EML RESEARCH REPORT NO. 2

FACILITY FORM 602
N67 16630 (ACCESSION NUMBER) (THRU)
19 (PAGES) 1 (CODE)
CR-81335 (NASA CR OR TMX OR AD NUMBER) 23 (CATEGORY)

THE EFFECT OF NON-LINEARITIES ON OPTICAL CORRELATION PROCESSING

J. P. Moffatt
F. J. Tischer

GPO PRICE $ ______
CFSTI PRICE(S) $ ______
Hard copy (HC) 3.00
Microfiche (MF) .65
ff 653 July 65

This research was supported by
The National Aeronautics and Space Administration
and partially funded under NGR-34-002-038

NORTH CAROLINA STATE UNIVERSITY
Raleigh, North Carolina

November 1966

THE EFFECT OF NON-LINEARITIES ON OPTICAL CORRELATION PROCESSING

Abstract

This paper presents a method of analytically determining the effect of internal non-linearities on the output of a coherent optical correlator operated as a matched filter. Such a device employs photographic transparencies or other media for the representation of the reference and signal functions. These include distortions due to the non-linearities of the processes involved.

The effect of the distortions on the output correlation functions is derived using a Taylor series expansion of the non-linearities. Three representative examples are considered to demonstrate the effect numerically.

THE EFFECT OF NON-LINEARITIES ON OPTICAL CORRELATION PROCESSING

I. Introduction

A Coherent optical correlator may be described in simple terms as an optical system consisting of a collimated coherent light source, a series of lenses which repeatedly transform amplitude distributions in one plane into their Fourier transforms in the following plane, transparency holders in the various image planes, and a series of detectors transversely distributed along a vertical line in the center of the terminal plane. Figure 1. shows a basic configuration of such a system. For more details reference is made to a description of a commercially available system [1]. By this system, cross-correlation processing can be carried out simultaneously for signal distributions in the horizontal direction in a number of channels distributed in the vertical direction.

Usually the assumption is made that the optical processes are linear so that the optical operations can be described by relationships as they are used for linear processing in electronic correlators. This assumption is not right away evident since non-linearities usually are more pronounced in optical systems than in the corresponding electronic devices. As a consequence the question arises to which extent these non-linearities will affect the operations in optical correlators. The question has two aspects, namely: (1) will undesired but practically unavoidable non-linearities make correlation processing inferior to other methods of filtering and (2) can requirements with regard to linearity be relaxed if the effects of non-linearities in processing are negligibly small.

Various sources may introduce non-linearities in the optical system. First, transparencies for the reference signal will not be an exact reproduction of the original signal. Two possible sources account for these distortions. The non-linear characteristics of the film including photo processing [2] represent one of these and the non-linear relationship between the output of the light source and the electric signal in producing the transparency is the other. Secondly, non-linearities resulting from the same sources may be present in the reproduction of the object signal. The non-linearities of the reference function and the signal function may be the same if they are produced by identical reproduction methods or they may be different. Additional sources of non-linearities are the detectors in the terminal plane. These last effects will be of minor concern at this time, though, since they may be compensated for by appropriate linearizing circuitry.

II. Model of a Non-Linear Processing System

For studying the effects of non-linearities on correlation processing, the following model will be used: All non-linearities introduced in the operations of transforming an electrical signal into an optical reference function given by the amplitude transmittance of the reference transparency will be lumped together. They are described by a characteristic curve which interrelates the amplitude transmittance and the electric reference signal which is usually the transmitted telemetry signal for a particular channel. A second characteristic curve will describe the transformation of the received telemetry signal including noise into a signal-function transparency.

Assuming a model of this type, the basic question becomes: What are the deviations of the crosscorrelation functions obtained by optical processing if the reference and signal functions of the respective transparencies become non-linear replicas of the corresponding electric sig ls. Based on this model, the crosscorrelation integral of the reference and signal functions will be determined where these functions are considered as output functions of non-linear circuit elements which are described by characteristic curves. The non-linearity of the characteristic curves is taken into account as an approximation by a Taylor series.

III. Essential Equations

Figure 2 shows a sample characteristic curve relating the output of a specific non-linear process to its input. We may consider the input signal represented by a variable y which varies about y_o and is limited to the region between y_{min} and y_{max}. Assuming that both f(y) and its derivatives are continuous at y_o we may write a Taylor series expansion of this function which will converge to f(y) in this region. Since the series is convergent, we may in practical applications approximate f(y) by the first several terms of this expansion. The Taylor series is written

$$x = f(y) = f(y_o) + \frac{f'(y_o)}{1!}(y-y_o) + \frac{f''(y_o)}{2!}(y-y_o)^2 + \frac{f'''(y_o)}{3!}(y-y_o)^3 + \cdots \qquad (1)$$

where the primes denote differentiation with respect to y. To simplify the notation we write

$$a_o = f(y_o)\ ,\ a_1 = \frac{f'(y_o)}{1!}\ ,\ a_2 = \frac{f''(y_o)}{2!}\ ,\ etc.,$$

since these quantities are characteristic constants of the particular process being considered. Letting

$$(y - y_o) = S(z)\ ,$$

we have

$$x = f(y) = a_0 + a_1 S(z) + a_2 S^2(z) + a_3 S^3(z) + \cdots \qquad (2)$$

Using this technique, we can describe the actual optical signal existing in a portion of the correlator in terms of the input signal $S(z)$ and the non-linearities of the particular component being studied. Repeated use of this procedure can be made in order to describe the performance of the system.

IV. Application to a Correlator

We will now consider a correlator as described in section I. for processing a signals of the form

$$S_n(z) = A \cos nz, \qquad (3)$$

where the factor n may take on various integer values and where A is a normalized amplitude. The correlator is to operate as a matched filter, thus both the reference and signal transparencies will contain replicas of $S_n(z)$. Ideally the reference and signal transparencies would have the signals recorded on them in a linear manner. For correlation filtering the reference transparency would contain a complete set of the reference functions, one in each channel, while the signal transparency would contain the received input signal only, in the form of a one-dimensional distribution of density. Due, however, to the non-linearities and biasing problems involved in producing the transparencies, the various signals will be modified. We will describe the processes of converting the signals $S_n(z)$ into the reference and signal transparencies by means of the non-linear model introduced in the preceding sections. Accordingly we may write for the light amplitudes leaving the references transparency

$$R_n(z) = a_0 + a_1 S_n(z) + a_2 S_n^2(z) + a_3 S_n^3(z) + a_4 S_n^4(z) + \cdots , \qquad (4)$$

where the factors a_n represent the coefficients of the Taylor series expansion of the characteristic curve of the reference transparency. Similarly we write for the signal transparency.

$$S_m(z) = b_0 + b_1 S_m(z) + b_2 S_m^2(z) + b_3 S_m^3(z) + b_4 S_m^4(z) + \cdots . \qquad (5)$$

For the signal transparency, the subscript m replaces n. Substituting expression (3) into (4) and (5) and expanding the various powers of the cosine function (see appendex) yields

$$R_n(z) = \left(a_0 + \frac{a_2 A^2}{2} + \frac{3a_4 A^4}{8}\right) + \left(a_1 A + \frac{3a_3 A^3}{4}\right)\cos nz + \left(\frac{a_2 A^2}{2} + \frac{a_4 A^4}{2}\right)\cos 2nz + \frac{a_3 A^3}{4}\cos 3nz + \frac{a_4 A^4}{8}\cos 4nz + \cdots ,$$

and

$$S_m(z) = \left(b_0 + \frac{b_2 A^2}{2} + \frac{3b_4 A^4}{8}\right) + \left(b_1 A + \frac{3b_3 A^3}{4}\right)\cos mz + \left(\frac{b_2 A^2}{2} + \frac{b_4 A^4}{2}\right)\cos 2mz + \frac{b_3 A^3}{4}\cos 3mz + \frac{b_4 A^4}{8}\cos 4mz + \cdots .$$

In order to simplify these expressions, we define

$$r_0 = \left(a_0 + \frac{a_2 A^2}{2} + \frac{3a_4 A^4}{8}\right), \quad r_1 = \left(a_1 A + \frac{3a_3 A^3}{4}\right), \text{ etc.},$$

$$s_0 = \left(b_0 + \frac{b_2 A^2}{2} + \frac{3b_4 A^4}{8}\right), \quad s_1 = \left(b_1 A + \frac{3b_3 A^3}{4}\right), \text{ etc.},$$

so that we may write

$$R_n(z) = r_0 + r_1 \cos nz + r_2 \cos 2nz + r_3 \cos 3nz + r_4 \cos 4nz + \cdots \quad (6)$$

$$S_m(z) = s_0 + s_1 \cos mz + s_2 \cos 2mz + s_3 \cos 3mz + s_4 \cos 4mz + \cdots \quad (7)$$

In the following discussion we shall consider only the first five terms of these expressions. The correlator then performs optically the operation of corss-correlation on the reference and signal functions (6) and (7). This operation may be written as

$$\phi_{mn}(\sigma) = \frac{1}{T}\int_{-T/2}^{T/2} S_m(z+\sigma)\, R_n(z)\, dz \qquad (8)$$

Where T is the aperture length of the correlator. We will assume for simplicity that T is such that it is an integer number of periods of each signal in the set. Under these conditions there will be a corss-correlation between a term of $R_n(z)$ and one of $S_m(z)$ only when they are of the same spatial frequency. For a given value of n there will be many values of m for which this is the case, but we will consider only 5 of these which typify the effect. These are:

$$\phi_{mn}(\sigma)\big]_{m=n/4} = r_0 s_0 + \frac{r_1 s_4}{2}\cos n\sigma,$$

$$\phi_{mn}(\sigma)\big]_{m=n/2} = r_0 s_0 + \frac{r_1 s_2}{2}\cos n\sigma + \frac{r_2 s_4}{2}\cos 2n\sigma,$$

$$\phi_{mn}(\sigma)\big]_{m=n} = r_0 s_0 + \frac{r_1 s_1}{2}\cos n\sigma + \frac{r_2 s_2}{2}\cos 2n\sigma + \frac{r_3 s_3}{2}\cos 3n\sigma + \frac{r_4 s_4}{2}\cos 4n\sigma,$$

$$\phi_{mn}(\sigma)\big]_{m=2n} = r_0 s_0 + \frac{r_2 s_1}{2}\cos 2n\sigma + \frac{r_4 s_2}{2}\cos 4n\sigma,$$

$$\phi_{mn}(\sigma)\big]_{m=4n} = r_0 s_0 + \frac{r_4 s_1}{2}\cos 4n\sigma. \qquad (9)$$

Note that expression C (where both frequencies are equal) corresponds to the case of the signal being detected. The other expressions correspond to "mixing" and erroneous "present" indications in the various channels. In the following section we shall investigate quantitatively the various terms of these correlation functions Eqs. (9) .

V. Examples

In an actual optical correlator many different schemes can be used for the production of signal and reference transparencies. In this section we will treat three examples which are representative of the various alternatives available. One of these will be the linear case which will also serve as a reference in comparing the results of the non-linear cases. In each example the input signal to the photographic process will be normalized so that the amplitude transmittance of the final transparencies will vary approximately between .2 and .8. This being chosen so as to eliminate fogging and saturation effects of the emulsions. This will be accomplished by appropriately adjusting the constant "A" in equation (3) for each case. In each case it will be assumed that the signal and reference transparencies are produced by identical photographic processes. Thus their Taylor series coefficients will be identical in each example

Case 1. Positive Processing, Linear Characteristic

Here we assume that the original signal is reproduced linearly and positive. This might be accomplished in two ways. If the electrical-light modulator used in producing the transparency is linear with respect to light amplitude, controlling of the photographic processing to attain a total gamma of 1.0 will result in a linear characteristic curve for the combined processes. If the electrical-light modulation is linear with respect to light intensity (such as a spot intensity modulation of a CRT) then a gamma value of 2.0 in the photographic processing will again yield a linear "characteristic". Figure 3 presents such a curve. For this case

$$S_n(z) = .3 \cos nz \, .$$

Expanding the curve x = y in a Taylor series about $x = .5(y_o - .5)$ we obtain

$$a_0 = .5,\ a_1 = 1.0,\ a_2 = 0,$$
$$a_3 = 0,\ a_4 = 0.$$

Correspondingly the coefficients of the reference and signal transparencies become

$$r_0 = s_0 = .5,\ r_1 = s_1 = .3,\ r_2 = s_2 = 0,$$
$$r_3 = s_3 = 0,\ r_4 = s_4 = 0.$$

The optical reference and signal functions are then

$$R_n^{(1)}(z) = .5 + .3 \cos nz,$$
$$S_m^{(1)}(z) = .5 + .3 \cos mz,$$

where the superscript (1) indicates that the functions are valid for case 1. The correlation functions obtained in section IV Eqs (9) become

$$\phi_{mn}^{(1)}(\sigma)\Big]_{m=n} = .25 + .045 \cos n\sigma,$$
$$\phi_{mn}^{(1)}(\sigma)\Big]_{m\neq n} = .25.$$

We thus observe that, in this case, there is no effect similar to "mixing" in non-linear electronic circuits.

Case 2: Square-Law Characteristic, Positive Processing

This case is presented primarily to determine the sensitivity of the correlation processing to variations in the gamma product of the developing process. If for instance the electrical-light modulator is linear with respect to light amplitude but the transparencies are developed with a gamma product of 2.0, a characteristic curve such as is shown in figure 4. will result. Here:

$$S_n(z) = .25 \cos nz.$$

The coefficients of the Taylor series for $x = y^2$ about $x = .5$ ($y_o = .707$) are

$$a_0 = .5,\ a_1 = 1.4,\ a_2 = 1.0,$$
$$a_3 = 0,\ a_4 = 0,$$

and the reference and signal transparency coefficients become

$$r_0 = s_0 \cong .53,\ r_2 = s_2 \cong .03,\ r_4 = s_4 = 0$$
$$r_1 = s_1 \cong .36,\ r_3 = s_3 \cong 0.$$

Then the optical reference and signal functions are

$$R_n^{(2)}(z) = .53 + .36 \cos nz + .03 \cos 2nz,$$
$$S_m^{(2)}(z) = .53 + .36 \cos mz + .03 \cos 2mz.$$

The correlation functions become

$$\Phi_{mn}^{(2)}(\sigma)\Big]_{m=n} = .28 + .06 \cos n\sigma + .0004 \cos 2n\sigma,$$
$$\Phi_{mn}^{(2)}(\sigma)\Big]_{m=n/2} = .28 + .005 \cos n\sigma,$$
$$\Phi_{mn}^{(2)}(\sigma)\Big]_{m=2n} = .28 + .005 \cos 2n\sigma,$$
$$\Phi_{mn}^{(2)}(\sigma)\Big]_{m=4n} = \Phi_{mn}^{(2)}(\sigma)\Big]_{m=n/4} = .28\ .$$

Case 3: Negative Processing

This case is considered since a considerable simplification of the photographic processing results when the signal may be represented as a negative. Figure 5 presents a characteristic curve that would result if the electrical-light modulator would be linear in light amplitude and the negative would be processed to a gamma of 1.0. Accordingly

$$S_n(z) = .8 \cos nz\ .$$

The Taylor series coefficients for $x = y^{-1}$ about $x = .5$ ($y_0 = 2.0$) are

$$a_0 = .5,\ a_1 = -.25,\ a_2 = .13,$$
$$a_3 = -.06,\ a_4 = .03.$$

The reference and signal transparency coefficients then become

$$r_0 = s_0 \cong .54,\ r_1 = s_1 \cong -.20,\ r_2 = s_2 \cong .01$$
$$r_3 = s_3 \cong .002,\ r_4 = s_4 \cong .0009.$$

Finally we have for the optical reference and signal functions

$$R_n^{(3)}(z) = .54 - .20 \cos nz + .01 \cos 2nz - .002 \cos 3nz + .00009 \cos 4nz$$

$$S_m^{(3)}(z) = .54 - .20 \cos mz + .01 \cos 2mz - .002 \cos 3mz + .00009 \cos 4mz$$

The correlation integrals are

$$\phi_{mn}^{(3)}(\sigma)\Big]_{m=n} = .29 + .02 \cos n\sigma + .5\times10^{-4} \cos 2n\sigma + .2\times10^{-5} \cos 3n\sigma + .4\times10^{-8} \cos 4n\sigma,$$

$$\phi_{mn}^{(3)}(\sigma)\Big]_{m=n/2} = .29 - .001 \cos n\sigma + .4\times10^{-6} \cos 2n\sigma,$$

$$\phi_{mn}^{(3)}(\sigma)\Big]_{m=2n} = .29 - .001 \cos 2n\sigma + .4\times10^{-6} \cos 4n\sigma,$$

$$\phi_{mn}^{(3)}(\sigma)\Big]_{m=n/4} = .29 - .9\times10^{-5} \cos n\sigma,$$

$$\phi_{mn}^{(3)}(\sigma)\Big]_{m=4n} = .29 - .9\times10^{-5} \cos 4n\sigma.$$

VI. Conclusions

A method was derived for the consideration on non-linearities in correlation processing. The method is based on describing the non-linearities of the signal and reference functions of the transparencies by their corresponding Taylor series coefficients. This permits computation of the correlation functions. A summary of the results of considering three representative cases, namely for linear, quadratic, and inverse characteristics is shown in Table 1.

The entries in the table are the correlation functions according to Eq. (8). The subscripts mn indicate the spatial frequencies of the signal and reference functions respectively. The frequency ratios considered in section IV and shown in the first column are 1/4, 1/2, 1, 2, and 4. The correlation functions for the three cases in the corresponding columns contain four types of terms. First there are the D.C. terms which are of minor interest since they can be eliminated by a D.C. stop or by adjustment of D.C. levels of the output detectors. There are then three types of terms varying with sigma. The ideal (linear) correlator will have an output varying with sigma only when $m = n$. This occurs in case one. Any harmonic term varying with sigma when $m \neq n$ corresponds to an error term indicating "mixing" between channels. The terms of the fourth type appear as harmonic distortions for $m = n$ but these do not affect the correlator operation. The ratio of the amplitudes of the error terms to the sigma varying terms for $m = n$ is then an indication of the detrimental effect of the non-linearities. We observe from Table 1 that although considerable non-linearities exist in the cases 2 and 3, this ratio remains below 0.1 in both cases. The correlation process thus tends to minimize the non-linear effects. Consequently, if a correlator is being operated in high signal-to-noise-ratio with only one signal processed at a time (time multiplex), the non-linear effects are negligible. In such cases it may be possible to relax the requirements with regard to linearity and to simplify photographic processing by using negative transparencies (case 3). If, however, the processor has to detect the presence of one of a number of signals received simultaneously, or is to operate in a large-noise environment, the effect of non-linearities may become appreciable. Under such conditions linear processing becomes desirable.

$$\phi_{mn}(\sigma)$$

	CASE 1.	CASE 2.	CASE 3.
$m = n/4$	$.25$	$.28$	$.29 - .9\times10^{-5} \cos n\sigma$
$m = n/2$	$.25$	$.28 + .005 \cos n\sigma$	$.29 - .001 \cos n\sigma$ $+ .4\times10^{-6} \cos 2n\sigma$
$m = n$	$.25 + .045 \cos n\sigma$	$.28 + .06 \cos n\sigma$ $+ .004 \cos 2n\sigma$	$.29 + .02 \cos n\sigma$ $+ .5\times10^{-4} \cos 2n\sigma$ $+ .2\times10^{-5} \cos 3n\sigma$ $+ .4\times10^{-8} \cos 4n\sigma$
$m = 2n$	$.25$	$.28 + .005 \cos 2n\sigma$	$.29 - .001 \cos 2n\sigma$ $+ .4\times10^{-6} \cos 4n\sigma$
$m = 4n$	$.25$	$.28$	$.29 - .9\times10^{-5} \cos 4n\sigma$

TABLE 1.

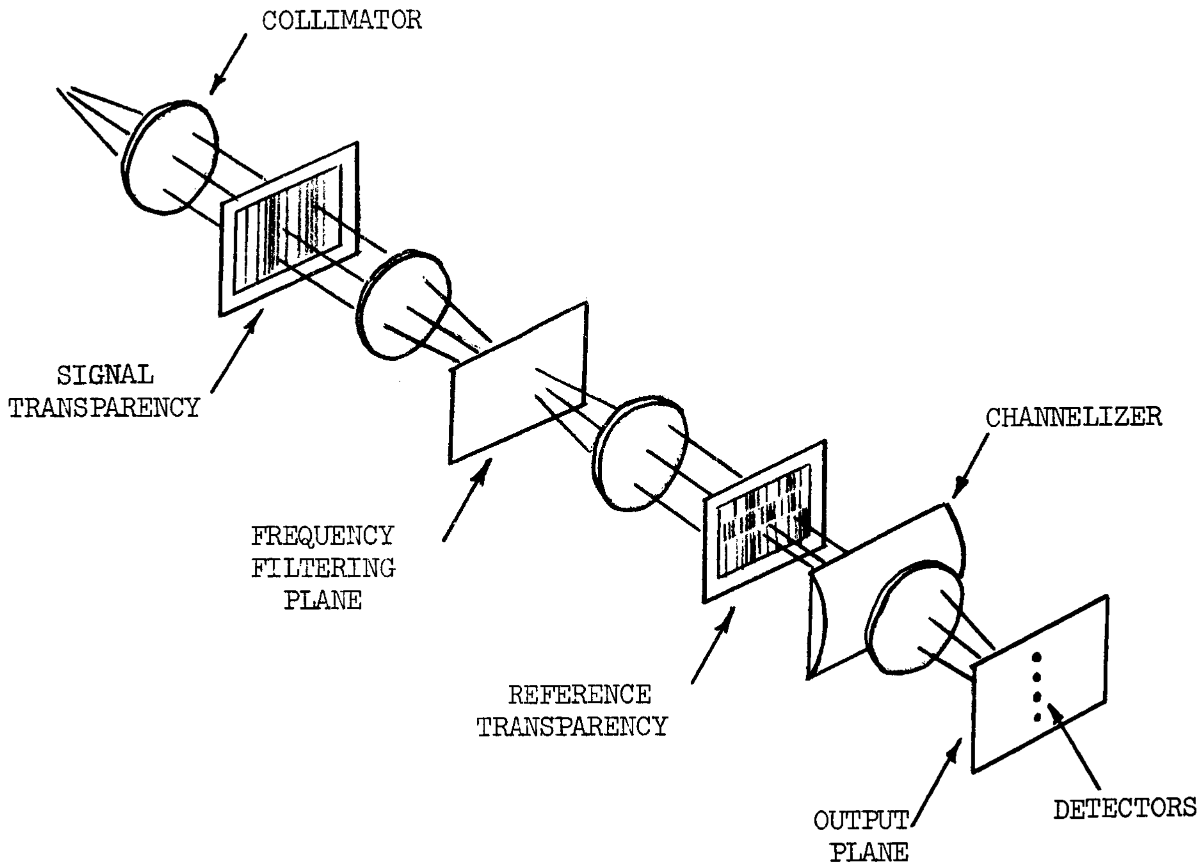
COLLIMATOR
SIGNAL TRANSPARENCY
FREQUENCY FILTERING PLANE
REFERENCE TRANSPARENCY
CHANNELIZER
OUTPUT PLANE
DETECTORS

Fig. 1.

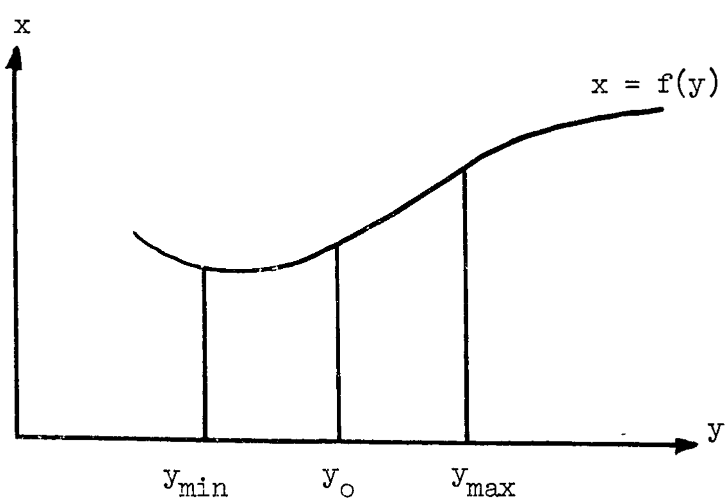
x
x = f(y)
y
y_{min}
y_o
y_{max}

Fig. 2.

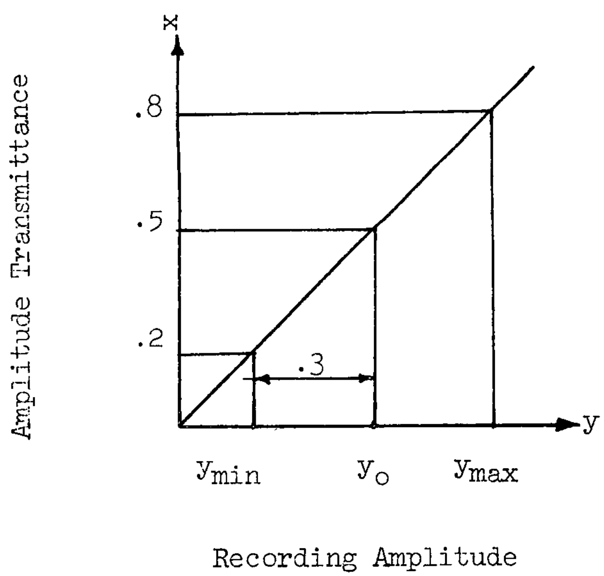

Fig. 3.

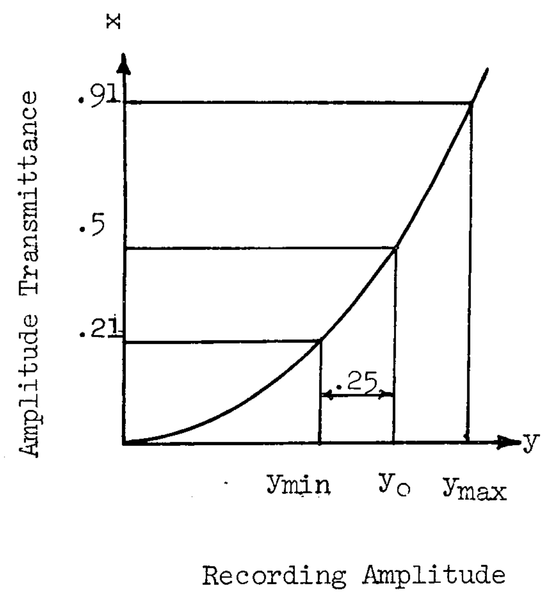

Fig. 4.

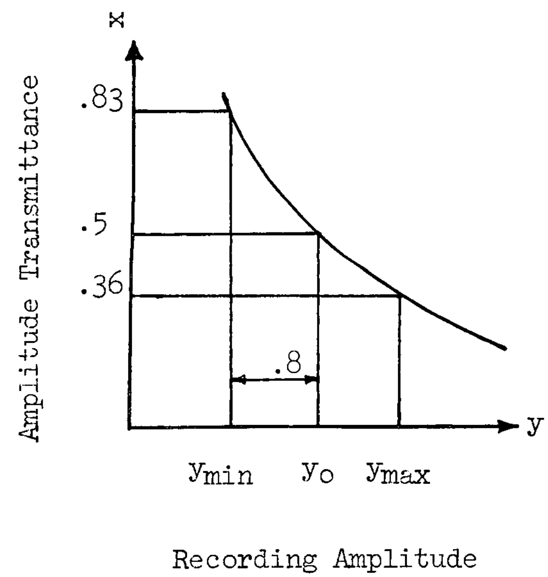

Fig. 5.

APPENDIX

A. Expansion of powers of cos θ

$$\cos^2 \theta = 1/2 \ (1 + \cos 2\theta)$$

$$\cos^3 \theta = 1/4 \ (3 \cos \theta + \cos 3\theta)$$

$$\cos^4 \theta = 1/8 \ (3 + 4 \cos 2\theta + \cos 4\theta)$$

$$\cos^5 \theta = 1/16 \ (10 \cos \theta + 5 \cos 3\theta + \cos 5\theta)$$

$$\cos^6 \theta = 1/32 \ (10 + 15 \cos 2\theta + 6 \cos 4\theta + \cos 6\theta)$$

B I B L I O G R A P H Y

[1] Instruction Manual for the C-100 Cross Correlator Conductron Corporation, Ann Arbor, Michigan.

[2] A. R. Shulman: Principles of Optical Data Processing for Engineers. Report No. X-521-66-434 Goddard Space Flight Center, Greenbelt, Maryland.