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ON THE POLARIZATION METHOD OF INVESTIGATION OF TWILIGHT PHENOMENA

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A method is proposed, which takes into account the tropospheric component (higher order scattering). It is based on observations of intensity and degree of polarization of the twilight sky in two symmetrical points of the solar vertical. The polarization ratio at these points must be known beforehand from preliminary calculations. Formulas for these calculations are derived and their application is given for a simplified model of a twilight segment.

Author

* * *

1. Before undertaking the application of twilight events for the study of optical properties of the upper atmosphere it is necessary to ascertain the intensity of primary twilight. To that effect I proposed a method of simultaneous observations of two points of the sky in the Sun's vertical, symmetrical relative to zenith and located at the height of $\sim 20^{\circ}$. The observed twilight brightness represents, generally speaking, the sum of three terms, which are: the primary twilight segment I_1, sufficiently remote from the observer and dependent on light scattering only of first order, the tropospheric component I_2, defined by light scattering of higher orders, nearly exclusively in the lower and denser air layers, and the general stellar background I_3, easily found from observations after the end of twilight. Therefore

$$I = I_1 + I_2 + I_3.$$

* O POLYARIZATSIONNOM METODE ISSLEDOVANIYA SUMERECHNYKH YAVLENIY.

If the second symmetrical point is not too high over the horizon, the primary twilight segment already ceases being superimposed to it at comparatively small dipping of the Sun under the horizon, for example, greater than 6^o; then

$$I' = I_2' + I_3'.$$

If we determine from appropriate theoretical calculations the ratio

$$K = I_2 / I_2',$$

(see [1]), we shall immediately have

$$I_1 = I - I_3 - KI_2'.$$

Theoretically, the brightness of primary twilight is represented in the form of integral equation containing a function of atmosphere's optical properties, which may hence be determined. It is desirable for a greater reliability to measure also the degree of polarization P, P' at the same symmetrical points of the sky, which will give us the additional correlations

$$IP = I_1P_1 + I_2P_2,$$
$$IP' = I_2'P_2'$$

(the night sky background is practically nonpolarized and its brightness is neglectingly small).

We find

$$I_1 = \frac{I(P - P_2) + I_3P_2}{P_1 - P_2} \qquad (1)$$

and

$$P_2' = IP' / I_2'.$$

The polarization P' of the primary twilight segment is quicky found in the function of angular distance ϑ from the Sun, provided the relative scattering indicatrix for the high atmosphere is known; but P_2 and P'_2, the polarizations at the same symmetrical points, may be found only by way of rather cumbersome calculations. It would be more reliable to compute for various Sun dippings under the horizon the ratio of these quantities

$$\alpha = P_2 / P_2'.$$

Then, knowing P_2' from direct observations, we shall immediately find P_2, and from (1) we shall also find I_1. Such may be the independent method of investigation, which might serve as a known control of the first. In the following we shall expound a method for computing the degree of polarization of the troposphere component at symmetrical points of the s.v.

2.- We shall consider at first the general case. The brightness of the daylight sky at a certain point M with zenithal distance z, located at the angular distance ϑ from a nonpolarized light source S, may be represented by the expression

$$I = kL[f_1(\vartheta) + f_2(\vartheta)]\varphi(z, \zeta, p);\ \varphi(z, \zeta, p) = \frac{p^{\sec\zeta} - p^{\sec z}}{\sec z - \sec\zeta}\sec z,$$

where f_1, f_2 are the components of the scattering indicatrix with oscillations respectively perpendicular and parallel to the grand circle SM; ζ is the zenithal distance of S and p is the atmosphere transparency index. The degree of polarization at M will be simply

$$P = (f_1 - f_2) / (f_1 + f_2).$$

Let us assume now that the light source of intensity L is partly polarized. Let the amplitudes of basic oscillations be a and b, so that $L = a^2 + b^2$. Assume also that the polarization angle at L will be such that the oscillations a constitute an angle α with the normal to the great circle SM. We thus shall obtain that the brightnesses of scattered light at the point M, conditioned by oscillations perpendicularly and parallelwise to the great circle SM, will be

$$\mathrm{I} = (a^2\cos^2\alpha + b^2\sin^2\alpha)f_1(\vartheta)\varphi(z, \zeta);$$
$$\mathrm{II} = (a^2\sin^2\alpha + b^2\cos^2\alpha)f_2(\vartheta)\varphi(z, \zeta).$$

Inasmuch as

$$a^2 = \frac{1}{2}L(1 + P_0);\quad b^2 = \frac{1}{2}L(1 - P_0);\quad P_0 = \frac{a^2 - b^2}{a^2 + b^2},$$

it follows that the total brightness at the point M, partly due to the polarized light source in S, will be I + II, which is easily brought up to the expression

$$I_M = k\frac{L}{2}(f_1 + f_2)\varphi(z, \zeta, p)\,(1 + PP_0\cos 2\alpha),$$

where, as earlier,

$$P = (f_1 - f_2) / (f_1 + f_2).$$

In the case when the light source in S with partial polarization P_0 has extended dimensions, we obtain

$$I_M = \iint B(\zeta, A)\,(f_1 + f_2)\varphi(z, \zeta, p)\ \ (1 + PP_0\cos 2\alpha)\sin\zeta\, d\zeta\, dA.$$

The polarization at the point M, induced by each element of the source S, is represented by the expression

$$P_M = \frac{(a^2\cos^2\alpha + b^2\sin^2\alpha)f_1 - (a^2\sin^2\alpha + b^2\cos^2\alpha)f_2}{(a^2\cos^2\alpha + b^2\sin^2\alpha)f_1 + (a^2\sin^2\alpha + b^2\cos^2\alpha)f_2}$$

or, after the necessary reductions,

$$P_M = \frac{P + P_0\cos 2\alpha}{1 + PP_0\cos 2\alpha}.$$

3. - We shall apply these considerations for the calculation of the troposphere component of the twilight sky for symmetrical points of solar vertical at zenithal distance $z = 70^\circ$. In the given case, the outer and partly polarized light source is the primary twilight segment, which, beginning with Sun's dipping by more than 6° under horizon is already sufficiently remote from the observer and the scattering troposphere layer above him. The brightness of the primary twilight segment at the point S (ζ, A) (the azimuth being counted from the solar vertical) may be computed with the help of the expression

$$I_0 = \frac{Lf^0(\vartheta_0)}{\sin\vartheta_0}\int_{h_{0\,min}}^{\infty} \mu(h)\,(abs)\,dh_0.$$

Here $\mu(h)$ characterizes the atmosphere's scattering capability as a function of height h above ground; (abs) is the value of solar ray's extinction, for a ray traversing the whole atmosphere at the minimum height of rapprochement with the terrestrial surface h_0; ϑ_0 is the angular distance of the considered element S of primary twilight from the Sun S_0 and $f^0(\vartheta)$ is the scattering indicatrix, characteristic of the upper atmosphere layers.

Inasmuch as the twilight segment is totally symmetrical relative to solar vertical and the polarization angle must be parallel to the horizon, we shall limit ourselves to the determination of the aggregate degree of polarization only.

For the constituents of polarization components perpendicular and parallel to the arc of great circle we have

$$a^2 = \frac{Lf_2^0(\vartheta a)}{\sin\vartheta_0}\int_{h_{0\,min}}^{\infty} \mu(h)\,(abs)\,dh_0; \quad b^2 = \frac{Lf_2^0(\vartheta_0)}{\sin\vartheta_0}\int_{h_{0\,min}}^{\infty} \mu(h)\,(abs)\,dh_0.$$

Bearing this in mind, we obtain, according to the above-said, for the total vertical component of oscillations at the point M of solar vertical, produced by all the elements of the primary twilight segment

$$\frac{1}{2}\sum[f_1(\vartheta)I_0(1+P_0\cos 2\alpha)\sin^2\beta + f_2(\vartheta)I_0(1-P_0\cos 2\alpha)\cos^2\beta]\,\varphi(z,\zeta,p)$$

and for the total horizontal oscillation component

$$\frac{1}{2}\sum[f_2(\vartheta)I_0(1+P_0\cos 2\alpha)\cos\beta + f_2(\vartheta)I_0(1-P_0\cos 2\alpha)\sin^2\beta]\,\varphi(z,\zeta,p),$$

whence we obtain the degree of polarization searched for :

$$\bar{P}=\frac{\int\int[f_1I_0(1+P_0\cos 2\alpha)\cos 2\beta - f_2I_0(1-P_0\cos 2\alpha)\cos 2\beta](\varphi(z,\zeta,p))\,d\sigma}{\int\int[f_1I_0(1+P_0\cos 2\alpha)+f_2I_2(1-P_0\cos 2\alpha)]\,\varphi(z,\zeta,p)\,d\sigma},$$

where

$$d\sigma=\sin\zeta\, d\zeta\, dA$$

is the element of primary twilight.

We finally have

$$\bar{P}=\frac{\int\int I_0(f_1+f_2)\varphi(z,\zeta,p)\cos 2\beta(P+P_0\cos 2\alpha)\,d\sigma}{\int\int I_0(f_1+f_2)\varphi(z,\zeta,p)\,(1+PP_0\cos 2\alpha)\,d\sigma}.$$

For the calculation of this expression we must first of all have the scattering indicatrix f^o for the high atmosphere (primary twilight segment) and f for the atmosphere as a whole, expanding them by polarization components. Inasmuch as determinations of f^o from altitude rockets have been unavailable to-date, the only thing to do is to adopt the scattering indicatrix, determined with its polarizing properties by Pyaskovskaya - Fesenkova in the Libyan Desert (Southern Egypt) in 1957, as the most convenient [2], for the Sun's position near horizon, when the effective height of the equivalent atmosphere layer was $\sim$ 15 km. The aerosol scattering indicatrix f^o was obtained from these observations upon introduction of appropriate corrections for higher order scattering and the Rayleigh scattering. The total indicatrix f has been found under the same conditions. Table 1 gives the respective values of f, P and f^o, P^o for different angular distances ϑ.

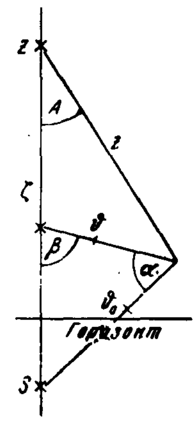
The angles α, β, and also ϑ and ϑ_0 (Fig. 1) may be found graphically with a sufficient precision, for example, by the V. V. Kavrayskiy network.

TABLE 1

ϑ	$f=f_1+f_2$	p	f°	p°	ϑ	$f=f_1+f_2$	p	f°	p°
10°	4.22	0.020	10.13	0.020	100°	1.000	0.715	0.934	0.425
15	3.183	0.035	6.60	0.040	110	1.068	0.633	0.890	0.405
20	2.675	0.054	4.95	0.062	120	1.177	0.520	0.894	0.380
30	2.075	0.125	3.153	0.120	130	1.266	0.383	0.898	0.335
40	1.707	0.220	2.235	0.180	140	1.362	0.270	0.900	0.280
50	1.452	0.333	1.733	0.240	150	1.432	0.130	0.894	0.222
60	1.279	0.475	1.408	0.308	160	1.496	0.050	0.890	0.153
70	1.152	0.607	1.246	0.360	170	1.540	0.010	0.890	0.075
80	1.046	0.710	1.333	0.405	180	1.567	0.000	0.890	0.000
90	1.000	0.750	1.000	0.424					

TABLE 2

z		A						
		0	10°	20°	30°	40°	50°	60°
90°	α	180°	109°	73	54.8	44.4	38.1	3.42
	β	0	27.3	47.0	59.4	68.0	74.1	70.0
	ϑ_0	10.0	14.3	22.5	31.8	41.1	51.0	60.8
	ϑ	20.0	22.2	28.2	35.8	44.3	53.1	62.2
85°	α	180	113.5	75.6	56.1	44.9	88.3	34.0
	β	0	34	55	66.9	74.6	80.0	84.4
	ϑ_0	15.0	18.0	25.0	33.4	42.6	52.1	61.8
	ϑ	15.0	17.8	24.8	32.9	41.8	50.9	60.3
80°	α	180	110.2	73.6	55.0	44.6	37.9	33.9
	β	0	45	65.8	75.6	81.6	86.4	90.0
	ϑ_0	20	22.0	28.0	35.6	44.1	53.3	62.8
	ϑ	10.0	13.8	21.7	30.4	39.8	49.0	58.5
75°	α	180	95.0	67.0	51.9	42.8	36.9	23.2
	β	0	64	78.3	85.1	89.3	92.6	95.8
	ϑ_0	25.0	23.9	31.8	38.8	46.7	55.4	64.5
	ϑ	5.0	10.7	19.8	29.0	38.6	47.8	57.2
70°	α	—	72.3	57.9	47.6	40.5	35.5	32.1
	β	0	92	93.7	95.3	97.2	99.2	01.4
	ϑ_0	30.0	31.6	35.9	42.1	49.4	57.7	66.2
	ϑ	0	9.5	19.0	28.2	37.8	46.9	56.2



horizon

Fig. 1

For the particular case when $\zeta = 100°$, $z = 70°$, we have the data brought out in Table 2. Finally, the function $\varphi(z, \zeta, p)$, which may be more accurately represented in the form

$$\varphi(z, z_0, p) = \frac{p^{m_0} - p^m}{m - m_0} m_0,$$

where m and m_0 are the atmosphere masses corresponding to zenithal distances z and z_0, is represented by numerical values for the transparency factor $p = 0.825$ and $z = 70°$, compiled in Table 3. Note that at $z = 90°$, the atmosphere mass is approximately 40 ($m = 40$).

TABLE 3

z	$\varphi(z, z_0)$	z	$\varphi(z, z_0)$	z	$\varphi(z, z_0)$
90°	0.01817	80	0.08460	60	0.1161
88	0.03345	75	0.09768	45	0.1225
85	0.05860	70	0.1068	0	0.1274

4. - The calculation of the aggregate polarization of the troposphere component is rather complicated, and the more so since the detailed calculations of primary twilight isophots for various dippings of the Sun under the horizon and various optical peculiarities of the upper atmosphere must precede it. It is appropriate to conduct such calculations on computers. This is why we shall limit ourselves here to a simplified scheme for an example, when the entire twilight segment is reduced only to the arc along the horizon of greater or lesser extension.

Noting that the function $\varphi(z, z_0, p)$ for two symmetrical points of the solar vertical is the same and is independent from the azimuth, we shall have a simplified expression for P_m, namely

$$P_m = \frac{\int_{-A}^{A} I_0 \cos 2\beta (f_1 + f_2)(P + P_0 \cos 2\alpha)\, dA}{\int_{-A}^{A} I_0 (f_1 + f_2)(1 + PP_0 \cos 2\alpha)\, dA}.$$

For the indicated symmetrical points of the solar vertical we have

$$\cos \vartheta_1 = \sin z \cos A, \quad \cos \vartheta_2 = -\sin z \cos A;$$

$$\cos \alpha_1 = \frac{\cos(\zeta - z) - \cos \vartheta_1 \cos \vartheta_0}{\sin \vartheta_1 \sin \vartheta_0}; \quad \cos \alpha_2 = \frac{\cos(\zeta + z) - \cos \vartheta_2 \cos \vartheta_0}{\sin \vartheta_2 - \sin \vartheta_0};$$

$$\mathrm{tg}\, \beta = \mathrm{tg}\, A \sec z; \quad \beta_1 = \beta_2.$$

For the first symmetrical point the computations give ($z_o = 70°$)

$$P_m = 1.44\%$$

and for the second

$$P_m = 0.615\%.$$

As may be seen, the corresponding degrees of polarization are rather small. For a better idea about the kind of influence the various properties of the primary twilight segment exert upon the result, analogous calculations were also performed for a scheme of arc along horizon of uniform brightness but different extension and, at the same time, nonpolarized, that is, when $I_0 = 0$ and $P_0 = 0$. In this case, for arcs from 0 to 50° extension we have (Table 4) :

TABLE 4

A	0	10°	20°	30°	40°	50°
P_{m_1} %	6.5	4.88	3.08	1.39	—1.098	— 4.63
P_{m_2} %	7.49	6.88	4.86	0.894	—4.66	—11.38

As may be seen from this, the parts of the twilight segment, more remote from the solar vertical, lead to negative polarization, though for a sufficient dipping of the Sun, the brightness of these parts is insignificant.

The method expounded calls for rather detailed preliminary calculations and measurements of polarization with an absolute precision of, say 0.01 percent.

*** THE END ***

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