AXISYMMETRIC DYNAMIC RESPONSE OF SPHERICAL AND CYLINDRICAL SHELLS

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Abstract

The mode-acceleration or Williams method is used to obtain the axisymmetric response, due to an arbitrary time-dependent loading, of a complete spherical shell, and of hemispherical shells with roller-hinged, roller-clamped and completely fixed edge. Numerical examples, where the shells are subjected to simple time-dependent loads, are presented. The same method is used in the study of a cylindrical shell with clamped-edge boundary conditions. A numerical example of a cylindrical shell subjected to a linearly decaying load, uniformly distributed over the shell, is included. The results here compare favorably with solutions obtained by means of a modal analysis in a previous investigation of the problem by the authors.

List of Symbols

a	Radius of sphere
b	Loading parameter
d	Duration of loading
D	Plate Rigidity
E	Modulus of Elasticity
g	Gravitational acceleration
h	Shell thickness
L	Shell length
p_α	Loading components
Q	Non-dimensional normal loading
R	Radius of Cylinder
t	Time
u_α	General displacement components
v, w	Meridional and normal displacements
x, φ	Coordinates
ρ	Mass density of shell
μ	Poisson's Ratio
ω_{1j} ω_{2j}	Frequencies
Ω_j	Frequency

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General Discussion

The general equations of motion of a shell, when damping is neglected, may be written as a system of three linear, coupled differential equations of the form

$$\left[L_{\alpha\beta}\right]\left\{u_\beta\right\} - \rho h \left\{\frac{\partial^2 u_\alpha}{\partial t^2}\right\} = \left\{p_\alpha\right\}^* \tag{1}$$

where $[L_{\alpha\beta}]$ is a matrix differential operator, and where $\{u_\alpha\}$ and $\{p_\alpha\}$ are the displacement and load column vectors respectively. Naturally, suitable boundary and initial conditions must be included.

Assume the displacement vector, according to Williams' method, in the form

$$\left\{u_\alpha(\xi_1,\xi_2,t)\right\} = \left\{\bar{u}_\alpha(\xi_1,\xi_2,t)\right\} + \sum_j \phi_j(t)\left\{u_{\alpha j}(\xi_1,\xi_2)\right\} \tag{2}$$

where $\{\bar{u}_\alpha\}$ and $\{u_{\alpha j}\}$ are solutions of

$$\left[L_{\alpha\beta}\right]\left\{\bar{u}_\beta\right\} = \left\{p_\alpha\right\} \tag{3}$$

and

$$\left[L_{\alpha\beta}\right]\left\{u_{\beta j}\right\} + \rho h \Omega^2{}_j \left\{u_{\alpha j}\right\} = 0 \tag{4}$$

respectively. Here, the $\bar{u}_\alpha$ represent the quasi-static response, and the $u_{\alpha j}$ are the normal modes of free vibration with corresponding eigenfrequencies Ω_j. The necessary coupling equations for the evaluation of the $\phi_j(t)$ are obtained by substituting (2) into equation (1) and simplifying by means of equations (3) and (4). The resultant coupled equation is

$$\sum_j (\ddot{\phi}_j + \Omega^2{}_j \phi_j)\left\{u_{\alpha j}\right\} = -\left\{\ddot{\bar{u}}_\alpha\right\} \tag{5}$$

Multiplication of both sides of equation (5) by the row vector $\lfloor u_{\alpha i} \rfloor$ and use of the orthogonality condition [1], i.e.,

$$\int_S \sum_{\alpha=1}^{3} u_{\alpha i} u_{\alpha j} dS = \delta_{ij} \int_S \sum_{\alpha=1}^{3} u^2_{\alpha i}\, dS \tag{6}$$

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*All Greek indices range from 1 to 3, and the Latin indices from 1 to ∞ unless specified otherwise.

results in the following expression for the ϕ_j

$$\ddot{\phi}_j + \Omega_j^2 \phi_j = - \frac{\int_S \sum_{\alpha=1}^{3} u_{\alpha j} \ddot{\bar{u}}_\alpha \, dS}{\int_S \sum_{\alpha=1}^{3} u_{\alpha j}^2 \, dS} \qquad (7)$$

where S denotes integration over the middle surface of the shell.

When the quasi-static solution can be obtained exactly, this will speed up the convergence of the total solution. In general, however, the quasi-static solution simply represents a quasi-equilibrium position about which the dynamic response of the shell is distributed. When an exact bending theory solution cannot be obtained for the quasi-static part, it is often feasible to use a membrane solution modified by corrections due to bending effects at the boundary.

More detailed discussion of the method and its application to shell problems may be found in [2] and [3]. The geometry of cylindrical and spherical shells and the coordinate systems used for investigation are shown in Fig. (1).

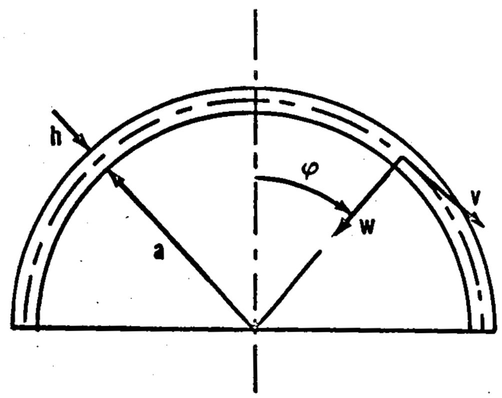

FIG. 1a. GEOMETRY OF HEMISPHERE.

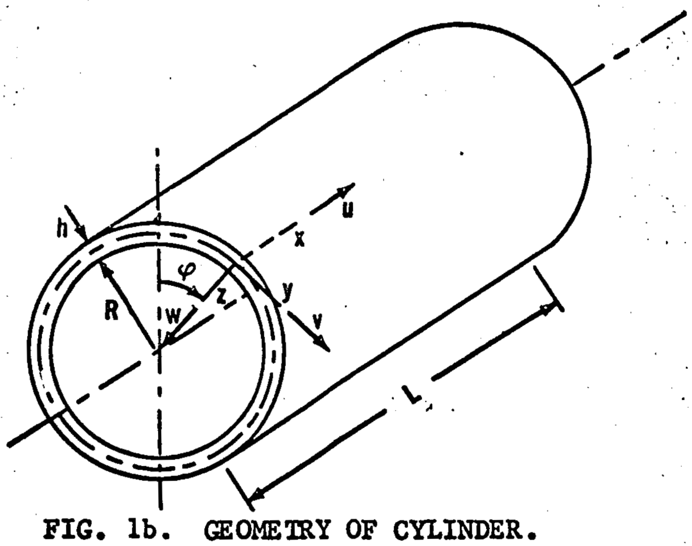

FIG. 1b. GEOMETRY OF CYLINDER.

The Axisymmetrically Loaded Cylindrical Shell

Under consideration is a cylindrical shell with an axisymmetric dynamic loading, normal to the surface of the shell. The shell is of finite length L with mean radius R and has clamped ends. For this problem the system of equations

$$\left[L_{\alpha\beta}\right]\left\{u_\beta\right\} - \rho h \left\{\frac{\partial^2 u_\alpha}{\partial t^2}\right\} = \left\{p_\alpha\right\}$$

in non-dimensional form, reduces to the single equation for the radial displacement w [4],

$$\frac{\partial^4 w}{\partial \xi^4} + a_1 w + a_2 \frac{\partial^2 w}{\partial \tau^2} = a_3 Q \tag{8}$$

where

$$\frac{u_3}{L} = w,\ \frac{x}{L} = \xi,\ \frac{t}{d} = \tau,\ a_1 = \frac{EhL^4}{DR^2};\ a_2 = \frac{\rho h L^4}{Dd^2};\ a_3 = \frac{EL^3}{D};\ Q = \frac{p_w}{E}. \tag{9}$$

To simplify calculations the loading is taken to be of the form

$$Q(\xi,\ \tau) = -\frac{p_o}{E}(1-\tau). \tag{10}$$

The boundary and initial conditions are

$$w(0,\tau) = w(1,\tau) = \frac{\partial w(0,\tau)}{\partial \xi} = \frac{\partial w(1,\tau)}{\partial \xi} = w(\xi,0) = \frac{\partial w(\xi,0)}{\partial \tau} = 0. \tag{11}$$

For the solution of Eq. (8), w is assumed in the form

$$w(\xi,\tau) = \bar{w}(\xi,\tau) + \sum_i \varphi_i(\tau) w_i(\xi). \tag{12}$$

As discussed previously $\bar{w}$ is the quasi-static part of the solution and satisfies the differential equation

$$\frac{\partial^4 \bar{w}}{\partial \xi^4} + a_1 \bar{w} = -\frac{a_3 p_o}{E}(1-\tau) \tag{13}$$

along with the boundary conditions (11). Hence,

$$\bar{w}(\xi,\tau) = \frac{R^2 p_o}{hLE}(1-\tau)\left\{\alpha(k\xi) + C_1\left[\beta(k\xi) - \gamma(k\xi)\right] + C_2\delta(k\xi) - 1\right\}, \tag{14}$$

where

$$k = \left(\frac{EhL^4}{4DR^2}\right)^{1/4}; \quad \begin{array}{l} \alpha(k\xi) = \cosh k\xi \cos k\xi;\ \beta(k\xi) = \cosh k\xi \sin k\xi; \\ \gamma(k\xi) = \sinh k\xi \cos k\xi;\ \delta(k\xi) = \sinh k\xi \sin k\xi, \end{array}$$

with

$$C_1 = \frac{(1-\alpha)(\gamma+\beta) - \delta(\beta-\gamma)}{\beta^2 - \gamma^2 - 2\delta^2}; \quad C_2 = \frac{(\beta-\gamma)^2 - 2\delta(1-\alpha)}{\beta^2 - \gamma^2 - 2\delta^2};$$

the quantities α, β, γ and δ all being evaluated at $\xi = 1$.

The w_i are the normal modes of free vibration and as such satisfy

$$\frac{d^4 w_i}{d\xi^4} - \lambda_i^4 w_i = 0, \tag{15}$$

subject to

$$w_i(0) = w_i(1) = \frac{dw_i(0)}{d\xi} = \frac{dw_i(1)}{d\xi} = 0,$$

with

$$\lambda_i^4 = a_2\Omega_i^2 - a_1. \tag{16}$$

These eigenfunctions are

$$w_i(\xi) = C_i\left[N(\lambda_i\xi) - \frac{N(\lambda_i)}{M(\lambda_i)} M(\lambda_i\xi)\right], \tag{17}$$

where

$$M(\lambda_i\xi) = \frac{1}{2}(\sinh \lambda_i\xi - \sin \lambda_i\xi),\ N(\lambda_i\xi) = \frac{1}{2}(\cosh \lambda_i\xi - \cos \lambda_i\xi).$$

The functions w_i are orthogonal on the basic interval (0,1) and they may be normalized by choosing

$$C_i = \left\{\int_0^1\left[N(\lambda_i\xi) - \frac{N(\lambda_i)}{M(\lambda_i)} M(\lambda_i\xi)\right]^2 d\xi\right\}^{-1/2}; \tag{18}$$

hence,

$$\int_0^1 w_i(\xi)\, w_j(\xi)\, d\xi = \delta_{ij}$$

where δ_{ij} is the Kronecker delta.

To evaluate the functions φ_i, Eq. (12) is substituted in Eq. (8), and after simplification by means of (13) and (15) the result is

$$\sum_i \left[\frac{d^2\varphi_i}{d\tau^2} + \Omega_i^2\varphi_i\right] w_i = - \frac{\partial^2 \bar{w}}{\partial\tau^2} . \tag{19}$$

In view of the orthogonality of the w_i and the fact that

$$\frac{\partial^2 w}{\partial\tau^2} = 0$$

for this problem, the φ_i are given by

$$\varphi_i(\tau) = A_i \cos \Omega_i\tau + B_i \sin \Omega_i\tau \tag{20}$$

with

$$A_i = - \int_0^1 \bar{w}(\xi,0)\, w_i(\xi)\, d\xi, \quad B_i = - \frac{1}{\Omega_i} \int_0^1 \frac{\partial\bar{w}(\xi,0)}{\partial\tau} w_i(\xi)\, d\xi$$

due to the initial conditions (11). The evaluation of these integrals is facilitated greatly if Eqs. (13) and (15) are used and the identity

$$\int_0^1 \frac{\partial^4\bar{w}}{\partial\xi^4} w_i\, d\xi = \left[\bar{w}''' w_i - \bar{w}'' w_i' + \bar{w}' w_i'' - \bar{w} w_i'''\right]_0^1 + \int_0^1 \bar{w} \frac{d^4 w_i}{d\xi^4} d\xi,$$

which is obtained by the repeated application of integration by parts. The primes denote differentiation with respect to ξ. The result of the evaluation is

$$A_i = \frac{1}{\Omega_i^2} \frac{a_3}{a_2} \frac{P_o}{E} K(\lambda_i), \quad B_i = - \frac{1}{\Omega_i^3} \frac{a_3}{a_2} \frac{P_o}{E} K(\lambda_i)$$

due to the fact that, in the present case

$$\frac{\partial\bar{w}(\xi,0)}{\partial\tau} = - \bar{w}(\xi,0).$$

The expression $K(\lambda_i)$ is given by

$$K(\lambda_i) = \frac{C_i}{\lambda_i} \left\{M(\lambda_i) - \frac{N(\lambda_i)}{M(\lambda_i)} \left[\frac{1}{2} \cosh \lambda_i + \frac{1}{2} \cos \lambda_i - 1\right]\right\}. \tag{21}$$

With these expressions in mind w may finally be written as

$$w(\xi,\tau) = \bar{w}(\xi,\tau) + \frac{p_o d^2}{\rho h L} \sum_i \frac{K(\lambda_i) w_i(\xi)}{\Omega_i^2} [\cos \Omega_i \tau - \frac{1}{\Omega_i} \sin \Omega_i \tau] \tag{22}$$

where $\bar{w}$ (ξ,τ) is given by (14).

To provide a numerical comparison with the results obtained in [4], the response was calculated at $\xi = \frac{1}{2}$, with

$$L = 10.0 \text{ in}; \ R = 5.0 \text{ in}; \ \mu = 0.3; \ E = 30 \times 10^6 \text{ psi};$$

$$h = 0.05 \text{ in}; \ d = 20 \text{ msec}; \ \rho = 0.0007298 \text{ lb - sec}^2/\text{in}^4.$$

For convenience $p_o = 1$ psi. The results are shown in Fig. 2. The value of the quasi-static solution at $\tau = 0$, namely, 1.67×10^{-6}, corresponds exactly to the results obtained by using the membrane theory. It should be noted that the extrema of the response are somewhat greater here than obtained in [4]. In [4], terms like cosh x - cos x, resulted in computorial round-off error for small values of x.

The Axisymmetrically Loaded Spherical Shells

The equations of motion for the axisymmetric response of spherical shells, used by the authors in a previous study [4], are:

$$L(\psi) = -\alpha L(w) + (1+\mu)w - (1-\mu)\psi + \lambda \frac{\partial^2 \psi}{\partial t^2} - \frac{a^2(1-\mu^2)}{Eh} \int_o^\varphi p_v d_\varphi, \tag{23}$$

$$LL(W) = -LL(\psi) + (1-\mu)L(w) + \frac{1+\mu}{\alpha} L(\psi) - \frac{2(1+\mu)}{\alpha} w - \frac{\lambda}{\alpha} \frac{\partial^2 w}{\partial t^2} + \frac{a^2(1-\mu^2)}{\alpha E h} p_w \tag{24}$$

where

$$\alpha = \frac{1}{12}\left(\frac{h}{a}\right)^2, \ \lambda = \rho \frac{a^2(1-\mu^2)}{E}, \ v = \frac{\partial \psi}{\partial \varphi}, \ L(\cdot) = (1-x^2)\frac{\partial^2(\cdot)}{\partial x^2} - 2x\frac{\partial(\cdot)}{\partial x} \text{ and } x = \cos\varphi.$$

The meaning of the geometric symbols may be inferred from Fig. 1. Equations (23) and (24) were derived in accordance with the elastic bending theory of thin shells. The related elasticity equations are given in [4]. As implied in the preliminary discussion the solutions are assumed in the form

$$\begin{aligned} \psi(x,t) &= \bar{\psi}(x,t) + \sum_j R_j(t)\, \psi_j(x), \\ w(x,t) &= \bar{w}(x,t) + \sum_j R_j(t)\, w_j(x). \end{aligned} \tag{25}$$

The functions $\bar{\psi}$ and $\bar{w}$ represent the quasi static part of the solution.

For the complete spherical and the hemi-spherical shell with roller-clamped or roller-hinged edge, the admissible solutions are given in terms of Legendre polynomials as

$$\bar{\psi} = \sum_n C_n(t)\, P_n(x) \;,\; \bar{w} = \sum_n D_n(t)\, P_n(x) \tag{26}$$

where n = 1, 3, 5 ... corresponds to the roller-hinged edge and n = 0, 2, 4 ... corresponds to the roller-clamped edge. The functions ψ_j and w_j are the normal modes of free vibration of the shell. Details of their construction may be found in [4]. The natural frequency Ω_j of the jth mode is obtained from the expression

$$\Omega_j^2 = \left\{ \begin{matrix} \omega_{1j}^2 \\ \omega_{2j}^2 \end{matrix} \right\} = \frac{1}{2\lambda} [A_{1j} \pm (A_{1j}^2 - 4A_{2j})^{\frac{1}{2}}] \tag{27}$$

the choice of sign corresponding to the desired frequency on the left side of the equation. A_{1j} and A_{2j} are given by

$$A_{1j} = [\alpha j(j+1)(j^2+j+1+\mu)] + [j(j+1) + (1+3\mu)], \tag{28}$$

$$A_{2j} = \{\alpha[j(j+1) - (1-\mu)][j(j+1)(j^2+j+1+\mu)] - \alpha^2 j^3 (j+1)^3 - 2\alpha j^2 (j+1)^2 (1+\mu)\}$$
$$+ (1-\mu^2)[j(j+1) - 2] \tag{29}$$

The subscripts 1 and 2 correspond respectively to the upper and lower branch frequencies of a complete spherical shell. When j is an even integer, equation (27) yields the eigenfrequencies of a hemispherical shell with a roller-clamped edge, when j is odd, the frequencies correspond to a hemispherical shell with roller-hinged edge. In each case the respective eigenfunctions are the Legendre polynomials $P_j(x)$.

When the bending theory of shells is used in obtaining the quasi-static part of the solution, no advantage is realized in applying the present method over the usual mode displacement method, unless the loading can be expressed in the form

$$p(x,t) = P_n(x)\, T(t). \tag{30}$$

However, in many cases the membrane theory of shells may be used to obtain $\bar{\psi}$ and $\bar{w}$ for an arbitrary loading. For a completely fixed hemispherical shell, assuming a separable loading function

$$p_\alpha(x,t) = q_\alpha(x)\, T(t), \tag{31}$$

the use of the membrane theory, including bending effects, results in quasi-static solutions of the form

$$\bar{v} = \frac{\partial \bar{\psi}}{\partial \varphi} = \frac{1+\mu}{Eh} T(t) \sin \varphi \int_0^{\varphi} \left\{ \frac{1}{\sin \eta} \left[q_w - \frac{2}{\sin^2 \eta} \int_0^{\eta} (q_w \cos \xi \right.\right.$$

$$\left.\left. + q_v \sin \xi) \sin \xi d\xi \right] - \frac{1}{\sin^2 \eta} \frac{dG}{d\eta} \right\} d\eta \tag{32}$$

$$\bar{w} = \bar{v} \cot \varphi + \frac{a}{Eh \sin \varphi} \left[\frac{dG}{d\varphi} - (1+\mu) G \cot \varphi \right] T(t) \tag{33}$$

where

$$G(\varphi) = \frac{\zeta}{a} \left[\left(2M_o - Q_o \frac{a}{\zeta} \right)^2 + \left(Q_o \frac{a}{\zeta} \right)^2 \right]^{\frac{1}{2}} e^{-\zeta(\frac{\pi}{2} - \varphi)} \sin \eta \cdot \sin \left[\zeta(\frac{\pi}{2} - \varphi) \right.$$

$$\left. + \tan^{-1} \left(\frac{aQ_o}{2\zeta M_o - aQ_o} \right) \right] ; \quad M_o = \frac{Eh}{2\zeta^2} f_o ; \quad Q_o = \frac{Eh}{\zeta a} f_o ; \quad \zeta^4 = 3(1-\mu^2)\left(\frac{a}{h}\right)^2 ;$$

$$f_o = \frac{a^2}{Eh} \left[q_w \Big|_{\frac{\pi}{2}} - (1+\mu) \int_0^{\frac{\pi}{2}} (q_w \cos \xi + q_v \sin \xi) \sin \xi d\xi \right] . \tag{34}$$

The eigenfrequencies Ω_j, as in [4], are given by the roots of the following expression

$$\sum_n \frac{(2n+1)[\lambda\Omega^2 - n(n+1) + (1-\mu)] P_n^2(0)}{\lambda^2 [\Omega^2 - \omega_{1n}^2][\Omega^2 - \omega_{2n}^2]} = 0, \tag{35}$$

the corresponding normal modes by

$$w_j = \sum_n \frac{(2n+1)[\lambda\Omega_j^2 - n(n+1) + (1-\mu)] P_n(0) P_n(x)}{\lambda^2 [\Omega_j^2 - \omega_{1n}^2][\Omega_j^2 - \omega_{2n}^2]} \tag{36}$$

The expression for v_j is similar.

Shells Subjected to Exponentially Decaying Loading Function

The solution for a complete spherical shell, a hemispherical shell with roller-clamped edge or a hemispherical shell with roller-hinged edge, subjected to the same type of loading as given by Eq. (30), may be derived by substituting Eq. (26) into equations of the form of Eq. (3), in conjunction with Eqs. (23) and (24). The result of this substitution is

$$C_n(t) = \frac{1}{\Delta}\begin{vmatrix} -\frac{a^2(1-\mu^2)}{Eh}(2n+1)\int_0^1\int_0^{\varphi(x)} q_v P_n(x)\,d\varphi\,dx & -\alpha n(n+1)-(1+\mu) \\ -\frac{a^2(1-\mu^2)}{\alpha Eh}(2n+1)\int_0^1 q_w P_n(x)\,dx & n^2(n+1)^2-(1-\mu)n(n+1) + \frac{2(1+\mu)}{\alpha} \end{vmatrix} \quad (37)$$

$$D_n(t) = \frac{1}{\Delta}\begin{vmatrix} -n(n+1)+(1-\mu) & -\frac{a^2(1-\mu^2)}{Eh}(2n+1)\int_0^1\int_0^{\varphi(x)} q_v P_n(x)\,d\varphi\,dx \\ n^2(n+1)^2+\frac{(1+\mu)n(n+1)}{\alpha} & -\frac{a^2(1-\mu^2)}{\alpha Eh}(2n+1)\int_0^1 q_w P_n(x)\,dx \end{vmatrix} \quad (38)$$

where

$$\Delta = \frac{\lambda^2}{\alpha}\,\omega_{1n}^2\omega_{2n}^2 \;.$$

With $\psi_n = w_n = P_n(x)$, the equation for the coupling function R_j becomes

$$\left(\frac{d^2}{dt^2}+\omega_{1j}^2\right)\left(\frac{d^2}{dt^2}+\omega_{2j}^2\right)R_j = -\ddot{C}_j - \ddot{D}_j \;. \quad (39)$$

The final solutions for a shell, which is considered to be at rest initially, and has a loading

$$p_w = q_w\,e^{-bt}\,P_n(x)\;,\; p_v = 0\;,$$

then become

$$\psi(x,t) = A[\alpha n(n+1)+(1+\mu)]\left\{B_1\left(\cos\omega_{1n}t - \frac{b}{\omega_{1n}}\sin\omega_{1n}t\right) + B_2\left(\cos\omega_{2n}t - \frac{b}{\omega_{2n}}\sin\omega_{2n}t\right) + e^{-bt}\right\}P_n(x)\;; \quad (40)$$

$$\psi(x,t) = A\left\{C_1\left(\cos\omega_{1n}t - \frac{b}{\omega_{1n}}\sin\omega_{1n}t\right) + C_2\left(\cos\omega_{2n}t - \frac{b}{\omega_{2n}}\sin\omega_{2n}t\right) - [n(n+1)-(1-\mu)]e^{-bt}\right\}P_n(x)\;, \quad (41)$$

where

$$A = \frac{a^2(1-\mu^2)}{Eh\lambda^2} \cdot \frac{\omega_{1n}^2 \omega_{2n}^2 - b^4}{\omega_{1n}^2 \omega_{2n}^2} \cdot \frac{1}{(b^2+\omega_{1n}^2)(b^2+\omega_{2n}^2)} ;$$

$$B_1 = \frac{b^2+\omega_{2n}^2}{\omega_{1n}^2-\omega_{2n}^2} , \qquad B_2 = \frac{b^2+\omega_{1n}^2}{\omega_{2n}^2-\omega_{1n}^2} ;$$

$$C_1 = B_1\{\lambda\omega_{1n}^2 - [n(n+1) - (1-\mu)] + \lambda b^2\} ;$$

$$C_2 = B_2\{\lambda\omega_{2n}^2 - [n(n+1) - (1-\mu)] + \lambda b^2\} .$$

To provide a quantitative evaluation, numerical examples for the response of a hemispherical shell at the apex, with a roller-clamped edge, and with

$$q_o = 1, \; b = 100 \text{ and } 0.25, \; a = 300 \text{ in},$$
$$E = 30 \times 10^6 \text{ psi}, \; \mu = 0.33, \text{ and } \rho = 0.7298 \times 10^{-3} \text{ lb-sec}^2/\text{in}^4,$$

are presented in Figures (3), (4), and (5).

Conclusion

1. When an exact solution can be obtained for the quasi-static part, the Williams method has a definite advantage over the usual modal analysis, in that the quasi-static part provides an immediate average response, and furthermore the spread in the individual modes is reduced.

2. When the quasi-static part of the solution cannot be obtained exactly by using bending theory, it suffices in many cases to use the membrane solution, including bending corrections at the boundaries.

3. In the numerical evaluation of the response of a cylindrical shell, as obtained by means of the modal analysis [4], terms of the form cosh x - cos x, and sinh x - sin x, resulted in smaller values of the response, than those obtained by means of the Williams method, since, for small x, these terms are close to zero. A large part of this error is eliminated by obtaining the quasi-static solution in its exact form.

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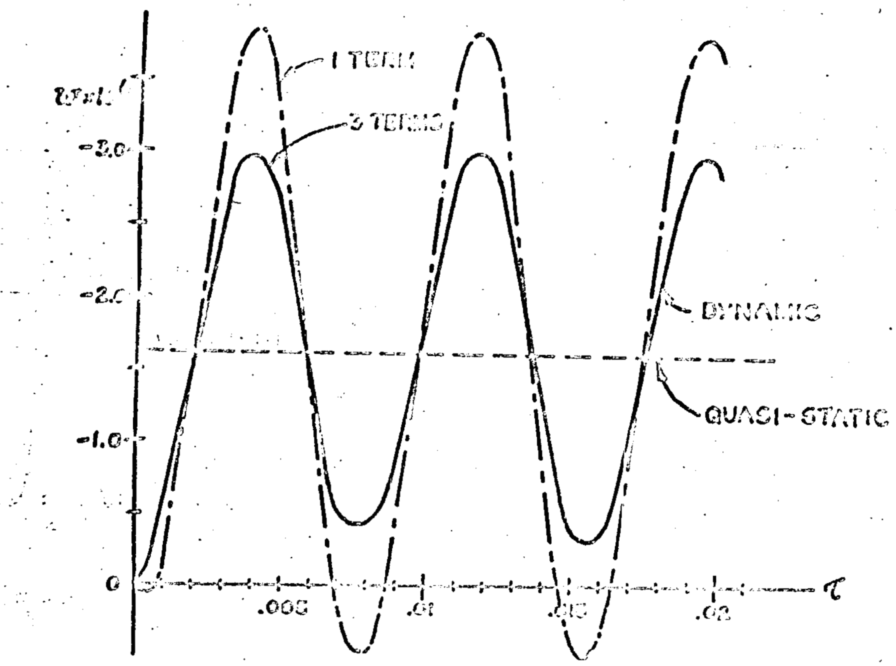

FIG. 2. CYLINDRICAL SHELL (CLAMPED), $Q=-\frac{P}{E}(1-\tau)$.

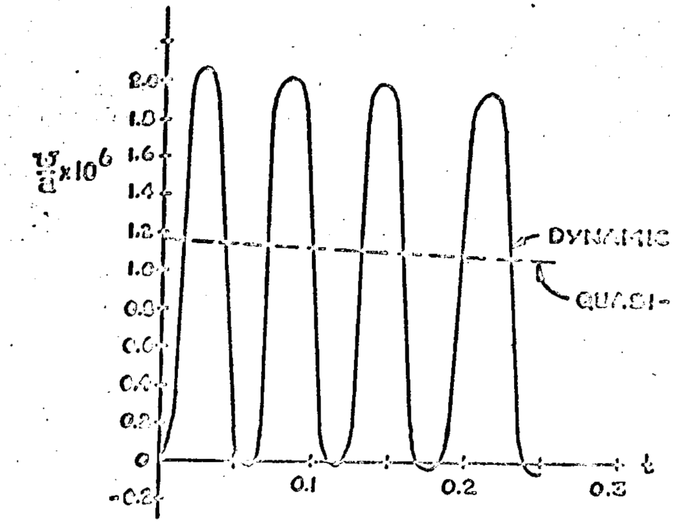

FIG. 3. HEMISPHERE (ROLLER-CLAMPED), b=0.25, $q_w(x)=1$.

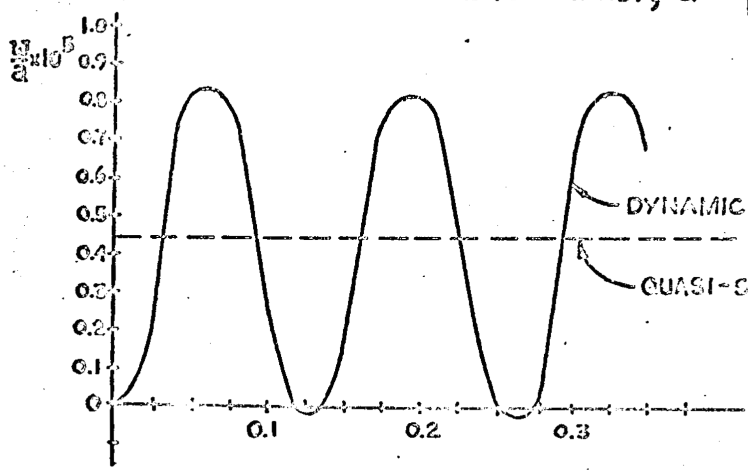

FIG. 4. HEMISPHERE (ROLLER-CLAMPED), b=0.25, $q_w(x)=P_2(x)$.

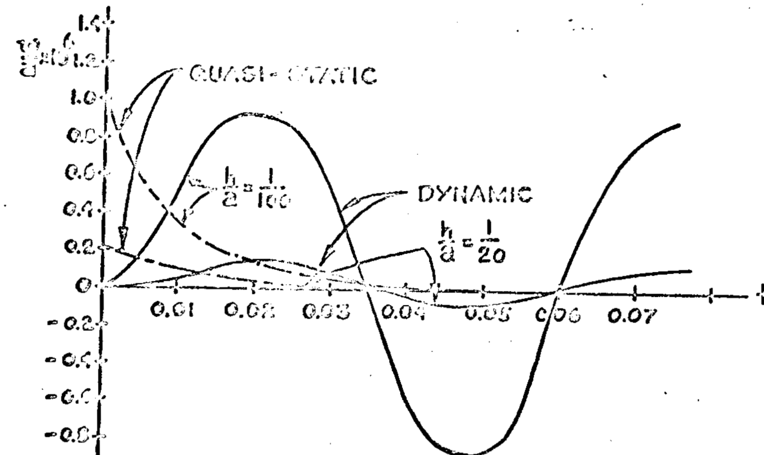

FIG. 5. HEMISPHERE (ROLLER-CLAMPED), b=100, $q_w(x)=1$.