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LONGITUDINAL DYNAMICS OF LIQUID FILLED ELASTIC SHELLS

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by

Daniel D. Kana

Summary Report
Contract No. NAS8-11045
Control No. TP 3-85175 & S-1 (IF)
SwRI Project No. 02-1391

Prepared for

National Aeronautics and Space Administration
George C. Marshall Space Flight Center
Huntsville, Alabama

September 30, 1966

SOUTHWEST RESEARCH INSTITUTE
SAN ANTONIO HOUSTON

SOUTHWEST RESEARCH INSTITUTE
8500 Culebra Road, San Antonio, Texas 78206

Department of Mechanical Sciences

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APPROVED:

H. Norman Ab

H. Norman Abramson, Director
Department of Mechanical Sciences

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The following summary of research conducted under NASA Contract NAS8-11045 is based on the direct technical effort of a number of investigators, both at Southwest Research Institute and at Marshall Space Flight Center.

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This entire program has been made possible only through the continuing efforts of Dr. Werner R. Eulitz and Dr. Rudolph F. Glaser of the Propulsion and Vehicle Engineering Laboratory at Marshall Space Flight Center. As technical monitors, both administratively and technically, they have made valuable contributions, without which the high level of technical significance could not have been achieved in the present program.

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I. INTRODUCTION

The behavior of liquid propellants in fuel tanks under dynamic flight conditions has long been recognized as a major factor in the overall stability of rockets and space vehicles. As a result, considerable effort has been extended over the last decade, to gain a better understanding of the various problems that arise in this area. Much of this work has been summarized recently in a comprehensive monograph [1B]*, and is concerned primarily with problems that are associated with the boost phase of flight.

The current trend in studies of liquid propellant dynamics has been that of a considerable broadening of the scope of knowledge required to cope with new problems that are becoming apparent, as more manned flight experience is gained. Many of the new liquid fuel dynamics problems have resulted from the use of larger, more flexible structures and associated systems, along with increasing requirements of orbital flight in low gravity environments. The liquid fuel dynamics analyst finds that he must now give a marked increase in attention to such subjects as structural dynamics, thermodynamics, liquid surface chemistry, dynamic stability and nonlinear response, etc. Thus, the state of the art has now advanced to where liquid behavior must be studied not only during the boost phase, but during an entire flight, including orbital maneuvers and their associated complexities.

Within the broad spectrum of research that now composes the study of liquid fuel dynamics, several categories of liquid behavior may conveniently be recognized; three such categories are the interaction of the liquid and the elastic structure, liquid surface instability, and bubble dynamics. These three types of liquid behavior become important under various dynamic input conditions, but can be particularly important for the case of longitudinal vibration. They are mutually related through the pressure response, which can be either linear or nonlinear, that occurs in the liquid shell system. This relationship is vividly displayed in the Flow Chart of Figure 1.

The purpose of the present report is to discuss how the above three types of liquid behavior fit into the overall liquid fuel dynamics problem, to review some of the investigations of this behavior that are currently being conducted at Southwest Research Institute, and to make recommendations for further research. For technical details of the results summarized herein, the referenced reports and publications should be consulted. This work has been directly sponsored, or is closely associated with work sponsored by Marshall Space Flight Center, Contract No. NAS8-11045.

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*Reference list appears at the end of this report.

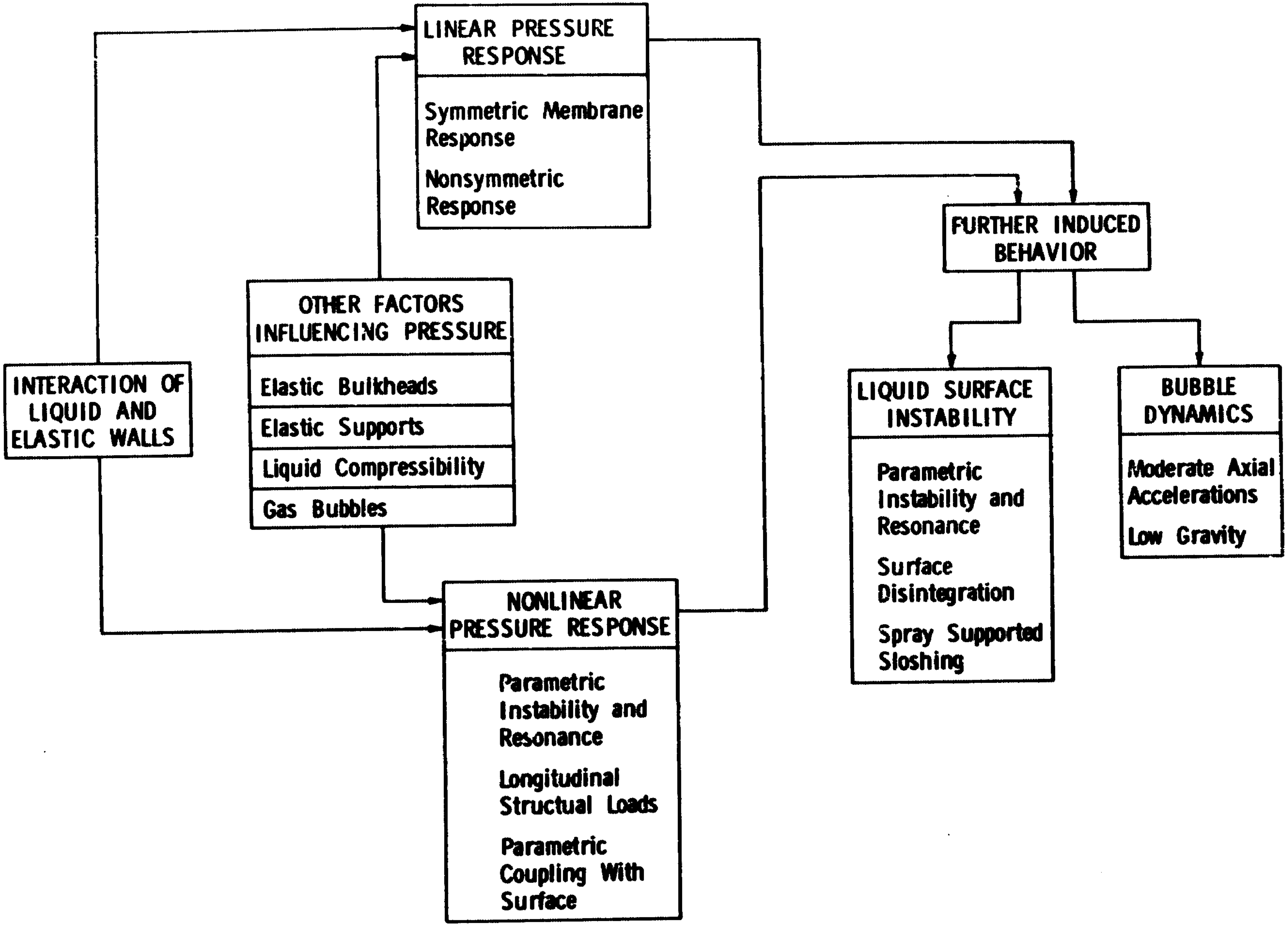

Figure 1. Relationship: Among Current Phases Of Research In Liquid Fuel Dynamics

II. INTERACTION OF LIQUID AND ELASTIC TANK

The liquid fuel, ullage gas under pressure, and elastic tank form a complicated part of the vehicle structure, and vibrational responses of this coupled unit influence the behavior of the entire vehicle. Recent surveys of investigations of this coupled liquid-elastic tank behavior have been reported by Abramson and Kana [1A, 2A]. The current emphasis for studies of this system under longitudinal vibration has been on the response in axisymmetric, longitudinal modes where the pressure distribution forms a maximum at the tank bottom. The importance of this pressure response is immediately obvious, for it affects, in one way or another, virtually all coupled fluid behavior that is important in the system. This interrelationship among some of the aspects of the coupled system was implied in Figure 1. Not only structural integrity, but POGO phenomena, bubble migration, and even liquid surface instability are directly related to this pressure response. Longitudinal excitation of this response may result from hold-down release at liftoff, rough combustion during the boost phase, sudden release of stored elastic structural energy at main engine cutoff (MECO), or even from low level inputs such as recirculating pumps, actuators, or docking maneuvers while in orbit. In this section, we will review recent studies of this type of linear response, as well as indicate nonlinear responses that can have equally important effects on the system.

A. Linear Pressure Response

1. Membrane Analysis of Shells

In recent investigations of the axisymmetric response of the liquid-shell system, the tank has been considered to be a membrane which can exert no longitudinal inertia or tension, and has a rigid flat bottom. This has been the approach of Bleich [2B], and Reissner [3B], in which the problem of forced vibration was studied directly, and also of Glaser [3A], who more explicitly studied the natural modes of the system. Some comparisons of experimental data [4A] with results from the above analyses are shown in Figures 2-4. The forced response at the bottom of the longitudinally vibrated tank, as shown in Figure 2, indicates that for sufficient accuracy with the membrane theory, a fairly large number of terms must be used in the series expression for the response pressure, particularly at lower depths. Figure 3 verifies that the axial pressure distribution is a multiple of a quarter cosine wave (relative to the tank bottom) for the axisymmetric natural modes in the flat bottom tank. Figure 4 shows the adequacy of using membrane theory for predicting the natural frequencies of the axisymmetric modes in

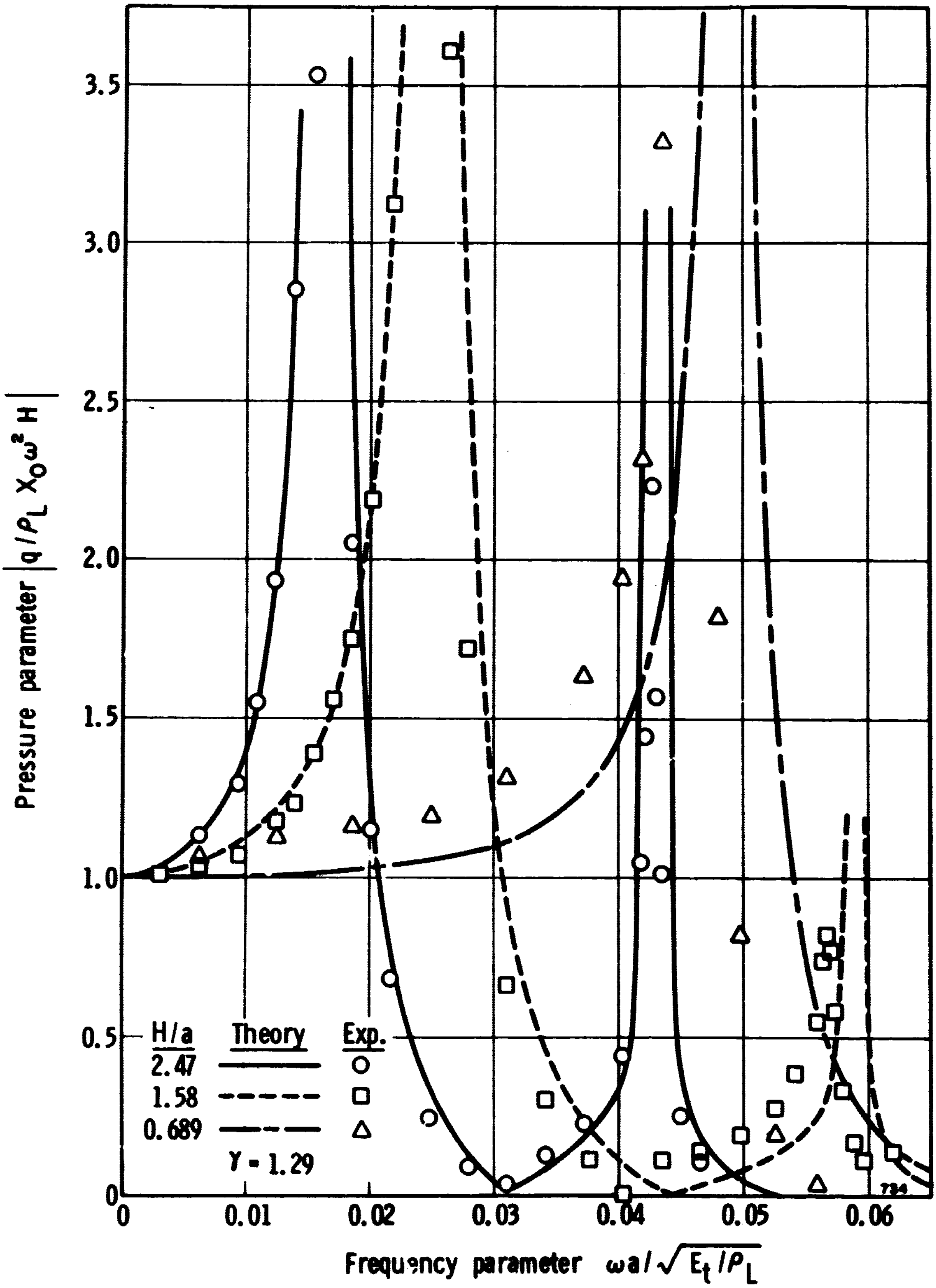

Figure 2. Linear Axisymmetric Pressure Response To Longitudinal Forced Vibration For Various Liquid Depths In A Ring Stiffened Elastic Cylinder With A Flat Rigid Bottom

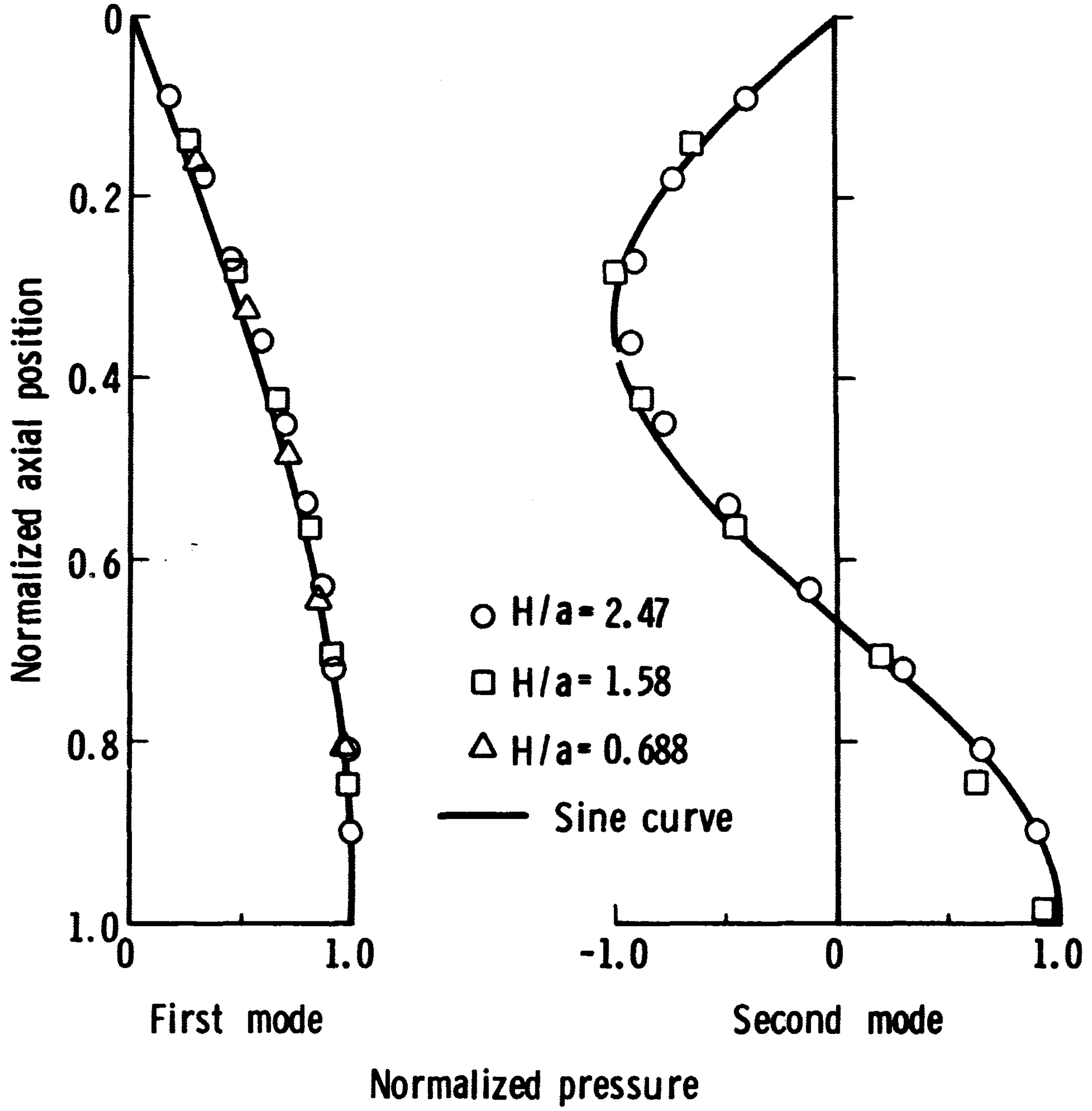

Figure 3. Normalized Axial Pressure Distributions For First Two Linear Axisymmetric Modes Of Liquid In A Longitudinally Excited Ring Stiffened Elastic Cylinder With A Flat Rigid Bottom

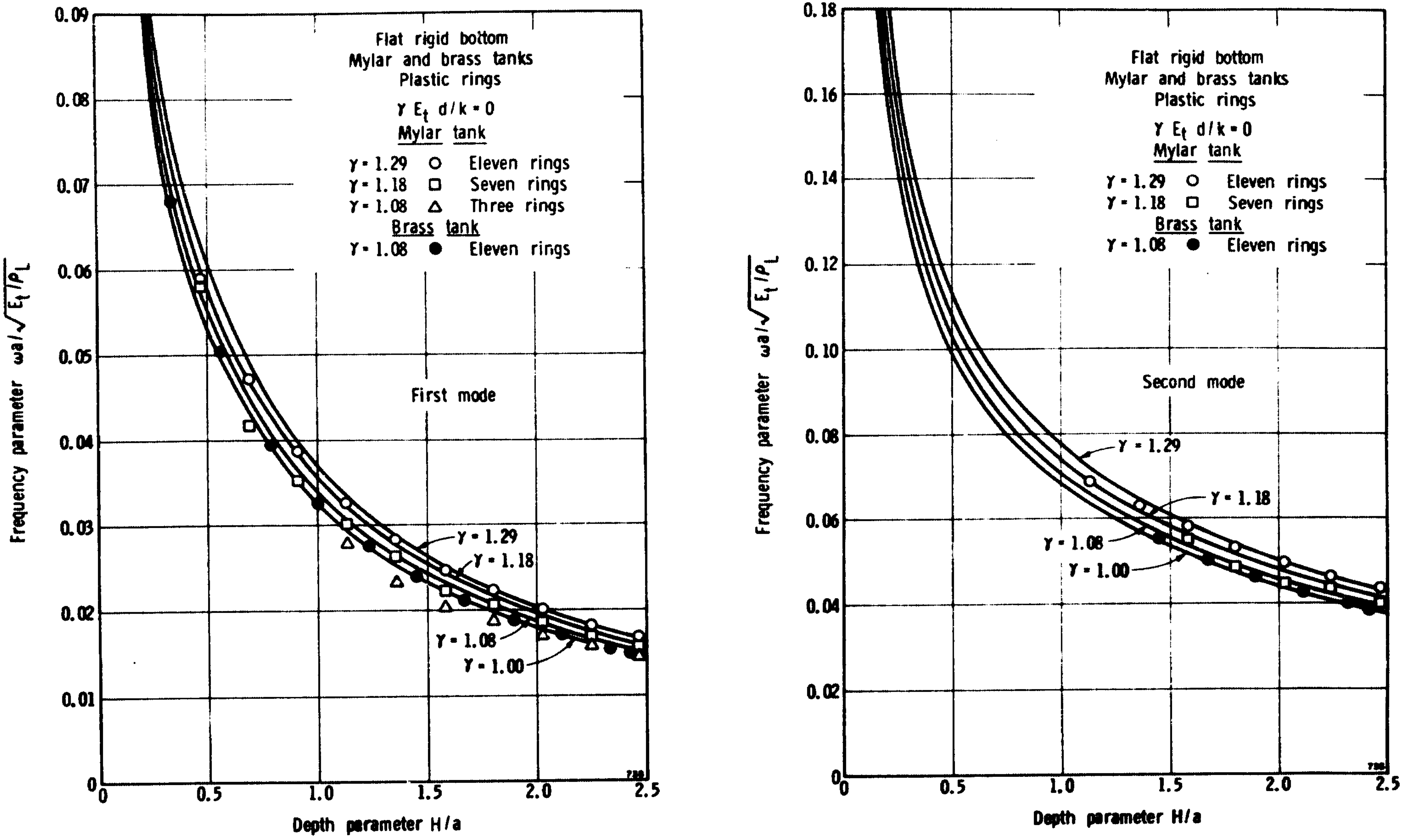

Figure 4. Variation Of Natural Frequency With Depth Of Liquid For Linear Axisymmetric Modes In A Ring Stiffened Elastic Cylinder With A Flat Rigid Bottom

a ring stiffened tank, provided that an approximate ring stiffener factor is utilized.

The above results indicate that bending effects have virtually no influence on the axisymmetric oscillations encountered for longitudinal excitation. This has been further shown in detail by Beal, et al. [4B], in which bending effects were found to be highly localized. Thus, membrane theory is useful for determining frequencies of vibration, but not for determining stresses in the areas of significant local bending.

The influence of stiffener rings on the axisymmetric modes is indicated in Figure 4. However, in that study [4A], the rings were bonded completely around the outside of the tank; hence, similar studies need to be performed for model tanks having more realistic stiffener systems. It is especially desirable to determine in what manner, if at all, a more realistic stiffener ring system would reduce linear nonaxisymmetric vibrational modes in the tank wall. These modes are discussed more in detail later.

2. Effects of Elastic Bulkheads

The above analyses were conducted for membrane shells having rigid flat bottoms, and are quite useful for indicating the general behavior of the liquid-elastic tank wall system. However, the effects of different bulkhead geometry, as well as bulkhead elasticity, must be considered in the study of a practical tank. Several investigations [5B, 6B, 7B] have considered these properties for representative tank models. Furthermore, Figure 5, taken from [4A], shows the effect of an elastic, ellipsoidal bottom on the corresponding modes given for the rigid flat bottom in Figure 4 (only the curves for $\gamma E_t d/k = 0$ are applicable here); the bulkhead was fabricated of the same material, but having 86% of the thickness of the tank walls. In general, the conclusion from this study is that the analysis of this more practical system is considerably more complicated than that for a flat bottom, and that the development of some form of simplified model applicable over a restricted frequency range is desirable. Such a development has been attempted by Pinson [7B]; some comparisons of results predicted by his model with experimental results are given in [5A]. Furthermore, a complete study of elastic ellipsoidal bottom effects should be carried out over a wide range of parameters since, to date, studies have been conducted more or less on tanks of specific design.

3. Effects of Elastic Supports

Very little work has thus far been performed on the effects of elastic supports on the longitudinal axisymmetric modes. Palmer and

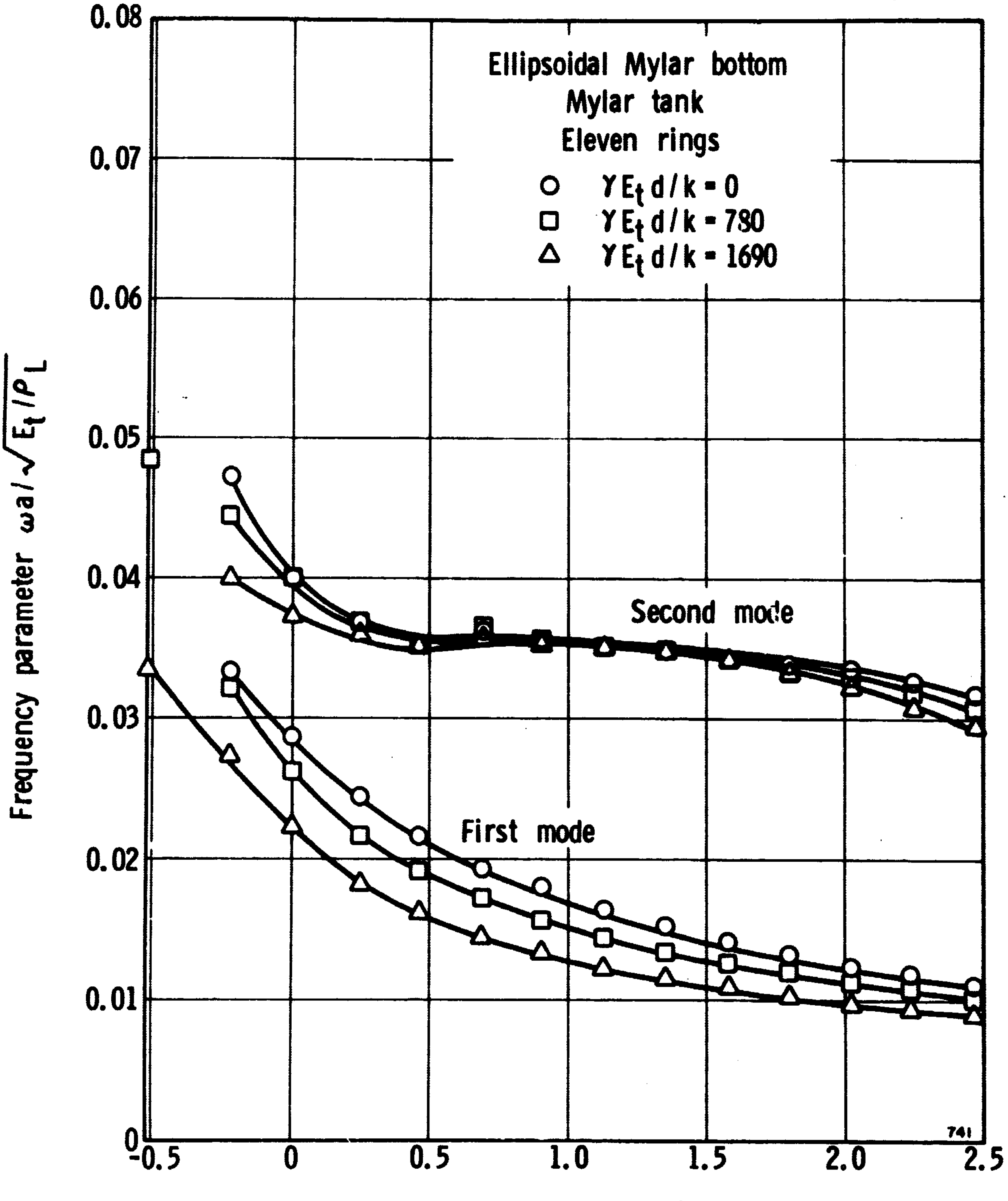

Figure 5. Variation Of Natural Frequency With Liquid Depth For Linear Axisymmetric Modes In A Ring Stiffened Elastic Cylinder Having An Elastic Ellipsoidal Bottom And Elastic Supports

Asher [5B] included an elastic support in their apparatus; however, the support was apparently so soft that little coupling between it and the longitudinal liquid-shell modes resulted. Such coupled effects for a relatively stiff spring support, for the elastic tank with a flexible ellipsoidal bottom, are given by the experimental data [4A] shown in Figure 5, where k denotes the stiffness of the support spring. Here, the parameters were such that the greatest effects occurred for the first mode.

Similar results for the effects of an elastic support for a tank with a rigid flat bottom are shown in Figure 6. Here, the results are compared with those of an approximation based on Bleich's theory [2B], as well as with the results of a membrane analysis applied to the tank [5A], in which ten terms were used for series expressions for the liquid pressure. Theoretical and experimental results correlate very well for this case.

The coupling of axisymmetric liquid-shell modes with elastic supports is a very important consideration in the representation of a vehicle structure as a lumped mass parameter system. The little data available indicate, however, that considerable error can result if liquids are considered to be only solid masses, and no consideration has been given to models representing the second, and higher coupled modes. Moreover, it can also be seen that additional consideration should be given to elastic support from above as well as from below in tanks that represent a given part of a vehicle structure. Thus, longitudinal forces must be considered in the shell walls, and studies should be conducted over a broad range of parameters in order to obtain a good understanding of the overall vibrational behavior of such a system. Ultimately, it would be highly desirable to study several such tanks in tandem, representing a composite vehicle system, and to allow for draining from the experimental system.

4. Effects of Liquid Compressibility

Most of the studies of longitudinal modes in the liquid-shell system have assumed the liquid fuel to be incompressible; this assumption is valid for very flexible tanks at low frequencies. Bleich [2B] and Glaser [3A] have discussed some of the effects of including liquid compressibility, but no investigation of these effects over a wide range of parameters is available. This subject is closely associated with wave propagation in conduits having elastic walls [8B]. The important fact remains that, in any tank including a compressible liquid, at some frequency, additional longitudinal symmetric oscillational modes can occur, resulting in large pressure amplitudes at the tank bottom. A parametric study of these modes over a wide range would show where they become important in a representative vehicle tank.

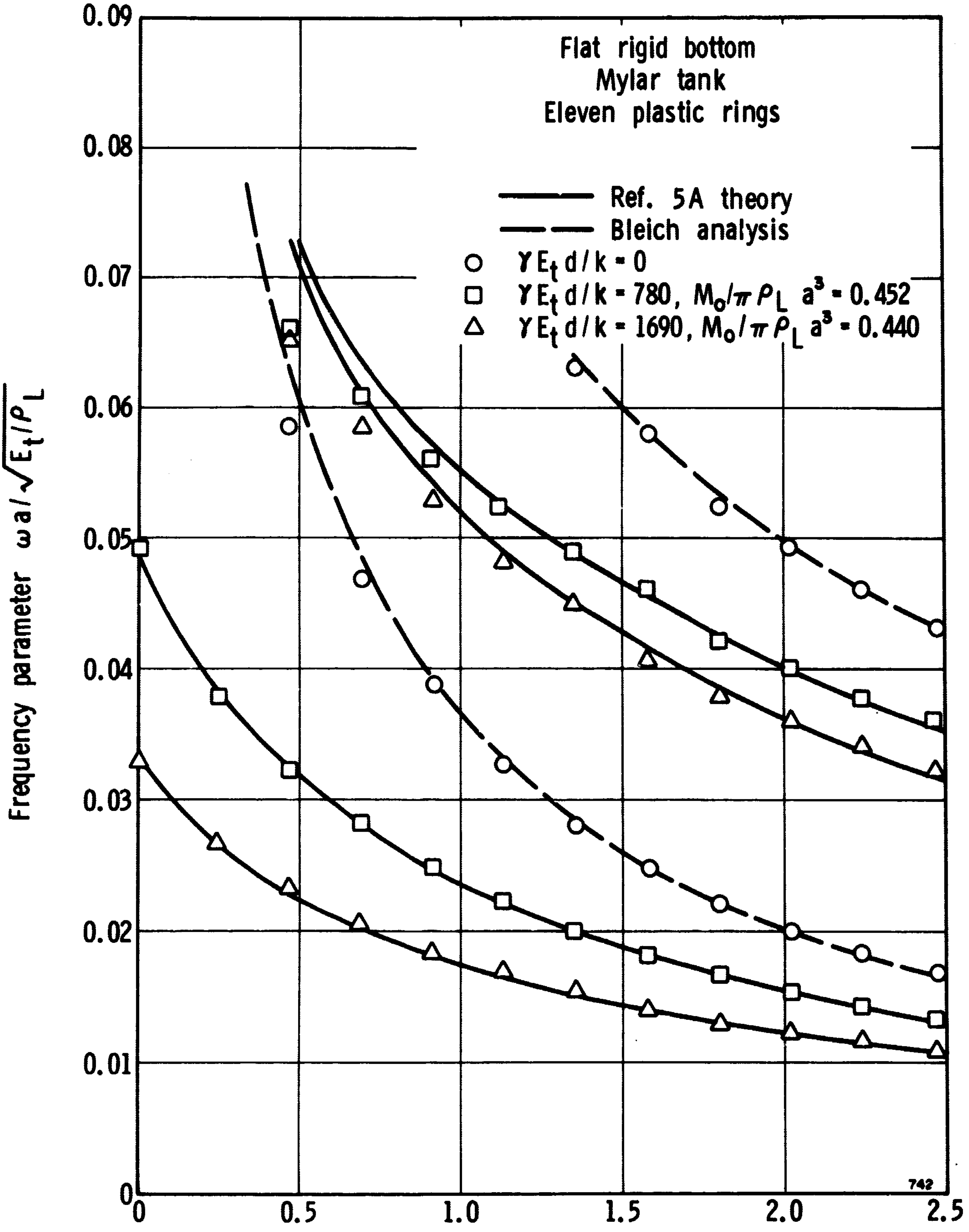

Figure 6. Variation Of Natural Frequency With Liquid Depth For Linear Axisymmetric Modes In A Ring Stiffened Elastic Cylinder Having A Flat Rigid Bottom And Elastic Supports

5. Nonaxisymmetric Linear Response

This type of vibrational mode in a cylindrical shell containing liquid has been studied quite extensively from the free natural mode point of view [1B]. However, only one study of the response of these modes to longitudinal forced oscillation is known to exist [9B], and no numerical data are given in this work. Recent experiments at SwRI indicate that such responses are very prominent in partially filled, bare wall cylinders containing an ullage pressure. However, these modes do not appear to result in excessive pressures in a tank having small damping at low amplitude excitational inputs. This behavior contrasts considerably with the case where the same modes cause dangerously large pressures when they appear as the dominant components in nonlinear responses to parametric excitation. The latter behavior will now be discussed.

B. Nonlinear Pressure Response

In the previous sections, we have considered linear responses of a liquid-shell system to longitudinal vibration. It was pointed out that axisymmetric modes are particularly important because of excessive pressure amplitudes that result at the tank bottom, even when small damping is present. Linear nonaxisymmetric shell modes were mentioned in which excessive pressures do not appear to occur. We now, however, discuss the case in which the above two types of modes couple nonlinearly in such a way that excessive pressures within the tank do occur at many frequencies other than those given by the natural, linear axisymmetric modes. In other words, nonlinear parametrically excited nonsymmetric shell modes result in large amplitude symmetric pressure oscillations at the tank bottom!

1. Parametric Instability and Resonance

Some of the most recent work at SwRI shows that parametric instability and nonlinear resonance in the longitudinally excited liquid-shell system can be just as significant as are the axisymmetric linear modes. This fact is vividly displayed in Figures 7-9, taken from [10B] which constitutes the only known work to date on this problem in this country*.

Figure 7 shows experimental verification of theoretically predicted stability boundaries for three nonsymmetric, principal parametric resonances in a liquid filled cylinder. The shell amplitude response in the

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*There is evidence of work on this problem in the Soviet Union [11B, 12B]; however, a solution for the complete problem, such as given in [10B], has apparently not been achieved.

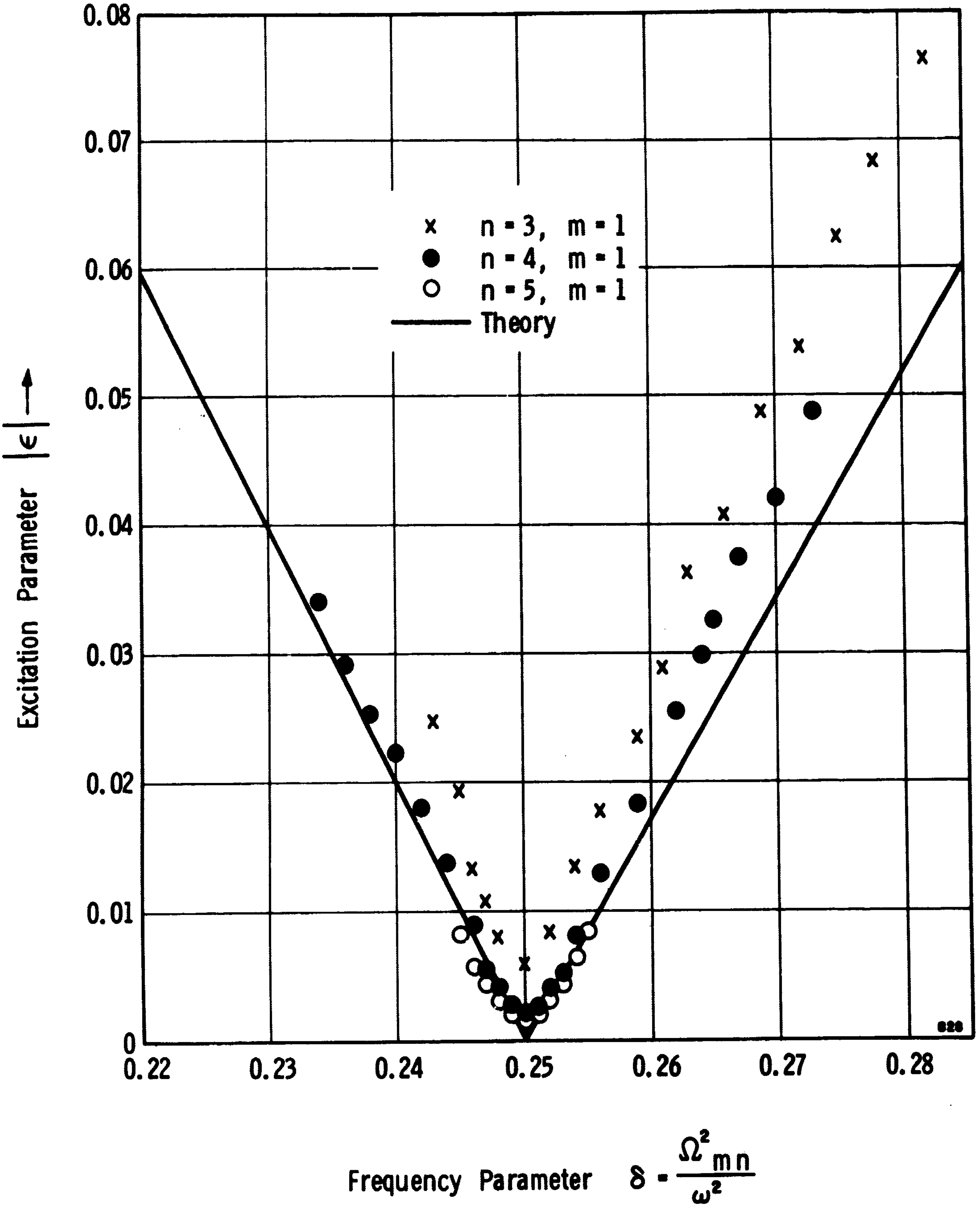


Figure 7. Stability Boundaries For Three Nonaxisymmetric Principal Parametric Modes In A Liquid Filled Elastic Cylinder Having A Rigid Flat Bottom

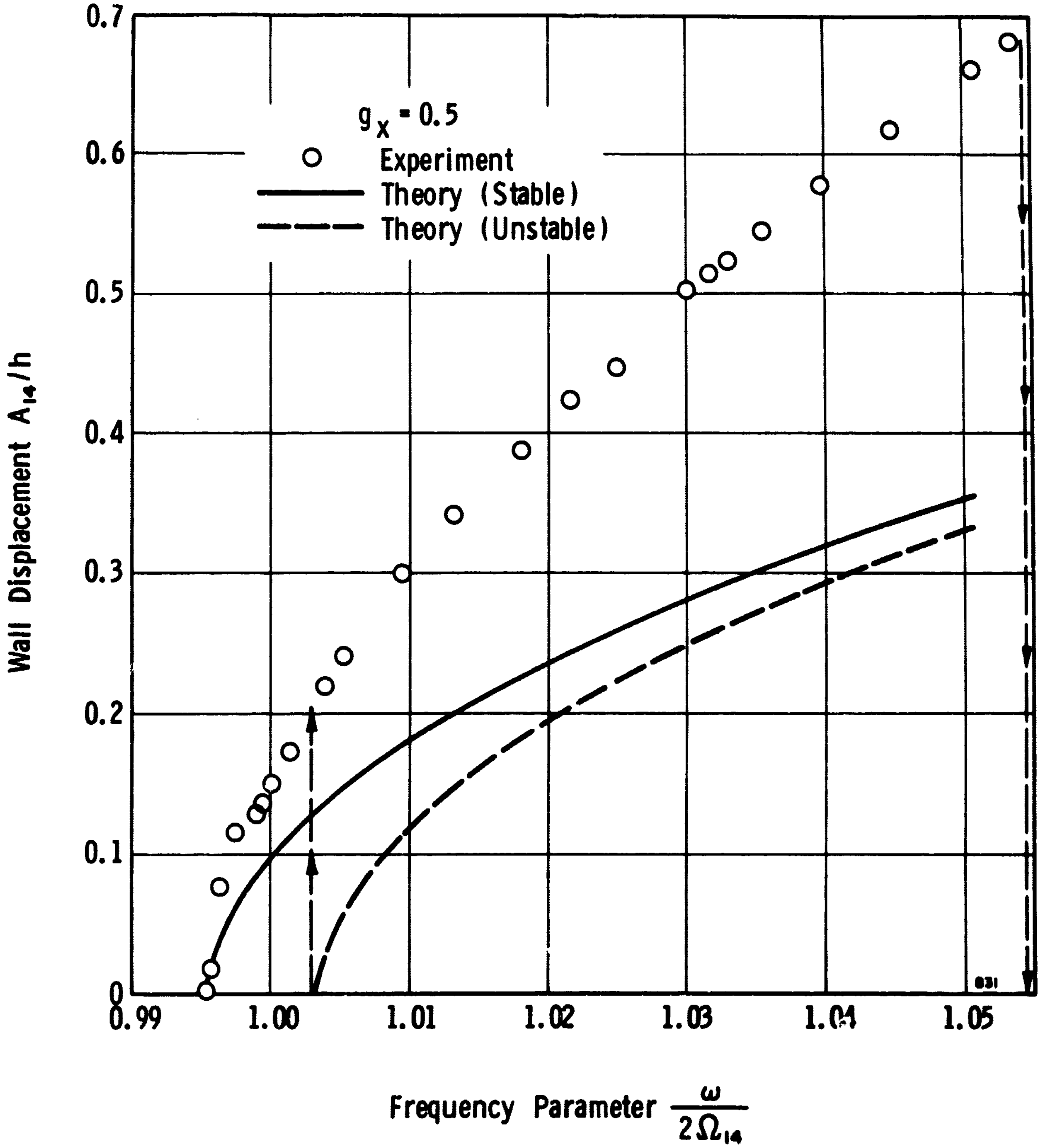


Figure 8. Nonlinear Lateral Wall Displacement Response Of A Liquid Filled Cylinder For The m = 1, n = 4 Principal Parametric Mode

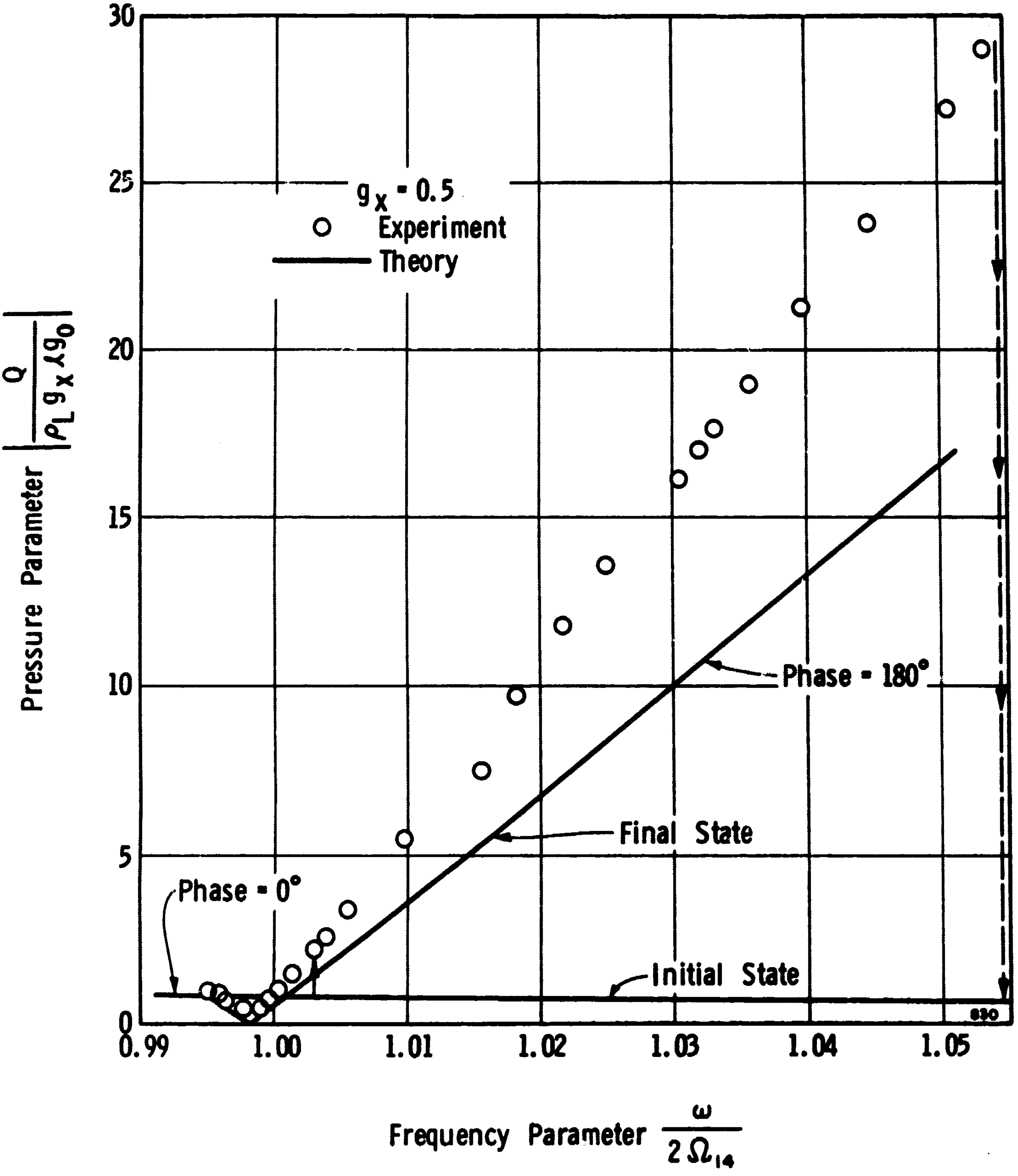


Figure 9. Nonlinear Axisymmetric Pressure Response In A Liquid Filled Cylinder For The m = 1, n = 4 Principal Parametric Mode

m = 1, n = 4, 1/2-subharmonic mode as a principal parametric resonance is shown in Figure 8, while the predicted and observed liquid pressure at the tank bottom for the same mode is shown in Figure 9.

The most important practical feature of such a nonlinear response resides not in the fact that the wall itself responds in such a mode of vibration, but that it simultaneously induces a large amplitude symmetric pressure in the liquid column, as can be seen from Figure 9. In this figure, the lower, relatively flat, curve represents the pressure anticipated as a consequence of the axisymmetric shell vibration, similar to the responses shown in Figure 2. The nonlinear pressure that is predicted and observed, as indicated, is not only far greater in amplitude, but can occur with sudden, hammerlike, jumps in response amplitude, which could impose very large stresses in a vehicle tank, and could certainly affect all aspects of longitudinal fluid behavior, similar to excitation near the symmetric linear modes! Thus, excessive pressures at low input accelerations can be experienced at many more frequencies than just those corresponding to the axisymmetric linear vibrational modes. This one distinguishing feature is what makes this type of nonlinear response so much more important than the linear response in the same nonsymmetric modes, as discussed in the previous section.

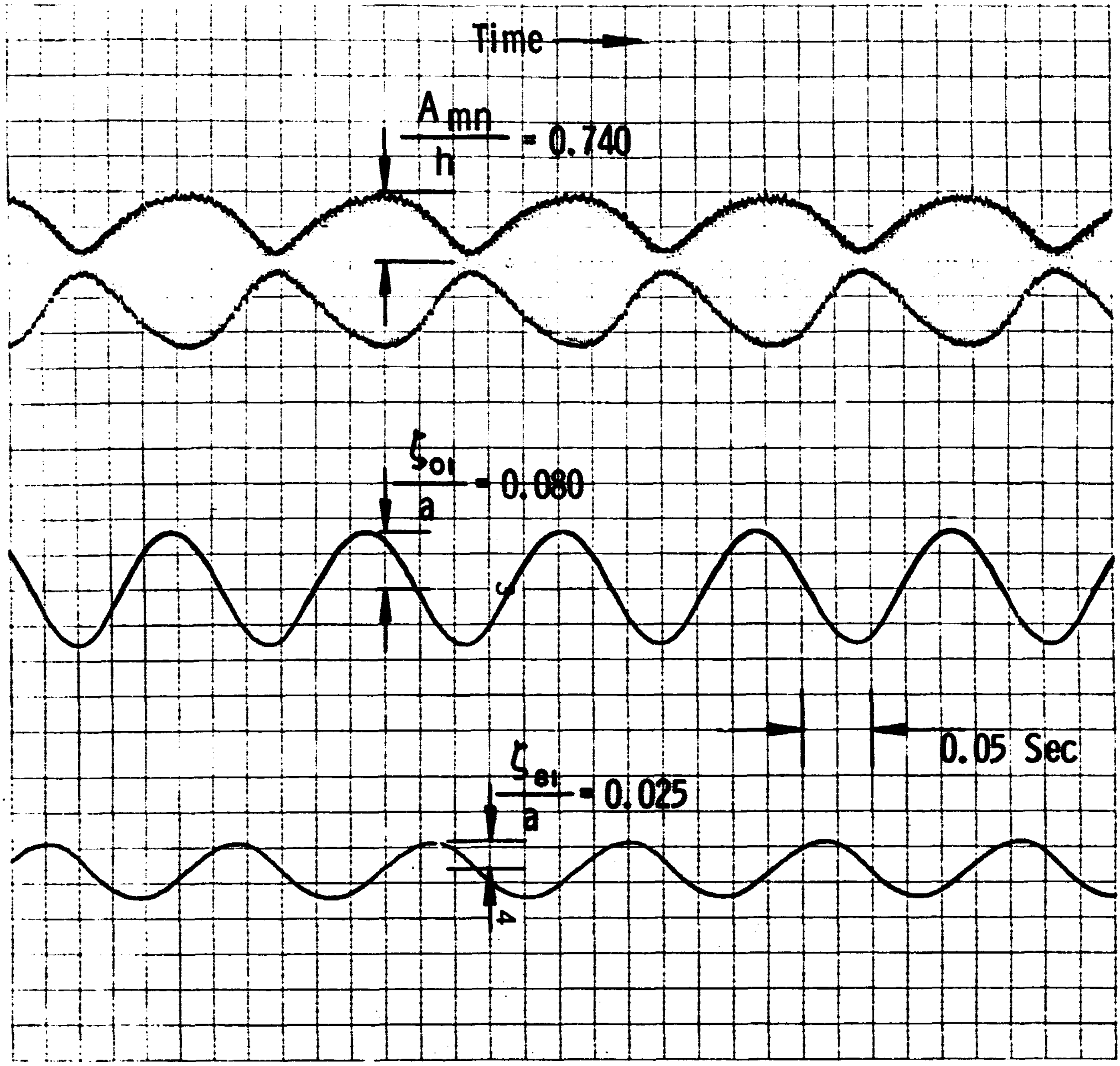
The current studies [10B] have been conducted only in a cylinder completely filled with liquid, but experiments have indicated similar behavior as well in partially filled cylinders. It is obvious that a considerable effort should be exerted immediately in order to determine the extent of the severity of this type of nonlinear pressure response in prototype vehicles. In particular, it must be determined how effectively, if at all, current representative ring stiffener designs reduce this type of response.

2. Longitudinal Structural Loadings

The preceding paragraphs have emphasized the significance of parametric resonance in causing excessive tank pressures. This behavior results primarily from radial, axisymmetric pressure loading of the liquid on the shell wall. Similar behavior, however, can be caused by longitudinal loadings of the tank walls, which can readily result from the inertia of other parts of the vehicle structure. In fact, this type of behavior leads to the possibility of loss of stability and parametric oscillation of the vehicle as a whole, similar to a longitudinally excited, free-free beam. No known work to date has been conducted on this fascinating aspect of the liquid-shell interaction problem.

C. Nonlinear Parametric Coupling

Parametric coupling in the longitudinally excited liquid-shell system can occur in yet another manner, other than that which has been described in the preceding paragraphs. Coupling of nonaxisymmetric shell breathing responses with the liquid surface has been found to occur experimentally in a partially filled tank [13B]. This coupling has been described analytically [14B], and a simplified analog [15B] has also been developed which displays the same behavior as the liquid-tank system. The parametrically coupled behavior is described in Figure 10, taken from [14B], where the shell response is shown (in the upper trace) to be a high frequency amplitude modulated response, while the liquid surface is simultaneously excited in a large amplitude, low frequency, nonlinear sloshing mode (two components of which are given in the second and third traces in Figure 10). These theoretical results obtained on an analog computer are very similar to the behavior observed experimentally in a nonsymmetrically vibrating shell that is force vibrated either laterally or longitudinally. This type of response to longitudinal vibration has not been investigated to any great extent.



$f = 671$ cps
$F^*_{42} = 0.0005$
$\delta_1 = 0.010$
$\delta_{01} = 0.010$
$\delta_{81} = 0.157$

Figure 10. Analog Computer Solution For Parametric Lateral Wall Displacement And Liquid Surface Coupling In A Partially Filled Elastic Cylinder

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III. LIQUID SURFACE INSTABILITY

The stability of the liquid-gas interface of propellants in tanks experiencing a low gravity environment is currently one of the most prominent problems in liquid fuel dynamics. For various reasons, it is highly desirable that the liquid remain in one large predictable mass at a given location within the tank. This condition is particularly desirable during venting operations in order to avoid the loss of liquid fuel which may be floating about in the form of small globules. Obviously, many disturbances of a transient character are capable of disrupting a stable interface under low gravity. Here, we consider primarily the effects of longitudinal vibration, which can occur for MECO or from noise inputs from onboard orbiting equipment.

Interface instability in a longitudinally excited tank manifests itself in the form of parametric instability, nonlinear response in the principal parametric resonance for some liquid surface mode, and subsequent surface disintegration into small liquid globules. Recently, considerable effort has been expended, mostly at SwRI, in studying this general behavior in a one-g environment. As a result, much knowledge has been gained, which can ultimately be extrapolated to predict the surface behavior in a low gravity environment.

It appears that the general behavior described above occurs throughout a wide range of excitational frequency. However, as frequency is increased, the parametric surface waves become shorter in wavelength and break up into spray droplets at much lower amplitudes. For a one-g environment, surface tension is important only for the shorter wavelengths, while, for low-g, surface tension is important throughout the frequency range.

A theoretical and experimental study of parametric instability and resonance in cylindrical tanks at low frequencies has been given in [6A]. Figure 11, taken from this study, shows a description of liquid surface behavior in a low frequency range. The frequency location of many natural modes, as well as stability boundaries of the four lowest principal parametric modes, is shown. Within the regions of instability, a corresponding 1/2-subharmonic parametric surface wave results, the amplitude of which depends on the excitation parameters. The first symmetric mode is shown in Figure 12; this photograph was taken at a response amplitude for which the presence of components of higher order symmetric modes is obvious, and the wave is on the verge of producing a spray droplet at its crest.

At present, it is believed that results similar to the above also occur for the shorter wavelength, higher order 1/2-subharmonic modes

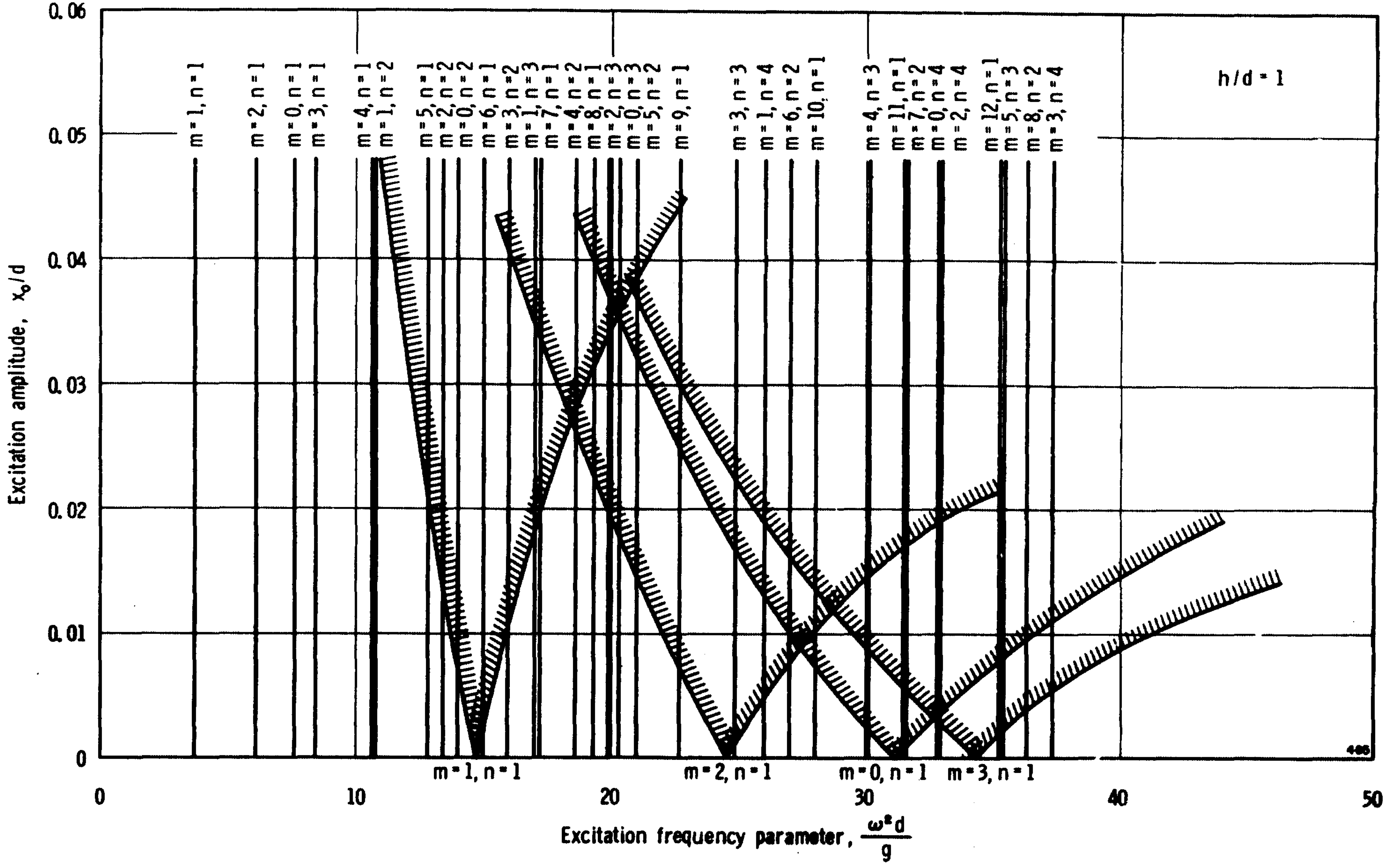

Figure 11. Modal Distribution And Stability Boundaries For Liquid Surface Modes In A Rigid Cylinder Having A Flat Rigid Bottom

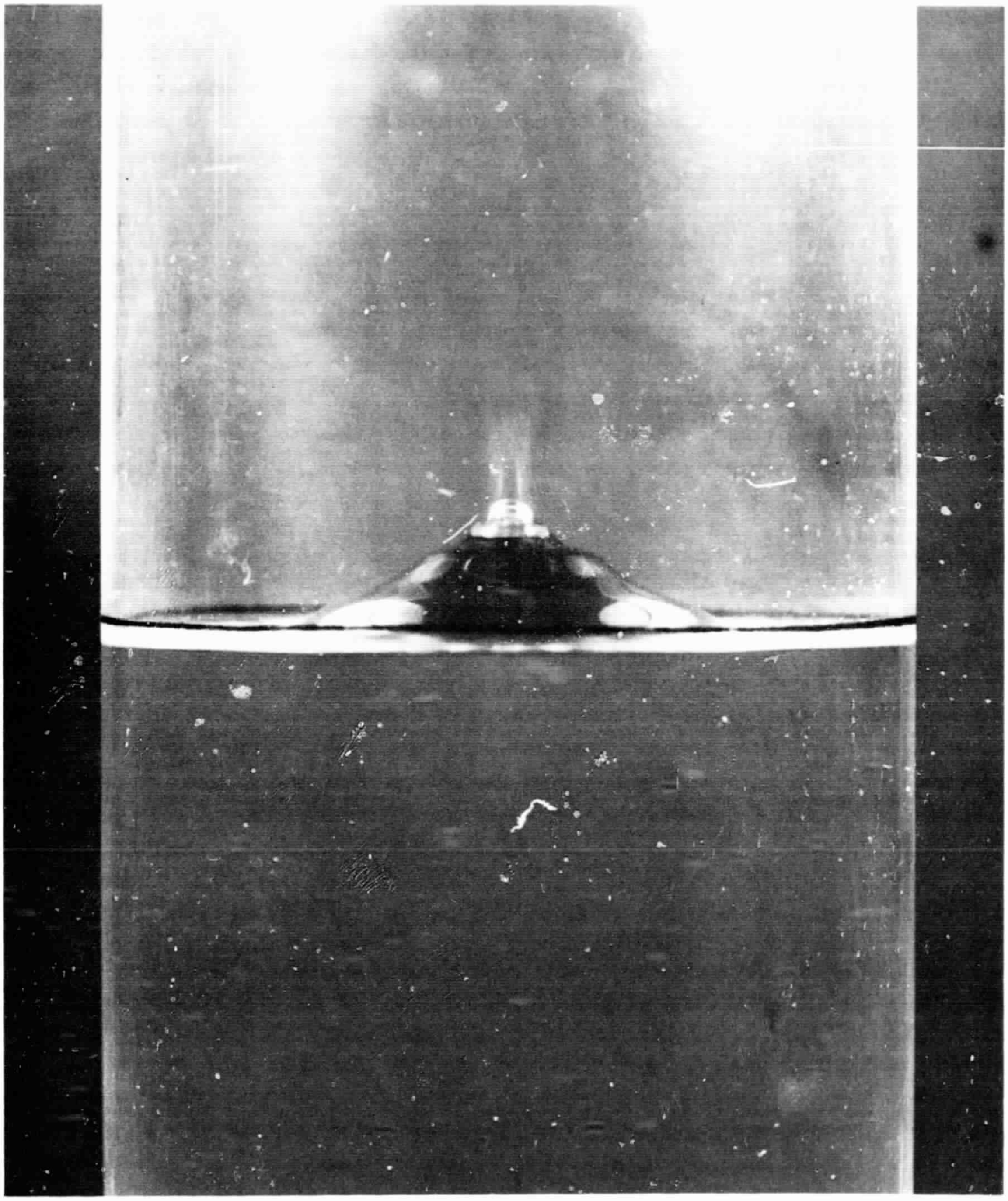

Figure 12. Typical Shape For Nonlinear First Symmetric Principal Parametric Liquid Surface Mode in A Longitudinally Excited Rigid Cylinder

at higher frequencies where, from the greater number of such waves that are present, a dense surface spray is produced. At the higher frequency range, the regions of instability become very dense and overlap one another to such an extent that a 1/2-subharmonic surface wave response is virtually assured at almost any input frequency, even for relatively small excitational amplitudes. Experimental data showing such behavior are given in Figure 13 in the form of a general stability boundary over a wide range of frequencies. Another, even more important, general boundary should be obtained for the parameters at which the waves begin to break up into individual spray droplets. Much work remains to be done before a good understanding of this form of surface behavior can be achieved.

Present experiments indicate that the ultimate breakup of parametric surface waves into floating spray droplets is not the only undesirable aspect of surface instability. Yarymovych [16B] originally found that such spray droplets, when returning to the agitated liquid surface under some axial acceleration, can produce a large amplitude, low frequency sloshing wave on which the spray production is superimposed. A wave of this type is shown in Figure 14. The occurrence of such a sloshing wave under low gravity conditions could be disasterous in an orbiting system.

The above described results were obtained in cylindrical tanks; similar studies have been conducted for tanks of other geometries. Experimental results for spherical and sectored tanks are presented in [7A], while a general theory for stability of liquid surfaces in arbitrary shaped tanks of revolution has also been developed [8A]. The extension of this theory to the case of low gravity is highly desirable. Delicate experimental observations of liquid surface behavior that are required for these studies were made possible by a special liquid surface displacement transducer [9A], also developed under the current program.

The present studies at SwRI [10A] are being concentrated on determining the conditions at which ultimate surface disintegration occurs, where floating liquid droplets are produced in a low gravity environment, as well as where low frequency spray supported sloshing is produced. It is obvious that further work is mandatory for learning more about this high priority problem. It is a real credit to the current program to realize that it is the source of virtually all available data on this subject.

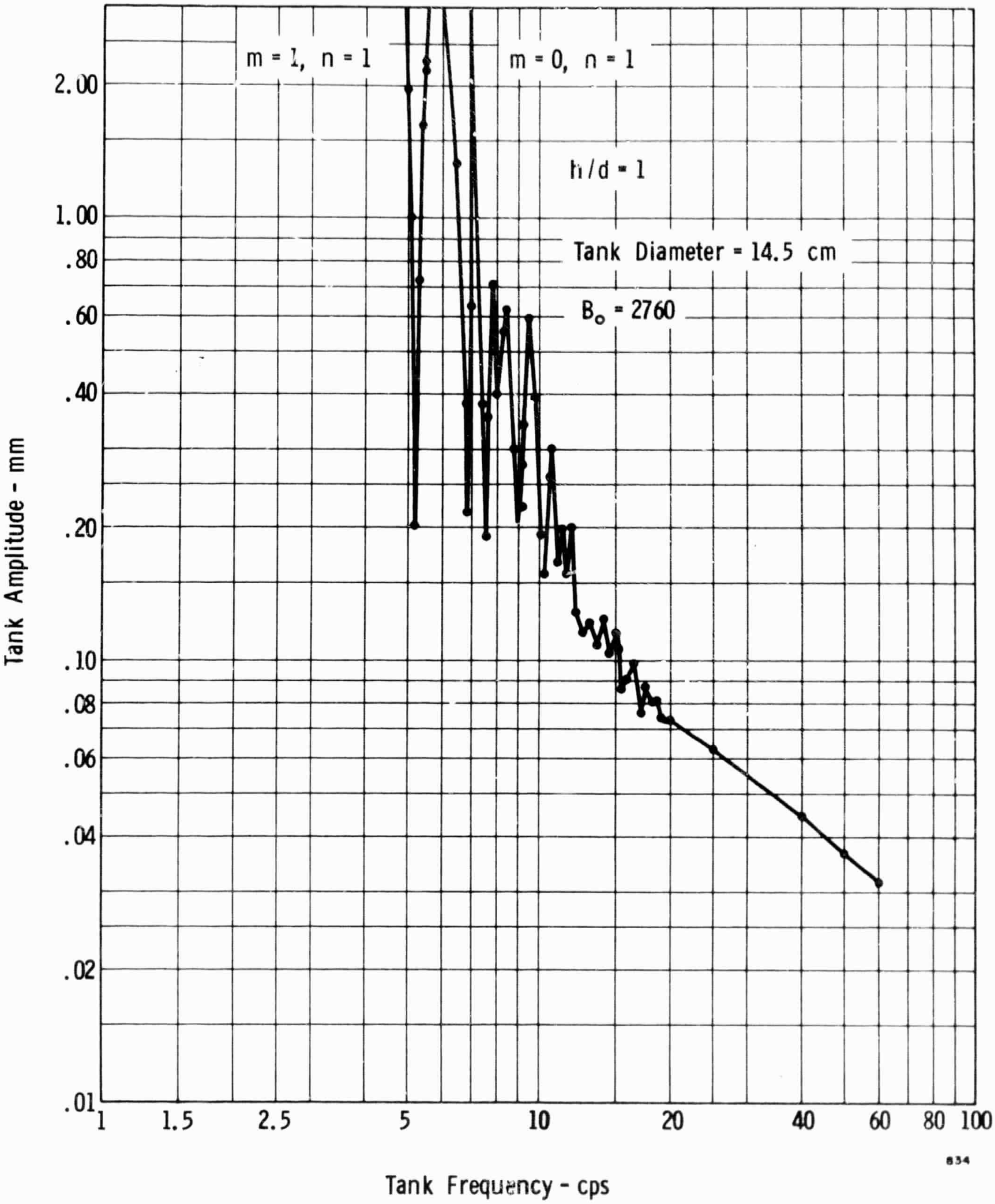

Figure 13. General Stability Boundary For Principal Parametric Resonance Of Liquid Surface In A Longitudinally Excited Rigid Cylinder

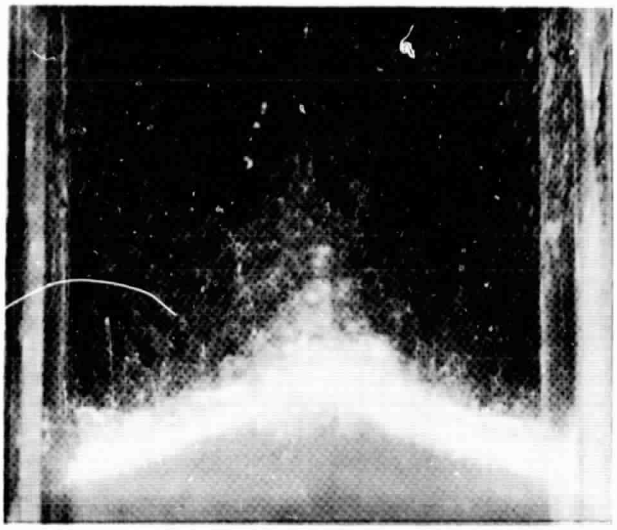

Figure 14. Low Frequency Liquid Surface Mode Excited Through Nonlinear Coupling Of High Frequency Surface Spray In A Longitudinally Excited Rigid Cylinder

Figure 15. Large Bubble Cluster Forming By Small Gas Bubbles Sinking To Tank Bottom Under The Influence Of Longitudinal Excitation

IV. BUBBLE DYNAMICS

The behavior of gas bubbles within the main mass of liquid propellant in a longitudinally excited tank is another prominent current problem in liquid fuel dynamics. The necessity of pure gas-free liquid supplied to vehicle systems is well recognized. The presence of small gas bubbles within the liquid propellant can be caused by surface instabilities, and even more readily by growth of bubbles from nucleation anywhere within the tank, when the ullage pressure over a saturated liquid is suddenly decreased during a venting operation. Regardless of how bubbles originate at a given point within the liquid, their movement to other parts of the tank under the influence of vibration is of major concern, for it can be seen that bubbles could readily collect in undesirable locations, such as in the vicinity of a drain outlet from the tank.

Average bubble motion in a vibrated liquid is influenced predominantly by two forces, the steady acceleration buoyancy force, and the induced average buoyancy force resulting from vibration. The forces are apparently independent of each other so that one can dominate over the other in different environments. The steady buoyancy force always acts opposite to the direction of the body force, while the average vibrational force can act in any direction in a given tank, depending on the oscillatory pressure distribution in the liquid, which, in turn, depends on the excitational conditions.

A. Moderate Acceleration Fields

All known investigations of bubble behavior under vibration have been for the case of a tank in a large gravity field (one g), i.e., large Bond numbers. Dodge [11A] has reported a recent survey of these investigations. Additional work in the current program has been reported in [12A], and Figures 15 and 16 show information taken from this work. Figure 15 shows a bubble cluster forming at the bottom of a longitudinally excited tank. The cluster is growing from bubbles that were produced at the liquid surface; however, the cluster could form just as well from bubbles injected at various locations in the tank. The direction of bubble migration, as well as the location at which clusters form, depends entirely on the pressure distribution within the gas-liquid mixture. This pressure distribution is the same axisymmetric pressure response that has been discussed in Section I and, as was pointed out, can arise from both linear and nonlinear behavior, although the additional effects of the gas bubbles must now be considered.

Figure 16 shows the input oscillatory acceleration required to cause small bubbles to sink from the indicated depth in a tank excited in a steady

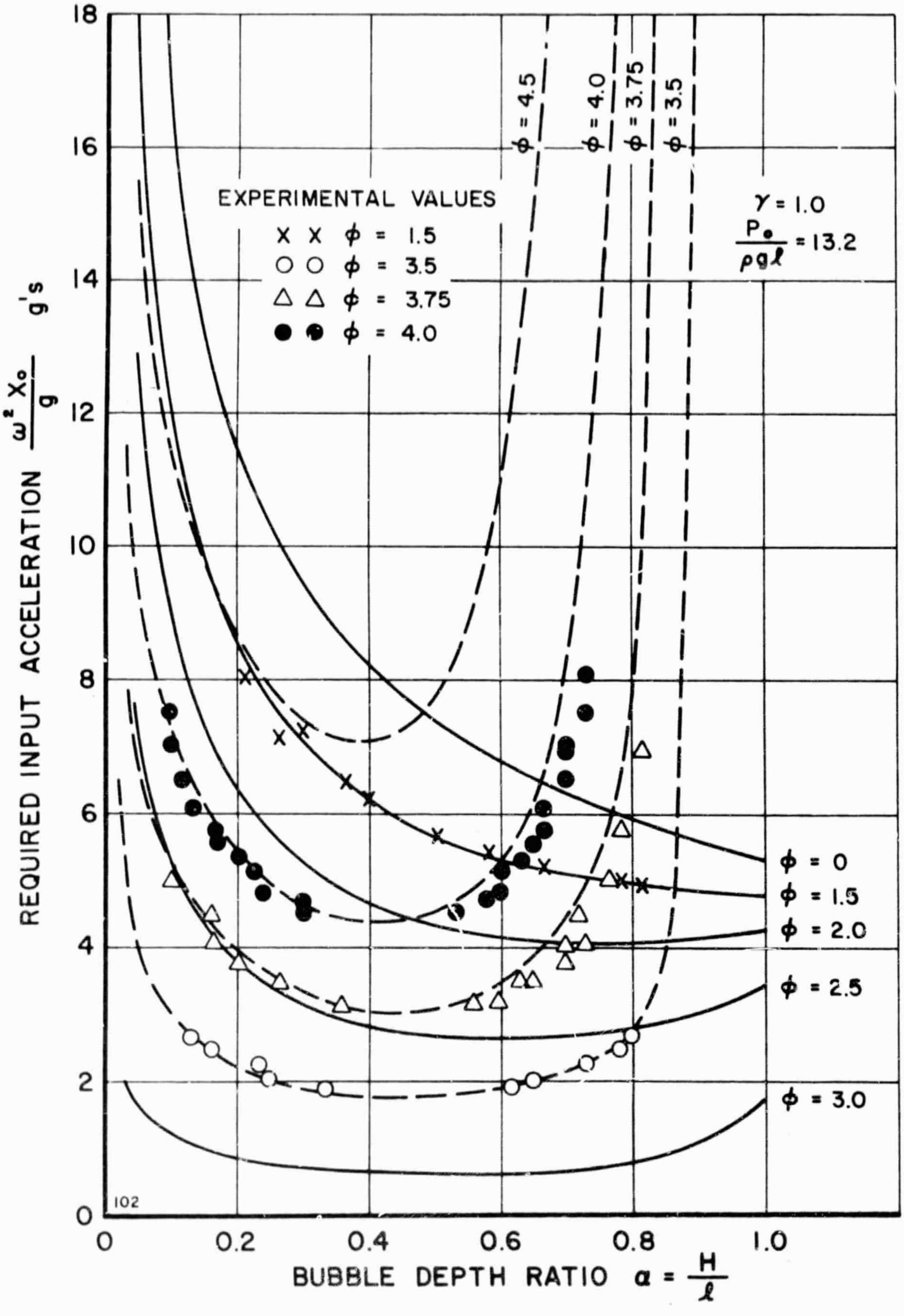

Figure 16. Variation Of Vibrational Input Acceleration Required To Induce Negative Buoyancy For Bubbles At Various Depths In A Longitudinally Excited Cylinder

one-g field. Experimental and theoretical results are seen to compare very well. It is obvious that rather strong oscillatory accelerations are required to overcome the static buoyancy force of the one-g acceleration field. Results similar to the above have been found by other investigators [17B, 18B] for the case of high Bond numbers.

B. Low Gravity Environment

The behavior of gas bubbles in tanks for a low gravity condition is of immediate importance for orbital flight. It has been pointed out above that the static buoyancy force predominates for large gravity environments. Conversely, it might be surmised that the average vibrational buoyancy force becomes dominant in low gravity environments. Experimental and theoretical efforts are being conducted in the current program to study this average vibrational buoyancy force on finite size bubbles for simulated low gravity conditions.

Figure 17, taken from recent work [13A], shows experimental data for the relationship between average vibrational force and input acceleration for a 3.8-cm bubble in a 46-cm column of water in a vertically oscillated tank. The effects of the static buoyancy force were eliminated by using a counterweighted captive bubble, while the average vibrational force was measured by means of a specially designed transducer. Each curve is for a constant frequency input, corresponding to the ratio of input frequency to natural frequency of the first axisymmetric pressure mode in the combined system. Thus, the importance of the pressure distribution on the overall coupled liquid dynamics behavior of the system is again emphasized. It can be seen that substantial average forces are imparted to the bubble for only slight excitational conditions. Thus, bubbles could be caused to move to various points in the tank for very low noise level inputs, such as can be experienced from recirculating pumps, etc., in the low gravity environment.

In general, it has been found that, even for larger size bubbles, the average buoyancy force tends to move the bubble toward the point of increasing oscillatory pressure gradient. Furthermore, current experiments show that a steady sinusoidal input is not required for this bubble migration to occur as a similar behavior has been observed in tanks subjected to random vibrational inputs. Much more work is required before the bubble behavior can be predicted in a given tank subjected to a variety of excitational conditions. Efforts in the current program are being directed toward obtaining a better understanding of such overall dynamic bubble behavior. It is believed that this behavior can be of paramount importance in the low gravity environment.

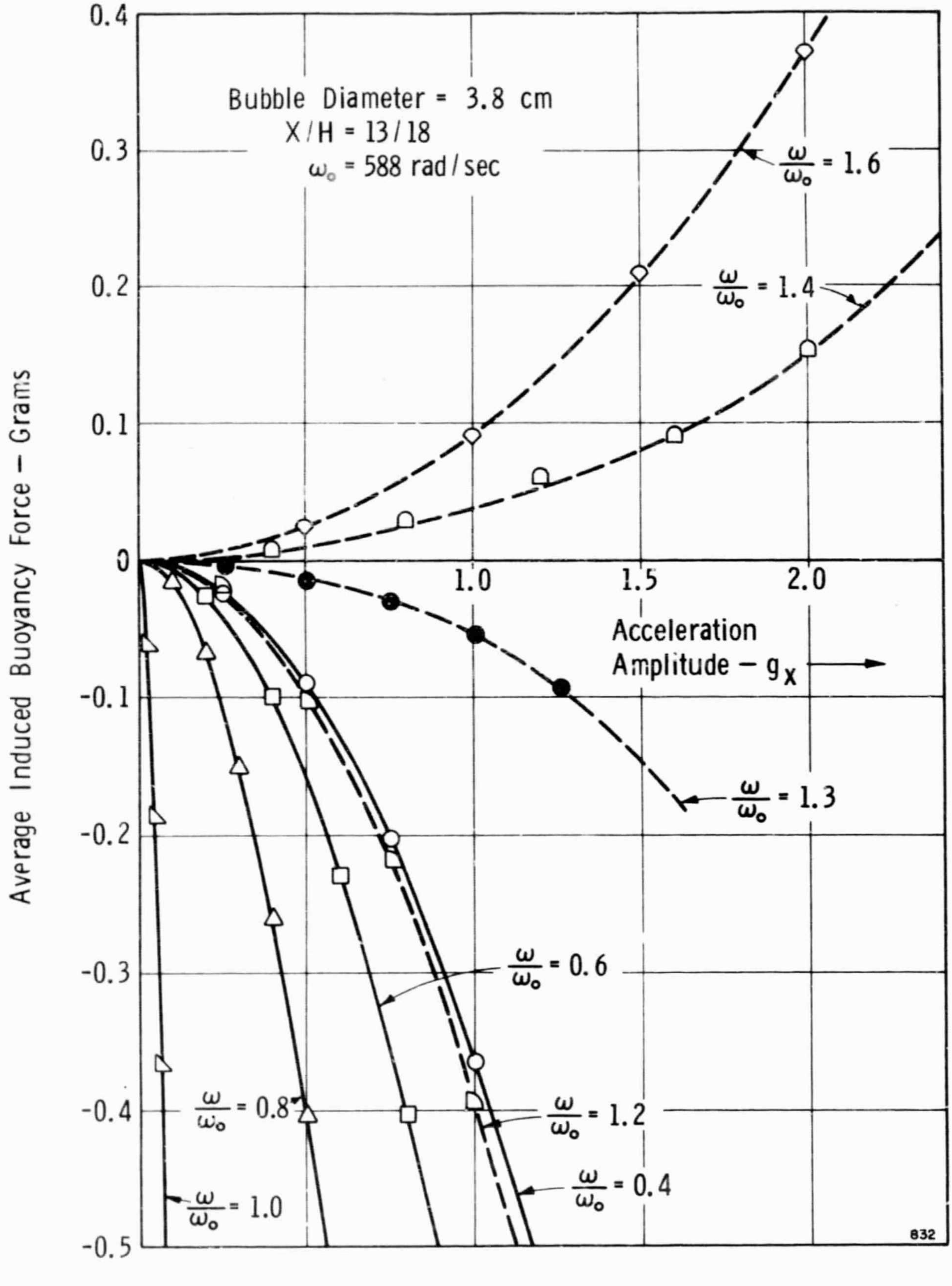

Figure 17. Variation Of Average Induced Buoyancy Force With Input Acceleration Amplitude For A Large Gas Bubble In A Longitudinally Excited Cylinder

V. SUMMARY

This report has discussed recent progress achieved in the liquid fuel dynamics program at SwRI, sponsored by NASA Marshall Space Flight Center under Contract NAS8-11045. Progress has been particularly noteworthy in the areas of the interaction of the liquid and the elastic structure, liquid surface instability, and bubble dynamics. These three areas of study, as well as other associated problems, are related by their common dependence on the axisymmetric pressure response within the liquid column in the liquid-elastic tank system, as shown in Figure 1. The most recent studies indicate that this pressure response can be influenced strongly by nonlinear as well as linear vibrations. In general, it can be said that increasing efforts will be required before the significance of laboratory observed behavior can be evaluated in prototype systems, although the potential importance of much of the observed behavior is already obvious.

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LIST OF SYMBOLS

a	tank mean radius
A_{mn}	shell displacement amplitude in mn-th mode
B_c	Bond number
C_e	equivalent sonic velocity
d	tank diameter
E_t	tank elastic modulus
f	frequency
F^*_{42}	nondimensional forcing amplitude
g_o, g	standard earth gravity
g_x	longitudinal excitational acceleration amplitude in g's
h	shell wall thickness
H	liquid depth
k	support spring constant
ℓ	shell length
m, n	axial and circumferential wave number for shell mode (Figures 7 through 10)
m, n	angular and radial wave number for liquid surface mode (Figures 11 through 13)
M_o	mass of tank bottom and support
P_o	ullage pressure
q, Q	oscillational pressure amplitude at tank bottom
X_o	longitudinal excitation displacement amplitude
x	bubble depth in liquid

α	liquid depth parameter
γ	ring stiffness factor (Figures 2 through 6)
γ	ratio of specific heats for gass bubble (Figure 16)
δ	frequency parameter $\frac{\omega^2}{\Omega_{mn}^2}$
$\delta_1, \delta_{01}, \delta_{81}$	damping factors for respective modes (Figure 10)
ϵ	excitational parameter
ζ_{01}, ζ_{81}	liquid amplitude factor for respective modes (Figure 10)
ρ_L, ρ	liquid mass density
ϕ	frequency parameter $\frac{2\omega\ell}{C_e}$
ω	excitational frequency
ω_o	natural frequency for first axisymmetric coupled mode of bubble and liquid-tank system
Ω_{mn}	natural frequency for mnth nonaxisymmetric transverse shell mode in a cylinder containing liquid

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