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WESTWARD DRIFT OF THE GEOMAGNETIC FIELD AND ITS RELATION TO MOTIONS OF THE EARTH'S CORE

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PREFACE

At the earth's surface the geomagnetic field drifts slowly westward with time. This drift has been observed for the past three centuries. It suggests that the earth's central core, the seat of the earth's field, rotates more slowly than the solid mantle and crust above.

The present study is one of a series intended to improve predictions of the strength of the geomagnetic field and predictions of the field patterns -- both of which dominate the distribution of the earth's radiation belts. The studies should also assist in the estimates of magnetic fields of other planets.

This work was supported by the National Aeronautics and Space Administration. The author, who is a consultant to The RAND Corporation, is currently studying on a Fulbright scholarship in Germany.

ABSTRACT

The rate of westward drift of the geomagnetic field for epoch 1960 is calculated by use of the method of a least-square fit. The drifts of the magnetic potential and of the radial magnetic field are calculated at the surface of the earth and at the surface of the earth's fluid core, and the discrepancies between the different values obtained are explained. It is shown that the value for the drift of the radial magnetic field at the earth's core (.155°/yr westward) should provide the best estimate of the drift of the fluid at the surface of the core.

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I. INTRODUCTION

Several writers have shown that the earth's magnetic field appears to drift westward. As early as 1692, Halley noted that the positions of isogonic lines on magnetic charts moved westward at about 0.5^{o}/yr. Bullard, et al. (1950) determined the apparent shift of the field along circles of latitude between the years 1907.5 and 1945, and found a resultant average drift rate of 0.18^{o}/yr westward. Vestine (1952) estimated 0.29^{o}/yr for westward motion of the eccentric dipole. Yukutake (1962) performed a detailed study of the latitudinal and longitudinal dependence of the westward drift and found that most of the secular change could be accounted for by the drift. Using the Y-component of the magnetic field, he found a mean drift of 0.20^{o}/yr westward.

Halley remarked that the core of the earth appeared to rotate more slowly than the surface of the earth. Similarly, Bullard, et al., Yukutake, and others who have calculated the drift of the magnetic field have inferred that the drift is directly related to a westward motion of the electrically conducting fluid at the surface of the earth's core.

It is my present purpose to examine more closely the relation between the westward drift of the magnetic field and the drift of fluid in the core. In Section II I calculate the drift of the field using the method of a least-square fit, and show how the value obtained depends on the distance from the earth's core at which the drift is calculated. In Section III I will consider how accurately the calculation of the drift of the magnetic field at the core surface represents the mean westward motion of the fluid at the surface of the core.

II. CALCULATIONS OF THE WESTWARD DRIFT

If a scalar quantity, such as the geomagnetic potential V, is defined on the surface of a sphere as a function of the colatitude θ, east longitude λ, and time t, a drift rate $W_v(\theta)$ (in radians/unit time) in the λ-direction can be found by using the method of least squares as follows:

$$V(\theta, \lambda - W_v dt, t) = V(\theta, \lambda, t) - W_v dt \frac{\partial V}{\partial \lambda}$$

$$V(\theta, \lambda, t + dt) = V(\theta, \lambda, t) + dt \frac{\partial V}{\partial t} \tag{1}$$

$$V(\theta, \lambda, t + dt) - V(\theta, \lambda - W_v dt, t) = dt \left[\frac{\partial V}{\partial t} + W_v \frac{\partial V}{\partial \lambda}\right]$$

We want to find $W_v(\theta)$ such that the expression in Eq. (1), when square-integrated around a circle of latitude, will be a minimum, i.e.,

$$\frac{d}{dW_v} \int_0^{2\pi} \left\{ dt^2 \left[\frac{\partial V}{\partial t} + W_v \frac{\partial V}{\partial \lambda}\right]^2 \right\} d\lambda = 0$$

This gives

$$W_v(\theta) = - \frac{\int_0^{2\pi} \frac{\partial V}{\partial t} \frac{\partial V}{\partial \lambda} d\lambda}{\int_0^{2\pi} \left(\frac{\partial V}{\partial \lambda}\right)^2 d\lambda} \tag{2}$$

If we want to find only a uniform (latitude-independent) drift rate for the whole sphere, D_v, it is easily seen that the integrals in Eq. (2) should be taken over the whole sphere instead of around a circle of latitude:

$$D_v = - \frac{\int_S \frac{\partial V}{\partial t} \frac{\partial V}{\partial \lambda} da}{\int_S \left(\frac{\partial V}{\partial \lambda}\right)^2 da} \tag{3}$$

The geomagnetic potential can be expanded in the series

$$V(r,\theta,\lambda,t) = a \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+1} [g_n^m(t) \cos m\lambda + h_n^m(t) \sin m\lambda] \, P_n^m(\cos \theta) \tag{4}$$

where a is the radius of the earth, r is the radial distance from the center of the earth, $P_n^m(\cos \theta)$ is a Schmidt-normalized associated Legendre polynomial of degree n and order m, and $g_n^m(t)$ and $h_m^n(t)$ are properly chosen coefficients. If the conductivity of the earth's crust and mantle is small enough, Eq. (4) is valid down to the surface of the core. The time derivative of Eq. (4) gives

$$\frac{\partial V}{\partial t} = a \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+1} [\dot{g}_n^m \cos m\lambda + \dot{h}_n^m \sin m\lambda] \, P_n^m \tag{5}$$

where $\dot{g}_n^m$ and $\dot{h}_n^m$ are the coefficients of secular change. Furthermore,

$$\frac{\partial V}{\partial \lambda} = a \sum_{n=1}^{\infty} \sum_{m=1}^{n} \left(\frac{a}{r}\right)^{n+1} m[-g_n^m \sin m\lambda + h_n^m \cos m\lambda] \, P_n^m \tag{6}$$

Substituting these series expansions for $\frac{\partial V}{\partial t}$ and $\frac{\partial V}{\partial \lambda}$ into Eq. (2), we obtain

$$W_v(\theta) = - \frac{\sum_{n=1}^{\infty} \sum_{n'=1}^{\infty} \sum_{m=1}^{N} \left(\frac{a}{r}\right)^{n+n'} m(\dot{g}_n^m h_{n'}^m - \dot{h}_n^m g_{n'}^m) \, P_n^m P_{n'}^m}{\sum_{n=1}^{\infty} \sum_{n'=1}^{\infty} \sum_{m=1}^{\infty} \left(\frac{a}{r}\right)^{n+n'} m^2(g_n^m g_{n'}^m + h_n^m h_{n'}^m) \, P_n^m P_{n'}^m} \tag{7}$$

where N is the smaller of n or n'. Yukutake (1962) has used this equation to calculate drifts at the earth's surface for various latitudes.

It is also of interest to calculate the drift by use of the radial component of the magnetic field, B_r, instead of the magnetic potential, V. The equations for this drift, analogous to Eqs. (2) and (3), are

$$W_B(\theta) = - \frac{\int_0^{2\pi} \frac{\partial B_r}{\partial t} \frac{\partial B_r}{\partial \lambda} d\lambda}{\int_0^{2\pi} \left(\frac{\partial B_r}{\partial \lambda}\right)^2 d\lambda} \tag{8}$$

and

$$D_B = - \frac{\int_S \frac{\partial B_r}{\partial t} \frac{\partial B_r}{\partial \lambda} da}{\int_S \left(\frac{\partial B_r}{\partial \lambda}\right)^2 da} \tag{9}$$

The series expansion for B_r is

$$B_r = - \frac{\partial V}{\partial r} = \sum_{n=1}^{\infty} \sum_{m=0}^{n} (n+1) \left(\frac{a}{r}\right)^{n+2} [g_n^m \cos m\lambda + h_n^m \sin m\lambda] P_n^m \tag{10}$$

so that

$$W_B(\theta) = - \frac{\sum_{n=1}^{\infty} \sum_{n'=1}^{\infty} \sum_{m=1}^{N} (n+1)(n'+1) \left(\frac{a}{r}\right)^{n+n'} m(\dot{g}_n^m h_{n'}^m - \dot{h}_n^m g_{n'}^m) P_n^m P_{n'}^m}{\sum_{n=1}^{\infty} \sum_{n'=1}^{\infty} \sum_{m=1}^{N} (n+1)(n'+1) \left(\frac{a}{r}\right)^{n+n'} m^2(g_n^m g_{n'}^m + h_n^m h_{n'}^m) P_n^m P_{n'}^m} \tag{11}$$

Equations (7) and (11) are rather complicated functions of r and θ. For my purposes, it is sufficient to calculate only the uniform latitude-independent drift, using Eqs. (3) and (9), which become

$$D_V = - \frac{\sum_{n=1}^{\infty} \sum_{m=1}^{n} \frac{1}{2n+1} \left(\frac{a}{r}\right)^{2n} m(\dot{g}_n^m h_n^m - \dot{h}_n^m g_n^m)}{\sum_{n=1}^{\infty} \sum_{m=1}^{n} \frac{1}{2n+1} \left(\frac{a}{r}\right)^{2n} m^2(g_n^{m\,2} + h_n^{m\,2})} \tag{12}$$

$$D_B = - \frac{\sum_{n=1}^{\infty} \sum_{m=1}^{n} \frac{(n+1)^2}{2n+1} \left(\frac{a}{r}\right)^{2n} m(\dot{g}_n^m h_n^m - \dot{h}_n^m g_n^m)}{\sum_{n=1}^{\infty} \sum_{m=1}^{n} \frac{(n+1)^2}{2n+1} \left(\frac{a}{r}\right)^{2n} m^2(g_n^{m\,2} + h_n^{m\,2})} \tag{13}$$

It is seen that Eq. (13) places more emphasis on higher-degree (larger n) terms in calculating the drift than does Eq. (12). It is also seen that the drifts D_V and D_B are dependent on the distance from the center of the earth, r. Instead of considering the drift of the whole field, we can look at the drifts of different components separately. Thus, the mean drift of all harmonics of an individual degree, n, can be found from Eqs. (12) or (13) by setting all g_n^m's, h_n^m's, $\dot{g}_n^m$'s, and $\dot{h}_n^m$'s equal to zero except those of the degree of n under consideration. This drift, d(n), is the same whether Eq. (12) or (13) is used:

$$d(n) = - \frac{\sum_{m=1}^{n} m(\dot{g}_n^m h_n^m - \dot{h}_n^m g_n^m)}{\sum_{m=1}^{n} m^2(g_n^{m\,2} + h_n^{m\,2})} \tag{14}$$

This drift is independent of r, as would be expected, since it represents a mean rotation of n multipoles of the same degree, the fields of which all have the same r-dependence.

In Eqs. (12) and (13) it is seen that because of the factor $(a/r)^{2n}$, higher-degree terms will become more important in determining D_V and D_B as we approach the earth's core. In fact, because of this factor, it is not permissible to assume that the series in Eqs. (12) and (13) will converge at the surface of the core.

In order to obtain quantitative relations showing which degrees of V or B_r are most important in determining the drift at different levels, we can define weighting factors i_n such that

$$D = \sum_n i_n \, d(n) \tag{15}$$

where $$\sum_n i_n = 1$$

It is easily seen that if V is used to determine the drift,

$$i_n = \frac{\frac{1}{2n+1} (\frac{a}{r})^{2n} \sum_{m=1}^{n} m^2(g_n^{m\,2} + h_n^{m\,2})}{\sum_n \left[\frac{1}{2n+1} (\frac{a}{r})^{2n} \sum_{m=1}^{n} m^2(g_n^{m\,2} + h_n^{m\,2})\right]} \tag{16}$$

and if B_r is used to determine the drift,

$$i_n = \frac{\frac{(n+1)^2}{2n+1} \left(\frac{a}{r}\right)^{2n} \sum_{m=1}^{n} m^2 \left(g_n^{m\,2} + h_n^{m\,2}\right)}{\sum_n \left[\frac{(n+1)^2}{2n+1} \left(\frac{a}{r}\right)^{2n} \sum_{m=1}^{n} m^2 \left(g_n^{m\,2} + h_n^{m\,2}\right)\right]} \tag{17}$$

These weighting factors are dependent on r and on the magnetic field coefficients, but are independent of the secular change coefficients. Since it would not be expected that

$$\sum_{m=1}^{n} m^2 \left(g_n^{m\,2} + h_n^{m\,2}\right)$$

will vary greatly with time, neither should the weighting factors vary appreciably over a period of time. As previously stated, the series in Eqs. (12) and (13) does not necessarily converge, and so the usefulness of these equations might be questioned. Physical considerations would lead us to expect, however, that the high-degree components of the secular change are due mainly to localized motions in the core, and thus the information given by these coefficients regarding overall drifts of the core fluid is obscured. It seems justifiable, therefore, to terminate the series at a finite value of n in order to exclude secular change components that are mainly due to these localized motions. Since data for the secular change coefficients are available only for degrees to n = 6, I have terminated the series in Eqs. (12) and (13) at this value in my calculations.

The various values of i_n, d(n), and D listed in Table 1 were obtained from the data of Hendricks and Cain (1966) for epoch 1960. The values of the drifts d(n) and D obtained by use of Eqs. (12), (13), and (14) are multiplied by -57.3, so that they are expressed as degrees/yr westward, instead of radians/yr eastward. Because the secular change coefficients for n = 5 and n = 6 are poorly known, the drifts of the n = 5 and n = 6 components of the field are rather uncertain. Table 1 also includes calculated values of the drift in the case where B_r is

Table 1

WEIGHTING FACTORS AND VALUES OF THE WESTWARD DRIFT*

		i_n									
		1	2	3	4	5	6	7	8	9	10
		a/r = 1				a/r = 1.8355					
		Using V		Using B_r		Using V		Using B_r			
n	d(n) (in °/yr westward)	Whole field to n=6	Non-dipole	Whole field to n=6	Non-dipole	Whole field to n=6	Non-dipole	Whole field to n=6	Non-dipole	Whole field to n=5	Whole field to n=4
1	.0395	.6132		.3226		.0958		.0143		.0277	.0408
2	.2205	.2227	.5757	.2636	.3892	.1172	.1296	.0392	.0398	.0763	.1119
3	.1116	.1243	.3213	.2616	.3861	.2203	.2436	.1312	.1331	.2549	.3744
4	.1843	.0298	.0771	.0981	.1448	.1781	.1969	.1657	.1681	.3220	.4729
5	-.0055	.0061	.0157	.0288	.0426	.1226	.1355	.1642	.1666	.3191	
6	.0975	.0039	.0102	.0253	.0374	.2661	.2943	.4854	.4924		
D (in °/yr westward)		.0930	.1779	.1204	.1590	.1123	.1200	.1008	.1017	.1040	.1552

*Data of Hendricks and Cain, epoch 1960.

used at the core surface, first leaving out terms in n = 6 (column 9), and next leaving out terms in both n = 5 and n = 6 (column 10). It has been observed that at the surface of the earth the dipole component of the field drifts much more slowly than the nondipole component. To facilitate comparison of my calculated drifts with those calculated by others for the nondipole field, I have repeated my calculations for the nondipole components of the field by leaving out terms in n = 1 in all summations over n (columns 2, 4, 6, and 8).

Two observations can be made on the basis of Table 1. First, the drifts d(n) of the different components of the field are not in good agreement. This suggests that types of motion other than a simple westward drift are important in the earth's core. The small drift of the dipole component could be explained if the actual drift were around some axis close to the geomagnetic axis of the earth, instead of around the geographic axis. However, a calculation of the drift of the various nondipole components about the geomagnetic axis gave results not significantly different from those in Table 1, and the drifts were in no better agreement than those in Table 1. Second, it is seen that the weighting factors for the higher-degree components become quite large at the core surface, more so when B_r is used than when V is used in finding the drift. As will be clear from Section III, the calculation of the drift where B_r is used at the core surface (columns 7, 8, 9, and 10) is physically the most meaningful of the drifts in Table 1. And yet the calculation of this drift relies most heavily on the poorly known data for the higher-degree secular change coefficients. Thus, if the $\dot{g}_5^m$'s, $\dot{h}_5^m$'s, $\dot{g}_6^m$'s, and $\dot{h}_6^m$'s were well known, the value for the drift given at the bottom of column 7 (.101°/yr) should be our best estimate of the drift of fluid in the core; actually our best estimate may be the value in column 10 (.155°/yr), which is based on only the better-known data.

III. RELATION OF THE DRIFT OF THE GEOMAGNETIC FIELD TO THE DRIFT OF FLUID IN THE CORE

Westward drifts such as those calculated above should provide us with some information about motions of the fluid in the core of the earth. These motions are governed by the equation

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \frac{1}{4\pi\sigma} \nabla^2 \vec{B} \qquad (18)$$

where σ is the electrical conductivity of the fluid. If the diffusion term of this equation, $\nabla^2\vec{B}/4\pi\sigma$, is small in comparison with the other two terms,* the fluid motions are approximately governed by the equation

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) \qquad (19)$$

The radial component of Eq. (19) at the surface of the core (where $v_r = 0$) is

$$\frac{\partial B_r}{\partial t} = - \left[\frac{\cot \theta}{r} v_\theta + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{1}{r \sin \theta}\frac{\partial v_\lambda}{\partial \lambda}\right] B_r - \frac{v_\theta}{r}\frac{\partial B_r}{\partial \theta} - \frac{v_\lambda}{r \sin \theta}\frac{\partial B_r}{\partial \lambda} \qquad (20)$$

For an incompressible fluid, $\nabla \cdot \underline{v} = 0$, and Eq. (20) can be written

$$\frac{\partial B_r}{\partial t} = B_r \frac{\partial v_r}{\partial r} - \frac{v_\theta}{r}\frac{\partial B_r}{\partial \theta} - \frac{v_\lambda}{r \sin \theta}\frac{\partial B_r}{\partial \lambda} \qquad (21)$$

In Eq. (20), a zonal velocity u_λ, which is independent of longitude, can be separated from the rest of the velocity pattern. Thus if we define

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*Fluid motions will strongly distort the magnetic field within the core, so that $\nabla^2\vec{B}/4\pi\sigma$ may be comparable in magnitude to the other two terms in Eq. (18). But retention of this diffusion term severely complicates the analysis, and so it is ignored.

$$v_\lambda' = v_\lambda - u_\lambda \tag{22}$$

Eq. (20) can be written

$$\frac{\partial B_r}{\partial t} + \frac{u_\lambda}{r \sin\theta}\frac{\partial B_r}{\partial \lambda} = -\left[\frac{\cot\theta}{r} v_\theta + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial v_\lambda'}{\partial \lambda}\right] B_r - \frac{v_\theta}{r}\frac{\partial B_r}{\partial \theta} - \frac{v_\lambda'}{r\sin\theta}\frac{\partial B_r}{\partial \lambda} \tag{23}$$

which, for an incompressible fluid, becomes

$$\frac{\partial B_r}{\partial t} + \frac{u_\lambda}{r \sin\theta}\frac{\partial B_r}{\partial \lambda} = B_r \frac{\partial v_r}{\partial r} - \frac{v_\theta}{r}\frac{\partial B_r}{\partial \theta} - \frac{v_\lambda'}{r\sin\theta}\frac{\partial B_r}{\partial \lambda} \tag{24}$$

A drift rate $s(\theta)$ (in radians/unit time) can be defined as

$$s(\theta) = \frac{u_\lambda}{r \sin\theta} \tag{25}$$

where u_λ is chosen to make

$$\int_0^{2\pi} v_\lambda' \, d\lambda = 0 \tag{26}$$

Then

$$s(\theta) = \frac{1}{2\pi}\int_0^{2\pi} \frac{v_\lambda}{r\sin\theta}\, d\lambda \tag{27}$$

Equation (24) is now

$$\frac{\partial B_r}{\partial t} + s(\theta)\frac{\partial B_r}{\partial \lambda} = B_r \frac{\partial v_r}{\partial r} - \frac{v_\theta}{r}\frac{\partial B_r}{\partial \theta} - \frac{v_\lambda'}{r\sin\theta}\frac{\partial B_r}{\partial \lambda} \tag{28}$$

It is desired to find $s(\theta)$ in terms of known quantities (e.g., the coefficients g_n^m, h_n^m, $\dot{g}_n^m$, and $\dot{h}_n^m$) without going through the trouble of finding v_θ and v_λ'. An approximate solution of this sort is possible. Multiplying both sides of Eq. (28) by $\partial B_r/\partial\lambda$ and integrating around a circle of latitude, we get

$$\int_0^{2\pi} \frac{\partial B_r}{\partial t}\frac{\partial B_r}{\partial \lambda}\, d\lambda + \int_0^{2\pi} s(\theta)\left(\frac{\partial B_r}{\partial \lambda}\right)^2 d\lambda$$

$$= \int_0^{2\pi} \frac{1}{2}\frac{\partial (B_r^2)}{\partial \lambda}\frac{\partial v_r}{\partial r}\, d\lambda - \int_0^{2\pi} \frac{v_\theta}{r}\frac{\partial B_r}{\partial \theta}\frac{\partial B_r}{\partial \lambda}\, d\lambda - \int_0^{2\pi} \frac{v_\lambda'}{r \sin\theta}\left(\frac{\partial B_r}{\partial \lambda}\right)^2 d\lambda \tag{29}$$

Consider the first integral on the right-hand side of Eq. (29). Since B_r^2 is a single-valued quantity on the surface of the core,

$$\int_0^{2\pi} \frac{1}{2}\frac{\partial (B_r^2)}{\partial \lambda}\, d\lambda = 0$$

At present we have no physical reason to expect $\partial v_r/\partial\lambda$ to be correlated with $\partial(B_r^2)/\partial\lambda$, so that the "most expected" value of the integral

$$\int_0^{2\pi} \frac{1}{2}\frac{\partial (B_r^2)}{\partial \lambda}\frac{\partial v_r}{\partial r}\, d\lambda$$

is zero. Similar arguments show that the "most expected" values of the other two integrals on the right-hand side of Eq. (29) will also be zero,* when it is noted that

$$\int_0^{2\pi} \frac{\partial B_r}{\partial \lambda}\, d\lambda = 0$$

*In making this statement, it is assumed that $\partial B_r/\partial\theta$, $\partial B_r/\partial\lambda$, and v_θ are uncorrelated. However, if hydromagnetic oscillations such as those proposed by Hide (1966) are important in the earth's core, $\partial B_r/\partial\theta$, $\partial B_r/\partial\lambda$, and v_θ may be interrelated to some degree, and the statement that the "most expected" value of $\int_0^{2\pi} \frac{v_\theta}{r}\frac{\partial B_r}{\partial \theta}\frac{\partial B_r}{\partial \lambda}\, d\lambda$ is zero would not necessarily be correct.

$$\int_0^{2\pi} \frac{v_\lambda'}{r \sin\theta}\, d\lambda = 0$$

However, such arguments cannot be used for the integrals on the left-hand side of Eq. (29). Physical observations show that $\partial B_r/\partial t$ and $\partial B_r/\partial_\lambda$ are correlated; it is this correlation that produces the "westward drift." And $(\partial B_r/\partial\lambda)^2$ is always a positive quantity, so that the integral

$$\int_0^{2\pi} s(\theta)\left(\frac{\partial B_r}{\partial\lambda}\right)^2 d\lambda = s(\theta)\int_0^{2\pi}\left(\frac{\partial B_r}{\partial\lambda}\right)^2 d\lambda$$

will not tend to be zero unless $s(\theta)$ is zero.

Kahle, Vestine, and Ball (1966) have found velocity patterns that satisfy Eq. (20), and their numerical results show that typical values of the terms in Eq. (28) are of the same order of magnitude. Then from the considerations above, we would expect the integrals on the right-hand side of Eq. (29) to be appreciably smaller than the integrals on the left-hand side. We can thus write

$$s(\theta) \approx W_B(\theta) = -\frac{\int_0^{2\pi} \frac{\partial B_r}{\partial t}\frac{\partial B_r}{\partial\lambda}\, d\lambda}{\int_0^{2\pi}\left(\frac{\partial B_r}{\partial\lambda}\right)^2 d\lambda} \tag{30}$$

This is essentially the same as Eq. (8). To find the mean (latitude-independent) drift for the whole sphere, S, u_λ can be redefined so that

$$u_\lambda = S\, r \sin\theta \tag{31}$$

while requiring that

$$\int_S \frac{v_\lambda'}{r\sin\theta}\, da = 0 \tag{32}$$

so that

$$S = \frac{1}{4\pi r^2}\int_S \frac{v_\lambda}{r\sin\theta}\, da \tag{33}$$

It is easily seen, using arguments similar to those used when finding an approximate solution for $s(\theta)$, that

$$S \approx D_B = - \frac{\int_S \frac{\partial B_r}{\partial t} \frac{\partial B_r}{\partial \lambda} \, da}{\int_S \left(\frac{\partial B_r}{\partial \lambda} \right)^2 da} \tag{34}$$

This is essentially identical to Eq. (9), which was used to calculate the drifts in columns 7, 8, 9, and 10 of Table 1. The expression for S in Eq. (4) should be much more accurate than the expression for $s(\theta)$ in Eq. (30) because integrals over the entire surface of the core of quantities such as

$$\frac{1}{2} \frac{\partial (B_r^2)}{\partial \lambda} \frac{\partial v_r}{\partial r}$$

will have a much stronger tendency to be small or zero than do integrals around a circle of latitude.

IV. SUMMARY AND DISCUSSION OF RESULTS

The westward drift of the geomagnetic field has been calculated by the method of a least-square fit, applied to both the geomagnetic potential, V, and the radial component of the magnetic field, B_r. It was shown quantitatively how the calculation of the drift will yield different values, depending on what value of a/r is used. It is seen that if the drift of the nondipole field is calculated at the surface of the earth, as Bullard, et al. (1950) and Yukutake (1962) have done, the value obtained will be dominated by the drift of harmonics of degree n = 2. Because the mean drift of harmonics of degree n = 2 is larger than the mean drift of harmonics of any other degree, the calculation of the drift of the nondipole field at the surface of the earth will be unrepresentative of the drift of the field as a whole. On the other hand, if the drift of the field is calculated at the surface of the core, higher-degree components of the field tend to dominate. Because of uncertainties in the data of the higher-degree secular change coefficients, the drift thus found may also be unrepresentative. A compromise was made by calculating the drift at the core with B_r, using degrees only through n = 4.

It was also shown, by use of the "frozen field" concept, that the drift at the core of the radial component of the magnetic field provides a good estimate of the drift of the electrically conducting fluid at the core surface.

It was noted, however, that the effects of a finite conductivity in the earth's mantle, diffusion of the magnetic field within the core, and large-scale hydromagnetic oscillations will modify the accuracy of calculated values for the drift of this fluid.

On the assumption that the large-scale drift features can be found by considering magnetic field and secular change components of degrees only through n = 4, the best estimate of the westward drift of the core was found to be 0.155°/yr.

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