A COMPARISON BETWEEN THE VARIATIONAL SOLUTION AND THE EXPERIMENTAL DATA

A Supplement to the Scientific Report No. 7, "An Experimental Study of the Dipole Antenna with Nonreflecting Resistive Loading."

by

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ABSTRACT

With the help of the variational principle, a simple trial function has been found for the current distribution on the antenna with nonreflecting resistive loading so that it agrees with the experimental data of the input admittance, the amplitude of the current distribution, and the radiation field pattern in the entire frequency range 450 to 900 MHz.

Storer [14] formulated a stationary expression for the input impedance of a cylindrical antenna in terms of the current distribution on the antenna. This variational formulation of the antenna problem can also be seen in a book by King [15] in a summarized version. With a particular trial function for the current distribution for the case of a brass antenna less than one wavelength long, the variational solution of the input impedance was found to be very close to that obtained from the iteration theory of King and Middleton [15] and to the experimental result. Tai [16] introduced another trial function to deal with antennas that are longer than a wavelength.

Although in theory the input impedance expressed by the stationary formula is usually assumed to be not critically dependent on the trial function, as the name of the formula suggests, in practice a successful result still relies more or less heavily on the degree of accuracy of the trial function with respect to the exact solution. The fact that it is necessary to replace Storer's trial function by Tai's for long antennas should discourage any one who hopes for too much from the stationary formula.

It should be noted that the trial functions employed by Storer and Tai were selected with the full knowledge of a well-approximated solution obtained by King and Middleton as well as experimental results. It is felt that as long as similar information is available, the variational method is useful since it provides a check on the existing theory and sometimes it can simplify the expression of the solution of the current distribution by means of a carefully selected trial function.

With all this in mind, the variational method is used in this supplement for a cylindrical antenna with nonreflecting resistive loading. The zero-order theory of Wu and King and the experiments reported in the Scientific Report No. 7 will serve as guide-lines for selecting the trial function. Since a higher-order solution for this antenna is not available so far, the choice of a trial function can only be justified by its agreement with experiment.

The stationary formula for the input impedance due to Storer is as follows with the notation $I(z) = I_z$.

$$Z_o = \frac{1}{I_o^2} \left[\int_{-h}^{h} dz\, z^i\, I_z^2 + \frac{\zeta_o}{4\pi i k} \int_{-h}^{h} dz\; I_z \int_{-h}^{h} dz'\; I_{z'}\; D(z - z') \right] \qquad (10)$$

where $D(z - z') = \left(\frac{\partial^2}{\partial z^2} + k^2 \right) K(z - z')$

It has been proved that when I_z is the exact solution, the first variation of Z_o with respect to I_z is zero. Note that Z_o is not affected by the normalization of I_z.

If the zero-order current is used as the trial function, the stationary formula (10) gives Z_1 which is

$$Z_1 = \int_{-h}^{h} dz\, z^i\, I_{1z}^2 + \frac{\zeta_o}{4\pi i k} \int_{-h}^{h} dz\, I_{1z} \int_{-h}^{h} dz'\, D(z - z')\, I_{1z'} \qquad (11)$$

where $I_{1z} = e^{ik|z|}\,(1 - |z|/h)$.

The double integral can be reduced to single integrals by integrations by parts, and if the terms $(ka)^2$ and a/h are negligible as compared to 1, Z_1 becomes

$$Z_1 = \frac{\zeta_o}{\pi}\left\{(1 - \frac{1}{2k^2h^2})\left[\sinh^{-1}(\frac{h}{a}) - C + iS - ika\right] + \frac{e^{2ikh}}{2k^2h^2}\sinh^{-1}(\frac{h}{a}) + \frac{i(e^{2ikh}-2)}{2kh} + \frac{1-2e^{2ikh}}{8k^2h^2} + \frac{|\psi|}{2kh}\left(i + \frac{1-e^{2ikh}}{2kh}\right)\right\} \quad (12)$$

where $C = C(O, 2kh)$, $S = S(O, 2kh)$ and $|\psi|$ is given by (3) .

The input admittance Y_1 of a monopole antenna over a ground plane is calculated and shown in Fig. 33 (curves marked with $\beta = \infty$), together with the experimental data. It is seen that considerable improvement has been achieved as compared with the zero-order admittance (see Fig. 16). Normalized values of I_{1z} such that $I_{10} = Y_1$ are plotted in Figs. 36, 37 and 38 together with the measured current distributions for antennas No. 2, 3, 4 and 5 which are normalized with respect to their measured input admittances with end corrections. The agreement between the current distributions are found to be fairly good. The end correction B_T that is necessary to be substracted out from the measured susceptance is based on Duff's measurement [10] where it is found that $B_T = -0.72$ millimho at 600 MHz for a traveling-wave antenna with a resistor located at quarter wavelength from the end and driven by a coaxial line of the same dimensions as that used in this experiment. Since the end effect is a localized phenomenon, it seems reasonable to assume that Duff's result is applicable here.

So far the trial function with only one term has been used. In order to apply the variational method, a second term is to be added. The examination of the experimental data suggests the following function:

$$I_z = I_{1z} + A\ I_{2z} \tag{13}$$

where $I_{2z} = e^{ik|z|}e^{-\beta|z|} - e^{ikh}e^{-\beta h}$ and I_{1z} is given in (11). "A" is a constant parameter to be determined by imposing on Z_o the condition that the first variation of Z_o with respect to "A" should be zero. The attenuation constant β is assumed to be real, and from the experimental data its order of magnitude is estimated to be in the range of 15 to 100 nepers per meter. Admittedly, the choice of I_{2z} is rather subjective since only the amplitude of the current distribution is known experimentally, the only justification for it is the fact that it gives a very satisfactory variational solution which is very close to the available experimental result.

If the trial function (13) is used, (10) gives Z_a which is

$$Z_a = (Z_1 + 2A\ Z_2 + A^2\ Z_3)/(1 + A)^2 \tag{14}$$

where

$$Z_{\substack{2\\3}} = \int_{-h}^{h} dz\ z^i\ I_{\substack{1z\\2z}} I_{2z} + \frac{\zeta_o}{4\pi ik} \int_{-h}^{h} dz\ I_{\substack{1z\\2z}} \int_{-h}^{h} dz'\ D(z-z') I_{2z'} \tag{15}$$

and Z_1 is given by (12). Let the first variation of Z_a with respect to A be put to zero, then the following equations are obtained

$$A = (Z_2 - Z_1)/(Z_2 - Z_3) \tag{16a}$$

$$Z_a = Z_1 + (Z_2 - Z_1)^2 / (2Z_2 - Z_1 - Z_3) \tag{16b}$$

The double integrals in the expressions of Z_2 and Z_3 in (15) can be reduced to single integrals by integrations by parts and if the terms $(ka)^2$, a/h, and $e^{-\beta h}$ are negligible as compared to 1, Z_2 and Z_3 can be put into the following approximate forms:

$$Z_2 = \frac{\zeta_o}{\pi}\left\{\frac{|\psi|}{h(\beta-2ik)} - \frac{1}{h}\left(\frac{1}{\beta-2ik}+\frac{1}{\beta}\right)\left[\sinh^{-1}\left(\frac{h}{a}\right) - C + iS - ika\right] + p_o\left(1+\frac{i}{kh}+\frac{1}{\beta h}-\frac{1}{h(\beta-2ik)}+\frac{e^{2ikh}}{\beta h}\left(\frac{1}{ikh}+\frac{1}{h(\beta-2ik)}-\frac{1}{\beta h}\right)\right)\right\} \tag{17}$$

$$Z_3 = \frac{\zeta_o}{\pi}\left\{\frac{|\psi|}{2h(\beta-ik)} + \frac{1}{2ik}\left[\beta(\beta-2ik)p_1 + \frac{[ik(2k^2+\beta^2)-\beta^3]\,p_o}{\beta^2+k^2}\right]\right\} \tag{18}$$

where

$$p_m = \int_o^h dz\, z^m K(z) I_{2z}, \quad m = 0, 1$$

Z_a given by (16b) is calculated for β ranging from 15 to 90 nepers per meter and the frequency ranging from 450 to 900 MHz, and the value $2/Z_a$ is shown in Fig. 33. It is found that when $\beta \simeq 60$ nepers per meter, both the real and imaginary parts of the input admittance agree very well with the experimental data. Also, the current distribution (13) with parameter A given by (16a) agrees satisfactorily with the experiment at this value of β and the current is not sensitive to β. The parameter A of the current distribution as a function of β is shown in Fig. 34 and the current distributions

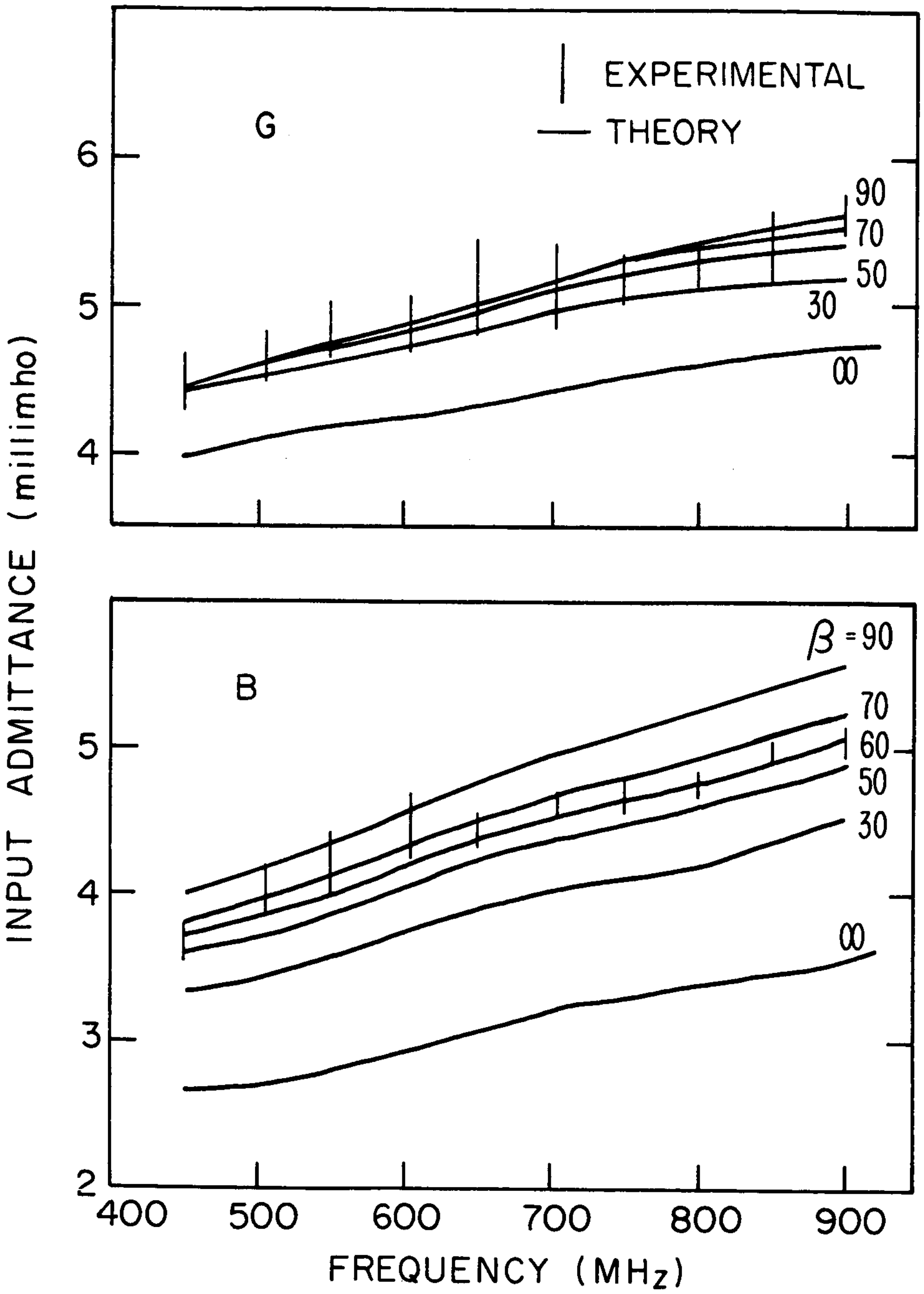

FIG. 33 INPUT ADMITTANCES, VARIATIONAL SOLUTIONS VS. EXPERIMENTAL DATA OF ANTENNAS No. 2, 3, 4, AND 5.

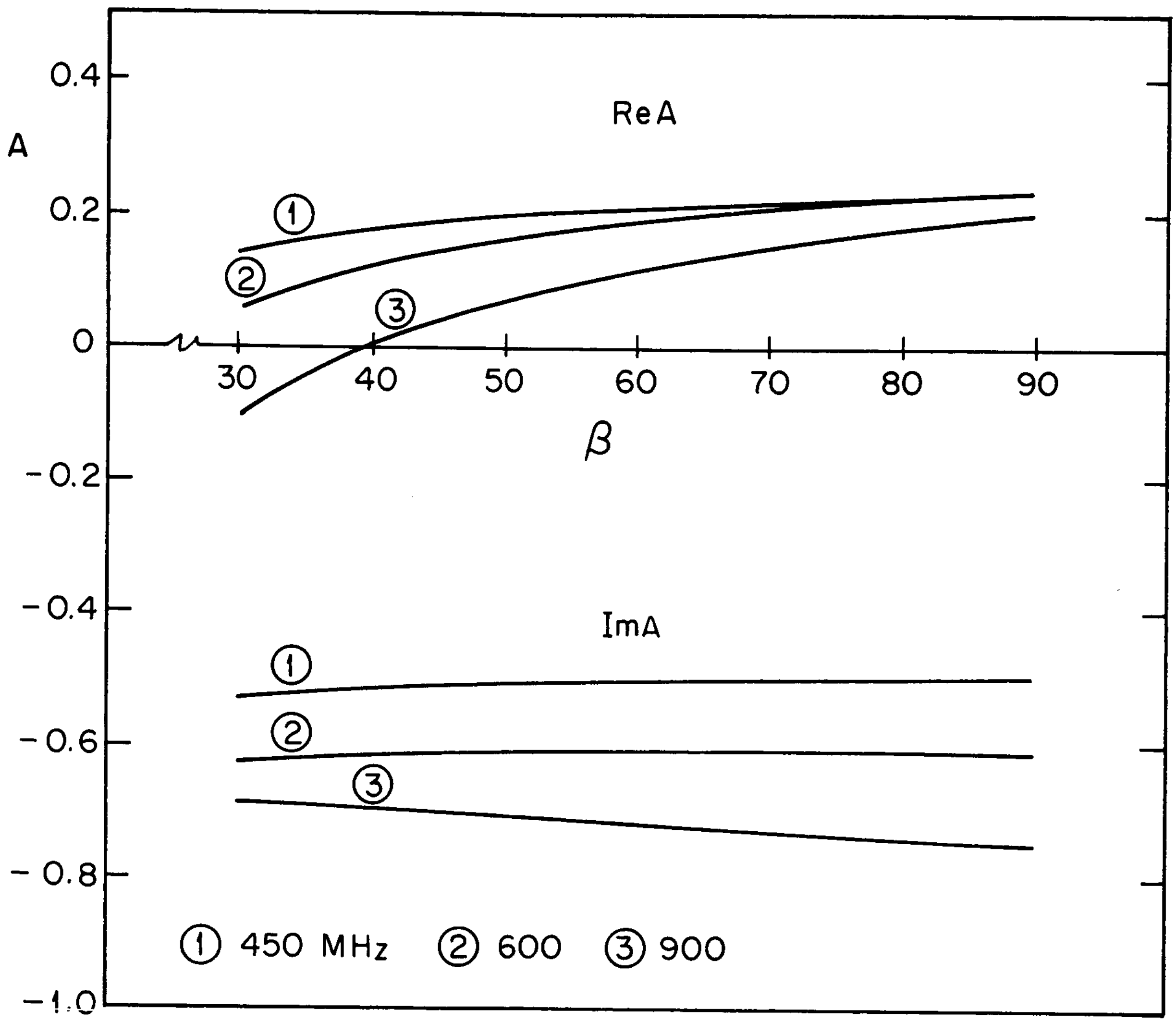

FIG. 34 CURRENT DISTRIBUTION PARAMETER A

are shown in Figs. 35 through 38. Since I_{2z} is a rapidly decaying function, its contribution to the radiation field pattern is small. In fact, it is found that the addition of the AI_{2z} term to the zero-order current I_{1z} changes the radiation field pattern by no more than 1 db and the agreement between the theory and the experiment as shown in Figs. 23 through 27 is not affected.

Simpson [17] calculated the current distribution for this resistive antenna with z^i varying like antenna No. 4 (z^i in five equal-length steps) at 600 MHz using a numerical method. In his method the integral equation of this antenna problem becomes a summation equation when the antenna is divided into a number of segments and the current in each segment is considered to be uniform. The curent distribution obtained by this method agrees very well with the experimental data except near the driving point where the accuracy of this numerical method is questionable (see Fig. 37). The input conductance is calculated to be equal to 4.909 millimho as compared with 4.841 millimho obtained from variational method. The input susceptance, unlike the conductance, is sensitive to N, which is the number of divisions that is taken in the numerical method. For N equal to 20, the input susceptance is equal to 2.864 millimho as compared with 4.179 millimho obtained from the variational solution. The susceptance increases as N does without a limit as a result of the assumption of a delta-function generator. This is the basic difficulty which remains to be overcome.

As far as the agreement between the experiment and the theory is concerned, it is seen that the stationary formula improves the zero-order theory by using the zero-order current as the trial function, and the trial function (13) which is obtained by adding another term to the zero-order current improves

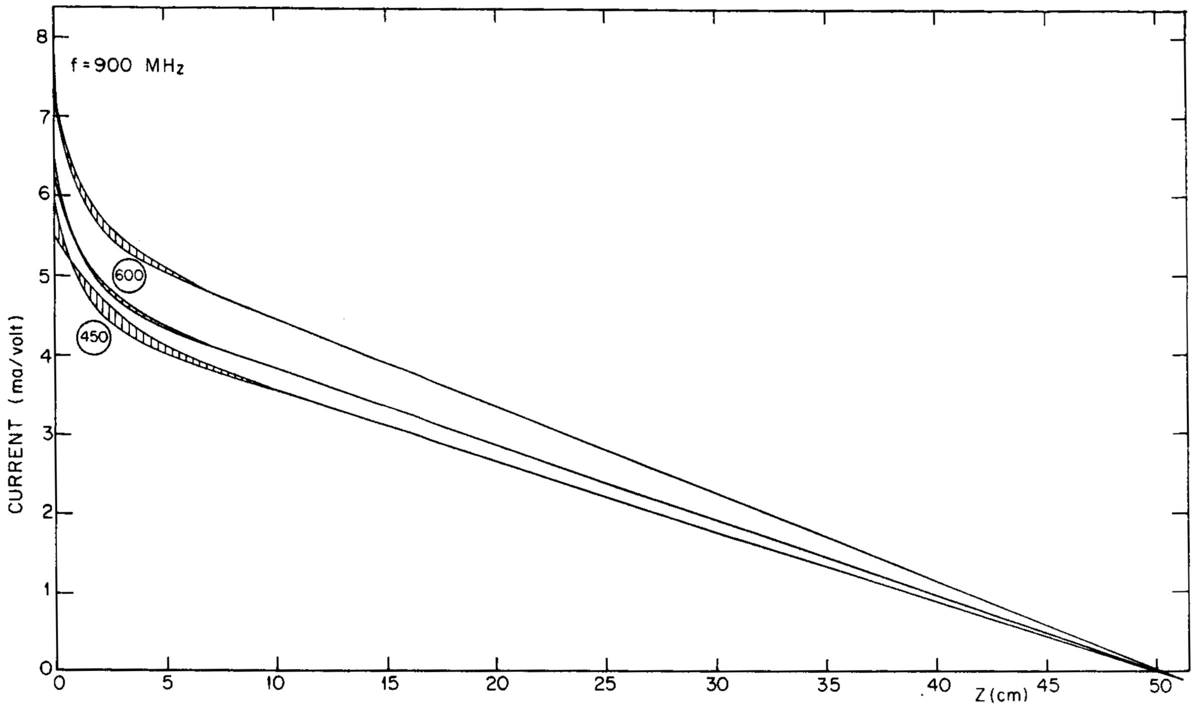

FIG. 35 VARIATIONAL SOLUTION OF THE CURRENT DISTRIBUTION, $\beta = 50$ TO 70 nepers/meter

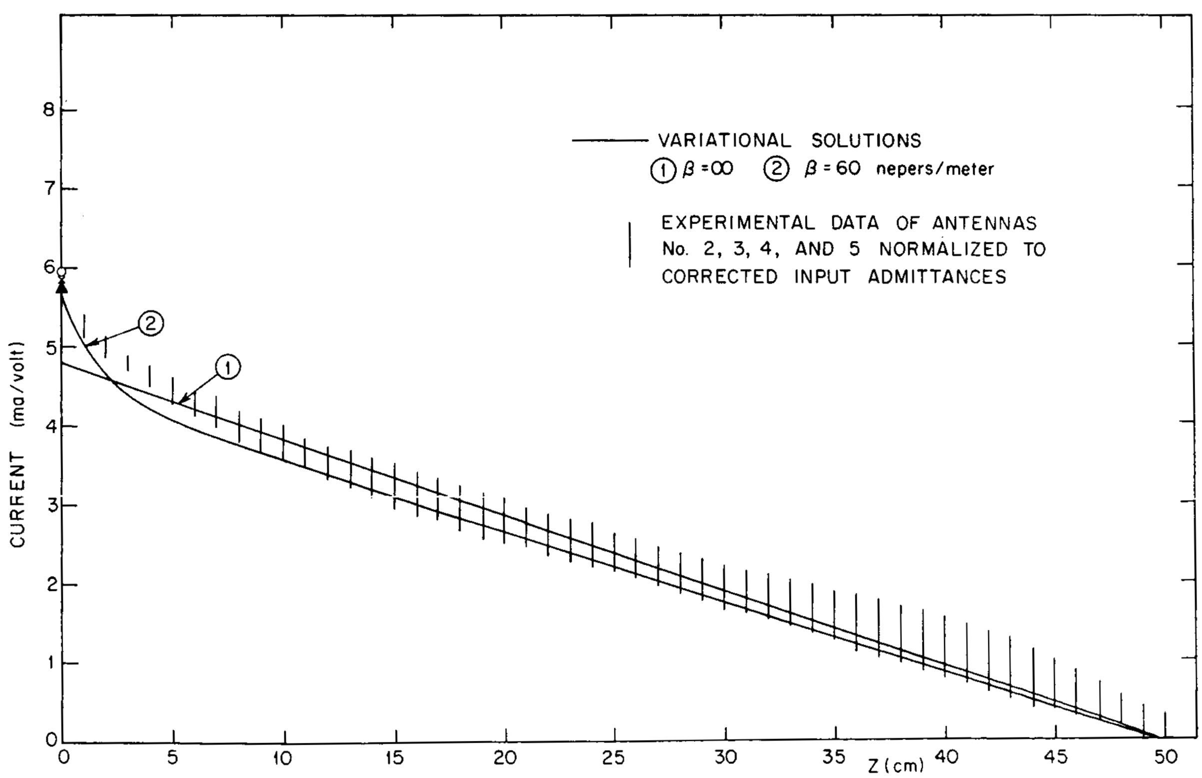

FIG. 36 CURRENT DISTRIBUTIONS, VARIATIONAL SOLUTIONS VS. EXPERIMENTAL DATA, FREQUENCY = 450 MHz

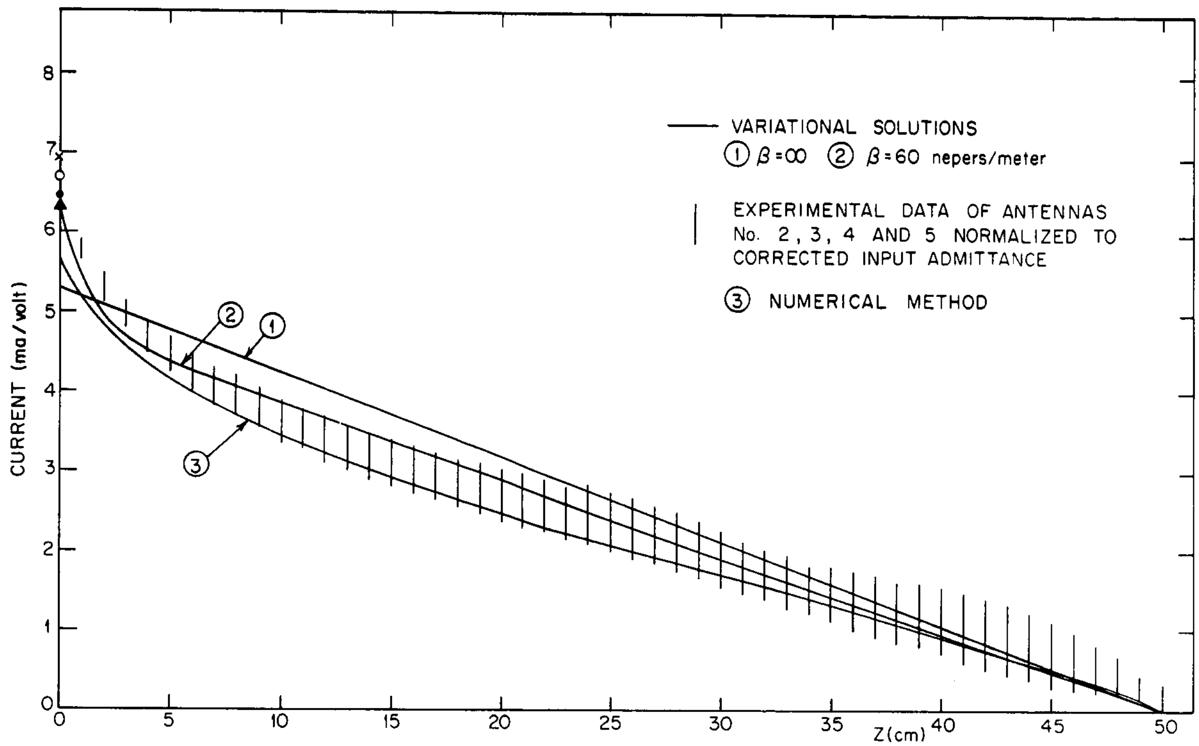

FIG. 37 CURRENT DISTRIBUTIONS, VARIATIONAL SOLUTIONS VS. EXPERIMENTAL DATA, FREQUENCY = 600 MHz

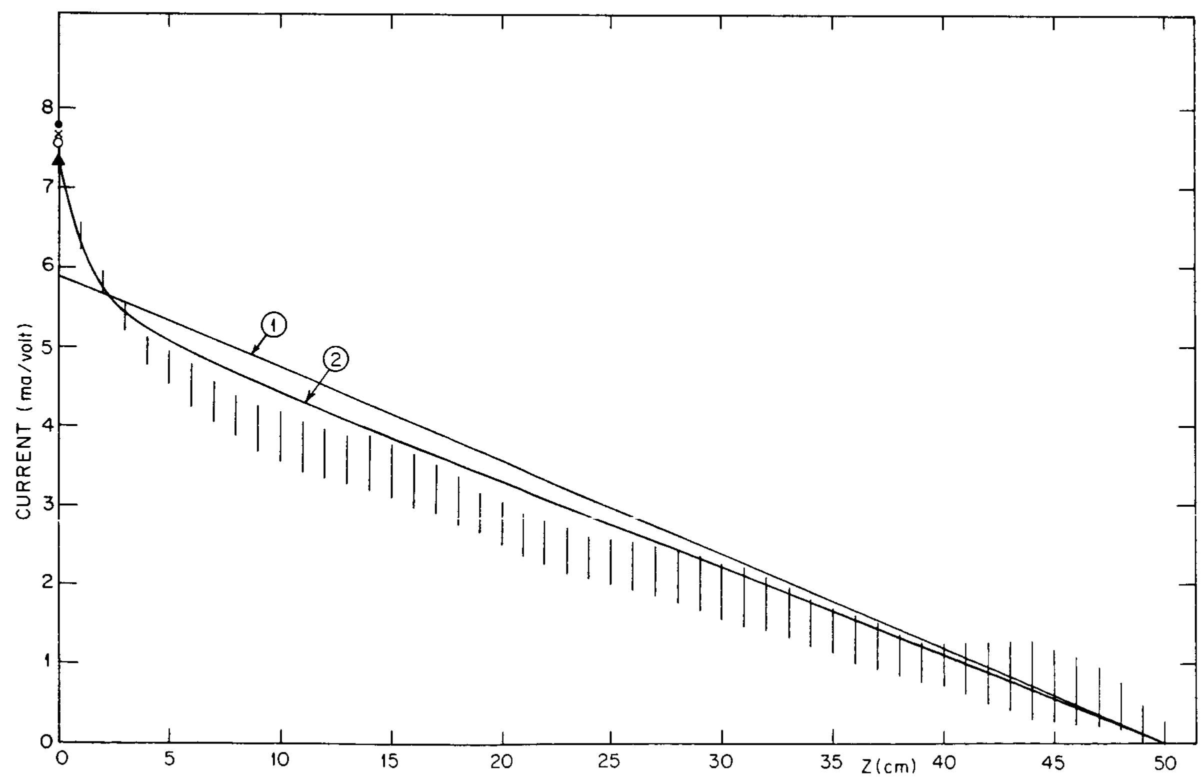

FIG. 38 CURRENT DISTRIBUTIONS, VARIATIONAL SOLUTIONS VS. EXPERIMENTAL DATA, FREQUENCY = 900 MHz

the agreement still further. It can be concluded that with the help of the variational principle, a simple trial function, namely (13), has been found for the current distribution on the antenna with nonreflecting resistive loading so that it agrees with the experimental data of the input admittance, the amplitude of the current distribution, and the radiation field pattern in the entire frequency range 450 to 900 MHz.

However, even the trial function (13) gives the correct input admittance, amplitude of current, and the field pattern, strictly speaking it still cannot be concluded that (13) is a good approximation to the antenna problem since the information about the phase of the antenna is not known experimentally. For engineering purposes, on the other hand, the major concern should be the input admittance, the field pattern and the frequency band of the antenna for which the trial function has been demonstrated to have given a very satisfactory result. In this sense, the trial function (13) can be thought as a good approximation to the exact solution of this antenna.

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