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MAXIMUM LIFT-TO-DRAG RATIO OF A SLENDER, FLAT-TOP, HYPERSONIC BODY

by

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SUMMARY

An investigation of the lift-to-drag ratio attainable by a slender, flat-top, homothetic body flying at hypersonic speeds is presented under the assumptions that the pressure distribution is modified Newtonian and the surface-averaged skin-friction coefficient is constant.

It is shown that a value of the thickness ratio exists such that the lift-to-drag ratio is a maximum; this particular value is such that the friction drag is one-third of the total drag. The subsequent optimization of the longitudinal and transversal contours is reduced to the extremization of products of powers of integrals related to the lift, the pressure drag, and the skin-friction drag. With regard to the longitudinal contour, a conical solution is the best. With regard to the transversal contour,

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the optimum solution is triangular without or with a vertical keel at midsection depending on whether the cross-sectional elongation ratio μ is smaller or larger than the critical value $\mu = 0.206$.

The lift-to-drag ratio of the optimum body increases as the elongation ratio of the cross section decreases; for a Newtonian pressure distribution and a surface-averaged skin-friction coefficient $C_f = 10^{-3}$, the highest attainable lift-to-drag ratio is $E = 5.29$.

1. INTRODUCTION

In a previous paper (Ref. 2), an investigation of the lift-to-drag ratio attainable by a slender, flat-top, homothetic body at hypersonic speeds was presented under the assumptions that the pressure distribution is Newtonian and the surface-averaged skin-friction coefficient is constant. Direct methods were employed, and the analysis was confined to the class of bodies whose longitudinal contour is a power law and whose transversal contour is semielliptical or triangular. For these special bodies, the lift-to-drag ratio depends on three parameters: the thickness ratio, the exponent of the power law, and the elongation ratio of the cross section. Therefore, by means of the theory of maxima and minima, the combination of parameters maximizing the lift-to-drag ratio can be found.

In this paper, the limitations set forth in Ref. 2 are removed, and the indirect methods of the calculus of variations are employed in order to determine the optimum longitudinal and transversal contours. The hypotheses employed are as follows: (a) a plane of symmetry exists between the left-hand and right-hand sides of the body; (b) the upper surface of the body is flat (reference plane); (c) the base plane is perpendicular to both the plane of symmetry and the reference plane; (d) the body is slender in the longitudinal sense, that is, the square of the slope of any meridian contour is small with respect to one; (e) the body is homothetic, in the sense that each cross section is geometrically similar to the base cross section and has the same orientation; (f) the free-stream velocity is perpendicular to the base plane and, therefore, is parallel to the line of intersection of the plane of symmetry and the reference plane; (g) the pressure coefficient is proportional to the cosine squared of the angle formed by the free-stream velocity and the normal to each surface element; (h) the surface-averaged skin-friction coefficient is constant; (i) the contribution of the tangential forces to the lift is negligible with respect to the contribution of the normal forces; and (j) the base drag is neglected.

2. DRAG AND LIFT

We consider the class of flat-top bodies and define two coordinate systems (Fig. 1): a Cartesian coordinate system Oxyz and a cylindrical coordinate system Oxrθ. For the Cartesian coordinate system, the origin O is the apex of the body; the x-axis is the intersection of the plane of symmetry and the reference plane, positive toward the base; the z-axis is contained in the plane of symmetry, perpendicular to the x-axis, and positive downward; and the y-axis is such that the xyz-system is right-handed. For the cylindrical coordinate system, r is the distance of any point from the x-axis, and θ measures the angular position of the vector $\vec{r}$ with respect to the xy-plane.

Next, we focus our attention on those bodies $r(x, \theta)$ such that any transversal contour is geometrically similar to that of the base and has the same orientation. The geometry of these homothetic bodies is given by (Ref. 2)

$$r = \ell(\tau/\mu) A(\xi) B(\theta) \tag{1}$$

where ℓ denotes the length and where

$$\tau = \frac{r(\ell, \pi/2)}{\ell} \quad , \quad \mu = \frac{r(\ell, \pi/2)}{r(\ell, 0)} \tag{2}$$

are the longitudinal thickness ratio and the elongation ratio of the cross section, respectively; also, $\xi = x/\ell$ is a nondimensional abscissa, $A(\xi)$ a function describing the longitudinal contour such that

$$A(0) = 0 \quad , \quad A(1) = 1 \tag{3}$$

and $B(\theta)$ a function describing the transversal contour such that

$$B(0) = 1 \quad , \quad B(\pi/2) = \mu \tag{4}$$

With this understanding and in the light of the hypotheses of the introduction, the drag D and the lift L can be rewritten as (Ref. 2)

$$\begin{aligned} D/q\ell^2 &= n\tau^4 I_1 J_1 + C_f \tau I_2 J_2 \\ L/q\ell^2 &= n\tau^3 I_3 J_3 \end{aligned} \tag{5}$$

where q is the free-stream dynamic pressure, n a factor modifying the Newtonian pressure distribution, and C_f the surface-averaged skin-friction coefficient. In Eqs. (5), the positive quantities I_1, I_2, I_3 are defined as

$$I_1 = \int_0^1 A\dot{A}^3 d\xi \quad , \quad I_2 = \int_0^1 A\, d\xi \quad , \quad I_3 = \int_0^1 A\dot{A}^2 d\xi \tag{6}$$

where $\dot{A} = dA/d\xi$. Also, the positive quantities J_1, J_2, J_3 are defined as

$$J_1 = (4/\mu^4)K_1 \quad , \quad J_2 = (2/\mu)K_2 \quad , \quad J_3 = (4/\mu^3)K_3 \tag{7}$$

where

$$\begin{aligned} K_1 &= \int_0^{\pi/2} \left[B^6/(B^2 + \dot{B}^2)\right] d\theta \\ K_2 &= \int_0^{\pi/2} \left[2/\pi + (B^2 + \dot{B}^2)^{1/2}\right] d\theta \\ K_3 &= \int_0^{\pi/2} \left[B^4/(B^2 + \dot{B}^2)\right](B \sin\theta - \dot{B}\cos\theta)\, d\theta \end{aligned} \tag{8}$$

and $\dot{B} = dB/d\theta$.

3. LIFT-TO-DRAG RATIO

From the previous formulas, it appears that--if the length ℓ, the thickness ratio τ, the longitudinal contour $A(\xi)$, and the transversal contour $B(\theta)$ are given--the drag and the lift can be evaluated from Eqs. (5) through (8). Once these quantities are known, one can determine the aerodynamic efficiency or lift-to-drag ratio

$$E = L/D \tag{9}$$

which, in the light of Eqs. (5), can be written as

$$E = n\tau^2 I_3 J_3/(n\tau^3 I_1 J_1 + C_f I_2 J_2) \tag{10}$$

4. OPTIMUM THICKNESS RATIO

We now assume that the longitudinal contour $A(\xi)$ and the transversal contour $B(\theta)$ are arbitrarily prescribed, and study the effect of the thickness ratio τ on the lift-to-drag ratio (10). Clearly, the lift-to-drag ratio is an extremum when the thickness ratio satisfies the relationship

$$E_\tau = 0 \tag{11}$$

whose explicit form

$$\tau\sqrt[3]{n/C_f} = \sqrt[3]{2(I_2/I_1)(J_2/J_1)} = \mu\sqrt[3]{(I_2/I_1)(K_2/K_1)} \tag{12}$$

means that the friction drag is one-third of the total drag. The associated lift-to-drag ratio is given by

$$E\sqrt[3]{C_f/n} = \sqrt[3]{(4/27)(I_3^3/I_1^2 I_2)(J_3^3/J_1^2 J_2)} = (2/3)\sqrt[3]{(I_3^3/I_1^2 I_2)(K_3^3/K_1^2 K_2)} \tag{13}$$

and is a maximum with respect to weak variations of the thickness ratio τ owing to the fact that $E_{\tau\tau} < 0$. Also, it is a maximum with respect to strong variations since $\Delta E < 0$ for every change $\Delta\tau$.

5. OPTIMUM LONGITUDINAL CONTOUR

Next, we consider bodies optimized with respect to the thickness ratio τ, assume that the transversal contour $B(\theta)$ is arbitrarily prescribed, and study the effect of the longitudinal contour $A(\xi)$ on the lift-to-drag ratio (13). Since the lift-to-drag ratio depends on the longitudinal contour through the expression

$$I = I_3^3/I_1^2 I_2 \tag{14}$$

we formulate the following problem: "In the class of functions $A(\xi)$ which satisfy the end conditions (3), find that particular function which extremizes the functional (14), where the integrals I_1, I_2, I_3 are defined by Eqs. (6)."

The functional (14) is a product of powers of integrals whose end points are fixed and is governed by the theory set forth in Ref. 3. In this reference, it is shown that the previous problem is equivalent to that of extremizing the integral

$$\tilde{I} = \int_0^1 F(A, \dot{A}, \lambda_1, \lambda_2)\, d\xi \tag{15}$$

where the fundamental function is defined as

$$F = A(\dot{A}^2 - \lambda_1 A^3 - \lambda_2) \tag{16}$$

and the undetermined, constant Lagrange multipliers are given by

$$\lambda_1 = 2I_3/3I_1 \quad , \quad \lambda_2 = I_3/3I_2 \tag{17}$$

Since the fundamental function does not contain the independent variable explicitly, standard methods of the calculus of variations show that the Euler equation

$$dF_{\dot{A}}/d\xi - F_A = 0 \tag{18}$$

admits the following first integral (see, for instance, Chapter 1 of Ref. 4):

$$F - \dot{A}F_{\dot{A}} = C \tag{19}$$

whose explicit form is

$$A(2\lambda_1\dot{A}^3 - \dot{A}^2 - \lambda_2) = C \tag{20}$$

Upon integrating Eq. (20) over the range 0, 1 and accounting for the definitions (6), we obtain the relationship

$$2\lambda_1 I_1 - I_3 - \lambda_2 I_2 = C \tag{21}$$

which is consistent with Eqs. (17) providing the integration constant has the value

$$C = 0 \tag{22}$$

Consequently, the differential equation of the extremal arc (20) becomes

$$2\lambda_1\dot{A}^3 - \dot{A}^2 - \lambda_2 = 0 \tag{23}$$

and implies that

$$\dot{A} = C_1 \tag{24}$$

where C_1 is a constant. Upon integrating this differential equation, we obtain the relationship

$$A = C_1 \xi + C_2 \tag{25}$$

where, because of the end conditions (3), the constants take the values

$$C_1 = 1 \quad , \quad C_2 = 0 \tag{26}$$

In conclusion, the optimum longitudinal contour is described by

$$A = \xi \tag{27}$$

and, therefore, is conical. For this cone, the integrals (6) take the values

$$I_1 = I_2 = I_3 = 1/2 \tag{28}$$

and the Lagrange multipliers (17) are given by

$$\lambda_1 = 2/3 \quad , \quad \lambda_2 = 1/3 \tag{29}$$

Finally, the optimum thickness ratio (12) and the lift-to-drag ratio (13) become

$$\tau \sqrt[3]{n/C_f} = u \sqrt[3]{K_2/K_1} \qquad , \qquad E \sqrt[3]{C_f/n} = (2/3) \sqrt[3]{K_3^3/K_1^2 K_2} \tag{30}$$

The lift-to-drag ratio (30-2) is a maximum with respect to weak variations of the slope $\dot{A}$ owing to the fact that the Legendre condition $F_{\dot{A}\dot{A}} \leq 0$ is satisfied. Also, it is a maximum with respect to strong variations since the Weierstrass condition $\Delta F - F_{\dot{A}} \Delta \dot{A} \leq 0$ is satisfied for every change $\Delta\dot{A}$ leading from the extremal slope to any comparison slope such that $0 \leq \dot{A} \leq \infty$.

6. OPTIMUM TRANSVERSAL CONTOUR

Finally, we consider configurations optimized with respect to the thickness ratio τ and the longitudinal contour $A(\xi)$, and study the effect of the transversal contour $B(\theta)$ on the lift-to-drag ratio (30-2). Since the lift-to-drag ratio depends on the transversal contour through the expression

$$K = K_3{}^3/K_1{}^2K_2 \tag{31}$$

we formulate the following problem: "In the class of functions $B(\theta)$ which satisfy the end conditions (4), find that particular function which extremizes the functional (31), where the integrals K_1, K_2, K_3 are defined by Eqs. (8)."

For each given elongation ratio μ, the functional (31) is a product of powers of integrals whose end points are fixed and is governed by the theory set forth in Ref. 3. Therefore, the previous problem is equivalent to that of extremizing the integral

$$\tilde{K} = \int_0^{\pi/2} F(\theta, B, \dot{B}, \lambda_1, \lambda_2)\, d\theta \tag{32}$$

where the fundamental function is defined as

$$F = \left[B^4/(B^2 + \dot{B}^2)\right](B \sin\theta - \dot{B}\cos\theta) - \lambda_1\left[B^6/(B^2+\dot{B}^2)\right] - \lambda_2\left[2/\pi + (B^2+\dot{B}^2)^{1/2}\right] \tag{33}$$

and the undetermined, constant Lagrange multipliers are given by

$$\lambda_1 = 2K_3/3K_1 \quad , \quad \lambda_2 = K_3/3K_2 \tag{34}$$

The extremal solution is described by the Euler equation (see, for instance, Chapter 1 of Ref. 4)

$$dF_{\dot{B}}/d\theta - F_B = 0 \tag{35}$$

which, in explicit form, is given by

$$\ddot{B} = \frac{\lambda_1 P_1(B,\dot{B}) + \lambda_2 P_2(B,\dot{B}) + P_3(\theta,B,\dot{B})}{\lambda_1 Q_1(B,\dot{B}) + \lambda_2 Q_2(B,\dot{B}) + Q_3(\theta,B,\dot{B})} \tag{36}$$

where

$$\begin{aligned} P_1 &= 2B^4(2B^4 + 7B^2\dot{B}^2 + 9\dot{B}^4) \\ P_2 &= (B^2 + 2\dot{B}^2)(B^2 + \dot{B}^2)^{3/2} \\ P_3 &= -2B^2\left[(B^4 + 5B^2\dot{B}^2 + 8\dot{B}^4)\,B\sin\theta + (B^4 + B^2\dot{B}^2 - 4\dot{B}^4)\,\dot{B}\cos\theta\right] \end{aligned} \tag{37}$$

and

$$\begin{aligned} Q_1 &= 2B^5(3\dot{B}^2 - B^2) \\ Q_2 &= B(B^2 + \dot{B}^2)^{3/2} \\ Q_3 &= 2B^3\left[(B^2 - 3\dot{B}^2)B\sin\theta - (3B^2 - \dot{B}^2)\dot{B}\cos\theta\right] \end{aligned} \tag{38}$$

There is no method known to these authors for obtaining the general integral of the differential equation and, as a consequence, numerical integration is necessary. Prior to undertaking this task, these authors have investigated the possibility that the triangular contour described by (Fig. 2-top)

$$B = \mu/(\sin\theta + \mu\cos\theta) \tag{39}$$

might be a particular solution(*). That this is the case can be shown with the following reasoning. First of all, the triangular contours (39) satisfy the end conditions (4). Next, the evaluation of the integrals (8) yields the relationships

$$
\begin{aligned}
K_1 &= \mu^3/(1+\mu^2) \\
K_2 &= 1 + \sqrt{1+\mu^2} \\
K_3 &= \mu^2/(1+\mu^2)
\end{aligned}
\tag{40}
$$

so that the Lagrange multipliers (34) are given by

$$
\begin{aligned}
\lambda_1 &= 2/3\mu \\
\lambda_2 &= \mu^2/3(1+\mu^2)(1+\sqrt{1+\mu^2})
\end{aligned}
\tag{41}
$$

Then, by direct substitution into Eq. (36), it can be verified that the assumed optimum contour (39) and the associated multipliers (41) reduce this Euler equation to an identity regardless of the cross-sectional elongation ratio μ. Consequently, the thickness ratio (30-1) and the lift-to-drag ratio (30-2) become (Figs. 3 and 4)

$$
\begin{aligned}
\tau\sqrt[3]{n/C_f} &= \sqrt[3]{(1+\mu^2)(1+\sqrt{1+\mu^2})} \\
E\sqrt[3]{C_f/n} &= 2/3\sqrt[3]{(1+\mu^2)(1+\sqrt{1+\mu^2})}
\end{aligned}
\tag{42}
$$

(*) The excellent aerodynamic qualities of bodies of triangular cross section are suggested by the theoretical analysis of Ref. 2 and the experimental results of Ref. 5.

The lift-to-drag ratio (42-2) is a maximum with respect to weak variations of the slope $\dot{B}$ providing the Legendre condition $F_{\dot{B}\dot{B}} \leq 0$ is satisfied; this is precisely the case for $\mu \leq 0.651$. Also, it is a maximum with respect to strong variations providing the Weierstrass condition $\Delta F - F_{\dot{B}} \Delta\dot{B} \leq 0$ is satisfied for every change $\Delta\dot{B}$ leading from the extremal slope to any comparison slope in the range $-\infty \leq \dot{B} \leq \infty$; this is the case for $\mu \leq 0.206$. Therefore, we conclude that the triangular solution (39) optimizes the transversal contour providing the cross-sectional elongation ratio is such that $\mu \leq 0.206$.

In order to find the optimum transversal contour for $\mu > 0.206$, we reformulate the variational problem by requiring that the extremal solution $B(\theta)$ be internal to the rectangle formed by the straight lines

$$\theta = 0 \quad , \quad \theta = \pi/2 \quad , \quad B \cos\theta = 1 \quad , \quad B \sin\theta = \mu \qquad (43)$$

If this restriction is communicated to the variational problem through the two-sided inequality constraints

$$0 \leq B \cos\theta \leq 1 \quad , \quad 0 \leq B \sin\theta \leq \mu \qquad (44)$$

we find that the extremal arc may include subarcs defined by the Eq. (36) and subarcs defined by Eqs. (43). While several ways exist for combining these subarcs, the subsequent investigation of the corner condition, the Legendre condition, and the Weierstrass condition allows one to exclude all possibilities except one, that of a triangular contour of effective elongation ratio $\mu_e < \mu$ with a vertical keel at midsection (Fig. 2-bottom). Analytically, this contour includes the subarc

$$B = \mu_e/(\sin\theta + \mu_e \cos\theta) \quad , \quad 0 \le B \sin\theta \le \mu_e \tag{45}$$

and the subarc

$$\theta = \pi/2 \quad , \quad \mu_e \le B \sin\theta \le \mu \tag{46}$$

As a consequence, the nondimensional integrals (8) become

$$\begin{aligned} K_1 &= \mu_e^3/(1+\mu_e^2) \\ K_2 &= 1 + \mu - \mu_e + \sqrt{1+\mu_e^2} \\ K_3 &= \mu_e^2/(1+\mu_e^2) \end{aligned} \tag{47}$$

the Lagrange multipliers (34) are given by

$$\begin{aligned} \lambda_1 &= 2/3\mu_e \\ \lambda_2 &= \mu_e^2/3(1+\mu_e^2)(1+\mu-\mu_e+\sqrt{1+\mu_e^2}) \end{aligned} \tag{48}$$

while the thickness ratio (30-1) and the lift-to-drag ratio (30-2) become

$$\begin{aligned} \tau\sqrt[3]{n/C_f} &= (\mu/\mu_e)\sqrt[3]{(1+\mu_e^2)(1+\mu-\mu_e+\sqrt{1+\mu_e^2})} \\ E\sqrt[3]{C_f/n} &= 2/3\sqrt[3]{(1+\mu_e^2)(1+\mu-\mu_e+\sqrt{1+\mu_e^2})} \end{aligned} \tag{49}$$

In these relationships, the effective elongation ratio μ_e is unknown and must be determined so that the corner condition

$$\Delta(F - \dot{B}F_{\dot{B}})\,\delta\theta + \Delta F_{\dot{B}}\,\delta B = 0 \tag{50}$$

is satisfied at the junction of the subarcs (45) and (46). Since the value of θ is specified, the variation $\delta\theta$ vanishes, and Eq. (50) supplies the condition

$$\Delta F_{\dot{B}} = 0 \tag{51}$$

which is equivalent to

$$2\mu_e^3(1 - \lambda_1/\mu_e) - \lambda_2(1 + \mu_e^2)^2(1 - \mu_e/\sqrt{1 + \mu_e^2}) = 0 \tag{52}$$

Upon combining Eqs. (48) and (52) and eliminating the Lagrange multipliers, we obtain the relationship (Fig. 5)

$$\mu = (\sqrt{1 + \mu_e^2} - \mu_e)(\sqrt{1 + \mu_e^2} - 2\mu_e)/2\mu_e - 1 \tag{53}$$

which implicitly determines the function $\mu_e(\mu)$. Once this function is known, the optimum thickness ratio (Fig. 3) and the maximum lift-to-drag ratio (Fig. 4) can be calculated using Eqs. (49-1) and (49-2)(*). Incidentally, the solution represented by Eqs. (45) and (46) satisfies both the Legendre condition and the Weierstrass condition as long as $\mu > 0.206$.

(*) The thickness ratio τ plotted in Fig. 3 is that of the body-keel combination. The effective thickness ratio of the body alone τ_e is related to τ through the relationship $\tau_e = (\mu_e/\mu)\tau$.

7. DISCUSSION AND CONCLUSIONS

In the previous sections, the optimization of the lift-to-drag ratio of a slender, flat-top, homothetic body flying at hypersonic speeds is presented under the assumptions that the pressure distribution is modified Newtonian and the surface-averaged skin-friction coefficient is constant.

It is shown that a value of the thickness ratio exists which maximizes the lift-to-drag ratio; this particular value is such that the friction drag is one-third of the total drag. The subsequent optimization of the longitudinal and transversal contours is reduced to the extremization of products of powers of integrals related to the lift, the pressure drag, and the skin-friction drag. With regard to the longitudinal contour, a conical solution is the best. With regard to the transversal contour, the optimum solution is triangular without or with a vertical keel at midsection depending on whether the elongation ratio of the cross section is smaller or larger than the critical value $\mu = 0.206$.

The lift-to-drag ratio of the optimum body depends on the factor modifying the Newtonian pressure distribution, the surface-averaged skin-friction coefficient, and the elongation ratio of the cross section. For $n = 1$ and $C_f = 10^{-3}$, the highest value of the lift-to-drag ratio is $E = 5.29$ and occurs for $\mu = 0$. As μ increases, the lift-to-drag ratio decreases reaching the value $E = 5.19$ at $\mu = 0.206$, the upper limit for which the optimum transversal contour is a simple triangle. Any further increase in μ causes a rapid decrease in the lift-to-drag ratio attainable with a triangular cross section, which can be partially offset by the addition of a vertical keel at midsection. Thus, for $\mu = 1$, the lift-to-drag ratio associated with the simple triangle is $E = 3.94$ while that of the triangle-keel combination is $E = 4.66$.

In closing, the following comments are pertinent:

(a) Since the present optimum bodies exhibit sharp corners at $\theta = 0$ and $\theta = \pi/2$, their main drawback is the severe heat transfer occurring at the lines of intersection between the surfaces composing the body. Consequently, the present sharp-edge configurations must be replaced by faired configurations in which the transition from one surface to another occurs with a finite curvature. If this is done, lift-to-drag ratios smaller than those predicted here are to be expected.

(b) While the slender body approximation has been employed in every meridian plane, the resulting optimum bodies are such that this approximation is violated in the meridian planes farther away from the plane of symmetry. In spite of this, the authors believe that these solutions approximate closely those which can be obtained without the slender body approximation. The reason is that, for a surface-averaged skin-friction coefficient $C_f = 10^{-3}$ and an elongation ratio in the range $0 \leq \mu \leq 1$, the lift-to-drag ratios calculated with and without the slender body approximation differ from one another by less than 1%.

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LIST OF CAPTIONS

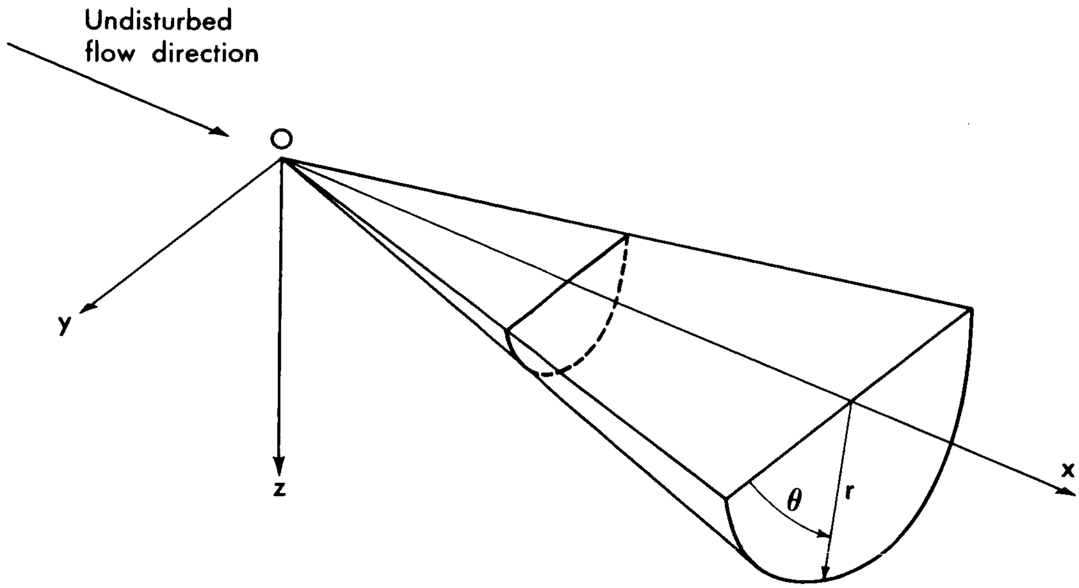

Fig. 1 Coordinate system.

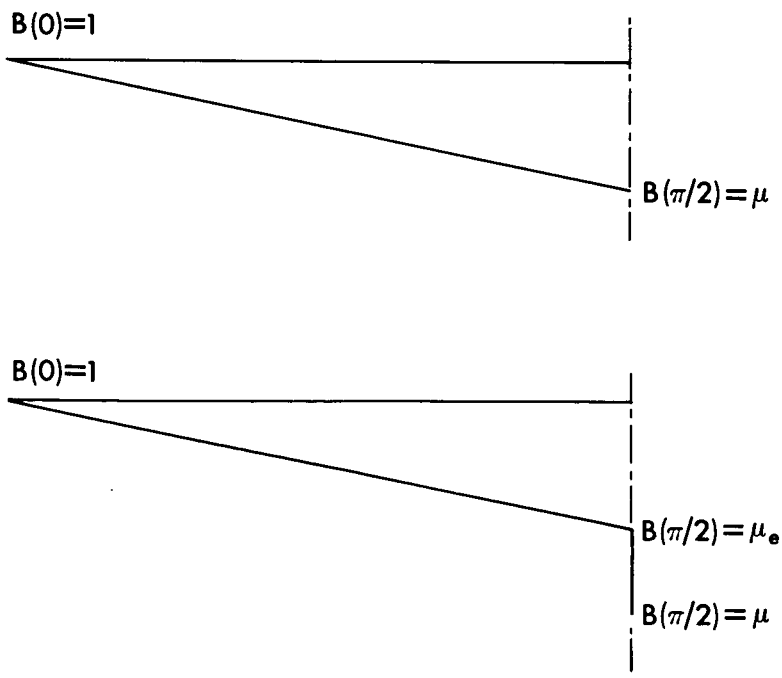

Fig. 2 Optimum transversal contour.

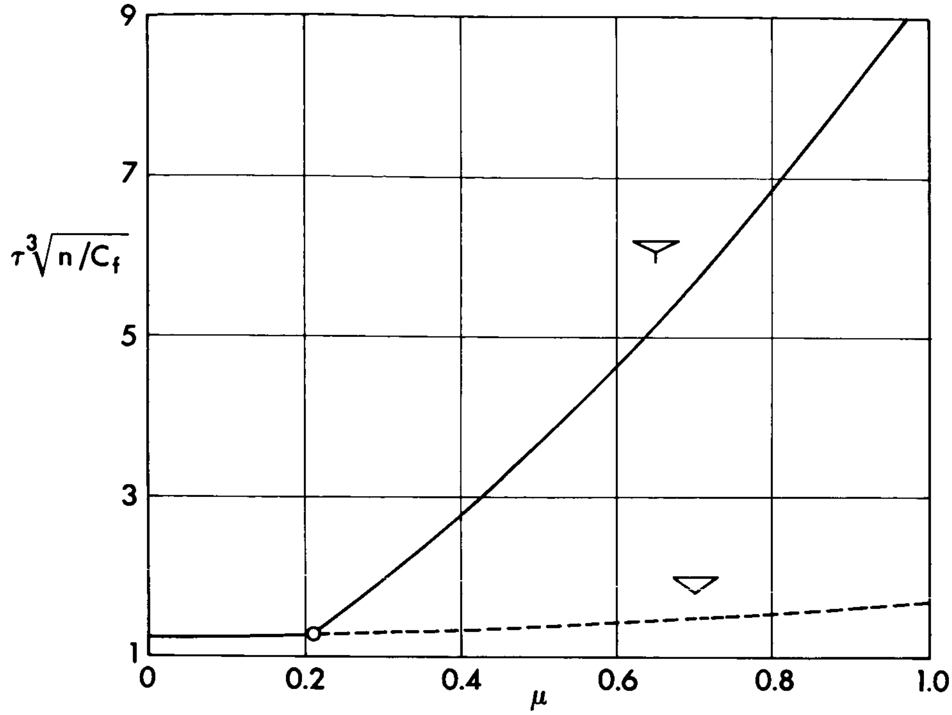

Fig. 3 Optimum thickness ratio.

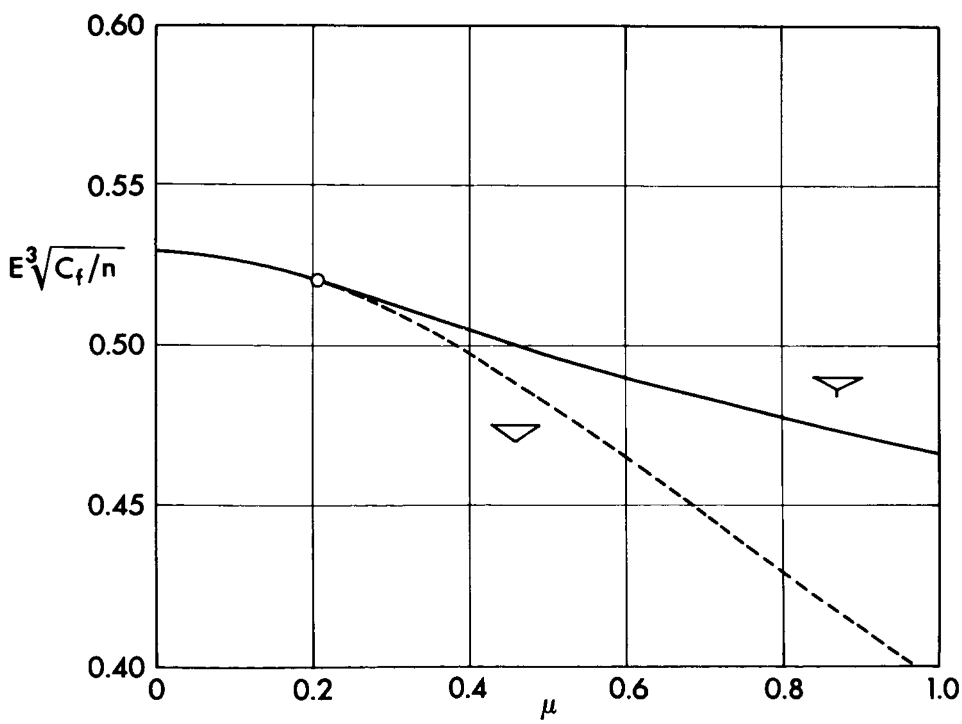

Fig. 4 Maximum lift-to-drag ratio.

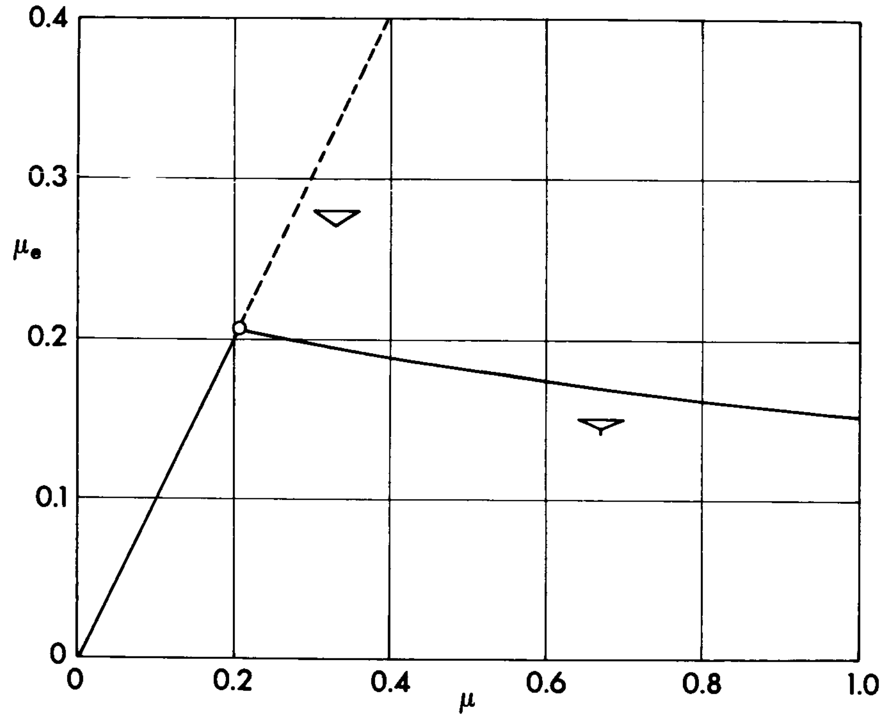

Fig. 5 Effective elongation ratio.