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# AN EXPLORATORY STUDY OF THE VORTEX SHEETS SHED FROM THE LEADING EDGES OF SLENDER WINGS

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## INTRODUCTION

The present report concerns efforts to extend slender-body theory for the purpose of accurately predicting the forces and moments associated with flow separation from the wing leading edges. The theory of Reference 1, which was developed to predict the flow field associated with leading-edge separation, suffers from two major shortcomings. First, the separated normal force is overpredicted, apparently because the local shedding rates are overpredicted, and second, the predicted vortex sheet shapes are quite unrealistic, producing highly irregular curves, particularly when large numbers of vortices are introduced.

The rate at which vorticity is shed from the wing leading edges is predicted in Reference 1 on the basis of the lateral velocity at the side edge of a two-dimensional plate. It would appear that such a model does not properly account for the fact that the span of the plate is a function of $x$ (or time), since the velocity potential in the cross-flow plane is unaffected by the lateral growth of the plate. That is, the potential depends only upon the local span at that station, not upon its rate of expansion. Therefore, the first two sections of the analysis deal with the vortex sheet shed from a two-dimensional plate whose width is a function of time.

The third portion of the analysis will be concerned with removing the irregularities in the vortex sheet shape which are produced as a result of the discrete vortex approximation. So long as one represents a continuous vortex sheet by a number of discrete vortices, high velocities will result when two vortices come close together, and the resulting sheet shape will be distorted. Furthermore, the larger the number of vortices used, the more difficult this problem becomes. Thus, convergence with number of vortices may be impaired. Therefore, a "smoothing" technique will be employed by which one forces the discrete vortices onto a smooth spiral at each step in the rolling-up process.

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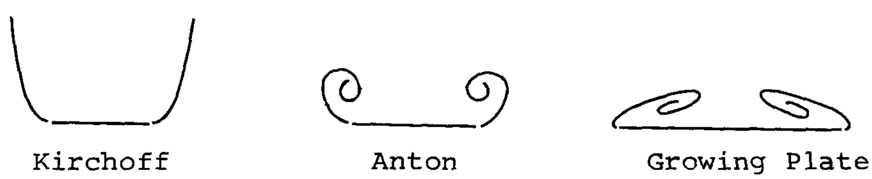
# 1. ANALYSIS

## 1.1 Force on a Growing Two-Dimensional Plate

After initial attempts to extend the classical wake solutions of Kirchoff (see Ref. 2) and Anton (Ref. 3) to the case of a two-dimensional growing plate, it became apparent that neither of these solutions is applicable to the present problem. In the case of the free-streamline (Kirchoff) solution, the appropriate assumption for a growing plate would be a growing dead-water region above the plate. This assumption leads to an increased shedding velocity and an increased force on the plate. A similar result is obtained from the solution of Anton (Ref. 3) for a pair of spiral vortex sheets. But in both cases, the assumed wake form is inconsistent with the physical picture of the wake behind a growing plate and the increased force is a direct consequence of the constraints placed on the wake shape. That is, both of the above models lead to the concept of a wake which expands with the plate. Actually, the growth of the plate will cause the vortex sheets to be flattened toward the leeward side of the plate, thus producing a smaller wake (see sketch).




In view of these findings, a more fundamental approach is required, as outlined in the following section.

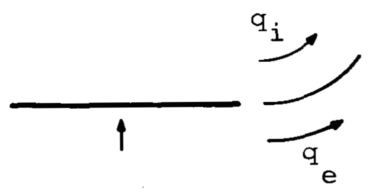
## 1.2 Unsteady Pressure Relation

If we wish to investigate the shedding of vorticity from a two-dimensional plate whose width is a function of time, the appropriate pressure relation is the unsteady Bernoulli equation; that is,

$$\phi_t + \frac{p}{\rho} + \frac{1}{2} q^2 + \Omega = f(t) \tag{1}$$

where $\phi_t$ is the time derivative of the potential, $q$ is the local fluid velocity, and $\Omega$ is a constant.

The function $f(t)$ is the same function everywhere in the flow field, provided that the region is simply connected; that is, so long as the wake is neither closed nor infinite in extent. Thus, applying Equation (1) to the internal and external sides of the vortex sheet (see sketch), we find




$$\left(\phi_{t_i} - \phi_{t_e}\right) + \frac{(p_i - p_e)}{\rho} + \frac{1}{2}\left(q_i^{\,2} - q_e^{\,2}\right) = 0 \tag{2}$$

But since the pressures on the two sides of the sheet must be equal, the second term vanishes and we have

$$\begin{aligned} \phi_{t_i} - \phi_{t_e} &= \frac{1}{2}\left(q_e^{\,2} - q_i^{\,2}\right) \\ &= \frac{1}{2}\,(q_e + q_i)(q_e - q_i) \end{aligned} \tag{3}$$

Further, we recognize $(q_e + q_i)/2$ as the tangential velocity of the sheet and $(q_e - q_i)$ as the vorticity of the sheet. If we denote these by $v_s$ and $\gamma$, respectively, we can write

$$\phi_{t_i} - \phi_{t_e} = v_s \cdot \gamma \tag{4}$$

Finally, the difference in $\phi_t$ across the sheet is equal to the time derivative of the jump in potential, which is the circulation. That is,

$$\phi_{t_i} - \phi_{t_e} = \Delta\phi_t = \frac{\partial}{\partial t}\,(\Delta\phi) = \frac{d\Gamma}{dt} \tag{5}$$

Hence, the final relation for the vortex sheet is

$$\frac{d\Gamma}{dt} = v_s \cdot \gamma \tag{6}$$

which relates the shedding rate to the shedding velocity and the vorticity of the sheet.

It will be noted in the above derivation that no assumption was made regarding the change of plate width with time, and that $q_i$ and $q_e$ refer to the absolute fluid velocities immediately above and below the sheet; that is, the fluid velocities are not relative to the expanding side edge of the plate. Furthermore, Equation(6) is precisely the relation used in the anal-ysis of Reference 1 in which $v_s$ was calculated directly from the potential, and $\gamma$ was calculated by assuming a flat vortex sheet of uniform strength which satisfies the Kutta condition at the local wing edge. Therefore, since the potential $\phi$ depends only on the local width of the plate (not on its rate of expansion), and since the complex velocity from which $v_s$ is extracted is obtained by differentiation in the plane $x$ = constant, it is concluded that, within the framework of slender-body theory, the analysis of Reference 1 does, in fact, properly account for the fact that the plate width is variable.

grammar

anal-y-sis

It will be recalled that the numerical calculations of Reference 1 were initiated by assuming that the shedding velocity of the first vortex pair is given by that of the steady free-streamline flow, namely,

$$v_s = V \frac{\alpha}{2}$$

One might expect that a better starting value would be obtained from a solution of the unsteady free-streamline flow for a growing plate, using Equation (1). However, it was found in the study of Reference 1 that the solution for slender wing-body combinations is quite insensitive to the starting value and therefore does not appear to warrant such a step.

1.3 Smoothing the Vortex Sheet with a Spiral Curve Fit

Several experimental investigations using various visual flow techniques (e.g., Refs. 4 and 5) have indicated that the primary vortex shed from the leading edge of a slender wing assumes the shape of a smooth spiral curve. This shape is closely approximated by the spiral equation (see Fig. 1).

$$R = \frac{A}{(\theta - \theta_o)^m} \quad (7)$$

where $m$ is a dimensionless exponent reflecting the rate at which the spiral approaches its center $(y_o, z_o)$ and $\theta_o$ is the asymptotic angle the spiral curve makes with the y-axis as $R$ becomes infinite.

The spiral form given by Equation (7) is also the form which arises in the theoretical treatments of Kaden (Ref. 6) and Anton (Ref. 3) which deal with the rolling-up of continuous vortex sheets. It would therefore seem desirable to pass a least-squares spiral curve of this form through the calculated vortex positions for the discrete vortex model of Reference 1. That is, at each chordwise station at which a vortex is introduced, one would fit all of the previously shed vortices onto a least-squares spiral curve by displacing them slightly before proceeding to the next station. This would prohibit the rather large distortions observed in the sheet shapes calculated in Reference 1, and might significantly affect the calculated forces due to separation.

In order to accomplish this "smoothing" of the vortex sheet, we consider the $n^{th}$ vortex to lie along a ray $\theta_n$ at a distance $r_n$ from the origin of the spiral. Then, the least-squares radial error from the mathematical spiral of Equation (7) is given by (Ref. 7)

$$I = \sum_{n=1}^{k} (r_n - R_n)^2 \quad (8)$$

where

$$r_n = \sqrt{(y_n - y_o)^2 + (z_n - z_o)^2}$$

and $k$ is the number of vortices (which must exceed the number of constants involved). The point on the spiral curve corresponding to the $n^{th}$ vortex is given by

$$R_n = \frac{A}{(\theta_n - \theta_o)^m} \quad (9)$$

where

$$\theta_n = \tan^{-1}\left(\frac{z_n - z_o}{y_n - y_o}\right)$$

Note that the quantity $(r_n - R_n)$ is a function of the five free parameters A, m, $y_o$, $z_o$, $\theta_o$. We must therefore minimize the error I, with respect to each of these and determine their values by a standard least-squares procedure (see Ref. 7). This involves differentiating the error with respect to each parameter, setting the derivatives all to zero, linearizing the resulting equations, and solving by iteration.

The number of iterations required in this procedure depends strongly on the accuracy of the first guess, particularly for the parameter m. If the first guess is not within a reasonable tolerance (say $\pm$25 to 50 percent), then solutions cannot be expected. Furthermore, it is possible to have more than one solution in such nonlinear problems. For these reasons, four sample cases were run (using six vortices) on the IBM 1620 which bracket the range of interest (aspect ratio 1 and 2 for $\alpha = 10^\circ$ and $20^\circ$). This was done to ensure suitable starting values beginning with six vortices in the field (k = number of parameters plus one). These truncated cases indicated that convergence was exceedingly difficult to achieve if $\theta_o$ is treated as an unknown. It was therefore considered expedient to fix its value and solve for the other four parameters. Best results were obtained by setting $\theta_o = -\pi/2$ (see Fig. 1).

The initial guesses for the remaining four parameters at each chordwise station thereafter $(k > 6)$ were taken to be the converged values at the previous station, except for the parameter A. The initial guess for A at each chordwise station was determined by assuming that the spiral passes through the vortex just shed at that station.

## 2. CALCULATIONS

In order to investigate the effect of the curve-fitting or "smoothing" of the vortex sheet shape on the calculated forces and moments on slender wings, a sample calculation was carried out on the IBM 7094 for a delta wing of aspect ratio 1.0 at $20^\circ$ angle of attack. The calculation was performed using 48 vortices, with the curve fit applied at each chordwise station to all vortices shed ahead of that station. A comparison of the calculated vortex sheet shapes with and without the spiral curve fit is shown in Figure 2, and the improvement is apparent. On the other hand, neither the calculated normal force nor the rate of convergence is significantly affected (see Fig. 3).

## 3. CONCLUSIONS

The present investigation has been carried out in an attempt to improve the theory of Reference 1 for the prediction of normal force and pitching moment on slender wing-body combinations exhibiting leading-edge separation. Two apparent shortcomings of the theory have been investigated. The first is the prediction of vorticity shed from a growing plate, and the second is the prediction of a smooth spiral vortex sheet.

A detailed derivation of the shedding rate from a growing two-dimensional plate indicates that the analysis of Reference 1 does, in fact, properly account for the rate of change of wing span with $x$, subject to the two assumptions made in Reference 1; namely: (1) a very slender configuration, and (2) a flat vortex sheet segment of uniform vorticity shed at each station.

A least-squares spiral curve fit has been applied to the calculated positions of the shed vortices at each chordwise station, with the result that the predicted vortex sheet shape closely resembles the smooth spiral curve observed experimentally. However, the effect on the calculated normal force and center of pressure on slender wings appears to be insignificant. The rate of convergence with number of vortices is similarly unaffected by this modification.

It is therefore concluded that the overprediction of the shedding rate and the corresponding overprediction of the normal force for wings of finite aspect ratio is brought about by three-dimensional effects which cannot be handled within the framework of slender-body theory. One must evidently, therefore, resort to a truly three-dimensional theory if one is to achieve more accurate predictions than those of Reference 1 for the forces produced by leading-edge separation.

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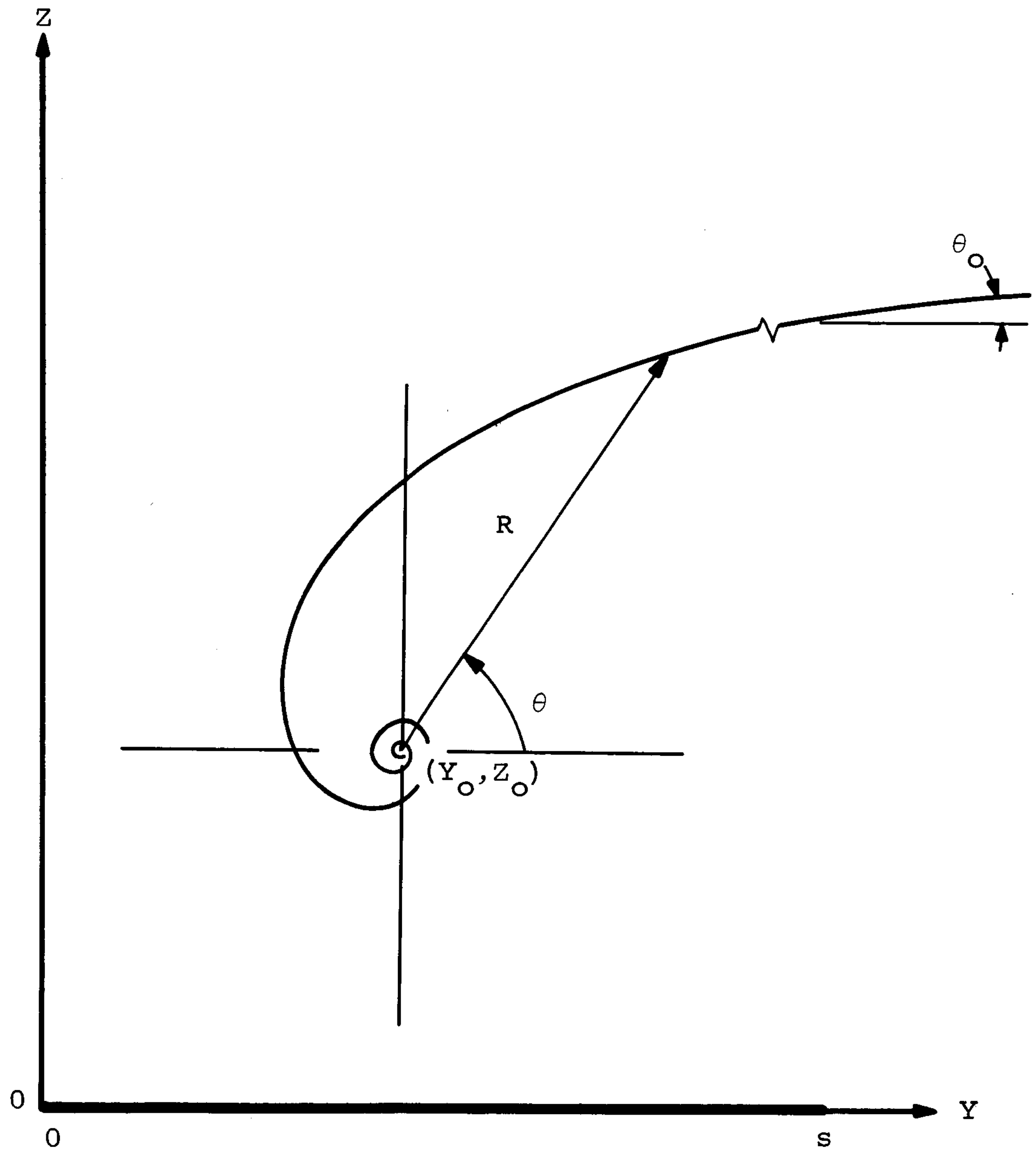
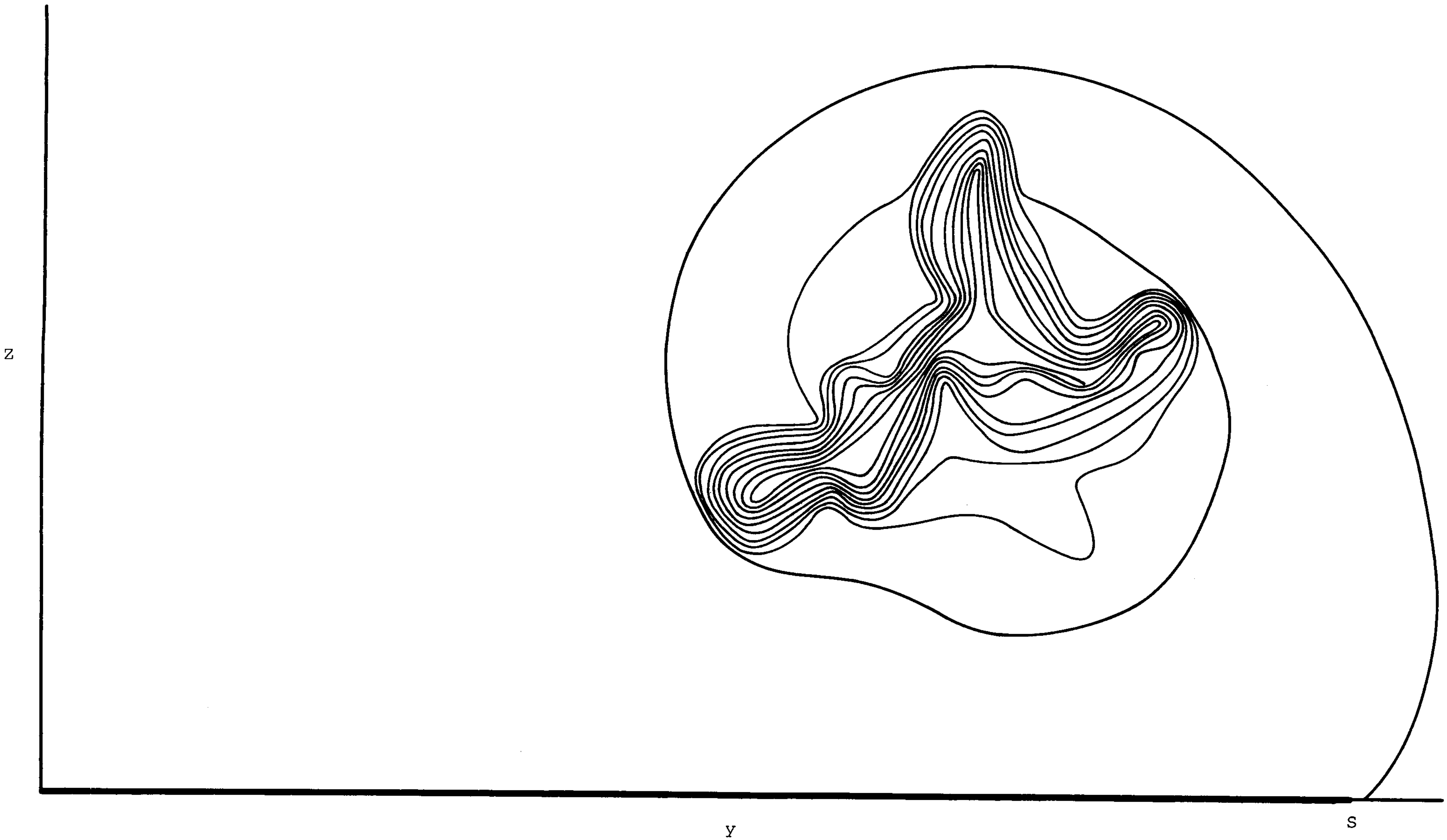
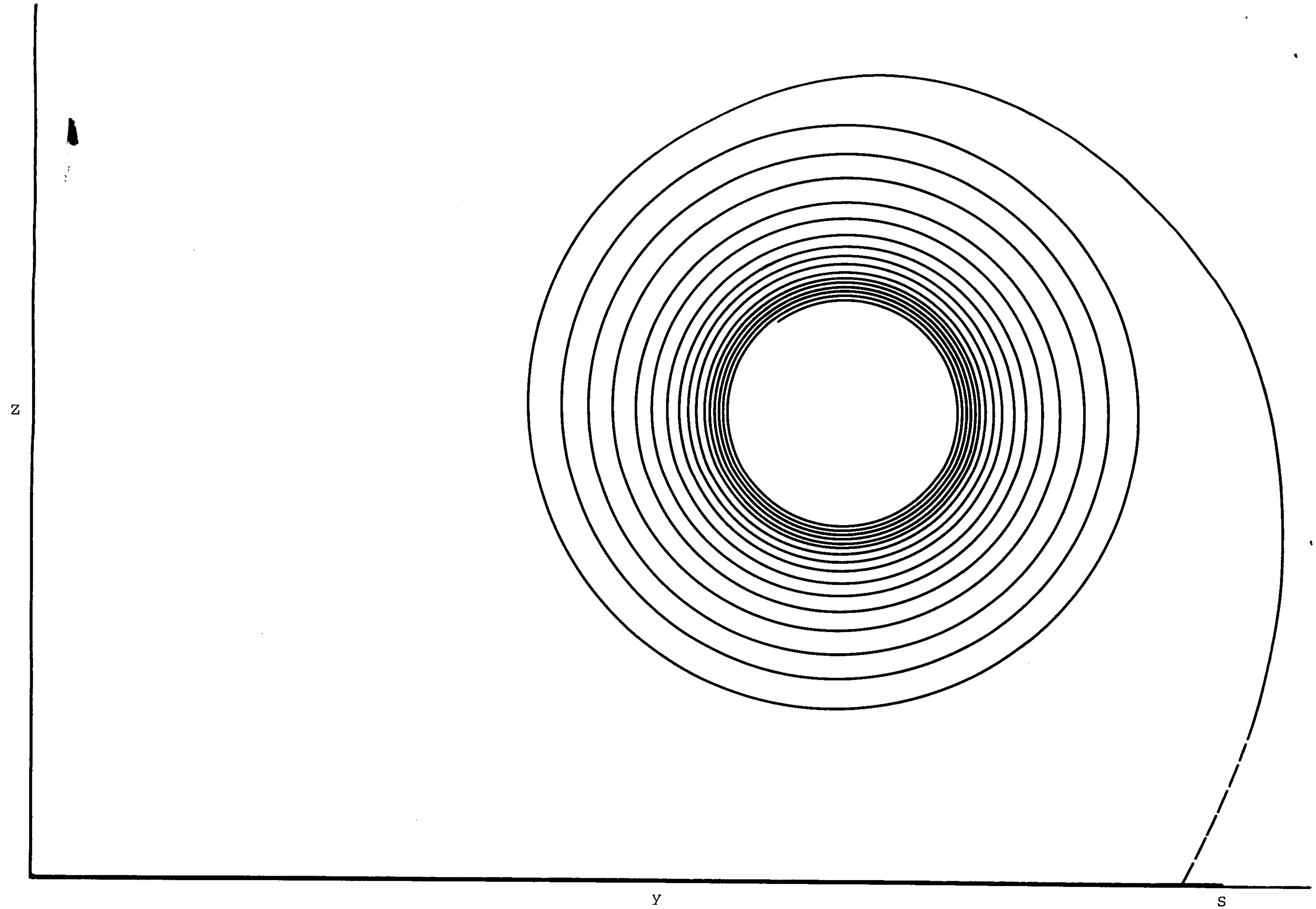



Figure 1.- Mathematical spiral given by Equation (7).




(a) Uncorrected.

Figure 2.- Trailing-edge vortex sheet shape for aspect ratio 1.0 delta wing at $\alpha = 20^{\circ}$.




(b) Corrected by least-squares spiral curve fit.

Figure 2.- Concluded.

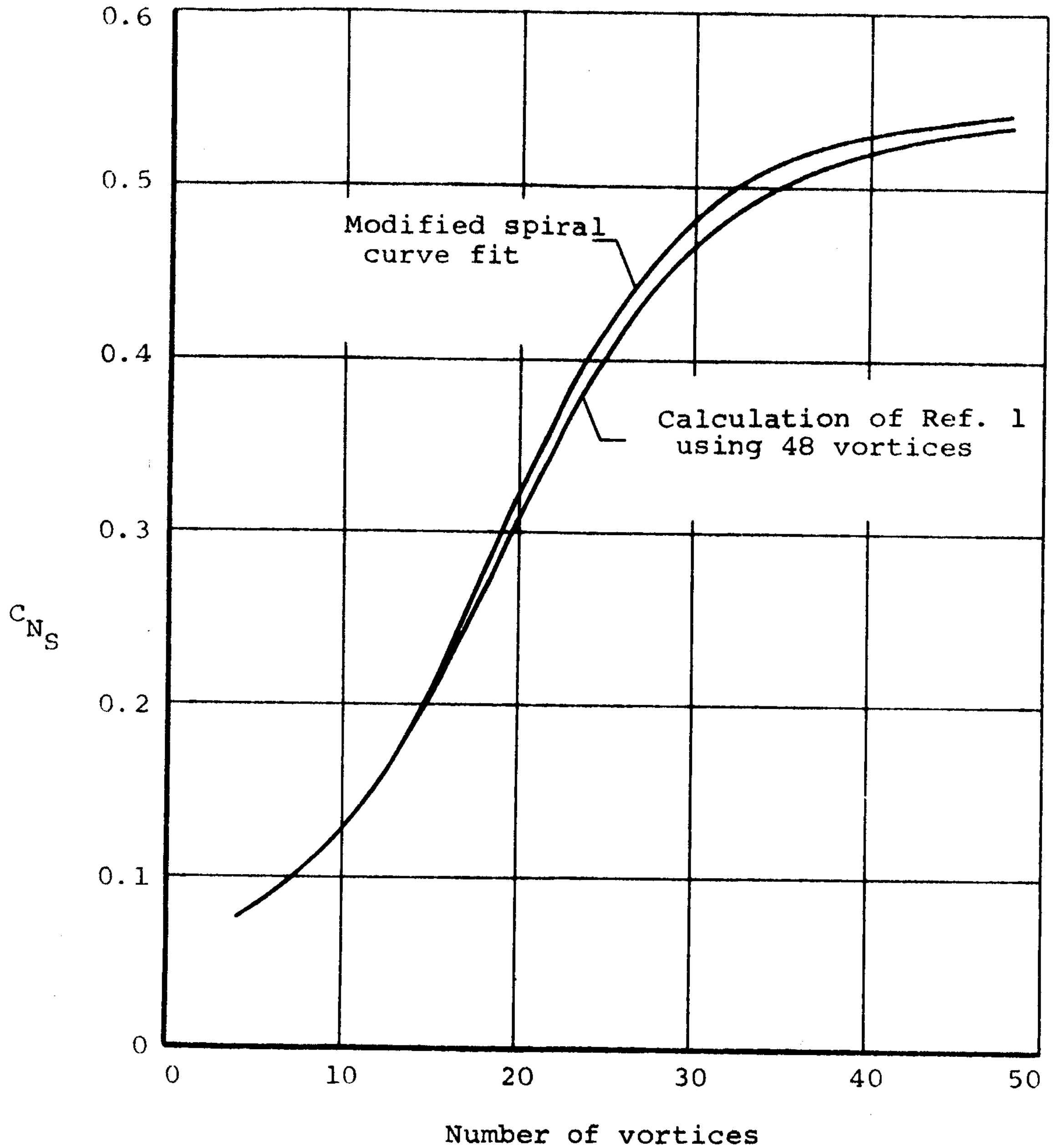



Figure 3.- Effect of spiral curve fit on calculated separation normal force for a delta wing of aspect ratio 1.0 at $\alpha = 20^{\circ}$.