Space Sciences Laboratory
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Berkeley, California 94720

Semi-Annual Report on

OPTIMIZATION OF DESIGN OF SPACE EXPERIMENTS FROM THE STANDPOINT OF DATA PROCESSING

Supported by

NASA Grant
NGR 05-003-143

for the period

May 1 through October 31, 1966

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Space Sciences Laboratory Series No. 7, Issue No. 48
September 30, 1966

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The present phase of this project is concerned with the development of techniques for processing the results of space experiments before their transmission to the ground. Taking into account some engineering constraints, it is desired to obtain a low data rate while preserving the features of the data of interest to the experimenters. The theoretical aspects stem from the work of Shannon on the relation of the information rate to the distortion in the transmission of analog messages by digital means.

The following engineering conditions also define and limit our work:

(1) the availability of a small on-board digital computer for the storage and the processing of data;

(2) the possibility of modifying the program of the computer by instructions from the ground; and

(3) the limited knowledge of the statistics of the data.

CONSIDERATION OF DATA SOURCES

In the initial stage of the project, principal consideration was given to the types of experiments being carried out now, and their properties as data sources. In particular, attention was given to the exact information sought by the experimenter and whether the information rate involved was sufficiently high to warrant the use of source coding or "data compression" techniques. After spending approximately two months considering these points, it was decided to focus attention for the balance of the first year on some particle-counting data obtained by Mozer and others[1] during the flight of the French rocket, Dragon. This decision was made for the following reasons:

(1) Particle-counting experiments comprise the majority of present experiments; the basic properties of the data source model for such experiments depend only to a minor extent on the differing physical aspects of the experiment. Hence, any conclusions based on the study of these data will have wide applicability.

(2) The experimenter is considering a future experiment involving 20 counting channels, each sampled at a rate of 1,000 samples per second; the resulting data rate is too high to go onboard most satellites, hence source coding could contribute substantially to the success of this experiment.

The experiment already started by Moser and others[1] has the objective of determining the origin of charged particles in the auroral regions by observing the temporal behavior of the counting rates of detectors with varying energy thresholds. The important feature of the detector output that must be preserved accurately is the occurrence of rather rapid fluctuations that take place in the mean counting rate. These changes can be a fraction or several times the average counting rate and can occur within 5 to 50 milliseconds.

VARIABLE RATE SAMPLING METHODS

Shannon's Rate Distortion Function[2] gives the absolute minimum telemetry rate necessary to transmit data from a certain type of source with a given level of distortion. It is known[2] that a gaussian process with a given spectrum requires a larger rate than any other process with the same spectrum; further, the rate-versus-distortion curve for a gaussian process with a given spectrum can be evaluated easily.[3] Although there is no reason to believe that the mean counting rates of the kinds of experiments in question are gaussian processes, the above two points indicate that the rate distortion curve for a gaussian process can be used as a meaningful standard by which to evaluate a source encoding or "data compression" method; for, by means of a suitable (perhaps quite complex) encoding method, one can achieve a given distortion level with the rate indicated by this curve. The work of Goblick and Holsinger[3] shows that for a stationary process, optimal source encoding methods can achieve a saving of only about a factor of 2.5 in the data rate over a simple low-pass filter, sample, and quantize system, provided the sampling rate and quantization levels are chosen properly. The data sources encountered in space experiments are highly non-stationary, hence an "adaptive" or variable sampling rate and quantization level scheme could provide performance reasonably close to that of an optimum encoding system. Since such a system could be implemented reasonably easily with a modest amount of special-purpose equipment, this approach seemed a logical one to investigate first.

This method was simulated on a computer using the data from the Dragon rocket flight.[1] Ideal low-pass filters with sinx/x impulse responses were used to filter the raw data. The bandwidths of these filters were picked so that the entire output of the filter could be reconstructed using only every n^{th} sample (thus affording a "compression ratio" of n). Values of n = 2, 3, 6, and 12 were used in the simulation. The reconstruction data indicated that although the reconstruction might be satisfactory when gauged in terms of mean-square error, the reconstruction was unacceptable because the sinx/x filter introduced a structure into the reconstructed data that was not present in the original data. Since such an artificial structure could lead an experimenter into drawing false conclusions from the reconstructed data, this method of data processing was abandoned and a more complicated method was investigated.

PRESENT DATA PROCESSING ALGORITHM

After abandoning variable-rate sampling methods, we expanded the majority of our effort in developing an encoding algorithm suitable for implementation on a small general-purpose computer. In describing the properties of the algorithm that was developed, it will be expedient to consider several aspects of source encoding of random data processes.

Distortion Measures Applicable to Space Experiments

Before discussing specific data-processing schemes, an acceptable tolerance in the reproduction of the data must be determined. This tolerance will affect greatly the ultimate data rate required. We shall start by recalling some of Shannon's results on coding with a fidelity criterion, which will provide us with a reference system and suitable terminology.

Let s(t) be a stationary data process and $\hat{\hat{s}}(t)$ denote the encoded version of s(t). The error between the transmitted process and the original process we then denote by

$$e(t) = s(t) - \hat{\hat{s}}(t)$$

We now consider a numerical measure of the distortion that occurs in the transmission of the signal. Let f( ) denote some non-negative valued function and let T denote some time duration, which, for the moment, we leave arbitrary. The <u>distortion</u> involved in transmission is then taken to be

$$D = E \frac{1}{T} \int_0^T f[e(t)] \, dt$$

in which E denotes expectation or an average over the ensemble of possible message signals. The function f and the time interval T can be chosen to express best the fidelity criterion of interest to the experimenter. Shannon has found a <u>Rate-Distortion Function</u> R(D), which gives the minimum rate that is required to transmit with a distortion level of value D. Unfortunately, this function is easy to evaluate only if s(t) is a gaussian process, and for $f(e) = e^2$, and in the limit as T approaches infinity. This particular choice of distortion measure is not applicable to the type of data generated by the counting process used by Moser. We refer to Figure 1, which shows a stretch of typical data. The features of interest to the experimenter are the occasional rises and falls in the particular count. It is possible to smooth beyond recognition these features of interest and still obtain a small value for the quantity:

$$\lim_{T \to \infty} \frac{1}{T} \int_0^T e^2(t) \, dt$$

In fact, most coding schemes that one might envision, which would yield a fixed value for this integral squared error, would yield a small error when the message process s(t) was changing only gradually and yield large errors during the infrequent sudden transitions — exactly those periods of the data that are of the greatest interest to the experimenter.

One fidelity criterion that would be quite suitable for our application would be to pick the function f as follows: let ε denote a satisfactory value for the instantaneous error; we then let $f[e(t)]$ be unity if

$$\sup_{0 \leq t \leq T} e(t) > \varepsilon$$

and let $f[e(t)]$ be zero otherwise. In this case, D represents the probability that the error never gets beyond an acceptable value anywhere in the interval of duration T.

Although this definition of distortion adequately expresses the experimenter's view of fidelity, it is virtually impossible to handle mathematically. Instead, we shall keep the integral square error over some finite time interval at a suitably low value. We define

$$\epsilon^2 = \frac{1}{T} \int_0^T e^2(t)dt$$

and we note that we do not take an average of ϵ^2 over an ensemble of data signals.

It is important to observe that holding ϵ^2 at a small value does not guarantee that e(t) will be small for all values of time within the interval T since the error will vary with time. This non-stationary character of the error makes ϵ^2 unsuitable as a criterion for large values of T since the error can be large during the rare events of interest. We will handle this problem by keeping the time interval T comparable to the duration of typical events, so that the error cannot be large during a sudden rise (event) without appreciably increasing the integral square error for the interval of duration T. For the data shown in Figure 1, we use T = 100 ms, which corresponds to 100 data points.

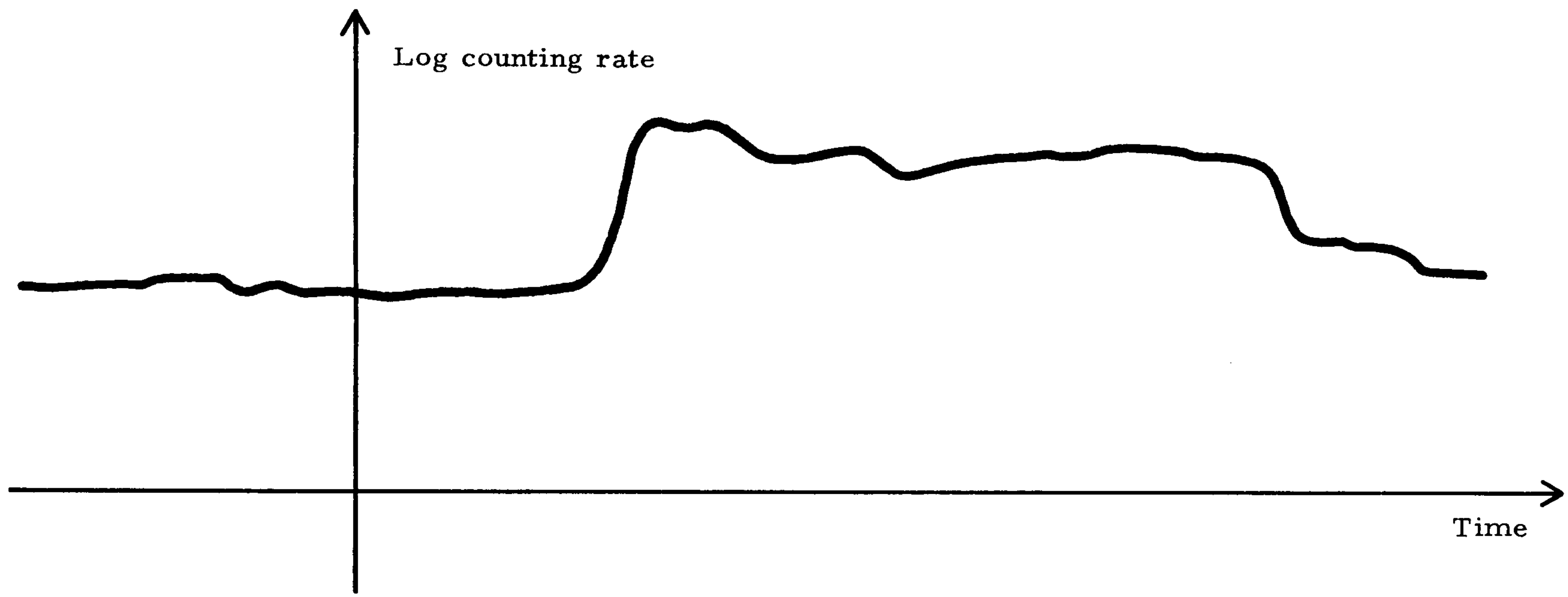

FIGURE 1: Character of the counting process

When the data observed are noisy our objective is still a good reproduction of the signal at any time, but the presence of noise may make this unreasonable or impossible. Whenever the noise is large, all one can hope for is to obtain the general trends of the signal. In that case we shall keep the expected value of ϵ^2, $E\{\epsilon^2\}$ at a value small compared with the amount of noise. Whenever the noise is small, the noisy data can be filtered to give a good detailed reproduction of the signal. In that case we shall treat the filtered data as if they were noise-free data and require that ϵ^2 be kept less than some set value for all possible message signals (except perhaps for a set of highly improbable signals).. Note that even when the noise is large and we are using a mean integral square error measure, we have to keep the representation interval T small for the reason mentioned earlier.

The conclusion of our discussion provides the type of definition of distortion that will be acceptable for experiments: the integral square error over a small time interval (or its expected value, in the case of noisy data) is maintained at some suitably small level. We did, in fact, obtain agreement on this point from Moser. We consider now the representation of signals of finite duration by digital means.

Efficient Discrete Description of Signals of Finite Duration

We would like to describe a signal of duration T by a small number of binary digits. This binary description should allow the reconstruction of the signal with an integral square error ϵ^2. We shall divide the process of going from the signal to its digital description into two steps. First, we shall describe the signal by a discrete set of numbers, then, the binary representation of this discrete set of numbers will be considered.

We restrict our attention to the representation of the signal in terms of an orthonormal set. That is to say we write:

$$s(t) = \sum_{k=1}^{\infty} s_k \varphi_k(t)$$

in which $\{\varphi_k(t)\}$ is a set of functions orthornormal over the interval T

$$\int_0^T \varphi_k(t)\,\varphi_j(t)\,dt = \begin{cases} 0 & k \neq j \\ 1 & k = j \end{cases}$$

We do not use a weighting function in this definition of orthogonality, because all instants of time in the interval are of equal interest. The s_k that minimize the integral square error are given by

$$s_k = \int_0^T s(t)\, \varphi_k(t)\, dt$$

A large number of orthonormal sets (complete sets) would be suitable to describe s(t) over the interval T. In accordance with our objectives we would like the set that will require the least number of coefficients s_k, on the average, for a given tolerance in the representation. Although the set of orthonormal functions possessing this property is not known, the set of functions that yield minimum mean integral square error for any fixed number of coefficients is known. This latter set of functions is known as the Karhunen-Loeve[4] expansion of the process; determination of this set of orthonormal functions requires only a knowledge of the covariance function of the message process. Thus, this set of functions constitutes a reasonable basis for expressing the message process on the interval of duration T. Infrequently, the number of coefficients required to keep the representation error within a suitable value will be large; but, on the average, the number of coefficients will be kept small.

Once we have obtained a discrete set of coefficients $\{s_k\}$ that describe the data, we have to represent these coefficients, within some signal representation error ϵ^2, by a small number of binary digits.

Digital Representation of Random Variables

Let us assume that we have N coefficients s_k and that they allow an error-free reconstruction of the signal s(t). If we describe each of them in binary form with an error ϵ_k^2, then the signal representation error will be

$$\epsilon^2 = \sum_{k=1}^{N} (\epsilon_k)^2$$

Now the problem is to choose the ϵ_k such that an acceptable representation error is obtained while the average number of binary digits needed in the description is minimized. This problem, related to Shannon's rate distortion function mentioned before, has been considered specifically by Posner[5] at the Jet Propulsion Laboratory. This is a difficult problem, for which no general solution is available. An optimum solution is to divide the N-dimensional space of the coefficients into non-overlapping exhaustive cells, each of which can be contained within a hypersphere of diameter ϵ, and then to determine which of these possible subdivisions has the smallest entropy. This subdivision would require, therefore, the least number of transmitted binary digits on the average. A solution to the problem would undoubtedly be of little practical interest because of the difficulty of implementing a quantization scheme that depends on the joint values of N different coefficients. A simpler approach is to choose the errors ϵ_k^2 independently of each other (corresponding to rectangular cells), and for each ϵ_k to determine the entropy of the corresponding random variable s_k. Let $H_k(\epsilon_k)$ be this entropy. We have then the formal minimization problem: By suitable choice of the ϵ_k, minimize

$$H = \sum_{k=1}^{N} H_k(\epsilon_k)$$

Under the constraint

$$\sum_{k=1}^{N} \epsilon_k^2 \leq \epsilon^2$$

This problem has been solved under the assumption that the s_k's have a gaussian distribution and that the variances σ_k^2 are known. An interesting feature of the solution is that the error ϵ_k on each coefficient is the same as long as

$$\frac{\epsilon_k}{\sigma_k} \leq 2$$

Whenever the variance σ_k^2 becomes very small then the error ϵ_k will also decrease, but with high probability we shall quantize s_k at the value zero. This result suggests that the approximate scheme used in our encoding algorithm, described later, will be quite close to optimum for most cases.

Once we have obtained a digital description of the coefficients s_k by quantization, an efficient binary form can be obtained by a Huffman code.[6] We use the probability of occurrence of the coefficients in digital form to determine the binary number assigned to each.

Effect of Noisy Observations

The preceding discussion has, for the most part, been predicated upon the assumption that the raw data signal on board the satellite represents an uncorrupted observation of the quantity of direct interest. In space experimentation, this is most often not the case. In the situation at hand, one observes a Poisson counting process and wishes to estimate the time-varying mean (ensemble average) counting rate. The number of counts occurring in the past millisecond gives an estimate of the mean counting rate, but this estimate contains statistical fluctuations that may well be of the same order of magnitude as the mean rate. In what follows, we will refer to corrupted observations as noisy observations, even though the source of corruption is not additive noise but of a more general cause, such as in the Poisson counting process. The difference between what one wishes to observe and what one actually observes, will be referred to as "noise".

The fact that one's observations represent a corrupted version of the process one wishes to relay to the ground observer changes both the theoretical and practical aspects of source encoding. The essence of this change may be summarized by the statement that it is pointless to use a high transmission rate to achieve a level of transmission distortion considerably less than the distortion inherent in the corrupted observation.

The rationale that we have employed in our source-encoding algorithm is as follows. Suppose we use the observable data on board the satellite to form the best estimate possible of the signal we wish to transmit to the ground. If the signal-to-noise ratio in this estimate is large compared to unity, then with a very large probability, any fluctuations in the estimate that are large compared to the rms noise level represent actual changes in the information-bearing signal. These fluctuations should thus be transmitted with some tolerable distortion level to the ground-based experimenter. Since the experimenter does not wish to miss any significant event (even if it is one that occurs only infrequently) it is logical to design an encoding method that, in the high signal-to-noise ratio case, is capable of transmitting all fluctuation shapes (except perhaps those that are extremely unlikely) with a fixed (independent of the fluctuation shape) distortion level. Thus, in the high signal-to-noise ratio situation, it is reasonable to form a good estimate of the information-bearing signal, and then encode this estimate, using a code that is capable of transmitting all fluctuation shapes in the estimate within a fixed distortion level.

Let us now consider the low signal-to-noise ratio situation. In this case, fluctuations in the estimate of the information-bearing signal may very likely be due to the noise and not to the information-bearing signal. It is then somewhat pointless to demand that we use a code that is capable of transmitting all estimate fluctuations with a fixed distortion level. It would seem more reasonable to require only that the mean (ensemble average) distortion level be less than some reasonable level (this level being based on the noise level). We have been able to show (see section on Information Theoretic Results, page 19, for a slightly expanded discussion) that in the case of mean square error, a processing method that first forms the minimum mean square error estimate of the information-bearing signal and then optimally encodes this estimate as if it were an uncorrupted message, performs just as well as a code that operates on the observed signal with the objective of minimizing the rate while constraining the mean square error between the information-bearing signal and the transmitted (encoded) signal.

Thus in either the high or low signal-to-noise ratio situation, it is reasonable (or optimal) to form first the best estimate of the information-bearing signal from the observation, and then to encode the estimate as if it were an uncorrupted message signal. In the high signal-to-noise-ratio case, this code is to be capable of transmitting all (or almost all) estimate signal shapes with a fixed distortion level. In the low signal-to-noise ratio case, the code need only be capable of keeping the mean transmission distortion to some reasonable level. This is the basis for the processing method to be described in the following subsection.

Final Coding Algorithm

The preceding subsections have discussed properties of optimal encoding or digitizing of a random process under a distortion or fidelity criterion suitable for the counting process considered. Rather than trying to construct an encoding algorithm that is statistically optimum, we have designed an algorithm with properties that are based on certain properties of optimum codes. One reason for this is that the problem of constructing an optimum code is analytically untractable, at least at present. Moreover, there are two practical reasons for not designing an optimum code:

(1) Ease of implementation. An optimum code seeks to minimize the transmission rate for a given fidelity criterion with no regard for the complexity of the required equipment or computation time. These factors are as important as minimizing the rate in a practical code; hence, our encoding algorithm incorporates those features that are computationally manageable.

(2) Statistical stability. An optimum code is optimum only for encoding a random process with the exact statistical properties for which the code was designed; yet this code may do very poorly when used on a process with different statistical properties. In space experimentation, invariably one is uncertain about the exact statistical properties of the data source involved, hence our encoding algorithm incorporates those properties of an optimum code that are reasonably universal, while rejecting those properties that require detailed statistical knowledge of the data source.

Our encoding algorithm is designed to yield a fixed but programmable fidelity criterion, so that when the properties of the source change, a corresponding change in the average bit rate occurs. It should also be pointed out that our code was designed on the assumption that a short amount of raw data would be transmitted unaltered over the telemetry link so that some of the properties of the data source could be observed and used in designing the encoding algorithm.

With the above comments in mind, we now discuss the different functions performed by our encoding algorithm. These different functions are shown in a block diagram (Figure 2).

(1) The Filter. In view of the second point given above, the filter is designed simply on the assumption that the noise has a flat spectrum and the message-bearing signal a low-pass spectrum. No particular form is assumed for the message spectrum, and the filter used is a simple symmetric geometric filter with delay. The time constant is picked to reduce the noise as much as possible without introducing what the experimenter regarded as undesirable lag or delay. This choice is based on the section of raw data available for ground observation. Although to date the filtering in our simulations has been done on a general purpose digital computer, this procedure would be wasteful in practice because the filtering could be carried out in real time by extremely simple, special-purpose digital circuitry.

As stated in Effect of Noisy Observations, page 10, the filtered signal is encoded as if it were a corruption-free message process. The difference between the filtered process and the raw data is squared and summed over a 100-point interval and this quantity is used as an estimate of the noise power in the filter output for that input. If the signal-to-noise ratio is greater than 4, then the objective of the encoding operation is taken to be transmission of the filtered 100-point interval with a sum square transmission error that is always kept below a specified constant. If the signal-to-noise ratio is less than 4, then the objective of the encoding operation is taken to be transmission with a mean (ensemble average) sum square error less than C times the estimated sum square noise power (C was normally takent to be somewhat less than one).

(2) Fourier Expansion. To describe the sample functions that can occur on a 100-point interval, we use the Karhunen-Loeve set of orthogonal functions. These functions are the eigenfunctions of the process covariance. An interval of 1300 points of filtered data is used

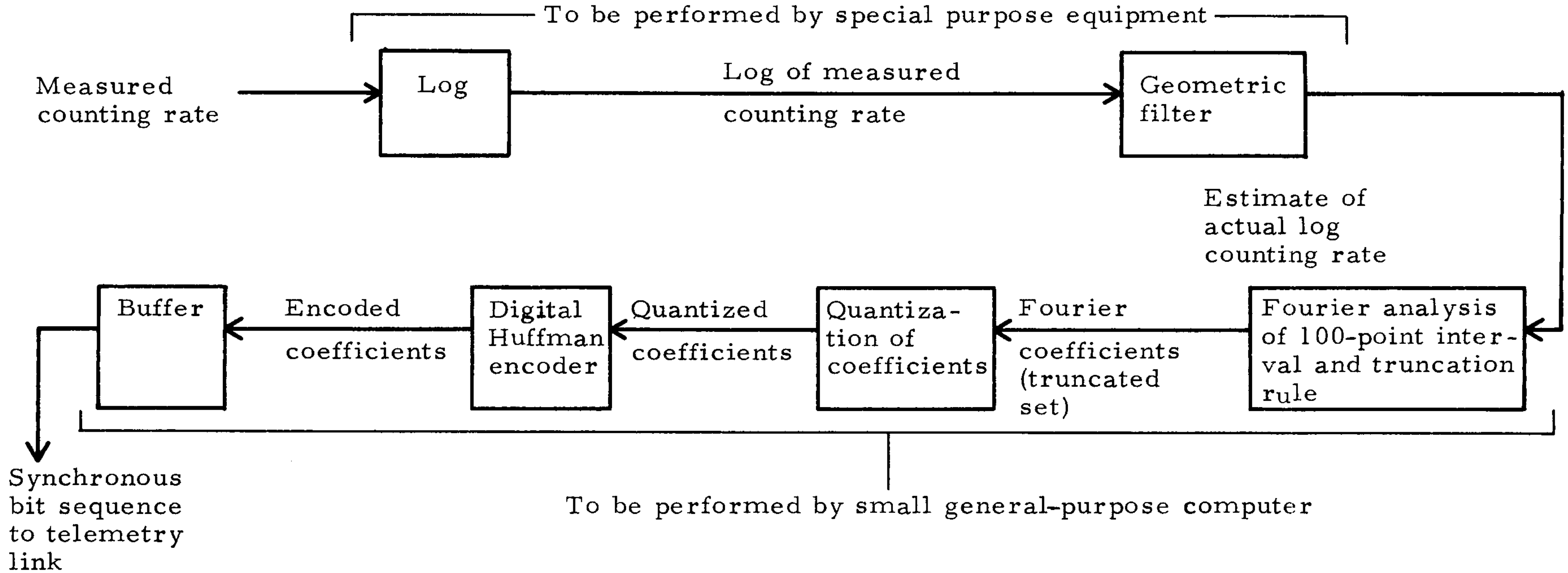

FIGURE 2: Functional Operations Performed by the Encoding Algorithm

to estimate the covariance function of the filtered data (this requires direct transmission of 1300 raw or filtered data points at the start of the experiment). The eigenfunctions of the 100 x 100 covariance matrix corresponding to the 20 largest eigenvalues are then found. In the encoding algorithm, these 20 orthogonal functions are stored in a general-purpose computer. As a block of 100 filtered data points is generated, it is also stored in the computer. The computer then finds the Fourier coefficients for each of these orthogonal functions by taking the inner (dot) product of the sample with each of the orthogonal functions. Only as many coefficients are computed as are necessary to represent the 100-point sample with the required accuracy. Note that the Karhunen-Loeve expansion gives Fourier coefficients that are uncorrelated random variables.

(3) Quantization. Once a 100-point sample has been expressed in terms of a number of Fourier coefficients, the numerical values of these coefficients must be quantized for telemetry transmission. It was pointed out earlier that we are interested in either transmission within a fixed tolerance with high probability, or in transmission with some specified mean square error. In both cases a good approximation to optimum encoding consists of quantization with the same quantization error of all coefficients up to a certain number and no transmission of the remaining coefficients. Thus, our algorithm quantizes all coefficients to be transmitted at the same level.

The total error occurring from truncation (using less than all 100 Fourier coefficients) and from quantizing those coefficients used must be kept less than the desired distortion level. As the number of coefficients used is increased and the truncation error decreases, the quantization error may increase with a resulting decrease of the number of binary digits needed per coefficient. An approximate rule is used in the algorithm to pick the number of coefficients transmitted for each 100-point interval so that the total number of binary digits required is minimum. The details of the procedure will not be described further here.

(4) Digital Encoding. Once a number of coefficients for a 100-point block has been chosen and quantized to the number of binary digits required for acceptable distortion, the binary M-tuple representing each coefficient (M is the largest number of binary digits required for suitable

quantization) can be encoded into a more efficient form by using a Huffman code.[6] The function of this code is to represent the more frequently occurring binary M-tuples by shorter binary words, so that the average number of binary digits required is minimized. Although this portion of the algorithm has not as yet been simulated on the digital computer, it merely requires looking up the codeword for any of the possible 2^M binary M-tuples (in our work M=7 is sufficient), and then generating this codeword. It appears that this function can be performed easily and quickly on a general purpose machine.

(5) Buffer Storage. As the coded words that represent the quantized coefficients are generated, they must be stored for transmission. This storage is necessary because the encoded binary digits are generated at a varying rate, while the telemetry channel transmits at a fixed rate. Simulation will be done in the immediate future to determine the storage capacity that is required by our encoding algorithm.

Preliminary Results

A first indication of the "compression ratio" that could be achieved was obtained by reconstruction of an acceptable signal from a truncated orthornormal expansion. We form an approximation s(t) every 100 ms

$$s(t) = \sum_{j=1}^{K} s_j \varphi_j(t)$$

The number of terms, K, is chosen so as to give an acceptable reproduction of the 100 data points. Two examples of the data and their reproduction are shown in Figures 3a and 3b. The number of coefficients needed for the reconstruction varies from 2 to 17 with an average of about 9 for the stretch of data used. The coefficients are efficiently quantized, and the quantized coefficients encoded in a Huffman code employing a block length of 7 binary digits. The final encoded version requires an average of 52.5 binary digits for a 100-point interval. This should be compared with with a rate of about 600 - 800 binary digits per 100-point interval that would be required if the experimenter were to transmit the raw data. We have not as yet compared the encoded rate against information theoretic bounds.

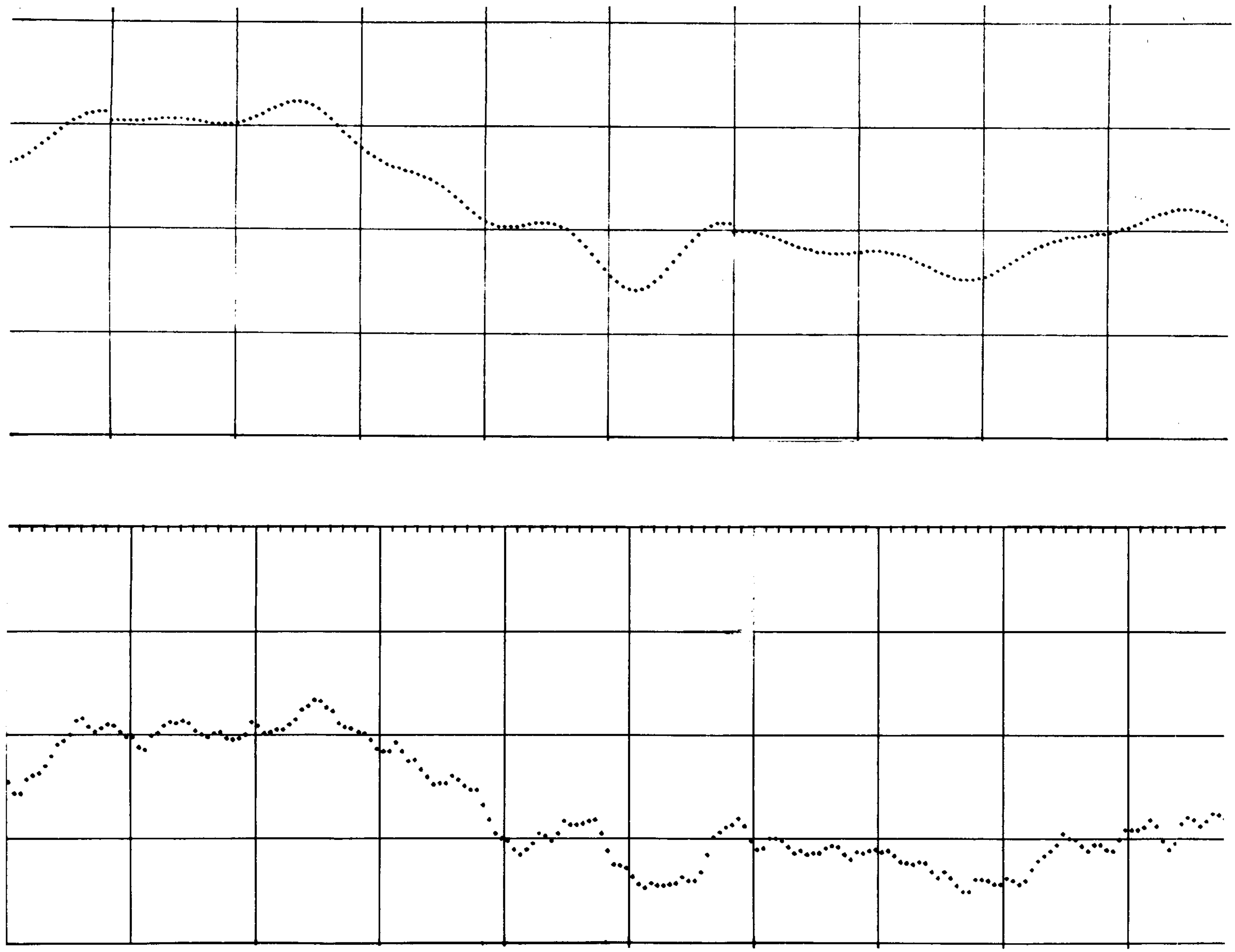

FIGURE 3-a. The filtered data (lower trace) and the data reconstructed from the "compressed" coefficients (upper trace).

Time: t = 430.630 to 430.830

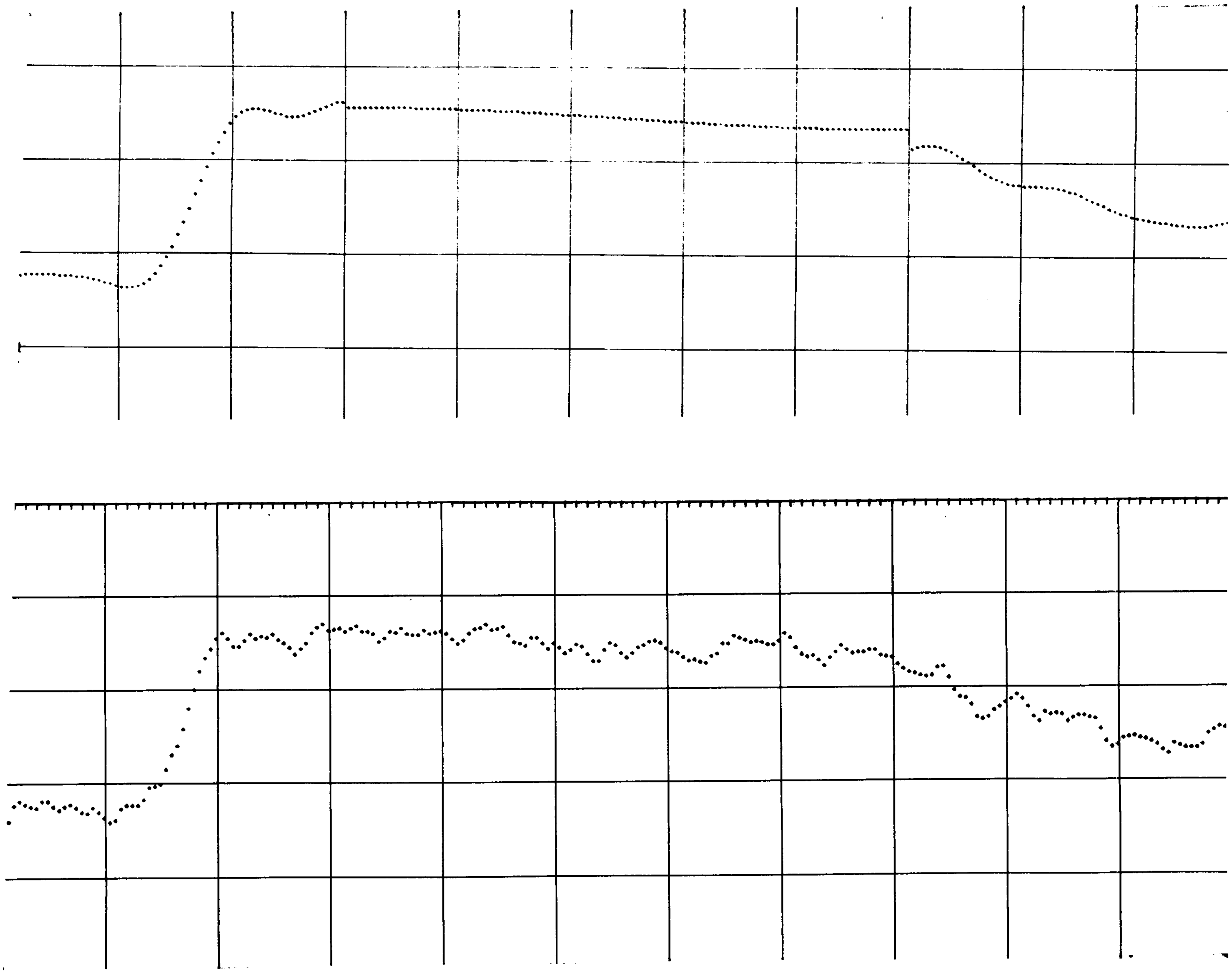

FIGURE 3-b. The filtered data (lower trace) and the data reconstructed from the "compressed" coefficients (upper trace).

Time: t = 430.990 to 431.210

INFORMATION THEORETIC RESULTS

In the process of considering how to encode Moser's counting-rate data, we have come across two basic information-theoretic problems concerning source encoding that have not been solved previously.

In all of Shannon's work[2, 7] it has been assumed that the message process to be encoded at the transmitter can be observed in an uncorrupted form. In many applications and, in particular, when the data source is a space experiment, this is not the case. It is desirable to make applicable to this more difficult problem as much of the existing theory of source encoding as is possible. We have been able to demonstrate the following: suppose we wish to transmit a random vector $\underset{\sim}{s}$, when we can observe a corrupted version of $\underset{\sim}{s}$, namely $\underset{\sim}{x}$. The vectors $\underset{\sim}{x}$ and $\underset{\sim}{s}$ can represent time functions or finite dimensional vectors. Let us assume that the distortion criterion is the mean square value of the square of the norm of the difference between $\underset{\sim}{s}$ and its encoded version $\underset{\sim}{\hat{\hat{s}}}$. Let $\underset{\sim}{\hat{s}}$ denote the best (mean square) estimate of $\underset{\sim}{s}$, based on observation of $\underset{\sim}{x}$. It has been shown that the problem of finding a code (mapping between $\underset{\sim}{x}$ and $\underset{\sim}{\hat{\hat{s}}}$) that minimizes the transmission rate for a fixed distortion, $E\left\{\|\underset{\sim}{s} - \underset{\sim}{\hat{\hat{s}}}\|^2\right\}$, can be reduced in <u>all</u> aspects to finding a mapping between $\underset{\sim}{\hat{s}}$ and $\underset{\sim}{\hat{\hat{s}}}$, which minimizes the transmission rate for a fixed value of $E\left\{\|\underset{\sim}{s} - \underset{\sim}{\hat{\hat{s}}}\|^2\right\}$. The only change that occurs in the case of a corrupted observation is the additional distortion occurring in estimation; i.e.,

$$E\left\{\|\underset{\approx}{s} - \underset{\approx}{\hat{\hat{s}}}\|^2\right\} = E\left\{\|\underset{\approx}{s} - \underset{\approx}{\hat{s}}\|^2\right\} + E\left\{\|\underset{\approx}{\hat{s}} - \underset{\approx}{\hat{\hat{s}}}\|^2\right\}$$

In all past work, the only distortion criterion that has received detailed attention for random processes is the mean integral square error. In many cases, such as Moser's counting-rate process or a video process, it is also crucial to preserve the slope of the signal accurately. We have found a rigorous derivation for the rate distortion function when the distortion measure is the mean integral square of an arbitrary linear operation on the error.

These results are being written and will be submitted for publication to a technical journal. Copies of the completed manuscript will be sent to NASA as soon as they are available.

FUTURE WORK

Short-Range Objectives

We plan to evaluate further the performance and the computing requirements of the present algorithm. It will be used on a long section of Moser's available data to check further the acceptability of the obtained reproduction.

Encoding the data for this entire long interval will also yield a more exact estimate of the average rate required by the scheme and of the "data compression" achieved. The average rate will be compared to the rate-distortion function (the minimum possible transmission rate for a given distortion level), calculated under the restrictive assumption that the process is gaussian and that the Fourier coefficients can be quantized independently of each other. To obtain a better estimate of the computing requirements, the algorithm will be used on a digital computer (IBM 1800, 8000 word storage, 16 bit word, 2 μsec cycle time), which is similar to what is currently being planned by NASA for use as an on-board computer. We shall then time the various parts of the algorithm in operation and assess the feasibility of implementing the algorithm on a small general-purpose machine.

Long-Range Objectives

(1) The current approach transmits to the ground an acceptable reproduction of the data output of the sensors. A further step would be on-board abstraction from the data of the information that is of direct interest to the experimenter, and transmission only of that information to the ground. This step would transfer to the satellite part of the decision-making process that is currently controlled by experimenters on the ground.

(2) The types of digital operations performed in the data-processing scheme are quite limited. Most of the computations are inner products of vectors. Further, the vectors are used in a specific order for which a direct access memory is not important. Inner products are handled clumsily by a general-purpose machine. It seems worthwhile to consider what kind of machine organization would be efficient in handling the types of computations that are involved in the encoding algorithm.

(3) Several basic information-theoretic problems require investigation. In particle-counting experiments, the information of interest is the rate of a time-varying Poisson process. Further work should be done on the estimation of this rate, the physical implementation of the estimation scheme, and the performance that can be thus achieved.

For the distortion measures encountered in space experimentation, a good lower bound to the rate-distortion function is of interest. Although the proposed scheme should be close to optimum, definite knowledge about the best scheme possible would be valuable.

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Professor D. Sakrison - Principal Investigators
Professor V. R. Algazi

Barry Haskell - Research Assistant