} = \mathbf{b}^{(2)T} \left[\sum_i J_i \boldsymbol{\rho}_i\right] \tag{97}$$

$$\mathcal{F}_{IIz} = \mathbf{b}^{(3)T} \left[\sum_i J_i \boldsymbol{\rho}_i\right] \tag{98}$$

$$\mathcal{F}_{II\theta} = \left[\sum_i J_i (\mathbf{r}'_{IIi} \times \boldsymbol{\rho}_i)\right] \cdot \mathbf{e}'_{II\theta} \tag{99}$$

$$\mathcal{F}_{II\phi} = \left[\sum_i J_i (\mathbf{r}'_{IIi} \times \boldsymbol{\rho}_i)\right] \cdot \mathbf{e}'_{II\phi} \tag{100}$$

$$\mathcal{F}_{II\psi} = \left[\sum_i J_i (\mathbf{r}'_{IIi} \times \boldsymbol{\rho}_i)\right] \cdot \mathbf{e}'_{II\psi} \tag{101}$$

where

$$\mathbf{e}'_\theta = \begin{bmatrix} (c\psi) \\ -(s\psi) \\ 0 \end{bmatrix}$$

$$\mathbf{e}'_\phi = \mathbf{c}^{(3)}$$

$$\mathbf{e}'_\psi = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{102}$$

E. Pneumatics

Pneumatic ejection systems are modeled as a number of piston-cylinder assemblies fixed in body II and connected to a central reservoir (with or without regulation). It is assumed that the gas process is adiabatic, and that the gas is ideal.

1. Symbols

A	area of piston face
J	regulator existence index: if 0, there is no regulator; if 1, there is a regulator that remains closed after $p_b = p_p$; if 2, there is a regulator that remains open after $p_b = p_p$
j_i	displacement of i^{th} piston from closed position (see Appendix B)
$j_{eqp,\,i}$	value of j_i such that $p_b = p_p$
$j_{\max,\,i}$	distance at which i^{th} piston separates
N_i	magnitude of the i^{th} pneumatic force
$\mathbf{N}_i$	pneumatic force vector of the i^{th} piston
p	pressure
V_b	volume of gas bottle, including all lines upstream of the faces of the closed pistons
b (subscript)	gas bottle
i (subscript)	refers to the i^{th} piston assembly or a part of the assembly
p (subscript)	piston chamber
0 (subscript)	initial conditions
γ	ratio of specific heats for working fluids
$\rho_{ix}, \rho_{iy}, \rho_{iz}$	cosines of angles between piston rod i and the $x'_{II}\, y'_{II}\, z'_{II}$ axes

2. Forces

Using the results of Appendix H, the forces are calculated as shown in Eq. (103).

$$\mathbf{N}_i = N_i \left(\rho_{ix}\,\mathbf{e}_{IIx'} + \rho_{iy}\,\mathbf{e}_{IIy'} + \rho_{iz}\,\mathbf{e}_{IIz'}\right) \tag{103}$$

The factors that must be taken into consideration when calculating the value of N_i are: whether or not there is a pressure regulator and, if there is, whether or not it stays open after $p_b = p_p$.

$$j_{eqp,\,i} = \frac{V_b}{\gamma A_i}\left(\frac{p_{b0}}{p_{pi}} - 1\right) \tag{104}$$

Where J = 0,

if

$$j_i < j_{\max,\,i}$$

then

$$N_i = p_{b0}\,A_i \left(1 + \frac{\sum_i A_i\, j_i}{V_b}\right)^{-\gamma} \tag{105}$$

if

$$j_i \geq j_{\max,\,i}$$

then

$$N_i = 0 \tag{106}$$

Where J = 1,

if

$$j_i \leqq j_{eqp,\,i}$$

then

$$N_i = p_{pi}\,A_i \tag{107}$$

if

$$j_{eqp,\,i} < j_i < j_{\max,\,i}$$

then

$$N_i = p_{pi}\,A_i \left(\frac{j_{eqp,\,i}}{j_i}\right)^{\gamma} \tag{108}$$

if

$$j_{\max,\,i} \leqq j_i$$

then

$$N_i = 0 \tag{109}$$

Where J = 2,

if

$$j_i \leqq j_{eqp,\,i}$$

then

$$N_i = p_{pi}\,A_i \tag{110}$$

if

$$j_{eqp,\,i} < j_i < j_{\max,\,i}$$

then

$$N_i = p_{pi}\,A_i \left(1 + \frac{\sum_k A_k\,b_k}{V_b \sum_k A_k\,C_k}\right)^{-\gamma} \tag{111}$$

if

$$j_{\max,\,i} \leqq j_i$$

then

$$N_i = 0 \tag{112}$$

Where

$$b_k = j_k \qquad \text{if} \qquad j_k < j_{eqp,\,k} \tag{113}$$

$$b_k = j_k - j_{eqp,\,k} \qquad \text{if} \qquad j_{eqp,\,k} \leqq j_k \tag{114}$$

and

$$C_k = j_k \qquad \text{if} \qquad j_k < j_{eqp,\,k} \tag{115}$$

$$C_k = j_{eqp,\,k} \qquad \text{if} \qquad j_{eqp,\,k} \leqq j_k \tag{116}$$

The generalized forces are given in Eqs. (117) through (126).

$$\mathcal{N}_{\mathrm{I}x} = \mathbf{b}^{(1)\mathrm{T}} \cdot \left[\sum_i N_i \boldsymbol{\rho}_i\right] \frac{x_{\mathrm{I}} - x_{\mathrm{II}}}{|x_{\mathrm{I}} - x_{\mathrm{II}}|} = -\mathcal{N}_{\mathrm{II}x} \tag{117}$$

$$\mathcal{N}_{\mathrm{I}y} = \mathbf{b}^{(2)\mathrm{T}} \cdot \left[\sum_i N_i \boldsymbol{\rho}_i\right] \frac{y_{\mathrm{I}} - y_{\mathrm{II}}}{|y_{\mathrm{I}} - y_{\mathrm{II}}|} = -\mathcal{N}_{\mathrm{II}y} \tag{118}$$

$$\mathcal{N}_{\mathrm{I}z} = \mathbf{b}^{(3)\mathrm{T}} \cdot \left[\sum_i N_i \boldsymbol{\rho}_i\right] \frac{z_{\mathrm{I}} - z_{\mathrm{II}}}{|z_{\mathrm{I}} - z_{\mathrm{II}}|} = -\mathcal{N}_{\mathrm{II}z} \tag{119}$$

$$\mathcal{N}_{\mathrm{I}\theta} = \left[\sum_i N_i \left(\mathbf{A}_{\mathrm{I}}^{\mathrm{T}}\, \mathbf{r}'_{\mathrm{I}i}\right) \times \left(\mathbf{A}_{\mathrm{II}}^{\mathrm{T}}\, \boldsymbol{\rho}_i\right)\right] \cdot \mathbf{e}_{\mathrm{I}\theta} \tag{120}$$

$$\mathcal{N}_{\mathrm{I}\phi} = \left[\sum_i N_i \left(\mathbf{A}_{\mathrm{I}}^{\mathrm{T}}\, \mathbf{r}'_{\mathrm{I}i}\right) \times \left(\mathbf{A}_{\mathrm{II}}^{\mathrm{T}}\, \boldsymbol{\rho}_i\right)\right] \cdot \mathbf{e}_{\mathrm{I}\phi} \tag{121}$$

$$\mathcal{N}_{\mathrm{I}\psi} = \left[\sum_i N_i \left(\mathbf{A}_{\mathrm{I}}^{\mathrm{T}}\, \mathbf{r}'_{\mathrm{I}i}\right) \times \left(\mathbf{A}_{\mathrm{II}}^{\mathrm{T}}\, \boldsymbol{\rho}_i\right)\right] \cdot \mathbf{e}_{\mathrm{I}\psi} \tag{122}$$

$$\mathcal{N}_{\mathrm{II}\theta} = -\left[\sum_i N_i \left(\mathbf{r}'_{\mathrm{II}i} \times \boldsymbol{\rho}_i\right)\right] \cdot \mathbf{e}'_{\mathrm{II}\theta} \tag{123}$$

$$\mathcal{N}_{\mathrm{II}\phi} = -\left[\sum_i N_i \left(\mathbf{r}'_{\mathrm{II}i} \times \boldsymbol{\rho}_i\right)\right] \cdot \mathbf{e}'_{\mathrm{II}\phi} \tag{124}$$

$$\mathcal{N}_{\mathrm{II}\psi} = -\left[\sum_i N_i \left(\mathbf{r}'_{\mathrm{II}i} \times \boldsymbol{\rho}_i\right)\right] \cdot \mathbf{e}'_{\mathrm{II}\psi} \tag{125}$$

Where

$$\mathbf{e}_{\mathrm{I}\theta} = \begin{bmatrix} (c\phi) \\ (s\phi) \\ 0 \end{bmatrix}, \qquad \mathbf{e}_{\mathrm{I}\phi} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \qquad \mathbf{e}_{\mathrm{I}\psi} = \mathbf{c}^{(3)} \tag{126}$$

The unit vectors with regard to the body II-fixed CS ($\mathbf{e}'_{\mathrm{II}\theta}$, $\mathbf{e}'_{\mathrm{II}\phi}$, $\mathbf{e}'_{\mathrm{II}\psi}$), are given in Eqs. (102).

F. Rockets

Provision is made for attaching body-fixed, constant-force rockets to either body I or body II.

1. Symbols

$\dot{\mathrm{I}}_{xx\mathrm{I}i}, \cdots, \dot{\mathrm{I}}_{zz\mathrm{II}i}$	moment of inertia degradation rates
$\dot{m}_{\mathrm{I}i}, \dot{m}_{\mathrm{II}i}$	mass degradation rates
$\mathbf{R}_i$	vector thrust developed by i^{th} rocket
R_i	magnitude of thrust developed by i^{th} rocket
R_i	rated thrust level of i^{th} rocket
t_{ie}	time at which i^{th} rocket starts firing
t_{if}	time at which i^{th} rocket ceases firing
$\lambda_{ix}, \lambda_{iy}, \lambda_{iz}$	cosines of angles between thrust vector for rocket i and the x'_{I}, y'_{I}, z'_{I} axes

2. Forces

The forces described in this subsection are of the same form as those described in Section VI D, except that the rocket forces are constant with time.

If t is not between t_{ie} and t_{if}, rocket i develops no thrust; however, if t is between t_{ie} and t_{if}, it is assumed to develop full thrust. Therefore, the thrust magnitude is as described by Eq. (127)

$$\begin{aligned}
&\text{If} \quad t < t_{ie} \quad &\text{then} \quad R_i = 0 \\
&\text{If} \quad t_{ie} \leq t \leq t_{if} \quad &\text{then} \quad R_i = \mathrm{R}_i \\
&\text{If} \quad t_{if} < t \quad &\text{then} \quad R_i = 0
\end{aligned} \tag{127}$$

Consequently, the components of the rocket contribution to the generalized forces are as given in Eqs. (128) through (139).

$$\mathscr{R}_{Ix} = \mathbf{a}^{(1)T} \cdot \left[\sum_i R_i \boldsymbol{\lambda}_i\right] \tag{128}$$

$$\mathscr{R}_{Iy} = \mathbf{a}^{(2)T} \cdot \left[\sum_i R_i \boldsymbol{\lambda}_i\right] \tag{129}$$

$$\mathscr{R}_{Iz} = \mathbf{a}^{(3)T} \cdot \left[\sum_i R_i \boldsymbol{\lambda}_i\right] \tag{130}$$

$$\mathscr{R}_{I\theta} = \left[\sum_i R_i (\mathbf{r}'_{Ii} \times \boldsymbol{\lambda}_i)\right] \cdot \mathbf{e}'_{I\theta} \tag{131}$$

$$\mathscr{R}_{I\phi} = \left[\sum_i R_i (\mathbf{r}'_{Ii} \times \boldsymbol{\lambda}_i)\right] \cdot \mathbf{e}'_{I\phi} \tag{132}$$

$$\mathscr{R}_{I\psi} = \left[\sum_i R_i (\mathbf{r}'_{Ii} \times \boldsymbol{\lambda}_i)\right] \cdot \mathbf{e}'_{I\psi} \tag{133}$$

$$\mathscr{R}_{IIx} = \mathbf{b}^{(1)T} \cdot \left[\sum_i R_i \boldsymbol{\rho}_i\right] \tag{134}$$

$$\mathscr{R}_{IIy} = \mathbf{b}^{(2)T} \cdot \left[\sum_i R_i \boldsymbol{\rho}_i\right] \tag{135}$$

$$\mathscr{R}_{IIz} = \mathbf{b}^{(3)T} \cdot \left[\sum_i R_i \boldsymbol{\rho}_i\right] \tag{136}$$

$$\mathscr{R}_{II\theta} = \left[\sum_i R_i (\mathbf{r}'_{IIi} \times \boldsymbol{\rho}_i)\right] \cdot \mathbf{e}'_{II\theta} \tag{137}$$

$$\mathscr{R}_{II\phi} = \left[\sum_i R_i (\mathbf{r}'_{IIi} \times \boldsymbol{\rho}_i)\right] \cdot \mathbf{e}'_{II\phi} \tag{138}$$

$$\mathscr{R}_{II\psi} = \left[\sum_i R_i (\mathbf{r}'_{IIi} \times \boldsymbol{\rho}_i)\right] \cdot \mathbf{e}'_{II\psi} \tag{139}$$

3. Mass and Moment of Inertia Decreases

During the firing of the rockets, the mass and the moments of inertia (of the body on which the rocket is mounted) will decrease. The amount of mass remaining is shown in Eqs. (140) through (145).

If

$$t \leq t_{ie} \quad \text{then} \quad m_I = m_{I0} \tag{140}$$

$$t_{ie} \leq t < t_{if} \quad \text{then} \quad m_I = m_{I0} - (t - t_{ie}) \sum_i \dot{m}_{Ii} \tag{141}$$

$$t_{if} \leq t \quad \text{then} \quad m_I = m_{I0} - (t_{if} - t_{ie}) \sum_i \dot{m}_{Ii} \tag{142}$$

$$t \leq t_{ie} \quad \text{then} \quad m_{II} = m_{II0} \tag{143}$$

$$t_{ie} < t < t_{if} \qquad \text{then} \qquad m_{II} = m_{II0} - (t - t_{ie}) \sum_i m_{IIi} \tag{144}$$

$$t_{if} \leqq t \qquad \text{then} \qquad m_{II} = m_{II0} - (t_{if} - t_{ie}) \sum_i m_{IIi} \tag{145}$$

The principal moments of inertia will decrease in a similar fashion. It is assumed that the inertia decrease can be adequately approximated by a linearization. It is also assumed that the mass loss has no effect on the products of inertia.

G. Hard-Mounted Springs on Body II

The usual linear-spring assumption has been made in this section; however, there is provision for the use of an experimental spring efficiency factor. No provision has been made for the spring-tip sliding friction. In practice, the static coefficient of friction is usually several times higher than the necessary value to prevent slippage so that the constraint mode will almost always be present (see Appendix H).

1. Symbols

j_i length of extension of i^{th} spring

N restriction index (if N = 0, there is no constraint; if N > 1, the constraint is operating)

$\mathbf{r}_{IIi}$ location of tip of spring i in the inertial CS (see Appendix B)

S_i force in i^{th} spring

S_{if} residual spring force

S_{i0} initial force in i^{th} spring

η spring efficiency factor

$\rho_{ix}, \rho_{iy}, \rho_{iz}$ cosines of angles between spring i and body II axes

2. Longitudinal Forces in Springs

The vector representation of the i^{th} spring force is given by Eqs. (146) and (147).

If

$$0 \leqq k_i j_i < S_{i0} - S_{if}$$

then

$$\mathbf{S}_i = \eta^2 (S_{i0} - k_i j_i)(\rho_{ix} \mathbf{e}_{IIx'} + \rho_{iy} \mathbf{e}_{IIy'} + \rho_{iz} \mathbf{e}_{IIz'}) \tag{146}$$

If

$$k_i j_i \geqq S_{i0} - S_{if}$$

then

$$\mathbf{S}_i = 0 \tag{147}$$

3. Generalized Forces

Because the Lagrange multipliers automatically supply the forces necessary to prevent slippage, only the longitudinal forces in the springs need be considered.

Then

$$\mathcal{S}_{IIx} = -\mathbf{a}^{(1)T} \sum_i S_i \boldsymbol{\rho}_i = -\mathcal{S}_{Ix} \tag{148}$$

$$\mathcal{J}_{IIy} = -\mathbf{a}^{(2)T} \sum_i S_i \boldsymbol{\rho}_i = -\mathcal{J}_{Iy} \quad (149)$$

$$\mathcal{J}_{IIz} = -\mathbf{a}^{(3)T} \sum_i S_i \boldsymbol{\rho}_i = -\mathcal{J}_{Iz} \quad (150)$$

$$\mathcal{J}_{II\theta} = -\left[\sum_i S_i (\mathbf{r}'_{IIi} \times \boldsymbol{\rho}_i)\right] \cdot \mathbf{e}'_{II\theta} \quad (151)$$

$$\mathcal{J}_{II\phi} = -\left[\sum_i S_i (\mathbf{r}'_{IIi} \times \boldsymbol{\rho}_i)\right] \cdot \mathbf{e}'_{II\phi} \quad (152)$$

$$\mathcal{J}_{II\psi} = -\left[\sum_i S_i (\mathbf{r}'_{IIi} \times \boldsymbol{\rho}_i)\right] \cdot \mathbf{e}'_{II\psi} \quad (153)$$

$$\mathcal{J}_{I\theta} = \left\{\sum_i S_i \left[(\mathbf{r}'_{IIi} - \mathbf{r}_I) \times (\mathbf{A}_{II}^{-1} \boldsymbol{\rho}_i)\right]\right\} \cdot \mathbf{e}_{I\theta} \quad (154)$$

$$\mathcal{J}_{I\phi} = \left\{\sum_i S_i \left[(\mathbf{r}'_{IIi} - \mathbf{r}_I) \times (\mathbf{A}_{II}^{-1} \boldsymbol{\rho}_i)\right]\right\} \cdot \mathbf{e}_{I\phi} \quad (155)$$

$$\mathcal{J}_{I\psi} = \left\{\sum_i S_i \left[(\mathbf{r}'_{IIi} - \mathbf{r}_I) \times (\mathbf{A}_{II}^{-1} \boldsymbol{\rho}_i)\right]\right\} \cdot \mathbf{e}_{I\psi} \quad (156)$$

where the unit vectors have been previously defined.

The equation for $\mathbf{r}_{IIi}$ is derived in Appendix B:

$$\mathbf{r}_{IIi} = \mathbf{r}_{II} + \mathbf{A}_{II}^T \mathbf{r}'_{IIi} + j_i \mathbf{A}_{II}^T \boldsymbol{\rho}_i$$

Clearly, $\mathbf{r}_{IIi} - \mathbf{r}_I$ is the vector connecting body I's CM and the tip of spring i, but expressed in inertial coordinates.

H. Universally-Jointed Springs

In actual practice, universally-jointed springs are rarely used. They do, however, closely approximate hard-mounted springs when the guiding is poor, and the relative motion is small (see Appendix H).

1. Symbols

- $\mathbf{d}_i$ vector connecting spring attachment points
- k_i spring rate of i^{th} spring
- $\mathbf{W}_i$ force in i^{th} spring
- W_{if} residual spring force
- W_{i0} initial force in i^{th} spring
- w_i a temporary notation convention (see Eq. 159)
- η spring efficiency factor

2. Forces

With universal joints at both ends, the spring force lines up with the vector connecting the two attachment points.

If

$$0 \leq k_i (|d_i| - |d_{io}|) < W_{io} - W_{if}$$

then

$$\mathbf{W}_i = \eta^2 [W_{io} - k_i (|\mathbf{d}_i| - |\mathbf{d}_{io}|)] \frac{\mathbf{d}_i}{|\mathbf{d}_i|} \tag{157}$$

If

$$k_i d_i \geq W_{io} - W_{if}$$

then

$$\mathbf{W}_i = 0 \tag{158}$$

For simplicity in notation, let

$$w_i = \frac{\eta^2 [W_{io} - k_i (|d_i| - |d_{io}|)]}{|d_i|} \tag{159}$$

Then

$$\mathbf{W}_i = w_i \, \mathbf{d}_i \tag{160}$$

The generalized forces are given in Eqs. (161) through (166).

$$\mathcal{W}_{Ix} = \sum_i w_i \, d_{ix} = -\mathcal{W}_{IIx} \tag{161}$$

$$\mathcal{W}_{Iy} = \sum_i w_i \, d_{iy} = -\mathcal{W}_{IIy} \tag{162}$$

$$\mathcal{W}_{Iz} = \sum_i w_i \, d_{iz} = -\mathcal{W}_{IIz} \tag{163}$$

$$\mathcal{W}_{I\theta} = \left\{ \sum_i w_i [(\mathbf{A}_I^T \mathbf{r}'_{Ii}) \times \mathbf{d}_i] \right\} \cdot \mathbf{e}_{I\theta} \tag{164}$$

$$\mathcal{W}_{I\phi} = \left\{ \sum_i w_i [(\mathbf{A}_I^T \mathbf{r}'_{Ii}) \times \mathbf{d}_i] \right\} \cdot \mathbf{e}_{I\phi} \tag{165}$$

$$\mathcal{W}_{I\psi} = \left\{ \sum_i w_i [(\mathbf{A}_I^T \mathbf{r}'_{Ii}) \times \mathbf{d}_i] \right\} \cdot \mathbf{e}_{I\psi} \tag{166}$$

The values of $\mathcal{W}_{II\theta,\phi,\psi}$ are obtained by replacing I by II in Eqs. (164) through (166).

VII. EQUATIONS OF MOTION

A. Unconstrained Case

The unconstrained equations of motion are determined from the Lagrange formulation as outlined in Section VB. The details of the calculation of the terms may be found in Appendix I. The equations of motion are given in Eqs. (167) through (172).

$$m\ddot{x} = \mathcal{Q}'_x \equiv p'_x \qquad (\text{from } q = x) \tag{167}$$

$$m\ddot{y} = \mathcal{Q}'_y \equiv p'_y \qquad (\text{from } q = y) \tag{168}$$

$$m\ddot{z} = \mathcal{Q}'_z \equiv p'_z \qquad (\text{from } q = z) \tag{169}$$

$$m_{11}\ddot{\theta} + m_{12}\ddot{\phi} + m_{13}\ddot{\psi} = p_1 + \mathcal{Q}'_\theta \equiv p'_1 \qquad (\text{from } q = \theta) \tag{170}$$

$$m_{21}\ddot{\theta} + m_{22}\ddot{\phi} + m_{23}\ddot{\psi} = p_2 + \mathcal{Q}'_\phi \equiv p'_2 \qquad (\text{from } q = \phi) \tag{171}$$

$$m_{31}\ddot{\theta} + m_{32}\ddot{\phi} + m_{33}\ddot{\psi} = p_3 + \mathcal{Q}'_\psi \equiv p'_3 \qquad (\text{from } q = \psi) \tag{172}$$

where

$$m_{11} = I_{xx}(c^2\psi) - 2I_{xy}(s\psi\, c\psi) + I_{yy}(s^2\psi) \tag{173}$$

$$m_{12} = [(I_{xx} - I_{yy})(s\psi\, c\psi) + I_{xy}(c^2\psi - s^2\psi)](s\theta) + [I_{xz}(c\psi) - I_{yz}(s\psi)](c\theta) \tag{174}$$

$$m_{13} = I_{xz}(c\psi) - I_{yz}(s\psi) \tag{175}$$

$$m_{21} = m_{12} \tag{176}$$

$$m_{22} = (s^2\theta)[I_{xx}(s^2\psi) + 2I_{xy}(s\psi\, c\psi) + I_{yy}(c^2\psi)] + 2(s\theta\, c\theta)[I_{xz}(s\psi) + I_{yz}(c\psi)] + (c^2\theta) I_{zz} \tag{177}$$

$$m_{23} = (s\theta)[I_{xz}(s\psi) + I_{yz}(c\psi)] + (c\theta) I_{zz} \tag{178}$$

$$m_{31} = m_{13} \tag{179}$$

$$m_{32} = m_{23} \tag{180}$$

$$m_{33} = I_{zz} \tag{181}$$

$$\begin{aligned} p_1 = {} & (s\theta)\dot{\phi}\dot{\psi}[4I_{xy}(s\psi\, c\psi) - (I_{xx} - I_{yy})(c^2\psi - s^2\psi) - I_{zz}] + 2\dot{\theta}\dot{\psi}[(I_{xx} - I_{yy})(s\psi\, c\psi) + I_{xy}(c^2\psi - s^2\psi)] \\ & + \dot{\psi}^2[I_{xz}(s\psi) + I_{yz}(c\psi)] + \dot{\phi}^2(s\theta\, c\theta)[I_{xx}(s^2\psi) + 2I_{xy}(s\psi\, c\psi) + I_{yy}(c^2\psi) - I_{zz}] \\ & + \dot{\phi}^2(c^2\theta - s^2\theta)[I_{xz}(s\psi) + I_{yz}(c\psi)] + 2(c\theta)[I_{xz}(s\psi) + I_{yz}(c\psi)] \end{aligned} \tag{182}$$

$$\begin{aligned} p_2 = {} & -2\dot{\phi}\{\dot{\theta}(s\theta\, c\theta)[I_{xx}(s^2\psi) + 2I_{xy}(s\psi\, c\psi) + I_{yy}(c^2\psi) - I_{zz}] + \dot{\psi}(s^2\theta)[(I_{xx} - I_{yy})(s\psi\, c\psi) \\ & + I_{xy}(c^2\psi - s^2\psi)] + \dot{\theta}(c^2\theta - s^2\theta)[I_{xz}(s\psi) + I_{yz}(c\psi)] + \dot{\psi}(s\theta\, c\theta)[I_{xz}(c\psi) - I_{yz}(s\psi)]\} \\ & - \dot{\theta}\{\dot{\theta}(c\theta)[(I_{xx} - I_{yy})(s\psi\, c\psi) + I_{xy}(c^2\psi - s^2\psi) + \dot{\psi}(s\theta)[(I_{xx} - I_{yy})(c^2\psi - s^2\psi) - 4I_{xy}(s\psi\, c\psi)] \\ & - \dot{\theta}(s\theta)[I_{xz}(c\psi) - I_{yz}(s\psi)]\} - \dot{\psi}\{\dot{\psi}(s\theta)[I_{xz}(c\psi) - I_{yz}(s\psi)] - \dot{\theta}(s\theta) I_{zz}\} \end{aligned} \tag{183}$$

$$\begin{aligned} p_3 = {} & -2\dot{\phi}\{\dot{\theta}(c\theta)[I_{xz}(s\psi) + I_{yz}(c\psi)]\} + \dot{\phi}(s\theta)\{(c^2\psi - s^2\psi)[I_{xx} - I_{yy})\dot{\theta} + I_{xy}\phi(s\theta)] + \dot{\theta} I_{zz}\} \\ & + (I_{xx} - I_{yy})(s\psi\, c\psi)[\dot{\phi}^2(s^2\theta) - \dot{\theta}^2] - 4I_{xy}\dot{\theta}\dot{\phi}(s\theta\, s\psi\, c\psi) - [I_{xz}(c\psi) - I_{yz}(s\psi)]\dot{\phi}^2(s\theta\, c\theta) \\ & - I_{xy}\dot{\theta}^2(c^2\psi - s^2\psi) \end{aligned} \tag{184}$$

It is understood that Eqs. (167) through (184) are to be applied to both body I and body II.

$$\mathcal{Q}'_s = k\mathcal{Q}_s, \qquad \text{where } k = 385.7 \text{ in./sec}^2 \tag{185}$$

The unconstrained equations of motion can then be written as

$$\mathbf{M}\ddot{\mathbf{q}} = \mathbf{f} \tag{186}$$

where

$$\mathbf{M} = \begin{bmatrix}
m_{\mathrm{I}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & m_{\mathrm{I}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & m_{\mathrm{I}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & m_{11\mathrm{I}} & m_{12\mathrm{I}} & m_{13\mathrm{I}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & m_{21\mathrm{I}} & m_{22\mathrm{I}} & m_{23\mathrm{I}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & m_{31\mathrm{I}} & m_{32\mathrm{I}} & m_{33\mathrm{I}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & m_{\mathrm{II}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{\mathrm{II}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{\mathrm{II}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{11\mathrm{II}} & m_{12\mathrm{II}} & m_{13\mathrm{II}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{21\mathrm{II}} & m_{22\mathrm{II}} & m_{23\mathrm{II}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{31\mathrm{II}} & m_{32\mathrm{II}} & m_{33\mathrm{II}}
\end{bmatrix} \tag{187}$$

$$\ddot{\mathbf{q}} = \begin{bmatrix} \ddot{x}_{\mathrm{I}} \\ \ddot{y}_{\mathrm{I}} \\ \ddot{z}_{\mathrm{I}} \\ \ddot{\theta}_{\mathrm{I}} \\ \ddot{\phi}_{\mathrm{I}} \\ \ddot{\psi}_{\mathrm{I}} \\ \ddot{x}_{\mathrm{II}} \\ \ddot{y}_{\mathrm{II}} \\ \ddot{z}_{\mathrm{II}} \\ \ddot{\theta}_{\mathrm{II}} \\ \ddot{\phi}_{\mathrm{II}} \\ \ddot{\psi}_{\mathrm{II}} \end{bmatrix} \tag{188}$$

$$\mathbf{f} = \begin{bmatrix} p'_{x\mathrm{I}} \\ p'_{y\mathrm{I}} \\ p'_{z\mathrm{I}} \\ p'_{1\mathrm{I}} \\ p'_{2\mathrm{I}} \\ p'_{3\mathrm{I}} \\ p'_{x\mathrm{II}} \\ p'_{y\mathrm{II}} \\ p'_{z\mathrm{II}} \\ p'_{1\mathrm{II}} \\ p'_{2\mathrm{II}} \\ p'_{3\mathrm{II}} \end{bmatrix} \tag{189}$$

B. Constrained Case

Since it is assumed that both constraint conditions will not be required at the same time, a set of constrained equations of motion has been derived for each constraint condition. These equations, with their attendant definitions, are given in Appendix J (see Appendixes K and L). Each set consists of 15 second-order differential equations.

C. Numerical Solution

In either the constrained or unconstrained case, the equations of motion are solved for the accelerations by Cramer's rule. The accelerations are then doubly integrated by the Adams–Moulton technique.

VIII. LIMITATIONS ON THE ANALYSIS AND SOLUTION ACCURACY

The limitations on the use of this program are established by the mathematical model described in this report. Considerable flexibility within the model can be obtained by ingenious use of its facets; the force equations and constraint modes need not be used for their nominal purposes. For example, the pin-puller delay can be used in conjunction with a very large m_I to obtain a fixed-point constraint on body II. A large number of other potentialities exist.

In view of the complexity of the program and the wide range of possible problem situations, computational accuracy is difficult to define. With the generalized coordinates given, the forces are obtained by straightforward noniterative procedures, so that the only errors introduced in computing these forces would be because of truncation. The relatively unknown area is then the integration of the equations of motion. As detailed in the computer program portion of this report (Sections IX through XIII), the equations are integrated with a routine that changes step size in an effort to maintain a constant integration error that is a function of the initial step size selected. It is recommended that different step sizes be tried on a new problem; a comparison of the results will indicate approximately the accuracy of integration.

IX. DETAILED DESCRIPTION OF THE COMPUTER PROGRAM: INPUT LINK, INTEGRATION LINK, AND PLOTTING LINK

The program is written in the FORTRAN/FAP system, and is divided into three chain links (input, integration, and plotting). The first link reads and prints the input data; the second link integrates the differential equations; the third link prepares output data for plotting (see Figs. 5 and 6). The detailed description of the integration technique (FMARK) is relegated to Appendix M. Rather than the conventional flow-chart approach, the discussion of the links is referenced to the actual FORTRAN listing (see Appendix N).

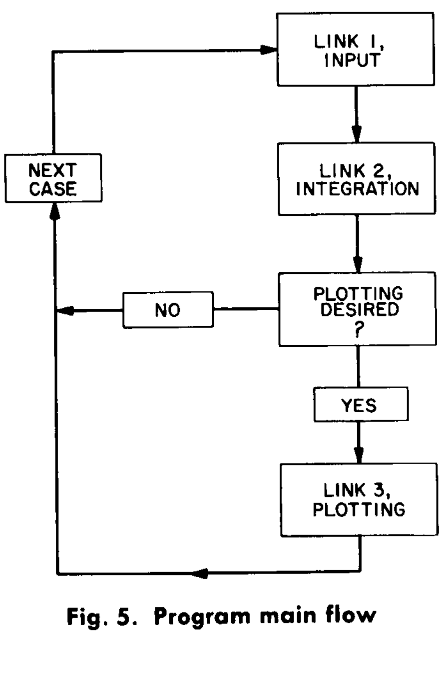

Fig. 5. Program main flow

A. Input Link

All appropriate input data are read and printed at the beginning of this link. Force input and initialization begins after FORMAT statement 845, and is controlled by the values of the indicators. These indicators are: ICD, coulomb drag; IPR, pyrotechnic; ICG, cold gas; IPN, pneumatic; IRK, rockets; ISP, hard-mounted springs; ISU, universally jointed springs; and IPN, pin-puller constraint. Input for a force is bypassed if its indicator is *zero*. If the indicator is *non-zero*, input/output is performed along with the initial calculations on the forces and conversion from drawing board CS to body-fixed CS. All parameters needed for the force calculations are transmitted to the integration link through COMMON storage. No special subroutines are used in the input link.

Of special note is the sentinel ICLAG, which, at the beginning of the link = 1. If the hard-mounted spring constraint is used, ICLAG = −1 (statement 6552); however, if the pin-puller constraint is used, ICLAG = 0 (above statement 43). Both constraints cannot be working at the same time, but the spring constraint can be used after the pin-puller constraint terminates.

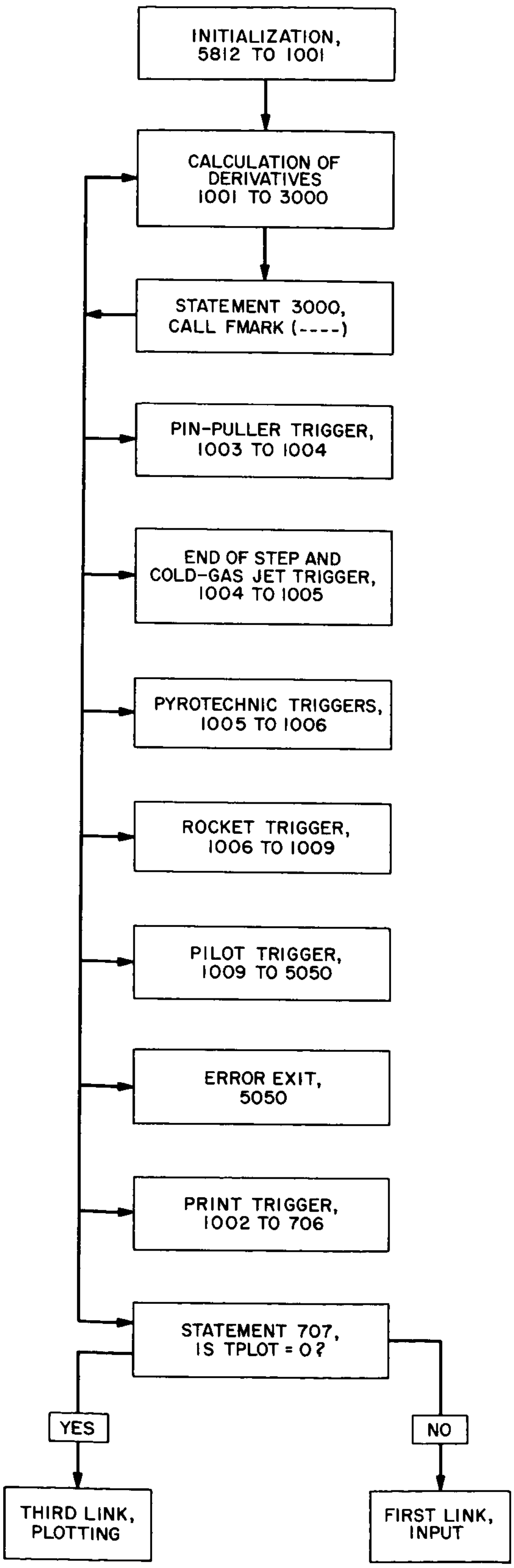

Fig. 6. Flow of integration link

B. Integration Link

1. Integration Subroutine, FMARK

The FMARK integration subroutine (statement 3000) applies the Adams–Moulton method with automatic error control. The computed GO TO statement, following the *call* to FMARK, is an integral part of the subroutine. The variables in the calling sequence are defined as follows:

KIK	fixed-point variable that must correspond to the variable of the computed GO TO statement
HB	an array of dimension 6 [i.e., HB (1), · · · , HB (6)] that contains parameters that monitor the Adams–Moulton automatic error control
HB (1) = Δt	step size at the start of integration
HB (2) = 1	used to prevent unnecessary reduction in the step size
HB (3) = 0.00025	minimum allowable step size
HB (4) = 0.01	maximum allowable step size
HB (5) = 10^{-8}	relative lower bound on truncation error
HB (6) = 10^{-6}	relative upper bound on truncation error
NH	an array of dimension 6 [i.e., NH (1), · · · , NH (6)] that supplies FMARK with needed parameters
NH (1) = 24	total number of first order differential equations
NH (2) = NH (1)	
NH (3)	not used
NH (4)	not used
NH (5) = 4	number of backward differences used
NH (6)	not used
IVP	set to zero, denoting that the independent variable is carried in single precision
IPHI	set to 4 denoting Adams–Moulton automatic error control mode
1,1	first value of 1, in the calling sequence, used to supply the location

	(to FMARK) of the portion of the program where the derivatives are calculated (FMARK sets KIK = 1, then executes the computed GO TO statement); second value of 1 needed to suppress option in FMARK
2,2	first value of 2, in the calling sequence, used to supply the location (to FMARK) of the *end of step area* (FMARK sets KIK = 2 to transfer to the *end of step area*); second value of 2 is used to initialize triggers
(I, TRG)	8 sets of variables containing a fixed point and a floating point number that define a *trigger* (when T = TRG, KIK set = to I; computed GO TO statement is executed)
0	denotes end of calling sequence

When FMARK is entered (i.e., when statement 3000 is executed) it excepts the *initial* value for the independent variable T stored in the *highest* location in COMMON. Since T could be carried in double precision, a dummy variable (ZERO) is adjacent to T. FMARK expects the 24 initial values of the dependent variables to be stored in COMMON just below T and ZERO. FMARK will transfer control to the area of the program where the derivatives are to be calculated; it expects these derivatives supplied in COMMON just below the values of the independent variables. Thus, the first COMMON of this link is

COMMON T, ZERO, V1, V2, YDOT1, YDOT2

DIMENSION V1 (18), V2 (6), YDOT1 (18), YDOT2 (6)

where YDOT1 (I) is the derivative of V1 (I) and the derivative area of the program computes YDOT1, YDOT2 using V1, V2. After the derivatives have been calculated and placed in COMMON, control is returned to FMARK by

CALL ROUT (0)

Triggers can be either *on* or *off*. Turning a trigger on can be accomplished by

CALL TRMOD (I,1)

where I is the number of the trigger in the calling sequence (the first trigger is number 0). A trigger is turned off by

CALL TRMOD (I,0)

Control is returned to FMARK from a trigger by

CALL ROUT (0)

The following is a list of the triggers:

Trigger No.	Calling sequence designation	Program location	Function
0	2,TRIG3	1004	Cold gas on
1	3,TRIG2	1003	Pin-puller constraint off
2	4,TRIG1	1002	Print output
3	5,TRIG4	1005	First pyrotechnic shot
4	6,TROC	1006	Rockets on and off
5	7,PYRO2	1007	Second pyrotechnic shot
6	8,PYRO3	1008	Third pyrotechnic shot
7	9,PLOT	1009	Plot output

In certain cases, control is not returned to FMARK by calling ROUT (0). If a discontinuity has occurred in a derivative (e.g., pyrotechnic), the program again calls FMARK. This is done to recompute the backward differences.

The main function of the end of step area is to verify that θ is not near zero. If θ is near zero, the transformation is made and the integration continues.

2. Main Program

The DO loop that ends at statement 5812 puts the hard-mounted spring locations and directions in a more convenient form for later computation. From statement 5812 to statement 6677, the maximum integration time (WIL1) is tested to see if the integration will be terminated when the forces become zero (yes IFORCE = -1, no IFORCE = 0). Triggers are set up and turned on from 55 to 704. The program, up to the call to subroutine GET (1001), sets up initial conditions for the dependent variables and the arrays of HB and NH along with miscellaneous sentinels.

The area of the link beginning with statement 1001 and ending at statement 6007 calculates the derivatives for FMARK. Just before 3000, there is logic to either CALL ROUT (0), the normal return to FMARK from the derivative area, or CALL FMARK (· · ·). The latter branch is taken the first time through the derivative area and when a discontinuity has occurred in the derivatives.

The function of GET (1001) (which communicates to the link through COMMON) is to compute the transformation matrixes $\mathbf{A}_{\mathrm{I}}, \mathbf{A}_{\mathrm{II}}$, and $\dot{\mathbf{A}}_{\mathrm{I}}, \dot{\mathbf{A}}_{\mathrm{II}}$, coefficients m_{ij} and terms p_i for the unconstrained equations of motion and, if necessary, the matrix **F** and terms r_i for the constrained equations of motion. After GET, the link checks the force indicators (3032 to 67) and computes the necessary forces. The twelve force components are computed from the particular force components in the DO loop starting at 69. If ICLAG $\leqq 0$, the constrained case must be solved (414). This is done from statements 74 up to 75. The subroutine SOLVE finds the solution for the 15 linear equations of the constrained case. The derivatives for the unconstrained case are computed from 75 to 6007.

The remainder of the link (the area below statement 3000) contains the triggers and the end of step area. Control is transferred to 1003 when the pin-puller constraint terminates. In some cases, the spring constraint will be used after the pin-puller constraint. Thus, a test is made for the spring constraint (N $\geqq$ 2 yes). ICLAG is set to the proper value and a printout is made.

Statement 1004 serves as the cold-gas jet trigger and the end of step area. The array TCO contains the time cold-gas jets are turned on—in ascending order. The fixed-point variable (ICOLD) denotes the next jet to be turned on; thus ICOLD = 1,ICG. If the trigger has just been activated [i.e., T = TC (ICOLD)], control is transferred to 8010. Statement 8007 is executed after the last jet has been turned on, otherwise 8006 is executed. In either case, LDER = 1 for a restart. If LDER < 0 (200), a force has just ceased and a restart has to be made. If ISS (1) + ISS (2) $\neq$ O (6519), the spring constraint is acting and a spring must be switched. The area of the program from statement 6522 up to 1005 checks the size of θ_{I} and θ_{II}. If either value of θ is close to zero, a rotation of the body axes is made. Exit to FMARK is made when statement 426 transfers to 403.

The first pyrotechnic trigger starts at 1005, the second at 1007, and the third at 1008. The logic assumes that the triggers are fixed in chronological order. The firing of the triggers is completed in Δt sec (Runge-Kutta integration is used over this interval). The rocket trigger is placed at statement 1006, and the plot trigger is at 1009. For the plot trigger, all possible variables are ouput on tape B1; they will be read in for plotting in the next link. Statement 5050 is executed if an error occurs in FMARK. A typical error would be an activated trigger that has an execution time less that of the current value of the independent variable.

The output trigger is placed at 1002. The subroutine STEP enters FMARK and finds the current step size. Body axis rates are computed and placed in the array CB (301). Other variables computed are speed (V), rate about instantaneous axis (W1), and magnitude of body axis rate vector (WP1). Separation distance and velocity are computed below 350. If a constraint is working, the constraint values FQQ1, FQQ2, and FQQ3 are printed (6663). The constraint values correspond to d_{ix}, d_{iy}, and d_{iz} for the pin-puller and g_x, g_z, $e_{az} e_{bx} - e_{ax} e_{bz}$ for the spring constraint. If IHY $\neq 0$, the transformation matrixes $(\mathbf{A}_{\mathrm{I}}, \mathbf{A}_{\mathrm{II}})$ are output (6670). The logic from 8001 to 6501 controls the termination of the program. If IFORCE = 0, the job terminates only when the independent variable, T, is greater than WIL1 or when all forces cease. A force must be on (SUN $\neq 0$) before termination for null force (SUN = 0). If TPLOT = 0 (706), plotting is not done, and control returns to the input link. Statement 5000 prints input errors. Control goes to 5002 if a 3×3 matrix used to find the rotational acceleration is singular.

3. Other Subroutines

a. Constraint subroutine. The subroutine GET calculates those parameters needed to find the acceleration components, the rotational matrixes $(\mathbf{A}_{\mathrm{I}}, \mathbf{A}_{\mathrm{II}}, \dot{\mathbf{A}}_{\mathrm{I}}, \dot{\mathbf{A}}_{\mathrm{II}})$ and, if necessary, the 15×15 constraint matrix F.

Initially, the subroutine sets IM = 0 to denote calculations for body I; IM = 1 denotes calculations for body II. After computations for body I trigonometric functions (after 5004), A and AD are computed—note that IM2 = IM + 1 stores the $\mathbf{A}_{\mathrm{I}}, \dot{\mathbf{A}}_{\mathrm{I}}$ and $\mathbf{A}_{\mathrm{II}}, \dot{\mathbf{A}}_{\mathrm{II}}$ in different areas. A test is made just before 48, and m_{ij} and p_i for either body I or II are computed. The control then goes to statement 41. Between 101 and 3, matrixes needed for the constrained case (ICLAG $\leqq 0$) are computed (see Table 1). At 104, IM is tested to see if body II computations have been made. If the computations have been made, control goes to 43 to test whether the constrained or unconstrained case is running. Calculations needed for both pin-puller and spring constraints are made after statement 106, then a test is made to determine which constraint is acting. The subroutine from 111 up to 110 calculates elements of the F matrix and r_1, r_2, and r_3 – stored in CA (1), CA (2), CA (3) – for the pin-puller. All code below 110 calculates the same values for the spring constraint.

b. Force subroutines. Almost all the routines use the variable LDER, which controls integration restart. At each entry, a test is made in these routines to see if a force terminates. If a force does terminate, a discontinuity can occur in the force function, and an integration restart

Table 1. Identification of matrixes

Matrix	Statement
$\mathbf{A}_{\mathrm{I}}$	A (1,1,1)
$\mathbf{A}_{\mathrm{II}}$	A (1,1,2)
$\dot{\mathbf{A}}_{\mathrm{I}}$	AD (1,1,1)
$\dot{\mathbf{A}}_{\mathrm{II}}$	AD (1,1,2)
$\frac{\partial \mathbf{A}_{\mathrm{I}}}{\partial \mathbf{q}_k}$ $k = 4,5,6$	WAP (1,1, K-3)
$\frac{\partial \mathbf{A}_{\mathrm{II}}}{\partial \mathbf{q}_k}$ $k = 10,11,12$	WAP (1,1, K-6)
$\frac{d}{dt}\frac{\partial \mathbf{A}_{\mathrm{I}}}{\partial \mathbf{q}_k}$ $k = 4,5,6$	WAPP (1,1, K-3)
$\frac{d}{dt}\frac{\partial \mathbf{A}_{\mathrm{II}}}{\partial \mathbf{q}_k}$ $k = 10,11,12$	WAPP (1,1, K-6)
$r_{\mathrm{I}l}'$	WRP1 (1,1)
$r_{\mathrm{I}m}'$	WRP1 (1,2)
$r_{\mathrm{II}l}'$	WRP2 (1,1)
$r_{\mathrm{II}m}'$	WRP2 (1,2)
ρ_l	WRH (1,1)
ρ_m	WRH (1,2)
$\mathbf{T}$	TEE (1,1)
$\dot{\mathbf{T}}$	TD (1,1)

must be made. In this case, LDER is set to -1; restart is made when the end of step area is entered.

The input and output calling sequence for the force subroutines is presented for a clearer understanding of the program.

Coulomb drag, input

D (I) — magnitude of i^{th} drag force (D_i)

DF (I) — separation distance beyond which i^{th} drag force ceases (d_{if})

XC1 (I), YC1 (I), ZC1 (I) — location of i^{th} drag force on body I in body I CS $(x'_{\mathrm{I}i})$, $(y'_{\mathrm{I}i})$, $(z'_{\mathrm{I}i})$

XC2 (I), YC2 (I), ZC2 (I) — location of i^{th} drag force on body II in body II CS $(x'_{\mathrm{II}i})$, $(y'_{\mathrm{II}i})$, $(z'_{\mathrm{II}i})$

ICD — number of drag forces

Coulomb drag, output

DD (J) — component of Q_j due to coulomb drag forces $(\mathcal{D}_j)$

LDER $= 0$, no restart

< 0, restart to be made after end of step

ICD1 — number of drag forces that have been turned off

Pyrotechnics, input

PI (I) — total impulse of i^{th} pyrotechnic device (I_i)

TP (I) — firing time of i^{th} device (t_i)

XP2 (I), YP2 (I), ZP2 (I) — location of i^{th} device on body II in body II CS $(x'_{\mathrm{II}i})$, $(y'_{\mathrm{II}i})$, $(z'_{\mathrm{II}i})$

UPX (I), UPY (I), UPZ (I) — direction cosine vector of the i^{th} device in body II CS (ρ_{xi}), (ρ_{yi}), (ρ_{zi})

IPR — number of devices

Pyrotechnics, output

EP (J) — component of Q_j due to pyrotechnics $(\mathcal{E}_j)$

Cold-gas jets, input

FG1 (I), FG2 (I) — characteristic force constants of jet i (f_{i1}), (f_{i2})

FG3 — gas-system characteristic (f_3)

UGX (I), UGY (I), UGZ (I) — cosines of angles between thrust vector for jet i and body I axis (λ_{ix}), (λ_{iy}), (λ_{iz})

VGX (I), VGY (I), VGZ (I) — cosines of angles between thrust vector for jet i and body II axis (ρ_{ix}), (ρ_{iy}), (ρ_{iz})

XG1 (I), YG1 (I), ZG1 (I) — location of i^{th} jet on body I in body I CS $(x'_{\mathrm{I}i})$, $(y'_{\mathrm{I}i})$, $(z'_{\mathrm{I}i})$

XG2 (I), YG2 (I), ZG2 (I) — location of i^{th} jet on body II in body II CS $(x'_{\mathrm{II}i})$, $(y'_{\mathrm{II}i})$, $(z'_{\mathrm{II}i})$

GST — value of jet force below which J_i may be considered zero (J_{st})

TCG (I) — time which i^{th} jet cuts off (t_{if})

TCO (I) — time which i^{th} jet begins (t_{ie})

ICG — number of jets

Cold-gas jets, output

GJ (J) component of Q_j due to cold-gas forces ($\mathcal{G}_j$)

LDER =0, no restart

<0, restart to be made after end of step

ICGI number of jets that have been turned off

Pneumatic forces, input

JN	regulator existence index: if 0, there is no regulator; if 1, there is a regulator that remains closed after $p_b = p_p$; if 2, there is a regulator that remains open after $p_b = p_p$ (J)
PBO	initial bottle pressure (p_{bo})
VBN	volume of gas bottle, including all lines upstream of the faces of the closed pistons (V_b)
GAMN	ratio of specific heats for working fluid (γ)
AN (I)	area of i^{th} piston face (A_i)
RAX (I)	distance at which i^{th} piston separates ($j_{\max, i}$)
VNX (I), VNY (I), VNZ (I)	cosines of angles between piston rod i and the x'_{II}, y'_{II}, z'_{II} axes (ρ_{ix}), (ρ_{iy}), (ρ_{iz})
XN1 (I), YN1 (I), ZN1 (I)	location of i^{th} piston on body I in body I CS (x'_{Ii}), (y'_{Ii}), (z'_{Ii})
XN2 (I), YN2 (I), ZN2 (I)	location of i^{th} piston on body II in body II CS (x'_{IIi}), (y'_{IIi}), (z'_{IIi})
PPN (I)	(p_{pi}) A_i
EQP (I)	$j_{eqp, i}$
PNEM (I)	($j_{eqp, i}$) A_i
IPN	number of pneumatic forces

Pneumatic forces, output

XN (J) component of $\mathcal{Q}_j$ due to pneumatic forces ($\mathcal{N}_j$)

LDER =0, no restart

<0, restart to be made after end of step

IPN1 number of pneumatics that have been turned off

Rockets, input

IRK	number of rockets
RR (I)	magnitude of thrust developed by i^{th} rocket (R_i)
TE (I)	time at which i^{th} rocket starts firing (t_{ie})
TF (I)	time at which i^{th} rocket ceases firing (t_{if})
RUM	the sum $\Sigma\, \dot{m}_{Ii}$
PUM	the sum $\Sigma\, \dot{m}_{IIi}$
URX (I), URY (I), URZ (I)	cosines of angles between thrust vector for rocket i and the x'_I, y'_I, z'_I axes (λ_{ix}), (λ_{iy}), (λ_{iz})
VRX (I), VRY (I), VRZ (I)	cosines of angles between thrust vector for rocket i and the x'_{II}, y'_{II}, z'_{II} axes (ρ_{ix}), (ρ_{iy}), (ρ_{iz})
FM10, FM20	initial values of mass
XR1 (I), YR1 (I), ZR1 (I)	location of i^{th} rocket on body I in body I CS (x'_{Ii}), (y'_{Ii}), (z'_{Ii})
XR2 (I), YR2 (I), ZR2 (I)	location of i^{th} rocket on body II in body II CS (x'_{IIi}), (y'_{IIi}), (z'_{IIi})
SUN1	the sum $\Sigma\, \dot{I}_{xxIi}$
SUN2	the sum $\Sigma\, \dot{I}_{yyIi}$
SUN3	the sum $\Sigma\, \dot{I}_{zzIi}$
SAN1	the sum $\Sigma\, \dot{I}_{xxIIi}$

SAN2	the sum $\Sigma\, \dot{I}_{yyIIi}$
SAN3	the sum $\Sigma\, \dot{I}_{zzIIi}$
XX10, YY10, ZZ10	initial values of moments of inertia, body I
XX20, YY20, ZZ20	initial values of moments of inertia, body II

Rockets, output

XX1, YY1, ZZ1	computed values of moments of inertia, body I
XX2, YY2, ZZ2	computed values of moments of inertia, body II
FM1, FM2	computed values of mass (m_{I}), (m_{II})
R (J)	component of $\mathcal{Q}_j$ due to rocket forces $(\mathcal{R}_j)$

Triggers, rather than the variable LDER, are used to initiate restarts for the rocket forces.

Hard-mounted springs, input

WRP1 (1,I), WRP1 (2,I), WRP1 (3,I)	location of i^{th} spring on body I in body I CS (x'_{Ii}), (y'_{Ii}), (z'_{Ii})
WRP2 (1,I), WRP2 (2,I), WRP2 (3,I)	location of i^{th} spring on body II in body II CS (x'_{IIi}), (y'_{IIi}), (z'_{IIi})
WRH (1,I), WRH (2,I), WRH (3,I)	direction cosines between spring i and body II axes (ρ_{ix}), (ρ_{iy}), (ρ_{iz})
SO (I), SK (I), SIOQ	initial force, spring constant, and residual force for i^{th} spring (corrected for spring efficiency—$\eta^2 S_{i0}$, $\eta^2 k_i$, $\eta^2 S_{if}$)
ISP	number of springs
N	restriction index

Hard-mounted springs, output

ICLAG	set to 1 when constraint goes off
S (J)	component of $\mathcal{Q}_j$ due to body II hard-mounted springs $(\mathcal{S}_j)$
LDER	=0, no restart
	<0, restart to be made after end of step
ISP1	number of springs that have come off
ISS (1), ISS (2)	values controlling interchange of springs in constraint condition, in the case that the springs being used for constraint are not the last two to cease acting; if ISS (1) = ISS (2) = 0, no transfer required—if ISS (1) or ISS (2) is non-zero, transfer made by subroutine SWITCH

Universally-jointed springs, input

ISU	number of springs
WO (I), UK (I), WF (I)	initial force, spring constant, and final force of spring i (corrected for spring efficiency—$\eta^2 W_{i0}$, $\eta^2 k_i$, $\eta^2 W_{if}$)
XU1 (I), YU1 (I), ZU1 (I)	spring location on body I in body I CS (x'_{Ii}), (y'_{Ii}), (z'_{Ii})
XU2 (I), YU2 (I), ZU2 (I)	spring location on body II in body II CS (x'_{IIi}), (y'_{IIi}), (z'_{IIi})

Universally-jointed springs, output

W (J)	component of $\mathcal{Q}_j$ due to universally-jointed springs $(\mathcal{W}_j)$
LDER	=0, no restart <0, restart to be made after end of step
ISU1	number of springs that have come off

c. Matrix algebra subroutines. To aid in these calculations, the following subroutines are used:

MULT1 (A,B,C,M1,M2,M3)	calculates $A^TB = C$ where A is M1 × M2, B is M2 × M3
MULT2 (A,B,C,M1,M2,M3)	calculates $AB^T = C$ where A is M1 × M2, B is M2 × M3
MULT3 (A,B,C,M1,M2,M3)	calculates $AB = C$ where A is M1 × M2, B is M2 × M3

MCR (A,B,C,J) — forms the dot product of the 3×1 vector B with the j^{th} row of the 3×3 matrix A and stores the result in C

MCS (A,B,C,J) — forms the dot product of the 3×1 vector B with the j^{th} column of the 3×3 matrix B and stores the result in C

MADD (A,B,C,M1,M2) — forms $A + B = C$ with dimension $M1 \times M2$

MSUB (A,B,C,M1,M2) — forms $A - B = C$ with dimension $M1 \times M2$

MSCAL (A,B,C) — forms $AB = C$ where A is 1×1, B is 3×1

MCROS (A,B,C) — forms $A \times B = C$ where A,B,C are dimensioned 3×1 (15×15 linear set for the constraint is solved by subroutine SOLVE[2])

C. Plotting Link

All of the values that can be plotted are written on tape B1. At the beginning of the plotting link, plot indicators are written into the array LOT. If variable I is to be plotted, LOT (I) is nonzero.

[2]Moler, C. B., *Numerical Matrix Inversion With Iterative Improvement*, Technical Report No. 32-394, Jet Propulsion Laboratory, Pasadena, California, April 15, 1963.

X. INPUT DESCRIPTION AND FORMAT

The input format is given in the form of a series of tables, some of which may be omitted, depending on whether or not the corresponding type of force is to act. A complete list of input symbols and dimensions is collected here for reference.

A. Input List With Dimensions

1. General Inertial and Geometrical Characteristics

$I_{xxI}, I_{xyI}, \cdots, I_{zzII}$ — elements of inertia matrixes in the drawing board CS: lbm–in.^{2}

i_A — transformation matrix indicator: $=1$, A_I, A_{II} printed; $=0$, A_I, A_{II} not printed

m_I, m_{II} — masses: lbm

Δt — initial integration interval: sec

t_{PR} — printout interval: sec

t_0 — time at which program begins: sec

$\varepsilon''_{Ix}, \cdots, \varepsilon''_{IIz}$ — coordinates in the drawing board CS of the CM of bodies I and II: in.

$\omega''_x, \omega''_y, \omega''_z$ — rates about drawing board axes just before separation: deg/sec

$\omega_{Iy'0}$ — spinup roll of body I relative to body II: deg/sec

2. Forces

i_{CD} — coulomb drag force indicator, number of drag forces present: up to 8

i_{PY} — pyrotechnic force indicator, number of pyrotechnic devices present: up to 3

i_{CG} — cold-gas jet indicator, number of cold-gas jets: up to 8

i_{PN} — pneumatics indicator, number of pneumatic pistons: up to 8

i_{RK} — rockets indicator, number of rockets: up to 8

i_{SP} — spring indicator, number of hard-mounted springs: up to 8

i_{SU} — universally-jointed spring indicator, number of springs: up to 8

i_{CN} — constraint index: if $=0$, no constraint; if $=1$, pin-puller constraint is present

a. Coulomb drag

D_i	magnitude of i^{th} drag force: lbf
d_{if}	separation distance beyond which i^{th} drag force ceases: in.
x_i'', y_i'', z_i''	drawing board locations of i^{th} drag force: in.

b. Pyrotechnics

I_i	total impulse of i^{th} device: lbf-sec
t_i	firing time of i^{th} device: sec (program requires that $t_1 \leq t_2 \leq t_3$)
x_i'', y_i'', z_i''	pyrotechnic locations: in.
$\mu_{ix}', \mu_{iy}', \mu_{iz}'$	cosines of the acute angles between the drawing board axes and the i^{th} device, except that μ_{iy}' is negative

c. Cold-gas jets

f_{i1}, f_{i2}, f_3	constants for jet i depending on initial conditions, nozzle geometry, and properties of working fluid: lbf, 1/sec, 1
J_{ST}	value of jet force below which J_i may be considered zero: lbf
t_{i1}	time at which i^{th} jet turns on: sec
t_{i2}	time at which i^{th} jet cuts off: sec
x_i'', y_i'', z_i''	cold-gas jet locations: in.
$\mu_{ix}, \mu_{iy}, \mu_{iz}$	cosines of angles between thrust vector for jet i and body I axes
$\mu_{ix}', \mu_{iy}', \mu_{iz}'$	cosines of angles between thrust vector for jet i and body II axes

d. Pneumatics

A_i	area of i^{th} piston face: in.[2]
J	regulator existence index: if 0, there is no regulator; if 1, there is a regulator that remains closed after $p_b = p_p$; if 2, there is a regulator that remains open after $p_b = p_p$
$j_{\max, i}$	distance at which i^{th} piston separates: in.
p_{b0}	initial pressure in gas bottle: lbf (absolute)/in.[2]
p_{pi}	pressure beyond regulator (if any) for i^{th} piston: lbf (absolute)/in.[2]
V_b	volume of gas bottle, including all lines upstream of the faces of the closed pistons: in.[3]
x_i'', y_i'', z_i''	pneumatics locations: in.
γ	ratio of specific heats for working fluid
$\mu_{ix}, \mu_{iy}, \mu_{iz}$	cosines of angles between piston rod i and the $x_{II}', y_{II}', z_{II}'$ axes

e. Rockets

$\dot{I}_{xxIi}, \cdots, \dot{I}_{zzIIi}$	rate of decrease of principal moments of inertia due to rocket i: lbm in.[2]/sec
$\dot{m}_{Ii}, \dot{m}_{IIi}$	mass decrease rate due to i^{th} rocket: lbm/sec
R_i	magnitude of thrust developed by i^{th} rocket: lbf
t_{ie}	time at which i^{th} rocket starts firing: sec
t_{if}	time at which i^{th} rocket ceases firing: sec
x_i'', y_i'', z_i''	rocket locations: in.
$\mu_{ix}, \mu_{iy}, \mu_{iz}$ $\mu_{ix}', \mu_{iy}', \mu_{iz}'$	cosines of angles between thrust vector for rocket i and the drawing board axes (μ if on body I, μ' if on body II; if on body I, all $\mu_i' = 0$, if on body II, all $\mu_i = 0$)

f. Hard-mounted springs

k_i	i^{th} spring constant: lbf/in.
N	restriction index (if N = 0, there is no constraint; if N > 1, the constraint is operating)
S_{i0}	initial force in i^{th} spring: lbf
x_i'', y_i'', z_i''	spring location: in.
η	spring efficiency factor
$\mu_{ix}', \mu_{iy}', \mu_{iz}'$	cosines of angles between spring i and body II axes

g. Universally-jointed springs

k_i	spring rate of i^{th} spring: lbf/in.

W_{i0}	initial force in i^{th} spring: lbf
$x''_{Ii}, y''_{Ii}, z''_{Ii}$	location of end of spring i in body I: in.
$x''_{IIi}, y''_{IIi}, z''_{IIi}$	location of end of spring i in body II: in.
η	spring efficiency factor

h. Constraints

t_G	time of firing of delayed pin-puller: sec
x_P'', y_P'', z_P''	location of delayed pin-puller: in.

i. Plots

TPLOT	time increment for plotting: sec
LOT_i	plot indicators: a nonzero value calls for plot i
NCAMERA	camera 1 requires photographic processing; camera 2 is a fast-copy device

B. Input Format

Input data takes the form of a data card deck. To simplify input procedure, only three types of card format are used (with the exception of the first two cards).

For format type 1, the FORTRAN code is 6I12. These cards will contain up to six numbers per card, and each number is allotted 12 columns. The numbers must be integers (without a decimal point) and right adjusted.

For format type 2, the FORTRAN code is 6E12.8. These cards also will contain up to six numbers per card, with each number allotted 12 columns. These numbers are not integers (must have a decimal point), need not be right adjusted, and may have an exponent.

For format type 3, the FORTRAN code is 14I5. These cards are similar to those of format type 1, except that each number is allotted only 5 columns.

The input arrays for each group of input data are given in Tables 2 through 12. Each group is to be ordered as given in its corresponding table, and the groups are to be stacked according to the order of the tables. Table 2 gives the mass, geometry, and indicator input, and Tables 3 through 9 give the force inputs. If a particular force is not to act(i.e., its indicator number is zero) its table and corresponding cards are to be omitted. Table 10 gives the pin-puller constraint input. Table 11 gives the plotting directions and completes the input, and Table 12 defines the plot indicators.

Table 2. Mass, geometry, indicators

Format type	Input					
(2X, A70)	Comment (up to 70 characters)					
(E12.6, 2I12)	t_{CUT}	Case No.	i_A			
2	I_{xxI}	I_{xyI}	I_{xzI}	I_{yyI}	I_{yzI}	I_{zzI}
2	I_{xxII}	I_{xyII}	I_{xzII}	I_{yyII}	I_{yzII}	I_{zzII}
2	m_I	m_{II}	Δt	t_{PR}	t_0	
2	$\varepsilon_{x''I}$	$\varepsilon_{y''I}$	$\varepsilon_{z''I}$	$\varepsilon_{x''II}$	$\varepsilon_{y''II}$	$\varepsilon_{z''II}$
2	$\omega_{x''}$	$\omega_{y''}$	$\omega_{z''}$	$\omega_{Iy'0}$		
1	i_{CD}	i_{PY}	i_{CG}	i_{PN}	i_{RK}	i_{SP}
1	i_{SU}	i_{CN}				

Table 3. Coulomb drag

Format type	Input[a]					
2	d_{1f}	D_1	x_1''	y_1''	z_1''	
2	d_{2f}	D_2	x_2''	y_2''	z_2''	
.						
.						
.						
2	d_{kf}	D_k	x_k''	y_k''	z_k''	

[a] $k = i_{CD}$

Table 4. Pyrotechnics

Format type	Input[a]					
2	I_1	t_1				
2	μ_{1x}'	μ_{1y}'	μ_{1z}'	x_1''	y_1''	z_1''
.						
.						
.						
2	I_k	t_k				
2	μ_{kx}'	μ_{ky}'	μ_{kz}'	x_k''	y_k''	z_k''

[a] $k = i_{PY}$

Table 5. Cold-gas jets

Format type	Input[a] (body I)					
1	1	k				
2	f_{11}	f_{12}	t_{11}	t_{12}		
2	μ_{1x}	μ_{1y}	μ_{1z}	x_1''	y_1''	z_1''
2	f_{21}	f_{22}	t_{21}	t_{22}		
2	μ_{2x}	μ_{2y}	μ_{2z}	x_2''	y_2''	z_2''
.						
.						
.						
2	f_{k1}	f_{k2}	t_{k1}	t_{k2}		
2	μ_{kx}	μ_{ky}	μ_{kz}	x_k''	y_k''	z_k''
	(body II)					
1	2	l				
2	$f_{(k+1)1}$	$f_{(k+1)2}$	$t_{(k+1),1}$	$t_{(k+1),2}$		
2	$\mu_{(k+1)x}$	$\mu_{(k+1)y}$	$\mu_{(k+1)z}$	x_{k+1}''	y_{k+1}''	z_{k+1}''
.						
.						
.						
2	$f_{(k+l)1}$	$f_{(k+l)2}$	$t_{(k+l),1}$	$t_{(k+l),2}$		
2	$\mu_{(k+l)x}'$	$\mu_{(k+l)y}'$	$\mu_{(k+l)z}'$	x_{k+l}''	y_{k+l}''	z_{k+l}''

[a] k = number of jets on body I; l = number of jets on body II

Table 6. Pneumatics

Format type	Input[a]					
1	J					
2	p_{b0}	V_b	γ			
2	A_1	$j_{max,1}$	p_{p1}			
2	μ_{1x}'	μ_{1y}'	μ_{1z}'	x_1''	y_1''	z_1''
2	A_2	$j_{max,2}$	p_{p2}			
2	μ_{2x}'	μ_{2y}'	μ_{2z}'	x_2''	y_2''	z_2''
.						
.						
.						
2	A_k	$j_{max,k}$	p_{pk}			
2	μ_{kx}'	μ_{ky}'	μ_{kz}'	x_k''	y_k''	z_k''

[a] $k = i_{PN}$

Table 7. Rockets

Format type	Input[a]					
2	μ_{1x}	μ_{1y}	μ_{1z}	μ_{1x}'	μ_{1y}'	μ_{1z}'
2	μ_{2x}	μ_{2y}	μ_{2z}	μ_{2x}'	μ_{2y}'	μ_{2z}'
.						
.						
.						
2	μ_{kx}	μ_{ky}	μ_{kz}	μ_{kx}'	μ_{ky}'	μ_{kz}'
2	t_{1e}	t_{1f}				
2	t_{2e}	t_{2f}				
.						
.						
.						
2	t_{ke}	t_{kf}				
2	$\dot{I}_{xxI1}$	$\dot{I}_{yyI1}$	$\dot{I}_{zzI1}$	$\dot{I}_{xxII1}$	$\dot{I}_{yyII1}$	$\dot{I}_{zzII1}$
2	$\dot{I}_{xxI2}$	$\dot{I}_{yyI2}$	$\dot{I}_{zzI2}$	$\dot{I}_{xxII2}$	$\dot{I}_{yyII2}$	$\dot{I}_{zzII2}$
.						
.						
.						
2	$\dot{I}_{xxIk}$	$\dot{I}_{yyIk}$	$\dot{I}_{zzIk}$	$\dot{I}_{xxIIk}$	$\dot{I}_{yyIIk}$	$\dot{I}_{zzIIk}$
2	$\dot{m}_{1I}$	$\dot{m}_{1II}$	R_1	x_1''	y_1''	z_1''
2	$\dot{m}_{2I}$	$\dot{m}_{2II}$	R_2	x_2''	y_2''	z_2''
.						
.						
.						
2	$\dot{m}_{kI}$	$\dot{m}_{kII}$	R_k	x_k''	y_k''	z_k''

[a] $k = i_{RK}$

Table 8. Hard-mounted springs on body II

Format type	Input[a]					
2	η					
1	N					
2	μ_{1x}'	μ_{1y}'	μ_{1z}'	x_1''	y_1''	z_1''
2	S_{10}	k_1	S_{1f}			
2	μ_{2x}'	μ_{2y}'	μ_{2z}'	x_2''	y_2''	z_2''
2	S_{20}	k_2	S_{2f}			
.						
.						
.						
2	μ_{ix}'	μ_{iy}'	μ_{iz}'	x_i''	y_i''	z_i''
2	S_{i0}	k_i	S_{if}			

[a] $i = i_{SP}$

Table 9. Universally-jointed springs

Format type	Input[a]					
1	η					
2	x_{I1}''	y_{I1}''	z_{I2}''	x_{II1}''	y_{II1}''	z_{II1}''
2	W_{10}	k_1	W_{1f}			
.						
.						
.						
2	x_{Ii}''	y_{Ii}''	z_{Ii}	x_{IIi}''	y_{IIi}''	z_{IIi}''
2	W_{i0}	k_i	W_{if}			

[a] $i = i_{SU}$

Table 10. Pin-puller constraint

Format type	Input					
2	t_G	x_p''	y_p''	z_p''		

Table 11. Plot

Format type	Input[a]
2	TPLOT
3	LOT_1 LOT_2, · · · , LOT_{14}

[a]Enter TPLOT = 0. if no plots are desired

Table 12. Plots—indicators and arguments

Plot No. i	Type of plot[a]
1	x_I, y_I, z_I vs t
2	x_{II}, y_{II}, z_{II} vs t
3	$\dot{x}_I, \dot{y}_I, \dot{z}_I$ vs t
4	$\dot{x}_{II}, \dot{y}_{II}, \dot{z}_{II}$ vs t
5	$\ddot{x}_I, \ddot{y}_I, \ddot{z}_I$ vs t
6	$\ddot{x}_{II}, \ddot{y}_{II}, \ddot{z}_{II}$ vs t
7	θ_I, ϕ_I, ψ_I vs t
8	$\theta_{II}, \phi_{II}, \psi_{II}$ vs t
9	$\dot{\theta}_I, \dot{\phi}_I, \dot{\psi}_I$ vs t
10	$\dot{\theta}_{II}, \dot{\phi}_{II}, \dot{\psi}_{II}$ vs t
11	$\ddot{\theta}_I, \ddot{\phi}_I, \ddot{\psi}_I$ vs t
12	$\ddot{\theta}_{II}, \ddot{\phi}_{II}, \ddot{\psi}_{II}$ vs t
13	$\dot{\theta}_I$ vs θ_I
	$\dot{\phi}_I$ vs ϕ_I
	$\dot{\psi}_I$ vs ψ_I
14	$\dot{\theta}_{II}$ vs θ_{II}
	$\dot{\phi}_{II}$ vs ϕ_{II}
	$\dot{\psi}_{II}$ vs ψ_{II}

[a]Each type of plot is identified by an indicator LOT_i. If LOT_i is not zero, plot type i will be included in the output.

XI. OUTPUT DESCRIPTION AND FORMAT

Program output consists of the generalized coordinates, velocities and useful auxiliary quantities. A listing of all output quantities and dimensions is given below.

A. Output List with Dimensions

d_s separation distance $=[(x_I-x_{II})^2+(y_I-y_{II})^2+(z_I-z_{II})^2]^{1/2}$: in.

t time: sec

v_s separation velocity $=[(\dot{x}_I-\dot{x}_{II})^2+(\dot{y}_I-\dot{y}_{II})^2+(\dot{z}_I-\dot{z}_{II})^2]^{1/2}$: in./sec

v_I speed of CM: in./sec

x_I, y_I, z_I inertial coordinates: in.

$\dot{x}_I, \dot{y}_I, \dot{z}_I$ inertial components of velocity of CM: in./sec

θ_I, ϕ_I, ψ_I Eulerian angles: deg

$\dot{\theta}_I, \dot{\phi}_I, \dot{\psi}_I$ rates of change of Eulerian angles: deg/sec

$\omega_{x''I}, \omega_{y''I}, \omega_{z''I}$ rotation rates about drawing board axes: deg/sec

ω_I rotation rate about instantaneous axis: deg/sec

$\omega_{PY\,I}$ magnitude of vector sum of pitch and yaw rates: deg/sec

All of the output listed for body I also applies for body II.

B. Output Format

All quantities in the output are self-explanatory; the format is illustrated in the sample problem of Section XIII.

XII. ERROR DIAGNOSIS AND CHECKOUT

Because of the flexibility of the program, there are few error diagnoses that can be made internally. Should an error occur within the integration routine FMARK, the statement "ERROR OCCURS IN FMARK" is printed. Should the (12 × 12) or (15 × 15) matrix currently being inverted to solve for the generalized accelerations become singular, an appropriate statement is printed.

XIII. SAMPLE PROBLEM

The purpose of the sample problem is to illustrate the operational use of the computer program. For this reason, the system has been presented in idealized form.

A. Problem Statement

A capsule is to be separated from a spacecraft by means of four springs hard mounted on the spacecraft. The spring tips are constrained not to slip on their bearing surfaces. After separation, the capsule is spun up by means of 3 cold-gas jets. Subsequent to spinup, a retrorocket is fired. The properties of the bodies and the force mechanisms are as follows (see Fig. 7):

Spacecraft, body II

$$m = 1000 \text{ lbm}$$

$$I_{xx} = 200 \text{ slug-ft}^2$$

$$I_{yy} = 205 \text{ slug-ft}^2$$

$$I_{zz} = 100 \text{ slug-ft}^2$$

$$I_{xy} = I_{xz} = I_{yz} = 0$$

Capsule, body I

$$m = 1800 \text{ lbm}$$

$$I_{xx} = I_{yy} = 250 \text{ slug-ft}^2$$

$$I_{zz} = 100 \text{ slug-ft}^2$$

$$I_{xy} = I_{xz} = I_{yz} = 0$$

Rocket motor

Thrust = 100 lbf (constant)

Starting time = separation + 10 sec

Duration = 5 sec

Springs

$$k_1 = k_2 = k_3 = 100 \text{ lbf/in.}$$

$$k_4 = 80 \text{ lbf/in.}$$

Initial compression = 4 in.

Final force = 0

The springs are parallel to the z'' axis; they have 100% efficiency. Initial angular velocities are zero.

Gas jets

Each nozzle has its own reservoir and is well insulated. From Eqs. (82) through (84), the properties of the nozzles and the working fluid yield the force law

$$J_i = \frac{50}{(1+t)^{1.5}}, \quad i = 1, 2, 3$$

The jets are to fire at separation +2 sec and continue for 5 sec.

B. Choice of the Drawing Board CS

Since the hard mounted spring constraint is present, the z'' axis must be chosen perpendicular to the separation plane (see Fig. 8).

C. Output

The computer printout for the sample problem is presented in Appendix N, pp. N-79–N-84. Because of the length of the complete printout, only a few typical output data sets are shown.

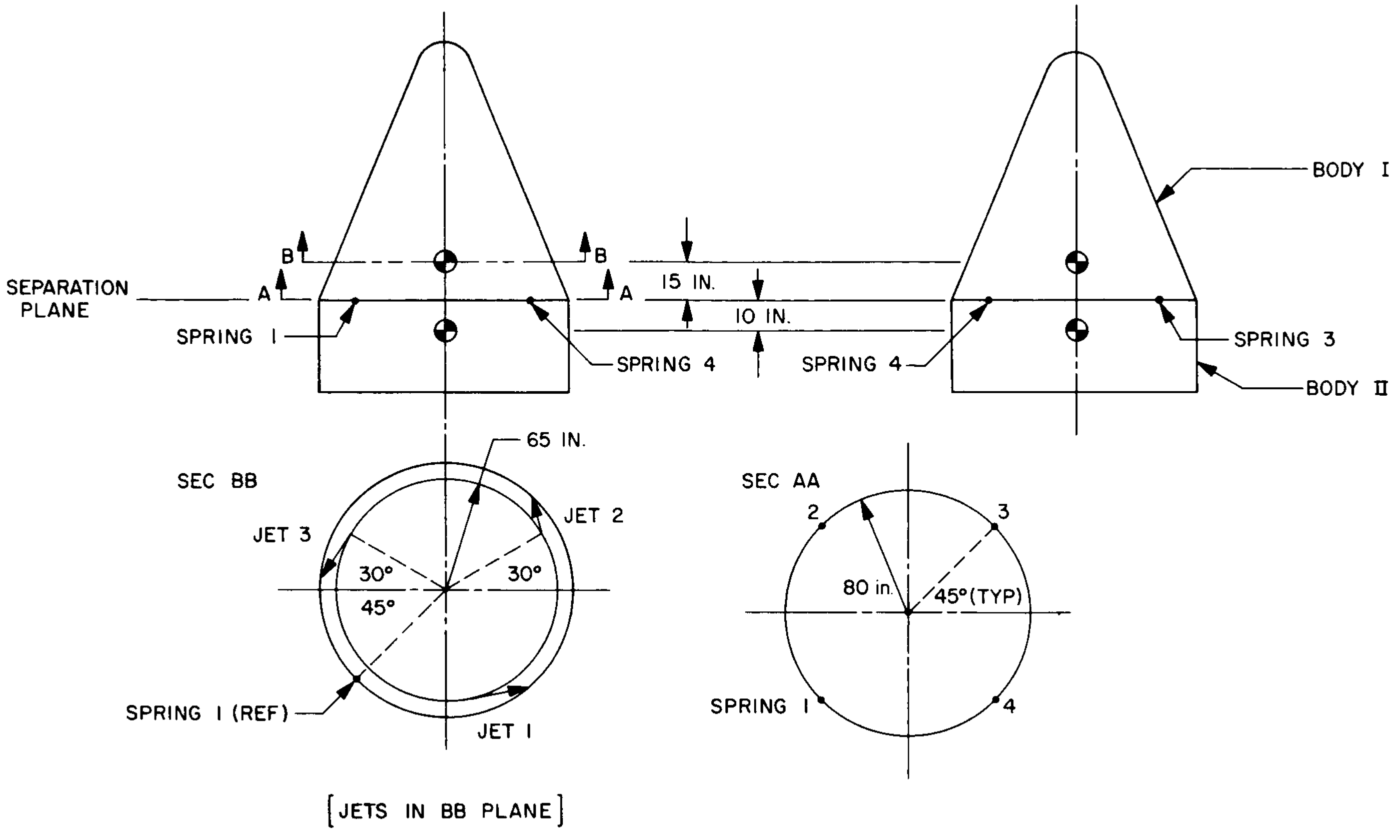

Fig. 7. Problem geometry

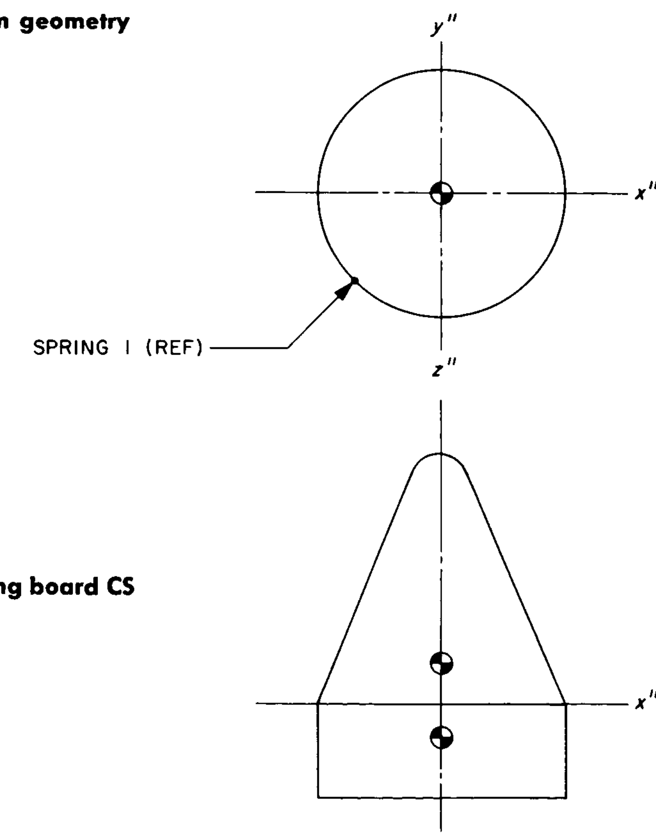

Fig. 8. Selection of drawing board CS

APPENDIX A

Eulerian Angles and Vector Component Transformations

To use the Lagrange equations it is necessary to define three independent parameters that specify the orientation of each body. A common choice for these parameters is the set of Eulerian angles. There is not complete uniformity in the definitions of the Eulerian angles; the definitions used in this report are those of Goldstein.[1]

A. Symbols

$\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$	rotation matrixes used in describing Eulerian angle generation
$\mathbf{A}^{-1}, \mathbf{B}^{-1}, \mathbf{C}^{-1}, \mathbf{D}^{-1}$	inverse of the $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ matrixes
a_{ij}	elements of the $\mathbf{A}_{\mathrm{I}}$ matrix (i^{th} row, j^{th} column)
b_{ij}	elements of the $\mathbf{A}_{\mathrm{II}}$ matrix (i^{th} row, j^{th} column)
c_{ij}	elements of the $\mathbf{A}_{\mathrm{I}}$ or $\mathbf{A}_{\mathrm{II}}$ matrix (i^{th} row, j^{th} column)
$\mathbf{r}_r$	location vector of the point whose index is i
$\mathbf{v}_a$	a vector in the xyz CS
v_x, v_y, v_z	components of the $\mathbf{v}_a$ in the xyz CS
$\mathbf{v}_{a'}$	a vector in the $x'y'z'$ CS
$v_{x'}, v_{y'}, v_{z'}$	components of $\mathbf{v}_{a'}$ in the $x'y'z'$ CS
$\mathbf{v}_\alpha$	a vector in the $\xi\eta\zeta$ CS
v_ξ, v_η, v_ζ	components of $\mathbf{v}_\alpha$ in the $\xi\eta\zeta$ CS
$\mathbf{v}_{\alpha'}$	a vector in the $\xi'\eta'\zeta'$ CS
$v_{\xi'}, v_{\eta'}, v_{\zeta'}$	components of $\mathbf{v}_{\alpha'}$ in the $\xi'\eta'\zeta'$ CS
x, y, z	the inertial CS
x', y', z'	the body-fixed CS
ξ, η, ζ	an intermediate CS used in the definition of the Eulerian angles
ξ', η', ζ'	another intermediate CS used in the definition of the Eulerian angles

B. Generation of Eulerian Angles

First, the x, y, z system (in which a vector is denoted $\mathbf{v}_a$ with components v_x, v_y, v_z) is rotated by an angle ϕ, counterclockwise, about the z axis (Fig. A-1a). The resultant CS is the $\xi\eta\zeta$ system; a vector in these coordinates is denoted $\mathbf{v}_\alpha$ with components v_ξ, v_η, v_ζ.

Second, the intermediate axes, $\xi\eta\zeta$, are rotated about the ξ axis, counterclockwise, by the angle θ to produce the $\xi'\eta'\zeta'$ axes (see Fig. A-1b). A vector in this CS, denoted $\mathbf{v}_{\alpha'}$, has components $v_{\xi'}, v_{\eta'}, v_{\zeta'}$. The ξ' axis also is known as the line of nodes.

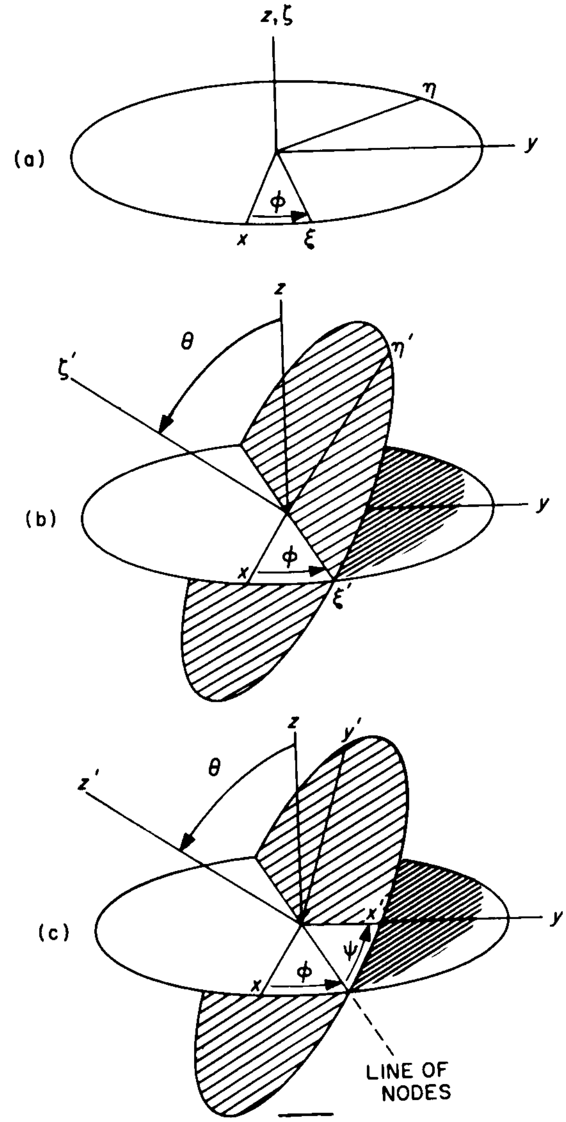

Fig. A-1. Generation of Eulerian angles

[1]Goldstein, H., *Classical Mechanics*, Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1959, pp. 107–109.

Third, the $\xi'\eta'\zeta'$ axes are rotated counterclockwise by an angle ψ about the ζ' axis to produce the $x'y'z'$ system of axes (see Fig. A-1c). The $x'y'z'$ system is the body-fixed system, and the xyz system is the space-fixed (or inertial) CS. A vector in the $x'y'z'$ system is denoted $\mathbf{v}_{a'}$ with components $v_{x'}, v_{y'}, v_{z'}$.

C. Vector Component Transformations

The elements of the complete rotation matrix **A** can be obtained by writing the matrix as the product of the separate rotation matrixes.

Thus

$$\mathbf{v}_{\alpha} = \mathbf{D}\,\mathbf{v}_{a} \qquad \text{where} \qquad \mathbf{D} = \begin{bmatrix} c\phi & s\phi & 0 \\ -s\phi & c\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{A-1}$$

$$\mathbf{v}_{\alpha'} = \mathbf{C}\,\mathbf{v}_{\alpha} \qquad \text{where} \qquad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta & s\theta \\ 0 & -s\theta & c\theta \end{bmatrix} \tag{A-2}$$

$$\mathbf{v}_{a'} = \mathbf{B}\,\mathbf{v}_{\alpha'} \qquad \text{where} \qquad \mathbf{B} = \begin{bmatrix} c\psi & s\psi & 0 \\ -s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{A-3}$$

$$\mathbf{v}_{a'} = \mathbf{BCD}\,\mathbf{v}_{a} = \mathbf{A}\,\mathbf{v}_{a} \qquad \text{where} \qquad \mathbf{A} = \mathbf{BCD} \tag{A-4}$$

$$\mathbf{A} = \begin{bmatrix} c\phi\,c\psi - c\theta\,s\phi\,s\psi & s\phi\,c\psi + c\theta\,c\phi\,s\psi & s\theta\,s\psi \\ -c\phi\,s\psi - c\theta\,s\phi\,c\psi & -s\phi\,s\psi + c\theta\,c\phi\,c\psi & s\theta\,c\psi \\ s\theta\,s\phi & -s\theta\,c\phi & c\theta \end{bmatrix} \tag{A-5}$$

For simplicity of notation, the elements of the $\mathbf{A}_{\mathrm{I}}$ and $\mathbf{A}_{\mathrm{II}}$ matrixes are denoted a_{ij} and b_{ij}, respectively. The elements of a nonspecific **A** matrix are denoted c_{ij}.

The inverse transformation is

$$\mathbf{v}_{a} = \mathbf{A}^{-1}\,\mathbf{v}_{a'} \tag{A-6}$$

Since the **BCD** matrixes are orthogonal, their product, **A**, is orthogonal, and the inverse of **A** is its transpose.

$$\mathbf{A}_{\mathrm{I}}^{-1} = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \tag{A-7}$$

$$\mathbf{A}_{\mathrm{II}}^{-1} = \begin{bmatrix} b_{11} & b_{21} & b_{31} \\ b_{12} & b_{22} & b_{32} \\ b_{13} & b_{23} & b_{33} \end{bmatrix} \tag{A-8}$$

The complete transformation for location vectors is

$$\mathbf{r}_i = \mathbf{r} + \mathbf{A}^{-1}\,\mathbf{r}'_i \tag{A-9}$$

$$\mathbf{r}'_i = \mathbf{A}\,(\mathbf{r}_i - \mathbf{r}) \tag{A-10}$$

APPENDIX B

Length of Extension of a Spring Hard-Mounted on Body II

To determine the length of extension of a hard-mounted spring on body II, it is necessary to define the bearing plane in the body I CS, express $\bar{\mathbf{r}}_i$ in the body I CS, and then solve for j_i.

A. Symbols

$\bar{\mathbf{r}}'_{IIi}$ location of the tip of spring i in body II CS

$\mathbf{T}$ matrix product $\mathbf{A}_I \mathbf{A}_{II}^T$

Because of the redefinition of the initial CS, the bearing plane is perpendicular to the y'_I axis, and $\mathbf{r}'_{Ii}$ is in the plane. The equation for the plane is

$$y'_I = y'_{Ii} \tag{B-1}$$

Clearly

$$\bar{\mathbf{r}}'_{IIi} = \mathbf{r}'_{IIi} + j_i \boldsymbol{\rho}_i$$

In the inertial CS

$$\bar{\mathbf{r}}_{IIi} = \mathbf{r}_{II} + \mathbf{A}_{II}^T \mathbf{r}'_{IIi} + j_i \mathbf{A}_{II}^T \boldsymbol{\rho}_i \tag{B-3}$$

In the body I CS

$$\bar{\mathbf{r}}'_{Ii} = \mathbf{A}_I (\mathbf{r}_{II} - \mathbf{r}_I) + \mathbf{A}_I \mathbf{A}_{II}^T \mathbf{r}'_{IIi} + j_i \mathbf{A}_I \mathbf{A}_{II}^T \boldsymbol{\rho}_i \tag{B-4}$$

$$\bar{\mathbf{r}}'_{Ii} = -\mathbf{A}_I \mathbf{r} + \mathbf{A}_I \mathbf{A}_{II}^T \mathbf{r}'_{IIi} + j_i \mathbf{T}^T \boldsymbol{\rho}_i \tag{B-5}$$

where a substitution of $\mathbf{T}$ has been used selectively for later simplicity.

Now the constraint condition is that $\bar{\mathbf{r}}'_{Ii}$ must have its tip in the bearing plane. Thus

$$y'_{Ii} = -\mathbf{a}_{(2)}^T \mathbf{r} + \mathbf{a}_{(2)}^T \mathbf{A}_{II}^T \mathbf{r}'_{IIi} + j_i \mathbf{t}^{(2)T} \boldsymbol{\rho}_i \tag{B-6}$$

and

$$j_i \mathbf{t}^{(2)T} \boldsymbol{\rho}_i = +y'_{Ii} + \mathbf{a}_{(2)}^T \mathbf{r} - \mathbf{a}_{(2)}^T \mathbf{A}_{II}^T \mathbf{r}'_{IIi} \tag{B-7}$$

The left-hand side of the above equation is simply $\mathbf{a}_2^T \cdot \mathbf{d}_i$ (see Coulomb Drag Section, Eq. 59). Thus

$$j_i = \frac{\mathbf{a}_{(2)}^T \cdot \mathbf{d}_i}{\mathbf{t}^{(2)T} \cdot \boldsymbol{\rho}_i} \tag{B-8}$$

APPENDIX C

Conversions Between Rates About Eulerian Axes and Rates About Body Axes

Since initial angular rates will normally be reported as body-axis angular rates, and since body-axis rates are often needed as the final result, conversions between Euler angle rates and body rates must be established.

A. Symbols

With the exception of those listed below, the symbols used in this appendix are the same as in Appendix A.

$\boldsymbol{\omega}$ angular velocity

$\boldsymbol{\omega}_\theta$ Eulerian θ component of $\boldsymbol{\omega}$

$(\boldsymbol{\omega}_\theta)_{x'}$ component along the body x' axis due to $\boldsymbol{\omega}_\theta$

$\boldsymbol{\omega}_{x'}$ body x' component of $\boldsymbol{\omega}$

$\mathbf{E}$ identity matrix

From the definition of $(\boldsymbol{\omega}_\theta)_{x'}$, etc., there is

$$\omega_{x'} = (\boldsymbol{\omega}_\theta)_{x'} + (\boldsymbol{\omega}_\phi)_{x'} + (\boldsymbol{\omega}_\psi)_{x'} \tag{C-1}$$

$$\omega_{y'} = (\boldsymbol{\omega}_\theta)_{y'} + (\boldsymbol{\omega}_\phi)_{y'} + (\boldsymbol{\omega}_\psi)_{y'} \tag{C-2}$$

$$\omega_{z'} = (\boldsymbol{\omega}_\theta)_{z'} + (\boldsymbol{\omega}_\phi)_{z'} + (\boldsymbol{\omega}_\psi)_{z'} \tag{C-3}$$

From geometry, it is evident that

$$\begin{bmatrix} (\boldsymbol{\omega}_\phi)_{x'} \\ (\boldsymbol{\omega}_\phi)_{y'} \\ (\boldsymbol{\omega}_\phi)_{z'} \end{bmatrix} = \mathbf{A} \begin{bmatrix} 0 \\ 0 \\ \dot{\phi} \end{bmatrix} \tag{C-4}$$

$$\begin{bmatrix} (\boldsymbol{\omega}_\theta)_{x'} \\ (\boldsymbol{\omega}_\theta)_{y'} \\ (\boldsymbol{\omega}_\theta)_{z'} \end{bmatrix} = \mathbf{B} \begin{bmatrix} \dot{\theta} \\ 0 \\ 0 \end{bmatrix} \tag{C-5}$$

$$\begin{bmatrix} (\omega_\psi)_{x'} \\ (\omega_\psi)_{y'} \\ (\omega_\psi)_{z'} \end{bmatrix} = \mathbf{E} \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} \tag{C-6}$$

Putting Eqs. (C-4), (C-5), (C-6) into Eqs. (C-1), (C-2), and (C-3) gives the components of $\boldsymbol{\omega}$ with respect to the body axes in terms of the rates about the Eulerian axes.

$$\omega_{x'} = \dot{\phi} \sin\theta \sin\psi + \dot{\theta} \cos\psi \tag{C-7}$$

$$\omega_{y'} = \dot{\phi} \sin\theta \cos\psi - \dot{\theta} \sin\psi \tag{C-8}$$

$$\omega_{z'} = \dot{\phi} \cos\theta + \psi \tag{C-9}$$

The inverse transformations may be found by a linear inversion (such as the use of Cramer's rule) of Eqs. (C-7) through (C-9). The result of this inversion is

$$\dot{\theta} = \omega_{x'} \cos\psi - \omega_{y'} \sin\psi \tag{C-10}$$

$$\dot{\phi} = \frac{1}{\sin\theta} [\omega_{x'} \sin\psi + \omega_{y'} \cos\psi] \tag{C-11}$$

$$\dot{\psi} = \omega_{z'} - \frac{\cos\theta}{\sin\theta} [\omega_{x'} \sin\psi + \omega_{y'} \cos\psi] \tag{C-12}$$

APPENDIX D

Determination of Generalized Forces

The generalized forces are defined by means of a virtual work principle: the virtual work done by a force, $\mathbf{f}$, acting through a virtual displacement, $\delta\mathbf{r}$, is equal to the virtual work done by the generalized forces, $\mathcal{Q}_k$, acting through the corresponding virtual generalized displacements, δq_k. That is

$$\mathbf{f}^T (\delta\mathbf{r}) = \sum_k \mathcal{Q}_k \, \delta q_k \tag{D-1}$$

Consider some point i on body I and let this be the point of action for a force $\mathbf{f}_i$. In inertial coordinates

$$\mathbf{r}_i = \mathbf{r}_\mathrm{I} + \mathbf{A}_\mathrm{I}^T \mathbf{r}'_{\mathrm{I}i}$$

and

$$\delta\mathbf{r}_i = \delta\mathbf{r}_\mathrm{I} + \delta\, [\, \mathbf{A}_\mathrm{I}^T \mathbf{r}'_{\mathrm{I}i}] \tag{D-2}$$

$$\delta\mathbf{r}_i = \delta\mathbf{r}_\mathrm{I} + \left[\frac{\partial \mathbf{A}_\mathrm{I}^T}{\partial \theta_\mathrm{I}} \delta\theta_\mathrm{I} + \frac{\partial \mathbf{A}_\mathrm{I}^T}{\partial \phi_\mathrm{I}} \delta\phi_\mathrm{I} + \frac{\partial \mathbf{A}_\mathrm{I}^T}{\partial \psi_\mathrm{I}} \delta\psi_\mathrm{I} \right] \mathbf{r}'_{\mathrm{I}i} \tag{D-3}$$

$$\delta\mathbf{r}_i = \delta\mathbf{r}_\mathrm{I} + \left[\frac{\partial \mathbf{A}_\mathrm{I}^T}{\partial \theta_\mathrm{I}} \mathbf{A}_\mathrm{I}\, \delta\theta_\mathrm{I} + \frac{\partial \mathbf{A}_\mathrm{I}^T}{\partial \phi_\mathrm{I}} \mathbf{A}_\mathrm{I}\, \delta\phi_\mathrm{I} + \frac{\partial \mathbf{A}_\mathrm{I}^T}{\partial \psi_\mathrm{I}} \mathbf{A}_\mathrm{I}\, \delta\psi_\mathrm{I} \right] \mathbf{r}_{\mathrm{I}i} \tag{D-4}$$

where $\mathbf{r}_{\mathrm{I}i}$ is $\mathbf{r}'_{\mathrm{I}i}$ expressed in the inertial CS.

Applying the virtual work principle

$$\sum_k \mathcal{Q}_k \, \delta q_k = \mathbf{f}_i^T \left\{ \left[\frac{\partial \mathbf{A}_\mathrm{I}^T}{\partial \theta_\mathrm{I}} \mathbf{A}_\mathrm{I}\, \delta\theta_\mathrm{I} + \frac{\partial \mathbf{A}_\mathrm{I}^T}{\partial \phi_\mathrm{I}} \mathbf{A}_\mathrm{I}\, \delta\phi_\mathrm{I} + \frac{\partial \mathbf{A}_\mathrm{I}^T}{\partial \psi_\mathrm{I}} \mathbf{A}_\mathrm{I}\, \delta\psi_\mathrm{I} \right] \mathbf{r}_{\mathrm{I}i} + \delta\mathbf{r}_\mathrm{I} \right\} \tag{D-5}$$

Since the δq_k are independent, the generalized forces are

$$\mathcal{Q}_{x\mathrm{I}} = f_{ix}, \mathcal{Q}_{y\mathrm{I}} = f_{iy}, \mathcal{Q}_{z\mathrm{I}} = f_{iz} \tag{D-6}$$

$$\mathcal{Q}_{\theta \mathrm{I}} = \mathbf{f}_i^{\mathrm{T}} \frac{\partial \mathbf{A}_{\mathrm{I}}^{\mathrm{T}}}{\partial \theta_{\mathrm{I}}} \mathbf{A}_{\mathrm{I}} \mathbf{r}_{\mathrm{I}i} \tag{D-7}$$

$$\mathcal{Q}_{\phi \mathrm{I}} = \mathbf{f}_i^{\mathrm{T}} \frac{\partial \mathbf{A}_{\mathrm{I}}^{\mathrm{T}}}{\partial \phi_{\mathrm{I}}} \mathbf{A}_{\mathrm{I}} \mathbf{r}_{\mathrm{I}i} \tag{D-8}$$

$$\mathcal{Q}_{\psi \mathrm{I}} = \mathbf{f}_i^{\mathrm{T}} \frac{\partial \mathbf{A}_{\mathrm{I}}^{\mathrm{T}}}{\partial \psi_{\mathrm{I}}} \mathbf{A}_{\mathrm{I}} \mathbf{r}_{\mathrm{I}i} \tag{D-9}$$

Equation (D-6) establishes the desired result for the linear coordinates. The remainder of this Appendix establishes the results for the angular coordinates.

It will now be shown that the matrix product $\mathbf{A}^{\mathrm{T}} \dot{\mathbf{A}}$ is skew symmetric. Since $\mathbf{A}_{\mathrm{I}}$ is orthogonal

$$\mathbf{a}^{(i)\mathrm{T}} \mathbf{a}^{(j)} = \delta_{ij} \tag{D-10}$$

Let $\mathbf{H} = \mathbf{A}_{\mathrm{I}}^{\mathrm{T}} \dot{\mathbf{A}}_{\mathrm{I}}$, with elements h_{ij}

Then

$$\mathrm{h}_{ij} = \mathbf{a}^{(i)\mathrm{T}} \dot{\mathbf{a}}^{(j)} \tag{D-11}$$

$$\mathrm{h}_{ij} + \mathrm{h}_{ji} = \mathbf{a}^{(i)\mathrm{T}} \dot{\mathbf{a}}^{(j)} + \mathbf{a}^{(j)\mathrm{T}} \dot{\mathbf{a}}^{(i)}$$

$$\mathrm{h}_{ij} + \mathrm{h}_{ji} = \mathbf{a}^{(i)\mathrm{T}} \dot{\mathbf{a}}^{(j)} + \dot{\mathbf{a}}^{(i)\mathrm{T}} \mathbf{a}^{(j)} = \frac{d}{dt} [\mathbf{a}^{(i)\mathrm{T}} \mathbf{a}^{(j)}] = \frac{d}{dt} [\delta_{ij}] = 0$$

and

$$\mathrm{h}_{ij} = -\mathrm{h}_{ji} \tag{D-12}$$

It may be easily verified that the product of a skew-symmetric matrix and a column matrix is equivalent to the vector cross product of two corresponding vectors

$$\boldsymbol{\omega} \times \mathbf{r} = \mathbf{L}\mathbf{r}$$

where

$$\mathbf{L} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}, \qquad \boldsymbol{\omega} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \tag{D-13}$$

Now consider two coordinate systems, $\Sigma_1 (x_1, x_2, x_3)$ and $\Sigma_2 (y_1, y_2, y_3)$. $\mathbf{D}$ is the transformation matrix from Σ_2 to Σ_1. Then, for a vector $\mathbf{r}$

$$\mathbf{r}_1 = \mathbf{D}\mathbf{r}_2 \tag{D-14}$$

$$\dot{\mathbf{r}}_1 = \mathbf{D}\dot{\mathbf{r}}_2 + \dot{\mathbf{D}}\mathbf{r}_2 = \mathbf{D}[\dot{\mathbf{r}}_2 + \mathbf{D}^{\mathrm{T}} \dot{\mathbf{D}} \mathbf{r}_2] \tag{D-15}$$

where the subscript on $\mathbf{r}$ indicates the coordinate system in which it is expressed. By analogy to elementary kinematics, we have

$$\mathbf{D}^{\mathrm{T}} \dot{\mathbf{D}} \mathbf{r}_2 = \boldsymbol{\omega} \times \mathbf{r}_2$$

where

$$\boldsymbol{\omega} = \begin{bmatrix} \omega_{y1} \\ \omega_{y2} \\ \omega_{y3} \end{bmatrix} \tag{D-16}$$

is the angular velocity of Σ_2 relative to Σ_1 expressed in Σ_2.

Since $\mathbf{A}_I$ is the transformation from the inertial CS to the body I-fixed CS, it is clear, from the above discussion, that

$$\mathbf{A}_I^T \dot{\mathbf{A}}_I = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

where

$$\boldsymbol{\omega} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \tag{D-17}$$

is the angular velocity of the inertial CS relative to the body I CS, expressed in the inertial CS.

Then

$$\dot{\mathbf{A}}_I^T \mathbf{A}_I = \begin{bmatrix} 0 & \omega_z & -\omega_y \\ -\omega_z & 0 & \omega_x \\ \omega_y & -\omega_x & 0 \end{bmatrix} \tag{D-18}$$

By the total derivative rule

$$\dot{\mathbf{A}}_I^T \mathbf{A}_I = \left[\frac{\partial \mathbf{A}_I^T}{\partial \theta}\dot{\theta} + \frac{\partial \mathbf{A}_I^T}{\partial \phi}\dot{\phi} + \frac{\partial \mathbf{A}_I^T}{\partial \psi}\dot{\psi}\right]\mathbf{A}_I \tag{D-19}$$

Let

$$\mathbf{B}_\theta \equiv [b_{\theta ij}] = \frac{\partial \mathbf{A}_I^T}{\partial \theta}\mathbf{A}_I, \mathbf{B}_\phi \equiv [b_{\phi ij}] = \frac{\partial \mathbf{A}_I^T}{\partial \phi}\mathbf{A}_I, \mathbf{B}_\psi \equiv [b_{\psi ij}] = \frac{\partial \mathbf{A}_I^T}{\partial \psi}\mathbf{A}_I$$

By considering equality for the (2,3) elements in (D-19), we obtain

$$\omega_x = b_{\theta 23}\,\dot{\theta} + b_{\phi 23}\,\dot{\phi} + b_{\psi 23}\,\dot{\psi} \tag{D-20}$$

but from vector addition

$$\omega_x = (-e_{\theta x})\,\dot{\theta} + (-e_{\phi x})\,\dot{\phi} + (-e_{\psi x})\,\dot{\psi} \tag{D-21}$$

where

$$\mathbf{e}_\theta = \begin{bmatrix} e_{\theta x} \\ e_{\theta y} \\ e_{\theta z} \end{bmatrix}, \qquad \mathbf{e}_\phi = \begin{bmatrix} e_{\phi x} \\ e_{\phi y} \\ e_{\phi z} \end{bmatrix}, \qquad \mathbf{e}_\psi = \begin{bmatrix} e_{\psi x} \\ e_{\psi y} \\ e_{\psi z} \end{bmatrix}$$

are the unit vectors along the positive rotation axes for θ, ϕ, ψ when the inertial CS is considered fixed.

Now, since $\dot{\theta}$, $\dot{\phi}$, and $\dot{\psi}$ are independent, Eqs. (D-20) and (D-21) yield

$$b_{\theta 23} = -e_{\theta x} \qquad b_{\phi 23} = -e_{\phi x} \qquad b_{\psi 23} = -e_{\psi x} \tag{D-22}$$

By writing equations similar to (D-20) and (D-21) for ω_y and ω_z, it can be shown that

$$\mathbf{B}_\theta = \begin{bmatrix} 0 & -e_{\theta z} & e_{\theta y} \\ e_{\theta z} & 0 & -e_{\theta x} \\ -e_{\theta y} & e_{\theta x} & 0 \end{bmatrix}, \quad \mathbf{B}_\phi = \begin{bmatrix} 0 & -e_{\phi z} & e_{\phi y} \\ e_{\phi z} & 0 & -e_{\phi x} \\ -e_{\phi y} & e_{\phi x} & 0 \end{bmatrix}, \quad \mathbf{B}_\psi = \begin{bmatrix} 0 & -e_{\psi z} & e_{\psi y} \\ e_{\psi z} & 0 & -e_{\psi z} \\ -e_{\psi y} & e_{\psi z} & 0 \end{bmatrix} \tag{D-23}$$

Equation (D-23) into (D-7) gives

$$\mathcal{Q}_{\theta I} = \mathbf{f}_i^T\, \mathbf{B}_\theta\, \mathbf{r}_{Ii} \tag{D-24}$$

Application of Eqs. (D-13) through (D-24) gives

$$\mathcal{Q}_{\theta I} = \mathbf{f}_i \cdot (\mathbf{e}_\theta \times \mathbf{r}_{Ii})$$

$$\mathcal{Q}_{\theta I} = -\mathbf{f}_i \cdot (\mathbf{r}_{Ii} \times \mathbf{e}_\theta) = -(\mathbf{r}_{Ii} \times \mathbf{e}_\theta) \cdot \mathbf{f}_i$$

$$\mathcal{Q}_{\theta I} = (\mathbf{r}_{Ii} \times \mathbf{f}_i) \cdot \mathbf{e}_{\theta I}$$

$$\mathcal{Q}_{\theta I} = (\text{torque vector}) \cdot \mathbf{e}_\theta$$

$$\mathcal{Q}_{\theta I} = \text{component of torque along } \mathbf{e}_\theta \tag{D-25}$$

Similarly

$$\mathcal{Q}_{\phi I} = \text{component of torque along } \mathbf{e}_\phi \tag{D-26}$$

$$\mathcal{Q}_{\psi I} = \text{component of torque along } \mathbf{e}_\psi \tag{D-27}$$

It is valid to sum over i in Eqs. (D-25) and (D-6) to obtain the generalized forces for many $\mathbf{f}_k$. This analysis also applies to body II.

APPENDIX E

Velocity of Separation Between a Point on Body I and a Point on Body II

The velocity of separation between initially coincident points on body I and body II is required in order to compute the directions of coulomb drag forces. By definition

$$\mathbf{v}_i = \frac{d(\mathbf{d}_i)}{dt} \tag{E-1}$$

$$\mathbf{v}_i = \dot{\mathbf{r}} + \dot{\mathbf{A}}_I^T \mathbf{r}'_{Ii} - \dot{\mathbf{A}}_{II}^T \mathbf{r}'_{IIi} \tag{E-2}$$

The time derivatives of the elements of the $\mathbf{A}$ matrixes are

$$c_{11} = (-c\theta\, s\phi\, s\psi + c\phi\, c\psi) \tag{E-3}$$

$$\dot{c}_{11} = (s\theta\, s\phi\, s\psi)\,\dot{\theta} - \dot{c}_{12}\,\dot{\phi} + c_{21}\,\dot{\psi} \tag{E-4}$$

$$c_{12} = (c\theta\, c\phi\, s\psi + s\phi\, c\psi) \tag{E-5}$$

$$\dot{c}_{12} = -(s\theta\, c\phi\, s\psi)\,\dot{\theta} + \dot{c}_{11}\,\dot{\phi} + c_{22}\,\dot{\psi} \tag{E-6}$$

$$c_{13} = (s\theta\, c\psi) \tag{E-7}$$

$$\dot{c}_{13} = (c\theta\, s\psi)\,\dot{\theta} + c_{23}\,\dot{\psi} \tag{E-8}$$

$$c_{21} = (-c\theta\, s\phi\, c\psi - c\phi\, s\psi) \tag{E-9}$$

$$\dot{c}_{21} = (s\theta\, s\phi\, c\psi)\,\dot{\theta} - c_{22}\,\dot{\phi} - c_{11}\,\dot{\psi} \tag{E-10}$$

$$c_{22} = (c\theta\, c\phi\, c\psi - s\phi\, s\psi) \tag{E-11}$$

$$\dot{c}_{22} = -(s\theta\, c\phi\, c\psi)\,\dot{\theta} + c_{21}\,\dot{\phi} - c_{12}\,\dot{\psi} \tag{E-12}$$

$$c_{23} = (s\theta\, c\psi) \tag{E-13}$$

$$\dot{c}_{23} = (c\theta\, c\psi)\,\dot{\theta} - c_{13}\,\dot{\psi} \tag{E-14}$$

$$c_{31} = (s\theta\, s\phi) \tag{E-15}$$

$$\dot{c}_{31} = (c\theta\, s\phi)\,\dot{\theta} - c_{32}\,\dot{\phi} \tag{E-16}$$

$$c_{32} = -\,(s\theta\, c\phi) \tag{E-17}$$

$$\dot{c}_{32} = -(c\theta\, c\phi)\,\dot{\theta} + c_{31}\,\dot{\phi} \tag{E-18}$$

$$c_{33} = c\theta \tag{E-19}$$

$$\dot{c}_{33} = -(s\theta)\,\dot{\theta} \tag{E-20}$$

APPENDIX F

Derivation of Force vs Time Relation for an Adiabatic Compressed Gas Jet

The classical ideal gas relations are utilized to determine the force-time relation for the cold gas-jet model (Fig. F-1).

A. Symbols

A	area
a	speed of sound
C_1, C_2, C_3	constants depending on working fluid properties and/or nozzle geometry
F	thrust
M	mach number
m	mass of gas in tank
$\dot{m}$	mass flow rate
p	static pressure
p'	external static pressure
R	gas constant
T	absolute temperature
u	velocity
V	volume upstream of throat
α	nozzle divergence half-angle
γ	ratio of specific heats for working fluid
λ	nozzle divergence correction factor, $\frac{1}{2}(1 + \cos\alpha)$
ρ	density
τ	time
0 (subscript)	initial condition

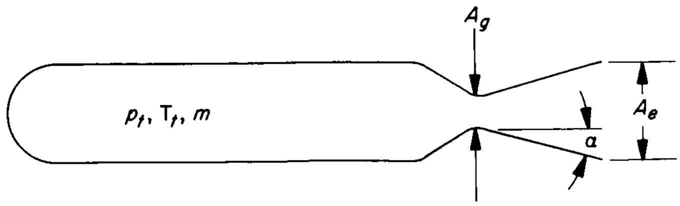

Fig. F-1. Compressed gas jet

B. Derivation

$$F = -\lambda u_e \dot{m} + (p_e - p') A_e \tag{F-1}$$

Assuming a perfect gas and an adiabatic, isentropic process

$$\dot{m} = \rho_g u_g A_g \tag{F-2}$$

$$a = (\gamma R T)^{1/2} \tag{F-3}$$

$$\rho = \rho_t \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1/(\gamma-1)} \tag{F-4}$$

$$\frac{u}{a_t} = M \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1/2} \tag{F-5}$$

$$p = \rho R T \tag{F-6}$$

$$\rho_g = \rho_t \left(\frac{2}{\gamma + 1}\right)^{1/\gamma - 1} = \frac{p_t}{R T_t} \left(\frac{2}{\gamma + 1}\right)^{-1/(\gamma-1)} \tag{F-7}$$

$$u_g = a_t \left(\frac{2}{\gamma + 1}\right)^{1/2} = \left(\frac{2}{\gamma + 1}\right)^{1/2} (\gamma R T_t)^{1/2} \tag{F-8}$$

$$\dot{m} = -\left(\frac{2}{\gamma + 1}\right)^{(\gamma+1)/2(\gamma-1)} \left(\frac{\gamma}{R T_t}\right)^{1/2} p_t A_g \equiv C_1 \frac{p_t}{(T_t)^{1/2}} \tag{F-9}$$

$$u_e = M_e \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{-1/2} (\gamma R T_t)^{1/2} \equiv C_2 (T_t)^{1/2} \tag{F-10}$$

M_e is implicitly determined by

$$\frac{A_g}{A_e} = \left(\frac{\gamma + 1}{2}\right)^{(\gamma+1)/[2(\gamma-1)]} M_e \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{-[(\gamma+1)/2(\gamma-1)]} \tag{F-11}$$

$$p_e = C_3 p_t$$

where

$$C_3 \equiv \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{-[\gamma/(\gamma-1)]} \tag{F-12}$$

$$F = -\lambda C_1 C_2 p_t + A_e C_3 p_t - A_e p' = (\lambda C_1 C_2 + A_e C_3) p_t - A_e p' \tag{F-13}$$

$$m = \rho_t V = \frac{V p_t}{R T_t} \tag{F-14}$$

Now

$$\mathrm{T}_t = \left(\frac{p_t}{p_{t0}}\right)^{(\gamma-1)/\gamma} \mathrm{T}_{t0} \tag{F-15}$$

so

$$m = \frac{Vp_t}{R\mathrm{T}_{t0}\left(\dfrac{p_t}{p_{t0}}\right)^{(\gamma-1)/\gamma}} = \frac{V\,(p_{t0})^{(\gamma-1)/\gamma}}{R\mathrm{T}_{t0}}\,p_t^{1/\gamma} \tag{F-16}$$

$$\dot{m} = C_1\,p_t\left(\frac{p_t}{p_{t0}}\right)^{-[(\gamma-1)/2\gamma]}\mathrm{T}_{t0}^{-½} = C_1\left(\frac{p_{t0}{}^{(\gamma-1)/\gamma}}{\mathrm{T}_{t0}}\right)^{½} p_t{}^{(\gamma+1)/2\gamma} \tag{F-17}$$

Using Eqs. (F-15) and (F-16) to eliminate p_t from Eq. (F-17) gives

$$\dot{m} = C_1\frac{\left(\dfrac{p_{t0}{}^{(\gamma-1)/\gamma}}{\mathrm{T}_{t0}}\right)^{½} p_t{}^{(\gamma+1)/2\gamma}}{\left[\dfrac{V}{R}\left(\dfrac{p_{t0}{}^{(\gamma-1)/\gamma}}{\mathrm{T}_{t0}}\right)p_t{}^{1/\gamma}\right]^{(\gamma+1)/2}}\,m^{(\gamma+1)/2} \tag{F-18}$$

$$\dot{m} = C_1\left(\frac{R}{V}\right)^{(\gamma+1)/2}\left(\frac{p_{t0}{}^{(\gamma-1)/\gamma}}{\mathrm{T}_{t0}}\right)^{-(\gamma/2)} m^{(\gamma+1)/2} \tag{F-19}$$

$$m = \left[m_0{}^{-[(\gamma-1)/2]} - C_1\left(\frac{\gamma-1}{2}\right)\left(\frac{R}{V}\right)^{(\gamma+1)/2}\left(\frac{\mathrm{T}_{t0}}{p_{t0}{}^{\gamma-1/\gamma}}\right)^{-(\gamma/2)}\tau\right]^{-[2/(\gamma-1)]} \tag{F-20}$$

$$m = \left[\left(\frac{R\mathrm{T}_{t0}}{Vp_{t0}}\right)^{(\gamma-1)/2} - C_1\left(\frac{\gamma-1}{2}\right)\left(\frac{R}{V}\right)^{(\gamma+1)/2}\left(\frac{T_{t0}}{p_{t0}{}^{\gamma-1/\gamma}}\right)^{\gamma/2}\tau\right]^{-[2/(\gamma-1)]} \tag{F-21}$$

$$p_t = \left(\frac{RT_{t0}}{Vp_{t0}}\right)^{\gamma} m^{\gamma} \tag{F-22}$$

Therefore, after combining terms

$$p_t = p_{t0}\left[1 + \left(\frac{\gamma-1}{2}\right)\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)}(\gamma R\mathrm{T}_{t0})^{½}\,\frac{A_g}{V}\,\tau\right]^{-[2\gamma/(\gamma-1)]} \tag{F-23}$$

Since $p' = 0$, the thrust is

$$F = p_{t0}\left[\frac{1}{2}(1+\cos\alpha)\,\gamma\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)}\left(1+\frac{\gamma-1}{2}M_e^2\right)^{-½} M_e A_g + \left(1+\frac{\gamma-1}{2}M_e^2\right)^{-[\gamma/(\gamma-1)]} A_e\right]$$
$$\left[1 + \left(\frac{\gamma-1}{2}\right)\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)}(\gamma R\mathrm{T}_{t0})^{½}\left(\frac{A_g}{V}\right)\tau\right]^{-[2\gamma/(\gamma-1)]} \tag{F-24}$$

APPENDIX G

Derivation of Force vs Distance Relation for an Adiabatic Pneumatic Ejection System

The ideal gas laws are used to derive the force-distance relation for a pneumatic separation system. The existence of a regulator that either stays open or stays closed after the regulation pressure has been reached is considered.

A. Symbols

A	area of piston face
b (subscript)	gas bottle
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
E	internal energy
j	displacement of piston from closed position
j (subscript)	as a function of j
j_{eqp}	The j at which $p_b = p_p$, and the regulator ceases to work
N_i	force due to the i^{th} pneumatic piston
p	pressure
p (subscript)	piston chamber
R	universal gas constant
T	absolute temperature
V_b	volume of gas bottle, including all lines upstream of the face of the closed piston
W	work
γ	ratio of specific heats, c_p/c_v
ρ	density
0	initial condition
′ (prime)	equivalent initial conditions for regulatorless motion when $j \geqq j_{eqp}$

As may be seen from Figs. G-1 and G-2, the force on the piston is

$$N_i = p_{pi} A_i \tag{G-1}$$

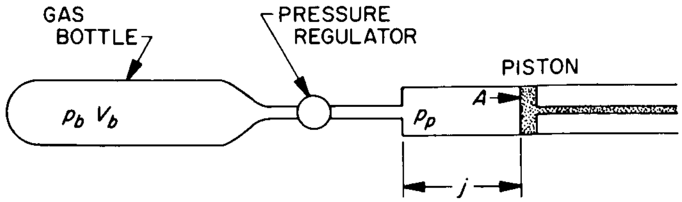

Fig. G-1. Pneumatic separation device with pressure regulator

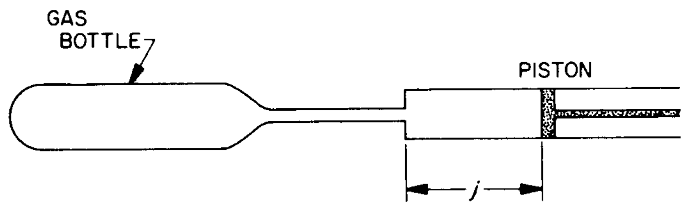

Fig. G-2. Pneumatic separation device without pressure regulator

The two cases to be considered are: (1) with a pressure regulator in the circuit, and (2) without a pressure regulator in the circuit. With a regulator, Eq. (G-2) is valid until $p_b = p_p$; without a regulator, Eq. (G-3) is valid.

$$p_p = \text{constant} \tag{G-2}$$

$$p_p = p_b = p \tag{G-3}$$

B. With Pressure Regulator

Using the law of conservation of energy on the gas contained within the system

$$E_0 = \rho_{b0} V_b c_v \mathrm{T}_{b0} \tag{G-4}$$

$$E_j = \rho_b V_b c_v \mathrm{T}_b + \rho_p c_v \mathrm{T}_p A j \tag{G-5}$$

$$W_j = p_p A j \tag{G-6}$$

Assuming adiabatic conditions

$$E_0 = E_j + W_j \tag{G-7}$$

$$\rho_{b0} V_b c_v \mathrm{T}_{b0} = \rho_b V_b c_v \mathrm{T}_b + \rho_p c_v \mathrm{T}_p A j + p_p A j \tag{G-8}$$

Assuming that the working fluid is a perfect gas, Eqs. (G-9), (G-10), and (G-11) are valid. Equations (G-9) through (G-11) will then yield Eq. (G-12) which, in turn, can be reduced to Eq. (G-13).

$$p = \rho RT \tag{G-9}$$

$$R = c_p - c_v \tag{G-10}$$

$$\frac{c_v}{R} = \frac{c_v}{c_p - c_v} = \frac{1}{\frac{c_p}{c_v} - 1} = \frac{1}{\gamma - 1} \tag{G-11}$$

$$\frac{c_v}{R} V_b p_{bo} = \frac{c_v}{R} V_b p_b + \frac{c_v}{R} p_p A j + p_p A j \tag{G-12}$$

$$V_b p_{bo} = V_b p_b + \gamma p_p A_j \tag{G-13}$$

Therefore

$$p_b = p_{bo} - \gamma p_b \frac{Aj}{V_b} \tag{G-14}$$

The force, N, is p_p A as long as $p_b \geqq p_p$, when $p_b = p_p$, which occurs at

$$j = j_{eqp} = \frac{V_b}{\gamma A}\left(\frac{p_{bo}}{p_p} - 1\right) \tag{G-15}$$

then this is no longer true.

When $j > j_{eqp}$, the system acts as if there were no regulator. The new initial conditions are

$$j' = j - j_{eqp} \tag{G-16}$$

$$p'_{bo} = p_p \tag{G-17}$$

and, if the pressure regulator stays closed

$$V'_b = A j_{eqp} \tag{G-18}$$

If the regulator stays open

$$V'_b = V_b + A j_{eqp} \tag{G-19}$$

C. *Without Pressure Regulator*

Using the law of conservation of energy gives

$$E_0 = \rho_0 V_b c_v T_0 \tag{G-20}$$

$$E_j = \rho c_v T (V_b + Aj) \tag{G-21}$$

$$W_j = \int_0^j pA\, dj \tag{G-22}$$

Assuming adiabatic conditions

$$E_0 = E_j + W_j \tag{G-23}$$

$$\rho_0 V_b c_v T_0 = \rho c_v T (V_b + Aj) + \int_0^j pA\, dj \tag{G-24}$$

Assuming a perfect gas

$$\frac{c_v}{R} V_b p_{bo} = \frac{c_v}{R}(V_b + Aj)\, p + \int_0^j pA\, dj \tag{G-25}$$

$$p_{bo} = \left(1 + \frac{Aj}{V_b}\right) p + (\gamma - 1)\frac{A}{V_b}\int_0^j p\, dj \tag{G-26}$$

Taking the derivative of Eq. (G-27) with respect to j results in

$$0 = \left(1 + \frac{Aj}{V_b}\right)\frac{dp}{dj} + \frac{A}{V_b} p + (\gamma - 1)\frac{A}{V_b} p \tag{G-27}$$

which reduces to

$$\frac{V_b}{\gamma A}\frac{dp}{p} = -\frac{dj}{1 + \frac{A}{V_b} j} \tag{G-28}$$

Integrating Eq. (G-28) gives

$$p = C\left(1 + \frac{Aj}{V_b}\right)^{-\gamma} \tag{G-29}$$

C is determined by setting $p = p_{bo}$ when $j = 0$.

$$p_{bo} = C, \text{ so} \tag{G-30}$$

$$p = p_{bo}\left(1 + \frac{Aj}{V_b}\right)^{-\gamma} \tag{G-31}$$

APPENDIX H

Derivation of Spring Efficiency Factor Relation

Experience has shown that separation velocities calculated by equating initial spring potential energy to eventual body kinetic energy do not agree with experimental results. Typical differences are large enough to warrant inclusion of this effect in the analysis.

A. Symbols

d_0 initial spring compression distance $= S_0/k$

d_f final spring compression distance $= S_f/k$

k true spring rate

k' artificial spring rate that gives correct velocity

S_0 initial spring force

S_f final spring force

S_0' modified initial spring force

S_f' modified final spring force

v_a actual separation velocity

v_p separation velocity predicted using k

η spring efficiency factor

B. Definition and Determination of η

Experimental determination of η does not lend itself to calculation of a different value for each spring; therefore, one value of η is used for all the springs. η is defined as

$$\eta = \frac{\text{actual separation velocity of bodies}}{\text{velocity predicted assuming perfectly efficient springs}} \tag{H-1}$$

In the analysis, η is included by calculating an artificial spring rate that yields the correct separation velocity.

It is assumed that the actual separation velocity is determined from a test in which all angular rates are zero (i.e., only rectilinear motion is involved), so that the separation process can be modeled as a simple 2-deg of freedom system. Since only relative motions between the two bodies need be considered, this 2-deg of freedom system degenerates to a 1-deg of freedom spring-mass system with effective mass m and composite spring constant k.

In the determination of k', it will be required that the initial spring compression distance, d_0, and the final spring compression distance, d_f, be preserved, rather than the initial and final forces.

C. Effect of η on Spring Rate

The differential equation representing the 1-deg of freedom model is

$$m\ddot{x} = k(d_0 - x) \tag{H-2}$$

where

$$x = (\text{separation distance between CM's}) - (\text{initial separation distance})$$

The initial values are

$$x(0) = \dot{x}(0) = 0$$

The solution to Eq. (H-2) is

$$\left.\begin{aligned} x(t) &= d_0\left[1 - \cos\left(\frac{k}{m}\right)^{1/2} t\right], \\ \dot{x}(t) &= d_0\left(\frac{k}{m}\right)^{1/2}\sin\left(\frac{k}{m}\right)^{1/2} t \end{aligned}\right\} \quad 0 \leq x \leq (d_i - d_f) \tag{H-3}$$

Let t_f be the time of complete separation ($x = d_0 - d_f$) for the perfectly efficient spring case, and let t_f' be the corresponding value for the actual case. Then, from Eqs. (H-1) and (H-3)

$$\eta = \frac{v_a}{v_p} = \left(\frac{k'}{k}\right)^{1/2} \frac{\sin\left(\frac{k'}{m}\right)^{1/2} t_f'}{\sin\left(\frac{k}{m}\right)^{1/2} t_f} \tag{H-4}$$

Now when $x = d_0 - d_f$, from Eq. (H-3)

$$d_0 - d_f = d_0\left[1 - \cos\left(\frac{k}{m}\right)^{1/2} t_f\right]$$

and

$$d_0 - d_f = d_0\left[1 - \cos\left(\frac{k'}{m}\right)^{1/2} t'_f\right]$$

Thus

$$\cos\left(\frac{k}{m}\right)^{1/2} t_f = \cos\left(\frac{k'}{m}\right)^{1/2} t'_f$$

which implies that

$$\sin\left(\frac{k}{m}\right)^{1/2} t_f = \pm\sin\left(\frac{k'}{m}\right)^{1/2} t'_f \tag{H-5}$$

since

$$v_a > 0 \qquad \text{and} \qquad v_p > 0$$

$$\sin\left(\frac{k}{m}\right)^{1/2} t_f = +\sin\left(\frac{k'}{m}\right)^{1/2} t'_f \tag{H-6}$$

and Eq. (H-4) becomes

$$\eta = \left(\frac{k'}{k}\right)^{1/2}$$

Then

$$k' = \eta^2 k \tag{H-7}$$

To preserve d_0 and d_f with this new spring constant, new values of S_0 and S_f must be used. These values are

$$S'_0 = \eta^2 S_0 \qquad S'_f = \eta^2 S_f \tag{H-8}$$

APPENDIX I

Calculation of Kinetic Energy, $\partial T/\partial q_k$, and $d/dt\,(\partial T/\partial \dot{q}_k)$ as Functions of the Generalized Coordinates

The quantities T, $\partial T/\partial q_k$, and $d/dt\,(\partial T/\partial \dot{q}_k)$ are required as explicit functions of the generalized coordinates in the equations of motion.

A. Symbols

$\mathbf{I}$ inertia matrix

I components of the inertia matrix

$\dot{\mathbf{I}}$ derivative of inertia matrix $[\dot{\mathrm{I}}_{jk} = \sum_i (\dot{\mathrm{I}}_{jk})_i]$

$\dot{m}$ $\sum_i \dot{m}_i$

T kinetic energy of the system

$\mathbf{v}$ velocity vector of CM, $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$, in inertial coordinates

$\boldsymbol{\omega}$ angular velocity about body-fixed axes, $\begin{bmatrix} \omega_{x'} \\ \omega_{y'} \\ \omega_{z'} \end{bmatrix}$

B. Calculation of $T(q_1, \cdots, q_{12})$

$$2T = \mathrm{m_I}\,\mathbf{v}_\mathrm{I}^\mathrm{T}\,\mathbf{v}_\mathrm{I} + \mathrm{m_{II}}\,\mathbf{v}_\mathrm{II}^\mathrm{T}\,\mathbf{v}_\mathrm{II} + \boldsymbol{\omega}_\mathrm{I}^\mathrm{T}\,\mathbf{I}_\mathrm{I}\,\boldsymbol{\omega}_\mathrm{I} + \boldsymbol{\omega}_\mathrm{II}^\mathrm{T}\,\mathbf{I}_\mathrm{II}\,\boldsymbol{\omega}_\mathrm{II} \tag{I-1}$$

Using the results of Appendix D

$$\boldsymbol{\omega} = \begin{bmatrix} \dot{\phi}\,(s\theta\, s\psi) + \dot{\theta}\,(c\psi) \\ \dot{\phi}\,(s\theta\, c\psi) - \dot{\theta}\,(s\psi) \\ \dot{\phi}\,(c\theta) + \dot{\psi} \end{bmatrix}$$

Thus we have

$$T\,(q_1, \cdots, q_{12}) \tag{I-2}$$

C. Calculation of $\partial T/\partial q_s$

Clearly

$$\frac{\partial T}{\partial q_s} = 0 \qquad \text{for } q_s = x_{\mathrm{I}}, x_{\mathrm{II}}, y_{\mathrm{I}}, y_{\mathrm{II}}, z_{\mathrm{I}}, z_{\mathrm{II}}, \phi_{\mathrm{I}}, \phi_{\mathrm{II}} \tag{I-3}$$

The derivatives with respect to the other coordinates are

$$\frac{\partial T}{\partial q_s} = \boldsymbol{\omega}_{\mathrm{I}}^{\mathrm{T}}\, \mathbf{I}_{\mathrm{I}} \frac{\partial \boldsymbol{\omega}_{\mathrm{I}}}{\partial q_s} + \boldsymbol{\omega}_{\mathrm{II}}^{\mathrm{T}}\, \mathbf{I}_{\mathrm{II}} \frac{\partial \boldsymbol{\omega}_{\mathrm{I}}}{\partial q_s} \tag{I-4}$$

where use has been made of the fact that the inertia matrixes are symmetric. Since the first term of Eq. (I-4) is only a function of θ_{I} and ψ_{I}, and the second term is only a function of θ_{II} and ψ_{II}

$$\frac{\partial T}{\partial \theta} = \boldsymbol{\omega}^{\mathrm{T}}\, \mathbf{I} \frac{\partial \boldsymbol{\omega}}{\partial \theta}, \frac{\partial T}{\partial \psi} = \boldsymbol{\omega}^{\mathrm{T}}\, \mathbf{I} \frac{\partial \boldsymbol{\omega}}{\partial \psi} \tag{I-5}$$

where

$$\frac{\partial \boldsymbol{\omega}}{\partial \theta} = \begin{bmatrix} \dot{\phi}\,(c\theta\, s\psi) \\ \dot{\phi}\,(c\theta\, c\psi) \\ -\dot{\phi}\,(s\theta) \end{bmatrix} \tag{I-6}$$

$$\frac{\partial \boldsymbol{\omega}}{\partial \psi} = \begin{bmatrix} \dot{\phi}\,(s\theta\, c\psi) + \dot{\theta}\,(s\psi) \\ -\dot{\phi}\,(s\theta\, s\psi) - \dot{\theta}\,(c\psi) \\ 0 \end{bmatrix} \tag{I-7}$$

The explicit forms are

$$\begin{aligned} \frac{\partial T}{\partial \theta} = {} & [\mathrm{I}_{xx}\,(s^2\,\psi) + 2\mathrm{I}_{xy}\,(s\psi\, c\psi) + \mathrm{I}_{yy}\,(c^2\,\psi) - I_{zz}]\, \dot{\phi}^2\,(s\theta\, c\theta) \\ & + (\mathrm{I}_{xx} - \mathrm{I}_{yy})\, \dot{\theta}\dot{\phi}\,(c\theta\, s\psi\, c\psi) + \mathrm{I}_{xy}\, \dot{\theta}\dot{\phi}\,(c\theta)\,(c^2\psi - s^2\,\psi) \\ & + \mathrm{I}_{xz}\, \dot{\phi}^2\,(s\psi)\,(c^2\,\theta - s^2\,\theta) + [\mathrm{I}_{xz}\,(s\psi) + \mathrm{I}_{yz}\,(c\psi)]\, \dot{\phi}\dot{\psi}\,(c\theta) \\ & + [\mathrm{I}_{yz}\,(s\psi) - \mathrm{I}_{xz}\,(c\psi)]\, \dot{\theta}\dot{\phi}\,(s\theta) + \mathrm{I}_{yz}\, \dot{\phi}^2\,(c\psi)\,(c^2\,\theta - s^2\theta) - \mathrm{I}_{zz}\, \dot{\phi}\dot{\psi}\,(s\theta) \end{aligned} \tag{I-8}$$

$$\begin{aligned} \frac{\partial T}{\partial \psi} = {} & (\mathrm{I}_{xx} - \mathrm{I}_{yy})\,(s\psi\, c\psi)\,[\dot{\phi}^2\,(s^2\,\theta) - \dot{\theta}^2] + [\mathrm{I}_{xx} - \mathrm{I}_{yy}]\, \dot{\theta}\dot{\phi}\,(s\theta)\,(c^2\,\psi - s^2\psi) \\ & + \mathrm{I}_{xy}\, \dot{\phi}^2\,(s^2\,\theta)\,(c^2\,\psi - s^2\,\psi) - 4\mathrm{I}_{xy}\, \dot{\theta}\dot{\phi}\,(s\theta\, s\psi\, c\psi) \\ & + [\mathrm{I}_{xz}\,(c\psi) - \mathrm{I}_{yz}\,(s\psi)]\,[\dot{\phi}\,(c\theta) + \dot{\psi}]\, \dot{\phi}\,(s\theta) - \mathrm{I}_{xy}\, \dot{\theta}^2\,(c^2\,\psi - s^2\,\psi) \\ & - \dot{\theta}\,[\dot{\phi}\,(c\theta) + \dot{\psi}]\,[\mathrm{I}_{xz}\,(s\psi) + \mathrm{I}_{yz}\,(c\psi)] \end{aligned} \tag{I-9}$$

D. Calculation of $\partial T/\partial \dot{q}_s$

From Eq. (I-1) and the definition of v

$$\frac{\partial T}{\partial \dot{x}} = m\dot{x} \qquad \text{(I-10)}$$

$$\frac{\partial T}{\partial \dot{y}} = m\dot{y} \qquad \text{(I-11)}$$

$$\frac{\partial T}{\partial \dot{z}} = m\dot{z} \qquad \text{(I-12)}$$

For the other coordinates

$$\frac{\partial T}{\partial \dot{q}_s} = \boldsymbol{\omega}_{\rm I}^{\rm T}\, \mathbf{I}_{\rm I}\, \frac{\partial \boldsymbol{\omega}_{\rm I}}{\partial \dot{q}_s} + \boldsymbol{\omega}_{\rm II}^{\rm T}\, \mathbf{I}_{\rm II}\, \frac{\partial \boldsymbol{\omega}_{\rm II}}{\partial \dot{q}_s} \qquad \text{(I-13)}$$

Because of the independence of the first and second terms

$$\frac{\partial T}{\partial \dot{\theta}} = \boldsymbol{\omega}^{\rm T}\, \mathbf{I} \frac{\partial \boldsymbol{\omega}}{\partial \dot{\theta}}, \quad \frac{\partial T}{\partial \dot{\phi}} = \boldsymbol{\omega}^{\rm T}\, \mathbf{I} \frac{\partial \boldsymbol{\omega}}{\partial \dot{\phi}}, \quad \frac{\partial T}{\partial \dot{\psi}} = \boldsymbol{\omega}^{\rm T}\, \mathbf{I} \frac{\partial \boldsymbol{\omega}}{\partial \dot{\psi}} \qquad \text{(I-14)}$$

where

$$\frac{\partial \boldsymbol{\omega}}{\partial \dot{\theta}} = \begin{bmatrix} (\mathrm{c}\psi) \\ -(\mathrm{s}\psi) \\ 0 \end{bmatrix}, \quad \frac{\partial \boldsymbol{\omega}}{\partial \dot{\phi}} = \begin{bmatrix} (\mathrm{s}\theta\, \mathrm{s}\psi) \\ (\mathrm{s}\theta\, \mathrm{c}\psi) \\ (\mathrm{c}\theta) \end{bmatrix}, \quad \frac{\partial \boldsymbol{\omega}}{\partial \dot{\psi}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad \text{(I-15)}$$

E. Calculation of $d/dt\,(\partial T/\partial \dot{q}_s)$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) = m\ddot{x} \qquad \text{(I-16)}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{y}}\right) = m\ddot{y} \qquad \text{(I-17)}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{z}}\right) = m\ddot{z} \qquad \text{(I-18)}$$

For the other coordinates

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_s}\right) = \dot{\boldsymbol{\omega}}_{\rm I}^{\rm T}\, \mathbf{I}_{\rm I} \frac{\partial \boldsymbol{\omega}_{\rm I}}{\partial \dot{q}_s} + \boldsymbol{\omega}_{\rm I}^{\rm T}\, \mathbf{I}_{\rm I} \frac{d}{dt}\left[\frac{\partial \boldsymbol{\omega}_{\rm I}}{\partial \dot{q}_s}\right] + \boldsymbol{\omega}_{\rm I}^{\rm T}\, \dot{\mathbf{I}}_{\rm I} \frac{\partial \boldsymbol{\omega}_{\rm I}}{\partial \dot{q}_s} + \dot{\boldsymbol{\omega}}_{\rm II}^{\rm T}\, \mathbf{I}_{\rm II} \frac{\partial \boldsymbol{\omega}_{\rm II}}{\partial \dot{q}_s} \qquad \text{(I-19)}$$

Because of the independence of the first terms in Eq. (I-19) from the last three

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_s}\right) = \dot{\boldsymbol{\omega}}^{\rm T}\, \mathbf{I} \frac{\partial \boldsymbol{\omega}}{\partial \dot{q}_s} + \boldsymbol{\omega}^{\rm T}\, \mathbf{I} \frac{d}{dt}\left[\frac{\partial \boldsymbol{\omega}}{\partial \dot{q}_s}\right] \qquad \text{(I-20)}$$

Equation (I-20) applies subscripted with a I when $q_s = \theta_{\rm I}, \phi_{\rm I}, \psi_{\rm I}$, and subscripted with a II when $q_s = \theta_{\rm II}, \phi_{\rm II}, \psi_{\rm II}$.

$$\dot{\boldsymbol{\omega}}_{\rm I} = \begin{bmatrix} \ddot{\phi}\,(\mathrm{s}\theta\, \mathrm{s}\psi) + \dot{\phi}\dot{\theta}\,(\mathrm{c}\theta\, \mathrm{s}\psi) + \dot{\phi}\dot{\psi}\,(\mathrm{s}\theta\, \mathrm{c}\psi) + \ddot{\theta}\,(\mathrm{c}\psi) - \dot{\theta}\dot{\psi}\,(\mathrm{s}\psi) \\ \ddot{\phi}\,(\mathrm{s}\theta\, \mathrm{c}\psi) + \dot{\phi}\dot{\theta}\,(\mathrm{c}\theta\, \mathrm{c}\psi) - \dot{\phi}\dot{\psi}\,(\mathrm{c}\theta\, \mathrm{s}\psi) - \ddot{\theta}\,(\mathrm{s}\psi) - \dot{\theta}\dot{\psi}\,(\mathrm{c}\psi) \\ \ddot{\phi}\,(\mathrm{c}\theta) - \dot{\phi}\dot{\theta}\,(\mathrm{s}\theta) + \ddot{\psi} \end{bmatrix}$$

$$\frac{d}{dt}\left[\frac{\partial \boldsymbol{\omega}}{\partial \dot{\theta}}\right] = \begin{bmatrix} (-\mathrm{s}\psi)\,\dot{\psi} \\ (-\mathrm{c}\psi)\,\dot{\psi} \\ 0 \end{bmatrix} \qquad \text{(I-21)}$$

$$\frac{d}{dt}\left[\frac{\partial \omega}{\partial \dot{\phi}}\right] = \begin{bmatrix} (c\theta\, s\psi)\,\dot{\theta} + (s\theta\, c\psi)\,\dot{\psi} \\ (c\theta\, c\psi)\,\dot{\theta} - (s\theta\, s\psi)\,\dot{\psi} \\ (-s\theta)\,\dot{\theta} \end{bmatrix} \tag{I-22}$$

$$\frac{d}{dt}\left[\frac{\partial \omega}{\partial \dot{\psi}}\right] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{I-23}$$

Therefore, the explicit forms of $d/dt\,[\partial T/\partial q_s]$ for $q_s = \theta, \phi, \psi$ are

$$\begin{aligned} \frac{d}{dt}\left(\frac{\partial T}{\partial \theta}\right) = {} & \ddot{\phi}\,\{(s\theta)\,[(I_{xx} - I_{yy})\,(s\psi\, c\psi) + I_{xy}\,(c^2\psi - s^2\psi)] + (c\theta)\,[I_{xz}\,(c\psi) - I_{yz}\,(s\psi)]\} \\ & + \ddot{\psi}\,\{I_{xz}\,(c\psi) - I_{yz}\,(s\psi)\} + \phi\,\{(c\theta)\,\theta\,[(I_{xx} - I_{yy})\,(s\psi\, c\psi) + I_{xy}\,(c^2\psi - s^2\psi)] \\ & + (s\theta)\,\dot{\psi}\,[(I_{xx} - I_{yy})\,(c^2\psi - s^2\psi) - 4I_{xy}\,(s\psi\, c\psi)] - (s\theta)\,\dot{\theta}\,[I_{xz}\,(c\psi) - I_{yz}\,(s\psi)] \\ & - (c\theta)\,\dot{\psi}\,[I_{xz}\,(s\psi) + I_{yz}\,(c\psi)]\} - 2\dot{\theta}\dot{\psi}\,\{[I_{xx} - I_{yy}]\,(s\psi\, c\psi) + I_{xy}\,(c^2\psi - s^2\psi)\} \\ & - \dot{\psi}^2\,\{I_{xz}\,(s\psi) + I_{yz}\,(c\psi)\} \end{aligned} \tag{I-24}$$

$$\begin{aligned} \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\phi}}\right) = {} & \ddot{\theta}\,\{(s\theta)\,[(I_{xx} - I_{yy})\,(s\psi\, c\psi) + I_{xy}\,(c^2\psi - s^2\psi)] + (c\theta)\,[I_{xz}\,(c\psi) - I_{yz}\,(s\psi)]\} \\ & + \ddot{\phi}\,\{(s^2\theta)\,[I_{xx}\,(s^2\psi) + 2I_{xy}\,(s\psi\, c\psi) + I_{yy}\,(c^2\psi)] + 2\,(s\theta\, c\theta)[I_{xz}\,(s\psi) + I_{yz}\,(c\psi)] + (c^2\theta)\,I_{zz}\} \\ & + \ddot{\psi}\,\{(s\theta)\,[I_{xz}\,(s\psi) + I_{yz}\,(c\psi)] + (c\theta)\,I_{zz}\} + 2\dot{\phi}\,\{\dot{\theta}\,(s\theta\, c\theta)\,[I_{xx}\,(s^2\psi) + 2I_{xy}\,(s\psi\, c\psi) + I_{yy}\,(c^2\psi) - I_{zz}] \\ & + \dot{\psi}\,(s^2\theta)\,[(I_{xx} - I_{yy})\,(s\psi\, c\psi) + I_{xy}\,(c^2\psi - s^2\psi)] + \dot{\theta}\,(c^2\theta - s^2\theta)\,[I_{xz}\,(s\psi) + I_{yz}\,(c\psi)] \\ & + \dot{\psi}\,(s\theta\, c\theta)\,[I_{xz}\,(c\psi) - I_{yz}\,(s\psi)]\} + \dot{\theta}\,\{\dot{\theta}\,(c\theta)\,[(I_{xx} - I_{yy})\,(s\psi c\psi) + I_{xy}\,(c^2\psi - s^2\psi)] \\ & + \dot{\psi}\,(s\theta)\,[(I_{xx} - I_{yy})\,(c^2\psi - s^2\psi) - 4I_{xy}\,(s\psi\, c\psi)] - \dot{\theta}\,(s\theta)\,[I_{xz}\,(c\psi) - I_{yz}\,(s\psi)]\} \\ & + \dot{\psi}\,\{\dot{\psi}\,(s\theta)\,[I_{xz}\,(s\psi) - I_{yz}\,(s\psi)] - \dot{\theta}\,(s\theta)\,I_{zz}\} \end{aligned} \tag{I-25}$$

$$\begin{aligned} \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\psi}}\right) = {} & \ddot{\theta}\,\{I_{xz}\,(c\psi) - I_{yz}\,(s\psi)\} + \ddot{\phi}\,\{(s\theta)\,[I_{xz}\,(s\psi) + I_{yz}\,(c\psi)] + (c\theta)\,I_{zz}\} \\ & + \ddot{\psi} I_{zz} + \dot{\phi}\,\{\dot{\theta}\,(c\theta)\,[I_{xz}\,(s\psi) + I_{yz}\,(c\psi)] + \dot{\psi}\,(s\theta)\,[I_{xz}\,(c\psi) - I_{yz}\,(s\psi)] - \dot{\theta}\,(s\theta)\,I_{zz}\} \\ & - \dot{\theta}\dot{\psi}\,\{I_{xz}\,(s\psi) + I_{yz}\,(c\psi)\} \end{aligned} \tag{I-26}$$

APPENDIX J

Constraints

As discussed in Section VII, the constraint conditions are incorporated by means of the Lagrange multiplier technique. It is assumed that the spring constraint and the pin-puller constraint do not act simultaneously.

The equations of motion for the constrained case are

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_s} - \frac{\partial T}{\partial q_s} = \mathcal{Q}'_s + \sum_l \lambda_l\, e_{ls} \qquad s = x_{\mathrm{I}}, y_{\mathrm{I}}, \cdots, \phi_{\mathrm{II}}, \psi_{\mathrm{II}} \tag{J-1}$$

$$\sum_s e_{ls}\ddot{q}_s = r_l \qquad l = 1, 2, \cdots, l_m \tag{J-2}$$

where the constraint equations are

$$\begin{aligned} f_1 &= 0 \\ f_2 &= 0 \\ &\cdots\cdots \\ f_{l_m} &= 0 \end{aligned} \tag{J-3}$$

and where

$$e_{ls} = \frac{\partial f_l}{\partial q_s} \tag{J-4}$$

$$e_{lt} = \frac{\partial f_l}{\partial t} = 0 \tag{J-5}$$

$$r_l = -\sum_s \dot{e}_{ls}\dot{q}_s \tag{J-6}$$

The pin-puller delay constraint is the condition that, until $t = t_i$, a point i on body I remains coincident with a point on body II. Then

$$d_i = 0 \Rightarrow \mathbf{d}_i = 0 \Rightarrow d_{ix} = 0, \qquad d_{iy} = 0, \qquad d_{iz} = 0$$

Let

$$f_1 = d_{ix}, \qquad f_2 = d_{iy}, \qquad f_3 = d_{iz}$$

and let

$$\mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \mathbf{r} = \begin{bmatrix} x_{\mathrm{I}} - x_{\mathrm{II}} \\ y_{\mathrm{I}} - y_{\mathrm{II}} \\ z_{\mathrm{I}} - z_{\mathrm{II}} \end{bmatrix} \mathbf{r}'_{\mathrm{I}i} = \begin{bmatrix} x'_{\mathrm{I}i} \\ y'_{\mathrm{I}i} \\ z'_{\mathrm{I}i} \end{bmatrix} \mathbf{r}'_{\mathrm{II}i} = \begin{bmatrix} x'_{\mathrm{II}i} \\ y'_{\mathrm{II}i} \\ z'_{\mathrm{II}i} \end{bmatrix} \mathbf{e}_s = \begin{bmatrix} e_{1s} \\ e_{2s} \\ e_{3s} \end{bmatrix} \tag{J-7}$$

Then

$$\mathbf{f} = \mathbf{A}_{\mathrm{II}}^{\mathrm{T}}\mathbf{r}'_{\mathrm{II}i} - \mathbf{A}_{\mathrm{I}}^{\mathrm{T}}\mathbf{r}'_{\mathrm{I}i} - \mathbf{r} \tag{J-8}$$

and

$$\mathbf{e}_s = \frac{\partial \mathbf{f}}{\partial q_s} = \frac{\partial \mathbf{A}_{\mathrm{II}}^{\mathrm{T}}}{\partial q_s}\mathbf{r}'_{\mathrm{II}i} = \frac{\partial \mathbf{A}_{\mathrm{I}}^{\mathrm{T}}}{\partial q_s}\mathbf{r}'_{\mathrm{I}i} - \frac{\partial \mathbf{r}}{\partial q_s} \tag{J-9}$$

where

$$\frac{\partial a_{ij}}{\partial \theta_{\mathrm{II}}} = \frac{\partial a_{ij}}{\partial \phi_{\mathrm{II}}} = \frac{\partial a_{ij}}{\partial \psi_{\mathrm{II}}} = \frac{\partial b_{ij}}{\partial \theta_{\mathrm{I}}} = \frac{\partial b_{ij}}{\partial \phi_{\mathrm{I}}} = \frac{\partial b_{ij}}{\partial \psi_{\mathrm{I}}} = \frac{\partial c_{ij}}{\partial x_{\mathrm{I}}} = \frac{\partial c_{ij}}{\partial y_{\mathrm{I}}} = \frac{\partial c_{ij}}{\partial z_{\mathrm{I}}} = \frac{\partial c_{ij}}{\partial x_{\mathrm{II}}} = \frac{\partial c_{ij}}{\partial y_{\mathrm{II}}} = \frac{\partial c_{ij}}{\partial z_{\mathrm{II}}} = 0 \tag{J-10}$$

for $i, j = 1, 2, 3$.

$$\frac{\partial \mathbf{A}}{\partial \theta} = \begin{bmatrix} (s\theta\, s\phi\, s\psi) & (-s\theta\, c\phi\, s\psi) & (c\theta\, s\psi) \\ (s\theta\, s\phi\, c\psi) & (-s\theta\, c\phi\, c\psi) & (c\theta\, c\psi) \\ (c\theta\, s\phi) & (-c\theta\, c\phi) & (-s\theta) \end{bmatrix} \tag{J-11}$$

$$\frac{\partial \mathbf{A}}{\partial \phi} = \begin{bmatrix} -c_{12} & c_{11} & 0 \\ -c_{22} & c_{21} & 0 \\ -c_{32} & c_{31} & 0 \end{bmatrix}$$

$$\frac{\partial \mathbf{A}}{\partial \psi} = \begin{bmatrix} c_{21} & c_{22} & c_{23} \\ -c_{11} & -c_{12} & -c_{13} \\ 0 & 0 & 0 \end{bmatrix} \tag{J-12}$$

and

$$\frac{\partial \mathbf{r}}{\partial \theta_{\mathrm{I}}} = \frac{\partial \mathbf{r}}{\partial \theta_{\mathrm{II}}} = \frac{\partial \mathbf{r}}{\partial \phi_{\mathrm{I}}} = \frac{\partial \mathbf{r}}{\partial \phi_{\mathrm{II}}} = \frac{\partial \mathbf{r}}{\partial \psi_{\mathrm{I}}} = \frac{\partial \mathbf{r}}{\partial \psi_{\mathrm{II}}} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{J-13}$$

$$\frac{\partial \mathbf{r}}{\partial x_{\mathrm{I}}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \frac{\partial \mathbf{r}}{\partial y_{\mathrm{I}}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \frac{\partial \mathbf{r}}{\partial z_{\mathrm{I}}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \frac{\partial \mathbf{r}}{\partial x_{\mathrm{II}}} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \quad \frac{\partial \mathbf{r}}{\partial y_{\mathrm{II}}} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \quad \frac{\partial \mathbf{r}}{\partial z_{\mathrm{II}}} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \tag{J-14}$$

Now

$$\dot{\mathbf{e}}_s = \frac{d}{dt}\left[\frac{\partial \mathbf{A}_{\mathrm{II}}^{\mathrm{T}}}{\partial q_s}\right]\mathbf{r}'_{\mathrm{II}i} - \frac{d}{dt}\left[\frac{\partial \mathbf{A}_{\mathrm{I}}^{\mathrm{T}}}{\partial q_s}\right]\mathbf{r}'_{\mathrm{I}i} - \frac{d}{dt}\left[\frac{\partial \mathbf{r}}{\partial q_s}\right] \tag{J-15}$$

where

$$\frac{d}{dt}\left[\frac{\partial \mathbf{A}}{\partial \theta}\right] = \begin{bmatrix} (\mathrm{c}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi)\,\dot{\theta} + (\mathrm{s}\theta\,\mathrm{c}\phi\,\mathrm{s}\psi)\,\dot{\phi} + (\mathrm{s}\theta\,\mathrm{s}\phi\,\mathrm{c}\psi)\,\dot{\psi}, & (-\mathrm{c}\theta\,\mathrm{c}\phi\,\mathrm{s}\psi)\,\dot{\theta} + (\mathrm{s}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi)\,\dot{\phi} - (\mathrm{s}\theta\,\mathrm{c}\phi\,\mathrm{c}\psi)\,\dot{\psi}, & -(\mathrm{s}\theta\,\mathrm{s}\psi)\,\dot{\theta} + (\mathrm{c}\theta\,\mathrm{c}\psi)\,\dot{\psi} \\ (\mathrm{c}\theta\,\mathrm{s}\phi\,\mathrm{c}\psi)\,\dot{\theta} + (\mathrm{s}\theta\,\mathrm{c}\phi\,\mathrm{c}\psi)\,\dot{\psi} - (\mathrm{s}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi)\,\dot{\psi}, & -(\mathrm{c}\theta\,\mathrm{c}\phi\,\mathrm{c}\psi)\,\dot{\theta} + (\mathrm{s}\theta\,\mathrm{s}\phi\,\mathrm{c}\psi)\,\dot{\phi} + (\mathrm{s}\theta\,\mathrm{c}\phi\,\mathrm{s}\psi)\,\dot{\phi}, & -(\mathrm{s}\theta\,\mathrm{c}\psi)\,\dot{\theta} - (\mathrm{c}\theta\,\mathrm{s}\psi)\,\dot{\psi} \\ -(\mathrm{s}\theta\,\mathrm{s}\phi)\,\dot{\theta} + (\mathrm{c}\theta\,\mathrm{c}\phi)\,\dot{\phi}, & (\mathrm{s}\theta\,\mathrm{c}\phi)\,\dot{\theta} + (\mathrm{c}\theta\,\mathrm{s}\phi)\,\dot{\phi}, & -(\mathrm{c}\theta)\,\dot{\theta} \end{bmatrix} \tag{J-16}$$

$$\frac{d}{dt}\left[\frac{\partial \mathbf{A}}{\partial \phi}\right] = \begin{bmatrix} -\dot{c}_{12} & \dot{c}_{11} & 0 \\ -\dot{c}_{22} & \dot{c}_{21} & 0 \\ -\dot{c}_{32} & \dot{c}_{31} & 0 \end{bmatrix} \tag{J-17}$$

$$\frac{d}{dt}\left[\frac{\partial \mathbf{A}}{\partial \psi}\right] = \begin{bmatrix} \dot{c}_{21} & \dot{c}_{22} & \dot{c}_{23} \\ -\dot{c}_{11} & -\dot{c}_{12} & -\dot{c}_{13} \\ 0 & 0 & 0 \end{bmatrix} \tag{J-18}$$

and the last term of Eq. (J-15) vanishes.

Then, with r_1, r_2, r_3 given by Eq. (J-6), the constrained equation of motion is

$$[\mathrm{F}]\begin{bmatrix} \ddot{q} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathscr{Q}' \\ r \end{bmatrix} \tag{J-19}$$

$$\begin{bmatrix} \ddot{q} \\ \lambda \end{bmatrix} = \begin{bmatrix} \ddot{x}_{\mathrm{I}} \\ \ddot{y}_{\mathrm{I}} \\ \ddot{z}_{\mathrm{I}} \\ \ddot{\theta}_{\mathrm{I}} \\ \ddot{\phi}_{\mathrm{I}} \\ \ddot{\psi}_{\mathrm{I}} \\ \ddot{x}_{\mathrm{II}} \\ \ddot{y}_{\mathrm{II}} \\ \ddot{z}_{\mathrm{II}} \\ \ddot{\theta}_{\mathrm{II}} \\ \ddot{\phi}_{\mathrm{II}} \\ \ddot{\psi}_{\mathrm{II}} \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} \tag{J-20}$$

$$\begin{bmatrix} \mathcal{Q}' \\ r \end{bmatrix} = \begin{bmatrix} \mathcal{Q}'_{x\mathrm{I}} \\ \mathcal{Q}'_{y\mathrm{I}} \\ \mathcal{Q}'_{z\mathrm{I}} \\ p'_{1\mathrm{I}} \\ p'_{2\mathrm{I}} \\ p'_{3\mathrm{I}} \\ \mathcal{Q}'_{x\mathrm{II}} \\ \mathcal{Q}'_{y\mathrm{II}} \\ \mathcal{Q}'_{z\mathrm{II}} \\ p'_{1\mathrm{II}} \\ p'_{2\mathrm{II}} \\ p'_{3\mathrm{II}} \\ r_1 \\ r_2 \\ r_3 \end{bmatrix} \tag{J-21}$$

$$[\mathrm{F}] = \begin{bmatrix}
m_{\mathrm{I}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & m_{\mathrm{I}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & m_{\mathrm{I}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & m_{11\mathrm{I}} & m_{12\mathrm{I}} & m_{13\mathrm{I}} & 0 & 0 & 0 & 0 & 0 & 0 & e_{1\theta\mathrm{I}} & e_{2\theta\mathrm{I}} & e_{3\theta\mathrm{I}} \\
0 & 0 & 0 & m_{21\mathrm{I}} & m_{22\mathrm{I}} & m_{23\mathrm{I}} & 0 & 0 & 0 & 0 & 0 & 0 & e_{1\phi\mathrm{I}} & e_{2\phi\mathrm{I}} & 0 \\
0 & 0 & 0 & m_{31\mathrm{I}} & m_{32\mathrm{I}} & m_{33\mathrm{I}} & 0 & 0 & 0 & 0 & 0 & 0 & e_{1\psi\mathrm{I}} & e_{2\psi\mathrm{I}} & e_{3\psi\mathrm{I}} \\
0 & 0 & 0 & 0 & 0 & 0 & m_{\mathrm{II}} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{\mathrm{II}} & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{\mathrm{II}} & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{11\mathrm{II}} & m_{12\mathrm{II}} & m_{13\mathrm{II}} & e_{1\theta\mathrm{II}} & e_{2\theta\mathrm{II}} & e_{3\theta\mathrm{II}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{21\mathrm{II}} & m_{22\mathrm{II}} & m_{23\mathrm{II}} & e_{1\phi\mathrm{II}} & e_{2\phi\mathrm{II}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{31\mathrm{II}} & m_{32\mathrm{II}} & m_{33\mathrm{II}} & e_{1\psi\mathrm{II}} & e_{2\psi\mathrm{II}} & e_{3\psi\mathrm{II}} \\
-1 & 0 & 0 & e_{1\theta\mathrm{I}} & e_{1\phi\mathrm{I}} & e_{1\psi\mathrm{I}} & 1 & 0 & 0 & e_{1\theta\mathrm{II}} & e_{1\phi\mathrm{II}} & e_{1\psi\mathrm{II}} & 0 & 0 & 0 \\
0 & -1 & 0 & e_{2\theta\mathrm{I}} & e_{2\phi\mathrm{I}} & e_{2\psi\mathrm{I}} & 0 & 1 & 0 & e_{2\theta\mathrm{II}} & e_{2\phi\mathrm{II}} & e_{2\psi\mathrm{II}} & 0 & 0 & 0 \\
0 & 0 & -1 & e_{3\theta\mathrm{I}} & 0 & e_{3\psi\mathrm{I}} & 0 & 0 & 1 & e_{3\theta\mathrm{II}} & 0 & e_{3\psi\mathrm{II}} & 0 & 0 & 0
\end{bmatrix} \tag{J-22}$$

At each step of the integration during which the constraint is acting, Eq. (J-19) is solved for the accelerations $\ddot{q}_s$, which are then integrated.

The hard-mounted spring constraint is the condition that, at the tips of the hard-mounted springs, neither translation nor rotation in a plane perpendicular to the springs is permitted. Thus, $g_x = g_z = 0$ (see Appendix K) and $h = 0$ (see Appendix L).

Let

$$\mathbf{r} = \begin{bmatrix} x_{\mathrm{I}} - x_{\mathrm{II}} \\ y_{\mathrm{I}} - y_{\mathrm{II}} \\ z_{\mathrm{I}} - z_{\mathrm{II}} \end{bmatrix}, \qquad \mathbf{r}_{\mathrm{I}l} = \begin{bmatrix} x'_{\mathrm{I}l} \\ y'_{\mathrm{I}l} \\ z'_{\mathrm{I}l} \end{bmatrix}, \qquad \mathbf{r}_{\mathrm{I}m} = \begin{bmatrix} x'_{\mathrm{I}m} \\ y'_{\mathrm{I}m} \\ z'_{\mathrm{I}m} \end{bmatrix}$$

$$\mathbf{v}_k = \begin{bmatrix} v_{kx} \\ v_{ky} \\ v_{kz} \end{bmatrix}, \qquad \mathbf{d}_k = \begin{bmatrix} d_{kx} \\ d_{ky} \\ d_{kz} \end{bmatrix}, \qquad \boldsymbol{\rho}_k = \begin{bmatrix} \rho_{kx} \\ \rho_{ky} \\ \rho_{kz} \end{bmatrix} \tag{J-23}$$

The symbols l, m refer to the first and second springs in the input, and k refers to either.

The constraint equations are then

$$f_1 = g_x = \mathbf{b}^{\mathrm{T}}_{(1)}\, \mathbf{r} + \mathbf{t}^{\mathrm{T}}_{(1)}\, \mathbf{r}'_{\mathrm{I}l} - x'_{\mathrm{II}l} - j_l\, \rho_{lx} = 0 \tag{J-24}$$

$$f_2 = g_z = \mathbf{b}^{\mathrm{T}}_{(3)}\, \mathbf{r} + \mathbf{t}^{\mathrm{T}}_{(3)}\, \mathbf{r}'_{\mathrm{I}l} - z'_{\mathrm{II}l} - j_l\, \rho_{lz} = 0 \tag{J-25}$$

Since $h = 0$ only when the numerator (Eq. L-13, Appendix L) vanishes

$$f_3 = \mathbf{t}^{\mathrm{T}}_{(3)}\, \mathbf{r}\, (x'_{\mathrm{II}l} - j_l\, \rho_{lx} - x'_{\mathrm{II}m} - j_m\, \rho_{mx}) - \mathbf{t}^{\mathrm{T}}_{(1)}\, \mathbf{r}\, (z'_{\mathrm{II}l} + j_l\, \rho_{lz} - z'_{\mathrm{II}m} - j_m\, \rho_{mz}) = 0 \tag{J-26}$$

We will have need for

$$\frac{d}{dt}\left[\frac{\partial \mathbf{A}_{\mathrm{II}}}{\partial q_s}\right]$$

and

$$\frac{d}{dt}\left[\frac{\partial \mathbf{A}_{\mathrm{I}}}{\partial q_s}\right], \qquad s = 1, \cdots, 12$$

For convenience, only

$$\frac{d}{dt}\left[\frac{\partial \mathbf{A}_{\mathrm{II}}}{\partial q_s}\right]$$

is computed; the rest are found by replacing II by I, and b by a.

$$\frac{d}{dt}\left(\frac{\partial b_{11}}{\partial \theta_{\mathrm{II}}}\right) = (\mathrm{c}\theta_{\mathrm{II}}\, \mathrm{s}\phi_{\mathrm{II}}\, \mathrm{s}\psi_{\mathrm{II}})\, \dot{\theta}_{\mathrm{II}} + (\mathrm{s}\theta_{\mathrm{II}}\, \mathrm{c}\phi_{\mathrm{II}}\, \mathrm{s}\psi_{\mathrm{II}})\, \dot{\phi}_{\mathrm{II}} + (\mathrm{s}\theta_{\mathrm{II}}\, \mathrm{s}\phi_{\mathrm{II}}\, \mathrm{c}\psi_{\mathrm{II}})\, \dot{\psi}_{\mathrm{II}} \tag{J-27}$$

$$\frac{d}{dt}\left(\frac{\partial b_{12}}{\partial \theta_{\mathrm{II}}}\right) = -\, (\mathrm{c}\theta_{\mathrm{II}}\, \mathrm{c}\phi_{\mathrm{II}}\, \mathrm{s}\phi_{\mathrm{II}})\, \dot{\theta}_{\mathrm{II}} + (\mathrm{s}\theta_{\mathrm{II}}\, \mathrm{s}\phi_{\mathrm{II}}\, \mathrm{s}\psi_{\mathrm{II}})\, \dot{\phi}_{\mathrm{II}} - (\mathrm{s}\theta_{\mathrm{II}}\, \mathrm{c}\phi_{\mathrm{II}}\, \mathrm{c}\psi_{\mathrm{II}})\, \dot{\psi}_{\mathrm{II}} \tag{J-28}$$

$$\frac{d}{dt}\left(\frac{\partial b_{13}}{\partial \theta_{\mathrm{II}}}\right) = -\, (\mathrm{s}\theta_{\mathrm{II}}\, \mathrm{s}\psi_{\mathrm{II}})\, \dot{\theta}_{\mathrm{II}} + \mathrm{c}\theta_{\mathrm{II}}\, \mathrm{c}\psi_{\mathrm{II}})\, \dot{\psi}_{\mathrm{II}} \tag{J-29}$$

$$\frac{d}{dt}\left(\frac{\partial b_{21}}{\partial \theta_{\mathrm{II}}}\right) = (\mathrm{c}\theta_{\mathrm{II}}\, \mathrm{s}\phi_{\mathrm{II}}\, \mathrm{c}\psi_{\mathrm{II}})\, \dot{\theta}_{\mathrm{II}} + (\mathrm{s}\theta_{\mathrm{II}}\, \mathrm{c}\phi_{\mathrm{II}}\, \mathrm{c}\psi_{\mathrm{II}})\, \dot{\phi}_{\mathrm{II}} - (\mathrm{s}\theta_{\mathrm{II}}\, \mathrm{s}\phi_{\mathrm{II}}\, \mathrm{s}\psi_{\mathrm{II}})\, \dot{\psi}_{\mathrm{II}} \tag{J-30}$$

$$\frac{d}{dt}\left(\frac{\partial b_{22}}{\partial \theta_{\mathrm{II}}}\right) = -\, (\mathrm{c}\theta_{\mathrm{II}}\, \mathrm{c}\phi_{\mathrm{II}}\, \mathrm{c}\psi_{\mathrm{II}})\, \dot{\theta}_{\mathrm{II}} + (\mathrm{s}\theta_{\mathrm{II}}\, \mathrm{s}\phi_{\mathrm{II}}\, \mathrm{c}\psi_{\mathrm{II}})\, \dot{\phi}_{\mathrm{II}} + (\mathrm{s}\theta_{\mathrm{II}}\, \mathrm{c}\phi_{\mathrm{II}}\, \mathrm{s}\psi_{\mathrm{II}})\, \dot{\psi}_{\mathrm{II}} \tag{J-31}$$

$$\frac{d}{dt}\left(\frac{\partial b_{23}}{\partial \theta_{\mathrm{II}}}\right) = -\, (\mathrm{s}\theta_{\mathrm{II}}\, \mathrm{c}\psi_{\mathrm{II}})\, \dot{\theta}_{\mathrm{II}} - (\mathrm{c}\theta_{\mathrm{II}}\, \mathrm{s}\psi_{\mathrm{II}})\, \dot{\psi}_{\mathrm{II}} \tag{J-32}$$

$$\frac{d}{dt}\left(\frac{\partial b_{31}}{\partial \theta_{\mathrm{II}}}\right) = -\, (\mathrm{s}\theta_{\mathrm{II}}\, \mathrm{s}\phi_{\mathrm{II}})\, \dot{\theta}_{\mathrm{II}} + (\mathrm{c}\theta_{\mathrm{II}}\, \mathrm{c}\phi_{\mathrm{II}})\, \dot{\phi}_{\mathrm{II}} \tag{J-33}$$

$$\frac{d}{dt}\left(\frac{\partial b_{32}}{\partial \theta_{\mathrm{II}}}\right) = (\mathrm{s}\theta_{\mathrm{II}}\, \mathrm{c}\phi_{\mathrm{II}})\, \dot{\theta}_{\mathrm{II}} + (\mathrm{c}\theta_{\mathrm{II}}\, \mathrm{s}\phi_{\mathrm{II}})\, \dot{\phi}_{\mathrm{II}} \tag{J-34}$$

$$\frac{d}{dt}\left(\frac{\partial b_{33}}{\partial \theta_{\mathrm{II}}}\right) = -\, (\mathrm{c}\theta_{\mathrm{II}})\, \dot{\theta}_{\mathrm{II}} \tag{J-35}$$

$$\frac{d}{dt}\left[\frac{\partial \mathbf{A}_{\mathrm{II}}}{\partial \phi_{\mathrm{II}}}\right] = \begin{bmatrix} -\dot{b}_{12} & \dot{b}_{11} & 0 \\ -\dot{b}_{22} & \dot{b}_{21} & 0 \\ -\dot{b}_{32} & \dot{b}_{31} & 0 \end{bmatrix} \tag{J-36}$$

$$\frac{d}{dt}\left[\frac{\partial \mathbf{A}_{\mathrm{II}}}{\partial \psi_{\mathrm{II}}}\right] = \begin{bmatrix} \dot{b}_{21} & \dot{b}_{22} & \dot{b}_{23} \\ -\dot{b}_{11} & -\dot{b}_{12} & -\dot{b}_{13} \\ 0 & 0 & 0 \end{bmatrix} \tag{J-37}$$

Clearly

$$\frac{d}{dt}\left[\frac{\partial \mathbf{A}_{\mathrm{II}}}{\partial q_s}\right] = 0 \qquad \text{for} \qquad s = \mathrm{I}_x, \cdots, \mathrm{II}_z \tag{J-38}$$

Now

$$\frac{\partial \mathbf{r}}{\partial x_{\mathrm{I}}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{J-39}$$

$$\frac{\partial \mathbf{r}}{\partial y_{\mathrm{I}}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{J-40}$$

$$\frac{\partial \mathbf{r}}{\partial z_{\mathrm{I}}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{J-41}$$

$$\frac{\partial \mathbf{r}}{\partial x_{\mathrm{II}}} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \tag{J-42}$$

$$\frac{\partial \mathbf{r}}{\partial y_{\mathrm{II}}} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \tag{J-43}$$

$$\frac{\partial \mathbf{r}}{\partial z_{\mathrm{II}}} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \tag{J-44}$$

$$\frac{\partial \mathbf{r}}{\partial q_s} = 0 \qquad \text{for all other } q_s \tag{J-45}$$

The following expressions also will be needed

$$\dot{\mathbf{T}} = \dot{\mathbf{A}}_{\mathrm{II}} \mathbf{A}_{\mathrm{I}}^{\mathrm{T}} + \mathbf{A}_{\mathrm{II}} \dot{\mathbf{A}}_{\mathrm{I}}^{\mathrm{T}} \tag{J-46}$$

$$\frac{\partial T}{\partial q_s} = \frac{\partial \mathbf{A}_{\mathrm{II}}}{\partial q_s} \mathbf{A}_{\mathrm{I}}^{\mathrm{T}} + \mathbf{A}_{\mathrm{II}} \frac{\partial \mathbf{A}_{\mathrm{I}}^{\mathrm{T}}}{\partial q_s} \tag{J-47}$$

$$\frac{d}{dt}\left[\frac{\partial T}{\partial q_s}\right] = \frac{d}{dt}\left[\frac{\partial \mathbf{A}_{\mathrm{II}}}{\partial q_s}\right] \mathbf{A}_{\mathrm{I}}^{\mathrm{T}} + \frac{\partial \mathbf{A}_{\mathrm{II}}}{\partial q_s} \dot{\mathbf{A}}_{\mathrm{I}}^{\mathrm{T}} + \dot{\mathbf{A}}_{\mathrm{II}} \frac{\partial \mathbf{A}_{\mathrm{I}}^{\mathrm{T}}}{\partial q_s} + \mathbf{A}_{\mathrm{II}} \frac{d}{dt}\left[\frac{\partial \mathbf{A}_{\mathrm{I}}^{\mathrm{T}}}{\partial q_s}\right] \tag{J-48}$$

Now let

$$f_k = \mathbf{a}^{(2)T}\,\mathbf{d}_k \tag{J-49}$$

$$g_k = \mathbf{a}^{(2)T}\,\mathbf{A}_{II}^{T}\,\rho_k \tag{J-50}$$

Then, from Appendix B

$$j_k = \frac{f_k}{g_k} \tag{J-51}$$

$$\dot{f}_k = \dot{\mathbf{a}}^{(2)T}\,\mathbf{d}_k + \mathbf{a}^{(2)T}\,\mathbf{v}_k \tag{J-52}$$

$$\dot{g}_k = \dot{\mathbf{a}}^{(2)T}\,\mathbf{A}_{II}^{T}\,\boldsymbol{\rho}_k + \mathbf{a}^{(2)T}\,\dot{\mathbf{A}}_{II}^{T}\,\boldsymbol{\rho}_k \tag{J-53}$$

and

$$\dot{j}_k = \frac{1}{g_k}\,(\dot{f}_k - j_k\,\dot{g}_k) \tag{J-54}$$

$$\frac{\partial f_k}{\partial q_s} = \frac{\partial \mathbf{a}^{(2)T}}{\partial q_s}\,\mathbf{d}_k + \mathbf{a}^{(2)T}\left(\frac{\partial \mathbf{r}}{\partial q_s} + \frac{\partial \mathbf{A}_{I}^{T}}{\partial q_s}\,\mathbf{r}'_{Ik} - \frac{\partial \mathbf{A}_{II}^{T}}{\partial q_s}\,\mathbf{r}'_{IIk}\right) \tag{J-55}$$

$$\frac{\partial g_k}{\partial q_s} = \frac{\partial \mathbf{a}^{(2)T}}{\partial q_s}\,\mathbf{A}_{II}^{T}\,\boldsymbol{\rho}_k + \mathbf{a}^{(2)T}\,\frac{\partial \mathbf{A}_{II}^{T}}{\partial q_s}\,\boldsymbol{\rho}_k \tag{J-56}$$

$$\frac{\partial j_k}{\partial q_s} = \frac{1}{g_k}\left[\frac{\partial f_k}{\partial q_s} - j_k\,\frac{\partial g_k}{\partial q_s}\right] \tag{J-57}$$

Now let

$$f'_k = \frac{\partial f_k}{\partial q_s} - j_k\,\frac{\partial g_k}{\partial q_s} \tag{J-58}$$

Then

$$\begin{aligned}\frac{d}{dt}\left[\frac{\partial f_k}{\partial q_s}\right] &= \frac{d}{dt}\left[\frac{\partial \mathbf{a}^{(2)T}}{\partial q_s}\right]\mathbf{d}_k + \frac{\partial \mathbf{a}^{(2)T}}{\partial q_s}\,\mathbf{v}_k + \dot{\mathbf{a}}^{(2)T}\left[\frac{\partial \mathbf{r}}{\partial q_s} + \frac{\partial \mathbf{A}_{I}^{T}}{\partial q_s}\,\mathbf{r}'_{Ik} - \frac{\partial \mathbf{A}_{II}^{T}}{\partial q_s}\,\mathbf{r}'_{IIk}\right] \\ &\quad + \mathbf{a}^{(2)T}\left[\frac{d}{dt}\left(\frac{\partial \mathbf{A}_{I}^{T}}{\partial q_s}\right)\mathbf{r}'_{Ik} - \frac{d}{dt}\left(\frac{\partial \mathbf{A}_{II}^{T}}{\partial q_s}\right)\mathbf{r}'_{IIk}\right]\end{aligned} \tag{J-59}$$

$$\begin{aligned}\frac{d}{dt}\left[\frac{\partial g_k}{\partial q_s}\right] &= \frac{d}{dt}\left[\frac{\partial \mathbf{a}^{(2)T}}{\partial q_s}\right]\mathbf{A}_{II}^{T}\,\boldsymbol{\rho}_k + \frac{\partial \mathbf{a}^{(2)T}}{\partial q_s}\,\dot{\mathbf{A}}_{II}^{T}\,\boldsymbol{\rho}_k \\ &\quad + \dot{\mathbf{a}}^{(2)T}\,\frac{\partial \mathbf{A}_{II}^{T}}{\partial q_s}\,\boldsymbol{\rho}_k + \mathbf{a}^{(2)T}\,\frac{d}{dt}\left[\frac{\partial \mathbf{A}_{II}^{T}}{\partial q_s}\right]\boldsymbol{\rho}_k\end{aligned} \tag{J-60}$$

Then

$$\dot{f}'_k = \frac{d}{dt}\left(\frac{\partial f_k}{\partial q_s}\right) - \dot{j}_k\left(\frac{\partial g_k}{\partial q_s}\right) - j_k\,\frac{d}{dt}\left(\frac{\partial g_k}{\partial q_s}\right) \tag{J-61}$$

and

$$\frac{d}{dt}\left(\frac{\partial j_k}{\partial q_s}\right) = \frac{1}{g_k}\left[\dot{f}'_k - \left(\frac{\partial j_k}{\partial q_s}\right)\frac{d}{dt}\left(\frac{\partial g_k}{\partial q_s}\right)\right] \tag{J-62}$$

Thus, for the coefficients e_{ls} we have

$$e_{15} = \frac{\partial \mathbf{b}^{\mathrm{T}}_{(1)}}{\partial q_s}\mathbf{r} + \mathbf{b}^{\mathrm{T}}_{(1)}\frac{\partial \mathbf{r}}{\partial q_s} + \frac{\partial \mathbf{t}^{\mathrm{T}}_{(1)}}{\partial q_s}\mathbf{r}'_{\mathrm{I}l} - \frac{\partial j_l}{\partial q_s}\rho_{lz} \tag{J-63}$$

$$e_{25} = \frac{\partial \mathbf{b}^{\mathrm{T}}_{(3)}}{\partial q_s}\mathbf{r} + \mathbf{b}^{\mathrm{T}}_{(3)}\frac{\partial \mathbf{r}}{\partial q_s} + \frac{\partial \mathbf{t}^{\mathrm{T}}_{(3)}}{\partial q_s}\mathbf{r}'_{\mathrm{I}l} - \frac{\partial j_l}{\partial q_s}\rho_{lz} \tag{J-64}$$

$$\begin{aligned} e_{35} = {} & \frac{\partial \mathbf{t}^{\mathrm{T}}_{(3)}}{\partial q_s}(\mathbf{r}'_{\mathrm{I}l} - \mathbf{r}'_{\mathrm{I}m})(x'_{\mathrm{II}l} + j_l\rho_{lx} - x'_{\mathrm{II}m} - j_m\rho_{mx}) + \mathbf{t}^{\mathrm{T}}_{(3)}(\mathbf{r}'_{\mathrm{I}l} - \mathbf{r}'_{\mathrm{I}m})\left(\frac{\partial j_l}{\partial q_s}\rho_{lx} - \frac{\partial j_m}{\partial q_s}\rho_{mx}\right) \\ & - \frac{\partial \mathbf{t}^{\mathrm{T}}_{(1)}}{\partial q_s}(\mathbf{r}'_{\mathrm{I}l} - \mathbf{r}'_{\mathrm{I}m})(z'_{\mathrm{II}l} + j_l\rho_{lz} - z'_{\mathrm{II}m} - j_m\rho_{mz}) - \mathbf{t}^{\mathrm{T}}_{(1)}(\mathbf{r}'_{\mathrm{I}l} - \mathbf{r}'_{\mathrm{I}m})\left(\frac{\partial j_l}{\partial q_s}\rho_{lz} - \frac{\partial j_m}{\partial q_s}\rho_{mz}\right) \end{aligned} \tag{J-65}$$

and

$$\dot e_{15} = \frac{d}{dt}\left(\frac{\partial \mathbf{b}^{\mathrm{T}}_{(1)}}{\partial q_s}\right)\mathbf{r} + \frac{\partial \mathbf{b}^{\mathrm{T}}_{(1)}}{\partial q_s}\dot{\mathbf{r}} + \dot{\mathbf{b}}^{\mathrm{T}}_{(1)}\frac{\partial \mathbf{r}}{\partial q_s} + \frac{d}{dt}\left(\frac{\partial \mathbf{t}^{\mathrm{T}}_{(1)}}{\partial q_s}\right)\mathbf{r}'_{\mathrm{I}l} - \frac{d}{dt}\left(\frac{\partial j_l}{\partial q_s}\right)\rho_{lz} \tag{J-66}$$

$$\dot e_{25} = \frac{d}{dt}\left(\frac{\partial \mathbf{b}^{\mathrm{T}}_{(3)}}{\partial q_s}\right)\mathbf{r} + \frac{\partial \mathbf{b}^{\mathrm{T}}_{(3)}}{\partial q_s}\dot{\mathbf{r}} + \dot{\mathbf{b}}^{\mathrm{T}}_{(3)}\frac{\partial \mathbf{r}}{\partial q_s} + \frac{d}{dt}\left(\frac{\partial \mathbf{t}^{\mathrm{T}}_{(3)}}{\partial q_s}\right)\mathbf{r}'_{\mathrm{I}l} - \frac{d}{dt}\left(\frac{\partial j_l}{\partial q_s}\right)\rho_{lz} \tag{J-67}$$

$$\begin{aligned} \dot e_{35} = {} & \frac{d}{dt}\left[\frac{\partial \mathbf{t}^{\mathrm{T}}_{(3)}}{\partial q_s}\right](\mathbf{r}'_{\mathrm{I}l} - \mathbf{r}'_{\mathrm{I}m})(x'_{\mathrm{II}l} + j_l\rho_{lx} - x'_{\mathrm{II}m} - j_m\rho_{mx}) + \frac{\partial \mathbf{t}^{\mathrm{T}}_{(3)}}{\partial q_s}(\mathbf{r}'_{\mathrm{I}l} - \mathbf{r}'_{\mathrm{I}m})(\dot j_l\rho_{lx} - \dot j_m\rho_{mx}) \\ & + \dot{\mathbf{t}}^{\mathrm{T}}_{(3)}(\mathbf{r}'_{\mathrm{I}l} - \mathbf{r}'_{\mathrm{I}m})\left(\frac{\partial j_l}{\partial q_s}\rho_{lx} - \frac{\partial j_m}{\partial q_s}\rho_{mx}\right) + \mathbf{t}^{\mathrm{T}}_{(3)}(\mathbf{r}'_{\mathrm{I}l} - \mathbf{r}'_{\mathrm{I}m})\left[\frac{d}{dt}\left(\frac{\partial j_l}{\partial q_s}\right)\rho_{lx} - \frac{d}{dt}\left(\frac{\partial j_m}{\partial q_s}\right)\rho_{mx}\right] \\ & - \frac{d}{dt}\left(\frac{\partial \mathbf{t}^{\mathrm{T}}_{(1)}}{\partial q_s}\right)(\mathbf{r}'_{\mathrm{I}l} - \mathbf{r}'_{\mathrm{I}m})(z'_{\mathrm{II}l} + j_l\rho_{lz} - z'_{\mathrm{II}m} - j_m\rho_{mz}) - \frac{\partial \mathbf{t}^{\mathrm{T}}_{(1)}}{\partial q_s}(\mathbf{r}'_{\mathrm{I}l} - \mathbf{r}'_{\mathrm{I}m})(\dot j_l\rho_{lz} - \dot j_m\rho_{mz}) \\ & - \dot{\mathbf{t}}^{\mathrm{T}}_{(1)}(\mathbf{r}'_{\mathrm{I}l} - \mathbf{r}'_{\mathrm{I}m})\left(\frac{\partial j_l}{\partial q_s}\rho_{lz} - \frac{\partial j_m}{\partial q_s}\rho_{mz}\right) - \mathbf{t}^{\mathrm{T}}_{(1)}(\mathbf{r}'_{\mathrm{I}l} - \mathbf{r}'_{\mathrm{I}m})\left[\frac{d}{dt}\left(\frac{\partial j_l}{\partial q_s}\right)\rho_{lz} - \frac{d}{dt}\left(\frac{\partial j_m}{\partial q_s}\right)\rho_{mz}\right] \end{aligned} \tag{J-68}$$

The constrained equations of motion are

$$[\mathrm{F}]\begin{bmatrix}\ddot q \\ \lambda\end{bmatrix} = \begin{bmatrix}\mathcal{Q}' \\ r\end{bmatrix} \tag{J-69}$$

where

$$\begin{bmatrix}\ddot q \\ \lambda\end{bmatrix} = \begin{bmatrix}\ddot x_{\mathrm{I}} \\ \ddot y_{\mathrm{I}} \\ \ddot z_{\mathrm{I}} \\ \ddot\theta_{\mathrm{I}} \\ \ddot\phi_{\mathrm{I}} \\ \ddot\psi_{\mathrm{I}} \\ \ddot x_{\mathrm{II}} \\ \ddot y_{\mathrm{II}} \\ \ddot z_{\mathrm{II}} \\ \ddot\theta_{\mathrm{II}} \\ \ddot\phi_{\mathrm{II}} \\ \ddot\psi_{\mathrm{II}} \\ \lambda_1 \\ \lambda_2 \\ \lambda_3\end{bmatrix} \qquad \begin{bmatrix}\mathcal{Q}' \\ r\end{bmatrix} = \begin{bmatrix}\mathcal{Q}'_{x\mathrm{I}} \\ \mathcal{Q}'_{y\mathrm{I}} \\ \mathcal{Q}'_{z\mathrm{I}} \\ \rho'_{1\mathrm{I}} \\ \rho'_{2\mathrm{I}} \\ \rho'_{3\mathrm{I}} \\ \mathcal{Q}'_{x\mathrm{II}} \\ \mathcal{Q}'_{y\mathrm{II}} \\ \mathcal{Q}'_{z\mathrm{II}} \\ \rho'_{1\mathrm{II}} \\ \rho'_{2\mathrm{II}} \\ \rho'_{3\mathrm{II}} \\ r_1 \\ r_2 \\ r_3\end{bmatrix} \tag{J-70}$$

$$[F] = \begin{bmatrix}
m_{\mathrm{I}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_{1x\mathrm{I}} & e_{2x\mathrm{I}} & e_{3x\mathrm{I}} \\
0 & m_{\mathrm{I}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_{1y\mathrm{I}} & e_{2y\mathrm{I}} & e_{3y\mathrm{I}} \\
0 & 0 & m_{\mathrm{I}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_{1z\mathrm{I}} & e_{2z\mathrm{I}} & e_{3z\mathrm{I}} \\
0 & 0 & 0 & m_{11\mathrm{I}} & m_{12\mathrm{I}} & m_{13\mathrm{I}} & 0 & 0 & 0 & 0 & 0 & 0 & e_{1\theta\mathrm{I}} & e_{2\theta\mathrm{I}} & e_{3\theta\mathrm{I}} \\
0 & 0 & 0 & m_{21\mathrm{I}} & m_{22\mathrm{I}} & m_{23\mathrm{I}} & 0 & 0 & 0 & 0 & 0 & 0 & e_{1\phi\mathrm{I}} & e_{2\phi\mathrm{I}} & e_{3\phi\mathrm{I}} \\
0 & 0 & 0 & m_{31\mathrm{I}} & m_{32\mathrm{I}} & m_{33\mathrm{I}} & 0 & 0 & 0 & 0 & 0 & 0 & e_{1\psi\mathrm{I}} & e_{2\psi\mathrm{I}} & e_{3\psi\mathrm{I}} \\
0 & 0 & 0 & 0 & 0 & 0 & m_{\mathrm{II}} & 0 & 0 & 0 & 0 & 0 & e_{1x\mathrm{II}} & e_{2x\mathrm{II}} & e_{3x\mathrm{II}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{\mathrm{II}} & 0 & 0 & 0 & 0 & e_{1y\mathrm{II}} & e_{2y\mathrm{II}} & e_{3y\mathrm{II}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{\mathrm{II}} & 0 & 0 & 0 & e_{1z\mathrm{II}} & e_{2z\mathrm{II}} & e_{3z\mathrm{II}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{11\mathrm{II}} & m_{12\mathrm{II}} & m_{13\mathrm{II}} & e_{1\theta\mathrm{II}} & e_{2\theta\mathrm{II}} & e_{3\theta\mathrm{II}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{21\mathrm{II}} & m_{22\mathrm{II}} & m_{23\mathrm{II}} & e_{1\phi\mathrm{II}} & e_{2\phi\mathrm{II}} & e_{3\phi\mathrm{II}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{31\mathrm{II}} & m_{32\mathrm{II}} & m_{33\mathrm{II}} & e_{1\psi\mathrm{II}} & e_{2\psi\mathrm{II}} & e_{3\psi\mathrm{II}} \\
e_{1x\mathrm{I}} & e_{1y\mathrm{I}} & e_{1z\mathrm{I}} & e_{1\theta\mathrm{I}} & e_{1\phi\mathrm{I}} & e_{1\psi\mathrm{I}} & e_{1x\mathrm{II}} & e_{1y\mathrm{II}} & e_{1z\mathrm{II}} & e_{1\theta\mathrm{II}} & e_{1\phi\mathrm{II}} & e_{1\psi\mathrm{II}} & 0 & 0 & 0 \\
e_{2x\mathrm{I}} & e_{2y\mathrm{I}} & e_{2z\mathrm{I}} & e_{2\theta\mathrm{I}} & e_{2\phi\mathrm{I}} & e_{2\psi\mathrm{I}} & e_{2x\mathrm{II}} & e_{2y\mathrm{II}} & e_{2z\mathrm{II}} & e_{2\theta\mathrm{II}} & e_{2\phi\mathrm{II}} & e_{2\psi\mathrm{II}} & 0 & 0 & 0 \\
e_{3x\mathrm{I}} & e_{3y\mathrm{I}} & e_{3z\mathrm{I}} & e_{3\theta\mathrm{I}} & e_{3\phi\mathrm{I}} & e_{3\psi\mathrm{I}} & e_{3x\mathrm{II}} & e_{3y\mathrm{II}} & e_{3z\mathrm{II}} & e_{3\theta\mathrm{II}} & e_{3\phi\mathrm{II}} & e_{3\psi\mathrm{II}} & 0 & 0 & 0
\end{bmatrix} \tag{J-71}$$

APPENDIX K

Distance in the $x_{\mathrm{II}}' z_{\mathrm{II}}'$ Plane Between the Initial Point of Contact on Body I of a Spring Hard-Mounted on Body II and the Tip of the Spring

The distance between the initial and current contact point for a hard-mounted spring tip is required to determine the no-slippage constraint condition.

A. Symbols

$\mathbf{d}_i$	vector distance between point i on body I and point i on body II
$\mathbf{g}$	vector distance (as defined in the title of this Appendix) $\mathbf{g} = g_x \mathbf{e}_{\mathrm{II}x'} + g_z \mathbf{e}_{\mathrm{II}z'}$
j_i	length of extension of spring i (see Appendix B)
l	subscript identifying first hard-mounted spring
$\bar{\mathbf{r}}_{\mathrm{II}i}$	location in inertial space of tip of spring i
$\bar{\mathbf{r}}'_{\mathrm{II}i}$	location in body II CS of the tip of spring i
$\mathbf{T}$	matrix product $\mathbf{A}_{\mathrm{II}} \mathbf{A}_{\mathrm{I}}^{-1}$
t_{ij}	elements of $\mathbf{T}$ matrix (i^{th} row, j^{th} column)

B. Determination of $\mathbf{g}_i$

The vector $\mathbf{g}$ consists of just the x and z components of $\mathbf{g}_i$ in the body II CS. Now

$$\mathbf{A}_{\mathrm{II}} \mathbf{d}_i = \mathbf{A}_{\mathrm{II}} \mathbf{r} + \mathbf{A}_{\mathrm{II}} \mathbf{A}_{\mathrm{I}}^{\mathrm{T}} \mathbf{r}'_{\mathrm{I}i} - \mathbf{A}_{\mathrm{II}} \mathbf{A}_{\mathrm{II}}^{\mathrm{T}} \bar{\mathbf{r}}'_{\mathrm{II}i} = \mathbf{A}_{\mathrm{II}} \mathbf{r} + \mathbf{T} \mathbf{r}'_{\mathrm{I}i} - \mathbf{r}'_{\mathrm{II}i} \tag{K-1}$$

Therefore

$$g_x = \mathbf{b}^{\mathrm{T}}_{(1)} \mathbf{r} + \mathbf{t}^{\mathrm{T}}_{(1)} \mathbf{r}'_{\mathrm{I}l} - \bar{x}'_{\mathrm{II}l}, \qquad g_z = \mathbf{b}^{\mathrm{T}}_{(3)} \mathbf{r} + \mathbf{t}^{\mathrm{T}}_{(3)} \mathbf{r}'_{\mathrm{I}l} - \bar{z}'_{\mathrm{II}l} \tag{K-2}$$

where the first spring has been specified.

Clearly

$$\bar{x}'_{IIl} = x'_{IIl} + j_l \rho_{lx} \tag{K-3}$$

$$\bar{z}'_{IIl} = z'_{IIl} + j_l \rho_{lz} \tag{K-4}$$

APPENDIX L

Angle of Rotation in the $x_{II}' z_{II}'$ Plane Between Body I Pads and Body II Spring Tips

The angle of rotation in the $x'_{II} z'_{II}$ plane is required to specify the constraint condition that no such rotation shall occur.

A. Symbols

$\mathbf{a}$	a vector on body I from the initial point of contact of spring tip l to the initial point of contact of spring tip m
$\mathbf{b}$	a vector joining the tip of spring l to the tip of spring m
$\mathbf{e}_a$	unit vector along the projection of $\mathbf{a}$ on the $x'_{II} z'_{II}$ plane
$\mathbf{e}_b$	unit vector along the projection of $\mathbf{b}$ on the $x'_{II} z'_{II}$ plane
h	angle of rotation in the $x'_{II} z'_{II}$ plane between body I pads and body II spring tips
l, m	subscripts identifying the first two hard-mounted springs
$\mathbf{p}_i$	location vector of tip of spring i (see Appendix B)
$\mathbf{T}$	matrix product $\mathbf{A}_{II} \mathbf{A}_I^{-1}$ (see Appendix K)
t_{ij}	elements of the $\mathbf{T}$ matrix (i^{th} row, j^{th} column)

B. Calculation of *h*

During any one integration interval, if the rotation is constrained, the rotation will be small, so it is not necessary to consider angles other than those in the first and fourth quadrants.

Expression of the definitions of $\mathbf{a}$ and $\mathbf{b}$ yields Eqs. (L-1) and (L-2).

$$\mathbf{a} = \mathbf{r}'_{Il} - \mathbf{r}'_{Im} \tag{L-1}$$

$$\mathbf{b} = \mathbf{p}_l - \mathbf{p}_m \tag{L-2}$$

Expressed in body II coordinates, Eqs. (L-1) and (L-2) become

$$\begin{aligned}
\mathbf{a} = {} & \mathbf{A}_{II} \mathbf{A}_I^{-1} [(x'_{Il} - x'_{Im}) \mathbf{e}_{Ix'} + (y'_{Il} - y'_{Im}) \mathbf{e}_{Iy'} + (z'_{Il} - z'_{Im}) \mathbf{e}_{Iz'}] \\
= {} & [t_{11} (x'_{Il} - x'_{Im}) + t_{12} (y'_{Il} - y'_{Im}) + t_{13} (z'_{Il} - z'_{Im})] \mathbf{e}_{IIx'} \\
& + [t_{21} (x'_{Il} - x'_{Im}) + t_{22} (y'_{Il} - y'_{Im}) + t_{23} (z'_{Il} - z'_{Im})] \mathbf{e}_{IIy'} \\
& + [t_{31} (x'_{Il} - x'_{Im}) + t_{32} (y'_{Il} - y'_{Im}) + t_{33} (z'_{Il} - z'_{Im})] \mathbf{e}_{IIz'}
\end{aligned} \tag{L-3}$$

$$\begin{aligned}\mathbf{b} = &(x'_{\mathrm{II}l} - x'_{\mathrm{II}m} + j_l\,\rho_{lx} - j_m\,\rho_{mx})\,\mathbf{e}_{\mathrm{II}x'}\\ &+ (y'_{\mathrm{II}l} - y'_{\mathrm{II}m} + j_l\,\rho_{ly} - j_m\,\rho_{my})\,\mathbf{e}_{\mathrm{II}y'}\\ &+ (z'_{\mathrm{II}l} - z'_{\mathrm{II}m} + j_l\,\rho_{lz} - j_m\,\rho_{mz})\,\mathbf{e}_{\mathrm{II}z'}\end{aligned} \tag{L-4}$$

Let

$$\begin{aligned}\Delta_a &= (a_{\mathrm{II}x'}{}^2 + a_{\mathrm{II}z'}{}^2)^{1/2}\\ &= \{[t_{11}(x'_{\mathrm{I}l} - x'_{\mathrm{I}m}) + t_{12}(y'_{\mathrm{I}l} - y'_{\mathrm{I}m}) + t_{13}(z'_{\mathrm{I}l} - z'_{\mathrm{I}m})]^2\\ &\quad + [t_{31}(x'_{\mathrm{I}l} - x'_{\mathrm{I}m}) + t_{32}(y'_{\mathrm{I}l} - y'_{\mathrm{I}m}) + t_{33}(z'_{\mathrm{I}l} - z'_{\mathrm{I}m})]^2\}^{1/2}\end{aligned} \tag{L-5}$$

$$\begin{aligned}\Delta_b &= (b_{\mathrm{II}x'}{}^2 + b_{\mathrm{II}z'}{}^2)^{1/2}\\ &= [(x'_{\mathrm{II}l} - x'_{\mathrm{II}m} + j_l\,\rho_{lx} - j_m\,\rho_{mx})^2 + (z'_{\mathrm{II}l} - z'_{\mathrm{II}m} + j_l\,\rho_{lz} - j_m\,\rho_{mz})^2]^{1/2}\end{aligned} \tag{L-6}$$

Then

$$\begin{aligned}\mathbf{e}_a &= \left(\frac{a_{\mathrm{II}x'}}{\Delta_a}\right)\mathbf{e}_{\mathrm{II}x'} + \left(\frac{a_{\mathrm{II}z'}}{\Delta_a}\right)\mathbf{e}_{\mathrm{II}z'} = e_{ax}\,\mathbf{e}_{\mathrm{II}x'} + e_{az}\,\mathbf{e}_{\mathrm{II}z'}\\ \mathbf{e}_b &= \left(\frac{b_{\mathrm{II}x'}}{\Delta_b}\right)\mathbf{e}_{\mathrm{II}x'} + \left(\frac{b_{\mathrm{II}z'}}{\Delta_b}\right)\mathbf{e}_{\mathrm{II}z'} = e_{bx}\,\mathbf{e}_{\mathrm{II}x'} + e_{bz}\,\mathbf{e}_{\mathrm{II}z'}\end{aligned} \tag{L-7}$$

and

$$\begin{aligned}e_{ax} &= \frac{[t_{11}(x'_{\mathrm{I}l} - x'_{\mathrm{I}m}) + t_{12}(y'_{\mathrm{I}l} - y'_{\mathrm{I}m}) + t_{13}(z'_{\mathrm{I}l} - z'_{\mathrm{I}m})]}{\Delta_a}\\ e_{az} &= \frac{[t_{11}(x'_{\mathrm{I}l} - x'_{\mathrm{I}m}) + t_{32}(y'_{\mathrm{I}l} - y'_{\mathrm{I}m}) + t_{33}(z'_{\mathrm{I}l} - z'_{\mathrm{I}m})]}{\Delta_a}\end{aligned} \tag{L-8}$$

$$e_{bx} = \frac{x'_{\mathrm{II}l} - x'_{\mathrm{II}m} + j_l\,\rho_{lx} - j_m\,\rho_{mx}}{\Delta_b} \tag{L-9}$$

$$e_{bz} = \frac{z'_{\mathrm{II}l} - z'_{\mathrm{II}m} + j_l\,\rho_{lz} - j_m\,\rho_{mx}}{\Delta_b} \tag{L-10}$$

Since $\mathbf{e}_a$ and $\mathbf{e}_b$ are unit vectors

$$\sin h = |\mathbf{e}_a \times \mathbf{e}_b| = e_{az}\,e_{bx} - e_{ax}\,e_{bz} \tag{L-11}$$

$$\cos h = \mathbf{e}_a \cdot \mathbf{e}_b = e_{ax}\,e_{bx} + e_{az}\,e_{bz} \tag{L-12}$$

Thus

$$h = \tan^{-1}\left(\frac{e_{az}\,e_{bx} - e_{ax}\,e_{bz}}{e_{ax}\,e_{bx} + e_{az}\,e_{bz}}\right) \tag{L-13}$$

APPENDIX M

FMARK Integration Method

The integration method used by FMARK is the fourth order Adams-Moulton predictor formula with one correction. Automatic error control is exercised by comparing the value of the predicted with the corrected result. Fourth order Runge-Kutta integration is used to build backward differences, initially or for a restart.

The classical fourth order Runge-Kutta equations follow. Let the differential equations to be solved be of the form

$$\dot{y}_j = f_j(t, \bar{y}) \qquad j = 1, n \tag{M-1}$$

where symbolically

$$\bar{y} = (y_1, \cdots, y_n) \tag{M-2}$$

Let

$$\begin{array}{ccccc} y_{j,\eta} & \text{and} & f_{j,\eta} & \text{denote} & \\ y_j & \text{and} & f_j & \text{at} & t = t_\eta \end{array} \tag{M-3}$$

Thus

$$\bar{y}_\eta = (y_{1,\eta}, \cdots, y_{n,\eta}) \tag{M-4}$$

Let

$$\bar{K}_i = (K_{1i}, \cdots, K_{ni}) \tag{M-5}$$

and

$$K_{j1} = hf_j(t, \bar{y}_\eta) \tag{M-6}$$

$$K_{j2} = hf_j\left(t + \frac{h}{2}, \quad \bar{y}_\eta + \frac{\bar{K}_2}{2}\right) \tag{M-7}$$

$$K_{j3} = hf_j\left(t + \frac{h}{2}, \quad \bar{y}_\eta + \frac{\bar{K}_2}{2}\right) \tag{M-8}$$

$$K_{j4} = hf_j(t + h, \quad \bar{y}_\eta + \bar{K}_3) \tag{M-9}$$

The numerical integration equation is then

$$\bar{y}_{\eta+1} = \bar{y}_\eta + \frac{1}{6}(\bar{K}_1 + 2\bar{K}_2 + 2\bar{K}_3 + K_4) \tag{M-10}$$

where $y_{j,\eta+1}$ denotes y_j at $t + h$.

The Adams-Moulton fourth order predictor-corrector equations follow, where $\bar{y}^P_{\eta+1}$ = the predicted values, and $\bar{y}^C_{\eta+1}$ = the corrected value

$$\bar{y}^P_{\eta+1} = \bar{y}_\eta + h\left(\sum_{i=0}^{4} a_i \nabla^i\right)\dot{\bar{y}}_\eta \tag{M-11}$$

where

$$\begin{aligned}\nabla^0 \dot{\bar{y}}_\eta &= \dot{\bar{y}}_\eta \\ \nabla^1 \dot{\bar{y}}_\eta &= \dot{\bar{y}}_\eta - \dot{\bar{y}}_{\eta-1} \\ \nabla^2 y_\eta &= \nabla^1 \dot{\bar{y}}_\eta - \nabla^1 \dot{\bar{y}}_{\eta-1} \\ &\text{etc.}\end{aligned} \tag{M-12}$$

and

$$a_0 = 1, \qquad a_1 = \frac{1}{2}, \qquad a_2 = \frac{5}{12}, \qquad a_3 = \frac{3}{8}, \qquad a_4 = \frac{251}{720} \tag{M-13}$$

Also

$$\dot{\bar{y}}^{P}_{j,\eta+1} = f_j(t_{\eta+1}, \bar{y}_{\eta+1}) \tag{M-14}$$

and the correction becomes

$$\dot{\bar{y}}^{C}_{j,\eta+1} = \dot{\bar{y}}_{j,\eta} + h\left(\sum_{i=0}^{4} b_i \nabla^i\right) \dot{\bar{y}}^{P}_{\eta+1} \tag{M-15}$$

where

$$b_0 = a_0, \quad b_{m+1} = a_{m+1} - a_m, \quad m = 1, 2, 3 \tag{M-16}$$

Let

$$E_{\eta+1} = \max{}_j \frac{|y^{P}_{j,\eta+1} - y^{C}_{j,\eta+1}|}{D_j} \tag{M-17}$$

where

$$D_j = \max|y^{C}_{j,\eta+1}, \hat{y}| \tag{M-18}$$

$E_{\eta+1}$ represents the maximum error in any of the dependent variables due to truncation error in the step t_η to $t_{\eta+1}$. The user, through the array HB, provides the

$\bar{E}$	upper bound on $E_{\eta+1}$
$\hat{E}$	lower bound on $E_{\eta+1}$
h_{max}	maximum allowable step size
h_{min}	minimum allowable step size
$\hat{y}$	constant used to prevent unnecessary reduction in h whenever $|y_{j,\eta+1}|$ is small

If $E_{\eta+1} \leq \hat{E}$ for four successive steps, h is doubled. If $\hat{E} < E_{\eta+1} \leq \bar{E}$, h is unchanged. If $E_{\eta+1} > \bar{E}$, h is halved.

If a discontinuity occurs in some $\dot{y}_j$ it is obvious that the Adams-Moulton technique with the backward differences can not be continued. In this case, a restart must be made (i.e., Runge-Kutta integration is used for four steps). With these points, Adams-Moulton integration is continued.

APPENDIX N

FORTRAN Listing and Sample Problem Printout

```
      GENERAL INPUT
C
    1 FORMAT(6E12.8)
    2 FORMAT(6I12)
      EQUIVALENCE (T,T),(ZERO,ZERO),
     1            (X1,V1(1)),(Y1,V1(2)),(Z1,V1(3)),(T1,V1(4)),(T2,V1(5))
     1,(P1,V1(6)),(P2,V1(7)),(S1,V1(8)),(S2,V1(9)),(X1X,V1(10)),(Y1Y,V1(
     111)),(Z1Z,V1(12)),(T1T,V1(13)),(T2T,V1(14)),(P1P,V1(15)),(P2P,V1(1
     16)),(S1S,V1(17)),(S2S,V1(18)),(X2,V2(1)),(Y2,V2(2)),(Z2,V2(3)),
     1(X2X,V2(4)),(Y2Y,V2(5)),(Z2Z,V2(6))
      COMMON T,ZERO,V1,V2,YDOT1,YDOT2
      COMMON A,B,AD,BD
      DIMENSION           YDOT2(6),YDOT1(18),V2(6),V1(18),NH(6),HB(6)
      DIMENSION A(3,3),B(3,3),AD(3,3),BD(3,3)
      DIMENSION DM(3,3),DN(3,3)
      COMMON WIL1,TCO
      DIMENSION TCG(8),TCO(8)
      DIMENSION FM(3,4),FEL(3),AONST(3),CM(3,3),DUD(3)
      DIMENSION DD(12),DF(8),D(8),XC1(8),YC1(8),ZC1(8),XC2(8),YC2(8),
     1ZC2(8)
      DIMENSION EP(12),PI(3),TP(6),XP1(3),YP1(3),ZP1(3),XP2(3),YP2(3),
     1ZP2(3),UPX(3),UPY(3),UPZ(3)
      DIMENSION G(12),GA(6),PA(6),VA(6),PAD(6),VAD(6),SGX(6),SGY(6),
     1SGZ(6),UAX(3),UAY(3),UAZ(3),UPAX(3),UPAY(3),UPAZ(3),VAX(3),VAY(3)
     1,VAZ(3),VPAX(3),VPAY(3),VPAZ(3)
     1,GG(6)
      DIMENSION BINV(15)
      DIMENSION H(12)
      DIMENSION GJ(12),FG1(8),FG2(8),XG1(8),YG1(8),ZG1(8),XG2(8),
     1YG2(8),ZG2(8),UGX(8),UGY(8),UGZ(8),VGX(8),VGY(8),VGZ(8)
      DIMENSION XN(12),AN(8),RAX(8),PPN(8),VNX(8),VNY(8),VNZ(8),XN1(8) ,
     1YN1(8),ZN1(8),XN2(8),YN2(8),ZN2(8),PNEM(8),EQP(8)
      DIMENSION R(12),RR(16),XR1(16),YR1(16),ZR1(16),XR2(16),YR2(16),
     1ZR2(16),URX(16),URY(16),URZ(16),VRX(16),VRY(16),VRZ(16),TE(16),
     1TF(16),TRK(32)
      DIMENSION SIOQ(12)
      DIMENSION S(12),XS1(8),YS1(8),ZS1(8),XS2(8),YS2(8),ZS2(8),USX(8),
     1 USY(8),USZ(8),SD(8),SK(8)
      DIMENSION W(12),WD(8),UK(8),XU1(8),YU1(8),ZU1(8),XU2(8),YU2(8),
     1  ZU2(8)
      DIMENSION Q(12)
      COMMON XX1,XY1,XZ1,YY1,YZ1,ZZ1,XX2,XY2,XZ2,YY2,YZ2,ZZ2,
     1FM10,FM20,DEL,TPR,TD,EX1,EY1,EZ1,EX2,EY2,EZ2,WX,WY,WZ,
     2W1YD,ICD,IPR,IAC,IHY,ICG,IPN,IRK,ISP,ISU,ICN
      COMMON DF,D,XC,YC ,ZC,XC1,XC2,YC1,YC2,ZC1,ZC2
      COMMON PI,TP,UX,UY,UZ,XP,YP,ZP,UPX,UPY,UPZ,XP1,XP2,YP1,YP2,
     1ZP1,ZP2
      COMMON TA,TB,TC,TD,IAC1,IAC2,MAC,NAC,IGLAG,GA,PA,VA,PAD,
     1VAD,SGX,SGY,SGZ,UX,UY,UZ,PUX,PUY,PUZ,VAX,VPAX,VAY,
     2VPAY,VAZ,VPAZ,UAX,UPAX,UAY,UPAY,UAZ,UPAZ,GG
      COMMON HD,HK,HETA,PPH,DHB,AH,PHBO,VHB,GAMH,JH,DEQP,HYDA,
     1HYDB,HYDC,HETA,DA,HYDD,HYDE,DB
      COMMON GJ,GST,FG3,IJLAG,MAXES,NAXES,FG1,FG2,TCO,TCG,
     1UX,UY,UZ,XG,YG,ZG,XG1,XG2,YG1,YG2,ZG1,ZG2,VGX,VGY,
     2VGZ,UGX,UGY,UGZ
      COMMON XN,JN,PBO,VBN,GAMN,AN,RAX,PPN,XXN,YN,ZN,VNX,VNY,
```

```
      GENERAL INPUT

     1VNZ,XN1,XN2,YN1,YN2,ZN1,ZN2,EQP,PPN,PNEM
      COMMON R,VRX,VRY,VRZ,URX,URY,URZ,TE,TF,TROC,SUN1,
     1SUN2,SUN3,SAN1,SAN2,SAN3,XX10,YY10,ZZ10,XX20,YY20,ZZ20,
     2FMD1,FMD2,RR,XR,YR,ZR,XR1,XR2 ,YR1,YR2,ZR1,ZR2,RUM,PUM,
     3TRK
      COMMON SIOQ
      COMMON S,SETA,XMU,EPSLN,DELTS,XS1,XS2,YS1,YS2,ZS1,ZS2,
     1SO,SK
      COMMON W,ETA,WO,UK,XU,YU,ZU,XU1,XU2,YU1,YU2,ZU1,ZU2,TG
      DIMENSION WIOQ(8)
      COMMON WIOQ
       COMMON DM,DN,R1,R2,P3,P21,P22,P23,ZTEST,F,CA,XI1,YI1,ZI1,XI2,
     XYI2,ZI2,USX,USY,USZ
      COMMON XP,YP,ZP,XI1,XI2,ZI1,ZI2,ICLAG,TRIG2,ELM,LIRK,N,XS,YS,ZS
      COMMON WL
      DIMENSION WL(8)
      DIMENSION F(15,15),CA(3)
      ICLAG=1
  449 FORMAT (1H0,28HBODY 1-FIXED AXES     X  Y  Z)
  450 FORMAT (1H0,28HBODY 2-FIXED AXES     X  Y  Z)
  451 FORMAT (1H ,30HDRAWING BOARD AXES    X  Z -Y)
      READ INPUT TAPE 5,1011,(G(I),I=1,12)
 1011 FORMAT(12A6)
      READ INPUT TAPE 5,1012,WIL1,IL1,IHY
 1012 FORMAT(E12.8,2I12)
      READ INPUT TAPE 5,1,XX1,XY1,XZ1,YY1,YZ1,ZZ1
      READ INPUT TAPE 5,1,XX2,XY2,XZ2,YY2,YZ2,ZZ2
      READ INPUT TAPE 5,1,FM10,FM20,DEL,TPR,TD
      READ INPUT TAPE 5,1,EX1,EY1,EZ1,EX2,EY2,EZ2
      READ INPUT TAPE 5,1,WX,WY,WZ,W1YO
      READ INPUT TAPE 5,2,ICD,IPR,ICG,IPN,IRK,ISP
      READ INPUT TAPE 5,2,ISU,ICN,IAC
      WRITE OUTPUT TAPE 6,308,(G(I),I=1,12),IL1
  308 FORMAT(1H1,12A6/5H0CASE,I5/11H0INPUT DATA/)
      WRITE OUTPUT TAPE 6,309,DEL,TPR,TD,W1YO
  309 FORMAT(1H0,13HDELTA TIME = E10.3, 4H SEC,3X,13HDELTA PRINT=
     1E10.3,4H SEC,3X,13HTIME(INITIAL)E10.3,4H SEC,3X,13HSPIN-UP RATE
     1E10.3, 8H DEG/SEC    )
      J=1
      WRITE OUTPUT TAPE 6,310,J,FM10
  310 FORMAT(1H0,14HINPUT FOR BODY I1,5X,7HMASS = E15.7,4H LBM)
      WRITE OUTPUT TAPE 6,311
  311 FORMAT(1H0,10X,25HINERTIA MATRIX LBM  IN SQ ,15X,19HRATES ABOUT DR
     1AWING, 6X,17HCOORDS IN DRAWING / 51X,18HBOARD AXIS DEG/SEC,7X,
     121HBOARD CS OF CM-INCHES   )
      WRITE OUTPUT TAPE 6,312,XX1,YY1,WX,EX1,XY1,YZ1,WY,EY1,XZ1,ZZ1,WZ,
     1 EZ1
  312 FORMAT(1H ,5HIXX  E15.7,5X,5HIYY  E15.7,5X,5HX    E15.7,5X,5HX
     1    E15.7/6H IXY  E15.7,5X,5HIYZ  E15.7,5X,5HY    E15.7,5X,5HY
     1    E15.7/6H IXZ  E15.7,5X,5HIZZ  E15.7,5X,5HZ    E15.7,5X,5HZ
     1    E15.7 //)
      J=2
      WRITE OUTPUT TAPE 6,310,J,FM20
      WRITE OUTPUT TAPE 6,311
      WRITE OUTPUT TAPE 6,312,XX2,YY2,WX,EX2,XY2,YZ2,WY,EY2,XZ2,ZZ2,WZ,
     1 EZ2
```

```
      GENERAL INPUT

      WRITE OUTPUT TAPE 6,313,ICD,IPR         ,ICG,IPN,IRK,ISP,ISU,ICN
  313 FORMAT(1H ,6HICD = I1,4X,6HIPR = I1,4X
     16HICG = I1,4X,6HIPN = I1,4X,6HIRK = I1,4X,6HISP = I1,4X,6HISU = I1
     1,4X,6HICN = I1 ////)
      WRITE OUTPUT TAPE 6,449
      WRITE OUTPUT TAPE 6,451
      WRITE OUTPUT TAPE 6,450
      WRITE OUTPUT TAPE 6,451
      WRITE OUTPUT TAPE 6,845
  845 FORMAT(1H1)
      DO 3 K=1,12
      DD(K)=0.0
    3 CONTINUE
      IF (ICD) 5000,4,5
C
C     COULOMB DRAG INPUT
    5 WRITE OUTPUT TAPE 6,314
  314 FORMAT(1H0,10HCOULOMB DRAG INPUT /5X,8HD(I)-LBF,13X,8HDF(I)-IN,18X
     1 ,40HDRAWING BOARD LOCATIONS OF DRAG FORCE-IN /49X,1HX,20X,1HY,
     1 20X,1HZ / )
      DO 6 I=1,ICD
      READ INPUT TAPE 5,1,DF(I),D(I),XC,YC,ZC
      WRITE OUTPUT TAPE 6,315,D(I),DF(I),XC,YC,ZC
  315 FORMAT(1H ,E15.7,6X,E15.7,6X,E15.7,6X,E15.7,6X,E15.7 )
      XC1(I)=XC-EX1
      XC2(I)=XC-EX2
      YC1(I)=ZC-EZ1
      YC2(I)=ZC-EZ2
      ZC1(I)=EY1-YC
      ZC2(I)=EY2-YC
    6 CONTINUE
C
C     PYROTECHNIC INPUT
    4 DO 7 K=1,12
      EP(K)=0.0
    7 CONTINUE
      IF (IPR) 5000,18,9
    9 WRITE OUTPUT TAPE 6,354
      TP(2)=0.0
      TP(3)=0.0
  354 FORMAT(1H0,17HPYROTECHNIC INPUT / 11X,37HCOSINE DRAW BOARD AXIS AN
     1D ITH DEVICE,30X,24HPYROTECHNIC LOCATIONS-IN / 6X,2HUX,19X,2HUY,
     119X,2HUZ,20X,1HX,20X,1HY,20X,1HZ  )
      DO 10 I=1,IPR
      READ INPUT TAPE 5,1,PI(I),TP(I)
      READ INPUT TAPE 5,1,UX,UY,UZ,XP,YP,ZP
      WRITE OUTPUT TAPE 6,355,UX,UY,UZ,XP,YP,ZP
  355 FORMAT(1H ,E15.7,6X,E15.7,6X,E15.7,6X,E15.7,6X,E15.7,6X,E15.7 )
      UPX(I)=UX
      UPY(I)=UZ
      UPZ(I)=-UY
      XP1(I)=XP-EX1
      XP2(I)=XP-EX2
      YP1(I)=ZP-EZ1
      YP2(I)=ZP-EZ2
      ZP1(I)=EY1-YP
```

```
      GENERAL INPUT

      ZP2(I)=EY2-YP
   10 CONTINUE
      WRITE OUTPUT TAPE 6,356
  356 FORMAT(1H ,1X,13HTOTAL IMPULSE,9X,11HFIRING TIME /  1X,
     111HITH DEVICE-LBF SEC,5X,12HITH PYRO-SEC )
      DO 357 I=1,IPR
      WRITE OUTPUT TAPE 6,355,PI(I),TP(I)
      PI(I)=PI(I)/DEL
  357 CONTINUE
C
C     COLD GAS JET INPUT
   18 DO 20 K=1,12
      GJ(K)=0.0
   20 CONTINUE
      IF (ICG) 5000,21,22
   22 READ INPUT TAPE 5,1,GST,FG3
  723 READ INPUT TAPE 5,2,IJLAG,K
      WRITE OUTPUT TAPE 6,369,FG3,GST
      WRITE OUTPUT TAPE 6,370,IJLAG
      IF (IJLAG-1) 720,721,720
  721 NAXES=1
      NAXES=K
      GO TO 722
  369 FORMAT(1H0,18HCOLD GAS JET INPUT /4H F3 E15.7,5X,28HVALUE JET FORC
     1E BECOMES ZERO  E15.7,4H LBF )
  370 FORMAT(1H ,8X,40HCOSINE ANGLE THRUST AND DRAW BOARD-BODY  I1,20X,
     125HCOLD GAS JET LOCATIONS-IN /
     16X,2HUX,19X,2HUY,19X,2HUZ,20X,1HX,20X,1HY,20X,1HZ )
  720 NAXES=ICG-K+1
      NAXES=ICG
  722 DO 23 I=NAXES,NAXES
      READ INPUT TAPE 5,1,FG1(I),FG2(I),TCO(I),TCS(I)
      READ INPUT TAPE 5,1,UX,UY,UZ,XG,YG,ZG
      WRITE OUTPUT TAPE 6,355,UX,UY,UZ,XG,YG,ZG
      XG1(I)=XG-EX1
      XG2(I)=XG-EX2
      YG1(I)=ZG-EZ1
      YG2(I)=ZG-EZ2
      ZG1(I)=EY1-YG
      ZG2(I)=EY2-YG
      IF (IJLAG-1) 24,25,24
   24 VGX(I)=UX
      VGY(I)=UZ
      VGZ(I)=-UY
      UGX(I)=0.0
      UGY(I)=0.0
      UGZ(I)=0.0
      GO TO 23
   25 VGX(I)=0.0
      VGY(I)=0.0
      VGZ(I)=0.0
      UGX(I)=UX
      UGY(I)=UZ
      UGZ(I)=-UY
   23 CONTINUE
      IF (NAXES-ICG)  723,8001,8001
```

```
      GENERAL INPUT

      SUN2=0.0
      SUN3=0.0
      SAN1=0.0
      SAN2=0.0
      SAN3=0.0
      DO 3024 I=1,IRK
      READ INPUT TAPE 5,1,UX,UY,UZ,VX,VY,VZ
      WRITE OUTPUT TAPE 6,355,UX,UY,UZ,VX,VY,VZ
      SUN1=SUN1+UX
      SUN2=SUN2+UY
      SUN3=SUN3+UZ
      SAN1=SAN1+VX
      SAN2=SAN2+VY
      SAN3=SAN3+VZ
 3024 CONTINUE
      XX10=XX1
      YY10=YY1
      ZZ10=ZZ1
      XX20=XX2
      YY20=YY2
      ZZ20=ZZ2
      WRITE OUTPUT TAPE 6,379
  379 FORMAT(1H ,6X,24HMASS FLOW RATE    LBM/SEC,10X,16HTHRUST MAGNITUDE,
     1 18X,16HROCKET LOCATIONS / 1H ,
     1 4X,6HBODY 1 ,15X,6HBODY 2 ,11X,3HLBF,19X,1HX,20X,1HY,20X,1HZ )
      RUM=0.0
      PUM=0.0
      DO 380 I=1,IRK
      READ INPUT TAPE 5,1,FMD1,FMD2,RR(I),XR,YR,ZR
      WRITE OUTPUT TAPE 6,355,FMD1,FMD2,RR(I),XR,YR,ZR
      XR1(I)=XR-EX1
      XR2(I)=XR-EX2
      YR1(I)=ZR-EZ1
      YR2(I)=ZR-EZ2
      ZR1(I)=EY1-YR
      ZR2(I)=EY2-YR
      RUM=RUM+FMD1
      PUM=PUM+FMD2
  380 CONTINUE
      DO 3025 I=1,IRK
      J=2*I-1
      TRK(J)=TE(I)
      TRK(J+1)=TF(I)
 3025 CONTINUE
      J=IRK+IRK
      DO 3026 I=1,J
      K=I
 3027 K=K+1
      IF (K-J  ) 3029,3029,3026
 3029 IF (TRK(I)-TRK(K)) 3027,3027,3028
 3028 TEMP=TRK(K)
      TRK(K)=TRK(I)
      TRK(I)=TEMP
      GO TO 3029
 3026 CONTINUE
      LIRK=1
```

```
      GENERAL INPUT
C
C     SPRING INPUT
   31 DO 36 K=1,12
      S(K)=0.0
   36 CONTINUE
      IF (ISP) 5000,37,38
   38 READ INPUT TAPE 5,1,SETA,XMU,EPSLN,DELTS
      IQ=1
      READ INPUT TAPE 5,2,N
      IF(N-2) 6551,6552,6552
 6552 ICLAG=-1
 6551 CONTINUE
      WRITE OUTPUT TAPE 6,381,SETA,N
  381 FORMAT(1H0,12HSPRING INPUT,/,2X,10HEFFICIENCY,5X,
     114HRESTRICT INDEX,/
     1E15.7,I10)
      SETA=SETA*SETA
      WRITE OUTPUT TAPE 6,382
  382 FORMAT(1H ,15X,28HCOSINE DRAW BOARD AND SPRING,39X,19HSPRING LOCAT
     1IONS-IN /6X,2HUX,19X,2HUY,19X,2HUZ,20X,1HX,20X,1HY,20X,1HZ )
      DO 39 I=1,ISP
      READ INPUT TAPE 5,1,UX,UY,UZ,XS,YS,ZS
      WRITE OUTPUT TAPE 6,355,UX,UY,UZ,XS,YS,ZS
      XS1(I)=XS-EX1
      XS2(I)=XS-EX2
      YS1(I)=ZS-EZ1
      YS2(I)=ZS-EZ2
      ZS1(I)=EY1-YS
      ZS2(I)=EY2-YS
      USX(I)=UX
      USY(I)=UZ
      USZ(I)=-UY
      READ INPUT TAPE 5,1,SO(I),SK(I),SIOQ(I)
   39 CONTINUE
      WRITE OUTPUT TAPE 6,383
  383 FORMAT(1H ,1X,13HINITIAL FORCE,7X,15HSPRING CONSTANT,
     17X,14HRESIDUAL FORCE/,
     26X,3HLBF,17X,6HLBF/IN,14X,3HLBF)
      DO 384 I=1,ISP
      WRITE OUTPUT TAPE 6,355,SO(I),SK(I),SIOQ(I)
      SK(I)=SK(I)*SETA
      SO(I)=SO(I)*SETA
      SIOQ(I)=SIOQ(I)*SETA
  384 CONTINUE
C
C     UNIVERSALLY JOINTED SPRING INPUT
   37 DO 40 K=1,12
      W(K)=0.0
   40 CONTINUE
      IF (ISU) 5000,41,42
   42 READ INPUT TAPE 5,1,ETA
      WRITE OUTPUT TAPE 6,385,ETA
  385 FORMAT(1H0,32HUNIVERSALLY JOINTED SPRING INPUT / 1X,
     118HEFFICIENCY FACTOR E15.7 /
     117X,23HLOCATIONS ITH SPRING-IN/,
     17X,2HX1,19X,2HY1,19X,2HZ1,19X,2HX2,19X,2HY2,19X,2HZ2)
```

```
      GENERAL INPUT

      ETA=ETA*ETA
      DO 43 I=1,ISU
      READ INPUT TAPE 5,1,XU,YU,ZU,XUD,YUD,ZUD
      WRITE OUTPUT TAPE 6,355,XU,YU,ZU,XUD,YUD,ZUD
      READ INPUT TAPE 5,1,WO(I),UK(I),WIOQ(I)
      WIOQ(I)=WIOQ(I)*ETA
      UK(I)=UK(I)* ETA
      WO(I)=WO(I)* ETA
      XU1(I)=XU-EX1
      XU2(I)=XUD-EX2
      YU1(I)=ZU-EZ1
      YU2(I)=ZUD-EZ2
      ZU1(I)=EY1-YU
      ZU2(I)=EY2-YUD
   43 CONTINUE
      TG=0
      WRITE OUTPUT TAPE 6,7312
 7312 FORMAT(16HO  INITIAL FORCE,9X,11HSPRING RATE,7X,14HRESIDUAL FORCE/
     1,8X,3HLBF,16X,6HLBF/IN,16X,3HLBF)
      DO 7311 I=1,ISU
 7311 WRITE OUTPUT TAPE 6,355  ,WO(I),UK(I),WIOQ(I)
C
   41 IF(ICN)5000,44,45
   45 READ INPUT TAPE 5,1,TG,      XP,YP,ZP
      IQ=-1
      WRITE OUTPUT TAPE 6,386,XP,YP,ZP,TG
  386 FORMAT(1HO,16HPIN-PULLER INPUT / 13X,30HLOCATION DELAYED PIN PULLE
     1R-IN,23X,11HFIRING TIME ,10X               /7X,1HX,20X,1HY,20X,
     11HZ,19X,3HSEC,18X/1X,
     1                         E15.7,6X,E15.7,6X,E15.7,6X,
     1 E15.7,6X,E15.7)
      XI1=XP-EX1
      XI2=XP-EX2
      YI1=ZP-EZ1
      YI2=ZP-EZ2
      ZI1=EY1-YP
      ZI2=EY2-YP
      ICLAG=0
      TRIG2=TG
   44 CONTINUE
   46 CALL CHAIN(2,3)
C
C     ERROR
 5000 WRITE OUTPUT TAPE 6,5001
 5001 FORMAT(1HO,16HERROR INPUT DATA )
 4000 CALL EXIT
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

SUBROUTINE SW(I,J,A)

```
SUBROUTINE SW(I,J,A)
DIMENSION A(1)
B=A(I)
A(I)=A(J)
A(J)=B
RETURN
END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```
 DIMENSION BA(3,3)     ,WR(3),WT1(3),WT2(3)   ,WT3(3)

 DIMENSION BA(3,3)     ,WR(3),WT1(3),WT2(3)   ,WT3(3)
 DIMENSION FOIE(8)
 EQUIVALENCE (T,T),(ZERO,ZERO),
1            (X1,V1(1)),(Y1,V1(2)),(Z1,V1(3)),(T1,V1(4)),(T2,V1(5))
1,(P1,V1(6)),(P2,V1(7)),(S1,V1(8)),(S2,V1(9)),(X1X,V1(10)),(Y1Y,V1(
111)),(Z1Z,V1(12)),(T1T,V1(13)),(T2T,V1(14)),(P1P,V1(15)),(P2P,V1(1
16)),(S1S,V1(17)),(S2S,V1(18)),(X2,V2(1)),(Y2,V2(2)),(Z2,V2(3)),
1(X2X,V2(4)),(Y2Y,V2(5)),(Z2Z,V2(6))
 COMMON T,ZERO,V1,V2,YDOT1,YDOT2
 COMMON A,B,AD,BD
 DIMENSION              YDOT2(6),YDOT1(18),V2(6),V1(18),NH(6),HB(6)
 DIMENSION A(3,3,1),B(3,3),AD(3,3),BD(3,3)
 DIMENSION DM(3,3),DN(3,3)
 COMMON WIL1,TCO
 DIMENSION TCG(8),TCO(8)
 DIMENSION FM(3,4),FEL(3),AONST(3),CM(3,3),DUD(3)
 DIMENSION DD(12),DF(8),D(8),XC1(8),YC1(8),ZC1(8),XC2(8),YC2(8),
1ZC2(8)
 DIMENSION EP(12),PI(3),TP(6),XP1(3),YP1(3),ZP1(3),XP2(3),YP2(3),
1ZP2(3),UPX(3),UPY(3),UPZ(3)
 DIMENSION G(12),GA(6),PA(6),VA(6),PAD(6),VAD(6),SGX(6),SGY(6),
1SGZ(6),UAX(3),UAY(3),UAZ(3),UPAX(3),UPAY(3),UPAZ(3),VAX(3),VAY(3)
1,VAZ(3),VPAX(3),VPAY(3),VPAZ(3)
1,GG(6)
 DIMENSION BINV(15)
 DIMENSION H(12)
 DIMENSION GJ(12),FG1(8),FG2(8),XG1(8),YG1(8),ZG1(8),XG2(8),
1YG2(8),ZG2(8),UGX(8),UGY(8),UGZ(8),VGX(8),VGY(8),VGZ(8)
 DIMENSION XN(12),AN(8),RAX(8),PPN(8),VNX(8),VNY(8),VNZ(8),XN1(8)  ,
1YN1(8),ZN1(8),XN2(8),YN2(8),ZN2(8),PNEM(8),EQP(8)
 DIMENSION R(12),RR(16),XR1(16),YR1(16),ZR1(16),XR2(16),YR2(16),
1ZR2(16),URX(16),URY(16),URZ(16),VRX(16),VRY(16),VRZ(16),TE(16),
1TF(16),TRK(32)
 DIMENSION SIOQ(12)
 DIMENSION S(12),XS1(8),YS1(8),ZS1(8),XS2(8),YS2(8),ZS2(8),USX(8),
1 USY(8),USZ(8),SO(8),SK(8)
 DIMENSION W(12),WO(8),UK(8),XU1(8),YU1(8),ZU1(8),XU2(8),YU2(8),
1  ZU2(8)
 DIMENSION Q(12)
 COMMON XX1,XY1,XZ1,YY1,YZ1,ZZ1,XX2,XY2,XZ2,YY2,YZ2,ZZ2,
1FM10,FM20,DEL,TPR,TO,EX1,EY1,EZ1,EX2,EY2,EZ2,WX,WY,WZ,
2W1YO,ICD,IPR,IAC,IHY,ICG,IPN,IRK,ISP,ISU,ICN
 COMMON DF,D,XC,YC ,ZC,XC1,XC2,YC1,YC2,ZC1,ZC2
 COMMON PI,TP,UX,UY,UZ,XP,YP,ZP,UPX,UPY,UPZ,XP1,XP2,YP1,YP2,
1ZP1,ZP2
 COMMON TA,TB,TC,TD,IAC1,IAC2,MAC,NAC,IGLAG,GA,PA,VA,PAD,
1VAD,SGX,SGY,SGZ,UX,UY,UZ,PUX,PUY,PUZ,VAX,VPAX,VAY,
2VPAY,VAZ,VPAZ,UAX,UPAX,UAY,UPAY,UAZ,UPAZ,GG
 COMMON HO,HK,HETA,PPH,DHB,AH,PHBO,VHB,GAMH,JH,DEQP,HYDA,
1HYDB,HYDC,HETA,DA,HYDD,HYDE,DB
 COMMON GJ,GST,FG3,IJLAG,MAXES,NAXES,FG1,FG2,TCO,TCG,
1UX,UY,UZ,XG,YG,ZG,XG1,XG2,YG1,YG2,ZG1,ZG2,VGX,VGY,
2VGZ,UGX,UGY,UGZ
 COMMON XN,JN,PBO,VBN,GAMN,AN,RAX,PPN,XXN,YN,ZN,VNX,VNY,
1VNZ,XN1,XN2,YN1,YN2,ZN1,ZN2,EQP,PPN,PNEM
 COMMON R,VRX,VRY,VRZ,URX,URY,URZ,TE,TF,TROC,SUN1,
```

```
      DIMENSION BA(3,3)     ,WR(3),WT1(3),WT2(3)   ,WT3(3)
     1SUN2,SUN3,SAN1,SAN2,SAN3,XX10,YY10,ZZ10,XX20,YY20,ZZ20,
     2FMD1,FMD2,RR,XR,YR,ZR,XR1,XR2 ,YR1,YR2,ZR1,ZR2,RUM,PUM,
     3TRK
      COMMON SIOQ
      COMMON S,SETA,XMU,EPSLN,DELTS,XS1,XS2,YS1,YS2,ZS1,ZS2,
     1SD,SK
      COMMON W,ETA,WD,UK,XU,YU,ZU,XU1,XU2,YU1,YU2,ZU1,ZU2,TG
      DIMENSION WIOQ(8)
      COMMON WIOQ
       COMMON DM,DN,R1,R2,P3,P21,P22,P23,ZTEST,F,CA,XI1,YI1,ZI1,XI2,
     XYI2,ZI2,USX,USY,USZ
      COMMON          XI1,XI2,ZI1,ZI2,ICLAG,TRIG2,ELM,LIRK,N,XS,YS,ZS
      COMMON WL
      COMMON SAVE,BINV,E,NH,HB,H,Q
      COMMON FQQ1,FQQ2,FQQ3
      DIMENSION WRP1(3,8),WRP2(3,8),WRH(3,8)
      COMMON WRP1,WRP2,WRH
      DIMENSION ISP1(8)
      DIMENSION ISS(2),FJ(8),WPP1(3)
    2 FORMAT(6I12)
    1 FORMAT(6E12.8)
      DO 5812 I=1,ISP
      WRP1(1,I)=XS1(I)
      WRP1(2,I)=YS1(I)
      WRP1(3,I)=ZS1(I)
      WRP2(1,I)=XS2(I)
      WRP2(2,I)=YS2(I)
      WRP2(3,I)=ZS2(I)
      WRH(1,I)=USX(I)
      WRH(2,I)=USY(I)
      WRH(3,I)=USZ(I)
 5812 CONTINUE
      IF(WIL1)6675,6675,6676
 6675 WIL1=-WIL1
      IFORCE=-1
      GO TO 6677
 6676 IFORCE=0
 6677 CONTINUE
      FSHT=0.
      IF(IAC)54,55,54
   54 FSHT=-FM10
   55 CONTINUE
      IQ=0
      TRIG1=TD
      TRIG2=TG
      CALL TRMOD (7,0)
      CALL TRMOD(0,0)
      CALL TRMOD(1,0)
      CALL TRMOD(2,1)
      CALL TRMOD (3,0)
      CALL TRMOD(4,0)
      CALL TRMOD (5,0)
      CALL TRMOD (6,0)
      IF(ICG)8005,8004,8005
 8005 TRIG3=TCO(1)
      CALL TRMOD(0,1)
```

```
      DIMENSION BA(3,3)      ,WR(3),WT1(3),WT2(3)    ,WT3(3)
      ICOLD=1
 8004 CONTINUE
      IF(IRK)5000,31,32
   32 CALL TRMOD(4,1)
      LIRK=1
   31 CONTINUE
      IF(ICN)5000,44,45
   45 CALL TRMOD(1,1)
   44 CONTINUE
      IF(IPR)46,46,47
   47 IF(IPR-2)469,470,471
  471 CALL TRMOD (6,1)
      PYRO3=TP(3)-DEL/2.0
      TP(3)=PYRO3
      TP(6)=TP(3)+DEL
  470 CALL TRMOD (5,1)
      PYRO2=TP(2)-DEL/2.0
      TP(2)=PYRO2
      TP(5)=TP(2)+DEL
  469 CALL TRMOD (3,1)
      TRIG4=TP(1)-DEL/2.0
      TP(1)=TRIG4
      TP(4)=TP(1)+DEL
C
   46 TEMP=XZ1
      READ INPUT TAPE 5,1,TPLOT
      IF (TPLOT) 705,704,705
  705 NT2=8
      REWIND NT2
      PLOT=TO
      CALL TRMOD (7,1)
      XPTS=0.0
      CALL CAMERA(18,1)
      CALL SET(500)
      WRITE OUTPUT TAPE 38,6263,(WL(I),I=1,8)
 6263 FORMAT(40X,8A6)
  704 XZ1=-XY1
      WRITE OUTPUT TAPE 6,899
  899 FORMAT(1H1)
      XY1=TEMP
      TEMP=ZZ1
      ZZ1=YY1
      YY1=TEMP
      TEMP=XZ2
      XZ2=-XY2
      XY2=TEMP
      TEMP=ZZ2
      ZZ2=YY2
      YY2=TEMP
      YZ1=-YZ1
      YZ2=-YZ2
      DIMENSION CBQ(3),QAQ1(3,3,3)
C     INERTIA MATRIX TRANSFORMED
      MAXES=1
      NAXES=1
      ELK=57.2957795131
```

```
      DIMENSION BA(3,3)     ,WR(3),WT1(3),WT2(3)   ,WT3(3)
      TEMP=WZ/ELK
      WZ=-WY/ELK
      WY=TEMP
      WX=WX/ELK
C     INITIAL VALUES
      DOL=DEL
      IMOD=0
      LAM=0
      L1=0
      L2=0
      L3=0
      LX=25
      SUM=FM10+FM20
      FM1=FM10
      FM2=FM20
      ECX=(FM1*EX1+FM2*EX2)/SUM
      ECY=(FM1*EY1+FM2*EY2)/SUM
      ECZ=(FM1*EZ1+FM2*EZ2)/SUM
      YMU=0.99*XMU
      IFINI=0
      JKUT=0
      LKUT=0
      HB(1)=DEL
      HB(2)=1.0
      HB(4)=MIN1F(.01,TPR*.2)
      HB(3)=HB(4)*.08
      HB(5)=1.0E-8
      HB(6)=1.0E-6
      DO 511 K=1,6
      NH(K)=0.0
  511 CONTINUE
      NH(5)=4
      DIMENSION CA(3),E(3,3),X(3)
     1F(15,15)  ,WL(30)
      NH(1)=24
   78 NH(2)=24
   80 V1(4)=1.5707963
      V1(5)=V1(4)
      V1(6)=0.0
      V1(7)=0.0
      V1(8)=0.0
      V1(9)=0.0
      W1YO=W1YO/ELK
      V1(13)=WX
      V1(14)=WX
      V1(15)=WY+W1YO
      V1(16)=WY
      V1(17)=WZ
      V1(18)=WZ
      IPHI=4
      CONST=385.7
      JIK=1
      ITER=1
      LOW=0
      IILAG=0
      X1O=EX1-ECX
```

```
      DIMENSION BA(3,3)      ,WR(3),WT1(3),WT2(3)   ,WT3(3)
      X20=EX2-ECX
      Y10=EY1-ECY
      Y20=EY2-ECY
      Z10=EZ1-ECZ
      Z20=EZ2-ECZ
      V1(1)=X10
      V2(1)=X20
      V1(2)=Y10
      V2(2)=Y20
      V1(3)=Z10
      V2(3)=Z20
      V1(10)=-WZ*V1(3)-(WY+W1Y0)*V1(2)
      V1(11)=(WY+W1Y0)*V1(1)-WX*V1(3)
      V1(12)=WX*V1(2)+WZ*V1(1)
      V2(4)=-WZ*V2(3)-WY*V2(2)
      V2(5)=WY*V2(1)-WX*V2(3)
      V2(6)=WX*V2(2)+WZ*V2(1)
      DO 512 I=1,9
      YDOT1(I)=V1(I+9)
      YDOT1(I+9)=0.0
  512 CONTINUE
      DO 513 I=1,3
      YDOT2(I)=V2(I+3)
      YDOT2(I+3)=0.0
  513 CONTINUE
      IVP=0
      T=T0
      ZERO=0.0
      IKUT=0
      SMALL=0.0001
      DIMENSION CX(3),CZ(3),SAVE(68),CB(3),CC(3),GAMMA(3)
      HLL=180.0/3.14159265
      SM=0.0
  279 DO 277 J=1,18
      SAVE(J)=V1(J)
      SAVE(J+24)=YDOT1(J)
  277 CONTINUE
      DO 278 J=1,6
      SAVE(J+61)=GA(J)
      SAVE(J+54)=NH(J)
      SAVE(J+48)=HB(J)
      SAVE(J+18)=V2(J)
      SAVE(J+42)=YDOT2(J)
  278 CONTINUE
      SAVE(61)=T
      ZTEST=0.64E-07
      LDER=1
      IWIL1=1
      IXFT=0
      HAP=DEL*.5
      FMAS1=FM1/CONST
      FMAS2=FM2/CONST
      XX10=XX1
      YY10=YY1
      ZZ10=ZZ1
      XX20=XX2
```

```
      DIMENSION BA(3,3)     ,WR(3),WT1(3),WT2(3)   ,WT3(3)

      YY20=YY2
      ZZ20=ZZ2
      ICD1=0
      ICG1=0
      IPN1=0
      ISP1=0
      ISU1=0
      ISS(1)=0
      ISS(2)=0
C
C     DER1
C
C     DER2
C
 1001 CALL GET
 3032 IF(ICD) 50,51,50
   50 CALL DRAG(D,DF,DD,XC1,YC1,ZC1,XC2,YC2,ZC2,ICD,LDER,ICD1)
   51 IF (IPR) 52,57,52
   52 CALL PYRO(PI,TP,XP1,YP1,ZP1,XP2,YP2,ZP2,UPX,UPY,UPZ,EP,IPR)
   57 IF(ICG) 58,59,58
   58 CALL COLGAS(ICG,GJ,FG1,FG2,FG3,UGX,UGY,UGZ,VGX,VGY,VGZ,XG1,YG1,
     1ZG1,XG2,YG2,ZG2,GST,TCG,LDER,ICG1)
   59 IF(IPN) 60,61,60
   60 CALL PNEUM(IPN,JN,PBO,VBN,GAMN,AN,RAX,VNX,VNY,VNZ,XN1,YN1,ZN1,XN2,
     1YN2,ZN2,XN,PPN,EQP,PNEM,LDER,IPN1)
   61 IF(IRK) 62,63,62
   62 CALL ROCKET(IRK,R,RR,TE,TF,RUM,PUM,FM1,FM2,URX,URY,URZ,FM10,FM20,X
     1R1,YR1,ZR1,XR2,YR2,ZR2,VRX,VRY,VRZ,SUN1,SUN2,SUN3,SAN1,SAN2,SAN3,
     2XX10,YY10,ZZ10,XX20,YY20,ZZ20,XX1,YY1,ZZ1,XX2,YY2,ZZ2)
   63 IF(ISP) 66,67,66
   66 CALL SPRING(WRP1,WRP2,WRH,SO,SK,SIOQ,ISP,N,ICLAG,S,FJ,LDER,ISP1,
     1ISS)
   67 IF(ISU) 68,69,68
   68 CALL UNIVSL(ISU,WO,UK,WIOF,W,XU1,YU1,ZU1,XU2,YU2,ZU2,LDER,ISU1)
   69 DO 70 K=1,12
      Q(K)=DD(K)+EP(K)+GJ(K)+XN(K)+R(K)+S(K)+W(K)
   70 CONTINUE
      Q(3)=Q(3)+FSHT
      R1=R1+Q(7)*CONST
      R2=R2+Q(9)*CONST
      P3=P3+Q(11)*CONST
      P21=P21+Q(8)*CONST
      P22=P22+Q(10)*CONST
      P23=P23+Q(12)*CONST
  414 IF(ICLAG)74,74,75
   74 CONTINUE
      F(1,1)=FM1
      F(2,2)=FM1
      F(3,3)=FM1
      F(7,7)=FM2
      F(8,8)=FM2
      F(9,9)=FM2
 3034 J=15
 6010 FORMAT(8E15.5)
      IN=1
      DQ=(2.)**(-26)
```

```
      DIMENSION BA(3,3)     ,WR(3),WT1(3),WT2(3)   ,WT3(3)
  700 DO 701 I=1,3
      YDOT2(I+3)=0.0
      K=2*I
      WL(I)=Q(K-1)*CONST
      WL(I+6)=Q(K)*CONST
      WL(I+12)=CA(I)
  701 CONTINUE
      WL(4)=R1
      WL(5)=R2
      WL(6)=P3
      WL(10)=P21
      WL(11)=P22
      WL(12)=P23
      CALL SOLVE(15,F,WL,IN,DQ,10,BINV,IN3)
      IF(IN3)6005,6006,6002
C     F MATRIX IS SINGULAR
 6005 WRITE OUTPUT TAPE 6,6003
 6003 FORMAT(22H1 F MATRIX IS SINGULAR)
      CALL DUMP
C     OVERFLOW IN F MATRIX INVERSION
 6006 WRITE OUTPUT TAPE 6,6004
 6004 FORMAT(37H1 F MATRIX INVERSION DID NOT CONVERGE)
      CALL DUMP
 6002 CONTINUE
 3037 DO 702 I=1,9
      YDOT1(I+9)=0.0
  702 CONTINUE
      DO 703 J=1,3
      K=J*2
      YDOT1(K+12)=BINV(J+9)
      YDOT1(J+9)=BINV(J)
      YDOT1(K+11)=BINV(J+3)
      YDOT2(J+3)=BINV(J+6)
  703 CONTINUE
      YDOT2(1)=V2(4)
      YDOT2(2)=V2(5)
      YDOT2(3)=V2(6)
      YDOT1(1)=V1(10)
      YDOT1(2)=V1(11)
      YDOT1(3)=V1(12)
      YDOT1(4)=V1(13)
      YDOT1(5)=V1(14)
      YDOT1(6)=V1(15)
      YDOT1(7)=V1(16)
      YDOT1(8)=V1(17)
      YDOT1(9)=V1(18)
      GO TO 6007
   75 CONST2=CONST/FM2
      YDOT2(4)=Q(2)*CONST2
      YDOT2(5)=Q(4)*CONST2
      YDOT2(6)=Q(6)*CONST2
      YDOT2(1)=V2(4)
      YDOT2(2)=V2(5)
      YDOT2(3)=V2(6)
      D2233=DM(2,2)*DM(3,3)
      D23=DM(2,3)*DM(2,3)
```

```
      DIMENSION BA(3,3)     ,WR(3),WT1(3),WT2(3)   ,WT3(3)
      D1323=DM(1,3)*DM(2,3)
      D1223=DM(1,2)*DM(2,3)
      D1233=DM(1,2)*DM(3,3)
      D1322=DM(1,3)*DM(2,2)
      CONS1=CONST/FM1
      DET=DM(1,1)*(D2233-D23)+DM(1,2)*(D1323-D1233)+DM(1,3)*(D1223-D1322
     1)
      IF (DET) 214,5002,214
  214 YDOT1(10)=Q(1)*CONS1
      YDOT1(11)=Q(3)*CONS1
      YDOT1(12)=Q(5)*CONS1
      YDOT1(2)=V1(11)
      YDOT1(3)=V1(12)
      YDOT1(1)=V1(10)
      YDOT1(13)=R1*(D2233-D23)+R2*(D1323-D1233)+P3*(D1223-D1322)
      YDOT1(13)=YDOT1(13)/DET
      YDOT1(15)=R1*(DM(2,3)*DM(3,1)-DM(2,1)*DM(3,3))
     1          +R2*(DM(1,1)*DM(3,3)-DM(1,3)*DM(1,3))
     1          +P3*(DM(1,3)*DM(2,1)-DM(1,1)*DM(2,3))
      YDOT1(15)=YDOT1(15)/DET
      YDOT1(17)=R1*(DM(2,1)*DM(3,2)-DM(2,2)*DM(3,1))
     1          +R2*(DM(1,2)*DM(3,1)-DM(1,1)*DM(3,2))
     1          +P3*(DM(1,1)*DM(2,2)-DM(1,2)*DM(1,2))
      YDOT1(17)=YDOT1(17)/DET
      D2233=DN(2,2)*DN(3,3)
      D23=DN(2,3)*DN(2,3)
      D1323=DN(1,3)*DN(2,3)
      D1233=DN(1,2)*DN(3,3)
      D1223=DN(1,2)*DN(2,3)
      D1322=DN(1,3)*DN(2,2)
      DET=DN(1,1)*(D2233-D23)+DN(1,2)*(D1323-D1233)+DN(1,3)*(D1223-D1322
     1)
      IF (DET) 213,5002,213
  213 CONTINUE
      YDOT1(14)=P21*(D2233-D23)+P22*(D1323-D1233)+P23*(D1223-D1322)
      YDOT1(14)=YDOT1(14)/DET
      YDOT1(16)=P21*(DN(2,3)*DN(3,1)-DN(2,1)*DN(3,3))
     1          +P22*(DN(1,1)*DN(3,3)-DN(1,3)*DN(1,3))
     1          +P23*(DN(1,3)*DN(2,1)-DN(1,1)*DN(2,3))
      YDOT1(16)=YDOT1(16)/DET
      YDOT1(18)=P21*(DN(2,1)*DN(3,2)-DN(2,2)*DN(3,1))
     1          +P22*(DN(1,2)*DN(3,1)-DN(1,1)*DN(3,2))
     1          +P23*(DN(1,1)*DN(2,2)-DN(1,2)*DN(1,2))
      YDOT1(18)=YDOT1(18)/DET
      YDOT1(4)=V1(13)
      YDOT1(5)=V1(14)
      YDOT1(6)=V1(15)
      YDOT1(7)=V1(16)
      YDOT1(8)=V1(17)
      YDOT1(9)=V1(18)
 6007 CONTINUE
      IF (LDER) 636,636,631
  636 CALL ROUT (0)
  631 LDER=0
C
 3000 CALL FMARK (KIK,HB,NH,IVP,IPHI,1,1,2,2,TRIG3,3,TRIG2,4,TRIG1,
```

```
      DIMENSION BA(3,3)     ,WR(3),WT1(3),WT2(3)   ,WT3(3)

     15,TRIG4,6,TROC,7,PYRO2,8,PYRO3,9,PLOT,D)
      GO TO (1001,1004,1003,1002,1005,1006,1007,1008,1009,5050),KIK
C
C     DER2 STARTS AT 1001
C     PRINT-OUT STARTS AT 1002
C     PIN-PULLER CONSTRAINT OFF AT 1003
C     EOS AND COLD GAS JET AT 1004
C     PYROTECHNIC TRIGGER STARTS AT 1005
C     SECOND PYROTECHNIC AT 1007
C     THIRD PYROTECHNIC AT 1008
C     ROCKET TRIGGER AT 1006
C     PLOT TRIGGER AT 1009
C
 1003 CALL TRMOD (1,D)
      IF(N-2)7337,7334,7334
 7337 ICLAG=1
 7338 FORMAT(30HOPIN-PULLER CONSTRAINT OFF   T=,E15.7)
      WRITE OUTPUT TAPE 6,7338,T
      GO TO 1001
 7334 ICLAG=-1
      WRITE OUTPUT TAPE 6,7333,T
 7333 FORMAT(52HOPIN-PULLER CONSTRAINT OFF   SPRING CONSTRAINT ON   T=
     1E15.7)
      GO TO 1001
 1004 IF(T-TCO(ICOLD))200,8010,200
 8010 IF(ICOLD-ICG) 8006,8007,8006
 8006 ICOLD=ICOLD+1
      TRIG3=TCO(ICOLD)
      LDER=1
      GO TO 1001
 8007 CALL TRMOD(0,D)
      TCO(ICG)=-3.
      LDER=1
      GO TO 1001
C     THETA TEST
C
  200 IF(LDER)6518,6519,6519
 6518 LDER=1
 6519 IF(ISS(1)+ISS(2))6521,6522,6521
 6521 DO 3232 I=1,2
      CT=COSF(V1(I+3))
      ST=SINF(V1(I+3))
      CP=COSF(V1(I+5))
      SP=SINF(V1(I+5))
      CS=COSF(V1(I+7))
      SS=SINF(V1(I+7))
      A(1,1,I)=-CT*SP*SS+CP*CS
      A(1,2,I)=CT*CP*SS+SP*CS
      A(1,3,I)=ST*SS
      A(2,1,I)=-CT*SP*CS-CP*SS
      A(2,2,I)=CT*CP*CS-SP*SS
      A(2,3,I)=ST*CS
      A(3,1,I)=ST*SP
      A(3,2,I)=-ST*CP
 3232 A(3,3,I)=CT
      DO 6523 J=1,2
```

```
      DIMENSION BA(3,3)     ,WR(3),WT1(3),WT2(3)   ,WT3(3)

      I=ISS(J)
      IF(I)6524,6523,6524
 6524 CONTINUE
      CALL MULT2 (B,A,BA,3,3,3)
      WR(1)=V1(1)-V2(1)
      WR(2)=V1(2)-V2(2)
      WR(3)=V1(3)-V2(3)
      CALL MULT1 (A,WRP1(1,I),WT1,3,3,1)
      CALL MULT1 (B,WRP2(1,I),WT2,3,3,1)
      CALL MSUB (WT1,WT2,WT3,3,1)
      CALL MADD (WT3,WR,WT1,3,1)
      CALL MSR (A,WT1,WT2(1),2)
      CALL MSC (BA,WRH(1,I),WT2(2),2)
      FJ(I)=WT2(1)/WT2(2)
      CALL MSCAL(FJ(I),WRH(1,I),WT1)
      CALL MADD(WT1,WRP2(1,I),WT1,3,1)
      CALL MULT1(BA,WT1,WT2,3,3,1)
      CALL MULT3(A,WR,WT1,3,3,1)
      CALL MSUB(WT2,WT1,WPP1,3,1)
      CALL SWITCH(WRP1,WRP2,WRH,SO,SK,SIOQ,J,I,T)
      ISS(J)=0
      DO 6525 K=1,3
 6525 WRP1(K,J)=WPP1(K)
      CALL MSR(B,WR,WT1,1)
      CALL MSR(BA,WRP1(1,J),WT1(2),1)
 6523 CONTINUE
 6522 CONTINUE
      ITHETA=0
      TH=T1
      THDOT=T1T
      PH=P1
      PDOT=P1P
      SI=S1
      SDOT=S1S
  234 LPHI=0
      PHSI=PH
      DOT=PDOT
  235 IF(DOT) 236,260,260
  264 PHSI=PHSI+6.2831853
      LDER=1
  236 IF(PHSI) 264,256,256
  258 PHSI=PHSI-6.2831853
      LDER=1
  260 IF(PHSI-6.2831853) 256,256,258
  256 IF(LPHI) 261,257,261
  257 LPHI=1
      PH=PHSI
      PHSI=SI
      DOT=SDOT
      GO TO 235
  261 LPHI=0
      SI=PHSI
      IF (THDOT) 259,263,263
  259 IF (TH-0.1) 237,413,413
  263 IF (TH-3.04) 413,413,237
  237 IF (ITHETA) 240,239,240
```

```
      DIMENSION BA(3,3)     ,WR(3),WT1(3),WT2(3)   ,WT3(3)
  239 DO 241 I=1,ICD
      TEMP=YC1(I)
      YC1(I)=-ZC1(I)
      ZC1(I)=TEMP
  241 CONTINUE
      TEMP=XY1
      XY1=XZ1
      XZ1=-TEMP
      TEMP=YY1
      YY1=ZZ1
      ZZ1=TEMP
      YZ1=-YZ1
      DO 242 I=1,IPR
      TEMP=YP1(I)
      YP1(I)=-ZP1(I)
      ZP1(I)=TEMP
  242 CONTINUE
      TEMP=YY1D
      YY1D=ZZ1D
      ZZ1D=TEMP
      DO 243 I=1,ICG
      TEMP=UGZ(I)
      UGZ(I)=-UGY(I)
      UGY(I)=TEMP
      TEMP=YG1(I)
      YG1(I)=-ZG1(I)
      ZG1(I)=TEMP
  243 CONTINUE
      DO 244 I=1,IPN
      TEMP=YN1(I)
      YN1(I)=-ZN1(I)
      ZN1(I)=TEMP
  244 CONTINUE
      DO 245 I=1,IRK
      TEMP=URZ(I)
      URZ(I)=-URY(I)
      URY(I)=TEMP
      TEMP=YR1(I)
      YR1(I)=-ZR1(I)
      ZR1(I)=TEMP
  245 CONTINUE
      DO 514 I=1,IAC1
      TEMP=UAZ(I)
      UAZ(I)=-UAY(I)
      UAY(I)=TEMP
      TEMP=UPAZ(I)
      UPAZ(I)=-UPAY(I)
      UPAY(I)=TEMP
  514 CONTINUE
      DO 246 I=1,ISP
      TEMP=YS1(I)
      YS1(I)=-ZS1(I)
      ZS1(I)=TEMP
  246 CONTINUE
      DO 247 I=1,ISU
      TEMP=YU1(I)
```

```
    DIMENSION BA(3,3)      ,WR(3),WT1(3),WT2(3)    ,WT3(3'
    YU1(I)=-ZU1(I)
    ZU1(I)=TEMP
247 CONTINUE
    TEMP=YI1
    YI1=-ZI1
    ZI1=TEMP
    WRITE OUTPUT TAPE 6,449
    MAXES=MAXES+1
    IF(MAXES-4) 461,461,460
460 MAXES=1
461 GO TO (455,456,457,458),MAXES
455 WRITE OUTPUT TAPE 6,451
    GO TO 248
456 WRITE OUTPUT TAPE 6,452
    GO TO 248
457 WRITE OUTPUT TAPE 6,453
    GO TO 248
458 WRITE OUTPUT TAPE 6,454
    GO TO 248
240 DO 249 I=1,ICD
    TEMP=YC2(I)
    YC2(I)=-ZC2(I)
    ZC2(I)=TEMP
249 CONTINUE
    TEMP=XY2
    XY2=XZ2
    XZ2=-TEMP
    TEMP=YY2
    YY2=ZZ2
    ZZ2=TEMP
    YZ2=-YZ2
    TEMP=YY2D
    YY2D=ZZ2D
    ZZ2D=TEMP
    DO 250 I=1,IPR
    TEMP=UPZ(I)
    UPZ(I)=-UPY(I)
    UPY(I)=TEMP
    TEMP=YP2(I)
    YP2(I)=-ZP2(I)
    ZP2(I)=TEMP
250 CONTINUE
    DO 251 I=1,ICG
    TEMP=VGZ(I)
    VGZ(I)=-VGY(I)
    VGY(I)=TEMP
    TEMP=YG2(I)
    YG2(I)=-ZG2(I)
    ZG2(I)=TEMP
251 CONTINUE
    DO 252 I=1,IPN
    TEMP=VNZ(I)
    VNZ(I)=-VNY(I)
    VNY(I)=TEMP
    TEMP=YN2(I)
    YN2(I)=-ZN2(I)
```

```
      DIMENSION BA(3,3)     ,WR(3),WT1(3),WT2(3)   ,WT3(3)

      ZN2(I)=TEMP
  252 CONTINUE
      DO 253 I=1,IRK
      TEMP=VRZ(I)
      VRZ(I)=-VRY(I)
      VRY(I)=TEMP
      TEMP=YR2(I)
      YR2(I)=-ZR2(I)
      ZR2(I)=TEMP
  253 CONTINUE
      DO 254 I=1,ISP
      TEMP=USZ(I)
      USZ(I)=-USY(I)
      USY(I)=TEMP
      TEMP=YS2(I)
      YS2(I)=-ZS2(I)
      ZS2(I)=TEMP
  254 CONTINUE
      J=IAC1+1
      DO 515 I=J,IAC2
      TEMP=VAZ(I)
      VAZ(I)=-VAY(I)
      VAY(I)=TEMP
      TEMP=VPAZ(I)
      VPAZ(I)=-VPAY(I)
      VPAY(I)=TEMP
  515 CONTINUE
      DO 255 I=1,ISU
      TEMP=YU2(I)
      YU2(I)=-ZU2(I)
      ZU2(I)=TEMP
  255 CONTINUE
      TEMP=YI2
      YI2=-ZI2
      ZI2=TEMP
      NAXES=NAXES+1
      IF (NAXES-4) 462,462,463
  463 NAXES=1
  462 WRITE OUTPUT TAPE 6,450
      GO TO (455,456,457,458),NAXES
  248 TEMP=SINF(TH)*COSF(SI)
      IF (TEMP) 850,851,850
  851 TNEW=1.57079633
      GO TO 852
  850 CONTINUE
      TNEW=ATANF( ABSF((SQRTF(1.0-TEMP**2))/TEMP) )
  852 FNEW=ATANF( ABSF(TANF(SI)/COSF(TH)) )
      SNEW=ATANF( ABSF(TANF(TH)*SINF(SI)) )
      IF (COSF(SI)) 265,267,268
  267 TNEW=1.57079633
      GO TO 265
  268 TNEW=3.14159265-TNEW
  265 IF (COSF(TH)) 269,266,266
  266 L=1
      ANGLE=SI
      GO TO 270
```

```
      DIMENSION BA(3,3)     ,WR(3),WT1(3),WT2(3)   ,WT3(3)

  269 L=1
      ANGLE=6.2831853-SI
  270 IF (ANGLE-3.14159265) 271,262,262
  271 IF (ANGLE-1.57079633) 430,430,431
  430 K=1
      GO TO 432
  431 K=2
  432 GO TO (433,434),L
  262 IF (ANGLE-4.712388981) 437,437,438
  437 K=3
      GO TO 432
  438 K=4
      GO TO 432
  433 GO TO (439,440,441,442),K
  440 PNEW=3.14159265-PNEW
      GO TO 439
  441 PNEW=3.14159265+PNEW
      GO TO 439
  442 PNEW=6.2831853-PNEW
  439 PH=PNEW
      IF (SINF(SI)) 443,444,444
  443 L=2
      ANGLE=TH
      GO TO 270
  444 L=2
      ANGLE=6.2831853-TH
      GO TO 270
  434 GO TO (445,446,447,448),K
  446 SNEW=3.14159265-SNEW
      GO TO 445
  447 SNEW=3.14159265+SNEW
      GO TO 445
  448 SNEW=6.2831853-SNEW
  445 SI=SNEW
      TH=TNEW
      WRITE OUTPUT TAPE 6,8000,TH,ITHETA
 8000 FORMAT(1H ,E15.7,I6)
      LDER=1
  413 IF (ITHETA) 272,273,272
  273 T1=TH
      TH=T2
      S1=SI
      P1=PH
      PDOT=P2P
      SDOT=S2S
      PH=P2
      THDOT=T2T
      SI=S2
      ITHETA=1
      GO TO 234
C     THETA MODIFIED RETURN TO FMARK
  272 TRIG3=TRIG3+DEL
      T2=TH
      S2=SI
      P2=PH
  449 FORMAT(1H0,28HBODY 1-FIXED AXES    X  Y  Z )
```

```
      DIMENSION BA(3,3)     ,WR(3),WT1(3),WT2(3)   ,WT3(3)
  450 FORMAT(1H0,28HBODY 2-FIXED AXES     X  Y  Z )
  451 FORMAT(1H ,28HDRAWING BOARD AXES    X  Z -Y )
  452 FORMAT(1H ,28HDRAWING BOARD AXES    X -Y -Z )
  453 FORMAT(1H ,28HDRAWING BOARD AXES    X -Z  Y )
  454 FORMAT(1H ,28HDRAWING BOARD AXES    X  Y  Z )
      DO 317 J=1,6
      SAVE(J+18)=V2(J)
      SAVE(J+42)=YDOT2(J)
  317 CONTINUE
      DO 318 J=1,18
      SAVE(J)=V1(J)
      SAVE(J+24)=YDOT1(J)
  318 CONTINUE
      SAVE(61)=T
      ZERO=0.0
      IF(IMOD) 425,426,425
  426 IF(LDER)5425,403,5425
  425 DO 427 J=1,6
      NH(J)=SAVE(J+54)
      HB(J)=SAVE(J+48)
  427 CONTINUE
 5425 CONTINUE
      IMOD=0
      LDER=1
      GO TO 1001
C
C     PYROTECHNIC TRIGGER
C
 1005 IF(IKUT) 423,424,423
  424 TRIG4=TRIG4+DEL
      TP(1)=T
      TP(4)=TRIG4
      IKUT=1
 3001 IPHI=2
      HB(1)=DEL*.2
  632 LDER=1
      GO TO 1001
  423 CALL TRMOD(3,0)
  415 IKUT=0
      JPHI=4
      IF(TP(2)) 633,600,633
  633 TP(4)=0.0
      DEP=TP(2)
      IF (TRIG4-DEP) 602,634,634
  634 JPHI=2
      GO TO 602
C     FORCE CEASES TO ACT  - - - SET INDEX = ZERO
  600 IPR=0
  601 HB(1)=DEL
      DO 420 J=1,12
      EP(J)=0.0
  420 CONTINUE
  602 IPHI=JPHI
      GO TO 632
C
C      2ND PYROTECHNIC TRIGGER
```

```
      DIMENSION BA(3,3)     ,WR(3),WT1(3),WT2(3)   ,WT3(3)
C
 1007 IF (JKUT) 465,466,465
  466 PYRO2=PYRO2+DEL
      TP(2)=T
      TP(5)=PYRO2
      JKUT=1
      GO TO 3001
  465 CALL TRMOD (5,0)
      JPHI=4
      IF (TP(3)) 635,600,635
  635 TP(5)=0.0
      DEP=TP(3)
      IF (PYRO2-DEP) 602,634,634
C
C     3RD PYROTECHNIC TRIGGER
C
 1008 IF (LKUT) 467,468,467
  468 PYRO3=PYRO3+DEL
      TP(3)=T
      TP(6)=PYRO3
      LKUT=1
      GO TO 3001
  467 CALL TRMOD (6,0)
      JPHI=4
      GO TO 600
C
C     ROCKET TRIGGER
C
 1006 I=IRK+IRK
      IF(TROC-TRK(I)) 428,429,429
  428 LIRK=LIRK+1
      TROC=TRK(LIRK)
      LDER=1
      GO TO 1001
  429 CALL TRMOD (4,0)
      IRK=0
      LDER=1
      GO TO 1001
C
C     PLOT TRIGGER
C
 1009 DO 4050 K=1,3
      J=2*K
      WL(K)=V1(J+2)*HLL
      WL(K+3)=V1(J+3)*HLL
      WL(K+6)=V1(J+11)*HLL
      WL(K+9)=V1(J+12)*HLL
      WL(K+12)=YDOT1(J+11)*HLL
      WL(K+15)=YDOT1(J+12)*HLL
 4050 CONTINUE
      WRITE TAPE NT2,V1(1),V1(2),V1(3),V2(1),V2(2),V2(3),
     1 V1(10),V1(11),V1(12),V2(4),V2(5),V2(6),YDOT1(10),
     1 YDOT1(11),YDOT1(12),YDOT2(4),YDOT2(5),YDOT2(6),
     1 (WL(K),K=1,18)
      PLOT=PLOT+TPLOT
      XPTS=XPTS+1.0
```

```
      DIMENSION BA(3,3)     ,WR(3),WT1(3),WT2(3)   ,WT3(3)

      TEND=T
      CALL ROUT(0)
 5050 CALL ERR(DEL,TPR,TP,TG,TA,TB,TC,TD,TRIG1,TRIG2,TRIG3,
     1 TRIG4,TROC,PYRO2,PYRO3,PLOT )
C
C     PRINT ROUTINE
C
 1002 TRIG1=TRIG1+TPR
      CALL STEP(DELP)
      WRITE OUTPUT TAPE 6,300,T,DELP
  300 FORMAT(1H0,//8H TIME = E15.7,1X,7HSECONDS,5X,10HSTEP SIZE=E15.7/)
      P=X1
      Y=Y1
      Z=Z1
      XDOT=X1X
      YDOT=Y1Y
      ZDOT=Z1Z
      TH=T1
      PH=P1
      SI=S1
      TDOT=T1T
      SDOT=S1S
      PDOT=P1P
      J=1
  301 CA(1)=PDOT*SINF(TH)
      CA(2)=SINF(SI)
      CA(3)=COSF(SI)
      CB(1)=(CA(1)*CA(2)+TDOT*CA(3))*ELK
      CB(2)=-(PDOT*COSF(TH)+SDOT) *ELK
      CB(3)=(CA(1)*CA(3)-TDOT*CA(2))*ELK
      WPY=CB(1)*CB(1)+CB(2)*CB(2)
      W1=SQRTF(WPY+CB(3)*CB(3))
      WPY=SQRTF(WPY)
C     CONVERT RADIANS TO DEGREES
      TH=TH*ELK
      PH=PH*ELK
      SI=SI*ELK
      TDOT=TDOT*ELK
      SDOT=SDOT*ELK
      PDOT=PDOT*ELK
      V=SQRTF(XDOT*XDOT+YDOT*YDOT+ZDOT*ZDOT)
      CA(1)=CA(1)*ELK
      CA(2)=CA(2)*ELK
      CA(3)=CA(3)*ELK
      WRITE OUTPUT TAPE 6,302,J
  302 FORMAT(1H , 5HBODY I1)
      WRITE OUTPUT TAPE 6,352,V
  352 FORMAT(1H ,14HSPEED OF CM = E15.7, 7H IN/SEC )
      WRITE OUTPUT TAPE 6,303,W1,WPY
  303 FORMAT(1H ,31HRATE ABOUT INSTANTANEOUS AXIS= E15.7,1X,7HDEG/SEC,
     1 7X,41HMAGNITUDE VECTOR SUM OF PITCH-YAW RATES= E15.7 )
      WRITE OUTPUT TAPE 6,304
  304 FORMAT(1H0,22HINERTIAL COORDS-INCHES,3X,19H1ST DERIVATIVE /SEC,
     16X,20HEULER ANGLES-DEGREES,5X,19H1ST DERIVATIVE /SEC ,6X,
     14X,10HRATE ABOUT/102X,17HBODY AXES DEG/SEC)
      WRITE OUTPUT TAPE 6,305,P,XDOT,TH,TDOT,CB(1),Y,YDOT,PH,PDOT,CB(2),
```

```
      DIMENSION BA(3,3)     ,WR(3),WT1(3),WT2(3)   ,WT3(3)
     1 Z,ZDOT,SI,SDOT,CB(3)
  305 FORMAT(1H ,1HX,4X,E15.7,5X,1HX,4X,E15.7,5X,5HTHETA,E15.7,5X,5HTHET
     1A,E15.7,5X,1HX,4X,E15.7/
     1 1X,      1HY,4X,E15.7,5X,1HY,4X,E15.7,5X,5HPHI   ,E15.7,5X,5HPHI
     1 ,E15.7,5X,1HY,4X,E15.7/
     1 1X,      1HZ,4X,E15.7,5X,1HZ,4X,E15.7,5X,5HPSI   ,E15.7,5X,5HPSI
     1 ,E15.7,5X,1HZ,4X,E15.7 )
      IF (J-1) 306,307,306
  307 TH=T2
      PH=P2
      SI=S2
      TDOT=T2T
      PDOT=P2P
      SDOT=S2S
      P=X2
      Y=Y2
      Z=Z2
      XDOT=X2X
      YDOT=Y2Y
      ZDOT=Z2Z
      J=2
      GO TO 301
  306 DO 350 J=1,6
      NH(J)=SAVE(J+54)
      HB(J)=SAVE(J+48)
  350 CONTINUE
      CA(1)=SQRTF((X1-X2)*(X1-X2)+(Y1-Y2)*(Y1-Y2)+(Z1-Z2)*(Z1-Z2))
      CA(2)=SQRTF((X1X-X2X)*(X1X-X2X)+(Y1Y-Y2Y)*(Y1Y-Y2Y)+(Z1Z-Z2Z)*
     1  (Z1Z-Z2Z))
      WRITE OUTPUT TAPE 6,353,CA(1),CA(2)
  353 FORMAT(1H0,22HSEPARATION DISTANCE = E15.7, 3H IN ,20X,
     1 22HSEPARATION VELOCITY = E15.7, 7H IN/SEC  )
      IF(ICLAG)6663,6663,6662
 6663 WRITE OUTPUT TAPE 6,6661,FQQ1,FQQ2,FQQ3
 6661 FORMAT(1H0,17HCONSTRAINT VALUES,3E16.7)
10171 FORMAT(1X,1P13E10.3/)
 6662 CONTINUE
      IF(IHY)6670,8001,6670
 6670 WRITE OUTPUT TAPE 6,6671,((A(I,J,1),J=1,3),(B(I,J),J=1,3),I=1,3)
 6671 FORMAT(1H0,22X,2HA1,48X,2HA2/(3E15.6,5X,3E15.6))
 8001 CONTINUE
      SUN=0.0
      DO 232 K=1,12
      SUN=SUN+ABSF(Q(K))
  232 CONTINUE
      IF(IFORCE)6678,405,6678
 6678 IF(SUN)404,404,6680
 6680 IFORCE=1
      GO TO 403
  404 IF(IFORCE)405,405,706
  405 IF(WIL1-T) 706,6501,6501
 6501 CONTINUE
  403 CALL ROUT(0)
  706 IF(TPLOT) 707,708,707
  707 YDOT1(12)=TEND
      YDOT1(11)=XPTS
```

```
      DIMENSION BA(3,3)     ,WR(3),WT1(3),WT2(3)   ,WT3(3)

      YDOT1(13)=TPLOT
      CALL CHAIN(3,3)
  708 CALL CHAIN(1,3)
C
C     ERROR
 5000 WRITE OUTPUT TAPE 6,5001
 5001 FORMAT(1H0,16HERROR INPUT DATA )
 4000 CALL EXIT
C
 5002 WRITE OUTPUT TAPE 6,5003
 5003 FORMAT(1H ,9HDELTA = 0,5X,48HSEE EQUATION 241,SECTION VII-EQUATION
     1S OF MOTION )
      CALL DUMP
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

SUBROUTINE GET

```
 SUBROUTINE GET
 EQUIVALENCE (T,T),(ZERO,ZERO),
1            (X1,V1(1)),(Y1,V1(2)),(Z1,V1(3)),(T1,V1(4)),(T2,V1(5))
1,(P1,V1(6)),(P2,V1(7)),(S1,V1(8)),(S2,V1(9)),(X1X,V1(10)),(Y1Y,V1(
111)),(Z1Z,V1(12)),(T1T,V1(13)),(T2T,V1(14)),(P1P,V1(15)),(P2P,V1(1
16)),(S1S,V1(17)),(S2S,V1(18)),(X2,V2(1)),(Y2,V2(2)),(Z2,V2(3)),
1(X2X,V2(4)),(Y2Y,V2(5)),(Z2Z,V2(6))
 COMMON T,ZERO,V1,V2,YDOT1,YDOT2
 COMMON A,AD
 DIMENSION          YDOT2(6),YDOT1(18),V2(6),V1(18),NH(6),HB(6)
 DIMENSION DM(3,3),DN(3,3)
 COMMON WIL1,TCO
 DIMENSION TCG(8),TCO(8)
 DIMENSION FM(3,4),FEL(3),ACNST(3),CM(3,3),DUD(3)
 DIMENSION DD(12),DF(8),D(8),XC1(8),YC1(8),ZC1(8),XC2(8),YC2(8),
1ZC2(8)
 DIMENSION EP(12),PI(3),TP(6),XP1(3),YP1(3),ZP1(3),XP2(3),YP2(3),
1ZP2(3),UPX(3),UPY(3),UPZ(3)
 DIMENSION G(12),GA(6),PA(6),VA(6),PAD(6),VAD(6),SGX(6),SGY(6),
1SGZ(6),UAX(3),UAY(3),UAZ(3),UPAX(3),UPAY(3),UPAZ(3),VAX(3),VAY(3)
1,VAZ(3),VPAX(3),VPAY(3),VPAZ(3)
1,GG(6)
 DIMENSION BINV(15)
 DIMENSION H(12)
 DIMENSION GJ(12),FG1(8),FG2(8),XG1(8),YG1(8),ZG1(8),XG2(8),
1YG2(8),ZG2(8),UGX(8),UGY(8),UGZ(8),VGX(8),VGY(8),VGZ(8)
 DIMENSION XN(12),AN(8),RAX(8),PPN(8),VNX(8),VNY(8),VNZ(8),XN1(8) ,
1YN1(8),ZN1(8),XN2(8),YN2(8),ZN2(8),PNEM(8),EQP(8)
 DIMENSION R(12),RR(16),XR1(16),YR1(16),ZR1(16),XR2(16),YR2(16),
1ZR2(16),URX(16),URY(16),URZ(16),VRX(16),VRY(16),VRZ(16),TE(16),
1TF(16),TRK(32)
 DIMENSION SIOQ(12)
 DIMENSION S(12),XS1(8),YS1(8),ZS1(8),XS2(8),YS2(8),ZS2(8),USX(8),
1 USY(8),USZ(8),SD(8),SK(8)
 DIMENSION W(12),WO(8),UK(8),XU1(8),YU1(8),ZU1(8),XU2(8),YU2(8),
1  ZU2(8)
 DIMENSION Q(12)
 COMMON XX1,XY1,XZ1,YY1,YZ1,ZZ1,XX2,XY2,XZ2,YY2,YZ2,ZZ2,
1FM10,FM20,DEL,TPR,TD,EX1,EY1,EZ1,EX2,EY2,EZ2,WX,WY,WZ,
2W1YD,ICD,IPR,IAC,IHY,ICG,IPN,IRK,ISP,ISU,ION
 COMMON DF,D,XC,YC ,ZC,XC1,XC2,YC1,YC2,ZC1,ZC2
 COMMON PI,TP,UX,UY,UZ,XP,YP,ZP,UPX,UPY,UPZ,XP1,XP2,YP1,YP2,
1ZP1,ZP2
 COMMON TA,TB,TC,TD,IAC1,IAC2,MAC,NAC,IGLAG,GA,PA,VA,PAD,
1VAD,SGX,SGY,SGZ,UX,UY,UZ,PUX,PUY,PUZ,VAX,VPAX,VAY,
2VPAY,VAZ,VPAZ,UAX,UPAX,UAY,UPAY,UAZ,UPAZ,GG
 COMMON HD,HK,HETA,PFH,DHS,AH,PHBO,VHB,GAMH,JH,DEQP,HYDA,
1HYDB,HYDC,HETA,DA,HYDD,HYDE,DB
 COMMON GJ,GST,FG3,IJLAG,MAXES,NAXES,FG1,FG2,TCO,TCG,
1UX,UY,UZ,XG,YG,ZG,XG1,XG2,YG1,YG2,ZG1,ZG2,VGX,VGY,
2VGZ,UGX,UGY,UGZ
 COMMON XN,JN,PBO,VBN,GAMN,AN,RAX,PPN,XXN,YN,ZN,VNX,VNY,
1VNZ,XN1,XN2,YN1,YN2,ZN1,ZN2,EQP,PPN,PNEM
 COMMON R,VRX,VRY,VRZ,URX,URY,URZ,TE,TF,TROC,SUN1,
1SUN2,SUN3,SAN1,SAN2,SAN3,XX10,YY10,ZZ10,XX20,YY20,ZZ20,
2FMD1,FMD2,RR,XR,YR,ZR,XR1,XR2 ,YR1,YR2,ZR1,ZR2,RUM,PUM,
```

```
      SUBROUTINE GET
     3TRK
      COMMON SIOQ
      COMMON S,SETA,XMU,EPSLN,DELTS,XS1,XS2,YS1,YS2,ZS1,ZS2,
     1SD,SK
      COMMON W,ETA,WD,UK,XU,YU,ZU,XU1,XU2,YU1,YU2,ZU1,ZU2,TG
      DIMENSION WIOQ(8)
      COMMON WIOQ
       COMMON DM,DN,R1,R2,P3,P21,P22,P23,ZTEST,F,CA,XI1,YI1,ZI1,XI2,
     XYI2,ZI2,USX,USY,USZ
      COMMON          XI1,XI2,ZI1,ZI2,ICLAG,TRIG2,ELM,LIRK,N,XS,YS,ZS
      COMMON WL
      COMMON SAVE,BINV,E,NH,HB,H,Q
      COMMON FQQ1,FQQ2,FQQ3
      DIMENSION SAVE(68),E(3,3),CA(3),F(15,15),WL(30)
      DIMENSION WR(3),WRD(3)                          ,WAP(3,3,6),
     1WAPP(3,3,6),WTP(3,3,6),WTPP(3,3,6),WD(3,2),WV(3,2),WJ(2),WJP(12,2)
     2,WJD(2),WJPP(12,2),A(3,3,2),AD(3,3,2),WRP(3,6)
      DIMENSION WT1(3),WT2(3),WT3(3,3),WT4(3,3),WT5(6),WT6(12),WT7(12),
     1WT8(12)
      DIMENSION TEE(3,3),RIL(3),RIK(3),WTD(3,3)
      DIMENSION SHT(12)
      DIMENSION GED(2)
      DIMENSION WRP1(3,8),WRP2(3,8),WRH(3,8)
      COMMON WRP1,WRP2,WRH
      DIMENSION WRPQ(3,2)
      DO 63 I=1,15
      DO 63 J=1,15
      F(I,J)=0.0
   63 CONTINUE
      DO 36 I=1,3
      SHT(I)=V1(I+9)
      SHT(I+6)=V2(I+3)
      J=2 *I
      SHT(I+3)=V1(J+11)
      SHT(I+9)=V1(J+12)
   36 CONTINUE
 5004 IM=0
      ST=SINF(T1)
      CT=COSF(T1)
      CP=COSF(P1)
      SP=SINF(P1)
      CS=COSF(S1)
      SS=SINF(S1)
   49 IF (ABSF(CT)-ZTEST) 50,50,51
   50 CT=0.0
   51 IF (ABSF(ST)-ZTEST) 52,52,53
   52 ST=0.0
   53 IF (ABSF(CP)-ZTEST) 54,54,55
   54 CP=0.0
   55 IF (ABSF(SP)-ZTEST) 56,56,57
   56 SP=0.0
   57 IF (ABSF(CS)-ZTEST) 58,58,59
   58 CS=0.0
   59 IF (ABSF(SS)-ZTEST) 60,60,61
   60 SS=0.0
   61 CONTINUE
```

```
      SUBROUTINE GET
      CTSP=CT*SP
      CTCP=CT*CP
      CTSS=CT*SS
      CTCS=CT*CS
      CPCS=CP*CS
      SPCS=SP*CS
      STSS=ST*SS
      CPSS=CP*SS
      SPSS=SP*SS
      STCS=ST*CS
      CSPSS=CT*SPSS
      CCPSS=CT*CPSS
      CSPCS=CT*SPCS
      CCPCS=CT*CPCS
      STSP=ST*SP
      STCP=ST*CP
C
      CT2=CT*CT
      ST2=ST*ST
      CS2=CS*CS
      SS2=SS*SS
      CSS=CS*SS
      STCT=ST*CT
      CT2=CT*CT
      CO1=CS2-SS2
      THD=V1(IM+13)
      PHD=V1(IM+15)
      PSD=V1(IM+17)
      IM2=IM+1
      A(1,1,IM2)=-CSPSS+CPCS
      A(1,2,IM2)=CCPSS+SPCS
      A(1,3,IM2)=STSS
      A(2,1,IM2)=-CSPCS-CPSS
      A(2,2,IM2)=CCPCS-SPSS
      A(2,3,IM2)=STCS
      A(3,1,IM2)=STSP
      A(3,2,IM2)=-STCP
      A(3,3,IM2)=CT
      AD(1,1,IM2)=ST*SPSS*THD-(CCPSS+SPCS)*PHD-(CSPCS+CPSS)*PSD
      AD(1,2,IM2)=-ST*CPSS*THD-(CSPSS-CPCS)*PHD+(CCPCS-SPSS)*PSD
      AD(1,3,IM2)=CT*SS*THD+STCS*PSD
      AD(2,1,IM2)=ST*SPCS*THD-(CCPCS-SPSS)*PHD+(CSPSS-CPCS)*PSD
      AD(2,2,IM2)=-ST*CPCS*THD-(CSPCS+CPSS)*PHD-(CCPSS+SPCS)*PSD
      AD(2,3,IM2)=CTCS*THD-STSS*PSD
      AD(3,1,IM2)=CT*SP*THD+STCP*PHD
      AD(3,2,IM2)=-CT*CP*THD+STSP*PHD
      AD(3,3,IM2)=-ST*THD
      IF (IM) 47,48,47
C
C     FORM DM MATRIX AND P1,P2,P3 FOR BODY 1
   48 DIFI=(XX1-YY1)*CSS
      DM(1,1)=XX1*CS2-2.0*XY1*CSS+YY1*SS2
      TER=XX1*SS2+2.0*XY1*CSS+YY1*CS2
      DIFM=XZ1*CS-YZ1*SS
      DIFP=XZ1*SS+YZ1*CS
      DIFT=DIFP*ST
```

```
      SUBROUTINE GET
      CO2=CO1*XY1
      DH(1,2)=(DIFI+CO2)*ST+DIFM*CT
      DH(1,3)=DIFM
      DH(3,1)=DH(1,3)
      DH(2,1)=DH(1,2)
      DH(2,2)=ST2*TER+2.D*DIFT*CT+ZZ1*CT2
      DH(2,3)=ST*DIFP+ZZ1*CT
      DH(3,2)=DH(2,3)
      DH(3,3)=ZZ1
      R1=ST*P1P*S1S*(4.D*CSS*XY1-(XX1-YY1)*CO1-ZZ1)+2.D*T1T*S1S*(DIFI+
     1CO2)
     1  +S1S*S1S*DIFP+P1P*P1P*STCT*(TER-ZZ1)+P1P*P1P*(CT2-ST2)*DIFP
     1+2.*P1P*S1S*CT*(XZ1*SS+YZ1*CS)
      R2=-2.D*P1P*(T1T*STCT*(TER-ZZ1)+S1S*ST2*(DIFI+CO2)+T1T*(CT2-ST2)*
     1   DIFP+S1S*STCT*DIFM)-T1T* (T1T*CT*(DIFI+CO2)+S1S*ST*((XX1-YY1)*
     1   CO1-4.D*XY1*CSS)-T1T*ST*DIFM)-S1S*(S1S*ST*DIFM-T1T*ST*ZZ1)
      P3=-2.D*P1P*(T1T*CT*DIFP)              +P1P*ST*(CO1*((XX1-YY1)*T1T
     1    +XY1*P1P*ST)+T1T*ZZ1)+DIFI*(P1P*P1P*ST2-T1T*T1T)
     1 -4.D*XY1*T1T*P1P*ST*CSS+DIFM*P1P*P1P*STCT-CO2*T1T*T1T
      GO TO 41
C
C     FORM DN MATRIX AND P21,P22,P23 FOR BODY2
   47 DIFI=(XX2-YY2)*CSS
      TER=XX2*CS2+2.D*XY2*CSS+YY2*CS2
      DIFP=XZ2*SS+YZ2*CS
      DIFM=XZ2*CS-YZ2*SS
      DIFT=DIFP*ST
      CO2=CO1*XY2
      DN(1,1)=XX2*CS2-2.D*XY2*CSS+YY2*SS2
      DN(1,2)=(DIFI+CO2)*ST+DIFM*CT
      DN(1,3)=DIFM
      DN(2,1)=DN(1,2)
      DN(2,2)=ST2*TER+2.D*DIFT*CT+ZZ2*CT2
      DN(2,3)=ST*DIFP+ZZ2*CT
      DN(3,1)=DN(1,3)
      DN(3,2)=DN(2,3)
      DN(3,3)=ZZ2
      P21=ST*P2P*S2S*(4.D*CSS*XY2-(XX2-YY2)*CO1-ZZ2)+2.D*T2T*S2S*(DIFI+
     1CO2)
     1   +S2S*S2S*DIFP+P2P*P2P*STCT*(TER-ZZ2)+P2P*P2P*(CT2-ST2)*DIFP
     1+2.*P2P*S2S*CT*(XZ2*SS+YZ2*CS)
      P22=-2.D*P2P*(T2T*STCT*(TER-ZZ2)+S2S*ST2*(DIFI+CO2)+T2T*(CT2-ST2)*
     1   DIFP+S2S*STCT*DIFM)-T2T* (T2T*CT*(DIFI+CO2)+S2S*ST*((XX2-YY2)*
     1    CO1-4.D*XY2*CSS)-T2T*ST*DIFM)-S2S*(S2S*ST*DIFM-T2T*ST*ZZ2)
      P23=-2.D*P2P*(T2T*CT*DIFP)              +P2P*ST*(CO1*((XX2-YY2)*T2T
     1     +XY2*P2P*ST)+T2T*ZZ2)+DIFI*(P2P*P2P*ST2-T2T*T2T)
     1 -4.D*XY2*T2T*P2P*ST*CSS+DIFM*P2P*P2P*STCT-CO2*T2T*T2T
   41 IF(ICLAG)101,101,104
C
C     CONSTRUCT R PARTIAL MATRIX
C
  101 DO 1 I=1,3
      DO 1 J=1,6
      WRP(I,J)=0.
    1 CONTINUE
      DO12 I=1,3
```

```
      SUBROUTINE GET

      WGP(I,I)=1.
      WGP(I,I+3)=-1.
   12 CONTINUE
      IM1=IM*3+1
      STSPSS=STSS*SP
      WAP(1,1,IM1)=STSPSS
      WAP(1,2,IM1)=-CPSS*ST
      WAP(1,3,IM1)=CTSS
      WAP(2,1,IM1)=STSP*CS
      WAP(2,2,IM1)=-STCP*CS
      WAP(2,3,IM1)=CTCS
      WAP(3,1,IM1)=CTSP
      WAP(3,2,IM1)=-CTCP
      WAP(3,3,IM1)=-ST
      DO 2 J=1,3
      WAP(J,1,IM1+1)=-A(J,2,IM2)
      WAP(J,2,IM1+1)=A(J,1,IM2)
      WAP(1,J,IM1+2)=A(2,J,IM2)
      WAP(J,3,IM1+1)=0.
      WAP(2,J,IM1+2)=-A(1,J,IM2)
      WAP(3,J,IM1+2)=0.
    2 CONTINUE
      WAPP(1,1,IM1)=CSPSS*THD+STSS*CP*PHD+SPCS*ST*PSD
      WAPP(1,2,IM1)=-CCPSS*THD+ST*SPSS*PHD-STCS*CP*PSD
      WAPP(1,3,IM1)=-STSS*THD+CTCS*PSD
      WAPP(2,1,IM1)=CSPCS*THD+CPCS*ST*PHD-STSS*SP*PSD
      WAPP(2,2,IM1)=-CCPCS*THD+STSP*CS*PHD+STCP*SS*PSD
      WAPP(2,3,IM1)=-STCS*THD-CTSS*PSD
      WAPP(3,1,IM1)=-STSP*THD+CTCP*PHD
      WAPP(3,2,IM1)=STCP*THD+CTSP*PHD
      WAPP(3,3,IM1)=-CT*THD
      DO 3 I=1,3
      WAPP(I,1,IM1+1)=-AD(I,2,IM2)
      WAPP(I,2,IM1+1)=AD(I,1,IM2)
      WAPP(I,3,IM1+1)=0.
      WAPP(1,I,IM1+2)=AD(2,I,IM2)
      WAPP(2,I,IM1+2)=-AD(1,I,IM2)
      WAPP(3,I,IM1+2)=0.
    3 CONTINUE
  104 IF(IM)43,44,43
C
C     MATRICES B AND BD
   44 ST=SINF(T2)
      CT=COSF(T2)
      CP=COSF(P2)
      SP=SINF(P2)
      CS=COSF(S2)
      SS=SINF(S2)
      IM=1
      GO TO 49
   43 IF(ICLAG)106,106,105
  105 RETURN
  106 DO 102 K=1,3
      DO 102 J=1,3
      F(K+9,J+9)=DN(K,J)
  102 CONTINUE
```

```
      SUBROUTINE GET
      DO 100 K=1,3
      DO 100 J=1,3
      F(K+3,J+3)=DM(K,J)
  100 CONTINUE
      DO13 I=1,3
      WR(I)=V1(I)-V2(I)
      WRD(I)=V1(I+9)-V2(I+3)
   13 CONTINUE
      IF(ICLAG)110,111,111
  111 WRPQ(1,1)=XI1
      WRPQ(2,1)=YI1
      WRPQ(3,1)=ZI1
      WRPQ(1,2)=XI2
      WRPQ(2,2)=YI2
      WRPQ(3,2)=ZI2
      DO 115 I=1,3
      CA(I)=0.
      F(I+12,I)=-WRP(I,I)
      F(I+12,I+6)=1.
      J=I+3
      CALL MULT1(WAP(1,1,J),WRPQ(1,2),F(13,I+9),3,3,1)
      CALL MULT1(WAP(1,1,I),WRPQ(1,1),F(13,I+3),3,3,1)
      DO 117 K=1,3
  117 F(K+12,I+3)=-F(K+12,I+3)
  115 CONTINUE
      DO 116 I=1,3
      J=I+3
      CALL MULT1(WAPP(1,1,J),WRPQ(1,2),WT1(1),3,3,1)
      CALL MULT1(WAPP(1,1,I),WRPQ(1,1),WT2(1),3,3,1)
      F(I,I+12)=F(I+12,I)
      F(I+6,I+12)=F(I+12,I+6)
      DO 116 K=1,3
      F(I+3,K+12)=F(K+12,I+3)
      F(I+9,K+12)=F(K+12,I+9)
  116 CA(K)=CA(K)-SHT(I+9)*WT1(K)+SHT(I+3)*WT2(K)
      CALL MULT1(A(1,1,1),WRPQ(1,1),WT1(1),3,3,1)
      CALL MULT1(A(1,1,2),WRPQ(1,2),WT2(1),3,3,1)
      FQQ1=WR(1)+WT1(1)-WT2(1)
      FQQ2=WR(2)+WT1(2)-WT2(2)
      FQQ3=WR(3)+WT1(3)-WT2(3)
      RETURN
  110 CONTINUE
C
C     COMPUTE T
C
      CALL MULT2(A(1,1,2),A,TEE,3,3,3)
C
C     THE FOLLOWING COMPUTES THE PARTIAL OF T WITH RESPECT TO Q
C     AND ITS TIME DERIVITIVE
C
      DO 10 I=1,3
      CALL MULT2(A(1,1,2),WAP(1,1,I),WTP(1,1,I),3,3,3)
      CALL MULT2(AD(1,1,2),WAP(1,1,I),WT3,3,3,3)
      CALL MULT2(A(1,1,2),WAPP(1,1,I),WT4,3,3,3)
      CALL MADD(WT3,WT4,WTPP(1,1,I),3,3)
      J=I+3
```

```
      SUBROUTINE GET

      CALL MULT2(WAP(1,1,J),A,WTP(1,1,J),3,3,3)
      CALL MULT2(WAPP(1,1,J),A,WT3,3,3,3)
      CALL MULT2(WAP(1,1,J),AD,WT4,3,3,3)
      CALL MADD(WT3,WT4,WTPP(1,1,J),3,3)
   10 CONTINUE
C
C     COMPUTE T DOT
C
      CALL MULT2(AD(1,1,2),A,WT3,3,3,3)
      CALL MULT2(A(1,1,2),AD,WT4,3,3,3)
      CALL MADD(WT3,WT4,WTD,3,3)
      DO 25 I=1,2
C
C     COMPUTE D
C
      CALL MULT1(A,WRP1(1,I),WT1,3,3,1)
      CALL MULT1(A(1,1,2),WRP2(1,I),WT2,3,3,1)
      CALL MADD(WT1,WR,WT1,3,1)
      CALL MSUB(WT1,WT2,WD(1,I),3,1)
C
C     COMPUTE V
C
      CALL MULT1(AD,WRP1(1,I),WT1,3,3,1)
      CALL MULT1(AD(1,1,2),WRP2(1,I),WT2,3,3,1)
      CALL MADD(WT1,WRD,WT1,3,1)
      CALL MSUB(WT1,WT2,WV(1,I),3,1)
C
C     FORM JI
C
      CALL MSR (A,WD(1,I),WF,2)
      CALL MULT1(A(1,1,2),WRH(1,I),WT1,3,3,1)
      CALL MSR (A,WT1,WG,2)
      WJ(I)=WF/WG
C
C     FORM THE PARTIALS OF JI WITH RESPECT TO QS
C
      DO 15 J=1,3
      WT8(J)=0.
      CALL MSR (A,WRP(1,J),WT1,2)
      CALL MSR (A,WRP(1,J+3),WT1(2),2)
      WJP (J,I)=WT1/WG
      WJP (J+6,I)=WT1(2)/WG
      WT8(J+6)=0.
      CALL MSR (WAP(1,1,J),WD(1,I),WT1,2)
      CALL MULT1(WAP(1,1,J),WRP1(1,I),WT2,3,3,1)
      CALL MSR (A,WT2,WT1(2),2)
      CALL MULT1(A(1,1,2),WRH(1   ,I),WT2,3,3,1)
      CALL MSR (WAP(1,1,J),WT2,WT1(3),2)
      WT8(J+3)=WT1(3)
      WJP (J+3,I)=(WT1(1)+WT1(2)-WT1(3)*WJ(1))/WG
      CALL MULT1(WAP(1,1,J+3),WRP2(1,I),WT1,3,3,1)
      CALL MSR (A,WT1,WT2,2)
      CALL MULT1(WAP(1,1,J+3),WRH(1,I),WT1,3,3,1
      CALL MSR (A,WT1,WT2(2),2)
      WT8(J+9)=WT2
      WJP (J+9,I)=-(WT2(1)+WT2(2)*WJ(I))/WG
```

```
      SUBROUTINE GET
   15 CONTINUE
C
C     COMPUTE J1 DOT
C
      CALL MSR (A,WV(1,I),WT5(2),2)
      CALL MSR (AD,WD(1,I),WT5,2)
      CALL MULT1(A(1,1,2),WRH(1,I),WT1,3,3,1)
      CALL MSR (AD,WT1,WT5(3),2)
      CALL MULT1(AD(1,1,2),WRH(1,I),WT1,3,3,1)
      CALL MSR (A,WT1,WT5(4),2)
      WJD(I)=(WT5(1)+WT5(2)-WJ(I)*(WT5(3)+WT5(4)))/WG
      GED(I)=WT5(3)+WT5(4)
C
C     FORM THE FIRST TIME DERIVITIVE OF JI
C     WITH RESPECT TO Q8
C
      DO 17 J=1,3
      CALL MSR (AD,WRP(1,J),WT6(J),2)
      WT7(J)=0.
      CALL MSR (AD,WRP(1,J+3),WT6(J+6),2)
      WT7(J+6)=0.
      CALL MSR (WAPP(1,1,J),WD(1,I),WT5,2)
      CALL MSR (WAP(1,1,J),WV(1,I),WT5(2),2)
      CALL MULT1(WAP(1,1,J),WRP1(1,I),WT1,3,3,1)
      CALL MSR (AD,WT1,WT5(3),2)
      CALL MULT1(WAPP(1,1,J),WRP1(1,I),WT1,3,3,1)
      CALL MSR (A,WT1,WT5(4),2)
      WT6(J+3)=WT5(1)+WT5(2)+WT5(3)+WT5(4)
      CALL MULT1(A(1,1,2),WRH(1,I),WT1,3,3,1)
      CALL MSR (WAPP(1,1,J),WT1,WT5,2)
      CALL MULT1(AD(1,1,2),WRH(1,I),WT1,3,3,1)
      CALL MSR (WAP(1,1,J),WT1,WT5(2),2)
      WT7(J+3)=WT5(1)+WT5(2)
      CALL MULT1(WAP(1,1,J+3),WRP2(1,I),WT1,3,3,1)
      CALL MSR (AD,WT1,WT5(1),2)
      CALL MULT1(WAPP(1,1,J+3),WRP2(1,I),WT1,3,3,1)
      CALL MSR (A,WT1,WT5(2),2)
      WT6(J+9)=-WT5(1)-WT5(2)
      CALL MULT1(WAP(1,1,J+3),WRH(1,I),WT1,3,3,1)
      CALL MSR (AD,WT1,WT5(1),2)
      CALL MULT1(WAPP(1,1,J+3),WRH(1,I),WT1,3,3,1)
      CALL MSR (A,WT1,WT5(2),2)
      WT7(J+9)=WT5(1)+WT5(2)
   17 CONTINUE
      DO 18 J=1,12
      FDK=WT6(J)-WJD(I)*WT8(J)-WJ(I)*WT7(J)
      WJPP(J,I)=(FDK-WJP(J,I)*GED(I))/WG
   18 CONTINUE
   25 CONTINUE
      DO 24 I=1,3
   24 RIL(I)=0.
      CALL MSUB(WRP1(1,1),WRP1(1,2),WT1,3,1)
      DO 27 I=1,12
      F(13,I)=-WJP(I,1)*WRH(1,1)
      F(14,I)=-WJP(I,1)*WRH(3,1)
      CALL MSR(TEE,WT1,WT5(4),3)
```

```
      SUBROUTINE GET
      WT5(1)=WJP(I,1)*WRH(1,1)-WJP(I,2)*WRH(1,2)
      WT2(1)=WT5(4)*WT5(1)
      CALL MSR(TEE,WT1,WT5(5),1)
      WT5(2)=WJP(I,1)*WRH(3,1)-WJP(I,2)*WRH(3,2)
      WT2(2)=WT5(5)*WT5(2)
      F(15,I)=WT2(1)-WT2(2)
      RIK(1)=-WJPP(I,1)*WRH(1,1)
      RIK(2)=-WJPP(I,1)*WRH(3,1)
      CALL MSR(WTD(1,1),WT1,WT2(1),3)
      CALL MSR(WTD(1,1),WT1,WT2(2),1)
      RIK(3)=WT2(1)*WT5(1)+WT5(4)*(WJPP(I,1)*WRH(1,1)-WJPP(I,2)*WRH(1,2)
     1)-WT2(2)*WT5(2)-WT5(5)*(WJPP(I,1)*WRH(3,1)-WJPP(I,2)*WRH(3,2))
      DO 27 J=1,3
      RIL(J)=RIL(J)+RIK(J)*SHT(I)
   27 CONTINUE
      DO 30 I=1,3
      DO 31 J=1,3,2
      J1=12+(J+1)/2
      CALL MSR (A(1,1,2),WRP(1,I),WT2,J)
      F(J1,I)=F(J1,I)+WT2(1)
      CALL MSR (A(1,1,2),WRP(1,I+3),WT2,J)
      F(J1,I+6)=F(J1,I+6)+WT2(1)
      CALL MSR (WAP(1,1,I+3),WR,WT2(2),J)
      F(J1,I+9)=F(J1,I+9)+WT2(2)
      CALL MSR(WTP(1,1,I),WRP1,WT2(3),J)
      F(J1,I+3)=F(J1,I+3)+WT2(3)
      CALL MSR(WTP(1,1,I+3),WRP1,WT2,J)
      F(J1,I+9)=F(J1,I+9)+WT2(1)
   31 CONTINUE
      CALL MSUB(WRP1(1,1),WRP1(1,2),WT1,3,1)
      DO 32 J=1,3,2
      WT5(J)=WRP2(J,1)+WJ(1)*WRH(J,1)-WRP2(J,2)-WJ(2)*WRH(J,2)
      WT6(J)=WT5(J)
      CALLMSCAL(WT5(J),WT1,WT3(1,J))
      J1=4-J
      CALL MSR (WTP(1,1,I),WT3(1,J),WT2,J1)
      CALL MSR (WTP(1,1,I+3),WT3(1,J),WT2(2),J1)
      GRG=FLOATF(2-J)
      F(15,I+3)=F(15,I+3)+GRG*WT2(1)
      F(15,I+9)=F(15,I+9)+GRG*WT2(2)
   32 CONTINUE
      DO 33 J=1,3,2
      CALL MSR (WAPP(1,1,I+3),WR,WT5,J)
      CALL MSR (WAP(1,1,I+3),WRD,WT5(2),J)
      CALL MSR (WTPP(1,1,I+3),WRP1,WT5(3),J)
      J1=(J+1)/2
      RIL(J1)=RIL(J1)+(WT5(1)+WT5(2)+WT5(3))*SHT(I+9)
      CALL MSR (WTPP(1,1,I),WRP1,WT5(4),J)
      RIL(J1)=RIL(J1)+SHT(I+3)*WT5(4)
      CALL MSR (AD(1,1,2),WRP(1,I),WT5(5),J)
      RIL(J1)=RIL(J1)+WT5(5)*SHT(I)
      CALL MSR(AD(1,1,2),WRP(1,I+3),WT5(6),J)
      RIL(J1)=RIL(J1)+WT5(6)*SHT(I+6)
   33 CONTINUE
      DO 34 J=1,3,2
      J1=4-J
```

```
      SUBROUTINE GET

      QQQ=WJD(1)*WRH(J,1)-WJD(2)*WRH(J,2)
      CALL MSCAL(QQQ    ,WT1,WT2)
      CALL MSR  (WTP(1,1,I),WT2,WT5,J1)
      GRQ=FLOATF(2-J)
      CALL MSR(WTPP(1,1,I),WT3(1,J),WT5(2),J1)
      RIL(3)=RIL(3)+SHT(I+3)*GRQ*(WT5(1)+WT5(2))
      CALL MSR  (WTP(1,1,I+3),WT2,WT5,J1)
      CALL MSR(WTPP(1,1,I+3),WT3(1,J),WT5(2),J1)
      RIL(3)=RIL(3)+SHT(I+9)*GRQ*(WT5(1)+WT5(2))
   34 CONTINUE
   30 CONTINUE
      CA(1)=-RIL(1)
      CA(2)=-RIL(2)
      CA(3)=-RIL(3)
      DO 37 I=1,3
      DO 37 J=1,12
   37 F(J,I+12)=F(I+12,J)
      DO 65 I=1,3,2
      CALL MSR(A(1,1,2),WR,WT5(1),I)
      CALL MSR(TEE,WRP1,WT5(2),I)
      WT5(3)=-WRP2(I,1)
      WT5(4)=-WJ(1)*WRH(I,1)
      FQ=WT5(1)+WT5(2)+WT5(3)+WT5(4)
      IF(I-2)66,65,67
   66 FQQ1=FQ
 1234 FORMAT(6E20.7)
      GO TO 65
   67 FQQ2=FQ
   65 CONTINUE
 1007 FORMAT(6E20.7)
      FQQ3=0.
      DO 68 J=1,3,2
      J1=4-J
      CALL MSCAL(WT6(J),WT1,WT5(1))
      CALL MSR(TEE,WT5(1)    ,CUF,J1)
      FQQ3=FQQ3+FLOATF(2-J)*CUF
   68 CONTINUE
      RETURN
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```
      SUBROUTINE SPRING(WRP1,WRP2,WRH,SD,SK,SIOQ,ISP,N,ICLAG,S,
```

```
      SUBROUTINE SPRING(WRP1,WRP2,WRH,SD,SK,SIOQ,ISP,N,ICLAG,S,
     1FJ,LDER,LSP1,ISS)
      DIMENSION ISS(1),FJ(1)
       DIMENSION WPP1(3,8)
      DIMENSION ISP1(8)
      DIMENSION ICRP(12)
      COMMON T,ZERO,V1,V2,YDOT1,YDOT2
      DIMENSION           YDOT2(6),YDOT1(18),V2(6),V1(18),NH(6),HB(6)
      EQUIVALENCE (T,T),(ZERO,ZERO),
     1   (X1,V1(1)),(Y1,V1(2)),(Z1,V1(3)),(T1,V1(4)),(T2,V1(5))
     1,(P1,V1(6)),(P2,V1(7)),(S1,V1(8)),(S2,V1(9)),(X1X,V1(10)),(Y1Y,V1(
     111)),(Z1Z,V1(12)),(T1T,V1(13)),(T2T,V1(14)),(P1P,V1(15)),(P2P,V1(1
     16)),(S1S,V1(17)),(S2S,V1(18)),(X2,V2(1)),(Y2,V2(2)),(Z2,V2(3)),
     1(X2X,V2(4)),(Y2Y,V2(5)),(Z2Z,V2(6))
      COMMON A,B,AD,BD
      DIMENSION A(3,3),B(3,3),AD(3,3),BD(3,3)
      DIMENSION EEK(3,4)
      DIMENSION WT1(3),WT2(3),WT3(3),BA(3,3),WT4(3),WT5(3),WT6(3)
      DIMENSION WRP1(3,1),WRP2(3,1),WRH(3,1),SD(1),SK(1),SIOQ(1)
     XS(1),FOIE(1),WR(3)
      DO 1 I=1,12
      ICRP(I)=0
      S(I)=0.0
    1 CONTINUE
      TUF=0.
      CP=COSF(P1)
      SP=SINF(P1)
      EEK(1,1)=CP
      EEK(2,1)=SP
      EEK(3,1)=0.
      ST=SINF(T1)
      CT=COSF(T1)
      EEK(1,2)=ST*SP
      EEK(2,2)=-ST*CP
      EEK(3,2)=CT
      EEK(1,3)=COSF(S2)
      EEK(2,3)=-SINF(S2)
      EEK(3,3)=0.
      LSP=0
      DO 200 I=1,3
      WR(I)=V1(I)-V2(I)
      WT4(I)=0.
      WT5(I)=0.
      WT6(I)=0.
  200 CONTINUE
      CALL MULT2 (B,A,BA,3,3,3)
      DO 100 I=1,ISP
      CALL MULT1 (A,WRP1(1,I),WT1,3,3,1)
      CALL MULT1 (B,WRP2(1,I),WT2,3,3,1)
      CALL MSUB (WT1,WT2,WT3,3,1)
      CALL MADD (WT3,WR,WT1,3,1)
      CALL MSR (A,WT1,WT2(1),2)
      CALL MSC (BA,WRH(1,I),WT2(2),2)
      FJ(I)=WT2(1)/WT2(2)
      PUT=SK(I)*FJ(I)
      PUT1 =PUT+SIOQ(I)
```

```
      SUBROUTINE SPRING(WRP1,WRP2,WRH,SO,SK,SIOQ,ISP,N,ICLAG,S,

      FOIE    =FUT1-SO(I)
      IF(FOIE    )3,3,6
    6 LSP=LSP+1
      ICRP(I)=1
      GO TO 100
    3 XS=(SO(I)-FUT)*SQRTF(WRH(1,I)**2+WRH(2,I)**2+WRH(3,I)**2)
C
      CALL MSCAL(XS,WRH(1,I),WT1)
      CALL MADD(WT4,WT1,WT4,3,1)
C
      CALL MCROS (WRP2(1,I),WT1(1),WT2)
      CALL MSUB (WT5,WT2,WT5,3,1)
C
      CALL MULT1 (B,WRP2(1,I),WT2,3,3,1)
      CALL MULT1 (B,WRH(1,I),WT3,3,3,1)
      CALL MSCAL(FJ(I),WT3,WT3)
      CALL MADD (WT2,WT3,WT2,3,1)
      CALL MSUB (WT2,WR,WT2,3,1)
      CALL MULT1(B,WT1,WT3,3,3,1)
      CALL MCROS (WT2,WT3,WT1)
      CALL MADD (WT1,WT6,WT6,3,1)
  100 CONTINUE
      DO 201 J=1,3
      J1=2*J-1
      CALL MSC(A,WT4,S(J1),J)
      S(J1+1)=-S(J1)
  201 CONTINUE
      S(12)=WT5(3)
      CALL MSC(WT6,EEK(1,2),S(11),1)
      CALL MSC(WT6,EEK(1,1),S(7),1)
      S(9)=WT6(3)
      CALL MSC(WT5,EEK(1,3),S(8),1)
      CALL MSC(B,WT5,S(10),3)
      IF(N-2)11,20,20
   20 IF(ISP-LSP-2)11,12,12
   12 KL=3
      IF(ICRP(1))13,14,13
   13 DO 15 L=3,ISP
      IF(ICRP(L))15,16,15
   16 ISS(1)=L
      KL=L+1
      GO TO 14
   15 CONTINUE
   14 IF(ICRP(2))17, 8,17
   17 IF(KL-ISP)33,33,7
   33 DO 19 L1=KL,ISP
      IF(ICRP(L1))19,21,19
   21 ISS(2)=L1
      GO TO 8
   19 CONTINUE
   11 ICLAG=1
      IF(LSP-ISP)8,7,8
C     FORCE CEASES TO ACT  - - - SET INDEX = ZERO
    7 ISP=0
      ICLAG=1
    8 IF(LSP-LSP1)18,18,25
```

```
      SUBROUTINE SPRING(WRP1,WRP2,WRH,SO,SK,SIOQ,ISP,N,ICLAG,S,

   25 LDER=-1
      LSP1=LSP
   18 RETURN
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

SUBROUTINE PYRO (PI,TP,XP1,YP1,ZP1,XP2,YP2,ZP2,UPX,UPY,UPZ,EP,IPR)

```
      SUBROUTINE PYRO (PI,TP,XP1,YP1,ZP1,XP2,YP2,ZP2,UPX,UPY,UPZ,EP,IPR)
      EQUIVALENCE (T,T),(ZERO,ZERO),
     1            (X1,V1(1)),(Y1,V1(2)),(Z1,V1(3)),(T1,V1(4)),(T2,V1(5))
     1,(P1,V1(6)),(P2,V1(7)),(S1,V1(8)),(S2,V1(9)),(X1X,V1(10)),(Y1Y,V1(
     111)),(Z1Z,V1(12)),(T1T,V1(13)),(T2T,V1(14)),(P1P,V1(15)),(P2P,V1(1
     16)),(S1S,V1(17)),(S2S,V1(18)),(X2,V2(1)),(Y2,V2(2)),(Z2,V2(3)),
     1(X2X,V2(4)),(Y2Y,V2(5)),(Z2Z,V2(6))
      COMMON T,ZERO,V1,V2,YDOT1,YDOT2
      COMMON A,AD,BD
      DIMENSION A(3,3,2),AD(3,3),BD(3,3)
      DIMENSION            YDOT2(6),YDOT1(18),V2(6),V1(18),NH(6),HB(6)
      DIMENSION PI(3),TP(6),EP(12),XP1(3),YP1(3),ZP1(3),XP2(3),YP2(3),
     1   ZP2(3),UPX(3),UPY(3),UPZ(3),CA(3),CB(3)
      DIMENSION RHO(3)  ,WT1(3),RP1(3)  ,RP2(3)  ,WT2(3,2),WT3(3,2),WT4(
     *3)
      E=0.
      DO 4 J=1,12
      EP(J)=0.0
    4 CONTINUE
      CONX=1.0
      CONY=1.0
      CONZ=1.0
      IF (X1-X2) 8,9,9
    8 CONX=-1.0
    9 IF (Y1-Y2) 10,11,11
   10 CONY=-1.0
   11 IF (Z1-Z2) 12,13,13
   12 CONZ=-1
   13 DO 100 I=1,IPR
       DIMENSION IT(3)
      IF(T-TP(I+3)) 1,20,22
   20 IT(I)=0
      GO TO 25
   22 IF(IT(I)) 25,100,25
    1 IF(T-TP(I)) 100,24,23
   23 IF(IT(I)) 25,100,25
   24 IT(I)=1
   25 E=PI(I)
      RP1(1)  =XP1(I)
      RP1(2)  =YP1(I)
      RP1(3)  =ZP1(I)
      RP2(1)  =XP2(I)
      RP2(2)  =YP2(I)
      RP2(3)  =ZP2(I)
      RHO(1)  =UPX(I)
      RHO(2)  =UPY(I)
      RHO(3)  =UPZ(I)
      CALL MULT1(A(1,1,2),RHO,WT1,3,3,1)
      DO 30 J=1,3
      WT2(J,1)=0.
      WT2(J,2)=0.
   30 EP(2*J-1)=EP(2*J-1)+E*WT1(J)
      CALL MCROS(RP2,RHO,WT3)
      CALL MCROS (RP1,RHO,WT3(1,2))
      DO 31 J=1,3
      WT2(J,1)=WT2(J,1)+E*WT3(J,1)
```

```
      SUBROUTINE PYRO (PI,TP,XP1,YP1,ZP1,XP2,YP2,ZP2,UPX,UPY,UPZ,EP,IPR)

  31  WT2(J,2)=WT2(J,2)+E*WT3(J,2)
 100  CONTINUE
      IF(E)34,6,34
  34  DO 32 J=1,2
      CP=COSF(V1(J+5))
      SP=SINF(V1(J+5))
      CT=COSF(V1(J+3))
      ST=SINF(V1(J+3))
      SNG=(-1)**J
      J1=3-J
      CALL MULT1(A(1,1,J1),WT2(1,J),WT1,3,3,1)
      EP(J+6)=EP(J+6)+SNG*(WT1(1)*CP+WT1(2)*SP)
      EP(J+8)=EP(J+8)+SNG*WT1(3)
      EP(J+10)=EP(J+10)+SNG*(ST*SP+WT1(1)-ST*CP*WT1(2)
     1+CT*WT1(3))
  32  CONTINUE
      CONX=-CONX
      CONZ=-CONZ
      CONY=-CONY
      EP(1)=EP(1)*CONX
      EP(3)=EP(3)*CONY
      EP(5)=EP(5)*CONZ
   7  EP(2)=-EP(1)
      EP(4)=-EP(3)
      EP(6)=-EP(5)
   6  RETURN
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```
      SUBROUTINE FNEUM(IPN,JN,FBO,VBN,GAMN,AN,RAX,UNX,UNY,UNZ,
```

```
      SUBROUTINE FNEUM(IPN,JN,FBO,VBN,GAMN,AN,RAX,UNX,UNY,UNZ,
     1XN1,YN1,ZN1,XN2,YN2,ZN2,XN,FPN,EQF,PNEM,LDER,IPN1)
      COMMON T,ZERO,V1,V2,YDOT1,YDOT2
      DIMENSION            YDOT2(6),YDOT1(18),V2(6),V1(18),NH(6),HB(6)
      EQUIVALENCE (T,T),(ZERO,ZERO),
     1   (X1,V1(1)),(Y1,V1(2)),(Z1,V1(3)),(T1,V1(4)),(T2,V1(5))
     1,(F1,V1(6)),(F2,V1(7)),(S1,V1(8)),(S2,V1(9)),(X1X,V1(10)),(Y1Y,V1(
     111)),(Z1Z,V1(12)),(T1T,V1(13)),(T2T,V1(14)),(F1F,V1(15)),(F2F,V1(1
     16)),(S1S,V1(17)),(S2S,V1(18)),(X2,V2(1)),(Y2,V2(2)),(Z2,V2(3)),
     1(X2X,V2(4)),(Y2Y,V2(5)),(Z2Z,V2(6))
      COMMON A,B,AD,BD
      DIMENSION A(3,3),B(3,3),AD(3,3),BD(3,3)
      DIMENSION AN(8),RAX(8),UNX(8),UNY(8),UNZ(8),XN1(8),XN2(8),YN1(8),
     1 YN2(8),ZN1(8),ZN2(8),FPN(8),XN(12),EQF(8),CA(3),CB(3),CC(3),DEX(8
     1),PNEM(8)
      DO 17 K=1,12
      XN(K)=0.0
   17 CONTINUE
      CONX=1.0
      CONY=1.0
      CONZ=1.0
      IF (X1-X2) 19,20,20
   19 CONX=-1.0
   20 IF (Y1-Y2) 21,22,22
   21 CONY=-1.0
   22 IF (Z1-Z2) 23,24,24
   23 CONZ=-1.0
   24 LPM=0
      SUM=0.0
      DO 1 I=1,IPN
      DO 2 K=1,3
      CA(K)=A(1,K)*XN1(I)+A(2,K)*YN1(I)+A(3,K)*ZN1(I)
      CB(K)=B(1,K)*UNX(I)+B(2,K)*UNY(I)+B(3,K)*UNZ(I)
      CC(K)=B(1,K)*XN2(I)+B(2,K)*YN2(I)+B(3,K)*ZN2(I)
    2 CONTINUE
      DEX(I)=-(A(1,2)*(X2+CC(1)-X1-CA(1))+A(2,2)*(Y2+CC(2)-Y1-CA(2))
     1         +A(3,2)*(Z2+CC(3)-Z1-CA(3)))/(A(1,2)*CB(1)+A(2,2)*CB(2)
     1+A(3,2)*CB(3))
      DEX(I)=ABSF(DEX(I))
      SUM=SUM+AN(I)*DEX(I)
    1 CONTINUE
      DO 100 I=1,IPN
      IF (JN-1) 3,5,9
    3 IF (DEX(I)-RAX(I)) 4,15,15
   15 LPM=LPM+1
      GO TO 100
    4 YN=FBO*AN(I)*((1.0+SUM/VBN)**GAMN)
      GO TO 50
    5 IF (DEX(I)-EQF(I)) 6,6,7
    6 YN=FPN(I)
      GO TO 50
    7 IF (DEX(I)-RAX(I)) 8,15,15
    8 YN=FPN(I)*(EQF(I)/DEX(I))**GAMN
      GO TO 50
    9 IF (DEX(I)-EQF(I)) 6,6,10
   10 IF (DEX(I)-RAX(I)) 11,15,15
```

```
      SUBROUTINE FNEUM(IFN,JN,FBO,VBN,GAMN,AN,RAX,UNX,UNY,UNZ,

   11 SUMN=0.0
      SUMD=0.0
      DO 12 J=1,IFN
      IF (DEX(J)-EQF(J)) 13,14,14
   13 SUMN=SUMN+AN(J)*DEX(J)
      SUMD=SUMD+AN(J)*DEX(J)
      GO TO 12
   14 SUMN=SUMN+AN(J)*DEX(J)-FNEM(J)
      SUMD=SUMD+FNEM(J)
   12 CONTINUE
      YN=FFN(I)/(1.0+SUMN/(VBN+SUMD))**GAMN
   50 DO 51 K=1,3
      CA(K)=A(1,K)*XN1(I)+A(2,K)*YN1(I)+A(3,K)*ZN1(I)
      CB(K)=B(1,K)*UNX(I)+B(2,K)*UNY(I)+B(3,K)*UNZ(I)
   51 CONTINUE
      XN(1)=XN(1)+YN*CB(1)
      XN(3)=XN(3)+YN*CB(2)
      XN(5)=XN(5)+YN*CB(3)
      XN(7)=XN(7)+YN*((CB(1)*CA(3)-CB(3)*CA(1))*SINF(P1)
     1                +(CB(3)*CA(2)-CB(2)*CA(3))*COSF(P1))
      XN(9)=XN(9)+YN*(CB(2)*CA(1)-CB(1)*CA(2))
      XN(11)=XN(11)+YN*((CB(3)*CA(2)-CB(2)*CA(3))*A(3,1)+(CB(1)*CA(3)-
     1 CB(3)*CA(1))*A(3,2)+(CB(2)*CA(1)-CB(1)*CA(2))*A(3,3))
      CB(1)=UNX(I)*ZN2(I)-UNZ(I)*XN2(I)
      CA(1)=UNZ(I)*YN2(I)-UNY(I)*ZN2(I)
      CA(2)=UNZ(I)*XN2(I)-UNX(I)*ZN2(I)
      CA(3)=UNY(I)*XN2(I)-UNX(I)*YN2(I)
      XN(8)=XN(8)-YN*(CA(1)*COSF(S2)+CA(2)*SINF(S2))
      XN(10)=XN(10)-YN*(CA(1)*B(1,3)+CB(1)*B(2,3)+CA(3)*B(3,3))
      XN(12)=XN(12)-YN*CA(3)
  100 CONTINUE
      CONX=-CONX
      CONY=-CONY
      CONZ=-CONZ
      XN(1)=XN(1)*CONX
      XN(3)=XN(3)*CONY
      XN(5)=XN(5)*CONZ
   53 XN(2)=-XN(1)
      XN(4)=-XN(3)
      XN(6)=-XN(5)
   54 IF(LPM-IFN) 18,16,18
C     FORCE CEASES TO ACT  - - - SET INDEX = ZERO
   16 IFN=0
   18 IF(LPM-IFN1)31,31,30
   30 LDER=-1
      IFN1=LPM
   31 RETURN
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```
      SUBROUTINE DRAG(D,DF,DD,XC1,YC1,ZC1,XC2,YC2,ZC2,ICD,LDER,ICD1)

      SUBROUTINE DRAG(D,DF,DD,XC1,YC1,ZC1,XC2,YC2,ZC2,ICD,LDER,ICD1)
      COMMON T,ZERO,V1,V2,YDOT1,YDOT2
      DIMENSION             YDOT2(6),YDOT1(18),V2(6),V1(18),NH(6),HB(6)
      EQUIVALENCE (T,T),(ZERO,ZERO),
     1   (X1,V1(1)),(Y1,V1(2)),(Z1,V1(3)),(T1,V1(4)),(T2,V1(5))
     1,(F1,V1(6)),(F2,V1(7)),(S1,V1(8)),(S2,V1(9)),(X1X,V1(10)),(Y1Y,V1(
     111)),(Z1Z,V1(12)),(T1T,V1(13)),(T2T,V1(14)),(P1F,V1(15)),(P2P,V1(1
     16)),(S1S,V1(17)),(S2S,V1(18)),(X2,V2(1)),(Y2,V2(2)),(Z2,V2(3)),
     1(X2X,V2(4)),(Y2Y,V2(5)),(Z2Z,V2(6))
      COMMON A,B,AD,BD
      DIMENSION A(3,3),B(3,3),AD(3,3),BD(3,3)
      DIMENSION  D(8),DD(12),DF(8),XC1(8),YC1(8),ZC1(8),XC2(8),YC2(8),
     1ZC2(8),CA(3),CB(3),CC(3)
      LDRAG=0
      DO 4 J=1,12
      DD(J)=0.0
    4 CONTINUE
      DO 100 I=1,ICD
      DO 1 J=1,3
      CA(J)=A(1,J)*XC1(I)+A(2,J)*YC1(I)+A(3,J)*ZC1(I)
      CB(J)=B(1,J)*XC2(I)+B(2,J)*YC2(I)+B(3,J)*ZC2(I)
    1 CONTINUE
      DIX=X1-X2+CA(1)-CB(1)
      DIY=Y1-Y2+CA(2)-CB(2)
      DIZ=Z1-Z2+CA(3)-CB(3)
      XD=SQRTF(DIX**2+DIY**2+DIZ**2)
      IF(XD-DF(I)) 2,7,7
    7 LDRAG=LDRAG+1
      WRITE OUTPUT TAPE 6,101,T,X1,Y1,Z1,X2,Y2,Z2,XD,DF(I)
  101 FORMAT(9F14.6)
      GO TO 100
    2 DO 3 J=1,3
      CC(J)=AD(1,J)*XC1(I)+AD(2,J)*YC1(I)+AD(3,J)*ZC1(I)
     1     -(BD(1,J)*XC2(I)+BD(2,J)*YC2(I)+BD(3,J)*ZC2(I))
    3 CONTINUE
      VIX=X1X-X2X+CC(1)
      VIY=Y1Y-Y2Y+CC(2)
      VIZ=Z1Z-Z2Z+CC(3)
      V=SQRTF(VIX**2+VIY**2+VIZ**2)
      V=D(I)/V
      DD(1)=DD(1)-VIX*V
      DD(3)=DD(3)-VIY*V
      DD(5)=DD(5)-VIZ*V
      DD(7)=DD(7)-V*((VIX*CA(3)-VIZ*CA(1))*SINF(P1)
     1              +(VIZ*CA(2)-VIY*CA(3))*COSF(P1))
      DD(8)=DD(8)+V*((VIX*CB(3)-VIZ*CB(1))*SINF(P2)
     1              +(VIZ*CB(2)-VIY*CB(3))*COSF(P2))
      DD(9)=DD(9)-V*(VIY*CA(1)-VIX*CA(2))
      DD(10)=DD(10)+V*(VIY*CB(1)-VIX*CB(2))
      DD(11)=DD(11)-V*((VIZ*CA(2)-VIY*CA(3))*A(1,3)+(VIX*CA(3)-VIZ*CA(1)
     1)*A(2,3)+(VIY*CA(1)-VIX*CA(2))*A(3,3))
      DD(12)=DD(12)+V*((VIZ*CB(2)-VIY*CB(3))*B(1,3)+(VIX*CB(3)-VIZ*CB(1)
     1)*B(2,3)+(VIY*CB(1)-VIX*CB(2))*B(3,3))
  100 CONTINUE
    6 DD(2)=-DD(1)
      DD(4)=-DD(3)
```

```
      SUBROUTINE DRAG(D,DF,DD,XC1,YC1,ZC1,XC2,YC2,ZC2,ICD,LDER,ICD1)

      DD(6)=-DD(5)
    5 IF(LDRAG-ICD) 9,8,9
C     FORCE CEASES TO ACT  - - - SET INDEX = ZERO
    8 ICD=0
    9 IF(LDRAG-ICD1)11,11,10
   10 LDER=-1
      ICD1=LDRAG
   11 RETURN
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```
      SUBROUTINE COLGAS(ICG,GJ,FG1,FG2,FG3,VGX,VGY,VGZ,UGX,UGY,UGZ,

      SUBROUTINE COLGAS(ICG,GJ,FG1,FG2,FG3,VGX,VGY,VGZ,UGX,UGY,UGZ,
     1XG1,YG1,ZG1,XG2,YG2,ZG2,GST,TCG,LDER,ICG1)
      COMMON T,ZERO,V1,V2,YDOT1,YDOT2
      DIMENSION            YDOT2(6),YDOT1(18),V2(6),V1(18),NH(6),HB(6)
      EQUIVALENCE (T,T),(ZERO,ZERO),
     1   (X1,V1(1)),(Y1,V1(2)),(Z1,V1(3)),(T1,V1(4)),(T2,V1(5))
     1,(F1,V1(6)),(F2,V1(7)),(S1,V1(8)),(S2,V1(9)),(X1X,V1(10)),(Y1Y,V1(
     111)),(Z1Z,V1(12)),(T1T,V1(13)),(T2T,V1(14)),(F1F,V1(15)),(F2F,V1(1
     16)),(S1S,V1(17)),(S2S,V1(18)),(X2,V2(1)),(Y2,V2(2)),(Z2,V2(3)),
     1(X2X,V2(4)),(Y2Y,V2(5)),(Z2Z,V2(6))
      COMMON A,B,AD,BD
      DIMENSION A(3,3),B(3,3),AD(3,3),BD(3,3)
      DIMENSION GJ(12),FG1(8),FG2(8),VGX(8),VGY(8),VGZ(8),UGX(8),UGY(8),
     1 UGZ(8),XG1(8),YG1(8),ZG1(8),XG2(8),YG2(8),ZG2(8),CA(3),CB(3)
     1,TCG(8)
      COMMON WIL1,TCO
      DIMENSION TCO(8)
      LGAS=0
      DO 1 K=1,12
      GJ(K)=0.0
    1 CONTINUE
      DO 100 I=1,ICG
      IF(T-TCO(I))100,15,15
   15 CONTINUE
      IF (T-TCG(I)) 5,5,9
    9 LGAS=LGAS+1
      GO TO 100
    5 XJ=FG1(I)/(1.+FG2(I)*(T-TCO(I)))**FG3
      IF(ABSF(XJ)-GST) 9,6,6
    6 DO 2 J=1,3
      CA(J)=A(1,J)*VGX(I)+A(2,J)*VGY(I)+A(3,J)*VGZ(I)
    2 CONTINUE
    3 DO 8 J=1,3
    4 CB(J)=B(1,J)*UGX(I)+B(2,J)*UGY(I)+B(3,J)*UGZ(I)
    8 CONTINUE
      GJ(2)=GJ(2)+XJ*CB(1)
      GJ(4)=GJ(4)+XJ*CB(2)
      GJ(6)=GJ(6)+XJ*CB(3)
    7 GJ(1)=GJ(1)+XJ*CA(1)
      GJ(3)=GJ(3)+XJ*CA(2)
      GJ(5)=GJ(5)+XJ*CA(3)
      CA(1)=VGZ(I)*YG1(I)-VGY(I)*ZG1(I)
      CA(2)=VGZ(I)*XG1(I)-VGX(I)*ZG1(I)
      CA(3)=VGY(I)*XG1(I)-VGX(I)*YG1(I)
      CB(1)=UGZ(I)*YG2(I)-UGY(I)*ZG2(I)
      CB(2)=UGZ(I)*XG2(I)-UGX(I)*ZG2(I)
      CB(3)=UGY(I)*XG2(I)-UGX(I)*YG2(I)
      GJ(7)=GJ(7)+XJ*(CA(1)*COSF(S1)+CA(2)*SINF(S1))
      GJ(8)=GJ(8)+XJ*(CB(1)*COSF(S2)+CB(2)*SINF(S2))
      GJ(9)=GJ(9)+XJ*(CA(1)*A(1,3)-CA(2)*A(2,3)+CA(3)*A(3,3))
      GJ(10)=GJ(10)+XJ*(CB(1)*B(1,3)-CB(2)*B(2,3)+CB(3)*B(3,3))
      GJ(11)=GJ(11)+XJ*CA(3)
      GJ(12)=GJ(12)+XJ*CB(3)
  100 CONTINUE
      IF (LGAS-ICG) 11,10,11
C     FORCE CEASES TO ACT - - - SET INDEX = ZERO
```

```
      SUBROUTINE COLGAS(ICG,GJ,FG1,FG2,FG3,VGX,VGY,VGZ,UGX,UGY,UGZ,

   10 ICG=0
   11 IF(LGAS-ICG1)13,13,12
   12 LDER=-1
      ICG1=LGAS
   13 RETURN
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```
      SUBROUTINE ROCKET(IRK,R,RR,TE,TF,RUM,PUM,FM1,FM2,URX,URY,URZ,
```

```
      SUBROUTINE ROCKET(IRK,R,RR,TE,TF,RUM,PUM,FM1,FM2,URX,URY,URZ,
     1FM10,FM20,XR1,YR1,ZR1,XR2,YR2,ZR2,VRX,VRY,VRZ,SUN1,SUN2,SUN3,
     2SAN1,SAN2,SAN3,XX10,YY10,ZZ10,XX20,YY20,ZZ20,XX1,YY1,ZZ1,XX2,YY2,
     3ZZ2)
      COMMON T,ZERO,V1,V2,YDOT1,YDOT2
      DIMENSION              YDOT2(6),YDOT1(18),V2(6),V1(18),NH(6),HB(6)
      EQUIVALENCE (T,T),(ZERO,ZERO),
     1    (X1,V1(1)),(Y1,V1(2)),(Z1,V1(3)),(T1,V1(4)),(T2,V1(5))
     1,(P1,V1(6)),(P2,V1(7)),(S1,V1(8)),(S2,V1(9)),(X1X,V1(10)),(Y1Y,V1(
     111)),(Z1Z,V1(12)),(T1T,V1(13)),(T2T,V1(14)),(P1P,V1(15)),(P2P,V1(1
     16)),(S1S,V1(17)),(S2S,V1(18)),(X2,V2(1)),(Y2,V2(2)),(Z2,V2(3)),
     1(X2X,V2(4)),(Y2Y,V2(5)),(Z2Z,V2(6))
      COMMON A,B,AD,BD
      DIMENSION A(3,3),B(3,3),AD(3,3),BD(3,3)
      DIMENSION R(12),RR(16),URX(16),URY(16),URZ(16),VRX(16),VRY(16),
     1VRZ(16),XR1(16),YR1(16),ZR1(16),XR2(16),YR2(16),ZR2(16)
     1,TE(16),TF(16)  ,CA(3)
      LROC=0
      DO 1 I=1,12
      R(I)=0.0
    1 CONTINUE
      DO 30 I=1,IRK
      IF (T-TE(I)) 2,3,3
    3 IF (T-TF(I)) 8,8,5
    5 S=TF(I)-TE(I)
      FM1=FM10-RUM*S
      FM2=FM20-PUM*S
      XX1=XX10-SUN1*S
      YY1=YY10-SUN2*S
      ZZ1=ZZ10-SUN3*S
      XX2=XX20-SAN1*S
      YY2=YY20-SAN2*S
      ZZ2=ZZ20-SAN3*S
      LROC=LROC+1
      IF (LROC-IRK) 2,40,40
C     FORCE CEASES TO ACT  - - - SET INDEX = ZERO
   40 IRK=0
    2 GO TO 30
    8 S=T-TE(I)
      FM1=FM10-RUM*S
      FM2=FM20-PUM*S
      XX1=XX10-SUN1*S
      YY1=YY10-SUN2*S
      ZZ1=ZZ10-SUN3*S
      XX2=XX20-SAN1*S
      YY2=YY20-SAN2*S
      ZZ2=ZZ20-SAN3*S
      DO 21 K=1,3
      CA(K)=A(1,K)*URX(I)+A(2,K)*URY(I)+A(3,K)*URZ(I)
   21 CONTINUE
      R(1)=R(1)+RR(I)*CA(1)
      R(3)=R(3)+RR(I)*CA(2)
      R(5)=R(5)+RR(I)*CA(3)
      CA(1)=URZ(I)*YR1(I)-URY(I)*ZR1(I)
      CA(2)=URZ(I)*XR1(I)-URX(I)*ZR1(I)
      CA(3)=URY(I)*XR1(I)-URX(I)*YR1(I)
```

```
      SUBROUTINE ROCKET(IRK,R,RR,TE,TF,RUM,PUM,FM1,FM2,URX,URY,URZ,

      R(7)=R(7)+RR(I)*(CA(1)*COSF(S1)+CA(2)*SINF(S1))
      R(9)=R(9)+RR(I)*(CA(1)*A(1,3)-CA(2)*A(2,3)+CA(3)*A(3,3))
      R(11)=R(11)+RR(I)*CA(3)
   52 DO 51 K=1,3
      CA(K)=B(1,K)*VRX(I)+B(2,K)*VRY(I)+B(3,K)*VRZ(I)
   51 CONTINUE
      R(2)=R(2)+RR(I)*CA(1)
      R(4)=R(4)+RR(I)*CA(2)
      R(6)=R(6)+RR(I)*CA(3)
   53 CA(1)=VRZ(I)*YR2(I)-VRY(I)*ZR2(I)
      CA(2)=VRZ(I)*XR2(I)-VRX(I)*ZR2(I)
      CA(3)=VRY(I)*XR2(I)-VRX(I)*YR2(I)
      R(8)=R(8)+RR(I)*(CA(1)*COSF(S2)+CA(2)*SINF(S2))
      R(10)=R(10)+RR(I)*(CA(1)*B(1,3)-CA(2)*B(2,3)+CA(3)*B(3,3))
      R(12)=R(12)+RR(I)*CA(3)
   30 CONTINUE
   32 RETURN
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```
      SUBROUTINE UNIVSL(ISU,WO,UK,WIOQ,W,XU1,YU1,ZU1,XU2,YU2,ZU2,LDER,
```



```
      SUBROUTINE UNIVSL(ISU,WO,UK,WIOQ,W,XU1,YU1,ZU1,XU2,YU2,ZU2,LDER,
```



```
     1ISU1)
      DIMENSION WIOQ(1)
      COMMON T,ZERO,V1,V2,YDOT1,YDOT2
      DIMENSION             YDOT2(6),YDOT1(18),V2(6),V1(18),NH(6),HB(6)
      EQUIVALENCE (T,T),(ZERO,ZERO),
     1   (X1,V1(1)),(Y1,V1(2)),(Z1,V1(3)),(T1,V1(4)),(T2,V1(5))
     1,(F1,V1(6)),(F2,V1(7)),(S1,V1(8)),(S2,V1(9)),(X1X,V1(10)),(Y1Y,V1(
     111)),(Z1Z,V1(12)),(T1T,V1(13)),(T2T,V1(14)),(F1F,V1(15)),(F2F,V1(1
     16)),(S1S,V1(17)),(S2S,V1(18)),(X2,V2(1)),(Y2,V2(2)),(Z2,V2(3)),
     1(X2X,V2(4)),(Y2Y,V2(5)),(Z2Z,V2(6))
      COMMON A,B,AD,BD
      DIMENSION A(3,3),B(3,3),AD(3,3),BD(3,3)
      DIMENSION WO(8),W(12),XU1(8),YU1(8),ZU1(8),XU2(8),YU2(8),ZU2(8)
     1,UK(8),CA(3),CB(3)
   20 LUN=0
      DO 1 I=1,12
      W(I)=0.0
    1 CONTINUE
      DO 100 I=1,ISU
      DO 2 K=1,3
      CA(K)=A(1,K)*XU1(I)+A(2,K)*YU1(I)+A(3,K)*ZU1(I)
      CB(K)=B(1,K)*XU2(I)+B(2,K)*YU2(I)+B(3,K)*ZU2(I)
    2 CONTINUE
      DIX=X1+CA(1)-X2-CB(1)
      DIY=Y1+CA(2)-Y2-CB(2)
      DIZ=Z1+CA(3)-Z2-CB(3)
      DD=SQRTF(DIX**2+DIY**2+DIZ**2)
      FUT=UK(I)*DD
      IF(ABSF(DUT)-ABSF(WO(I)+WIOQ(I)))3,3,9
    9 LUN=LUN+1
      GO TO 100
    3 IF(DD) 6,7,6
    7 DIZ=1.0
      XW=WO(I)
      GO TO 8
    6 XW=WO(I)/DD-UK(I)
    8 W(1)=W(1)+XW*DIX
      W(3)=W(3)+XW*DIY
      W(5)=W(5)+XW*DIZ
      W(9)=W(9)+XW*(DIY*CA(1)-DIX*CA(2))
      W(7)=W(7)+XW*((DIX*CA(3)-DIZ*CA(1))*SINF(P1)+(DIZ*CA(2)-DIY*CA(3))
     1   *COSF(P1))
      W(11)=W(11)+XW*((DIZ*CA(2)-DIY*CA(3))*A(1,3)+(DIX*CA(3)-DIZ*CA(1))
     1   *A(2,3)+(DIY*CA(1)-DIX*CA(2))*A(3,3))
      W(8)=W(8)-XW*((DIX*CB(3)-DIZ*CB(1))*SINF(F2)+(DIZ*CB(2)-DIY*CB(3))
     1   *COSF(P2))
      W(10)=W(10)-XW*(DIY*CB(1)-DIX*CB(2))
      W(12)=W(12)-XW*((DIZ*CB(2)-DIY*CB(3))*B(1,3)+(DIX*CB(3)-DIZ*CB(1))
     1   *B(2,3)+(DIY*CB(1)-DIX*CB(2))*B(3,3))
  100 CONTINUE
    4 W(2)=-W(1)
      W(4)=-W(3)
      W(6)=-W(5)
    5 IF(LUN-ISU) 11,10,11
C     FORCE CEASES TO ACT  - - - SET INDEX = ZERO
```

```
      SUBROUTINE UNIVSL(ISU,WO,UK,WIOQ,W,XU1,YU1,ZU1,XU2,YU2,ZU2,LDER,

   10 ISU=0
   11 IF(LUN-ISU1)13,13,12
   12 LDER=-1
      ISU1=LUN
   13 RETURN
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```
SUBROUTINE MCROS (A,B,C)

SUBROUTINE MCROS (A,B,C)
DIMENSION A(1),B(1),C(1)
C(1)=A(2)*B(3)-A(3)*B(2)
C(2)=-A(1)*B(3)+A(3)*B(1)
C(3)=A(1)*B(2)-A(2)*B(1)
RETURN
END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

SUBROUTINE MSR(A,B,C,J)

```
      SUBROUTINE MSR(A,B,C,J)
      DIMENSION A(3,3),B(3)
      C=0.
      DO 1 I=1,3
    1 C=C+B(I)*A(J,I)
      RETURN
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```
      SUBROUTINE MSC (A,B,C,J)

      SUBROUTINE MSC (A,B,C,J)
      DIMENSION A(3,3),B(3)
      C=0.
      DO 1 I=1,3
    1 C=C+B(I)*A(I,J)
      RETURN
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```
      SUBROUTINE MSCAL(A,B,C)

      SUBROUTINE MSCAL(A,B,C)
      DIMENSION B(3),C(3)
      DO 1 I=1,3
    1 C(I)=A*B(I)
      RETURN
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```
      SUBROUTINE MADD(A,B,C,M1,M2)

      SUBROUTINE MADD(A,B,C,M1,M2)
      DIMENSION A(3,3),B(3,3),C(3,3)
      DO 1 I=1,M1
      DO 1 J=1,M2
    1 C(I,J)=A(I,J)+B(I,J)
      RETURN
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```
      SUBROUTINE MSUB(A,B,C,M1,M2)

      SUBROUTINE MSUB(A,B,C,M1,M2)
      DIMENSION A(3,3),B(3,3),C(3,3)
      DO 1 I=1,M1
      DO 1 J=1,M2
    1 C(I,J)=A(I,J)-B(I,J)
      RETURN
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

SUBROUTINE MULT3(A,B,C,M1,M2,M3)

```
      SUBROUTINE MULT3(A,B,C,M1,M2,M3)
      DIMENSION A(3,3),B(3,3),C(3,3)
      DO 2 I=1,M1
      DO 2 K=1,M3
      C(I,K)=0.
      DO 2 J=1,M2
    2 C(I,K)=C(I,K)+A(I,J)*B(J,K)
      RETURN
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

SUBROUTINE MULT2(A,B,C,M1,M2,M3)

```
      SUBROUTINE MULT2(A,B,C,M1,M2,M3)
      DIMENSION A(3,3),B(3,3),C(3,3)
      DO 2 I=1,M1
      DO 2 K=1,M3
      C(I,K)=0.
      DO 2 J=1,M2
    2 C(I,K)=A(I,J)*B(K,J)+C(I,K)
      RETURN
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```
      SUBROUTINE MULT1(A,B,C,M1,M2,M3)

      SUBROUTINE MULT1(A,B,C,M1,M2,M3)
      DIMENSION A(3,3),B(3,3),C(3,3)
      DO 2 I=1,M1
      DO 2 K=1,M3
      C(I,K)=0.
      DO 2 J=1,M2
    2 C(I,K)=A(J,I)*B(J,K)+C(I,K)
      RETURN
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

SUBROUTINE SWITCH(A1,A2,A3,A4,A5,A6,I,J,T)

```
      SUBROUTINE SWITCH(A1,A2,A3,A4,A5,A6,I,J,T)
      DIMENSION A1(3,1),A2(3,1),A3(3,1)
      DO 1 K=1,3
      CALL SW1(I,J,A1(K,1))
      CALL SW1(I,J,A2(K,1))
    1 CALL SW1(I,J,A3(K,1))
      CALL SW (I,J,A4)
      CALL SW(I,J,A5)
      CALL SW (I,J,A6)
      WRITE OUTPUT TAPE 6,100,I,J,T
  100 FORMAT (7H SPRING,I3,19H  INTERCHANGED WITH,I3,7H  AT T=E11.4)
      RETURN
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

SUBROUTINE SW(I,J,A)

```
SUBROUTINE SW(I,J,A)
DIMENSION A(1)
B=A(I)
A(I)=A(J)
A(J)=B
RETURN
END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```
SUBROUTINE SW1(I,J,A)

SUBROUTINE SW1(I,J,A)
 DIMENSION A(3,1)
Q=A(1,J)
A(1,J)=A(1,I)
A(1,I)=Q
RETURN
END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```
      SUBROUTINE ERR (DEL,TPR,TP,TG,TA,TB,TC,TD,TRIG1,TRIG2,TRIG3,
```

```
      SUBROUTINE ERR (DEL,TPR,TP,TG,TA,TB,TC,TD,TRIG1,TRIG2,TRIG3,
     1 TRIG4,TROC,PYRO2,PYRO3,PLOT )
      EQUIVALENCE (T,T),(ZERO,ZERO),
     1              (X1,V1(1)),(Y1,V1(2)),(Z1,V1(3)),(T1,V1(4)),(T2,V1(5))
     1,(P1,V1(6)),(P2,V1(7)),(S1,V1(8)),(S2,V1(9)),(X1X,V1(10)),(Y1Y,V1(
     111)),(Z1Z,V1(12)),(T1T,V1(13)),(T2T,V1(14)),(P1P,V1(15)),(P2P,V1(1
     16)),(S1S,V1(17)),(S2S,V1(18)),(X2,V2(1)),(Y2,V2(2)),(Z2,V2(3)),
     1(X2X,V2(4)),(Y2Y,V2(5)),(Z2Z,V2(6))
      COMMON T,ZERO,V1,V2,YDOT1,YDOT2
      DIMENSION          YDOT2(6),YDOT1(18),V2(6),V1(18),NH(6),HB(6)
      COMMON A,B,AD,BD
      DIMENSION A(3,3),B(3,3),AD(3,3),BD(3,3)
      WRITE OUTPUT TAPE 6,1
    1 FORMAT(1H1,21HERROR OCCURS IN FMARK )
      WRITE OUTPUT TAPE 6,2,T
    2 FORMAT(1H0,7HTIME = E15.7/)
      WRITE OUTPUT TAPE 6,7,DEL,TPR,TP
    7 FORMAT (1H0,9HSTEP SIZE,E16.5 /
     111H PRINT STEP,E16.5/
     118H PYRO FIRING TIMES,E16.5)
      WRITE OUTPUT TAPE 6,8,TG,TA,TB,TC,TD
    8 FORMAT (1H ,15HPIN PULLER TIME,E16.5/
     119H ROCKET TIMES--- TA,E16.5,2HTB,E16.5,2HTC,E16.5,2HTD,E16.5)
      IF (T-TRIG1)10,9,9
   10 WRITE OUTPUT TAPE 6,11,TRIG1
   11 FORMAT (1H ,13HPRINT TRIGGERE16.5)
    9 IF (T-TRIG2)12,13,13
   12 WRITE OUTPUT TAPE 6,14,TRIG2
   14 FORMAT (1H ,18HPIN PULLER TRIGGER E16.5)
   13 IF (T-TRIG3) 15,16,16
   15 WRITE OUTPUT TAPE 6,17,TRIG3
   17 FORMAT (1H ,19HEND OF STEP TRIGGER E16.5)
   16 IF (T-TRIG4) 18,19,19
   18 WRITE OUTPUT TAPE 6,20,TRIG4
   20 FORMAT (1H ,16H1ST PYRO TRIGGER  E16.5)
   19 IF (T-PYRO2) 21,22,22
   21 WRITE OUTPUT TAPE 6,23,PYRO2
   23 FORMAT (1H ,16H2ND PYRO TRIGGER  E16.5)
   22 IF (T-PYRO3) 24,25,25
   24 WRITE OUTPUT TAPE 6,26,PYRO3
   26 FORMAT (1H ,16H3RD PYRO TRIGGER  E16.5)
   25 IF (T-TROC) 27,28,28
   27 WRITE OUTPUT TAPE 6,29,TROC
   29 FORMAT (1H ,14HROCKET TRIGGER E16.5)
   28 IF (T-PLOT) 30,31,31
   30 WRITE OUTPUT TAPE 6,32,PLOT
   32 FORMAT (1H ,12HPLOT TRIGGER  E16.5)
   31 CONTINUE
      DO 3 I=1,18
      WRITE OUTPUT TAPE 6,4,I,V1(I),I,YDOT1(I)
    3 CONTINUE
    4 FORMAT(1H ,3HV1( I2,1H) E15.7,5X,6HYDOT1( I2,1H)E15.7)
      DO 5 I=1,6
      WRITE OUTPUT TAPE 6,6,I,V2(I),I,YDOT2(I)
    5 CONTINUE
    6 FORMAT(1H ,3HV2( I2,1H) E15.7,5X,6HYDOT2( I2,1H)E15.7)
```

```
SUBROUTINE ERR (DEL,TPR,TF,TG,TA,TB,TC,TD,TRIG1,TRIG2,TRIG3,

CALL DUMP
RETURN
END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

SOLVE   LINEAR EQUATION SOLVER WITH ITERATIVE IMPROVEMENT

```
      SUBROUTINE SOLVE(NN,AA,BB,IN,EPS,ITMAX,X,ITT)
C     SOLVES AX=B WHERE A IS NXN MATRIX AND B IS NX1 VECTOR
C     IN=
C         1 FOR FIRST ENTRY
C         2 FOR SUBSEQUENT ENTRIES WITH NEW B
C         3 TO RESTORE AA AND BB
C     EPS AND ITMAX ARE PARAMETERS IN THE ITERATION
C     ITT=
C         -1 IF AA IS SINGULAR
C         0 IF NOT CONVERGENT
C         NUMBER OF ITERATIONS IF CONVERGENT
C     CALLS FAP SUBROUTINES EQUIL,DOT AND DAD
C
      DIMENSION AA(15,15),A(15,15),KA(15,15),B(15),BB(15),X(15),Z(15),
     1CM(15),RM(15),JCP(15),IRP(15)
      EQUIVALENCE (A,KA)
C
C     MA MUST = DECLARED DIMENSION OF SYSTEM
C
      MA=15
C
C
C
      MA1=MA+1
      GO TO (1,2,3),IN
    1 N=NN
      IP=1
C
C     EQUILIBRATION
C
      CALL EQUIL1(AA,KA,N,MA)
      DO 510 I=1,N
          KT=KA(I,1)
          DO 503 J=2,N
          IF(KT-KA(I,J)) 502,503,503
  502     KT=KA(I,J)
  503      CONTINUE
           RM(I)=2.0**KT
          DO 509 J=1,N
  509     KA(I,J)=KA(I,J)-KT
  510 CONTINUE
      DO 520 J=1,N
          KT=KA(1,J)
          DO 513 I=2,N
          IF(KT-KA(I,J))512,513,513
  512     KT=KA(I,J)
  513     CONTINUE
          CM(J)=2.0**KT
          DO 519 I=1,N
  519     KA(I,J)=KA(I,J)-KT
  520 CONTINUE
      CALL EQUIL2
      DO 530 I=1,N
  530     BB(I)=BB(I)/RM(I)
C
```

SOLVE LINEAR EQUATION SOLVER WITH ITERATIVE IMPROVEMENT

```
C     SAVE EQUILIBRATED DATA
C
      DO 548 I=1,N
         DO 548 J=1,N
  548    A(I,J)=AA(I,J)
      IP=2
C
C     GAUSSIAN ELIMINATION WITH COMPLETE PIVOTING
C
      NM1=N-1
      DO 99 M=1,NM1
         TOP=ABSF(A(M,M))
         IMAX=M
         JMAX=M
         DO 12 I=M,N
         DO 12 J=M,N
            IF(TOP-ABSF(A(I,J)))10,12,12
   10       TOP=ABSF(A(I,J))
            IMAX=I
            JMAX=J
   12    CONTINUE
         IF(TOP)14,13,14
   13    IT=-1
         IP=10
         ITT=IT
         RETURN
   14    IRP(M)=IMAX
         JCP(M)=JMAX
         IF(JMAX-M)29,23,21
   21    DO 22 I=1,N
            TEMP=A(I,M)
            A(I,M)=A(I,JMAX)
   22       A(I,JMAX)=TEMP
   23    IF(IMAX-M)29,29,24
   24    DO 25 J=1,N
            TEMP=A(M,J)
            A(M,J)=A(IMAX,J)
   25       A(IMAX,J)=TEMP
   29    MP1=M+1
         DO 33 I=MP1,N
            EM=A(I,M)/A(M,M)
            A(I,M)=EM
            DO 32 J=MP1,N
   32       A(I,J)=A(I,J)-A(M,J)*EM
   33    CONTINUE
   99 CONTINUE
C     STORAGE FOR A NOW CONTAINS TRIANGULAR L AND U SO THAT (L+I)*U=A
      IF(A(N,N))101,93,101
   93    IT=-1
         IP=10
         ITT=IT
         RETURN
  101 IRP(N)=N
      JCP(N)=N
C
C     DUPLICATE INTERCHANGES IN DATA
```

```
SOLVE   LINEAR EQUATION SOLVER WITH ITERATIVE IMPROVEMENT
C
      DO 219 J=1,N
         IF(J-JP)211,219,211
  211    DO 212 I=1,N
            TEMP=AA(I,J)
            AA(I,J)=AA(I,JP)
  212       AA(I,JP)=TEMP
  219 CONTINUE
      DO 229 I=1,N
         IP=IRP(I)
         IF(I-IP)221,229,221
  221    DO 222 J=1,N
            TEMP=AA(I,J)
            AA(I,J)=AA(IP,J)
  222       AA(IP,J)=TEMP
         TEMP=BB(I)
         BB(I)=BB(IP)
         BB(IP)=TEMP
  229 CONTINUE
      IP=3
      GO TO 199
C
C     PROCESS NEW RIGHT HAND SIDE
C
    2 N=N
      IP=4
      DO 601 I=1,N
  601    BB(I)=BB(I)/RM(I)
      DO 609 M=1,NM1
      IP=IRP(M)
         TEMP=BB(M)
         BB(M)=BB(IP)
         BB(IP)=TEMP
  609 CONTINUE
      GO TO 199
C
C     SOLVE FOR FIRST APPROXIMATION TO X
C
  199 DO 200 I=1,N
         IM1=I-1
         BEE=BB(I)
  200    B(I)=-DOT(IM1,A,I,MA,B,1,1,BEE,1.D)
      DO 201 K=1,N
         L=K-1
         I=N-L
         IA=I*MA1
         IX=I+1
         BE=B(I)
         D=A(I,I)
  201    X(I)=-DOT(L,A,IA,MA,X,IX,1,BE,D)
      IP=5
C
C     ITERATIVE IMPROVEMENT
C
      IF(ITMAX)370,370,300
```

SOLVE   LINEAR EQUATION SOLVER WITH ITERATIVE IMPROVEMENT

```
  300 TOP=0.0
      DO 303 I=1,N
          IF(TOP-ABSF(X(I)))302,303,303
  302     TOP=ABSF(X(I))
  303 CONTINUE
      EPSX=EPS*TOP
      DO 369 IT=1,ITMAX
C     FIND RESIDUALS
          DO 319 I=1,N
              BEE=BB(I)
  319         Z(I)=-DOT(N,AA,I,MA,X,1,1,BEE,1.D)
          IF=11
C     FIND INCREMENT
          DO 329 I=1,N
              IM1=I-1
              ZEE=Z(I)
  329         B(I)=-DOT(IM1,A,I,MA,B,1,1,ZEE,1.D)
          DO 339 K=1,N
              L=K-1
              I=N-L
              IA=I*MA1
              IZ=I+1
              BE=B(I)
              D=A(I,I)
  339         Z(I)=-DOT(L,A,IA,MA,Z,IZ,1,BE,D)
          IF=6
C     INCREMENT AND TEST
          TOP=0.0
          DO 342 I=1,N
              TEMP=X(I)
              ZEE=Z(I)
              X(I)=DAD(TEMP,ZEE)
              DELX=ABSF(X(I)-TEMP)
              IF(TOP-DELX)341,342,342
  341         TOP=DELX
  342     CONTINUE
          IF=7
          IF(TOP-EPSX)381,381,369
  369 CONTINUE
  370 IT=0
  381 DO 383 K=1,N
          I=N-K+1
          IP=JCF(I)
          TEMP=X(I)
          X(I)=X(IP)
  383     X(IP)=TEMP
      DO 385 I=1,N
  385     X(I)=X(I)/CM(I)
      IF=9
      ITT=IT
      RETURN
C
C     RESTORE AA AND BB
C
    3 NP1=N+1
      DO 709 K=1,N
```

SOLVE   LINEAR EQUATION SOLVER WITH ITERATIVE IMPROVEMENT

```
          I=NP1-K
          IP=IRP(I)
          IF(I-IP)701,709,701
  701     DO 702 J=1,N
                TEMP=AA(I,J)
                AA(I,J)=AA(IP,J)
  702           AA(IP,J)=TEMP
          TEMP=BB(I)
          BB(I)=BB(IP)
          BB(IP)=TEMP
  709 CONTINUE
      DO 719 K=1,N
          J=NP1-K
          JP=JCP(J)
          IF(JP-J)711,719,711
  711     DO 712 I=1,N
                TEMP=AA(I,J)
                AA(I,J)=AA(I,JP)
  712           AA(I,JP)=TEMP
  719 CONTINUE
      DO 721 I=1,N
          R=RM(I)
          BB(I)=BB(I)*R
          DO 721 J=1,N
  721     AA(I,J)=AA(I,J)*R
      DO 722 J=1,N
          C=CM(J)
          DO 722 I=1,N
  722     AA(I,J)=AA(I,J)*C
      IP=12
      RETURN
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

```
        SEPARATION OF TWO BODIES IN SPACE
C
      COMMON Y,TPLOT
      DIMENSION Y(38),W(15 000),LOT(14),A(36),X(4),S(4),BUF(100)
      DO 5040 I=1,14
 5040 LOT(I)=0
      READ INPUT TAPE 5,1,(LOT(I),I=1,14)
    1 FORMAT (14I5)
      NT3=38
      NT2=8
      NT1=18
    2 FORMAT(E12.8,I12)
      NO=0
      PI2=360.0
      S(1)=119.0
      S(2)=903.0
      S(3)=1023.0
      S(4)=0.0
      X(1)=0.0
      NPTS=Y(37)
      MPTS=NPTS+NPTS
      ZERO=0.0
      REWIND NT2
      L=0
    3 L=L+1
      IF (L-15) 100,101,101
  100 IF (LOT(L)) 5,3,5
    5 CALL ADV(18)
      IF(LOT(L)-12)7,7,8
    7 I=3*L-3
      READ TAPE NT2,(A(K),K=1,36)
      IREAD=1
      DO 6 K=1,36
      Y(K)=ABSF(A(K))
    6 CONTINUE
      W(1)=A(I+1)
      W(2)=A(I+2)
      W(3)=A(I+3)
      J=0
      DO 9 II=2,NPTS
      READ TAPE NT2,(A(K),K=1,36)
      J=J+3
      DO 9 JJ=1,3
      K=I+JJ
      IF (ABSF(A(K))-Y(K)) 106,106,107
  107 Y(K)=ABSF(A(K))
  106 KK=J+JJ
      W(KK)=A(K)
    9 CONTINUE
      REWIND NT2
   10 NO=NO+1
      I=3*L-3+IREAD
      FINAL=Y(I)
      IF(FINAL-1.0) 606,605,605
  606 WRITE OUTPUT TAPE NT3,750,FINAL
  750 FORMAT(1H1,F13.5)
```

SEPARATION OF TWO BODIES IN SPACE

```
      GO TO 751
  605 WRITE OUTPUT TAPE NT3,500,FINAL
  500 FORMAT(1H1,F13.2)
  751 CALL GRID (NT1,20,20,119,903)
      IF(L-6) 26,26,11
   26 WRITE OUTPUT TAPE 6,660
  660 FORMAT(1H0)
      CALL SET(444)
      WRITE OUTPUT TAPE NT3,501
  501 FORMAT (1H ,6HINCHES,4X,3H0.0)
      IF (L-2) 90,19,20
   90 JJ=1
      GO TO 91
   19 JJ=2
      GO TO 91
   20 WRITE OUTPUT TAPE NT3,508
  508 FORMAT (1H ,5H/ SEC)
      IF (L-4) 90,19,93
   93 WRITE OUTPUT TAPE NT3,508
      IF(L-6) 90,19,11
   91 FINAL=-FINAL
      CALL SET(895)
      IF(FINAL+1.0) 780,780,781
  781 WRITE OUTPUT TAPE NT3,754,FINAL
      GO TO 782
  780 WRITE OUTPUT TAPE NT3,502,FINAL
  502 FORMAT(1H ,F13.2)
  782 CALL SET(911)
      WRITE OUTPUT TAPE NT3,503,Y(38)
  503 FORMAT (1H ,14X,3H0.0,48X,7HSECONDS,46X,F7.2)
      IF (L-2) 22,22,23
   23 IF (L-4) 24,24,25
   24 WRITE OUTPUT TAPE NT3,509,NO
  509 FORMAT (1H0,16HVELOCITY VS TIME,100X,7HPLOT NO,I2)
      GO TO 12
   25 WRITE OUTPUT TAPE NT3,510,NO
  510 FORMAT (1H0,20HACCELERATION VS TIME,96X,7HPLOT NO,I2)
      GO TO 12
   22 WRITE OUTPUT TAPE NT3,504,NO
  504 FORMAT (1H0,16HDISTANCE VS TIME,100X,7HPLOT NO,I2)
   12 IF (IREAD-2) 14,15,16
   14 WRITE OUTPUT TAPE NT3,505,JJ
  505 FORMAT (1H0,1HX,I2)
      GO TO 17
   15 WRITE OUTPUT TAPE NT3,506,JJ
  506 FORMAT (1H0,1HY,I2)
      GO TO 17
   16 WRITE OUTPUT TAPE NT3,507,JJ
  507 FORMAT (1H0,1HZ,I2)
   17 I=3*L-3+IREAD
      X(2)=-Y(I)
      X(3)=Y(38)
      X(4)=Y(I)
      IF(Y(I)) 600,601,600
  601 X(2)=-0.01
      X(4)=0.01
```

```
      SEPARATION OF TWO BODIES IN SPACE

  600 T=0.0
      CALL SDINIT (BUF,100,X,S,NT1)
      II=IREAD-3
      DO 94 J=1,NPTS
      II=II+3
      CALL SDNPUT (T,W(II),BUF,0)
      T=T+TPLOT
   94 CONTINUE
      CALL STERM (NT1,BUF)
      IREAD=IREAD+1
      IF (IREAD-3) 10,10,3
   55 J=1
  150 NO=NO+1
      IF(FINAL-1.0) 757,758,758
  757 WRITE OUTPUT TAPE NT3,750,FINAL
      GO TO 759
  758 WRITE OUTPUT TAPE NT3,500,FINAL
  759 CALL GRID (NT1,20,20,119,903)
   11 WRITE OUTPUT TAPE 6,660
      CALL SET(444)
      WRITE OUTPUT TAPE NT3,511
  511 FORMAT (1H ,7HDEGREES,3X,3H0.0)
      IF (L-9) 27,28,28
   28 WRITE OUTPUT TAPE NT3,508
      IF (L-11) 27,29,30
   30 IF (L-12) 27,29,27
   29 WRITE OUTPUT TAPE NT3,508
   27 CALL SET(895)
      FINAL=-FINAL
      IF(FINAL+1.0) 752,752,753
  753 WRITE OUTPUT TAPE NT3,754,FINAL
  754 FORMAT(1H ,F13.5)
      GO TO 755
  752 WRITE OUTPUT TAPE NT3,502,FINAL
  755 CALL SET(911)
      IF (L-12) 31,31,32
   31 WRITE OUTPUT TAPE NT3,503,Y(38)
      GO TO 33
   32 FINAL=-FINAL
609   WRITE OUTPUT TAPE NT3,512,FINAL
  512 FORMAT (1H ,14X,3H0.0,47X,7HDEGREES,48X,F6.1)
   33 IF (L-8) 36,34,35
   36 JJ=1
      GO TO 39
   34 JJ=2
   39 WRITE OUTPUT TAPE NT3,513,NO
  513 FORMAT (1H0,13HANGLE VS TIME,103X,7HPLOT NO,I2)
      GO TO 40
   35 IF (L-10) 41,37,38
   41 JJ=1
      GO TO 700
   37 JJ=2
  700 WRITE OUTPUT TAPE NT3,509,NO
      GO TO 40
   38 IF(L-12) 701,44,45
  701 JJ=1
```

```
      SEPARATION OF TWO BODIES IN SPACE

      GO TO 702
  44  JJ=2
 702  WRITE OUTPUT TAPE NT3,510,NO
  40  IF (IREAD-2) 46,47,48
  46  WRITE OUTPUT TAPE NT3,514,JJ
 514  FORMAT (1H0,5HTHETA,I2)
      GO TO 49
  47  WRITE OUTPUT TAPE NT3,515,JJ
 515  FORMAT (1H0,3HPHI,I2)
      GO TO 49
  48  WRITE OUTPUT TAPE NT3,516,JJ
 516  FORMAT (1H0,3HPSI,I2)
      GO TO 49
  45  WRITE OUTPUT TAPE NT3,517,NO
 517  FORMAT (1H0,17HVELOCITY VS ANGLE,99 X,7HPLOT NO,I2)
      IF(L-13) 770,771,770
 770  JJ=2
      GO TO 40
 771  JJ=1
      GO TO 40
  49  IF(L-12) 17,17,53
  53  X(3)=PI2
      CALL SDINIT (BUF,100,X,S,NT1)
      I=J
      IF( J-MPTS) 65,151,152
  65  CALL SDNPUT (W(J),W(J+1),BUF,0)
      I=J+2
 151  CALL SDNPUT (W(I),W(I+1),BUF,0)
  66  IF (W(I)-W(J)) 69,68,67
  68  J=I
      I=I+2
      IF(I-MPTS) 65,152,152
  69  K=I
      IGO=1
  70  IF (W(K+2)-W(K)) 71,71,72
  72  J=K+2
      CALL STERM (NT1,BUF)
      GO TO 150
  71  K=K+2
      CALL SDNPUT (W(K),W(K+1),BUF,0)
      IF(K-MPTS) 73,73,74
  73  GO TO (70,75),IGO
 152  CALL STERM(NT1,BUF)
  74  IREAD=IREAD+1
      IF (IREAD-3) 78,78,3
  67  K=I
      IGO=2
  75  IF (W(K+2)-W(K)) 72,71,71
   8  IREAD=1
  78  Y(19)=180.0
      Y(22)=180.0
      DO 105 I=1,2
      Y(I+19)=360.0
      Y(I+22)=360.0
 105  CONTINUE
      I=0
```

```
      SEPARATION OF TWO BODIES IN SPACE

      READ TAPE NT2,(A(K),K=1,36)
      IF (L-13) 102,103,102
  102 I=3
  103 K=18+IREAD+I
      X(2)=-Y(K)
      X(4)=Y(K)
      IF(Y(K)) 602,603,602
603   X(2)=-0.01
      X(4)=0.01
  602 FINAL=Y(K)
      W(1)=A(K)
      W(2)=A(K+6)
      Y(K+6)=ABSF(A(K+6))
      J=1
      DO 104 II=2,NPTS
      J=J+2
      READ TAPE NT2,(A(K),K=1,36)
      W(J)=A(K)
      W(J+1)=A(K+6)
      IF (ABSF(W(J+1))-Y(K+6)) 104,104,110
  110 Y(K+6)=ABSF(W(J+1))
  104 CONTINUE
      REWIND NT2
      GO TO 55
  101  CALL CHAIN(1,3)
      END(1,0,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

Sample Problem

```
CASE 23904

INPUT DATA

DELTA TIME =  1.000E-02 SEC   DELTA PRINT=  0.100E-00 SEC   TIME(INITIAL) 0.       SEC   SPIN-UP RATE  0.       DEG/SEC

INPUT FOR BODY1      MASS =   0.1000000E 04 LBM

           INERTIA MATRIX LBM  IN SQ                    RATES ABOUT DRAWING         COORDS IN DRAWING
                                                        BOARD AXIS DEG/SEC          BOARD CS OF CM-INCHES
IXX    0.9272000E 06     IYY    0.9504000E 06     X      0.                   X      0.
IXY    0.                IYZ    0.                Y      0.                   Y      0.
IXZ    0.                IZZ    0.4636000E 06     Z      0.                   Z      0.1500000E 02

INPUT FOR BODY2      MASS =   0.1800000E 04 LBM

           INERTIA MATRIX LBM  IN SQ                    RATES ABOUT DRAWING         COORDS IN DRAWING
                                                        BOARD AXIS DEG/SEC          BOARD CS OF CM-INCHES
IXX    0.1159000E 07     IYY    0.1159000E 07     X      0.                   X      0.
IXY    0.                IYZ    0.                Y      0.                   Y      0.
IXZ    0.                IZZ    0.4636000E 06     Z      0.                   Z     -0.1000000E 02

ICD = 0    IPR = 0    ICG = 3    IPN = 0    IRK = 1    ISP = 4    ISU = 0    ION = 0


BODY 1-FIXED AXES     X  Y  Z
DRAWING BOARD AXES    X  Z -Y

BODY 2-FIXED AXES     X  Y  Z
DRAWING BOARD AXES    X  Z -Y
```

COLD GAS JET INPUT

F3 0.1500000E 01 VALUE JET FORCE BECOMES ZERO 0. LBF

COSINE ANGLE THRUST AND DRAW BOARD-BODY 1			COLD GAS JET LOCATIONS-IN		
UX	UY	UZ	X	Y	Z
-0.1000000E 01	0.	0.	0.	-0.6500000E 02	0.1500000E 02
0.5000000E 00	-0.8660000E 00	0.	0.5640000E 02	0.3250000E 02	0.1500000E 02
0.5000000E 00	0.8660000E 00	0.	-0.5640000E 02	0.3250000E 02	0.1500000E 02

CONSTANTS JET I		TIME JET I GOES	
F1 LBF	F2 1/SEC	ON SEC	OFF SEC
0.5000000E 02	0.1000000E 01	0.2000000E 01	0.7000000E 01
0.5000000E 02	0.1000000E 01	0.2000000E 01	0.7000000E 01
0.5000000E 02	0.1000000E 01	0.2000000E 01	0.7000000E 01

ROCKETS INPUT

COSINE ANGLE THRUST AND DRAW BOARD

BODY 1			BODY 2		
UX	UY	UZ	UX	UY	UZ
0.	0.	0.1000000E 01	0.	0.	0.

TIME FIRING STARTS SEC	TIME FIRING CEASES SEC
0.1000000E 02	0.1500000E 02

DECREASES IN PRINCIPAL MOMENTS OF INERTIA LBM IN SQ/SEC

BODY 1			BODY 2		
XX	YY	ZZ	XX	YY	ZZ
0.	0.	0.	0.	0.	0.

MASS FLOW RATE LBM/SEC		THRUST MAGNITUDE	ROCKET LOCATIONS		
BODY 1	BODY 2	LBF	X	Y	Z
0.	0.	0.1000000E 03	0.	0.	0.

SPRING INPUT

EFFICIENCY	RESTRICT INDEX
0.1000000E 01	2

COSINE DRAW BOARD AND SPRING			SPRING LOCATIONS-IN		
UX	UY	UZ	X	Y	Z
0.	0.	0.1000000E 01	-0.5660000E 02	-0.5660000E 02	-0.
0.	0.	0.1000000E 01	-0.5660000E 02	0.5660000E 02	-0.
0.	0.	0.1000000E 01	0.5660000E 02	0.5660000E 02	-0.
0.	0.	0.1000000E 01	0.5660000E 02	-0.5660000E 02	-0.

INITIAL FORCE LBF	SPRING CONSTANT LBF/IN	RESIDUAL FORCE LBF
0.4000000E 03	0.1000000E 03	-0.
0.4000000E 03	0.1000000E 03	-0.
0.4000000E 03	0.1000000E 03	-0.
0.3200000E 03	0.8000000E 02	-0.

```
TIME =    0.              SECONDS     STEP SIZE=  1.0000000E-02
BODY 1
SPEED OF CM =   0.               IN/SEC
RATE ABOUT INSTANTANEOUS AXIS=   0.              DEG/SEC       MAGNITUDE VECTOR SUM OF PITCH-YAW RATES=   0.

INERTIAL COORDS-INCHES   1ST DERIVATIVE /SEC      EULER ANGLES-DEGREES     1ST DERIVATIVE /SEC          RATE ABOUT
                                                                                                    BODY AXES DEG/SEC
X       0.                X       0.              THETA  0.9000000E 02     THETA  0.                X        0.
Y       0.                Y       0.              PHI    0.                PHI    0.                Y       -0.
Z       0.1607143E 02     Z       0.              PSI    0.                PSI   -0.                Z        0.
BODY 2
SPEED OF CM =   0.               IN/SEC
RATE ABOUT INSTANTANEOUS AXIS=   0.              DEG/SEC       MAGNITUDE VECTOR SUM OF PITCH-YAW RATES=   0.

INERTIAL COORDS-INCHES   1ST DERIVATIVE /SEC      EULER ANGLES-DEGREES     1ST DERIVATIVE /SEC          RATE ABOUT
                                                                                                    BODY AXES DEG/SEC
X       0.                X      -0.              THETA  0.9000000E 02     THETA  0.                X        0.
Y       0.                Y       0.              PHI    0.                PHI    0.                Y       -0.
Z      -0.8928571E 01     Z       0.              PSI    0.                PSI   -0.                Z        0.

SEPARATION DISTANCE =   0.2500000E 02 IN                         SEPARATION VELOCITY =   0.         IN/SEC

CONSTRAINT VALUES   0.1907349E-05  -0.1907349E-05  -0.

                     A1                                                     A2
 1.000000E 00   0.               0.                   1.000000E 00   0.               0.
-0.             0.               1.000000E 00        -0.             0.               1.000000E 00
 0.            -1.000000E 00     0.                   0.            -1.000000E 00     0.
SPRING  2  INTERCHANGED WITH  3  AT T= 0.9000E-01


TIME =    0.1000000E-00 SECONDS     STEP SIZE=  0.1979060E-08
BODY 1
SPEED OF CM =   0.3856862E 02 IN/SEC
RATE ABOUT INSTANTANEOUS AXIS=   0.1558812E 01 DEG/SEC       MAGNITUDE VECTOR SUM OF PITCH-YAW RATES=   0.1558812E 01

INERTIAL COORDS-INCHES   1ST DERIVATIVE /SEC      EULER ANGLES-DEGREES     1ST DERIVATIVE /SEC          RATE ABOUT
                                                                                                    BODY AXES DEG/SEC
X       0.6388861E-02     X      -0.1005825E-00   THETA  0.9017740E 02     THETA -0.1139844E 01     X     -0.1139847E 01
Y      -0.6918635E-02     Y       0.1048707E-00   PHI    0.2023377E-03     PHI    0.2991826E-02     Y     -0.1063316E 01
Z       0.1847882E 02     Z       0.3856835E 02   PSI    0.3598242E 03     PSI    0.1063326E 01     Z     -0.5050041E-03
BODY 2
SPEED OF CM =   0.2142701E 02 IN/SEC
RATE ABOUT INSTANTANEOUS AXIS=   0.1657113E 01 DEG/SEC       MAGNITUDE VECTOR SUM OF PITCH-YAW RATES=   0.1657113E 01

INERTIAL COORDS-INCHES   1ST DERIVATIVE /SEC      EULER ANGLES-DEGREES     1ST DERIVATIVE /SEC          RATE ABOUT
                                                                                                    BODY AXES DEG/SEC
X      -0.3549367E-02     X       0.5587918E-01   THETA  0.8985526E 02     THETA  0.1195752E 01     X      0.1195756E 01
Y       0.3843686E-02     Y      -0.5826150E-01   PHI    0.3599993E 03     PHI    0.2933975E-02     Y      0.1147254E 01
Z      -0.1026601E 02     Z      -0.2142686E 02   PSI    0.1463798E-00     PSI   -0.1147261E 01     Z     -0.1209621E-03

SEPARATION DISTANCE =   0.2874483E 02 IN                         SEPARATION VELOCITY =   0.5999563E 02 IN/SEC

CONSTRAINT VALUES   0.1382828E-04   0.1192093E-04  -0.3784180E-02

                     A1                                                     A2
 0.999995E 00   0.130300E-04  -0.306778E-02          0.999997E 00  -0.576893E-05   0.255480E-02
 0.306780E-02  -0.309619E-02   0.999990E 00         -0.255478E-02   0.252612E-02   0.999994E 00
 0.353144E-05  -0.999995E 00  -0.309621E-02         -0.122226E-04  -0.999997E 00   0.252610E-02
```

```
TIME =   0.2000000E-00 SECONDS      STEP SIZE=  0.3841706E-08
BODY 1
SPEED OF CM =   0.3878719E 02 IN/SEC
RATE ABOUT INSTANTANEOUS AXIS=   0.2460264E 01 DEG/SEC        MAGNITUDE VECTOR SUM OF PITCH-YAW RATES=   0.2460264E 01

INERTIAL COORDS-INCHES    1ST DERIVATIVE /SEC       EULER ANGLES-DEGREES       1ST DERIVATIVE /SEC           RATE ABOUT
                                                                                                       BODY AXES DEG/SEC
X      -0.3952325E-02     X      -0.1033879E-00     THETA  0.9000176E 02      THETA -0.1785013E 01     X      -0.1785013E 01
Y       0.3893239E-02     Y       0.1080947E-00     PHI    0.4212135E-03      PHI   -0.4926649E-03     Y      -0.1693112E 01
Z       0.2235669E 02     Z       0.3878690E 02     PSI    0.3599907E 03      PSI    0.1693112E 01     Z      -0.7811086E-03
BODY 2
SPEED OF CM =   0.2154844E 02 IN/SEC
RATE ABOUT INSTANTANEOUS AXIS=   0.2395754E 01 DEG/SEC        MAGNITUDE VECTOR SUM OF PITCH-YAW RATES=   0.2395754E 01

INERTIAL COORDS-INCHES    1ST DERIVATIVE /SEC       EULER ANGLES-DEGREES       1ST DERIVATIVE /SEC           RATE ABOUT
                                                                                                       BODY AXES DEG/SEC
X       0.2195736E-02     X       0.5743775E-01     THETA  0.9002480E 02      THETA  0.1718265E 01     X       0.1718265E 01
Y      -0.2162911E-02     Y      -0.6005260E-01     PHI    0.3599995E 03      PHI   -0.6665016E-03     Y       0.1669491E 01
Z      -0.1242038E 02     Z      -0.2154828E 02     PSI    0.3599817E 03      PSI   -0.1669491E 01     Z      -0.1181567E-03

SEPARATION DISTANCE =    0.3477707E 02 IN                        SEPARATION VELOCITY =    0.6033563E 02 IN/SEC

                       A1                                                A2
  1.000000E 00    0.735653E-05   -0.161592E-03           1.000000E 00   -0.904473E-05   -0.319127E-03
  0.161592E-03   -0.307213E-04    1.000000E 00           0.319123E-03   -0.432788E-03    1.000000E 00
  0.735156E-05   -1.000000E 00   -0.307225E-04          -0.918284E-05   -1.000000E 00   -0.432786E-03


TIME =   0.3000000E-00 SECONDS      STEP SIZE=  1.0000000E-02
BODY 1
SPEED OF CM =   0.3878719E 02 IN/SEC
RATE ABOUT INSTANTANEOUS AXIS=   0.2460264E 01 DEG/SEC        MAGNITUDE VECTOR SUM OF PITCH-YAW RATES=   0.2460264E 01

INERTIAL COORDS-INCHES    1ST DERIVATIVE /SEC       EULER ANGLES-DEGREES       1ST DERIVATIVE /SEC           RATE ABOUT
                                                                                                       BODY AXES DEG/SEC
X      -0.1429112E-01     X      -0.1033879E-00     THETA  0.8982326E 02      THETA -0.1785002E 01     X      -0.1785012E 01
Y       0.1470271E-01     Y       0.1080947E-00     PHI    0.9500863E-04      PHI   -0.6031450E-02     Y      -0.1693114E 01
Z       0.2623538E 02     Z       0.3878690E 02     PSI    0.1600579E-00      PSI    0.1693132E 01     Z      -0.1044927E-02
BODY 2
SPEED OF CM =   0.2154844E 02 IN/SEC
RATE ABOUT INSTANTANEOUS AXIS=   0.2395754E 01 DEG/SEC        MAGNITUDE VECTOR SUM OF PITCH-YAW RATES=   0.2395754E 01

INERTIAL COORDS-INCHES    1ST DERIVATIVE /SEC       EULER ANGLES-DEGREES       1ST DERIVATIVE /SEC           RATE ABOUT
                                                                                                       BODY AXES DEG/SEC
X       0.7939511E-02     X       0.5743775E-01     THETA  0.9019662E 02      THETA  0.1718256E 01     X       0.1718265E 01
Y      -0.8168170E-02     Y      -0.6005260E-01     PHI    0.3599992E 03      PHI   -0.5673247E-02     Y       0.1669491E 01
Z      -0.1457521E 02     Z      -0.2154828E 02     PSI    0.3598148E 03      PSI   -0.1669511E 01     Z      -0.1180400E-03

SEPARATION DISTANCE =    0.4081060E 02 IN                        SEPARATION VELOCITY =    0.6033563E 02 IN/SEC

                       A1                                                A2
  0.999996E 00    0.102754E-04    0.279352E-02           0.999995E 00   -0.363127E-05   -0.323299E-02
 -0.279354E-02    0.308467E-02    0.999991E 00           0.323296E-02   -0.343173E-02    0.999989E 00
  0.165821E-05   -0.999995E 00    0.308468E-02          -0.147260E-04   -0.999994E 00   -0.343170E-02


TIME =   0.4000000E-00 SECONDS      STEP SIZE=  1.0000000E-02
BODY 1
SPEED OF CM =   0.3878719E 02 IN/SEC
RATE ABOUT INSTANTANEOUS AXIS=   0.2460264E 01 DEG/SEC        MAGNITUDE VECTOR SUM OF PITCH-YAW RATES=   0.2460264E 01
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                                                                                                             BODY AXES DEG/SEC
X      -0.7368218E 02     X      -0.9488960E 01     THETA  0.9558528E 02     THETA -0.8919962E 01     X      -0.2503493E 01
Y       0.1297472E 03     Y       0.1690635E 02     PHI    0.1971992E 03     PHI   -0.5528139E 03     Y      -0.5693560E-01
Z       0.2252492E 04     Z       0.2306667E 03     PSI    0.3593318E 03     PSI   -0.5375655E 02     Z      -0.5502550E 03
BODY 2
SPEED OF CM =   0.2154844E 02 IN/SEC
RATE ABOUT INSTANTANEOUS AXIS=   0.2395753E 01 DEG/SEC       MAGNITUDE VECTOR SUM OF PITCH-YAW RATES=   0.2395753E 01

INERTIAL COORDS-INCHES   1ST DERIVATIVE /SEC      EULER ANGLES-DEGREES     1ST DERIVATIVE /SEC         RATE ABOUT
                                                                                                    BODY AXES DEG/SEC
X       0.1145206E 01     X       0.5743775E-01     THETA  0.1220430E 03     THETA  0.1363998E 01     X       0.1718224E 01
Y      -0.1197209E 01     Y      -0.6005260E-01     PHI    0.3488800E 03     PHI   -0.1232695E 01     Y       0.1669532E 01
Z      -0.4412309E 03     Z      -0.2154828E 02     PSI    0.3225498E 03     PSI   -0.2323547E 01     Z      -0.1147725E-03

SEPARATION DISTANCE =   0.2697942E 04 IN                   SEPARATION VELOCITY =   0.2529652E 03 IN/SEC

                        A1                                                  A2
 -0.954882E 00  -0.296759E-00  -0.116058E-01       0.841199E 00   0.163447E-00  -0.515431E 00
 -0.399221E-01   0.895372E-01   0.995183E 00       0.515420E 00  -0.530567E 00   0.672934E 00
 -0.294290E-00   0.950746E 00  -0.973447E-01      -0.163482E-00  -0.831735E 00  -0.530556E 00
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TIME =   0.1989999E 02 SECONDS     STEP SIZE=  1.0000000E-02
BODY 1
SPEED OF CM =   0.2314800E 03 IN/SEC
RATE ABOUT INSTANTANEOUS AXIS=   0.5502608E 03 DEG/SEC      MAGNITUDE VECTOR SUM OF PITCH-YAW RATES=   0.2444964E 01

INERTIAL COORDS-INCHES   1ST DERIVATIVE /SEC      EULER ANGLES-DEGREES     1ST DERIVATIVE /SEC          RATE ABOUT
                                                                                                     BODY AXES DEG/SEC
X      -0.7178438E 02     X      -0.9488960E 01     THETA  0.8878715E 02     THETA  0.5557124E 02     X      -0.1459889E 01
Y       0.1263659E 03     Y       0.1690635E 02     PHI    0.3072290E 03     PHI   -0.5475667E 03     Y       0.1961268E 01
Z       0.2206359E 04     Z       0.2306667E 03     PSI    0.5948272E 01     PSI    0.9628836E 01     Z      -0.5502554E 03
BODY 2
SPEED OF CM =   0.2154844E 02 IN/SEC
RATE ABOUT INSTANTANEOUS AXIS=   0.2395753E 01 DEG/SEC      MAGNITUDE VECTOR SUM OF PITCH-YAW RATES=   0.2395753E 01

INERTIAL COORDS-INCHES   1ST DERIVATIVE /SEC      EULER ANGLES-DEGREES     1ST DERIVATIVE /SEC          RATE ABOUT
                                                                                                     BODY AXES DEG/SEC
X       0.1133719E 01     X       0.5743775E-01     THETA  0.1217694E 03     THETA  0.1372404E 01     X       0.1718225E 01
Y      -0.1185198E 01     Y      -0.6005260E-01     PHI    0.3491248E 03     PHI   -0.1216021E 01     Y       0.1669532E 01
Z      -0.4369213E 03     Z      -0.2154828E 02     PSI    0.3230132E 03     PSI   -0.2309770E 01     Z      -0.1147592E-03

SEPARATION DISTANCE =   0.2647360E 04 IN                      SEPARATION VELOCITY =   0.2529652E 03 IN/SEC

                      A1                                                   A2
  0.603492E 00  -0.790609E 00   0.103607E-00          0.844191E 00   0.160367E-00  -0.511491E 00
 -0.459342E-01   0.952499E-01   0.994393E 00          0.511480E 00  -0.526513E 00   0.679097E 00
 -0.796045E 00  -0.604867E 00   0.211666E-01         -0.160402E-00  -0.834905E 00  -0.526502E 00


TIME =   0.1999999E 02 SECONDS     STEP SIZE=  1.0000000E-02
BODY 1
SPEED OF CM =   0.2314800E 03 IN/SEC
RATE ABOUT INSTANTANEOUS AXIS=   0.5502607E 03 DEG/SEC      MAGNITUDE VECTOR SUM OF PITCH-YAW RATES=   0.2486550E 01

INERTIAL COORDS-INCHES   1ST DERIVATIVE /SEC      EULER ANGLES-DEGREES     1ST DERIVATIVE /SEC          RATE ABOUT
                                                                                                     BODY AXES DEG/SEC
X      -0.7273328E 02     X      -0.9488960E 01     THETA  0.9393443E 02     THETA  0.3960344E 02     X      -0.2241270E 01
Y       0.1280565E 03     Y       0.1690635E 02     PHI    0.2524090E 03     PHI   -0.5501292E 03     Y       0.1076865E 01
Z       0.2229426E 04     Z       0.2306667E 03     PSI    0.4360648E 01     PSI   -0.3882392E 02     Z      -0.5502551E 03
BODY 2
SPEED OF CM =   0.2154844E 02 IN/SEC
RATE ABOUT INSTANTANEOUS AXIS=   0.2395753E 01 DEG/SEC      MAGNITUDE VECTOR SUM OF PITCH-YAW RATES=   0.2395753E 01

INERTIAL COORDS-INCHES   1ST DERIVATIVE /SEC      EULER ANGLES-DEGREES     1ST DERIVATIVE /SEC          RATE ABOUT
                                                                                                     BODY AXES DEG/SEC
X       0.1139463E 01     X       0.5743775E-01     THETA  0.1219064E 03     THETA  0.1368219E 01     X       0.1718224E 01
Y      -0.1191204E 01     Y      -0.6005260E-01     PHI    0.3490028E 03     PHI   -0.1224345E 01     Y       0.1669532E 01
Z      -0.4390761E 03     Z      -0.2154828E 02     PSI    0.3227818E 03     PSI   -0.2316640E 01     Z      -0.1147325E-03

SEPARATION DISTANCE =   0.2672651E 04 IN                      SEPARATION VELOCITY =   0.2529652E 03 IN/SEC

                      A1                                                   A2
 -0.306318E-00  -0.948902E 00   0.758550E-01          0.842698E 00   0.161904E-00  -0.513466E 00
 -0.422379E-01   0.931555E-01   0.994755E 00          0.513455E 00  -0.528544E 00   0.676021E 00
 -0.950992E 00   0.301508E-00  -0.686149E-01         -0.161939E-00  -0.833323E 00  -0.528534E 00


TIME =   0.2009999E 02 SECONDS     STEP SIZE=  1.0000000E-02
BODY 1
SPEED OF CM =   0.2314800E 03 IN/SEC
RATE ABOUT INSTANTANEOUS AXIS=   0.5502607E 03 DEG/SEC      MAGNITUDE VECTOR SUM OF PITCH-YAW RATES=   0.2504141E 01

INERTIAL COORDS-INCHES   1ST DERIVATIVE /SEC      EULER ANGLES-DEGREES     1ST DERIVATIVE /SEC          RATE ABOUT
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