DIFFUSION AND CONVECTION OF ENERGETIC ELECTRONS BEHIND THE EARTH'S BOW SHOCK

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ABSTRACT

Effects of diffusion on the energy spectrum of a group of electrons being convected downwind in the Earth's bow shock transition region are analyzed. The recent observation that the energy spectra of energetic electron pulses soften with increasing L_{SEP} is readily explained if each electron pulse is caused by a region of electrons being blown past the satellite. Electrons escape from the region by an energy dependent random walk through the irregular magnetic field. The observed change in the spectrum requires that the mean random walk step length λ be about 10^7 cm, or of the order of the cyclotron radius, for 40 keV electrons in the transition region. It is argued that this result is physically reasonable.

Author

1. Introduction

Recent observations by Fan, Gloeckler and Simpson (1965) indicate that the energy spectra of > 30 keV electron pulses observed in the Earth's bow shock transition region (herein called transition region) soften with increasing Sun-Earth-probe angle (L_{SEP}). If the integral energy spectrum at the peak of a pulse is assumed to be of the form $J(>E) \simeq J_0 E^{-\gamma}$ for ~ 30 - 50 keV electrons, they find that $\gamma \simeq 2.5 - 3.5$ for $L_{SEP} \leq 45^\circ$ and $\gamma \simeq 3.0 - 4.0$ for $L_{SEP} \geq 80^\circ$. In addition, they report that the spectrum tends to harden with increasing radial distance from the magnetopause and present evidence that the electrons are not of magnetospheric origin. In this paper these recent observations are interpreted in terms of a model, proposed by Jokipii and Davis (1964), in which a transition region electron pulse is due to an extended volume of electrons, which are essentially tied to the magnetic lines of force, being blown past the satellite by the solar wind. Escape of electrons along the magnetic field is impeded by irregularities in the field; the particles slowly diffuse through the irregular field as they are blown downwind. It is initially assumed that further acceleration or deceleration of the electrons as they blow downwind is negligible. Nonetheless, softening of the energy spectrum is expected as a group of electrons is blown downwind since faster, more energetic electrons escape more rapidly. It is therefore suggested that the observed softening of the spectrum with increasing L_{SEP} is due to the pulses at large L_{SEP} being, on the average, further behind the bow shock. A similar argument can be made for the hardening of spectra with distance from the magnetosphere. The general configuration is illustrated in Figure 1.

It is found below that the observed increase of γ is quantitatively consistent with the model and makes possible a determination of the diffusion parameter for the motion of 30 – 50 keV electrons through the fluctuating magnetic field in the transition region. A simple extension of the model suggests that energetic electron events observed in the Earth's magnetic tail (Montgomery, et al., 1965; Anderson, 1965) may be in part explained by transition region electrons being injected into the neutral sheet (Ness, 1965).

II. The Transition Region Pulses

Much attention has recently been directed toward the energetic electron pulses observed in the transition region and beyond the bow shock (Fan, Gloeckler and Simpson, 1964, 1965; Anderson, Harris and Paoli, 1965; Frank and Van Allen, 1965). The observations consist of pulses of 30 keV or higher electrons in which the flux typically rises abruptly to values of 10^4 - 10^6 cm^{-2} sec^{-1}, substantially above the normal flux level, and which last only a few minutes at the satellite. Jokipii and Davis (1964) suggested a picture in which groups of electrons, occasionally accelerated at the bow shock, are directed into the disordered magnetic fields of the transition region (Sonett and Abrams, 1963; Ness, Scearce and Seek, 1964), where their motion is a combination of a slow (compared with individual particle velocities) random walk through the field and convection downwind with the solar wind plasma. Each pulse of energetic electrons lasting a few minutes in the transition region is then considered due to a volume occupied by these electrons of the order of 10^{10} cm, or a characteristic dimension of the transition region, across, being blown past the satellite. The problem of the origin of the electrons is not yet settled, although some possible models have been discussed (Scarf, Bernstein and Fredricks, 1965; Jokipii, 1965). None of the following discussion, though, depends on a specific origin for the electrons.

The random walk of charged particles behind a shock has previously been analysed in the diffusion limit for a plane shock geometry (Jokipii, 1965). However, application of this method to the bow shock, taking into account the three dimensional geometry and the flow around the magnetosphere, depends on a knowledge of the boundary conditions and the flow parameters and is scarcely worthwhile at the present. We here utilize a simpler approach which makes possible a rough, order of magnitude estimate of the parameters. Consider first the average motion of the particles. We expect the disordered magnetic field in the transition region to be carried, on the average, with the solar plasma. Energetic electrons thus tend to be convected downwind with the plasma. In addition the electrons will random walk through the irregular field. In general the random walk will be anisotropic; the particles tend to move more readily along the field then normal to it (see discussion in Parker, 1965). Let λ be the mean step length along the average magnetic field, which will be taken to define the x axis. Particles with velocity v_p will make approximately $\nu \simeq v_p / \lambda$ steps of length λ along the field per unit time, and if they are at x_1 at time t_1 they will travel a mean distance $\Delta x_m = \sqrt{\langle \Delta x^2 \rangle}$ along the field in time Δt, where

$$\begin{aligned} \langle \Delta x^2 \rangle &\simeq \nu \lambda^2 \Delta t \\ &\simeq v_p \lambda \Delta t \end{aligned} \qquad (1)$$

Motion normal to the field is more complicated. Let $\Omega = qB/m$ be the cyclotron frequency of the particle in the average magnetic field B. Following Parker (1965) we note that if $\nu \ll \Omega$, the mean step normal to the field is of the order of a cyclotron radius $r_c = V_p/\Omega$, whereas if $\nu \gg \Omega$ the step length should be λ. A sufficient representation of these effects is to write, in analogy with eq (1),

$$\langle \Delta y^2 \rangle = \langle \Delta z^2 \rangle \simeq \nu \lambda^2 \Delta t \left[\frac{1}{1 + \lambda^2/r_c^2} \right] \simeq v_p \lambda \Delta t \left[\frac{1}{1 + \lambda^2/r_c^2} \right] \tag{2}$$

and again

$$\Delta Y_m = \Delta Z_m = \sqrt{\langle \Delta y^2 \rangle} \tag{2'}$$

The effect of this random walk relevant to our purposes is that it causes the volume occupied by the electrons to expand as it is blown downwind. It follows from equations (1) and (2) that, barring the unlikely event that λ decreases with V_p as $1/V_p$ or faster, the rate of expansion increases with particle energy. The spectrum is expected to soften with time, or distance behind the shock, in qualitative agreement with observations of Fan, et al., (1965).

Consider now the change in γ due to the random walk. Following Fan, et al. (with slightly different notation), approximate the integral energy spectrum by $J(>E) \simeq J_0 E^{-\gamma}$; the differential spectrum may then be written $F(E)dE = F_0 E^{-(\gamma+1)} dE$. Even if the true spectrum is not purely power law, the parameter γ may be regarded as a measure of the steepness of the spectrum over a limited energy range. If $F_1(E)$ and $F_2(E)$ are the differential spectra at times t_1 and t_2,

$$\Delta\gamma = \gamma_2 - \gamma_1 = E\left(\frac{1}{F_1}\frac{\partial F_1}{\partial E} - \frac{1}{F_2}\frac{\partial F_2}{\partial E}\right) \tag{3}$$

Now assume the particles initially to occupy a volume V_1 with average flux F_1. After a time Δt they occupy a volume V_2 with average flux F_2. Since there are assumed to be no sources or sinks of particles, we have the relation

$$F_1 V_1 \simeq F_2 V_2 \tag{4}$$

Equations (3) and (4) readily yield

$$\Delta\gamma \simeq E\frac{\partial}{\partial E}\ln\left(\frac{V_2}{V_1}\right) \tag{5}$$

We now attempt to relate V_2 to V_1 and ΔX_m, ΔY_m and ΔZ_m. In general this depends on the detailed shape of the region. For the purposes of the present order of magnitude calculation, however, it is sufficient to assume V_1 independent of energy and roughly ellipsoidal in shape. It is then possible to write $V_1 \simeq C_1 [\delta X_1 \, \delta Y_1 \, \delta Z_1]$ and
$V_2 \simeq C_2 [(\delta X_1 + \Delta X_m)(\delta Y_1 + \Delta Y_m)(\delta Z_1 + \Delta Z_m)]$

where $C_1 \simeq C_2$ and $\delta X_1, \delta Y_1, \delta Z_1$ are the initial dimensions of the region. Utilizing equation (1), (2) and (5) one eventually obtains

$$\Delta \gamma \simeq \frac{1}{4}\left[\frac{\Delta X_m}{\delta X_1 + \Delta X_m} + \frac{\Delta Y_m}{\delta Y_1 + \Delta Y_m} + \frac{\Delta Z_m}{\delta Z_1 + \Delta Z_m}\right]\left[1 + 2\frac{\partial \ln \lambda}{\partial \ln E}\right]$$

$$+ \frac{1}{2}\left[\frac{\Delta Y_m}{\delta Y_1 + \Delta Y_m} + \frac{\Delta Z_m}{\delta Z_1 + \Delta Z_m}\right]\left[\frac{1 - 2\frac{\partial \ln \lambda}{\partial \ln E}}{1 + r_c^2/\lambda^2}\right] \quad (6)$$

It is noteworthy that in the limit of an instantaneous point source of particles ($\delta X_1 = \delta Y_1 = \delta Z_1 \Rightarrow 0$), equation (6) reduces to the exact change in γ at the origin obtained from the corresponding diffusion equation with diagonal diffusion tensor $D_{xx} = \langle \Delta x^2 \rangle / \Delta t$ and $D_{yy} = D_{zz} = \langle \Delta y^2 \rangle / \Delta t$

Equation (6) should give a reasonable estimate of the dependence of $\Delta \gamma$ on the relevant parameters. In particular, since $\Delta \gamma$ has been observed for a given V_p (or energy range) and magnetic field, and since the values of δX_1, δY_1, δZ_1, and Δt can be roughly inferred from the observations, equation (6) yields an estimate for λ. The consistency of the model thus hinges on the reasonableness of the required λ.

The average value of $\Delta \gamma$ reported by Fan, et al. is about 0.5 between $L_{SEP} \leq 45°$ to $L_{SEP} \geq 80°$ for electrons with energies of the order of 40 keV. For the 10 γ magnetic field in the transition region, $\Omega \simeq 2 \times 10^3$ sec^{-1} and $r_c \simeq 5 \times 10^6$ cm. The change in γ occurs in a time of a few hundred seconds; that is, in the length of time required for the solar wind to blow a field line around a substantial portion of the magnetosphere. Furthermore, we set $\delta X_1 \simeq \delta Y_1 \simeq \delta Z_1 \simeq 10^{10}$ cm since the pulse must take a few minutes to blow past the satellite at a few hundred kilometers per second. It is also not likely that the dimensions are much larger than 10^{10} cm since this is of the order of the dimensions of the transition region. $\frac{\partial \ln \lambda}{\partial \ln E}$ is not readily determined, but it is not likely to be much larger than unity or less than zero and may therefore be neglected in this crude analysis. Substituting these values into equation (6), we find that $\lambda \simeq 2 \times 10^7$ cm for $\simeq$ 40 keV electrons, an estimate which may be correct to an order of magnitude.

Is $\lambda \simeq 2 \times 10^7$ cm physically reasonable? We first note that it is of the same order or perhaps slightly larger than the cyclotron radius of the electrons. An analysis by Parker (1964) indicates that magnetic irregularities are most effective in scattering particles if their characteristic scale is of the order of the cyclotron radius of the scattered particle. Irregularities much larger or much smaller than the cyclotron radius are appreciably less effective. This suggests that in a medium having a broad spectrum of irregularities, such as the transition region, those most effective may be of the order of the cyclotron radius and λ may therefore be expected to be of the same order. We may conclude, therefore, that the required λ is indeed quite reasonable physically and the model is consistent with the observations.

Associated with the systematic increase in γ, the present model predicts a systematic decrease in peak intensity with L_{SEP}. The decrease expected from equation (4) is of the order of a factor of 2 from $L_{SEP} \lesssim 45^{\circ}$ to

$L_{SEP} \gtrsim 80^{\circ}$. Careful examination of the published observations of Anderson, et al. (1965) gives tentative confirmation of such an effect, although the statistics are quite poor. Further confirmation of this effect would be desirable.

For completeness it should be emphasized that any further acceleration or deceleration of the electrons as they drift downwind may also contribute to the change in γ and intensity. One likely possibility is compression or expansion of the solar plasma as it flows around the magnetosphere; the associated change in the magnetic field intensity may result in betatron acceleration or deceleration of the electrons. However, the flow of the solar wind in the transition region is not yet well enough understood to attempt a quantitative analysis of this effect. We may conclude from the above analysis, however, that the observed change in the spectrum is readily explained in terms of diffusion alone and that the model gives a reasonable interpretation of the observations.

III. The Further Evolution of the Pulses

In the light of the above discussion it is likely that each energetic electron pulse observed in the transition region is caused by a slowly dissipating region of electrons which is blown downwind by the solar wind. If λ remains essentially constant at a few times 10^7 cm, the characteristic time for dissipation of the electron can be estimated as the time for, say, $\Delta X_m >> \delta X_1 \simeq 10^{10}$ cm. Setting $\Delta X_m \simeq 3 \times 10^{10} - 10^{11}$ cm yields a characteristic dissipation time of $10^3 - 10^4$ seconds, or of the order of an hour. For a solar wind velocity of a few hundred km sec^{-1}, the region may therefore be expected to travel of the order of 100 R_E before merging into the background.

Figure (1) illustrates the expected evolution of a typical pulse. It is therefore expected that a satellite at large distances downwind from the bow shock should see somewhat broader and weaker electron pulses.

Consider a group of electrons generated in the vicinity of the subsolar point. As the electrons are carried downwind along the extended tail of the magnetosphere (observed by Ness, 1965, see also discussion in Dessler, 1964 and Axford, et al., 1965) they random walk through the transition region field as discussed in section II of this paper. It is expected that some will random walk to the boundary of the magnetic tail. Alternatively, we may say that, because of the random walk, the volume occupied by the electrons expands to the boundary of the magnetic tail. The general picture is as illustrated in Figure (2). Now, the boundary of the tail may be relatively impenetrable since the magnetic field inside is mainly parallel to the surface and probably quite uniform. It may therefore be a reflecting boundary. Energetic electrons which do manage to penetrate this boundary may provide a source of radiation in the tail as postulated by Dessler and Juday (1965). More attractive, however, is the possibility that the electrons find their way to the edge of the magnetically neutral sheet in the tail (Ness, 1965). Since the magnetic field in the sheet is nearly zero, there is no barrier to the further penetration of electrons into this region. It is tempting to suggest that energetic electrons from the transition region which are thus injected into the neutral sheet may provide a partial explanation of the electron events observed in the magnetic tail (Montgomery, et al., 1965 and Anderson, 1965). It should be noted that the presence of the magnetic tail with its reflecting boundary and neutral sheet will in general affect the dissipation of the pulse as discussed in section II. However, it is doubtful that the order of magnitude of the conclusions will be changed by this effect.

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Figure Captions

Fig. 1. Schematic illustration of the evolution of a typical region of electrons as it is blown downwind in the transition region. Shaded areas 1, 2, and 3 show the extent of the region at three different times.

Fig. 2. The general configuration in a plane normal to the sun-earth line looking toward the earth. As the region of electrons is blown downwind (out of the paper), it expands by diffusion as indicated by the arrows. Electrons may not be able to penetrate into the magnetic tail, but should get into the neutral sheet.

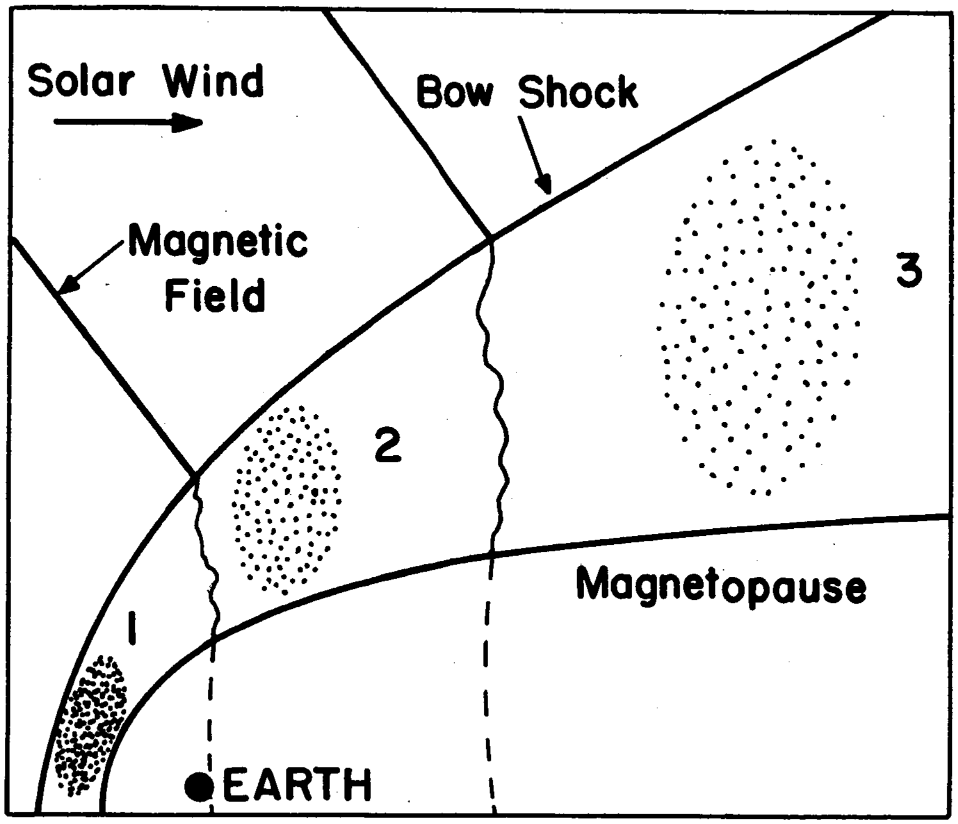
Solar Wind
Bow Shock
Magnetic
Field
3
2
Magnetopause
1
EARTH

Fig. 1.