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ON THE FLUCTUATIONS OF ENERGY FLUXES OF THE NUCLEAR-ACTIVE AND ELECTRON-PHOTON COMPONENTS OF AN EXTENSIVE AIR SHOWER

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SUMMARY

Experimental data are presented on the fluctuations of energy fluxes of the nuclear-active and electron-photon components of an extensive air - shower and on the mutual relationship of these fluctuations with those of the shower's age parameter s.

It is shown that the aggregate of experimental data is in contradiction with the model described in the work by Nymmik and Shestoperov [14]. The great significance of the experimentally established correlations between the parameter s and the other characteristics for a correct experimental setup regarding the extensive air showers is discussed.

* * *

The fluctuations of particle and energy fluxes of various components of an extensive air shower (EAS), having a given number of charged particles (N) at observer level, have been studied in a series of works [1 — 6], using the complex instrumentation of the Moscow State University (MSU).

The existence of fluctuations of energy fluxes and of the energy spectrum of nuclear-active (n.a.) particles has already been shown in the works [1, 2]. As to those of the electron-photon component (e.-ph.) it has been dealt with in the work [3]. Finally, a detailed investigation of fluctuations of μ — meson fluxes of various energies has been conducted in the works [4 — 6].

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* O FLUKTUATSIYAKH POTOKOV ENEGII YADERNO-AKTIVNOY I ELEKTRONNO-FOTONNOY KOMPONENT SHIROKOGO ATMOSFERNOGO LIVNYA.

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The possible correlation in the behavior of various EAS - components is also an important characteristic of EAS. The quantitative characteristics of such a correlation must evidently be determined by the character of shower development in the atmosphere. The experimental determination of fluctuations and of their correlation must offer great interest, for it provides new data for the choice among various models of EAS development. In the present work we bring forth the experimental data on fluctuations and on the correlation in the n.-a. and e.-ph. components of EAS and on their relationship with the age parameter s. Moreover, for the sake of comparison, we bring forth the results of theoretical calculations of these characteristics.

Note also that in the course of the last few years large experimental material on nuclear interactions at high energies has been gathered. This is why in the present work we again had recourse to the question of energy determination of n-a particles of EAS by the combined filter method. All the experimental data on the n-a component of EAS are given, taking into account these new experimental data.

The measurements were conducted during a series of years on the complex apparatus at the Moscow State University (MSU). This installation allows to determine simultaneously the position of the axis and the total number N of particles in each individual shower, and also the energy fluxes of n-a and e-ph.components. During the latest years the parameter s, characterizing the energy spectrum of e-ph component was determined for each shower.

The position of the axis r, the total number N of particles and the parameter s were determined with the help of a system made of a great number of counters included in the hodoscope and disposed over an area of several ha. The calculations of N, r, and s were performed with the aid of an M-20 electronic computer on the basis of empirical data. A detailed description of the installation and of the method of computations is given in the works [1, 2].

The energy fluxes of the n-a and e-ph components were determined with the help of a detector, made of two series of ionization chambers located under a lead or graphite filter [2]. The upper row, consisting of 12 chambers with an effective area of 3 m^2, disposed under a lead layer 2.5 cm thick, served for the determination of the energy flux of the e-ph component. The lower row, consisting of 128 chambers of 8 m^2 area and situated under a combined graphite-lead filter, served for the determination of the energy flux of the n-a component. The combined filter consisted of 5 cm lead, then further of 60 cm graphite and, finally, of 2.5 cm lead, and it was disposed directly under the ionization chambers. The lower part of the detector was well shielded laterally, for its protection from the e-ph component of EAS.

ON THE FLUCTUATIONS OF ENERGY FLUXES OF THE N-A AND E-PH COMPONENTS

The experimental distributions of the fluctuations of energy fluxes of the n-a and e-ph components may be distorted under the influence of some methodical factors.

Let us examine their influence on the distribution of the fluctuations of energy fluxes of the e-ph component. There are two causes for the distortions of interest to us.

First of all, the position of the shower axis r and the total number N of particles in it are determined with a certain error, as a consequence of which the distribution of energy fluxes of the e-ph component may be distorted if the energy flux is determined for a given distance r from the axis of the shower and for a given number N of particles.

The second methodical error consists in the following. The energy flux of the e-ph component F_{e-ph}, registered by the installation, is induced by a flux of electrons and photons distributed by a certain energy spectrum. A comparatively small number of electrons and photons of high energy, incident upon the apparatus, may significantly contribute to the total flux F_{e-ph}. The number of such electrons and photons undergoes relatively great Poisson fluctuations, which may precisely create significant fluctuations in the energy flux F_{e-ph}.

We have computed the distribution of the values of energy flux of F_{e-ph}, occurring on account of these methodical causes, for showers with the number of particles in the interval $N \div N + \Delta N$, whose axes pass at the distance $r - r + \Delta r$ from the center of the detector. It was assumed that for showers with the data for N and r, the energy of the e-ph component is constant. The spatial distribution function for particle fluxes of the e-ph component was taken the same for all the showers [1]. Considered were all the experimentally observable showers with particle number $N = 10^5 \div 3 \cdot 10^5$ and $r = 0 \div 2$ m, $r = 4 \div 6$ m. For each shower with the given experimental values of N and r, a drawing of possible values N' and r' was performed according to the distribution

$$W_1(r', N'/r, N) = \Phi(r', N')W_2(r, N/r', N'),$$

where $\Phi(r', N')$ is the apriori distribution for r' and N'; ; $W_2(r, N/r', N')$ is the distribution of possible locations of the shower axis r and of the number N of particles, determined with the help of the device used at the condition that the true position of the axis be r', and the number of particles — N'. The distribution W_2 (r, N/r', N') was obtained by the Monte-Carlo method in the work [7]. The function $\Phi(r', N')$ is well known for the sorting system used by us and it was computed for the shower spectrum index by the number of particles $\varkappa = 1.5$. As a result of the drawing, a set of values N' and r' was obtained instead of the earlier existing set of values N and r. Subsequently, the average number of particles incident upon the detector was determined for each shower with the obtained values N' and r'. Then, utilizing the energetic distribution of these particles (including photons), borrowed from [8], and taking into account the Poisson fluctuations in the number of particles of different energy, we obtained by way of drawing the energy flux of the e-ph component in each individual shower. The thus obtained distributions of F_{e-ph} fluxes were found to be substantially narrower than the experimental ones. The relative dispersions of the computed and experimental data on distributions are respectively equal to 0.7 and 3.0. The experimental distributions are plotted in Fig. 1.

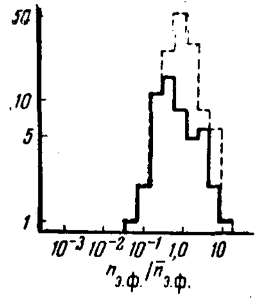

Fig. 1.- Experimental distribution of energy fluxes of the e-ph component in showers with the number of particles $N = 10^5 \div 3\ 10^5$ at various distances from the axis:

———— r = 0 ÷ 2 m.

– – – – r = 4 ÷ 6 m.

Thus, there is no doubt that in showers with a given number N of particles at various distances from the axis of the shower <u>r</u> there exist real fluctuations of energy fluxes of the e-ph component, evidently linked with the variation of the absolute value of the flux as a whole along the entire shower, as well as with the fluctuations of the spatial distribution of that flux.

Let us now pass to the consideration of the fluctuations of energy fluxes of the n-a component. The determination of the energy of the n-a particles by the combined filter method is somewhat difficult mainly because :

1) so far, the composition of the n-a component of the EAS, that is, the relative quantity of $\pi^{\pm}$- mesons, nucleons and possibly of other particles at various energies E of the considered particles has been unknown;

2) the energy spectrum of secondary particles, created in the filter by the interacting particle, is not known with sufficient precision.

At the same time numerous data have been obtained in the course of the past years on the interaction cross section and the distribution of the inelasticity modulus at interaction with light and heavy nuclei of n-a particles with energy $10^{11} - 10^{13}$ ev.

The relationship between the ionization $\underline{n}$, observed in chambers, and the energy E_{n-a} of the n-a particle bears a probabilistic character, which is basically defined by the following causes :

a) a greater role of fluctuations in the interaction events and in the development of the avalanche of n-a particles;

b) the difference in the character of n-a particle interaction in lead and grphite;

c) a possible difference in the character of the interaction with the nuclei of $\pi^{\pm}$-mesons and nucleons entering into the composition of the n-a component of EAS. That is why the general solution of the problem of relationship between $\underline{n}$ and E_{n-a} is quite cumbersome even with a comparatively small filter thickness (of the order of two nuclear path lengths).

The problem has been resolved by the Monte-Carlo method at the following assumptions: Interactions of n-a particles in the upper layer of lead and graphite were studied. The path length relative to nuclear interaction of nucleons and $\pi^{\pm}$-mesons was estimated identical and taken equal for lead at $\lambda_{int} = 170$ g/cm^2 and for graphite $\lambda_{int} = 80$ g/cm^2. In the case of nucleon interaction it was estimated that the distribution of inelasticity factors (moduli) in lead has the form $\delta(k - k)$ at $k = 1$, and for graphite the distribution brought up in [9] was utilized. For $\pi^{\pm}$- mesons the distribution of inelasticity moduli in lead and graphite was taken in the form $\delta(k - k)$ at $k = 1$. It was further assumed that in all interactions of n-a particles, one third of energy, lost by them for the formation of secondary particles is carried by π^{0}-mesons and two-thirds — by $\pi^{\pm}$- mesons; the latter, formed at interaction in graphite, transfer the energy to π^{0}-mesons according to the law $E_{\pi^0}(x) = E_{\pi^\pm}(1 - e^{-x/3})$, where $E_\pm$ is the energy of $\pi^\pm$-mesons, having formed at interaction, x is the length of the avalanche of $\pi^\pm$-mesons from the place of their formation to egress from graphite, expressed in units of $\lambda_{int} = 80\,g/cm^2$. The energy transfer to π^0-mesons from the $\pi^\pm$-avalanches occurring in the upper layer of lead, was computed by analogous formulas.

When computing the value of ionization occurring in the ionization chambers as a result of development of photon-electron avalanches from the π^0-mesons, formed in the upper layer of lead, we took into account the transitional effect lead - graphite. Computations have shown that if the interaction takes place at the beginning of the upper layer of lead, the number of electrons under the utilized filter decreases fivefold at the expense of the presence of graphite by comparison with the case when the graphite layer is substituted by a lead layer, equivalent by number to t units.

At transition from energy, transferred to π^0-mesons, to the value of the impulse in the ionization chambers, we assumed that all the energy E_{π^0} was concentrated in a single π^0-meson. We utilized in computations the cascade curves for lead obtained in [10]. As a result of drawing we obtained the distributions of impulses from n-a particles of fixed energy. Subsequently, we obtained the distributions by energies of nuclear-active particles E_{n-a}, contributing to ionization impulses situated in the given interval of values from n to n + Δn:*

$$W_2(E_{\text{я.а.}}/n)\,dE_{\text{я.а.}} = f(E_{\text{я.а.}})\,dE_{\text{я.а.}}W_1(n/E_{\text{я.а.}}),$$

where the spectrum $f(E_{\text{я.а.}})$ was taken in the form

$$f(E_{\text{я.а.}})\,dE_{\text{я.а.}} \sim E_{\text{я.а.}}^{-(\gamma+1)}\,dE_{\text{я.а.}}.$$

We brought out in Fig. 2 a series of distributions obtained for the case $\gamma = 1.5$. In the computations, presented below, we utilized the relation between the value n of ionization with the energy of the n-a particle, obtained by the method described.

Compiled in Table 1 are the values of the coupling factor <u>k</u> between the values n of the ionization and the mean energy of the n-a particle inducing that ionization ($kn = \bar{E}_{n-a}$) for two values of energy spectrum indicator of n-a particles : $\gamma = 1.0$ and $\gamma = 1.5$.

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* For uniformity of the text, we shall preserve from now on all the Russian denominations in the various formulas : я.а. for n-a, э.ф. for e-ph, and so forth.

The fluctuations of interest to us ($\Phi_{я.а.}$ stands for n.a.particle energy fluxes) are linked with the observed fluctuations of ionization by the following correlation :

$$f(n/\bar{n}) = \int F(\Phi_{я.а}/\bar{\Phi}_{я.а})\, W(n/\Phi_{я.а})\, d\Phi_{я.а},$$

at the same time, $f(n/\bar{n})$ is an experimental function determining the fluctuation of the ionization; $F(\Phi_{я.а.}/\Phi_{я.а.})$ is the function searched for, which determines the fluctuations of energy fluxes of n-a particles; $W(n/\Phi_{я.а.})$ is the probability of registration of ionization of various values n at incidence upon the installation of the given energy flux $\Phi_{я.а.}$. The function $W(n/\Phi_{я.а.})$ is evidently linked with the earlier introduced function $W_1(n/E_{я.а.})$; at the same time the distribution of $W(n/\Phi_{я.а.})$ is evidently already $W_1(n/E_{я.а.})$, since $W(n/\Phi_{я.а.})$ is not considered for a separate particle, but for their flux.

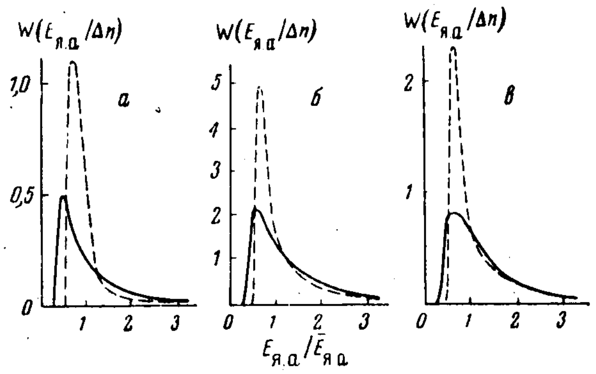

Fig. 2. - Distribution by energies of n.a. particles contributing to ionization impulses in the given interval of values —— n ÷ n + Δn : a —— : $n = 10^2 \div 1.5\cdot 10^2$; б — : $n = 10^3 \div 1.5\cdot 10^3$; в —: $n = 10^4 \div 1.5\cdot 10^4$. The solid line corresponds to nucleons, the dashed line — to a ± - meson. The probability is plotted in ordinates in arbitrary units. The functions are normalized by areas.

The distribution of $F(\Phi_{я.а.}/\Phi_{я.а.})$ was determined by us for a group of 35 showers lying in the interval of values $N = 3\ 10^5 \div 10^6$ and $r = 0 \div 2$ m. The calculation was conducted by the Monte-Carlo method using the following scheme.

TABLE 1

n	nucleon		π± - meson	
	γ = 1,0	γ = 1,5	γ = 1,0	γ = 1,5
10^2	$3,5\cdot 10^8$	$3,2\cdot 10^8$	$2,0\cdot 10^8$	$1,8\cdot 10^8$
10^3	$5,5\cdot 10^8$	$4,9\cdot 10^8$	$4\cdot 10^8$	$3,6\cdot 10^8$
10^4	$1,2\cdot 10^9$	$9,1\cdot 10^8$	$8,3\cdot 10^8$	

The ratio $n/\overline{n}$ was found experimentally for every shower. With the aid of the functions $\Phi_{\text{я.а.}}/\overline{\Phi}_{\text{я.а.}}$ computed for the case when the primary particle is a nucleon, we determined by way of drawing the value of $\Phi_{\text{я.а.}}/\overline{\Phi}_{\text{я.а.}}$, corresponding to it. The spectrum of the values of $\Phi_{\text{я.а.}}/\overline{\Phi}_{\text{я.а.}}$ obtained in the first approximation, was taken as the initial one when computing the second approximation utilizing the Bayes'theorem. Four approximations in all were considered.

The results of calculations show that the fourth approximation already hardly differs from the third. That is why we may estimate that the spectrum of the values of $\Phi_{\text{я.а.}}/\overline{\Phi}_{\text{я.а.}}$, obtained above, fluctuates to the same degree as does the experimental spectrum of the values of $n/\overline{n}$. Therefore, there exist real fluctuations of the values of energy fluxes of the n-a component in the observed showers. These fluctuations constitute a superimposition of the fluctuations of the spatial distribution of the energy fluxes of the n-a component on the fluctuations of the total energy flux of the n-a component as a whole by the shower. The experimental distribution for the values $N = 10^5 \div 3 \cdot 10^5$ and $r = 0 \div 2$ m is plotted in Fig. 3, 6.

The fluctuations of energy fluxes of the n-a component follow also from the form of the averaged energy spectrum of n-a particles.

The experimental spectra of ionization impulses from n-a particles were obtained at various assumptions of spatial distribution of ionization induced by each n-a particle in the system of ionization chambers. It was assumed that the ionization from one n-a particle may encompass the following areas $\delta_1 \sim 0{,}56$ м², $\delta_2 \sim 0{,}25$ м² and $\delta_3 \sim 0{,}06$ м². The thus obtained spectra do not practically differ and are characterized by one and the same value of the indicator $\gamma = 1{,}0 \div 1.2$, and this within the limits of errors.

The energy spectrum $F(>E_{\text{я.а.}})$ was obtained by the Monte-Carlo method from the spectrum of impulses $F(>n)$ on the basis of the correlation

$$F(>n) = \int F(E_{\text{я.а.}}) W_1(>n/E_{\text{я.а.}})\,dE_{\text{я.а.}}$$

It was assumed that the primary particles are nucleons. The energy spectrum of n-a particles was obtained for showers with $N = 2 \cdot 10^5 \div 3.5 \cdot 10^5$ in a circle of 6 m radius, drawn from the shower's axis.

Fig. 3. - Distribution of energy fluxes of the n-a component in showers with particle number $N = 3 \cdot 10^5 \div 10^6$ (a) & $N = 10^5 \div 3 \cdot 10^5$ (б) at distances from the axis $r = 0 \div 2$ m; the solid lines represents the experimental distributions; the dashed line — the theoretical. The number of cases when the flux is less than the registered are : a — : 2; б — : 11.

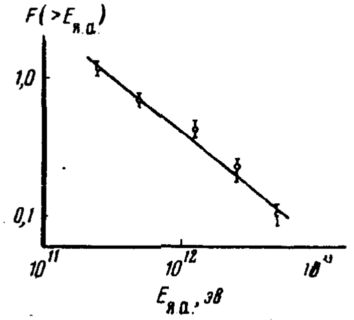

Fig. 4.- Energy spectrum of nuclear-active particles in showers with particle number $N = 2 \cdot 10^5 \div 3.5 \cdot 10^5$ in the central region of the shower ($r \leqslant 6$ m)

Plotted in Fig. 4 is the spectrum $F(>E_{я.а.})$ The obtained spectrum of n-a particles in the energy interval $3 \cdot 10^{11}$ ev $< E_{я.а.} < 10^{12}$ ev may be represented in the form

$$F(>E_{я.а.}) = \frac{8 \cdot 10^4}{E_{я.а.}^{\gamma_N}} \frac{N}{10^5},$$

where $\gamma_N = 1.0 \pm 0.2$. The analytical calculation was conducted in the assumption that the primary particles are $\pi^{\pm}$-mesons. In this case the spectrum of n-a particles in the region of energies $2.0 \cdot 10^{11}$ ev $< E_{я.а.} < 3.0 \cdot 10^{12}$ ev has the form

$$F(>E_{я.а.}) = \frac{4{,}7 \cdot 10^4}{E_{я.а.}^{\gamma_\pi}} \frac{N}{10^5},$$

with the spectrum indicator γ_π coinciding within the limits of errors with that obtained in the case when the primary particles are nucleons.

Basing ourselves on the data of the present work, let us point out that the dependence between the energy flux of the n-a component $\Phi_{я.а.}$ and N in the central region of the shower ($r \leqslant 6$ m) for showers with a number of particles $N \geqslant 10^5$, may be represented in the form $\Phi_{я.а.} = kN/10^5$, where $k = 3.7 \cdot 10^{12}$ in the case of $\pi^{\pm}$-mesons. This result agrees well with data brought up in [2, 11] (taking into account the conversion factor from n to E_{n-a} in them, and in the current one.

RELATIONSHIP BETWEEN THE FLUCTUATIONS OF THE NUCLEAR-ACTIVE, ELECTRON-PHOTON COMPONENTS AND THE AGE PARAMETER S

As already mentioned, it is possible to determine with the aid of the MSU installation the energy fluxes of the e-ph and n-a components of EAS, and also the parameter s characterizing the energy spectrum of the e-ph component. In the process of shower development the fluctuations of all these quantities must be specifically interrelated. Evidently, the following quantities may be interrelated : the energy flux of the n-a component and that of the e-ph component; the energy flux of the n.a.component and the parameter s; the energy flux of the e-ph component and the parameter s. The simplest characteristic of relationship between two random quantities x and y is the correlation factor defined as

$$K(x, y) = (\overline{xy} - \bar{x}\bar{y}) / [D(x)D(y)]^{1/2}.$$

The experimental results, presented below and characterizing the degree of correlation of the mentioned quantities, were obtained on the basis of analysis of 307 showers with a number of particles $N \gg 10^5$, whose axes lay within the limits of a 10 m radius drawn from the center of the detector. The distribution of the processed showers by s is plotted in Fig. 5.

It was found that the spatial distributions of the n-a and e-ph components in the showers considered are dependent on s. 52 showers were considered with s lying within the interval $0.8 \div 1.1$, and 60 showers with s within the interval $1.3 \div 1.5$. The showers were normalized to $N = 3 \cdot 10^5$. The determined spatial distributions of the density of energy fluxes are presented in Fig. 6. It was found that at $s = 0.8 \div 1.1$ the index of the spatial distribution function of the energy fluxes' density $\rho(r) \sim r^{-n}$ is equal to $n_1 = 1.88 \pm 0.22$, and for $s = 1.3 \div 1.5$ it is $n_2 = 1.25 \pm 0.25$. The respective indexes for the spatial distribution of the density of the energy fluxes of the e-ph component are $n_1 = 1.67 \pm 0.21$ and $n_2 = 1.06 \pm 0.15$. As may be seen from Fig. 7, the absolute values of fluxes are also strongly dependent on s. This fact of dependence of spatial distribution and absolute value of energy fluxes of the n-a and e-ph components on s is already in itself evidence of existence of a strong correlation between the parameter s and the energy fluxes of n-a and e-ph components. In order to characterize this correlation quantitatively, it is necessary to select a combination of showers with close values of N. With this in view we considered showers in the range of values $N = 2 \cdot 10^5 \div 3.5 \cdot 10^5$. The energy fluxes of the n-a and e-ph components were normalized in the selected showers to values $N = 3 \cdot 10^5$ and $r = 2$m. At the same time we took into account the dependence of the form of the spatial distribution function of energy fluxes' density on s.

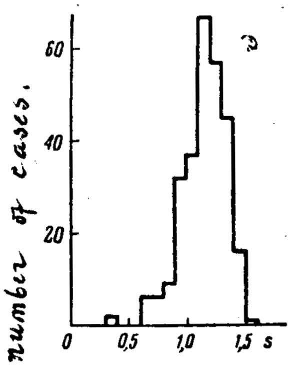

Fig. 5. - Distribution by s of showers with a number of particles $N \geqslant 10^5$.

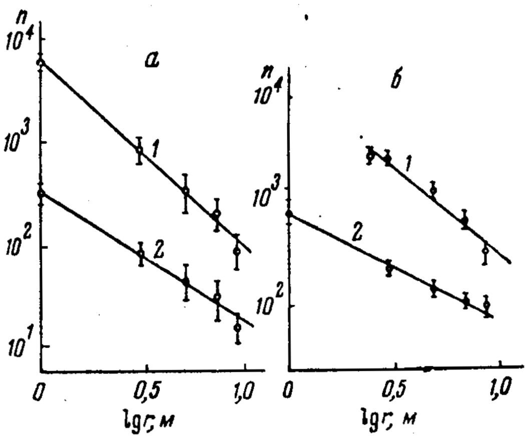

Fig. 6. - Spatial distributions of the density of energy fluxes of the n-a and e-ph components (respectively (a) and (б)) [in the number of relativistic particles separated in the ionization chambers]. Curv. 1— : s = 0.8 ÷ 1.1 ; curves 2— : s = = 1.3 ÷ 1.5

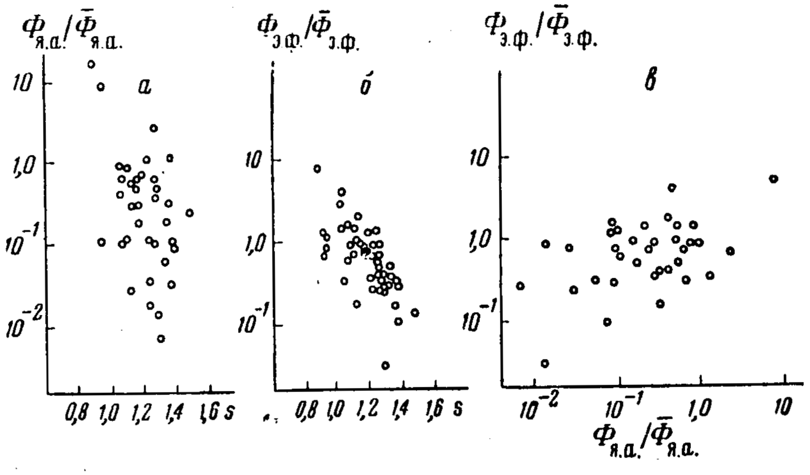

Fig. 7. - Experimental fields of the points

$\Phi_{я.а.}$, s; $\Phi_{э.ф.}$, s; $\Phi_{а.я.}$, $\Phi_{э.ф.}$

The experimental fields of the points $\Phi_{\text{я.а.}}, s$ and $\Phi_{\text{э.ф.}}, s$ are represented in Figs. 7, а, б . and in Fig. 7в — the field of the points $\Phi_{\text{я.а.}}$, $\Phi_{\text{э.ф.}}$. A **direct** application of the formula brought up for the correlation factor would be inaccurate, since in the experiment we obtain directly the value of the ionization, linked by some probabilistic fashion with the energy of the incident particles, and not the latter. If we utilize certain simplemost properties of the functions establishing a relationship between the observed quantities (ionization) and those of interest to us (energy), we may express the correlation factor of the quantities of interest to us through certain characteristics of distribution of the observable quantities.

Let $\Phi_1(x, y)\,dx\,dy$ be the joint distribution of the experimentally observable quantities x and y, and $W_2(y/v)\,dy$ — the distribution of the quantities of interest to us, u and v, the distributions $W_1(x/u)dx$ and $W_2(y/v)dy$ being known. Then

$$K(x, y) = \frac{\overline{xy} - \bar{x}\bar{y}}{[D(x)\,D(y)]^{1/2}} =$$

$$= \left\{\iint xy\,\Phi_1\,dx\,dy - \iint x\Phi_1\,dx\,dy \iint y\Phi_1\,dx\,dy\right\} \times$$

$$\times \left\{\left[\iint x^2\Phi_1\,dx\,dy - \left(\iint x\Phi_1\,dx\,dy\right)^2\right]\left[\iint y^2\Phi_1\,dx\,dy - \right.\right.$$

$$\left.\left. - \left(\iint y\Phi_1\,dx\,dy\right)^2\right]\right\}^{-1/2},$$

with, at the same time,

$$\Phi_1(x, y)\,dx\,dy = \iint W_1(x/u)\,W_2(y/v)\,\Phi_2(u, v)\,du\,dv\,dx\,dy.$$

Assume that the conditional distributions $W_1(x/u)$ and $W_2(y/v)$ are respectively functions of the relations $x/u = \xi$ and $y/v = \eta$. Changing the order and the variables of integration with the help of the substitution of $x = \xi u$ and $y = \eta v$, and taking into account the normalization of the functions W_1 and W_2,

$$\int W_1(\xi)\,d\xi = \frac{1}{u}, \quad \int W_2(\eta)\,d\eta = \frac{1}{v},$$

and the independence of ξ and η, we obtain for K(x, y) the following expression:

$$K(x,y)=\frac{\bar{\xi}\bar{\eta}(\overline{uv}-\bar{u}\bar{v})}{[\overline{\xi^2}\,\overline{u^2}-\bar{\xi}^2\bar{u}^2-\bar{\eta}^2\bar{v}^2)]^{1/2}}.$$

Hence, assuming $\overline{x^2}=\overline{u^2}\overline{\xi^2}$ and so forth, we obtain

$$K(u,v)=(\overline{xy}-\bar{x}\,\bar{y})\left[\left(\frac{\overline{\xi^2}}{\bar{\xi}^2}\overline{x^2}-\bar{x}^2\right)\left(\frac{\eta^2}{\bar{\eta}^2}\overline{y^2}-\bar{y}^2\right]^{-1/2}.$$

Therefore, in order to compute the correlation factor of the quantities u and v, it is necessary to know the covariation $\overline{xy}-\bar{x}\bar{y}$ of the observed quantities, and also the first and the second moments of distribution of the quantities x, y ξ, η. From the experimental data obtained according to the formula brought up we computed the correlation factors between the different parameters of the showers. At the same time we took into account the form of the function W_1 (see Fig. 4). The respective results are compiled in Table 2.

COMPARISON OF EXPERIMENTAL DATA WITH VARIOUS MODELS OF DEVELOPMENT OF EAS

In the work [12] a method was developed for the calculation of multidimensional distributions $\Phi_{E_0,A}(N,s,\Phi_{э.ф.},\Phi_{я.а.})$, representing the distribution of showers with fixed values of primary particle energy E_0 and its atomic number A, by the parameters N, s, $\Phi_{э.ф.}$ and $\Phi_{я.а.}$. For the two models I and III, considered in this work, we brought out the calculations of the distribution $\Phi_{E_0,A}(N,s,\Phi_{э.ф.},\Phi_{я.а.})$ at $E_0=10^{16}$ev and $A=1$. Two variants of the model I were considered, which differ from one another only by the mean value of the inelasticity modulus k: the variant Ia at $\bar{k}=0.5$ and the variant Iб at k = 0.4.

We brought out in Figs. 8, 9 and 10 the fields of the points $\Phi_{я.а.}$, $\Phi_{э.ф.}$; $\Phi_{я.а.}$, s; N, s; $\Phi_{э.ф.}$, s, corresponding to the different models Ia, Iб, III. The model I corresponds factually to such a model of elementary interaction event, when secondary particles are forming in a system of two centers (fire ball). The model III was taken according to [13].

TABLE 2

	$N = 3\cdot10^5$, $r \leqslant 6$ м				$\bar N = 3\cdot10^5$, $r = 1\div100$ м		$K(\Phi_{\text{я.а.}}, \Phi_{\text{э.ф.}})$		$K(\Phi_{\text{я.а.}}, s)$		$K(\Phi_{\text{э.ф.}}, s)$	
Primary particle	$\bar\Phi_{\text{я.а.}}$	$\frac{[D(\Phi_{\text{я.а.}})]^{1/2}}{\bar\Phi_{\text{я.а.}}}$	$\bar\Phi_{\text{э.ф.}}$	$\frac{[D(\Phi_{\text{э.ф.}})]^{1/2}}{\bar\Phi_{\text{э.ф.}}}$	$\bar\Phi_{\text{я.а.}}$	$\bar\Phi_{\text{э.ф.}}$	$K(x, y)$	$K(u, v)$	$K(x, y)$	$K(u, v)$	$K(x, y)$	$K(u, v)$
Nucleon	$1{,}8\cdot10^{13}$	1,5	$2{,}1\cdot10^{13}$	1,45	$3{,}2\cdot10^{13}$	$6{,}0\cdot10^{13}$	0,75±0,07	0,81±0,04	−0,52±0,11	−0,71±0,07	−0,41±0,12	−0,57±0,10
±-meson	$1{,}1\cdot10^{13}$				$1{,}9\cdot10^{13}$							

TABLE 3

	Type of Model				Type of Model		
	Ia	Iб	III		Ia	Iб	III
$K(\Phi_{\text{я.а.}}, \Phi_{\text{э.ф.}})$	0,61±0,1	—	—	$[D(\Phi_{\text{э.ф.}})]^{1/2}/\bar\Phi_{\text{э.ф.}}$	0,90	—	—
$K(\Phi_{\text{я.а.}}, s)$	−0,6±0,1	−0,7±0,08	−0,30±0,1	$\bar N$	$1{,}5\cdot10^6$	$1{,}9\cdot10^6$	$2{,}9\cdot10^6$
$K(\Phi_{\text{э.ф.}}, s)$	−0,68±0,08	—	—	$\bar s$	1,31	1,29	1,26
$\bar\Phi_{\text{я.а.}}$	$7{,}8\cdot10^{13}$ эв	$1{,}35\cdot10^{14}$ эв	$3{,}9\cdot10^{13}$ эв	$K(\Phi_{\text{я.а.}}, N)$	0,60±0,1	0,75±0,07	0,31±0,1
$\bar\Phi_{\text{э.ф.}}$	$3\cdot10^{14}$ эв	—	—	$K(N, s)$	−0,8±0,05	−0,80±0,05	−0,82±0,05
$[D(\Phi_{\text{я.а.}})]^{1/2}/\bar\Phi_{\text{я.а.}}$	1,75	1,4	1,75				

In Table 3 we compiled the correlation factors of the corrresponding quantities and also their mean values and the relative dispersions. When comparing the results of calculations with experimental data it is necessary to take into account first of all that the experimental data are related to showers with a fixed number of particles N and not with a fixed energy E_0. Because of that we are not in a position to draw a precise quantitative comparison of theoretical and experimental data, and we shall limit ourselves to their semiquantitative comparison. The fundamental distinction between

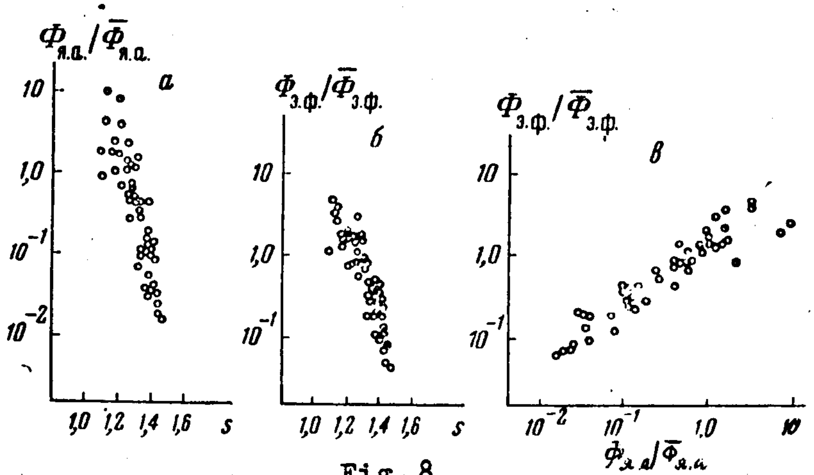

Fig. 8

Fields of points $\Phi_{я.а.}$, s; $\Phi_{э.ф.}$, s; $\Phi_{я.а.}$, $\Phi_{э.ф.}$, computed by the model Ia ($k = 0,5$)

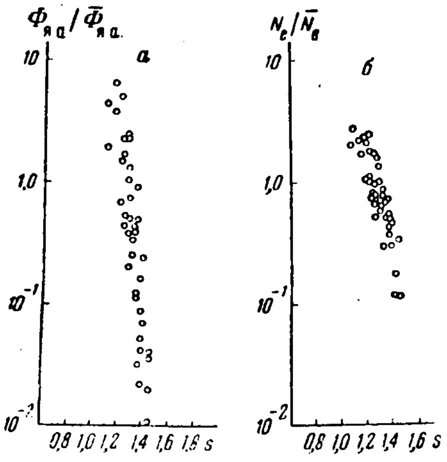

Fig. 9.- Fields of the points $\Phi_{я.а.}$, s; N, s, computed according to the model Iб (k =0.4).

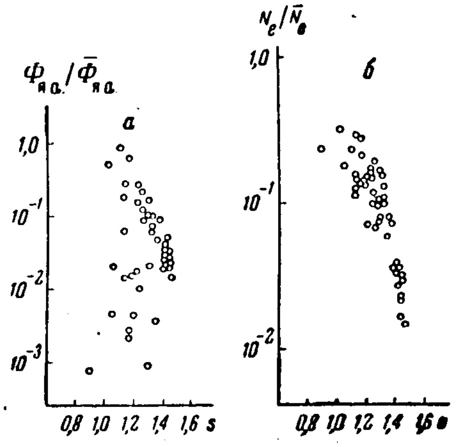

Fig. 10. - Fields of the points $\Phi_{я.а.}$, s; N, s, computed according to model III.

the models I and III consists in the comparatively low value of the energy flux $\bar{\Phi}_{я.а.}$, and also of the correlation factors $K(\Phi_{я.а.}, s)$ for the model III. It may be asserted that such a distinction between the models I and III will also be preserved when passing to the consideration of showers with a fixed N. Indeed, as was shown in the works [14,15], the mean value of energy $\bar{E}_0$, corresponding to the shower with a given number N of particles at sea level for the models of the types I and III, is by 1.5 to 2 times lower than the Energy E_0^*, determined as the primary energy, for which the mean value $\bar{N}$ equals the number N considered by us. This is why the basic contribution to the mean value $\bar{\Phi}_{я.а.}$ at given N will be made by showers with $N/\bar{N} > 1$, originating from primary particles with energy smaller than E_0^* (Note that, as was shown above, $\Phi_{я.а.} \sim N$. This fact is taken advantage of here for comparison with theory). Because of the first circumstance, the quantity $\bar{\Phi}_{я.а.}$ increases somewhat, and on account of the second — it decreases. The construction of regression showers for the models I and III shows that the increase in case of model I is by 1.5 times greater than the decrease, whereas for the case of model III the increase and decrease compensate one another. Therefore, the model I (and more particularly the variant $I\delta$) gives for $\Phi_{я.а.}$ a value significantly nearer that of the experiment than the model III at fixed N also. As to the correlation factor $K(\Phi_{я.а.}, s)$, for a fixed N it will be mainly determined by the relationship between the quantities $\Phi_{я.а.}$ and N and between $\underline{s}$ and $\underline{N}$ at fixed E_0. As may be seen from Table 3, the correlation factor $K(\Phi_{я.а.}, N)$ for the model III is small, That is why there is no basis to expect that the correlation factor $K(\Phi_{я.а.}, s)$ at fixed N will be greater than at fixed E_0.

Thus, the experimental value of $\bar{\Phi}_{я.а.}$ and $K(\Phi_{я.а.}, s)$ are in contradiction with the model III. At the same time, the experimental data are not in contradiction with models I (particularly I) either by the mean values of energy fluxes $\bar{\Phi}_{я.а.}$ and $\bar{\Phi}_{э.ф.}$, or by their dispersions and different correlation factors of these quantities.

We shall remark, in conclusion, that the correlation between the various parameters of EAS and, more particularly, the correlations of $\Phi_{я.а}$ and $\underline{s}$, $\bar{\Phi}_{э.ф.}$ and $\underline{s}$, $\underline{N}_\mu$ (number of μ-mesons) and $\underline{s}$, established in the present as well as in our former works [16], are of great significance for a correct setup of experiments for the study of extensive air showers. At present, in most of the works, groups of electron detectors, switched on coincidences are used for the sorting of studied EAS. At such a sorting the mean value of the parameter s remains invariable only in a specific range of distances from the center of the guiding system; at the same time, this range of distances decreases with the decrease of N. Obviously, for sufficiently small N only showers with small values of s will be sorted. It seems to us that the analysis of the experimental data on the variation of the characteristics of the elementary event [17], must be performed once more, taking into account the strong relationship between the parameter s and the other characteristics of EAS, revealed in our work.

*** THE END ***

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