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NEUTRAL (ZERO) POINTS OF MAGNETIC FIELDS

by

Yu. D. Zhugzhda

(USSR)

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SUMMARY

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The geometry of a magnetic field is investigated in the immediate vicinity of the neutral points. The complete classification of neutral points of a free field is described in detail.

It is shown that the neutral points of a force-free field in the immediate vicinity are reduced to the neutral points of a free field with a precision to the terms of higher orders. The simplest neutral points of a static field are considered.

* * *

Author

At present the most probable source of energy of a solar flare is estimated to be that of the magnetic field of sunspots. A series of experimental [1, 2] and theoretical works [3 – 5] relate the solar flare events with processes taking place at neutral points of a magnetic field, that is, in those points where the magnetic field becomes zero. In connection with this it is of interest to resort to the classification of all the possible types of neutral points, lines and surfaces and to the geometry of the field in their vicinity. Because the Sun's atmosphere has a high conduction and distributed currents may exist, considered in the work were not only the neutral points of a free field (field in vacuum), but also those of a force-free field and of a field in a state of hydrostatic equilibrium with the medium (static field). As far as we know a systematic investigation of these problems is not available in literature, though some of the particular questions have been considered [3, 6, 7].

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* NEYTRAL'NYYE (NULEVYYE) TOCHKI MAGNITNYKH POLEY.

METHOD OF INVESTIGATION. Within a sufficiently close neighborhood of a neutral point the magnetic field may be represented in the form of a series by powers of coordinates. The first terms of the series that are not zero describe a field characteristic for a given type of neutral point, whose geometry remains invariable within a wide range of parameter variations of field sources. If in the close vicinity of a neutral point the field depends linearly on coordinates, that is, if the first linear terms of the expansion in series are not zero, we shall describe it as a first order point. In the vicinity of a second order point the field is already a quadratic function of coordinates, etc. Therefore the problem of finding points of all types amounts to the problem of searching for the solution of field equations in the form of series by rising coordinate powers.

The method applied allowed to ascertain the character of field behavior in the neighborhood of the neutral point, digressing from the general complex pattern of an arbitrary magnetic field, and to establish all the basic types of neutral points.

FREE FIELD. The free field is described by the equations

$$\mathrm{rot}\,\mathbf{H} = 0, \qquad \mathrm{div}\,\mathbf{H} = 0 \tag{1}$$

and a scalar potential may be introduced for it in such a way that

$$\mathbf{H} = \mathrm{grad}\,\psi.$$

The scalar potential must satisfy the Laplace equation

$$\Delta\psi = 0. \tag{2}$$

In the neighborhood of the neutral point the scalar potential, as well as the magnetic field intensity, may be expanded in series by increasing coordinate powers. In the most general case such a series, satisfying Eq. (2), is expressed by spherical functions and in spherical coordinates, having the form

$$\psi = \sum_{n=2}^{\infty} r^n \left[\alpha_n P_n(\cos\theta) + \sum_{m=1}^{n} (\beta_{nm}\cos m\varphi + \gamma_{nm}\sin m\varphi) P_n^m(\cos\theta) \right] \tag{3}$$

where α_n, β_{nm} and γ_{nm} are arbitrary constants. In this solution to each spherical function corresponds a specific type of neutral point, distinct from other types. Evidently, the field geometry in the vicinity of each of the these types of neutral points does not depend on the arbitrary constants entering into the general equation. The free magnetic field near the arbitrary neutral point may be represented as the sum of the field corresponding to these simple types of neutral points.

The scalar potential of the field in the neighborhood of neutral points of the order p is expressed by means of spherical functions with $n = p + 1$.

Neutral Plane. In order that there exist a neutral plane (let us assume it to be coinciding with the plane $\theta = 90^{\circ}$), it is necessary that the potential (3) and its derivatives with respect to coordinates convert to zero on it. It is not difficult to see that these conditions are satsified only at unconditional equality to zero of the magnetic field about the entire space. Therefore, a neutral plane, and consequently, a neutral surface cannot exist in a free field. The obvious corrolary of that is the impossibility of existence of bounded configuration of a free field.

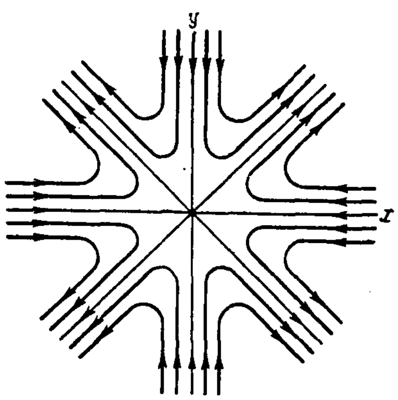

Fig. 1 a

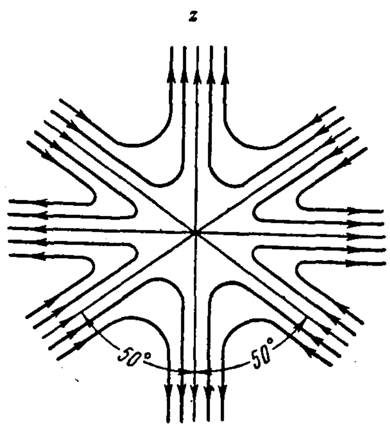

Fig. 1

Neutral Line. The scalar potential in the vicinity of a point lying on a neutral line coinciding with the axis z, is described by sectorial spherical functions of the form

$$r^n P_n{}^m (\cos\theta) \, {}^{\sin}_{\cos} n\varphi. \qquad (4)$$

Classifying these neutral points, we classify in fact the neutral lines to which these points belong. There is one type of neutral line of each order. Each type enters twice in the general solution and only the field patterns find themselves rotated by the angle $\varphi = \pi/2n$ around the axis **z**. A neutral line of third order corresponding to $n = 4$ is drawn in Fig. 1a. The neutral oine coincides with the axis **z**. The magnetic lines of force lay in planes parallel to the plane xy. In each such plane there are four asymptotes on which lie the lines of force passing through the neutral line. The pattern of the lines of force in the neighborhood of the neutral line of the order $p = n - 1$ differs from the above-considered only by the number of asymptotes, which is equal to **n**. The asymptotes are always symmetric relative to the axis **z**.

INTERSECTIONS OF NEUTRAL LINES

The cubic spherical functions of the form

$$r^n P_n^m (\cos\theta) \begin{smallmatrix}\sin\\ \cos\end{smallmatrix} m\varphi \qquad (5)$$

with $m \neq n$ give the expression for the magnetic field potential in the neighborhood of the neutral point lying at the intersection of neutral lines. A unit sphere with its center at the considered neutral point is shown in Fig. 2. The meridians and the parallels on the sphere are the intersection lines with the surfaces on which the potential $\psi = 0$. The parallels constitute the intersections of the sphere with infinite cones, of which the summits are at the neutral point. The family of these cones is described by the equation

$$P_n^m(\cos\theta) = 0. \qquad (6)$$

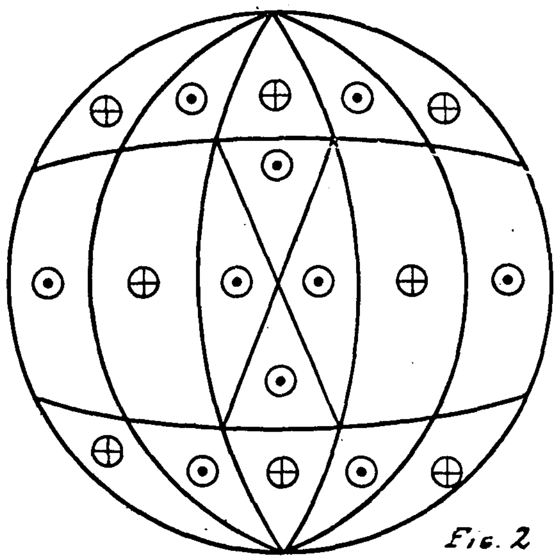

Fig. 2

The family consists of $(n - m)$ cones and at the same time $(n - m)$ is an odd number and the equatorial plane is part of the family. The cones are disposed symmetrically relative to the equatorial plane. Moreover, the equation has a root $\theta = 0$ of multiplicity **m**. The meridians are the intersections with the planes passing through the axis **z**. The family of the planes is described by the equation

$$\sin m\varphi = 0. \qquad (7)$$

It consists of planes symmetrically disposed relative to the axis **z**.

The magnetic lines of force are perpendicular to the above-considered equipotential surfaces. Consequently, the intersection lines of the planes with the cones are neutral lines. The axis $\underline{z}$ is also a neutral line when $m \geqslant 2$. Therefore, the neutral line of the order $(p = n - 1)$ constitutes the intersection of $2m(n - m) + 1$ neutral lines at $m \geqslant 2$, and by one less when $m = 1$.

Ascertaining the type of neutral lines intersecting at the considered points is of interest. Near the intersection of neutral lines the magnetic field is distorted in the neighborhood of everyone of them on account of the proximity of other neutral lines. As one draws farther away from the point of intersection, the "distortions" of the field pattern decrease to the extent that the determination of the type of each of the neutral lines becomes possible.

It is found that, except for the neutral line coinciding with the axis $\underline{z}$, all the other neutral lines are of first order. As to the one coinciding with the axis $\underline{z}$, it has the order $(m - 1)$.

In order to make out the geometry of the magnetic field in the neighborhood of the intersection point, we shall consider one of the solid angles by which all the space with surfaces with $\psi = 0$ is divided. In this solid angle all the lines of force approach the asymptote at infinity, the position of which being determined by the condition of attaining the function's $P_n^m(\cos\theta)$ extremum and satisfying the condition $\sin m\varphi = 1$. The planes drawn through that asymptote and each of the solid angle's ribs constituting neutral lines, divide the entire pyramid into several parts (by the number of sides). The lines of intersection of these planes with the unit sphere for one of the solid angles are shown in Fig. 2. All the lines of force belonging to one such part of the solid angle pass into the neighboring solid angle as one draws nearer the summit, intersecting the general face at a right angle. In the neighboring solid angle they also fill only that part of it, which is bounded by planes drawn through the edges of this solid angle and its asymptote. One of the two possible variants in the alternation of line of force directions in solid angles is shown in Fig. 2, where the crosses indicate that within the bounds of this solid angle the lines of force are directed inside the unit sphere, while the dots indicate the opposite direction. A second possible variant takes place when the directions of the lines of force are opposite to those indicated in Fig. 2.

The exact mutual disposition of all the described geometric objects for any neutral point among the described class is easy to find with the help of the tables of values of Legendre polynomials. Two neutral points of each of the described types enter into the general solution , with field patterns turned relative to one another by the angle $\varphi = \pi / 2m$ around the axis **z**.

Isolated Neutral Points. The zonal spherical functions $r^n P_n(\cos\theta)$, describing the field in the vicinity of the isolated neutral point, enter into the general solution of the Laplace equation for the potential. Only one type of such a neutral point of the given order exists. The magnetic lines of force in the neighborhood of such points form a family of patterns of revolution around the axis **z**.

The structure of the field in the plane passing through the axis **z** is plotted in Fig. 16 for the case of an isolated neutral point of third order. The number of asymptotes is always by one unit greater than the order of the point. The asymptote position is determined by the extrema of the corresponding spherical harmonic.

FORCE-FREE MAGNETIC FIELD.

A force-free magnetic field satisfies the equations

$$[\mathbf{H} \operatorname{rot} \mathbf{H}] = 0, \quad \operatorname{div} \mathbf{H} = 0. \qquad (8)$$

If we transform the first equation after taking the divergence from the left-hand part, we shall obtain

$$\mathbf{H} \operatorname{rot} \mathbf{j} = 4\pi / c (\mathbf{j})^2. \qquad (9)$$

The corollary of this equation is : in a neutral point of a force-free field the current $\mathbf{j} = (c/4\pi) \operatorname{rot} \mathbf{H}$ is zero. The first equation of a force-free field may be written in a different form

$$\operatorname{rot} \mathbf{H} = \alpha \mathbf{H}, \qquad (10)$$

where α is an arbitrary scalar function of coordinates. Since there is no current at the neutral point, α is finite at that point and, consequently may be expanded in series by coordinates

$$\alpha = \alpha_0 + \alpha_1 x + \alpha_2 y + \alpha_3 z + \dots \qquad (11)$$

When the considered point is the neutral point of n-th order, the field expansion in series by coordinates must begin from the terms of n-th order. At the same time the expansion in series for $\underline{\alpha}$ may begin from the terms of arbitrary order $\underline{m}$, and this would include $m = 0$, i. e. $\alpha_0 \neq 0$. When substituting these series into the free-force field equation (10), it is found that the terms of the order $n + m$ and higher will be in the right-hand part, while the terms of the order $n - 1$ and higher will be in the left-hand part. Consequently, in the neighborhood of the considered neutral point rot H is zero with a precision to terms of the $(n + m)$-th order, that is, the distinction between a force-free field and a free field is manifest only in the terms of the $(n + m)$-th order and higher. Thus in a force-free field, in a sufficiently small neighborhood the neutral points cannot be differentiated by the field geometry from the neutral points of a free field, provided in the field expansion only the first series terms which are not zero are taken into account. Consequently, the whole classification of neutral points of a free field is automatically transferred to the force-free field.

STATIC FIELD.

A field in a static equilibrium with a conducting medium satisfies the equations

$$[\mathrm{H}\,\mathrm{rot}\,\mathrm{H}] = -4\pi\nabla p, \quad \mathrm{div}\,\mathrm{H} = 0. \qquad (12)$$

In order for the equation to be satisfied at the neutral point itself, it is necessary that the pressure gradient convert to zero and, consequently, the expansion of pressure in series by coordinates must have the following form :

$$p = p_0 + p_1 x^2 + p_2 xy + p_3 y^2 + p_4 yz + p_5 z^2 + p_6 zx + \dots \qquad (13)$$

The expansion in series for the field components may be written in the tensor form

$$H_i = a_{ij} x_j + a_{ijk} x_i x_k + \dots + a_{ijk\dots l} x_j x_k \dots x_l + \dots \qquad (14)$$

The problem of search for the field structure near the neutral points amount to seeking the tensors $a_{ikj\dots l}$, satisfying the equation. In its general form the problem is not resolved because of equation's nonlinearity. Let us consider the neutral points of first order.

The Isolated Neutral Point.- The field equations (12) for the case of neutral point of first order is satisfied if the tensor a_{ij} describing the field's linear part is antisymmetric or symmetric. The antisymmetric tensor corresponds in reality to the neutral point lying on the neutral line, that is, such a tensor always has one intrinsic value equal to zero. The second solution corresponds to the case when the quadratic terms in the pressure expansion (13) are unconditionally zero and the linear part of the field is a free field, and the force character of the field must be manifest only in subsequent terms of the series. Consequently, in a static field there may exists isolated neutral points only on condition that this field is free with a precision to the terms of subsequent orders.

The Neutral Line. The magnetic field in the vicinity of a point lying on the neutral line of first order may, by coordinate rotation, always be brought to the form

$$H_x = a_{12}y, \quad H_y = a_{21}x. \qquad (15)$$

The field is in equilibrium with the medium in the case when the pressure is $p = p_0 + p_1x^2 + p_3y^2$, where $p_1 = -(1/8\pi)a_{21}(a_{21} - a_{12})$ and $p_3 = -(1/8\pi)a_{12}(a_{12} - a_{21})$. The line of force equation has the form

$$a_{12}y^2 - a_{21}x^2 = C. \qquad (16)$$

There exist two types of neutral lines. If a_{12} and a_{21} are of same sign, the lines of force constitute hyperbolae lying in planes parpendicular to the neutral line, and the neutral point constitutes a peculiar point of saddle type (Fig. 3a). Contrary to free field, the lines, on which lay the lines of force, passing through the neutral point may intersect at any angle. At drawing nearer the neutral point from the side of the sharp angle between these lines the pressure rises, and at drawing nearer from the side of the blunt angle — it drops.

If a_{12} and a_{21} have a different sign, the lines of force constitute ellipses with semiaxes $|a_{12}|^{-1/2}$ and $|a_{21}|^{-1/2}$ (Fig. 3). The pressure increases no matter which side the rapprochement with neutral line takes place from.

<u>The Neutral Surface.</u> - Contrary to the force-free and free fields there may existsin a static magnetic field a neutral surface of first order. In the vicinity of a point lying on such a surface the field depends on the coordinates in the following fashion :

$$H_x = a_{13}z, \quad H_y = a_{23}z. \qquad (17)$$

At the same time $p = p_0 + p_5 z^2$ and the field is in equilibrium with the medium so long as $a_{13}^2 + a_{23}^2 = -8\pi p_5$. The lines of force constitute a family of lines parallel to the neutral plane. On either side from the neutral plane a_{13}, a_{23} and p_5 can be different.

* * *

The described classification of the neutral points of a free field is complete, that is, either any neutral point is one of the described neutral points, or the field near that point may be represented as the sum of the fields of simplest neutral points.

The neutral points of a force-free field in a small neighborhood with a precision to terms of higher orders coincide with the corresponding neutral points of the free field. This is why the classification of neutral points of a free field extends also to a force-free field.

Because of nonlinearity of static field equations, only the neutral points of first order were considered. Contrary to free and force-free fields, a neutral surface may exist in a static field. Another peculiarity of the latter consists in the presence of two types of neutral lines of first order.

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***** THE END *****

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