NASA CONTRACTOR REPORT

NASA CR-61160

NASA CR-61160

VARIABILITY OF SOUND PROPAGATION PREDICTION DUE TO ATMOSPHERIC VARIABILITY

Prepared under Contract No. NAS 8-11348 by
C. Eugene Buell

KAMAN AIRCRAFT CORPORATION
Kaman Nuclear Division

For

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER
Huntsville, Alabama January 1967

January 1967 NASA CR-61160

VARIABILITY OF SOUND PROPAGATION PREDICTION DUE TO ATMOSPHERIC VARIABILITY

By

C. Eugene Buell

FINAL REPORT
(Dated June 10, 1966)

Prepared under Contract No. NAS 8-11348 by

KAMAN AIRCRAFT CORPORATION
Kaman Nuclear Division
Garden of the Gods Rd.
Colorado Springs, Colo.

NASA-GEROGE C. MARSHALL SPACE FLIGHT CENTER

TABLE OF CONTENTS

TABLE OF CONTENTS

TABLE OF CONTENTS

TABLE OF CONTENTS

TABLE OF CONTENTS

TABLE OF CONTENTS

TABLE OF CONTENTS

TABLE OF CONTENTS

SUMMARY

The effect of the variation of meteorological parameters on the estimates of sound intensity at distances from a few to several tens of kilometers is discussed. The meteorological parameters concerned are virtual temperature, wind components, and the humidity (water vapor pressure specifically). Sound intensity estimates are based on the results of a ray tracing technique.

The variations of intensity estimates as determined by Monte Carlo methods are discussed in Chapter II. It is found that the variability is in the neighborhood of 5 decibels regardless of whether the magnitude of atmospheric variability corresponds to that expected over a fraction of an hour or to a half a day. A significant factor is the determination of the probability of returning sound rays at a given location; in other words, whether the ray tracing method yields a sound intensity estimate or none at all.

The basic characteristics of the variability of wind, temperature, and vapor pressure are discussed in Chapter III. This information is required as an input for the calculations reported in the previous chapter.

The effect of atmospheric variability on sound attenuation is of a different character from that due to the effect on the geometry of the propagative pattern and may be determined by standard methods. The results of the analysis, in Chapter IV, indicate that the variability of atmospheric humidity is the most important factor. The small scale perturbations have little effect, but the large scale perturbations that enter into

variations expected over a period of half a day may account for intensity variability of several (near 5) decibels at 10 km.

The appendices include several topics that are of importance but which would interrupt the trend of the presentation if included previously. Appendix A is devoted to a discussion of the ray tracing technique. Several well known methods are presented in some detail. The physical assumptions behind ray tracing methods are discussed. It is found that the ray tracing procedure as applied to the problem at hand (sound (noise) with a broad spectrum and with appreciable intensity in the low frequency bands) seriously violates these physical assumptions. The situation may be eased somewhat (but not completely) by a modification of the interpretation of the atmospheric measurements. When the data points of a vertical sounding through the atmosphere are joined by straight lines, the description is quite adequate for most purposes, but the presence of "corners" is physically unacceptable for sound propagation estimates using ray tracing. A method is proposed in which the sounding is described by using parabolic arcs that join the data points in as "smooth" a way as possible.

The mathematical problems involved in a theoretical discussion of the variability of sound intensity estimates are discussed in Appendix B.

Appendix C is a summary of several methods of constructing sequences of random numbers with given second order properties (standard deviations and internal correlations) from independent random numbers.

CHAPTER I

INTRODUCTION

A. THE PROBLEM

It is required to estimate the variations expected in sound intensity level from an intense noise source located on the ground at other points on the ground at distances from a few to several tens of kilometers from the source which are due to meteorological factors.

To quote L. M. Brekhovskikh(1)* (p. 446), "....we shall investigate the field from a concentrated source situated in a layered, inhomogeneous medium. This is one of the foremost problems in modern radiophysics, acoustics, and the physics of the earth's crust." Needless to say, no pat solution has been reached.

The basic technique that suggests itself is the elementary ray tracing method. The ray tracing technique has been used to study various aspects of anomalous sound propagation. The early estimates of the temperature structure near the ozone layer were based on this method(2)(3). The procedure for estimating the possibility of sound damage from high energy explosions also used this method (4).

---

*Raised numbers in parentheses indicate the corresponding reference to be found just before the illustrations and following the text.

To delineate the problem a little more clearly, the reasonably well-behaved aspects of sound propagation in the atmosphere are excluded from consideration. In the first instance, much work has been done on the effect of the atmosphere on the propagation of sound at ultrasonic frequencies. In this case, the frequencies are sufficiently high that there is no question of using the limiting form or ray acoustic approximations. The path lengths are relatively short; say, of a few hundred meters down to the order of few meters. The depth of the atmosphere involved is either negligible or of the order of a few meters or tens of meters. As a consequence, the atmospheric parameters may be considered as highly simplified. Well-known relations may be used to estimate the "average" vertical distribution of wind and sound speed. The variation of these on still smaller scales then plays the predominant rôle in the propagation problem.

On the larger scale, the major causes for refraction earthward in the atmosphere lie at the ozone level (near 40 km in the vertical). The variation of parameters over layers of the order of magnitude of a few kilometers becomes relatively unimportant. The major anomalies of the sound propagation may then be traced without paying particular attention to details of the atmospheric parameters even though the variations of these details are easily measured -- in fact, regularly observed.

In both of the above "extreme cases," there is usually no question but that the methods of ray acoustics are applicable with some degree of accuracy.

When the ray method is applied to the problem at hand, this basic procedure for estimating sound intensity suffers from two distinct disadvantages. First, it does not always give an estimate of the sound intensity. Second, its application in several instances is physically unsound.

In those cases in which the ray method does not provide a sound intensity estimate, it is possible to make some kind of an estimate on the basis of the sound scattered and diffracted into the region concerned.

Where the ray method does give answers, it is possible to estimate the variations of sound intensity due to the variation of atmospheric parameters, at least in part; but it is not possible to determine what part of the observed variation of intensity is due to the variation of these parameters and how much was due to the fact that the ray method should not be applied at all.

B. THE RAY TRACE DESCRIPTION

The method of treatment in the following sections is based on the following quantification of sound intensity estimation. If the ray tracing method can be applied:

$$I = I_* f e^{-\alpha r} , \quad f = |r/(dr/d\psi_o)\tan\psi_o|$$

where

I = sound intensity estimate

I_* = sound intensity from spherical spreading

f = focusing factor

α = attenuation coefficient

r = source-to-receiver distance

ψ_o = initial inclination of a sound ray

The total intensity is then a sum of intensities over all the arriving rays

$$I_T = \Sigma g I$$

where the symbolic summation includes "direct" rays arriving at the receiver and all rays that might arrive at the receiver after one or more reflections from the ground.

Several situations are included in the preceding summation. In the first place, there may be two or more direct rays arriving at the receiver. All of these are included. A particular example is the case of a "focus." This occurs at that distance, r_f, for which $dr/d\psi_o = 0$ so that f is "infinite". (This case is not being discussed here since it requires special treatment.) For a distance somewhat larger than r_f, there are two rays arriving at the receiver. The intensity values associated with each are included in the summation. In these situations, the undetermined factor, g, is taken as unity.

Another situation included in the summation is that which occurs with a ground based "inversion" of the speed of sound and wind component. The value of $c + u$ increases with height to a maximum and thereafter decreases. In such a situation, there are rays which return to the ground associated with ray inclinations $\psi_o = 0$ to $\psi_o = \psi_{max}$. The ray ψ_{max} is that associated with the maximum value (need be local maximum only) of $c + u$. In such a case, the ray tracing procedure indicates that no rays will return in some interval or, if an appropriate approximation is used, will be associated with a very small focusing factor. In this situation, sound reflected from the ground from distances $r/2$, $r/3$, $r/4$, etc., may arrive at the receiver after 1, 2, 3, etc., reflections. These rays contribute to the summation, and the undetermined factor, g, includes the reflection coefficient for the ground, etc.

When the ray description fails to provide an estimate of the sound intensity as described above, the situation is one in which there is a decrease of the sum $c + u$ from the ground upward (to some level, at least). In this situation, the sound ray corresponding to $\psi_o = 0$ is bent upward and no direct sound rays are received (in the ray tracing technique) out to at least some distance away (or maybe not at all). The sound reaching the ground in this instance is considered to come from two sources, that diffracted into the region and that scattered into it.

C. COORDINATE SYSTEM

The coordinate system used throughout is a right-handed rectangular system tangent to the earth. The (x,y)-plane is considered as tangent to the earth with the sound source at the origin. The z-axis is directed vertically and is considered positive upwards. Since the propagation distance is less than 100 km, there are no corrections made for the curvature of the earth. Some departures from this system are made, but what is intended is generally clear (we hope). Such variations usually consist of a shift in origin that permits omission of an additive constant of no importance.

Since the sound propagation is nearly planar in radial planes through the source, the (x,z)-plane is taken in the direction of propagation. Reference to other directions requires a rotation of coordinates. Wind components (u,v,w) are resolved in the above system. Wind observations are, of course, made with respect to north for the reference wind direction and must be resolved into components in accordance with the particular plane of sound propagation being considered.

D. NOMENCLATURE AND TERMINOLOGY

The nomenclature and terminology used herein are strictly that of standard good practice in both meteorology and mathematics. This is particularly important in connection with such terms as gradient, lapse rate, and inversion. By the gradient, the signed derivative is always implied; such as df/dz for a scalar function of one coordinate or the vector whose components are $\partial f/\partial x$, $\partial f/\partial y$, $\partial f/\partial z$ for a scalar function of three coordinates. The term lapse rate is used in exactly the sense the words would have in ordinary usage (and in meteorological usage); i.e., the rate of "falling off" or of lapse. Invariably this rate is taken in the vertical direction only. The expression is used only in reference to temperature. The lapse rate (of temperature) is the negative of the gradient of temperature in the vertical direction. Its meteorological usage is easily justified to get rid of the incessantly present minus sign that would tag along with the temperature gradient. (This simple situation seems to have caused abundant confusion[5][6][7]. An "inversion" usually applies to the temperature structure in the vertical and indicates a situation in which, instead of decreasing, the temperature increases with height; a positive temperature gradient, or a negative lapse rate. The term has been used (perhaps badly) herein to indicate a condition in which $c + u$ increases with height up to a certain point (the "top" of the "inversion").* An "increasing" temperature gradient is used only to indicate a positive second derivative, $d^2f/dz^2 > 0$; and the term "increasing" is <u>never</u> used to indicate the sign of the gradient. When the temperature is increasing with altitude it is described as having a positive gradient (or negative lapse rate), etc.[5]

*Words that appear in quotation marks are being used in a special sense and should be interpreted in the particular context required.

E. OUTLINE OF TREATMENT

Empirical results on the variability of sound intensity estimates by the ray tracing method are described in Chapter II. The background material that was required for specification of atmospheric variability is discussed in Chapter III. Chapter IV contains an estimate of the variability of attenuation as dependent on variability of atmospheric parameters. The conclusions presented in Chapter V repeat in abbreviated form the results stated at some length in Chapters II and IV.

In order to keep the material of the chapters conveniently abbreviated, there is a sequence of appendices followed by the list of references. These contain the technical details which, if included where the questions initially come up, might confuse the trend of the treatment.

CHAPTER II

VARIABILITY OF SOUND INTENSITY ESTIMATES

A. RAY TRACING DESCRIPTION

Practical estimation of sound intensity at a distance is usually approached by the ray-tracing method. The application of this method dates from the time of Lord Rayleigh with many modifications by various authors(26)(32) depending on the basic physical assumptions and assumptions regarding the structure of the atmosphere. Although differences in the ray-tracing equations lead to somewhat different results, these differences are of relatively less importance than the way in which the atmosphere is described and the basic inaccuracy of measurement or variability of the atmospheric parameters that enter into these equations. The general ray-tracing technique is discussed in Appendix A.

The ray-tracing equations used herein are:

$$x_2 - x_1 = -R(\sin\psi_2 - \sin\psi_1) \qquad (1)$$

$$\tan\psi_1 = c_1 \sin\varphi_1/(c_1 \cos\varphi_1 + u_1)$$

$$\tan\psi_2 = c_2 \sin\varphi_2/(c_2 \cos\varphi_2 + u_2) \qquad (2)$$

$$\cos\varphi_1 = c_1 \cos\varphi_o / [c_o - (u_1 - u_o)\cos\varphi_o]$$
$$\cos\varphi_2 = c_2 \cos\varphi_o / [c_o - (u_2 - u_o)\cos\varphi_o] \qquad (3)$$

$$R^{-1} = [(c_2 - c_1)\cos\varphi^* + (u_2 - u_1)]/c^*(z_2 - z_1)$$
$$\cos\varphi^* = (\cos\varphi_1 + \cos\varphi_2)/2 \qquad (4)$$
$$c^* = (c_1 + c_2)/2$$

where

$x_2 - x_1$ = horizontal distance traveled in the layer from z_1 to z_2

ψ_1, ψ_2 = ray tangent angles at levels z_1, z_2

$\varphi_o, \varphi_1, \varphi_2$, = phase normal angles at the source and levels z_1, z_2

c_o, c_1, c_2, = speed of sound at the source and levels z_1, z_2

u_o, u_1, u_2 = horizontal wind speed components in the plane of propagation at the source and levels z_1, z_2

R = radius of curvature of the ray in the layer from z_1 to z_2

The derivation of these (and other) ray-tracing equations is discussed in Appendix A. The basic assumptions are that the speed of sound and wind component change linearly throughout layer and that the wind speed is small compared with the speed of sound. The distinction between phase normal and ray tangent is maintained, although a relatively minor point. These relations are subject to the basic objections that may be raised to any ray tracing method, that they do not adequately satisfy the basic physical assumptions. This point is rather important and is discussed in Appendix A, also. The variability of sound intensity estimates may be discussed reasonably without laboring this point here.

The above equations give the horizontal distance traversed as the ray passes through the layer from z_1 to z_2. For an atmosphere consisting of several such layers, the distances are added. If the ray returns to earth, the value of φ (and ψ) must become zero in some layer. The total horizontal distance traversed will be twice the distance traversed to reach the level where $\varphi = \psi = 0$;

$$r = 2 \sum_{i=o}^{n} (x_{i+1} - x_i) \tag{5}$$

where r is the total horizontal distance traveled by the ray, and the ray reaches its maximum altitude between the levels z_n and z_{n+1}.

The atmospheric parameters enter into the problem directly as u_1, u_2 (wind components along the plane of the ray) and indirectly through c_1 and c_2, $c = 20.0468\sqrt{273.15 + T^*}$ (mps) where T^* is the virtual air temperature in degrees centigrade.

The intensity of the sound is obtained from the relation

$$I = I_* f e^{-\alpha r} \tag{6}$$

where I_* is the sound intensity due to spherical spreading and f is the focusing factor given by

$$f = |r/(\partial r/\partial \psi_o) \tan\psi_o| \tag{7}$$

in which ψ_o is the initial inclination angle of the ray. The factor $e^{-\alpha r}$ accounts for atmospheric attenuation of various kinds. For the present purposes, this factor is neglected, and the value of 0 is assigned to α.

If more than one sound ray arrives at a point, the total sound intensity is obtained by adding the individual ray contributions

$$I = \sum_{i} g_i I_i \qquad (8)$$

where I_i is the intensity due to each "ray" and g_i is a function of how the sound ray arrives. For direct air transmission, $g_i = 1$. (Sound ducted along the ground would require that g_i include the ground attenuation, the number of ground reflections, etc.)

B. SPECIFICATION OF THE PROBLEM

The problem of estimating the variability of sound involves working backwards through the equations of the preceding section. The total intensity from (8) is dependent upon the focusing factor (7) for the rays that return to the point concerned. This factor (and whether any rays return at all at this point) is determined from the summation (5) of the quantities from (1) through (4).

A variational or perturbation treatment presents difficulties. These are described in Appendix B. The present analysis is confined to an empirical approach using equations (1) through (8). There are severe restrictions in this method, but the reliability of sound intensity variability estimates seems reasonably significant. The principal restriction lies in the basic assumption of horizontal homogeneity in the atmosphere. To relieve this restriction, the computation difficulties are increased many fold. In addition, the basic information on variability of the wind and temperature that would be required is not available.

The specification of the problem also involves specification of the variability of the atmospheric wind and temperature. Since this is reasonably important, the subject is discussed in some detail in Chapter III as it applies to this problem. Three levels of variability are considered for practical purposes.

First, small variability that would correspond to the changes to be expected over an interval of a fraction of an hour. This would correspond to estimating sound intensity from atmospheric measurements with a time lag that would allow only for completion of the intensity computations.

Second, moderate variability that would correspond to changes expected over a period of four to six hours. This would roughly correspond to a reasonable minimum planning time for test operations.

Third, a variability that would correspond to a time lag of eight to twelve hours.

(See Chapter III for more explicit specification in the last two cases.)

C. RESULTS ON VARIABILITY OF INTENSITY DUE TO DIRECT RAYS

1. Empirical Approach

The empirical approach used to obtain estimates of sound intensity variability was to compute the sound intensity for direct rays returning to the ground at various distances from a hypothetical source given a specific initial atmospheric sounding. The sounding was then perturbed by a specific amount to correspond to what might be expected at a later time. Such perturbations to the sounding were repeated 100 times, and the results were compiled in tabular form including the mean sound intensity level, the standard deviation of the intensity level, and the number of cases in which no direct rays returned at the point concerned.

Since the perturbations of the sounding were to correspond to realistic changes that might be expected in the atmosphere, care was taken to assure the proper interlevel correlation of the perturbations. For short time lags, the general character of the sounding remains unchanged, but small changes occur in temperature and wind with a correlation distance of only a few hundred meters. For longer time lags, changes have a correlation distance of seven kilometers. The proper perturbations were applied by starting with a set of independent random numbers (mean zero, variance one), and then forming a set of acceptably intercorrelated random numbers with the proper standard deviations by means of a linear transformation of the initial set. (See Appendix C for details.)

2. Results

a) Illustrative Cases

Figures 1 through 7 illustrate several case of the variability of sound intensity estimates (direct rays) due to the variability of atmospheric wind and temperature. Each of these figures is divided into four parts: a, b, c and d. For each figure, the parts illustrate the following information:

Part a - Graphical representation of c + u as a function of altitude. (c = speed of sound, u = wind component in the plane of propagation, both meters/second.) Altitude is plotted vertically in meters.

Part b - The percent of occasions when no ray returned directly at the distance shown by the abscissa (kilometers). Curves along the bottom indicate a ray returned to the ground in nearly all cases, near the top in almost none of the cases.

Part c - The standard deviation of the sound intensity level (in decibels) for those occasions when a ray returned at the distance shown by the abscissa (kilometers).

Part d - The average sound pressure level (decibels) for those occasions when a ray returned at the distance shown by the abscissa (kilometers). The sound pressure level indicated is that for the rays that return directly to the point indicated. No correction is made for attenuation. No correction is made for return of sound after one or more reflections from the ground between the sound source and the distance indicated.

In parts b, c and d, there are three curves shown corresponding to different levels of atmospheric variability:

Solid with dots: corresponds to variability expected over a very short time (materially less than an hour).

Dashed: corresponds to one-fourth the natural variability or to a time lapse of approximately four to six hours between sounding and sound transmission.

Solid: corresponds to one-half the natural seasonal variability or to a time lapse of eight to twelve hours between the meteorological sounding and the sound transmission.

i) Case I, Figures 1a, 1b, 1c and 1d

The variation of $c + u$ with height shown in Figure 1a indicates a rapid increase of $c + u$ with height from the ground to near 1000 m with irregular variations of $c + u$ above characterized by a very weak trend to larger values at higher elevations.

The fraction of rays returned to the ground at distances to near 12 km is nearly 100%. At distances from 15 to 20 km, few rays are returned due to the fact that the parameter $c + u$ (Figure 1a) shows a well defined local maximum near 1000 m. At larger distances, the proportion of returning rays increase steadily due to the increasing trend of $c + u$ with altitude at higher altitudes.

The standard deviation of intensity in the range from 13 to 30 km (Figure 1c) is unusually large in all cases (10 to 20 db) with wide variation between the three levels of atmospheric variability. This reflects primarily the fact that there were relatively few cases of returning rays.

ii) Case II, Figures 2a, 2b, 2c and 2d

Reference to Figure 2a indicates the presence of a reasonably strong "inversion" with the maximum of $c + u$ at 900 meters, but with a secondary small "inversion" at the ground. In the neighborhood of 4000 meters and above, the values of $c + u$ exceed these at lower levels and are increasing with altitude.

The trend of the three curves in Figure 2b clearly indicates (1) the effect of the small inversion near the ground by the presence of a high percentage of returning rays in the range from 0 to 4 km, (2) the effect of the peak of $c + u$ at 900 m in returning rays in the range from 9 to 14 km, and (3) the effect of the upper increase in $c + u$ at high levels in the returning rays at distances beyond 30 km.

iii) Case III, Figures 3a, 3b, 3c and 3d

Figure 3a indicates a steady decrease of $c + u$ from the ground to near 1000 m followed by a reasonably rapid increase to near 1800 m and by a slow increase from that level upward.

The fraction of cases with rays not returning as a function of distance (Figure 3b) is large to near 21 km due to the marked decrease of $c + u$ near the ground. In the case of smallest variability, only one case occurred with rays returning at distances of less than 21 km. The number of cases of returning rays in this interval increases with increasing basic variability magnitude. The local weak maximum of $c + u$ near 2000 m brings rays to the ground between 25 and 30 km in a majority of cases, the most for the small variability situations. Since the maximum near 2000 m is only slightly above $c + u$ at the ground and since $c + u$ increases weakly above the 2000 m level, the proportion of cases of returning rays beyond 30 km is only near 60%.

The large peak in the standard deviation of the sound pressure level near 22 km (Figure 3c) reflects the small data sample in the case of weak basic variability.

iv) Case IV, Figures 4a, 4b, 4c and 4d

The characteristics of $c + u$ in Figure 4a indicate elevated refraction with rays reaching their maximum above 1000 m. Since $c + u$ decreases steadily immediately above the ground, rays return to earth at some distance from the sound source.

The fraction of rays not returning earthward (Figure 4b) is nearly 100% up to near 13 km. Beyond 23 km, nearly all cases indicate rays returned to earth (the effect of the strong increase of $c + u$ at upper levels in Figure 4a). In the zone from 16 to 18 km, a large majority of cases showed ray returns.

The disconnectedness of the dashed curve and the termination of the dot curve in Figures 4c and 4d reflect the fact that none or only a few cases of returning rays occurred at distances less than 15 km.

v) Case V, Figures 5a, 5b, 5c and 5d

The values of $c + u$ decrease irregularly to near 1000 m (Figure 5a) and increase from that level upward, the increase above 2500 m becoming rapid. The situation is similar to Case IV.

Returning rays are generally absent at distances less than 21 km (Figure 5b), some cases occurring for the intermediate and larger variability levels, but none for the small variability level. At distances from 23 to 30 km, there are returning rays in all cases. Beyond 30 km, the fraction of returning rays decreases with increasing distance.

The absence of returning rays is reflected in Figures 5c and 5d by the fact that the dot curve for the smallest variability situation is not shown in the range from zero to 21 km.

vi) Case VI, Figures 6a, 6b, 6c and 6d

The values of $c + u$ increase irregularly from the surface to 2500 m (Figure 6a) and decrease above that level. The overall rate of change of $c + u$ with height throughout is very small, but in some levels, the rate of change is reasonably large.

The fraction of cases in which rays returned at given distances is near 100% beyond 17 km for the cases of small and moderate atmospheric variability, but near 80% for large atmospheric variability.

vii) Case VII, Figures 7a, 7b, 7c and 7d

The values of $c + u$ (Figure 7a) increase from the ground to near 3000 m with a secondary local maximum near 1500 m. Above 3000 m, the value of $c + u$ decreases steadily. The situation is similar to Case VI, (Figure 6a). The feature that might distinguish the two cases is the fact that, while in Case VI there were several minor fluctuations in $c + u$ with height, in Case VII the prominent secondary maximum near 1500 m is the only feature that disturbs a reasonably smooth trend of $c + u$ with height. The total magnitude of the range of $c + u$ is reasonably small.

The fraction of the time that rays returned to the ground at given distances never reaches 100% (Figure 7b), but in all cases, it is largest near 24 km, presumably due to refraction below the secondary maximum of $c + u$ near 1500 m.

b) Conclusions

The seven illustrative cases discussed above (Figures 1 through 7) cover a wide range of atmospheric conditions for which direct rays are returned earthward. The range of variability assigned to the atmospheric parameters samples their variability over a span of from a fraction of an hour to in the neighborhood of half a day.

There are several features of the variability of sound intensity estimates that characterize all of these illustrations. The first of these is the division of the intensity estimate problem into two distinct parts:

i) Whether or not there will be any rays returning directly to a given distance, and

ii) When there are rays returning directly at the given distance from the source, what will be the variability of the sound intensity?

In the first instance, under conditions where $c + u$ aloft is of the order of 10 to 20 m/sec larger than the values near the ground, there are zones where rays return to given distances in nearly all cases. On the other hand, in these and the other cases for any fixed distance, there are some instances of direct rays returning earthward. Consequently, the presence of directly returning rays at a given point is essentially a probability problem. The probability being large at some distances, small at others (and seldom zero in the cases shown).

There seems to be no consistent systematic connection between the size of the atmospheric variability imposed on wind and temperature and either the likelihood of direct rays returning at a given distance or the standard deviation of sound intensity when rays are returned.

The standard deviation of intensity generally lies in the neighborhood of 5 db. In a general way, it appears that if the likelihood of returning rays is not large, the standard deviation of intensity of what rays are returned may be somewhat larger than otherwise. The occurrence (number of cases) of the standard deviation of sound intensity estimates (direct returning rays, decibels) by class intervals against the fraction (percent) of cases in which direct rays failed to return at given distances is shown in Table I. Distances ranged from 5 to 50 km. When rays return in nearly all cases (0-20), the largest number of cases show a standard deviation of intensity estimate in the 2.5-5.0 db range. As the fraction of cases where rays fail to return increases, the largest number of cases moves into the 5.0-7.5 db range.

It is tempting to form a regression of "sigma" on the fraction of cases, but the scatter of the cases and irregularily of details of their distribution clearly indicate that it would be of little or no significance (with the possible exception of the case of the smallest variability level).

The distribution of cases in the column of "totals" indicates that the maximum for the standard deviation of sound intensity estimate (direct returning rays) moves from the 2.5-5.0 db range at the largest variability level. This is what

TABLE I

Occurrence of Standard Deviation of Sound Intensity Estimates (Decibels) as a Function of the Fraction of Cases in which Direct Rays Failed to Return to Given Distances (Percent). Distances Range from 5 to 50 km.

Standard Deviation	% Cases Direct Rays Failed To Return 0-20	20-40	40-60	60-80	80-98	Total
Smallest Variability Level						
0.0-2.5	51	5		2	2	60
2.5-5.0	68	14	2	1		85
5.0-7.5	31	21	21	7	5	85
7.5-10.0	1	3	13	5	11	33
10.0-12.5		3	4	1	3	11
12.5-15.0	1				13	14
15.0-17.5					1	1
Total	152	46	40	16	35	289
Moderate Variability Level						
00.0-2.5	36	3	4	3	18	64
2.5-5.0	72	11	7	7	22	119
5.0-7.5	17	20	28	14	2	81
7.5-10.0	1	6	5	8	2	22
10.0-12.5		2	4		3	9
12.5-15.0					1	1
15.0-17.5					1	1
17.5-20.0					8	8
20.0					6	6
Total	126	42	48	32	63	311
Largest Variability Level						
0.0-2.5	4	1	4	1	1	11
2.5-5.0	53	6	6	10	6	81
5.0-7.5	44	55	43	18	22	182
7.5-10.0		9	12	4	11	36
10.0-12.5			2	10		12
Total	101	71	67	43	40	322

one would expect. This shift is weakly indicated and the dispersion is large in all cases.

With regard to the number of cases in which rays fail to return at a given distance (raw totals for each variability case), the high percentage of returning rays (0-20 column) decreases steadily with increasing degree of atmospheric variability, but only from about half to one-third of the total number of cases. This is expected. The large number of cases in each of the other row categories clearly indicates a moderate probability of returning ray at any given distance.

CHAPTER III

VARIABILITY OF METEOROLOGICAL PARAMETERS

Several aspects of the basic variability of meteorological parameters with application to the problem of the variability of sound intensity are discussed in this chapter. The sound intensity estimates from ray tracing methods depend on the parameters c and u, c = speed of sound, u = wind speed in the plane of propagation. The speed of sound is dependent on several meteorological parameters, the most important of which is the temperature. These are discussed in the first section below. Since the azimuth of the sound propagation plane may be selected at will, the wind component, u, is determined from both u_{EW} and u_{NS}, the two orthogonal components of the wind vector.

The range of interest in the variability of the meteorological parameters extends from the climatological variability to the micrometeorological scale of a few minutes and a few miles in distance. It covers the entire range of the ray paths, which includes the region from the surface to above 20,000 feet (5 km).

In addition to variability in the strict sense, it is also necessary to have a rather clear idea of the errors of measurement of the atmospheric parameters together with the accuracy with which a method of describing atmospheric conditions actually represents the atmosphere.

A basic problem that is involved in ray tracing estimates is the space variation of atmospheric parameters over the ray path. It is supposed that the measurements made (near the source, say) actually represent conditions without error. The path may extend over a distance of many kilometers so that on the downward leg conditions may be different from those on the upward leg.

Measurements by radiosonde techniques are those of the parameter values along the path of the balloon, which may go in a direction quite different from that of the ray concerned. Consequently, the distance induced errors may be largest at the top of the ray path where errors are most critical.

There is always a time lag in a practical situation to permit the accumulation of data, computation of sound intensity estimates, and last minute decisions as required.

A. GENERAL CONSIDERATIONS

1. The Dependence of the Speed of Sound on Atmospheric Parameters

The speed of sound is related to the temperature of the air and its moisture content through the relation

$$c = (\gamma R_d T^*)^{\frac{1}{2}} \simeq 331.6(T^*/273)^{\frac{1}{2}}$$

γ = ratio of specific heats

R_d = gas constant for (dry) air

T^* = the virtual temperature (°K)

The virtual temperature is introduced to make possible the use of the gas constant for dry air instead of using a gas constant for moist air, a variable quantity depending on the amount of moisture. Thus,

$$T^* = T(R_w/R_d)$$

where T = absolute temperature, R_w = gas constant for moist air, or

$$T^* = T/(1 - 3e/8p)$$

($3/8 = 1 - 5/8 \simeq 1-0.6221 = 0.3779$ where 0.6221 is the specific gravity of water vapor as compared with that of dry air at the same temperature and pressure [7a]) where

e = partial pressure of the water vapor

p = total pressure of the damp air

The partial pressure of the water vapor is related to the relative humidity and the temperature

$$e = He_s(T)$$

where H is the relative humidity and $e_s(T)$ is the saturation pressure of the water vapor at the temperature of the air.

Variations in the speed of sound are related to variations in both temperature and relative humidity, thus

$$2\delta c/c = (\delta T/T)[1 + T^*H(3e_s/8p)(de_s/e_s dT)] + (T^*/T)(3e_s/8p)(\delta H).$$

Using nominal values, this is approximately

$$2\delta c/c \simeq (\delta T/T)(1 + 0.074H) + 0.0038(\delta H).$$

The change of relative humidity from 1.0 to 0.0 would bring about a change of the speed of sound equivalent to a 1° change of temperature. The effect of temperature change reflects a small correction due to the fact that saturation pressure increases with temperature, 7.4% at $H = 1$ (100% relative humidity).

In terms of vapor pressure changes

$$2\delta c/c = \delta T/T + (T^*/T)(3\delta e/8p).$$

Since $T^*/T \simeq 1$ and if $p \sim 1000$ mb, then for $\delta e \sim 10$ mb, (an extremely large change) the last term amounts to 0.003 or that due to about 1° C change in temperature.

2. Variability of Meteorological Parameters

The variability of meteorological parameters involves several considerations and may be made a complex subject. Some of the basic considerations are discussed in this section to clearly define the limitations and restrictions of the analysis that follows.

The variability of a meteorological parameter is loosely described as a measure of how and how much a parameter varies as a function of time difference, coordinate difference, level difference, etc. On the basis of these ideas, a fundamental characteristic of the problem is two values of a parameter, say U, at two points 1 and 2, the individual point values being denoted by U_1 and U_2. (We exclude the case where U_1 and U_2 may be different parameters as being outside our present area of applications.) The points 1 and 2 may differ in time or space coordinates or both. The problem then consists of the following -- If the value U_1 is known, what may be said about the probability distribution of U_2 when the coordinate values are specified?

It is readily seen that such a problem may be greatly extended and generalized. To keep the situation reasonably simple, we consider only the mean values and second moment parameters (standard deviations and correlation coefficients or variances and covariances). These quantities carry with themselves an implication that the probability density functions concerned are Gaussian (normal). This is by no means the case, but not enough is known about their non-Gaussian character to be intelligently applied to the situation at hand.

One more complicated situation will be considered. If U_1 is replaced by the ensemble of meteorological information of the past, how well does a "forecast" of U_2, say U_2^f, differ from U_2 itself?

In the following sections, some aspects of the variability of meteorological parameters in general are discussed. It is pointed out that two different quantities are required for the description of atmospheric variability in the simplest terms, the variances or standard deviations of the parameters at a specific level and the covariances or correlation coefficients of the parameter between levels.

a. Variation in Time or Distance

The variation of a wind component or temperature difference with time or distance or both may be expressed as a mean square difference:

$$\sigma^2 = \overline{(U_2 - U_1)^2} = \overline{u_2^2} - 2(\overline{u_1 u_2}) + \overline{u_1^2} = 2\overline{u^2}\ (1 - r)$$

and where $u_1 = U_1 - \overline{U}$ and $u_2 = U_2 - \overline{U}$ are departures from the mean $\overline{U}$, and where the values of time (or distance) 1 and 2 are indicated by subscripts. If it is assumed that

$$\overline{u_1^2} = \overline{u_2^2} = \overline{u^2} = \sigma_0^2$$

(i.e., the standard deviations are time or distance invariant), the last expression follows where r is the correlation coefficient. The correlation coefficient is a function of time (or distance) and has a definite functional form depending on the difference $\Delta t = t_2 - t_1$*. Consequently,

$$\sigma^2/\sigma_0^2 = 2(1 - r(\Delta t)).$$

The correlation coefficient function may be written in the form (7b)

$$r(\Delta t) = 1 - A|\Delta t|^\alpha + |\Delta t|^\alpha \Phi(\Delta t)$$

where A is a constant, α is a parameter such that $0 < \alpha \leq 2$ and $\Phi(\Delta t)$ is a function that converges uniformly to zero in Δt. Then

$$\sigma^2 = 2\sigma_0^2\,[A + \Phi(\Delta t)]\ |\Delta t|^\alpha .$$

---

* These specifications amount to considering $U(t)$ as a stationary homogeneous process. This is not exactly the case in the atmosphere.

If the last term in the above is neglected, then

$$\sigma \simeq \sigma_0 \sqrt{2A}\ |\Delta t|^{\alpha/2} .$$

The preceding form for the correlation coefficient is theoretical in that it is assumed that there are no basic errors of observation. A more realistic form is

$$r_{real} = kr(\Delta t)$$

where $0 < k < 1$ and k is a measure of the observational errors as compared with the natural variability of the parameter concerned. In this case, the variance of the difference is expressed as

$$\sigma^2 = 2\sigma_0^2 [(1 - k) + kA|\Delta t|^{\alpha} + ---] .$$

For reasonably accurate measurements, k is nearly 1 so that the first term in brackets is small (which in part explains why it is overlooked).

The values of α depend on the nature of the process that gives rise to variation of the parameter, u. It may be shown that if the process is differentiable in time, then, $\alpha = 2$. If the value of α is less than 2, considerable care should be taken to inspect the physical processes involved. For example, for $\alpha = 1$ the process may be one that is piece-wise constant with random jumps. The standard textbook example is that of a process that alternates at random between two arbitrary values.

b. Scales and Errors

The nature of the measurement conditions occasionally limits meteorological measuring technique so that the error of measurement cannot be ignored. In addition, the atmosphere is never in a steady state (constant parameters) so that detailed measurements are not repeatable. The size of the disturbances that bring about this constantly varying condition ranges in horizontal size from an appreciable fraction of the circumference of the earth to a few millimeters. The description of such a range of scale is properly done by spectrum analysis. The application of such a treatment to the problem at hand is awkward. In the interest of simplicity of treatment, the scale representation is thought of as dichotomous. Consequently, an atmospheric parameter is considered to consist of three additive parts

$$U = U_a + U_b + U_c$$

where U_a = large scale part, U_b = small scale part, U_c = error. The first will be considered to have a non-zero mean value $\overline{U} = \overline{U_a}$ and the other two will be thought of as having zero means. Then

$$U = \overline{U} + u_a + u_b + u_c$$

where the lower case letters represent departures from the mean. If the mean square difference is formed in this instance, then

$$\sigma^2 = \overline{(U_2 - U_1)^2} = \overline{[(\overline{U}_2 - \overline{U}_1) - (U_1 - \overline{U}_1) + (U_2 - \overline{U}_2)]^2}$$

$$= \overline{[(\overline{U}_2 - \overline{U}_1) + (u_{a_2} - u_{a_1}) + (u_{b_2} - u_{b_1}) + (u_{c_2} - u_{c_1})]^2}$$

so that

$$\sigma^2 = (\overline{U}_2 - \overline{U}_1)^2 + \sigma_a^2 + \sigma_b^2 + \sigma_c^2$$

where

$$\sigma_a^2 = \overline{(u_{a_2} - u_{a_1})^2} , \quad \sigma_b^2 = \overline{(u_{b_2} - u_{b_1})^2} , \quad \sigma_c^2 = \overline{(u_{c_2} - u_{c_1})^2}$$

The cross products with $\overline{U}_2 - \overline{U}_1$ drop because $(\overline{u_{a_2} - u_{a_1}}) = 0$, etc., and the cross products of lower case terms drop because it is assumed that the values in each category are independent of those in any other category.

This description of the variability of U leaves much to be desired, but accounts for more than is really known about atmospheric variability excluding a few highly specialized studies.

One particular weakness is the treatment of the error term. Such an error description actually describes only a small fraction of the error situation. For example, errors that result in a bias are not accounted for. This type of error is reasonably common in some meteorological data, particularly observations from atmospheric soundings to high altitudes. Fortunately, we are interested in only the lower 3 to 5 km of the atmosphere where such a bias is small or negligible.

c. Covariances and Correlation Coefficients

The basic correlation coefficient, r, of section a, is defined from the relation

$$\sigma_1 \sigma_2 r_o = \overline{(U_1 - \overline{U}_1)(U_2 - \overline{U}_2)}$$

$$\sigma_1^2 = \overline{(U_1 - \overline{U}_1)^2} , \quad \sigma_2^2 = \overline{(U_2 - \overline{U}_2)^2}$$

where the bar over the symbols indicates an appropriate mean value (expectation).

The subdivision of U of the previous paragraph leads to

$$\sigma_1{}^2 = \sigma_{a_1}{}^2 + \sigma_{b_1}{}^2 + \sigma_{c_1}{}^2 \,, \quad \sigma_2{}^2 = \sigma_{a_2}{}^{2'} + \sigma_{b_2}{}^2 + \sigma_{c_2}{}^2$$

and to

$$\sigma_1 \sigma_2 r_0 = \overline{u_{a_1} u_{a_2}} + \overline{u_{b_1} u_{b_2}} + \overline{u_{c_1} u_{c_2}} \quad .$$

The final term

$$\overline{u_{c_1} u_{c_2}}$$

is set equal to zero on the basis of the assumption that the errors in measuring U at points 1 and 2 are independent. Now let

$$\sigma_{a_1} \sigma_{a_2} r_a = \overline{u_{a_1} u_{a_2}}$$

$$\sigma_{b_1} \sigma_{b_2} r_b = \overline{u_{b_1} u_{b_2}}$$

define the correlation coefficient for the large scale part, r_a , and the small scale part, r_b . Then

$$\sigma_1 \sigma_2 r_0 = \sigma_{a_1} \sigma_{a_2} r_a + \sigma_{b_1} \sigma_{b_2} r_b$$

so that

$$r_0 = (\sigma_{a_1}/\sigma_1)(\sigma_{a_2}/\sigma_2) r_a + (\sigma_{b_1}/\sigma_1)(\sigma_{b_2}/\sigma_2) r_b \quad .$$

Each of the ratios (in parentheses on the right-hand side) has a form like

$$(\sigma_{a_1}/\sigma_1)^2 = \sigma_{a_1}{}^2/(\sigma_{a_1}{}^2 + \sigma_{b_1}{}^2 + \sigma_{c_1}{}^2)$$

which is numerically less than 1. If the observations had been made without error, $\sigma_{c_1} = \sigma_{c_2} = 0$, the corresponding correlation coefficient would be indicated by the symbol r_*.

d. Reconstruction of a Sounding

Let U_i represent a sequence of values of U as observed from a meteorological sounding. The serial values of $i = 1, 2, ---,$ indicate the levels at which the parameter U is observed, Z_i. It is required to construct a hypothetical sounding, $U_i{}^*$, that is similar to the given sounding but which differs from it in a way that allows for the basic variability of the atmosphere.

Let

$$U_i^* = U_i + u_i$$

where u_i is an increment added to U_i to give a new value U_i^*. The mean square value of this increment is

$$\overline{(U_i^* - U_i)^2} = \overline{u_i^2} = \sigma_i^2 \ .$$

If the variation from U_i is assumed to be described by a Gaussian (or normal) distribution, the addition of a random number at each level, i, with zero mean and standard deviation σ_i would be satisfactory. On the other hand, the numbers to be added at the different levels are not independent. In other words, the random increments, u_i, at the various levels must be correlated with each other in a way that describes the variability of the atmosphere. In other words, if i and j are two levels, then

$$\overline{u_i u_j} = \sigma_i \sigma_j r_{ij}$$

where r_{ij} is the coefficient of correlation between u_i and u_j. The problem is then to construct a sequence of values u_i in such a way that each has a standard deviation σ_i but so that they are interrelated with a given correlation matrix r_{ij}.

There are two common methods to construct sequences of numbers with the required properties from a sequence of independent random numbers with zero mean and unit variance; by

i) moving averages

ii) by linear equations

The method of moving averages is formalized by the statement

$$u_i/\sigma_i = \sum_{j=1}^{k} a_j \; x_{j+i}$$

where x_{k+i} are members of a sequence of k + N random numbers, (N = number of levels required, i = 1, ---, N). The values of k and of a_1, ---, a_k are to be determined in such a way that u_i and u_j are correlated in the specified manner. This method has the advantage of simplicity in application but the problem of finding the a_j's for a given correlation coefficient matrix, r_{ij}, is not easily solved in general. (Appendix C)

The linear equations method consists of expressing the values of u_i/σ_i in the form

$$u_i/\sigma_i = \sum_{j=1}^{N} a_{ij} x_j$$

where the x_i's are N independent random numbers with zero mean and unit varience. The coefficients, a_{ij}, are to be determined in such a way that u_i, u_j are properly correlated. Methods for determining the coefficients, a_{ij}, are discussed in Appendix C.

Matching levels of the sounding with those for which the correlation coefficient matrix are known must be made for practical applications. This is handled in a reasonable manner by forming the sequence u_i for the levels corresponding to the correlation coefficient matrix and then interpolating for the levels indicated in the sounding. In terms of correlation coefficients at the sounding levels, this procedure is equivalent to using correlation coefficients interpolated from the correlation coefficient matrix.

B. CLIMATOLOGICAL VARIABILITY, WIND AND TEMPERATURE

The climatological variability of an atmospheric parameter, U, consists of a measure of its standard deviation at a point together with the two point correlation coefficients (or covariances) relating the quantity at points 1 and 2, where 1 and 2 may differ in time, altitude, geographical coordinate, or any combination thereof as required. The details of the second group of parameters is generally very restricted for the atmosphere. The adjective "climatological" refers generally to the mean value with respect to which the quantities concerned are computed. The usual usage (adopted herein) is that the appropriate mean is based on a substantial period of record (several years).

The meteorological parameters do not have constant means so that the meaning of the terms is further restricted to eliminate or to adequately account for such items as the annual and diurnal changes that occur. The parameter mean is not only a function of level and location, but also a function of time (of day and hour). (Long term trends are neglected.)

1. Standard Deviations of Wind Components and Temperature

Standard deviations of meteorological parameters in the neighborhood of the Marshall Space Flight Center were extracted from standard climatological tabulations and are shown in the accompanying table. The climatological standard deviation of temperature must be modified to an approximate speed of sound value for comparison purposes. This may be estimated from

$$\Delta c \cong (c/2T)(\Delta T) \cong 0.61(\Delta T) .$$

The wind and temperature standard deviations display the same sort of variations with season, but show different variations with altitude. The standard deviations of wind increase with altitude while those of temperature decrease. The approximate

TABLE II

Standard Deviations of Wind Components and Temperatures Estimated in the Neighborhood of Huntsville, Alabama (8), (9). The Wind Observations Were Those of 1500 GMT While the Temperatures Were Those at 0300 GMT.

	Winter	Spring	Summer	Autumn
950 mb				
σ_T(°C)	7.1	5.3	2.7	4.9
850 mb				
σ_u (m sec^{-1})	6.2	6.3	5.1	7.5
σ_v (m sec^{-1})	8.3	7.9	4.4	6.4
σ_T(°C)	5.9	5.0	2.4	4.2
700 mb				
σ_u	12.1	8.3	5.7	8.5
σ_v	8.9	8.6	5.1	7.5
σ_T	5.0	3.9	2.0	3.6
500 mb				
σ_u	10.6	11.8	6.5	11.5
σ_v	12.0	10.4	5.7	9.8
σ_T	4.4	3.2	1.9	3.5

equality of the standard deviations of the two wind components is evident. (The individual component standard deviations are to be used and <u>not</u> the vector standard deviation.) Though the wind component and temperature standard deviations are of the same order of magnitude, the numerical value of the temperature "sigma" is roughly half those of wind and when adjusted by the factor 0.61 to convert to speed, they are reduced to $\frac{1}{4}$ to $\frac{1}{3}$ the wind values. Consequently, the wind may be expected to play the dominant role in introducing errors in intensity of sound estimates on a climatological or forecast basis.

2. Interlevel Wind Component Correlations

The correlation coefficients for wind components at various levels are reasonably available for stations throughout the United States either at standard pressure levels or at standard altitude levels. Tables IV and V illustrate values that may be reasonably representative of values pertaining to the area near Huntsville, Alabama. The correlation coefficients are given at standard pressure levels. The conversion of altitude levels on the basis of a standard atmosphere is shown in Table III.

3. Interlevel Temperature Correlations

Data on interlevel temperature correlations are not as readily available as for wind. The example shown in Table V pertains to combined data from Washington, D. C., and Tampa, Florida. The character of the interlevel temperature correlations differs radically from that of the wind components.

TABLE III

Standard Pressure Levels in Terms of Altitude Based on a Standard Atmosphere

Pressure Level (millibars)	Altitude (kilometers)	Pressure Level (millibars)	Altitude (kilometers)
1000	0.11	250	10.36
950	0.54	200	11.79
850	1.46	150	13.62
700	3.01	100	16.64
600	4.20	80	17.64
500	5.57	50	20.64
400	7.18	20	26.59
300	9.16	10	31.20

TABLE IV

Interlevel Wind Component Correlation Coefficients (Interpolated from Nashville and Montgomery[10]). The Level is Given in Millibars (Table III for Conversion to Kilometers), Standard Deviation in mps. The Interlevel Correlations for the East Pointing Component Are Shown Above the Diagonal (1.000) and for the North Pointing Wind Component Below the Diagonal.

SUMMER

LVL		950	850	700	500	400	300
	SD	5.4	5.8	6.6	8.4	9.7	12.0
950	5.4	1.000	.748	.494	.344	.244	.148
850	6.4	.782	1.000	.707	.534	.433	.230
700	7.0	.530	.738	1.000	.758	.661	.552
500	9.0	.430	.540	.728	1.000	.854	.740
400	10.5	.323	.444	.634	.846	1.000	.841
300	13.1	.231	.330	.532	.720	.830	1.000

WINTER

LVL		950	850	700	500	400	300
	SD	5.2	5.7	5.9	6.8	9.2	11.6
950	5.6	1.000	.776	.498	.276	.160	-.016
850	6.4	.770	1.000	.607	.427	.302	.180
700	7.4	.598	.774	1.000	.630	.547	.452
500	8.8	.424	.572	.716	1.000	.814	.660
400	10.8	.330	.498	.652	.824	1.000	.837
300	14.0	.232	.388	.550	.750	.860	1.000

TABLE V

Interlevel Temperature Correlations (Averaged Values for Washington, D. C., and Tampa, Florida[11]). Levels Are Given in Kilometers and Standard Deviations in Degrees Centigrade.

SUMMER

Level	SD	Sfc	1	2	4	6	8
Sfc	3.3	1.000					
1	1.9	.591	1.000				
2	1.7	.355	.800	1.000			
4	1.7	.266	.520	.658	1.000		
6	1.9	.278	.507	.655	.786	1.000	
8	2.3	.290	.504	.670	.664	.860	1.000

WINTER

Level	SD	Sfc	1	2	4	6	8
Sfc	5.8	1.000					
1	5.8	.731	1.000				
2	5.2	.590	.858	1.000			
4	4.7	.485	.700	.828	1.000		
6	4.6	.436	.636	.702	.878	1.000	
8	4.1	.413	.560	.638	.690	.835	1.000

C. CHANGES OF WIND OVER SHORT PERIODS

1. Wind Variability Over Shorter Periods

In a recent study by Nou[12], the changes in wind for periods of six to twelve hours are reported. The results are summarized in the following table for Shreveport, Louisanna. As is the case for nearly all tabulations on the variability of winds, the data are not what is needed, and what is needed can be deduced only by using drastic assumptions. The columns headed MD are the means of the deviations without regard to sign. The columns headed RMS are the corresponding root mean square values.

TABLE VI

Some Wind Variability Statistics for Shreveport, Louisanna[12]

		Direction (Degrees)		Speed (mps)		Vector (mps)	
		MD	RMS	MD	RMS	MD	RMS
			ALL LEVELS AND SEASONS				
6 hrs.		19	28	3.8	5.2	6.2	7.6
12 hrs.		25	36	5.0	6.6	8.4	10.1
			ALL LEVELS BY SEASONS				
6 hrs.	Jan	12	18	4.6	6.4	7.3	9.0
	Apr	14	20	4.5	6.1	7.3	8.9
	July	35*	51*	2.5	3.2	4.5	5.4
	Oct	16	25	3.5	4.9	5.6	7.0
12 hrs.	Jan	17	26	6.0	8.0	10.1	12.3
	Apr	19	26	6.0	7.9	9.9	11.8
	July	43*	59*	3.1	4.0	5.8	6.8
	Oct	22	32	6.5	6.5	7.9	9.6
			ALL SEASONS BY LEVELS				
6 hrs.	3 km	28	41	2.8	3.9	5.0	6.1
	6 km	22	32	3.5	4.8	6.0	7.4
12 hrs.	3 km	37	51	2.7	4.9	6.6	7.9
	6 km	28	40	4.7	6.2	8.0	9.7

*Unusually large values

TABLE VII

Wind Variability Measures[13]

Average Time Wind Direction and Speed Changes as Function of Wind Speed

	Average Speed (mph)			
	6.7	15.7	24.6	33.6
Direction (°/30 min)	12.0	6.0	5.0	3.0
Speed (mph/30 min)	0.7	1.1	2.2	2.7

(No significant changes with height.)

Average Horizontal Wind Speed and Direction Changes as Function of Height

	Height (10^3 ft)					
	3	6	10	15	24	36
Direction (°/11 mi)	12	10	6	4	4	3
Speed (mph/11 mi)	2.7	1.8	1.1	1.3	2.7	2.2

Wind Variation With Time and Distance

Time	Std. Dev.	Distance	Std. Dev.
1/2 min.	1.3 mph	1/2 miles	1.3 mph
5 min	1.7	3	1.9
30	3.5	5	2.5
1 hr.	4.75	70	7.75
2	6.0	112	10.25
4	7.25	300	18.75
8	9.75	375	19.75
12	12.0	450	22.25
24	18.0	570	26.5

TABLE VIII

Summary of σ_t (Vector Std. Dev.) (mi hr^{-1}) in Lower 500 Ft.[15]

Speed	Time Interval (min) 0	2	4	6	8
5 mph	0.8	2.4	2.9	3.2	3.4
10	1.3	3.5	4.2	4.7	5.0
15	1.8	4.6	5.5	6.2	6.6
20	2.3	5.6	6.7	7.6	8.1
25	2.8	6.7	8.0	9.1	9.5

The differences are for time intervals of six and twelve hours and are those of the wind direction, wind speed, and the magnitude of the vector wind difference. If we assume that the differences of the wind vectors is circularly and normally distributed (not true), then the root mean square component differences will be

$$\sqrt{2}/2 = (0.707)$$

times the value given in the last column (Vector, RMS). Unfortunately, the subdivision by level and season is not made. To get seasonal values by level, one must make assumptions about the seasonal variation. If the seasonal variation at 3 and 6 km is assumed to be like that at all levels combined, the following estimate seems reasonable:

$$\sigma_{\substack{\text{level}\\ \text{season}}} \cong \sigma_{\text{season}} / \sigma_{\text{year}} \left(\sqrt{2}/2 \right) \sigma_{\text{level}} \cdot$$

A comprehensive review of wind variability was made by Baginsky, et al.[13], some results of which are shown in Table VII. Durst[14] has described the wind variability in time as

$$\sigma_t^2 = 2\sigma^2(1-r_t)\ , \quad r_t = e^{-at}\ , \quad a = 6.9 \times 10^{-2}$$

where σ is the climatological vector standard deviation and σ_t is the root mean square vector difference after an elapsed time t. The formulation above holds reasonably well for 5 minutes $< t <$ 24 hours with little variation with altitude to above the 500 mb level. Some data on variability near the ground for short time periods is given in Table VIII from Bellucci[15].

a. Some Limitations of Wind Variability Data

Data on the variability of wind is usually given in terms of mean variability of direction and/or speed and the mean or root mean square vector variability. None of these nor any combination describe the variability of wind with distance in an adequate way. This is because no account is taken of the eddy structure of the atmospheric winds. This must be accounted for by considering the wind components <u>separately</u> in terms of suitable reference directions. This is illustrated in regard to distance variability in Figure 8, in which the mean square difference of components is shown as a function of the differene in distance for the longitudinal (along flight path) and transverse (across flight path) directions[16]. These data pertain to levels near 30,000 feet but <u>not</u> in the jet stream. The larger size of the transverse variation as compared with the longitudinal is independent of direction chosen and is a characteristic of atmospheric eddies. This means that the distribution of the wind vector differences is not circularly distributed, a fact that is ignored completely where only the mean or root mean square vector difference is tabulated. The RMS transverse component appear to be approximately $\sqrt{2}$ times the RMS longitudinal component.

In terms of the expressions at the first of this section for the mean square differences, the formulation of the correlation coefficient is different for the two components of the wind.

2. <u>Temperature Variability Over Shorter Periods</u>

Cox[17] indicates values of temperature variation from 1.0 to 3.5° C. over a half-hour period with observations made every 6 seconds.

The variation of temperature near the ground (standard meteorological exposure) is extremely complex. The removal of the diurnal variation helps to simplify the situation somewhat but great complexitities remain. These complexitities are brought out in the discussion of Godske[18] concerning the temperature at Bergen and Oslo, Norway, over a 25-30 year period. For example, the diurnal variation of temperature as measured by the standard deviation varies on an annual basis from 2.6° C. to 3.8° C. On the other hand, the amount of diurnal variation depends on the time of day used for the comparison. In April it is largest near midday (13 hours, 3.8° C.) and a minimum at midnight (01 hours, 3.0° C.), while in January it is least at midday (3.1°) and largest at night (3.7°). The correlation of temperature as a function of time lag has the same characteristics in that it depends not only on the time lag, Δt, but also on the time of day, t, from which the lag is measured, $r = r(\Delta t, t)$. As an example, for December, $r = 0.77 = r$ (1 day, t), rather uniformly through the day, but in April, r(1 day, 19 hours) = 0.75, while r(1 day, 06 hours) = 0.66, and in September, r(1 day, 06 hours) = 0.48, while r(1 day, 19 hours) = 0.67. If one roughs in an experimental correlation coefficent of the form $\exp(-at)$, then $1.5 < a < 0.65$ (days^{-1}). The surface temperature variation (even after removal of annual and diurnal effects) is quite nonstationary.

Gossard[19] indicates that at altitudes from 1,000 to 3,000 feet on the Southern California coast, the RMS temperature fluctuation over a four minute period is about 0.26° C. with a scale length of about 250 feet. The corresponding RMS vapor pressure fluctuation is about 0.4 mb. Crain, et al.[20], indicate scale size of a few hundred feet at altitude of near 1,000 feet decreasing to 5 to 15 feet near the surface. In the above, the scale size is used in two different senses. In Gossard's work on balloon measurements, the scale size is actually a time scale which is converted to distance on multiplication by the wind speed. It does, however, conform roughly to scale size obtained from aircraft measurements of Crain, et al.[20].

Definitive measurements of temperature variability in the upper atmosphere were treated by J. S. Sawyer[21] in such a way that the required variability parameters are explicitely given. The standard deviations of the temperature fluctuations were found to be 0.5° F. under normal lapse rate conditions and 0.75° F. for inversions. The correlation coefficients as a function of height separation were found to be 0.82 (400 ft), 0.57 (800 ft), 0.24 (1200 ft), 0.08 (1600 ft), 0.00 (2000 ft and larger).

D. DEPARTURE FROM EXPECTED CONDITIONS

The accuracy of expected wind and temperatures is measured in many ways. The most convenient measures for our purpose are those in terms of the "relative root mean square error," the ratio of the root mean square error of the forecast to the root mean square value of the climatic variability. The figure provided by such a measure gives immediately a standard deviation value that will permit the reconstruction of a reasonable covariance matrix for estimating the range of perturbation that might occur in an observed sounding of wind components and temperature.

A convenient norm for measuring the effectiveness of a forecast is "persistence." The persistence forecast is formalized by the assertion that the wind and temperature will remain as last observed. Another forecast norm is "climatology," formalized by the assertion that regardless of present conditions, expected conditions will be those of the climate mean values. Each of these forecast norms may be thought of as a "best" forecast in some domain of time or distance. The situation is illustrated graphically in Figure 9 where time lag is the abscissa and relative error (root mean square) is the ordinate. The relative residual error of the climate forecast is always 1. It is reasonably obvious that for estimates a long time in the future that an estimate based on the climate mean values cannot (at present) be improved upon. The error of the persistence forecast is given by

$$\sigma_p / \sigma_c = \sqrt{2(1 - r)}$$

where r = correlation coefficient relating present and future parameter values, σ_p = standard deviation of error (root mean square error), σ_c = standard deviation of the natural error or climate estimate. For time lags large enough, the present parameter value is uncorrelated with the future value so that r approaches zero, and the relative root mean square error approaches 1.414 = $\sqrt{2}$. Thus, the persistence forecast at long time lags is appreciably worse than the climatological forecast. The two are of equal value at such time that r = 0.5, i.e., while the correlation between present and future parameter values is still appreciably large.

A third forecast norm is the "statistical estimate" given by the relation

$$p = \overline{p} + r(p_o - \overline{p})$$

where p = the estimate of the parameter, $\overline{p}$ = the mean value of the parameter (the climate estimate), p_o = the observed value of the parameter, and r = correlation coefficient (function of time lag or distance). In another form

$$p = rp_o + (1 - r)\overline{p}$$

the statistical estimate is a value interpolated between the climate estimate ($\overline{p}$) and the persistence estimate (p_o); further, it is the best of all possible interpolated values. The relative root mean square error of the statistical estimate is given by

$$\sigma_s/\sigma_c = \sqrt{1 - r^2}$$

where σ_s = root mean square error of the statistical estimate. It is readily seen that

$$\sigma_s < \sigma_c \ , \quad \sigma_s < \sigma_p$$

for all time (or distance) lags.

Idealized curves for σ_p/σ_c and σ_s/σ_c , the relative root means square error of persistence and statistical forecast norms are shown in Figure 9.

The verification of forecasts has produced little information that gives explicitly the root mean square forecast error. Thompson[22] presents an illustration analogous to our Figure 9. Buell[23] has collected data from various sources. The scatter of forecast verification data is so large and verification is carried out by such widely different methods that it is impossible to plot points on Figure 9 to represent the situation. Two rather wide-hatched areas are indicated at 12 hours and 6 hours.

The lower part of the hatched areas indicates the skill of objective forecast techniques (numerical prediction) while the upper part of the range represents the skill of subjective forecasts. The skill of subjective forecasts is generally accepted as insufficient to warrant an effort over a shorter period than six hours.

The trend of the persistence and statistical forecast errors shown in Figure 9 applies particularly to wind forecasts. For such items as temperature, the general shape of the curves remains about the same, but the critical point where persistence equals climatology (shown at 24 hours with ordinate 1.0) moves to the neighborhood of 48 hours. The relative position of the forecast "areas" to the curves remains unchanged. The details of the curve labeled persistence, in the case of temperature at the surface (instrument shelter level), is particularly complex[18].

Even in applying Figure 9 to wind estimates, it is understood that σ and σ_c are "vector" standard deviations. In the case of individual wind component standard deviations, the components follow different curves for persistence. The reasons for this behavior lie beyond the scope of this study (see Buell[16]).

The shape of the curves for persistence and statistics near the $t = 0$ is that given by Durst[39] and is approximated by $\sqrt{t}$. For practical purposes, the curve should intersect the ordinate at $t = 0$ near the point marked A $(\sigma/\sigma_c \simeq 0.2)$. This is due to the fact that the small scale variability of atmospheric parameters such as wind and temperature are dominant influences near $t = 0$. The definition of the curve in this region requires special experimental techniques that are usually not compatible with operational requirements.

E. SOME SPECIFIC DETAILS

1. Short-term Variability

Temperature and wind fluctuations corresponding to a correlation function described by Sawyer[21] were used to describe the short period variability. The standard deviations assigned to such fluctuations were 0.3° C. and 0.6 mps for temperature and wind components, respectively. These standard deviations are on the small side as compared with values encountered over a few tens of minutes quoted in the previous sections. They were deliberately chosen so to present what might be considered the most conservative case that would be encountered in a practical situation.

The correlation function above was entered as a height lag correlation matrix of considerable size (mostly zeros except bordering the principal diagonal) since the spacing between levels was only 400 feet and a height range of several kilometers was required. Since this matrix corresponded to fixed levels while the soundings to be perturbed were recorded at variable levels, the perturbation of the sounding was accomplished by interpolation between perturbations at fixed levels.

2. Longer-term Variability

The interlevel correlation tables for wind components and temperature (Tables IV and V) were used as a basis for the interlevel correlation structure of the perturbations. The standard deviations used to model perturbations that might occur were taken as one-quarter and one-half the climatological standard deviation for the season concerned and conditions expected correspond to a departure from a forecast of about 6 hours and 12 hours after the sounding (with a small bias to a somewhat shorter time of 3 to 6 hours, respectively, if persistence only is considered).

Since the separation between levels in the above correlation coefficient tables is rather large, values were interpolated at convenient intervals (250 m) to roughly correspond to the spacing between levels in the meteorological soundings. The resulting correlation matrix was then treated as in (1) above for short-term variability.

The interpolation process gives reasonable correlation coefficients except for values that would border the principal diagonal of the matrix. The principal diagonal entries were assigned the value 1.000 and the interpolated values in diagonals bordering the principal diagonal were obtained by a process that took this into account.

The resulting matrix of correlation coefficients is somewhat unrealistic if considered from the viewpoint of the large scale atmospheric motions. On the other hand, it is intended not only to mimic the large scale motions but also the smaller scale eddies. The difficulty with the interpolated sounding lies in the fact that as separation becomes small the correlation coefficients approach 1.0 linearly. Even Sawyer's[21] small scale correlation structure does not behave in this way. The situation is automatically adjusted by the fact that when applied to the sounding, values are interpolated from fixed matrix heights to sounding heights as they may occur. This results in "smoothing" of the perturbations on application to the sounding. If one were to work backward from the perturbations applied to the sounding to the interlevel correlation of the perturbations, the "sharp" peak on the diagonal would not be present and instead there would be a "rounded" peak; just the approach to reality that we wish to obtain.

(The same situation applies to the case of the short-term variability. Here the matrix initially contains a "rounded" peak along the diagonal. The interpolation process tends to broaden this peak somewhat. Though the effect is present, it is minimized by the fact that the level spacing is smaller in this correlation coefficient matrix.)

The effect of time lag on the correlation coefficients has been ignored. This effect has been introduced through the control of the standard deviation of the perturbation of the temperature or wind component. Though some data exist for time lag correlations of wind, little is available for temperature in this more extended sense.

CHAPTER IV

VARIABILITY OF THE ATTENUATION COEFFICIENT

A. GENERAL DISCUSSION

The coefficient of attenuation is employed in the form

$$I = I_* \, e^{-\alpha r} \tag{1}$$

where I_* is the unattenuated intensity at a distance r and where α is the attenuation coefficient. The distance, r, is strictly the distance along the ray path, but is not appreciably greater than the distance along the ground. The attenuation coefficient in this form is more properly expressed as the average attenuation.

$$\alpha_* = (1/r)\int_0^r \alpha(s)ds \tag{2}$$

where $\alpha(s)$ is the attenuation at the distance s along the ray path.

Let the operator δ indicate a perturbation from nominal conditions. Then from the above

$$\delta\alpha_* = (1/r)\int_0^r (\delta\alpha)ds$$

in which the perturbation of the ray path itself is neglected.

The mean square variation of the attenuation coefficient is then given by a double integral

$$\overline{(\delta\alpha_*)^2} = (1/r^2)\int_0^r\int_0^r \overline{[(\delta\alpha')(\delta\alpha'')]^2}\, ds'\, ds''$$

where $\delta\alpha'$ is the perturbation of the attenuation coefficient at s′ while $\delta\alpha''$ is that at s″.

Let α be considered as dependent on an atmospheric parameter, p. Then $\delta\alpha \cong (\partial\alpha/\partial p)\delta p$ so that

$$\overline{(\delta\alpha_*)^2} = (1/r^2)\int_0^r\int_0^r (\partial\alpha'/\partial p)(\partial\alpha''/\partial p)\ \overline{(\delta p')(\delta p'')}\, ds'\, ds''$$

The product of partial derivatives under the integral can be evaluated adequately only in specific cases. These factors are taken outside the integral and assigned a common average value for the path so that

$$\overline{(\delta\alpha_*)^2} \cong [(\partial\alpha/\partial p)/r]^2\int_0^r\int_0^r \overline{(\delta p')(\delta p'')}\, ds'\, ds'' .$$

Let the correlation coefficient relating the perturbations at s′ and s″ be indicated by R(s′, s″). Then

$$\overline{(\delta\alpha_*)^2} \cong [(\partial\alpha/\partial p)/r]^2\ \overline{(\delta p)^2}\int_0^r\int_0^r R(s', s'')\, ds'\, ds'' .$$

To simplify further the integration, the correlation function is assumed to be function of only the separation between s′ and s″ along the path

$$\overline{(\delta\alpha_*)^2} \cong [(\partial\alpha/\partial p)/r]^2\,\overline{(\delta p)}\int_0^r\int_0^r R(s'' - s')\, ds'\, ds''$$

whence, writing $s = s'' - s'$,

$$\overline{(\delta\alpha_*)^2} \cong [(\partial\alpha/\partial p)/r]^2\,\overline{(\delta p)^2} \cdot 2\int_0^r (r-s)R(s)ds\ . \tag{3}$$

The value of the integral factor depends on the detailed structure of the correlation coefficient. This factor is not critical for our purposes. For the small scale, two examples indicate the magnitude of this factor. Let $R(s) = \exp(-s/\ell)$ where ℓ is a size parameter.

Then

$$2\int_0^r (r-s)R(s)ds = r\ell[1-\exp(-r/\ell)]$$

$$- \ell^2[1-(r/\ell + 1)\exp(-r/\ell)] \cong 2\ r\ell,\ r >> \ell. \tag{4}$$

As another example, let $R(s) = \exp(-s^2/2\ell^2)$ whence

$$2\int_0^r (r-s)R(s)ds = (\sqrt{\pi}\ \ell r)\mathrm{erf}(r/2\ell) - 2\ell^2[1-\exp(-r^2/2\ell^2)]\ ,$$

$$\cong 2\sqrt{\pi}\ r\ell\ , \qquad r >> \ell\ . \tag{5}$$

In the case of the large scale perturbations, the value of $R(s)$ may be taken as near 1 over the whole range so that

$$2\int_0^r (r-s)R(s)\ ds = r^2 \tag{6}$$

The mean square perturbation of the total attenuation coefficient then becomes, using (4) or (5) in (3)

$$(\delta\alpha_*)^2 \cong (\partial\alpha/\partial p)^2 (\delta p)^2 \cdot k \cdot (\ell/r) \qquad (7)$$

for small scale perturbations where k lies in the range from 2 to $2/\sqrt{\pi}$. For the large scale perturbations, using (6) in (3)

$$\overline{(\delta\alpha_*)^2} \cong (\partial\alpha/\partial p)^2 \overline{(\delta p)^2} \qquad (8)$$

It is readily seen that the small scale mean square perturbation of the total attenuation coefficient approaches zero for large distances (from (7)), while it remains constant (from (8)) for the large scale perturbations.

B. APPLICATION TO THE CLASSICAL ABSORPTION COEFFICIENTS

The absorption of sound in air may be divided into three parts

$$\alpha = \alpha_1 + \alpha_2 + \alpha_3$$

where

α_1 = classical absorption coefficients,

α_2 = intermolecular absorption,

α_3 = miscellaneous other absorption.

The classical absorption coefficients are given by

$$\alpha_1 = \alpha_v + \alpha_c + \alpha_d + \alpha_r$$

where

α_v = absorption due to viscosity

α_c = absorption due to conduction of heat

α_d = absorption due to diffusion of molecules

α_r = absorption due to radiation of heat .

Nominal values of the classical absorption coefficient are given[40] (db/km) in the short table inserted below. They have the variations

T	$\alpha_v + \alpha_c$	α_d	α_r
-50° C	$1.08 \times (f^2/p^*) \times 10^{-1}$	$1.21f^2 \times 10^{-2}$	1.68×10^{-2}
0° C	1.18	1.25	1.51
50° C	1.25	1.31	1.38

with the temperature of $+1.7(f^2/p^*) \times 10^{-4}$, $+ 1.0(f^2) \times 10^{-5}$ and -3.0×10^{-5} db km^{-1} $(°C)^{-1}$, respectively, where f is frequency in kilocycles and p* is pressure in atmospheres.

Using nominal values at 0° C with f = 1 kilocycle, p* = 1 atmosphere, then $\alpha_1 = .147$ db/km while $(\partial\alpha_1/\partial T) = 1.53 \times 10^{-4}$ db km$^{-1}(°C)^{-1}$. For small scale temperature variability in the atmosphere of 0.3°C, the root mean square variation of the classical attenuation coefficient is

$$\sqrt{(\delta\alpha_*)^2}\ \text{(small scale)} \cong 4.6 \times 10^4 \sqrt{k(\ell/r)}\ .$$

A nominal value of $\ell = 0.2$ km, and k = 2, then at 10 km the RMS variation of classical attenuation from small scale variations is 9×10^{-3} db, an amount too small to be considered. For large scale variability of 3°C, corresponding to a time lag of near 12 hours under worst (most highly variable) conditions, the RMS attenuation variability at 10 km is approximately 1.4×10^{-1} db, an order of magnitude larger, but still quite negligible.

C. APPLICATION TO INTERMOLECULAR ABSORPTION

The coefficient of attenuation due to intermolecular absorption may be written as

$$\alpha_2 = \alpha_m = \alpha_{max} \cdot w$$

$$\alpha_{max} \cong (18.9 + 0.45T)f \times 10^{-3}$$

$$w = 2x/(1 + x^2)$$

$$x = f_m/f$$

$$f_m = 1.0 \times 10^3 \ h^2$$

where

f = frequency in cycles
T = temperature in degrees Centigrade
h = absolute humidity (grams m^{-3})

and the α's are in units of decibels per kilometer. The numerical expression for α_{max} as a function of temperature is simplified from a more complex expression in terms of physical parameters[41]. The absolute humidity is expressed in terms of more accessible parameters as

$$h = (em_W/RT) \times 10^6$$

where

e = vapor pressure of the water vapor
T = absolute temperature
m_W = molecular weight of the water vapor
R = gas content .

The factor of 10^6 converts to grams per cubic meter, a more convenient unit than grams per cubic centimeter. An abbreviated table in terms of saturation vapor pressure and saturation absolute humidity as a function of temperature is given.

TABLE IX

SATURATION VAPOR PRESSURE AND ABSOLUTE HUMIDITY AS FUNCTION OF TEMPERATURE

T(°C)	e(mb)	h(gm^{-3})	T(°C)	e(mb)	h(gm^{-3})
-30	0.38*	0.342	5	8.72	6.757
-25	0.64*	0.559	10	12.28	9.401
-20	1.04*	0.894	15	17.06	12.832
-15	1.90	1.403	20	23.40	17.300
-10	2.86	2.158	25	31.70	23.049
-5	4.22	3.261	30	42.48	30.371
0	6.11	4.847	35	56.30	39.599

*With respect to a plane ice surface, others with respect to a plane water surface.

The value of α_{max} is linearly dependent on the frequency. The factor w is dependent on both the frequency and absolute humidity in such a way that w has the maximum value of 1 when $f = f_m = 1.01 \times 10^3\ h^2$.

Since the perturbation of the attenuation coefficient depends on both temperature and vapor pressure perturbations, we write instead of (7) or (8), the expression

$$\overline{(\delta\alpha_m)^2} = [(\partial\alpha_m/\partial T)^2\,\overline{(\delta T)^2} + (\partial\alpha_m/\partial e)^2\,\overline{(\delta e)^2}](k\ell/r) \qquad (9)$$

where the final factor is set equal to 1 for large scale fluctuation. In the above, the temperature and vapor pressure perturbations are considered as uncorrelated and to have the same scales and correlation functions.

The derivative expressions, coefficients of the perturbation variances, are

$$\partial\alpha_m/\partial T = \alpha_{max} \cdot w \left[\frac{0.45}{0.45T-104.1} - \frac{2}{T} \cdot \left(\frac{1-x^2}{1+x^2}\right)\right] \qquad (10)$$

$$\partial\alpha_m/\partial e = \alpha_{max} \cdot w \left(\frac{1-x^2}{1+x^2}\right)\left(\frac{2}{e}\right) \qquad (11)$$

where, in the first, T is in degrees absolute and the first term in brackets is confined to the range from 265°K to 310°K (the valid range for $\alpha_{max} = 18.5 + 0.45T(°C)$ and is the expression $(1/\alpha_{max})(\partial\alpha_{max}/\partial T)$). Aside from the linear dependence of α_{max} on frequency, the frequency enters these expressions in w and $(1-x^2)/(1+x^2)$. Selecting appropriate <u>maximum</u> values, then

$$|\partial\alpha_m/\partial T| < \alpha_{max}\left[0.45/(0.45T-104.1) + 1/T\right] \cong 0.028\alpha_{max} \qquad (12)$$

$$|\partial\alpha_m/\partial e| < \alpha_{max}/e \qquad (13)$$

and the resulting upper bound for the variance of the perturbations is given by

$$\overline{(\delta\alpha_m)^2} < \alpha^2{}_{max}\left((0.038)^2\,\overline{(\delta T)^2} + \overline{(\delta e)^2}/e^2\right)(k\ell/r) \;.$$

For small scale variations with $\alpha_{max} \cong 20$ db/km at one kilocycle $\sqrt{\overline{(\delta T)^2}} = 0.3°C$, the term due to temperature variations amounts to $.168\sqrt{k\ell/r}$ and with $k = 2$, $\ell = 0.2$, $r = 10$ km, the result at 10 km is 0.336 db root mean square variation. The part due to variation of water vapor pressure (humidity) (with nominal estimates of

$$\sqrt{\overline{(\delta e)^2}} = 0.4 \text{ mb.} (e = 8 \text{ mb.})$$

becomes 2 db, an appreciable amount. The combination of these two components into a total root mean square variability of the intermolecular attenuation is weighted so heavily on the side of the part due to vapor pressure variability that the temperature variability contribution is of little importance.

For large scale variability, the factor $k\ell/r$ is replaced by 1 and

$$(\delta T)^2 \cong 3.0°C \; , \; \sqrt{\overline{(\delta e)^2}} \cong 2 \text{ mb.}$$

Under the same conditions as before, the part due to temperature variation becomes 1.68 db/km while that due to vapor pressure variation becomes 5 db/km. These are, like before, values that are much too large.

The estimates (12) and (13) supply upper bounds that are unrealistically large. In arriving at the above estimate of the variability of the attenuation due to intermolecular absorption, the values of x used to approximate the derivatives are those giving maximum values of $w = 2x/(1 + x^2)$ and $w(1 - x^2)$ (1.0 and 0.5, respectively, though the second should be 0.515). Conservative, but frequency dependent, estimates may be obtained in the form

$$|\partial\alpha_m/\partial T| < \alpha_m[0.45/(0.45T-104.1) + 2/T] \cong 0.03\,\alpha_m \tag{14}$$

$$|\partial\alpha_m/\partial e| < \alpha_m/e \tag{15}$$

where α_m on the right is an average value of the attenuation coefficient at the frequency concerned. Since $\alpha_m = \alpha_{max} \cdot w$, $w = 2x/(1 + x^2)$, $x = f_m/f$, the value of α_m may be less than α_{max} by an order of magnitude or so. In this case a nominal value of α_m of 2 db/km replaces that for α_{max} (20 db/km). The resulting small scale variability is then about 0.2 db at 10 km. For large scale variability, the estimate is .5 db/km or 5 db at 10 km.

CHAPTER V

CONCLUSIONS

The variability of the meteorological parameters does not permit the estimation of sound intensity level more accurately than with a standard deviation of 5 db. This is the case regardless of whether a reasonably long or a very, very short time elapses between specification of the meteorological parameters from sounding observations and estimation of sound intensity. This amount of error is inherent in the small scale variability of the atmospheric parameters.

Though there are soundly based objections to using the ray tracing method for estimates of sound intensity (failure of the method to satisfy in detail the basic assumptions involved), it is not clear that any (physically) more satisfying method of intensity estimation would result in less variability of the estimates.

The effect of atmospheric variability on the absorption coefficients is relatively small except in the case of intermolecular effects that depend on the humidity. The small scale variability of the atmosphere contributes little to their variability, amounting to a less than 1 db at 10 km. The large scale variability is more effective since conditions along the whole ray path are changed. These may amount to as much as 5 db at 10 km.

Although the variability of sound intensity estimates (for direct rays) is near 5 db regardless of the variability parameters (over the range covered), the probability of rays returning to a fixed point is highly variable. It appears that this information would be of value for operational purposes. To provide this information, the ray tracing program may be augmented to provide a Monte Carlo perturbation of the sounding and the tabulation of these probabilities.

APPENDIX A

RAY TRACING

The fundamentals and some details of the ray tracing technique are discussed in this appendix. To begin with, we quote L. M. Brekhovskikh[1](p. 461), "Unfortunately, ray constructions which are helpful in elucidating the nature of the wave propagation rather frequently turn out to be completely useless for a quantitative description." This situation is to be kept in mind, since sound intensity estimates based on ray tracing methods are discussed and used almost exclusively in those regions of the sound field where they may be applied. The reason for using ray tracing (in spite of the above warning) lies quite simply in the fact that the range of atmospheric conditions which must be covered in estimating the sound intensity is so varied that estimates based on a more complete solution of the wave equations are beyond hope of attainment.

The section on fundamentals follows the treatment of Ingard[24] who in turn follows that of Blokhintsev[25] and is included to put into perspective the fundamental background of the ray tracing method.

Some details are discussed in the section on the ray tracing method with emphasis on the distinction between the ray tangent and the phase normal, though this difference is relatively unimportant for practical considerations.

The physical assumptions behind the ray tracing method are considered in detail in terms of their practical significance. The section on general consideration of the assumptions is followed by a discussion of the discontinuous character of $dr/d\psi_o$. This comes about through violation of the assumptions and comments on the gradient of the focusing factor which must be severely bounded if the assumptions are to be satisfied. The intensity at a "focus" has long been known to require special treatment since the ray tracing assumptions are not satisfied at such a point.

The use of "rounded maxima" offers at least a partial way out of the problems introduced by the violation of the assumptions in the usual "linear layer" treatment.

A. FUNDAMENTALS

The results of approximating the fundamental equations of acoustics in a moving, nonhomogeneous medium lead, in the zero'th approximation, to the "eikonal equation"

$$|\nabla \Theta|^2 = q^2/c^2 , \qquad q = c_0 - \nabla \Theta \cdot \mathbf{v} \tag{A.1}$$

where Θ is the phase of the wave, c_0 is the reference speed of sound, c the speed of sound as a function of location (coordinate) and $\mathbf{v}$ is the wind vector. The quantity $q/c = \mu$ is the generalized index of refraction. The surfaces $\Theta =$ constant of the partial differential equation (A.1) represent the expanding sound waves. The complete solution of the ray geometry eventually resolves itself into finding the solutions of this equation or of carrying out the equivalent processes (either exactly or approximately). In obtaining (A.1) it is assumed that: (a) the changes in the medium are small in a distance of a wavelength, and (b) that the wave number k ($k = 2\pi/\lambda$, λ = wave length) is large (wavelength is small).

The analysis of the implications of (A.1) lead to the association of two velocities with the propagation of the sound wave. The phase velocity is given by

$$V_f = c + v_n \ . \tag{A.2}$$

It is directed along the normal to the surfaces of constant phase (Θ = constant), and v_n is the projection of the wind speed vector on the normal to the wave. The sound energy is propagated in a somewhat different direction determined by the ray velocity

$$\mathbf{v}_s = c\,\mathbf{n} + \mathbf{v} \tag{A.3}$$

where $\mathbf{n}$ is the unit vector normal to the surface of constant phase and $\mathbf{V}$ is the wind vector. It is readily recognized that the projection of $\mathbf{V_s}$, the ray velocity, on the direction normal to the phase surface, $\mathbf{n}$, has the magnitude of the phase velocity

$$V_f = \mathbf{V_s} \cdot \mathbf{n} .$$

The intensity of the sound is conserved along a "ray tube" so that

$$\frac{p^2 V_s J}{\rho q c^2} = \frac{p_o^2 V_{s_o} J_o}{\rho_o q_o c_o^2}$$

where p = amplitude of the sound pressure

ρ = air density

J = area of the "ray tube"

V_s = ray velocity

q = refractive index

and where the unmodified quantities refer to a point P while those with subscript "$_o$" refer to some other point P_o on the same ray tube. The sound intensity is usually proportional to p^2. The intensity at P in terms of the intensity at P_o is

$$I = I_o(J_o/J)(V_{s_o}/V_s)(\rho/\rho_o)(q/q_o)(c/c_o)^2 .$$

When the points P and P_o are at essentially the same level, it is readily seen that all of the ratios are individually very near unity with the exception of the ray tube area ratio which may have undergone a considerable change during the propagation process. The approximate intensity at P is then given by

$$I \cong I_o(J_o/J).$$

It is readily seen from Figure 10 that the ray tube cross section PB at P is given by

$$PB = (PP')\sin\psi_p$$

and that the horizontal distance between rays separated by the initial angle $\Delta\psi$ is

$$PP' = (\partial r/\partial\psi_o)\Delta\psi$$

whence

$$PB = (\partial r/\partial\psi_o)\sin\psi_p(\Delta\psi).$$

Taking the cylindrical symmetry of the representation into account, the area of the ray tube is $2\pi r$ times the distance PB, so that

$$J = 2\pi r(\partial r/\partial\psi_o)\sin\psi_p(\Delta\psi).$$

The total energy emitted by the source is represented by W so that the most that is emitted in the interval $\Delta\psi$ is given by

$$\Delta W = I_o J_o = W\cos\psi_o(\Delta\psi)/2$$

which is identified with the total energy of the ray tube at unit distance, $I_o J_o$. Then from the above

$$I = W\cos\psi_o/\left[4\pi r(dr/d\psi_o)\sin\psi_p\right] .$$

If we let $I_* = W/4\pi R^2$ be the intensity that would prevail at a distance R with spherical spreading under homogeneous conditions, then

$$I = I_*\left[R\cos\psi_o/r(dr/d\psi_o)\sin\psi_p\right] .$$

Since, for our purposes, $\psi_p = \psi_o$ and $R = r$, we may write

$$I = I_* \cdot f, \qquad f = r/(dr/d\psi_o)\tan\psi_o ,$$

in which f is called the "focusing factor." When the source does not radiate uniformly in all directions it may be necessary to introduce additional dependence of I_* on the altitude and azimuth ψ_o, θ_o of the rays.

B. THE RAY TRACING METHOD

1. The Ray Equations

Methods of tracing sound rays in the atmosphere has a long history and dates, at least, from Lord Rayleigh[26] who credits Prof. James Thompson (1876) with the method of estimating ray curvature (for light rays) and Prof. Osborne Reynolds (1874) with pointing out the effect of temperature on the speed of sound. Since that time the literature on ray tracing has become so very extensive that only the most important references need be mentioned. R. Emden[27] and E. A. Milne[28] treat the problem with great care, particularly with regard to the generalization of Snell's Law to an atmosphere in motion. This particular topic, the generalization of Snell's Law, is treated with great care by Kornhauser[29]. Nearly all ray tracing techniques are based on approximations to the equations for the sound rays. However, an exact method for integrating the equations under reasonable assumptions is given by Rothwell[30]. The method of Milne is followed in the analysis used by Dorman and Brown[31].

The differential equations of the "phase normal" locus or of the "ray" may be written from the velocity expressions (A.2) and (A.3). For the "phase normals" from (A.2)

$$dx/dt = (c + v_n)\cos\alpha$$
$$dy/dt = (c + v_n)\cos\beta \qquad (A.4)$$
$$dz/dt = (c + v_n)\cos\gamma$$
$$v_n = u\cos\alpha + v\cos\beta + w\cos\gamma$$

where (α , β , γ) are the direction angles of the "phase normal" (the angle that the tangent of this curve makes with the reference axes) and (u, v, w) are the components of the wind.

For the sound "ray" the corresponding equations from (A.3) are

$$dx/dt = c\cos\alpha + u,$$
$$dy/dt = c\cos\beta + v, \qquad (A.5)$$
$$dz/dt = c\cos\gamma + w.$$

The differential equations of the "phase normals" and of the "rays," (A.4) and (A.5), respectively, amount to nothing more than a statement of the velocity relations, (A.2) and (A.3).

In view of the fact that along a ray the phase normals are parallel to a fixed plane (see Milne[(28)]), we may take coordinate axes so that this is the (x,z)-plane, i.e., $\cos\beta = 0$. In such a case we may let $\alpha = \varphi$, $\gamma = \pi/2 - \varphi$ so that the differential equations then become, for the "phase normals,"

$$dx/dt = (c + v_n)\cos\varphi ,$$
$$dy/dt = 0 ,$$
$$dz/dt = (c + v_n)\sin\varphi ,$$
$$v_n = u\cos\varphi + w\sin\varphi .$$

and for the "rays"

$$dx/dt = c \cos\varphi + u ,$$

$$dy/dt = v ,$$

$$dz/dt = c \sin\varphi + w .$$

For the applications at hand, the time dependence of the "ray" or "phase normal" coordinate is of little interest. The time may be eliminated to yield

$$dx/dz = \cot\varphi ,$$

for the "phase normals" and

$$dx/dz = (c \cos\varphi + u)/(c \sin\varphi + w) ,$$

$$dy/dz = v/(c \sin\varphi + w) ,$$

for the "rays".

The presence of the vertical component of the wind in the denominator above is troublesome, but may be handled by a subterfuge. One may write

$$dx/dz = (c \cos\varphi + u^*)/c \sin\varphi ,$$

$$dy/dz = v^*/c \sin\varphi,$$

where

$$u^* = u - w(c \cos\varphi + u)/(c \sin\varphi + w) ,$$

$$v^* = cv \sin\varphi/(c \sin\varphi + w) ,$$

so that the presence of the vertical velocity term is accounted for by a small perturbation or error in the evaluation of the horizontal wind components.

2. Integration of the Ray Equations

The ray tracing process consists essentially of selecting an appropriate method for the integration of the equations for the rays. Many techniques are available; the choice among them lies only in arriving at a solution of sufficient accuracy with a minimum of labor. The most commonly used method is that of a "constant curvature" or "circular arc" approximation. In this case, the ray curvature is estimated for each layer of the atmosphere and the ray is approximated by a circular arc through that layer. Rothwell[30] uses approximations after integrating the ray equations exactly. In all instances a fundamental crutch for the integration process is Snell's Law (the appropriate generalization to take care of the wind situation).

a) Snell's Law

Snell's Law in its usual form

$$c/\cos\varphi = c_o/\cos\varphi_o = \text{const.}$$

along a phase normal must be modified for application to a moving medium. The modifications that need to be made are somewhat subtle. They have been carried out by Milne[28] and later by Kornhauser[29] with the result that

$$(c + u\,\cos\varphi + w\,\sin\varphi)/\cos\varphi = (c_o + u_o\cos\varphi_o + w_o\sin\varphi_o)/\cos\varphi_o \qquad (A.6)$$

along a ray (rather than along a "phase normal") although the angles φ, φ_o refer to the inclination angle of the phase normal. An associated expression due to Milne[28] is that along a ray the ratio of the direction cosines associated with the x and y directions remains constant

$$\tan\theta = \cos\beta/\cos\alpha = \text{const},$$

so that the projection of the unit normal to the phase surfaces has a constant direction in the (x,y)-plane. To obtain the expression (A.6) above, the coordinate system is chosen so that the "phase normals" along the ray concerned lie in the (x,y) -plane. This requires that $\beta=\pi/2$ and consequently $\alpha = \varphi$, $\gamma = \pi/2 - \varphi$ where φ is the elevation angle of the "phase normal" along the ray.

The terms in w, w_0 of (A.6) are not included in the analysis by Milne[28], but his analysis may be extended to include them without difficulty. Kornhauser's[29] analysis may be construed as containing these terms though they are not specifically stated and his further analysis is confined to a special case in which they are not required.

The vertical velocity components, w, w_0, may be combined with the horizontal component in the formulation of the generalized Snell's Law by a kind of subterfuge. Thus

$$c/\cos\varphi + u^* = c_0/\cos\varphi_0 + u_0^* \tag{A.7}$$

where

$$u^* = u + w\tan\varphi, \quad u_0^* = u_0 + w_0 \tan\varphi_0 .$$

The vertical wind component is usually two or more orders of magnitude smaller than the horizontal component and, consequently, (except for exceptional cases) may be considered as a part of the error or of the variability of the horizontal component as far as its appearance in Snell's Law is concerned.

b) Circular Arc Approximation

From the equations of the ray path (A.6) and Snell's Law (A.7)

$$dx/dt = c\cos\varphi + u ,$$

$$dz/dt = c\sin\varphi ,$$

$$c/\cos\varphi + u = c_0/\cos\varphi_0 + u_0 .$$

The radius of curvature, R, of the ray may be calculated following Gutenberg[32]. The inclination of the ray is given by

$$\tan\psi = (dz/dt)/(dx/dt) = dz/dx$$

and since $R^{-1} = d\psi/ds$ where s is the arc length along the ray then

$$R^{-1} = \sin\psi \, \cos^2\psi \, \left[\, d(dz/dx)/dz \,\right].$$

The indicated derivative with respect to z involves the vertical rates of change of the inclination of the phase normal $d\varphi/dz$ which is obtained from Snell's Law in terms of the vertical rate of change of the speed of sound, dc/dz, and wind component, du/dz . The resulting expression for the ray curvature is

$$R^{-1} = -c\left[(c\cos\varphi + u\cos 2\varphi)(dc/dz) + (c + u\cos^3\varphi)(du/dz)\right] \div (c^2 + 2c\,u\cos\varphi + u^2)^{3/2}. \qquad (A.8)$$

Since the speed of sound is much larger than the wind component, this reduces to, in the zero order approximation,

$$R^{-1} = -\left[(dc/dz)\cos\varphi + (du/dz)\right] c^{-1} \qquad (A.9)$$

If the atmosphere is divided into layers through which the speed of sound and wind component are linear functions of altitude, i.e.,

$$dc/dz = \alpha \,,$$

$$du/dz = \beta \,,$$

then the ray may be considered as a circular arc in such a layer. The parametric equations for the ray through the layer, in terms of the parameter ψ, the angle of inclination of the ray, may be written as

$$x_2 - x_1 = -R(\sin\psi_2 - \sin\psi_1) \,, \qquad \text{(A.10)}$$
$$z_2 - z_1 = R(\cos\psi_2 - \cos\psi_1) \,,$$

where the subscript 1 indicates the conditions at the bottom and the subscript 2 those at the top of the layer.

The angles of inclination of the ray at the top and bottom of the layer are not known a priori and must be calculated from the equations of the ray and Snell's Law. Thus, from the equations of the ray

$$\tan\psi = c \sin\varphi/(c \cos\varphi + u),$$
$$\sin\psi = c \sin\varphi/(c^2 + 2cu \cos\varphi + u^2)^{1/2} \qquad \text{(A.11)}$$
$$\cos\psi = (c \cos\varphi + u)/(c + 2cu \cos\varphi + u^2)^{1/2}.$$

When the wind speed is small compared with the speed of sound, the first order approximation becomes

$$\sin\psi \cong \sin\varphi\left[1 - (u/c)\cos\varphi\right] \,, \qquad \text{(A.12)}$$
$$\cos\psi \cong \cos\varphi + (u/c)\sin^2\varphi \,.$$

The angle of inclination of the phase normal, φ , is known at each level and for each initial value, φ_0 , from Snell's Law

$$\cos\varphi = c \cos\varphi_0 \big/ \left[c_0 - (u - u_0)\cos\varphi_0\right] . \qquad \text{(A.13)}$$

The system of equations (A.8), (A.10), (A.11) and (A.13) provide a reasonably precise method of obtaining the ray path. The corresponding approximate system (A.9), (A.10), (A.12) and (A.13) provides a system which, though not so precise, requires much less arithmetic. The second of equations (A.10) in both cases serves little purpose, since the quantities concerned are somewhat redundant. For example, Snell's Law is nearly the same as the second of (A.10) when expressed in the same terms. One may write Snell's Law in the form

$$c_2/\cos\varphi_2 + u_2 = c_1/\cos\varphi_1 + u_1$$
$$= \left[c_1 + \alpha(z_2 - z_1)\right]/\cos\varphi_2 + u_1 + \beta(z_2 - z_1) \ .$$

Solving for the height difference across the layer, the expression

$$z_2 - z_1 = \left[c_1/\cos\varphi_1(\alpha + \beta\cos\varphi_2)\right](\cos\varphi_2 - \cos\varphi_1)$$

bears a very close resemblance to the second of equations (A.10). The first factor on the right resembles the radius of curvature while the second factor, the difference in cosines, corresponds but with cosines of somewhat different angles (the inclination of the phase normal instead of the inclination of the ray).

The parameters, $\cos\varphi$ and c, which appear in (A.9), vary through the layer. It is required to make a choice which represents average values in order to apply (A.9) to a layer. A suitable choice seems to be

$$\overline{\cos\varphi/c} = \cos\varphi_0/c_0 \quad ; \quad \bar{c} = (c_1 + c_2)/2$$

For layer applications, (A.9) becomes

$$R^{-1} = -\left[(dc/dz)\cos\varphi_0/c_0 + (du/dz)/\bar{c}\right] = -(\alpha\cos\varphi_0/c_0 + \beta/\bar{c}) \ .$$

It may also be seen from (A.11) that $\cos\varphi_0$ differs very little from $\cos\psi_0$. Consequently, for practical applications

$$R^{-1} = -(\alpha\cos\psi_0 + \beta)/c_0$$

where $\bar{c} = c_0$ has been used in connection with the β term. This last approximation is quite poor, but is justified in view of the great inaccuracies in determining β.

c) Rothwell's Method

Rothwell's[30] solution of the equations of motion has the virtue of being mathematically exact (within the framework of the physical assumptions). The following is an outline of his procedure. The basic differential equations and Snell's

Law are

$$dx/dt = c \cos\varphi + u,$$

$$dz/dt = c \sin\varphi ,$$

$$c/\cos\varphi + u = c_0/\cos\varphi_0 + u_0 = K.$$

Change to the inclination of the phase normal, φ , as the independent variable so that

$$dx/d\varphi = (dx/dt)/(d\varphi/dt), \quad dt/d\varphi = 1/(dz/dt)(d\varphi/dz),$$

and assume that in the layer concerned c and u are **linear** functions of height

$$c = c_1 + \alpha z, \quad u = u_1 + \beta z , \tag{A.14}$$

where z is the distance above the bottom of the layer. Snell's Law gives the relation between z and φ .

$$z = \left[(K - u_1)\cos\varphi - c_1\right]/(\alpha+\beta\cos\varphi). \tag{A.15}$$

Differentiating Snell's Law with respect to z

$$d\varphi/dz = -(\alpha + \beta\cos\varphi)\cos\varphi/(c \sin\varphi),$$

so that

$$dt/d\varphi = -1/\cos\varphi(\alpha + \beta\cos\varphi), \tag{A.16}$$

and

$$dx/d\varphi = -(c \cos\varphi + u)/\cos\varphi(\alpha + \beta\cos\varphi). \tag{A.17}$$

In (A.17) the quantities c and u are functions of φ which may be found by substituting (A.15) into each of (A.14)

$$c = \left[\alpha K - (u_1\alpha - c_1\beta)\right]\cos\varphi/(\alpha+\beta \cos\varphi) \tag{A.18}$$

$$u = \left[\beta K\cos\varphi + (u_1\alpha - c_1\beta)\right]/(\alpha + \beta\cos\varphi). \tag{A.19}$$

Divide (A.17) into two parts, one with the "c" term and the other with the "u" term so that

$$x = x' + x'' \tag{A.20}$$

where
$$dx'/d\varphi = c/(\alpha + \beta\cos\varphi), \tag{A.21}$$

$$dx''/d\varphi = u/\cos\varphi(\alpha + \beta\cos\varphi). \tag{A.22}$$

Using (A.18) and (A.19) in (A.21) and (A.22) respectively

$$dx'/d\varphi = \left[\alpha K + (c_1\beta - u_1\alpha)\right]\cos\varphi/(\alpha + \beta\cos\varphi)^2 \tag{A.23}$$

$$dx''/d\varphi = \left[(u_1\alpha - c_1\beta) + \beta K\cos\varphi\right]/\cos\varphi(\alpha + \beta\cos\varphi)^2 \tag{A.24}$$

Equations (A.16), (A.20), (A.23) and (A.24) then constitute the differential equations for the rays.

i) The time integral.

The results of integrating (A.18) are of minor interest but are included for completeness. There are four cases that need to be considered in each integration.

Case I: $\alpha^2>\beta^2$

$$t_2-t_1 = -(1/\alpha)\Big[\log\tan(\pi/4 + \varphi/2)\Big]_{\varphi_1}^{\varphi_2} + \left\{2\beta/\alpha(\alpha^2-\beta^2)^{\frac{1}{2}}\right\}\Big[\tan^{-1}\left\{(\alpha^2-\beta^2)^{\frac{1}{2}}\tan(\varphi/2)/(\alpha+\beta)\right\}\Big]_{\varphi_1}^{\varphi_2} \tag{A.25}$$

Case II: $\beta^2>\alpha^2$

$$t_2-t_1 = -(1/\alpha)\Big[\log\tan(\pi/4 + \varphi/2)\Big]_{\varphi_1}^{\varphi_2} + \left\{2\beta/\alpha(\beta^2-\alpha^2)^{\frac{1}{2}}\right\}\Big[\tanh^{-1}\left\{(\beta^2-\alpha^2)^{\frac{1}{2}}\tan(\varphi/2)/(\alpha+\beta)\right\}\Big]_{\varphi_1}^{\varphi_2} \tag{A.26}$$

Case III: $\alpha = \beta \neq 0$

$$t_2-t_1 = -(1/\alpha)\Big[\log\tan(\pi/4+\varphi/2)\Big]_{\varphi_1}^{\varphi_2} + (1/\alpha)\Big[\tan(\varphi/2)\Big]_{\varphi_1}^{\varphi_2} \tag{A.27}$$

Case IV:

$$t_2 - t_1 = -(1/\alpha)\Big[\log\tan(\pi/4 + \varphi/2)\Big]_{\varphi_1}^{\varphi_2} + (1/\alpha)\Big[\cot(\varphi/2)\Big]_{\varphi_1}^{\varphi_2} \tag{A.28}$$

ii) The major displacement integral.

The largest part of $x = x' + x''$ is that due to x' from (A.23). The integration of (A.23) yields the following:

Case I: $\alpha^2 > \beta^2$

$$x_2' - x_1' = \frac{\alpha K + (c_1\beta - u_1\alpha)}{\alpha^2 - \beta^2}\ \alpha\left[\frac{\sin\varphi}{\alpha + \beta\cos\varphi}\right]_{\varphi_1}^{\varphi_2} - \frac{2\beta}{(\alpha^2 - \beta^2)^{1/2}}\left[\tan^{-1}\left\{\frac{(\alpha^2-\beta^2)^{1/2}}{\alpha+\beta}\tan(\varphi/2)\right\}\right]_{\varphi_1}^{\varphi_2} . \tag{A.29}$$

Case II: $\beta^2 > \alpha^2$

$$x_2' - x_1' = \frac{\alpha K + (c_1\beta - u_1\alpha)}{\beta^2 - \alpha^2}\ \alpha\left[\frac{\sin\varphi}{\alpha + \beta\cos\varphi}\right]_{\varphi_1}^{\varphi_2} - \frac{2\beta}{(\beta^2 - \alpha^2)^{1/2}}\left[\tanh^{-1}\left\{\frac{(\beta^2-\alpha^2)^{1/2}}{\alpha+\beta}\tan(\varphi/2)\right\}\right]_{\varphi_1}^{\varphi_2} . \tag{A.30}$$

Case III: $\alpha = \beta \neq 0$

$$x_2' - x_1' = \Big[\alpha K + (c_1\beta - u_1\alpha)\Big]\Big[\alpha - \tan(\varphi/2)\Big]_{\varphi_1}^{\varphi_2} \Big/ \alpha\beta . \tag{A.31}$$

Case IV: $\alpha = -\beta \neq 0$

$$x_2' - x_1' = -\Big[aK + (c_1\beta - u_1\alpha\Big]\Big[\varphi - \cot(\varphi/2)\Big]_{\varphi_1}^{\varphi_2} \Big/ \alpha\beta . \tag{A.32}$$

iii) The minor displacement integral.

The integration of (A.24) for x" leads to somewhat more complex results. The first case is given here so that it may be "viewed with horror." This part of the displacement is always small and may be approximated quite easily.

Case I: $\alpha^2 > \beta^2$

$$x''_2 - x''_1 = \frac{u_1\alpha - c_1\beta}{\alpha}\Big[\log\tan(\pi/4 + \varphi/2)\Big]_{\varphi_1}^{\varphi_2} -$$

$$\frac{\beta^2\big[\alpha K + (c_1\beta - u_1\alpha)\big]}{\alpha(\alpha^2 - \beta^2)}\Big[\frac{\sin\varphi}{\alpha + \beta\cos\varphi}\Big]_{\varphi_1}^{\varphi_2} +$$

$$\frac{2\beta\big[\alpha^3 K + (2\alpha^2-\beta^2)(c_1\beta - u_1\alpha)\big]}{\alpha(a^2-\beta^2)^{3/2}}\Big[\tan^{-1}\Big\{\frac{(\alpha^2 - \beta^2)^{1/2}}{\alpha + \beta}\tan(\varphi/2)\Big]_{\varphi_1}^{\varphi_2} . \tag{A.33}$$

Case II: $\beta^2>\alpha^2$, is similar to (A.33) except that the last term is modified in the same way as (A.30) and (A.26).

The values of $x''_2 - x''_1$ may be approximated easily from the relation

$$x''_2 - x''_1 \cong (x'_2 - x'_1)(\bar{u}/\bar{c}) \tag{A.34}$$

where $\bar{u}$ and $\bar{c}$ are mean values for the layer concerned. Since $\bar{u}/\bar{c}$ is a small fraction, $x''_2 - x''_1$ is a small correction to $x'_2 - x'_1$.

iv) Short cuts

It has already been mentioned that the integration of the "minor displacement" due to the wind speed term of (A.17) may be estimated approximately by the expression (A.34) as a fraction of the "major displacement."

The major displacement itself may be estimated from (A.21) directly by assuming that the speed of sound is constant in the layer concerned. Thus

$$x_2' - x_1' \cong \frac{2\bar{c}}{(\alpha^2-\beta^2)^{1/2}} \left[\tan^{-1}\left\{\frac{(\alpha^2-\beta^2)^{1/2}}{\alpha+\beta}\tan(\varphi/2)\right\}\right]_{\varphi_1}^{\varphi_2}, \quad \alpha^2>\beta^2, \tag{A.35}$$

$$x_2' - x_1' \cong \frac{2\bar{c}}{(\beta^2-\alpha^2)^{1/2}} \left[\tanh^{-1}\left\{\frac{(\beta^2-\alpha^2)^{1/2}}{\alpha+\beta}\tan(\varphi/2)\right\}\right]_{\varphi_1}^{\varphi_2}, \quad \beta^2>\alpha^2, \tag{A.36}$$

$$x_2' - x_1' \cong (\bar{c}/\alpha)\left[\tan(\varphi/2)\right]_{\varphi_1}^{\varphi_2}, \quad \alpha = \beta \neq 0, \tag{A.37}$$

$$x_2' - x_1' \cong -(\bar{c}/\alpha)\left[\cot(\varphi/2)\right]_{\varphi_1}^{\varphi_2}, \quad \alpha = -\beta \neq 0. \tag{A.38}$$

For practical purposes, the value of $\bar{c}$ may be taken at the mid-point of the layer concerned. For a more accurate result the value of $\bar{c}$ may be estimated by equating the results of (A.35),---,(A.38) to those of (A.29),---,(A.32). The resulting value of $\bar{c}$ is given by

$$\bar{c} = c_1 + \alpha(z_2 - z_1)\left[(2\sin\varphi_1 + \sin\varphi_2)/6\sin\varphi_1\right].$$

It is readily seen that the parenthetical factor is 1/2 when φ_1 and φ_2 are nearly the same. This factor reduces to 1/3 for a minimum value when $\varphi_2 = 0$.

v) The transverse displacement.

The standard methods of ray tracing handle the situation as though the ray paths lay in planes through the sound source. Such is only an approximation, as pointed out by Milne[28] and Emden[27]. The third of the ray path equations may be integrated approximately to estimate the transverse displacement. From

$$dy/dt = v$$

one may assume that v is a constant in the layer concerned so that the displacement is approximately given by

$$y_2 - y_1 \cong \bar{v}\,(t_2 - t_1)\ ,$$

or

$$y_2 - y_1 \cong \bar{v}\,(x_2 - x_1)/\bar{c}\ ,$$

where $\bar{v}$ is the mean transverse component in the layer and $\bar{c}$ is the mean speed of sound in the layer. $t_2 - t_1$ is the time of traverse from (A.25),---,(A.28) or, more simply, $x_2 - x_1$ is the displacement from (A.29), ---, (A.32) or (A.35), ---, (A.39). The degree of approximation included is such that it makes little difference whether the major displacement or the total displacement is used.

vi) The focusing factor.

It was seen in the preceding section that the intensity might be expressed in terms of the intensity due to spherical spreading times a "focusing factor," where

$$f = r/(dr/d\psi_0)\tan\psi_0\ ,$$

with r = source to receiver distance and ψ_0 = initial inclination angle of the ray. The inclination angle of the ray in terms of the phase normal is given by

$$\tan\psi = c\ \sin\varphi/(c\ \cos\varphi + u)$$

so for the initial conditions

$$\tan\psi_0 = c_0\sin\varphi_0/(c_0\cos\varphi_0 + u_0)\ .$$

The focusing factor requires that $dr/d\psi_0$ be computed by some method. This may be done by computing $r = r(\psi_0)$ for several values of ψ_0 and performing a numerical differentiation of the results. It may also be simply computed (at least approximately) as a part of the computation of $r(\varphi_0)$.

First, express $dr/d\psi_0$ in terms of $dr/d\varphi_0$ using

$$dr/d\psi_0 = (dr/d\varphi_0)(d\varphi_0/d\psi_0)$$

so that

$$\frac{dr}{d\psi_0} = \frac{dr}{d\varphi_0} \tan\varphi_0 \left[\frac{d\varphi_0}{d\psi_0} \frac{\tan\psi_0}{\tan\varphi_0} \right].$$

The parenthetical factor may be evaluated as follows

$$(\tan\psi_0/\tan\varphi_0)(d\psi_0/d\varphi_0) \cong 1 + (u_0/c_0)\sin\varphi_0\tan\varphi_0 \ .$$

The second term in this final expression is exceedingly small since not only u_0/c_0 is small but, usually, $\sin\varphi_0$, $\tan\varphi_0$ are also small. Consequently, the focusing factor may be effectively expressed as

$$f = r\Big/\left[(dr/d\varphi_0)\tan\varphi_0\right].$$

The ray path equation

$$dx/dt = c\cos\varphi + u$$

may be differentiated with respect to φ_0 to give

$$d(dx/d\varphi_0)/dt = -c\sin\varphi(d\varphi/d\varphi_0)$$

and from Snell's Law

$$d(dx/d\varphi_0)/dt = -c_0\sin\varphi_0(\cos^2\varphi/\cos^2\varphi_0)$$

As before,

$$d(dx/d\varphi_0)/d\varphi = \left[d(dx/d\varphi_0)/dt\right](dt/d\varphi)$$

so that

$$d(dx/d\varphi_0)/d\varphi = (c_0\sin\varphi_0/\cos^2\varphi_0)\left[\cos\varphi/(\alpha+\beta\cos\varphi)\right]$$

where we are concerned with a layer in which α = constant, β = constant.

This expression integrates at once to give, $\beta \neq 0$

$$\left[\frac{dx}{d\varphi_0}\right]_2 - \left[\frac{dx}{d\varphi_0}\right]_1 = \frac{c_0 \sin\varphi_0}{\beta\cos^2\varphi_0}\left\{\left[\varphi\right]_{\varphi_1}^{\varphi_2} - \frac{2\alpha}{(\alpha^2-\beta^2)^{1/2}}\left[\tan^{-1}\left\{\frac{(\alpha^2-\beta^2)^{1/2}}{\alpha+\beta}\tan\frac{\varphi}{2}\right\}\right]_{\varphi_1}^{\varphi_2}\right\}$$

or, $\beta = 0$,

$$= (c_0 \sin\varphi_0/\alpha\cos^2\varphi_0)(\sin\varphi_2 - \sin\varphi_1) ,$$

in the first instance. The four cases, $\alpha^2>\beta^2$, $\beta^2>\alpha^2$, $\alpha + \beta \neq 0$, $\alpha = -\beta \neq 0$ are to be considered. The second relation forms only a special subcase of the first case.

Though this expression seems formidable, all the evaluations would have been performed in the computation of $x_2' - x_1'$ (even in the abbreviated form) so that the computational effort consists only of forming the required sums over the layers in a somewhat different way. Consequently, the computational effort is somewhat less in computing the focusing factor along a ray at the time of ray computation than to compute several rays and to determine $dr/d\psi_0$ by differentiation of the results.

C. PHYSICAL ASSUMPTIONS BEHIND THE RAY TRACING METHOD

1. Formulation in General Terms

The basic assumptions behind the ray tracing method are dependent on two major steps in the derivation of the physical equations. The first step consists of linearizing the equations of motion. This requires that the perturbations of the medium be small (i.e., that the intensity of the sound be not too great). The second step requires that the frequency of the sound be high; in fact, the limit at infinite frequency is the sound field that is referred to as that of the ray or geometrical acoustics (Ingard[24]). It is sometimes stated that the variation of the properties of the air over a distance of one wavelength be small. The first restriction on the amplitude of the sound is generally satisfied if the mean acoustic velocity amplitude, U_e is small compared with the wave velocity, c . The second approximation actually requires more than is stated in that also the variations must take place "smoothly." This restriction is effectively that

$$\lambda|\nabla^2 c| << |\nabla c|$$

where ∇c is the gradient of c and $\nabla^2 c$ is its Laplacian. (Morse and Ingard[33], p. 80). (The same also holds for the wind component.)

2. Formulation in Terms of a Ducting Problem

The above statements of the basic assumptions require some amplification to be meaningful. The specific form of the second assumption for some very similar refraction problems is given by Brekhovskikh[1] for an underwater sound channel consisting of a layer of uniform speed of sound of depth H below which the speed of sound has a positive gradient (increases with depth). The situation is almost exactly analogous to the case of a layer of air with constant temperature capped by an inversion.

In this instance the relation required is that

$$H \gg \left[\lambda^2 c_0/4\pi(dc/dz)\right]^{1/3} = H^*.$$

The situation is illustrated in Figure 11. It is readily seen that for the higher frequencies the assumption is readily fulfilled for most atmospheric cases, but that for frequencies of a few cycles per second, it is seldom (if ever) satisfied.

Another form of the assumptions is the statement that the initial ray inclination angle must not be too small:

$$\psi_0 \gg \left[\lambda(dc/dz)/2\pi\right]^{1/3} = \psi_0^*.$$

This formulation of the assumption is illustrated in Figure 12.

3. Formulation in Terms of Ray Tube Cross Section

The assumptions behind the ray tracing method are discussed at some length by Kerr(34) for the case of short radio waves. Though the waves are not sound waves, the mathematical arguments and assumptions are analogous in considerable detail when appropriate changes in the quantities concerned are made. The two conditions are that

$$(\lambda/2\pi n)\ (|\nabla n/n|) \ll 1,$$

and

$$(\lambda/2\pi n)\ (|\nabla J/J|) \ll 1,$$

where λ = wavelength

n = index of refraction = $c_0/(c+u)$

u = wind speed

$J = \partial(x,y,z)/\partial(\xi,\eta,\zeta)$; $\xi=f_1(x,y,z)$,

$\eta=f_2(x,y,z)$,

$\zeta=f_3(x,y,z)$,

(x,y,z) are the space coordinates and (ξ,η,ζ) are coordinates such that ξ,η determine the ray (from the intersection of the surfaces ξ = const, η = const) while ζ corresponds to a surface normal to the rays, a wave front. In somewhat different terms,

$$J = dA/n(d\xi d\eta)$$

where dA = element of area on a wave front cut by the ray tube determined by the parameters $\xi,\xi + d\xi,\eta,\eta + d\eta$. Thus, along a ray tube, J is proportional to the cross-section of the ray tube.

The first inequality simply states that the change in index of refraction in a wavelength must be small compared with unity. The second condition states that the relative change in cross section of a ray tube in a wavelength must also be small compared with unity. Unfortunately, the second criterion is not applicable a priori. It does, however, indicate already that the situation cannot be assessed by the path of a single ray and that the ray has meaning only when associated with the family of rays to which it belongs (Kerr[34], p. 54). This is particularly important in the case of a "focus," where the ray method gives a ridiculous answer for sound intensity and, consequently, other methods must be used for the intensity esitmate.

4. Formulation in Terms of Longest Wavelength Ducted

The return of sound waves to the earth is analogous to the problem of the ducting of short radio waves. One may use the analysis of Kerr[34] for the longest wavelength trapped in a duct to give a criterion for the applicability of the ray tracing method in the atmosphere:

$$\lambda_{max} = (8\sqrt{2}/3)\int_0^d \left[n(z)-n(d)\right]^{1/2} dz$$

where d is the height of the duct. The index of refraction may be represented as

$$n(z) = c_o/\left[c_o + u_o + (\alpha + \beta)z\right]$$

where u_o is wind speed at $z = 0$ and $\alpha = dc/dz$, $\beta = du/dz$, which are assumed constant. Performing the integration approximately, it follows that

$$\lambda_{max} = (16\sqrt{2}/9)\left[(\alpha+\beta)d^3/c_o\right]^{1/2}$$

which may be written in the form

$$d = \left[\lambda_{max}^2 c_o/(16\sqrt{2}/9)^2(\alpha+\beta)\right]^{1/3}$$

which corresponds to the strong inequality from Brekhovskikh quoted previously. Some minor changes are to be noted:

Brekhovskikh		Above
H	(1)	d
>>	(2)	$\cong$
λ	(3)	λ_{max}
$4\pi \cong 12.6$	(4)	$(16\sqrt{2}/9)^2 \cong 6.32$
dc/dz	(5)	dc/dz + du/dz

The important differences lie in items (2), where strong inequality is replaced by approximate equality; (3), where the interpretation of the wavelength is correspondingly different; and (4), where the numerical factors are somewhat different.

The expression for λ_{max} may be expressed differently if we let $\Delta c^* = (\alpha + \beta)\cdot d$ = change (including wind) over the distance d. Then

$$\lambda_{max} = 2.5d(\Delta c^*/c_o)^{1/2}$$

The equation is graphed in Figure 13.

Item (1) is essentially notation while item (5) is essentially the same in both cases. (Wind was included above since it is a part of the acoustic problem.)

One may interpret the Brekhovskikh strong inequality as the approximate equality above provided that the wavelength is decreased by a factor of approximately 0.71. Stated differently, given the ordinate (H or d) and abscissa (dc/dz or $\alpha+\beta$) in Figure 13, the wavelength indicated is one which, from Brekhovskikh's inequality, should be much larger than that required to satisfy the ray method assumptions; while from the approximate quality above, 0.71 times the wavelength indicated is the maximum wavelength ducted.

In this connection, Kerr(34), page 21, seems pertinent; "---radiation at several times these wavelengths may also be affected strongly by the duct."

D. DISCONTINUOUS CHARACTER OF $dr/d\psi_o$ IN THE "LINEAR LAYER" MODEL

The calculation procedures for the ray tracing method generally violate the second of the basic assumptions of the ray tracing technique. These assumptions were, essentially, that:

a) Changes in property were small over a wavelength or $\lambda|\nabla c| \ll c$; and that

b) Changes take place smoothly over a wavelength or $\lambda|\nabla^2 c| \ll |\nabla c|$.

In the "linear layer" model of the atmosphere (in which changes in speed of sound and wind are linear functions of height within reasonably small layers), gradient changes at the layer boundaries are discontinuous. The situation is illustrated in Figure 14, in which the "linear layer" model is indicated by the solid lines while a "smoothly varying" atmosphere is indicated by the dashed curves. The value of $\nabla^2 c \sim d^2c/dz^2$ is essentially zero everywhere except near the layer boundary but has a "hump" here in the smoothly varying case. As the smooth case approaches the linear case, the hump becomes larger. Thus, in the "linear layer" one may think of $\nabla^2 c$ as zero everywhere except on layer boundaries where it has an "infinite" discontinuity. Consequently, the "linear layer" model cannot satisfy the second assumption for any wavelength.

The failure to satisfy the second assumption shows up clearly in certain irregularities of the ray tracing results. We carry through this arithmetical exercise in the following. Consider a simplified case in which there is no wind* so that

---

* This assumption is made to simplify the arithmetic. In this case we need not quibble about the difference between "ray" inclination, ψ , and "phase normal", φ , since they are identical.

the ray tracing may be represented by the layer formula

$$r = 2\sum_{1}^{n-1} R_i(\sin\varphi_{i-1} - \sin\varphi_i) + 2R_n \sin\varphi_{n-1} \ ,$$

$$R_i \cong (c_o/\cos\varphi_o)/(dc/dz)_i = c_o/\mu_i\cos\varphi_o \ , \qquad (A.39)$$

$$c_i/\cos\varphi_i = c_o/\cos\varphi_o \ .$$

It is readily seen that

$$dr/d\varphi_o = (2c_o\sin\varphi_o/\cos^2\varphi_o)\left[1/\mu_1\sin\varphi_o + \sum_{1}^{n-1}(1/\mu_{i+1} - 1/\mu_i)/\sin\varphi_i\right]. \quad (A.40)$$

Consider now the ray, φ_o^k , which becomes tangent (horizontal) to the k'th interface. The value of $\sin\varphi_o^k$ is determined by Snell's Law

$$\cos\varphi_o^k = c_o/c_k$$

with $\cos\varphi_k = 1$. The value of r and of $dr/d\varphi_o$ from (A.39) and (A.40) respectively are nicely defined. The values of $\sin\varphi_k = 0$ are never reached since n = k, and the highest value is k-1 at the end of summation.

Consider the same ray, but as entering the layer above the k'th interface. The position of return from (A.39) works very nicely since the last term of the summation

$$(R_{k+1} - R_k)\sin\varphi_k \qquad (A.41)$$

is zero. The intensity relation, (A.40), is not valid since the term

$$(1/\mu_{k+1} - 1/\mu_k)/\sin\varphi_k \qquad (A.42)$$

is included and is undefined.

If we look at the situation from the point of view of angles φ_o slightly greater than φ_o^k, the situation is a little better understood. In this case the last terms of the sums to find r and $dr/d\varphi_o$ given by (A.41) and (A.42) are valid since $\sin\varphi_k \neq 0$. However, as $\varphi_o \to \varphi_o^k$ (from above) the final summation term in (A.41) becomes zero while that of (A.42) becomes large, its sign depending on the relative values of μ_{k+1}, μ_k as shown below and in Figure 15;

$$0 < \mu_k < \mu_{k+1}, \quad dr/d\varphi_o \to -\infty$$

$$0 < \mu_{k+1} < \mu_k, \quad dr/d\varphi_o \to +\infty$$

(the value of μ_k must be positive in order to have a ray tangent at the k'th interface.)

In the third case

$$\mu_{k+1} < 0 < \mu_k, \quad dr/d\varphi_o \to -\infty$$

there is not only a discontinuity in $dr/d\varphi_o$ since the ray, φ_o^k, is bent upward on the k'th interface and continues until the level is reached where again (if ever)

$$c \geq c_o/\cos\varphi_o^k.$$

In such a case several more (positive) terms are added to the summation (A.39).

A part of the situation is easily seen by looking into how the angle, φ_k, is affected by Snell's Law as a function of φ_o^k. For those angles such that $\varphi_o > \varphi_o^k$, say $\varphi_o = \varphi_o^k + \epsilon$, one has

$$\sin\varphi_k = \left[1 - (c_k/c_o)^2\cos^2(\varphi_o^k + \epsilon)\right]^{1/2}$$

from which

$$\sin\varphi_k \cong (c_k/c_o)(\sin 2\varphi_o^k)^{1/2} .$$

Thus, for $\epsilon \to 0$, then $\sin\varphi_k \to 0$, but $d(\sin\varphi_k)/d\varphi_k \to +\infty$ i.e., the derivative of $\sin\varphi_k$ has a discontinuity at $\varphi_o = \varphi_o^k$ (vertical tangent).

The properties of the function $r = r(\varphi_0)$ (radial distance of the returning ray as a function of the initial inclination angle) which have been discussed are characteristic of the "linear layer" approximation to atmospheric conditions and are not unique to the particular example related above. This selection was made only on the basis of its relatively simple arithmetic to illustrate the point involved. This point is that for all angles ψ_0 such that the ray becomes tangent to an horizontal interface between layers with constant gradient of wind and speed of sound, the basic assumption of "smooth" variation of the atmospheric parameters is violated for any wavelength and, consequently, the sound intensity calculated at these points (zero in the limit) cannot be considered as correct.

The question of what to do about this situation is difficult. There appear to be two alternatives. The first, to use a "smooth" variation of parameters c and u before integrating the equations for the ray, introduces great analytical difficulties unless one resorts to a systematic "numerical integration" of the equations of the ray. This may be done at the expense of computer time. A second alternative is to smooth the computed values of r and $dr/d\psi_0$ (or of f) afterwards. The second answer raises the question of "How?". This does not appear to be any more difficult than answering the same question with regard to the first alternative, how to smooth the atmospheric data to better (?) represent the atmospheric conditions.

Something along these lines is discussed by Anderson, Gocht, and Serota [35], though their work would require extensive modification for application to the atmospheric problem.

E. THE GRADIENT OF THE FOCUSING FACTOR

The focusing factor may be written in the form

$$f = kr^2 J_0/J$$

where J = cross section of the ray tube at distance r

J_0 = reference cross section of the ray tube

k = suitable proportionality factor.

It was seen that the second assumption could be expressed as

$$|\nabla J/J| << 2\pi n/\lambda$$

where ∇ = gradient operator. Considering distances along the ground

$$(1/f)(df/dr) = 2/r - (1/J)(dJ/dr)$$

so we must have

$$|2/r - (1/f)(df/dr)| << 2\pi n/\lambda.$$

Since we are interested in reasonably large distances such that $r >> \lambda$ we may neglect the term $2/r$ on the left so that

$$(1/f)(df/dr) << 2\pi/\lambda \ .$$

Consequently, the second assumption implies a reasonably smooth field of the focusing factor; the percentage change of focusing factor in a distance of a wavelength should be small. This kind of statement is somewhat indefinite, but if we take it to mean a ratio of 10^{-2}, or less, and wish to include wavelengths up to 100 m (3 cycles), we should have an upper boundary as 6% per 100 m. In somewhat another form

$$|d(10 \cdot \log_{10} f)/dr| << 3/\lambda$$

so that the intensity change should not exceed 0.03 db per hundred meters.

F. INTENSITY AT A FOCUS

The focusing factor was expressed

$$f = r/(dr/d\psi_\circ)\tan\psi_\circ$$

where

r = radial distance from source to receiver

$\psi_\circ$ = inclination angle of the ray (here assumed the same at both source and receiver). At those points, or those angles $\psi_\circ$, at which $dr/d\psi_\circ = 0$, the focusing factor becomes infinitely large. The resulting intensity

$$I = I_* f$$

where I_* = intensity for spherical spreading, becomes infinite at these locations. This result is more dramatic than factual and is brought about by the failure of the second basic assumption of the ray methods; i.e., at such points the relative rate of change of ray tube area per wavelength is no longer negligible. The analysis to determine the focusing factor must be redone to account for interference effects. It may be shown that (Brekhovskikh[1], page 483ff) the appropriate expression for the focusing factor becomes

$$f = (1.25/\tan\psi_\circ)\left[2\pi r\sin\psi_\circ/\lambda(d^2r/d\psi_\circ^2)\right]^{\frac{1}{3}} .$$

Wave interference which takes place at the focus gives rise to interference zones. The distance from the focus to the first zero is referred to as the "width" of the first maximum of intensity and is given by the expression

$$\Delta r \cong 1.86\left[4\pi^2(d^2r/d\psi_0^2)/(\lambda\sin\psi_0)^2\right]^{\frac{1}{3}} .$$

G. NOTES ON "ROUNDED MAXIMA"

It has been mentioned that the "linear layer" model of the atmosphere, though convenient, violates at least the second assumption behind the ray tracing method, that of the "smoothness" of the changes of speed of sound and wind as a function of coordinate.

In the following, some ray tracing relations are derived in some instances in which speed of sound changes smoothly in the vertical. The situation is most critical when the speed of sound has a maximum or secondary maximum. In such a case the vertical gradient of the speed of sound changes abruptly from positive to negative with the possibility of rather large changes. To simplify the arithmetic, the wind is ignored. This permits treating wave normals as ray tangents and eliminates the correction term for downwind ray displacement due to wind.

When a maximum or secondary maximum with a "corner", as in the linear layer model, occurs in the speed of sound vs height curve, the ray return to the ground shows a break or a shadow zone. When the corner at the maximum is rounded to have a zero gardient at the maximum value, the ray tracing picture is altered materially. The break in the function $r = r(\psi_0)$ is eliminated. There remains, however, a discontinuity in the sense that for $\psi_0 \to \psi_1$, $r \to +\infty$. This means that the shadow zone has been eliminated and the whole space becomes ensonified (however lightly).

1. Parabolic Maximum Instead of a Corner

Let the values of the speed of sound at two levels be c_1 and c_2 and let the heights be 0, H. The usual linear model is

$$c = c_1 + \left[(c_2 - c_1)/H\right]z, \quad 0 \le z \le H.$$

This may be replaced by a parametric linear-quadratic model of the form

$$c = c_1 + \alpha z \;, \qquad 0 \le z \le h,$$
$$c = c_2 - \gamma(H-z)^2, \qquad h \le z \le H,$$

with the formulation of α , γ depending on the parameter h. To obtain a match at $z = h$ of c and dc/dz,

$$c_1 + \alpha h = c_2 - \gamma(H-h)^2 \;,$$

$$\alpha = 2\gamma(H-h) \;,$$

whence

$$\alpha = 2(c_2 - c_1)/(H+h) \;,$$

$$\gamma = (c_2 - c_1)/(H^2-h^2) \;.$$

The final expressions are

$$c = c_1+2\left[(c_2-c_1)/(H+h)\right]z, \qquad 0\le z \le h \;,$$

$$c = c_2 -\left[(c_2-c_1)/(H^2-h^2)\right](H-z)^2, \qquad h\le z \le H \;.$$

The above provides a way of subdividing a layer with h to obtain a parabolic maximum. The curvature of the maximum is a function of h . For $h = 0$, the fit is quadratic; for $h = H$, the fit is linear, as shown in Figure 16.

In practical applications, a judicious choice of the parameter h is necessary. Through "rounding" the maximum, its use might increase the difficulties of the discontinuous derivative dc/dz at the junction with lower levels. The slope at the bottom level of the layer is $2(c_2-e_1)/(H+h)$ which may be made to vary from $2(c_2 - c_1)H$ to $(c_2 - c_1)/H$. If the slope of the speed of sound curve for the linear layer model in the layer below lies in this range, a value of h may be chosen so that dc/dz is the same in both layers at the junction. If the slope in the layer below exceeds the larger value, the choice of h=0 at least minimizes the slope discontinuity at the junction within the limits of this analysis (i.e., remaining confined to modifying this layer only). When the slope of the layer below is less than the lower limit, then any choice of $h < H$ increases the slope discontinuity; the least change at the juncture being for $h = H$. In this last case, the "best" choice produces an entirely linear layer again which is exactly what should be avoided. Some value of h near H seems reasonable but a criterion for selection is missing.

2. Refraction Earthward in a Parabolic Layer

Consider the layer with a maximum speed of sound at the top and let the interpolation form be quadratic

$$c = c_2 - \gamma(H - z)^2 , \quad \gamma = (c_2 - c_1)/H^2 .$$

Consider the case of downward refraction of rays within the layer. The distance traveled from entry to attaining a horizontal tangent is given by

$$x = \int_0^{z^*} \cot\varphi dz , \quad 0 < z^* \le H.$$

The inclination as a function of z depends on Snell's Law

$$\cos\varphi = (c/c_1)\cos\varphi_1$$

where c is given above and φ_1 is the inclination at $z = 0$.
Then

$$\sin\varphi = \left[c_1^2 - c_2^2\cos^2\varphi_1 + 2\gamma c_2(H-z)^2\cos^2\varphi_1 - \gamma^2(H-z)^4\cos^2\varphi_1\right]^{1/2} \Big/ c_1 .$$

The inclination will be horizontal for $\varphi = 0$ so that the top of the integration will be determined by

$$c_1{}^2 - c_2^2\cos^2\varphi_1 = -2\gamma c_2(H-z^*)^2\cos^2\varphi_1 + \gamma^2(H-z^*)^4\cos^2\varphi_1 .$$

Then

$$\sin\varphi = (\cos\varphi_1/c_1)(2\gamma c_2)^{1/2} \times \left\{(z^*-z)(2H-z^*-z)\left[1-(\gamma/2c_2)\left\{(H-z^*)^2+(H-z)^2\right\}\right]\right\}^{1/2} .$$

To simplify the integration, the factor in brackets is approximated as 1.

The displacement then becomes

$$x = \int_{0}^{z^*} \frac{c_2 - \gamma\ (H-z)^2}{(2\gamma c_2)^{1/2}\left[(z^*-z)(2H-z^*-z)\right]^{1/2}}\, dz\ .$$

or

$$(2\gamma c_2)^{1/2} x = (c_1 - \gamma\alpha^2/4)A - \gamma\alpha B - \gamma^2 C\ ,$$

where

$$\alpha = 2(H - z^*)$$

and, for $y = z^* - z$,

$$A = \int_{0}^{z^*}\left[y(\alpha+y)\right]^{-\frac{1}{2}} dy,\quad B = \int_{0}^{z^*} y(\alpha+y)\Big]^{-\frac{1}{2}} dy\ ,\quad C = \int_{0}^{z^*} y^2 \left[y(\alpha+y)\right]^{-\frac{1}{2}} dy\ .$$

The final result is

$$x = \frac{2c_2 - (c_2 - c_1)(1-z^*/H)^2}{\left[2c_1(c_2-c_1)\right]^{\frac{1}{2}}}\ H\sinh^{-1}\left[z^*/2(H-z^*)\right]^{\frac{1}{2}} -$$

$$\left\{\left[(c_2-c_1)/8c_2\right]z^*(2H-z^*)\right\}^{\frac{1}{2}}\ .$$

In the above, it is readily seen that the principal part is contained in

$$x \doteq H\left[2c_2/(c_2-c_1)\right]^{\frac{1}{2}} \sinh^{-1}\ \left[z^*/2(H-z^*)\right]^{\frac{1}{2}}\ .$$

Considering z^* as a ray parameter, those rays that penetrate only a small distance, $z^* \ll H$, travel on correspondingly short distance in the horizontal during the process. Those rays for which $z^* \to H$, i.e., penetrate nearly the top of the layer, travel an extremely large horizontal distance; in fact, for $z^* \to H$, then $x \to \infty$.

a) Digression on the Linear Layer for Comparison

In the case of a linear layer for which

$$c = c_1 + az, \qquad a = (c_2 - c_1)/H ,$$

then

$$x \cong \left[2c_1 z^* H/(c_2 - c_1)\right]^{\frac{1}{2}} .$$

It is readily seen from the above that for $z^* = 0$ then $x = 0$; while $z^* = H$ implies that

$$x \cong H\left[2c_1/(c_2 - c_1)\right]^{\frac{1}{2}}.$$

Consequently, all rays returned in this layer are displaced (on return to the bottom of the layer) a finite range, depending on their depth of penetration (or on their angle of inclination at entry through the bottom).

In comparing the linear layer model and the parabolic model, note that common values of z^* in each instance do <u>not</u> indicate corresponding rays (angles φ_1). They correspond at $z^* = 0$, $\varphi_1 = 0$ and at $z^* = H$, $\cos\varphi_1 = c_1/c_2$, but the depth of penetration is greater (same values of φ_1) in the case of the linear model for intermediate values of φ_1.

3. <u>Penetration of a Parabolic Layer</u>

The method of integration used in the examples is confined to the case of a ray reaching its maximum height within the layer concerned. For the rays that penetrate the layer, the displacement integral is

$$x = \int_0^H \frac{\left\{c_2 - \gamma(H-z)^2\right\}\cos\varphi_1}{\left[c_1^2 - \left\{c_2 - \gamma(H-z)^2\right\}^2\cos^2\varphi_1\right]^{\frac{1}{2}}}\,dz$$

so that

$$x = \frac{H/2}{(c_1 + \bar{c}\ \cos\varphi_1)^{\frac{1}{2}}} \left\{ \left[c_1 (1 - \cos\varphi_1) \right]^{\frac{1}{2}} + \right.$$

$$\left. \frac{(c_1 + c_2 \cos\varphi_1}{\left[(c_2 - c_1) \cos\varphi_1 \right]^{\frac{1}{2}}} \tanh^{-1} \left[\frac{(c_2 - c_1) \cos\varphi_1}{c_1 (1 - \cos\varphi_1)} \right]^{\frac{1}{2}} \right\}$$

where $c_1 < \bar{c} < c_2$ is an approximate average which was used to simplify the integration.

For logical consistency to trace the rays penetrating a layer topped by a rounded maximum of speed of sound, it should be paired with a layer above with similarly rounded "maximum" at the bottom.

In this instance

$$c = c_1 - \gamma z^2, \quad \gamma = (c_1 - c_2)/H^2,$$

and the displacement integral is

$$x = \int_0^H \frac{(c_1 - \gamma z^2) \cos\varphi_1 \, dz}{\left[c_2^2 - (c_1 - \gamma z^2)^2 \cos^2\varphi_1 \right]^{\frac{1}{2}}}$$

Integration yields

$$x = \frac{H(c_1 - c_2)^{\frac{1}{2}}}{2\left[c_1 + \bar{c}\ \cos\varphi_1 \right]^{\frac{1}{2}}} \left[(c_1 - c_2 \cos\varphi_1) + \frac{c_1 (1 + \cos\varphi_1)}{(c_1 - c_2)(\cos\varphi_1)^{\frac{1}{2}}} \tanh^{-1} \left\{ \frac{(c_1 - c_2) \cos\varphi_1}{(c_1 - c_2 \cos\varphi_1)} \right\}^{\frac{1}{2}} \right].$$

In this instance we are allowed $\varphi_1 \to 0$ and, from the second term (the principal term) it is seen that the displacement becomes infinite for $\varphi_1 \to 0$.

H. Sound Ray Tracing in an Atmosphere with "Parabolic" Layers

1. The Atmosphere Model and the Ray Model

The ray model for tracing sound rays in the atmosphere may be expressed by the ray (differential) equation:

$$\frac{dx}{dz} = \frac{c \cos\varphi + u}{c \sin\varphi} \tag{A.43}$$

and by Snell's Law:

$$\frac{c}{\cos\varphi + u} = \frac{c_o}{\cos\varphi_o} + u_o = K \tag{A.44}$$

The above differential equation (A.43) expresses the fact that the ray inclination ($\cot^{-1}(dx/dz)$) is not the same as the phase normal, φ, since the sound energy is transported horizontally by the wind component, u. It is assumed that the propagation is in the plane that contains the wind component. The effect of the wind component perpendicular to this plane is easily taken into consideration. Snell's Law (A.44) also contains the wind component, and along a "ray", the relation is constant as indicated by the initial values with subscript on the right-hand side.

The ray tracing procedure consists of solving (A.44) for $\cos\varphi$, substituting into (A.43), and integrating the resulting expression where the wind component, u, and speed of sound, c, are considered as functions of height, z. The result of eliminating the angle of the phase normal is the expression:

$$\begin{aligned}\frac{dx}{dz} &= \frac{c^2 \cos\varphi_o + u\left[c_o + (u_o - u)\cos\varphi_o\right]}{c\left\{\left[c_o + (u_o-u)\cos\varphi_o\right]^2 - \left[c\cos\varphi_o\right]^2\right\}^{\frac{1}{2}}} \\ &= \frac{c^2 + u(K-u)}{c\left[(K-u)^2 - c^2\right]^{\frac{1}{2}}}\end{aligned} \tag{A.45}$$

where use has been made of the right part of (A.44) above.

The radical in the denominator of (A.45) may be factored so that the expression becomes:

$$\frac{dx}{dz} = F/\left[K-(u+c)\right]^{\frac{1}{2}} \tag{A.46}$$

where

$$F = \left[c^2+u(K-u)\right]/c\left[K+c-u\right]^{\frac{1}{2}} \tag{A.47}$$

The numerator expression, F, is a function of altitude, but does not undergo large changes. Its magnitude is in the neighborhood of $\sqrt{c/2}$, neglecting small effects of u, $u \ll c$, and considering c and $c_0/\cos\varphi_0$ as being of the same order of magnitude. The denominator of (A.46) is important since for any ray, φ_0, which returns to the earth it becomes zero at some level, the level for which $\varphi = 0$, the crest or maximum altitude of the ray. The behavior of the integrated form of (A.46)

$$x - x_0 = \int \left\{F(z)/[K-(u+c)]^{\frac{1}{2}}\right\} dz \tag{A.48}$$

i.e., the ray displacement in the horizontal direction, is strongly affected by the behavior of the denominator near this level.

In view of the above, the integration of (A.48) may be approximated using the mean value theorem in the form

$$x - x_0 = F(\zeta)\int_{z_0}^{z} [K-(u+c)]^{-\frac{1}{2}} dz$$

where ζ is a suitable value in the range $z_0 < \zeta < z$. It is assumed that the layer z_0, z is not extremely large (usual practice in ray tracing methods).

The distinction between the many ray tracing techniques involve the way in which the radical in (A.45) is handled. In the form (A.45), the variation of c and u with z must be treated separately. For each represented by a linear variation throughout a layer, the integration is elementary (Rothwell[30]). If parabolic variation in a layer is considered, the integration leads to the well known Elliptic Integrals but not in a simple way. (The square root of a rather general quartic is involved and the integration requires detailed classification of its roots.) Even in this case, the other terms of the integrand are being ignored and must be accounted for. When the integrand is expressed as in (A.48), the use of a parabolic variation of $u+c=v$ with altitude is convenient and the distinction between the variation of u and c separately with altitude is no longer necessary as far as denominator is concerned. The mixture of u and c in $F(z)$ may be handled in a variety of ways without leading to undue complications. The methods of carrying out the integration will be discussed subsequently.

2. The Atmospheric Model

The usual model of the atmosphere is a "linear layer" model by which is meant that in layers the sum $c+u$ is assumed to be a linear function of height, represented by a sequence of points joined by straight lines. It has been pointed out that such a model does not satisfy the basic physical assumptions, particularly in that the second derivative with respect to height has infinite discontinuities at the layer boundaries.

If the linear layer model is abandoned, the question of what is a reasonable substitute is not readily answered. To some extent, the answer depends on how the atmospheric sounding to determine c and u is made.

The usual routine sounding for synoptic purposes is recorded in such a way that c is determined from temperature (virtual) information which is selected at "significant" levels. These levels are significant because, among other things, they represent the boundaries of layers through which the temperature varies linearly with height. Some of these points may be selected because they represent the occurrence of local maxima or minima in the sounding. In such a case, a local "rounding" at these points may be appropriate with the local maximum or minimum retained at the point itself.

Soundings made in such a way that data are recorded at specified time intervals or altitude intervals must be considered from a different point of view. In this case, the occurrence of a true local maximum or minimum at the data point would be quite accidental, the normal occurrence being between the data points. Consequently, joining the data points by straight lines in such regions may be inappropriate.

a) Parabolic Layer Model

The parabolic layer model considered here is appropriate for representation of a sounding in which data is taken at prescribed intervals of time or altitude. It is by no means a unique representation, but has some inherent advantages. Among others, it has the advantage of being a "smoothest" representation that is thoroughly consistent with the data. The ground rules for constructing such a representation are:

a) The function $v = u+c = v(z)$ must fit the data points exactly.

b) The slope dv/dz at the data points is assigned on the basis of the adjacent data points.

c) The parabolic arcs must join with a common tangent.

d) When a choice exists, it is made so that the discontinuity of the second derivative at join points is least.

The above is formalized in the following. Consider the quantity $v = u+c = v(z)$ as specified at data points z_{i-1}, z_i, z_{i+1}, etc., and consider the layer z_i to z_{i+1}. To reduce the subscript algebra let $v_1 = v(z_i)$, $v_2 = v(z_{i+1})$, $\Delta = z_{i+1}-z_i$, and let k be a parameter that is 0 at z_i and 1 at z_{i+1} so that $z = z_i+(z_{i+1}-z_i)k = z_i+\Delta k$. Consider the layer z_i, z_{i+1} as divided into two parts. In the lower part, let the function be represented by the parabola

$$y_1 = a_1+b_1\Delta k+c_1\Delta^2 k^2$$

and in the upper part of the parabola

$$y_2 = a_2+b_2\Delta k+c_2\Delta^2 k^2$$

The function values and derivatives are prescribed at $v_1(0) = v(z_i)$ and $v_2(1) = v(z_{i+1})$ from which

$$v_1(0) = v_1 = a_1$$

$$v_1'(0) = v_1' = b_1\Delta$$

$$v_2(1) = v_2 = a_2+b_2\Delta+c_2\Delta^2$$

$$v_2'(1) = v_2' = b_2\Delta+2c_2\Delta^2$$

where $v' = dv/dk$ (not dv/dz). It then follows that

$$a_1 = v_1$$

$$b_1\Delta = v_1'$$

$$a_2 = v_2-v_2'+c_2\Delta^2$$

$$b_2\Delta = v_2'-2c_2\Delta^2$$

Since the two expressions for the upper and lower parts of the layers must join with a common tangent, the conditions are

$$c_1 \Delta^2 k^2 - c_2 \Delta^2 (1-k)^2 = v_2 - v_1 - v_2'(1-k) - v_1'k$$

and

$$c_1 \Delta^2 k + c_2 \Delta^2 (1-k) = (v_2' - v_1')/2$$

from which it follows that

$$c_1 \Delta^2 k = v_2 - v_1 - v_2'(1-k)/2 - v_1'(1+k)/2$$

$$c_2 \Delta^2 (1-k) = v_1 - v_2 + v_2'(1-k)2 + v_1'(k/2)$$

so that c_1 and c_2 depend on the location of the join point k, $0 < k < 1$. The difference in second derivatives at the join point is measured by the difference, $c_2 - c_1$, and

$$\Delta^2 (c_2 - c_1) = [k(1-k)]^{-1} [(v_1 - v_2) + (v_2' + v_1')/2]$$

It is readily seen that this difference, $c_2 - c_1$, is a minimum at $k = \frac{1}{2}$, the midpoint of the layer, where the first factor on the right has the value 4.

If the parabola in each half layer is in the form $v = A + Bt + Ct^2$, $0 < t < 1$, the coefficients are given by the following. In the layer $z_n < z < (z_{n+1} + z_n)/2$

$$A = v_n$$

$$B = v_n'(z_{n+1} - z_n)/2$$

$$C = [(v_{n+1} - v_n) - (v_{n+1}' + 3v_n')(z_{n+1} - z_n)/4]/2$$

and in the layer $(z_{n+1} + z_n)/2 < z < z_{n+1}$

$$A = [(v_n + v_{n+1}) - (v'_{n+1} - v'_n)(z_{n+1} - z_n)/4]/2$$

$$B = (v_{n+1} - v_n) - (v'_{n+1} + v'_n)(z_{n+1} - z_n)/4$$

$$C = -[(v_{n+1} - v_n) - (3v'_{n+1} + v'_n)(z_{n+1} - z_n)/4]/2$$

where now v'_n, v'_{n+1} are derivatives with respect to z, dv/dz, at the indicated data points. The assignment of these derivatives is quite arbitrary. The standard Lagrange formula is available

$$f'(x_1) = (f_1 - f_o)/(x_1 - x_o) + (f_2 - f_1)/(x_2 - x_1) - (f_2 - f_o)/(x_2 - x_o)$$

or the secant approximation

$$f'(x_1) = (f_2 - f_o)/(x_2 - x_o)$$

or any other handy (and reasonable) form may be used.

3. Integration of the Ray Equation

The ray path integral is

$$x_2 - x_1 = \int_{z_1}^{z_2} [F(z)/(K-v)^{\frac{1}{2}}]dz$$

where

$$K = c_0/\cos\varphi_o + u_o$$

$$F(z) = [c^2 + u(K-u)]/c[K + c-u]^{\frac{1}{2}}$$

$$v = u + c.$$

The integration is approximated in the form

$$x_2 - x_1 = F(\zeta) \int_{z_1}^{z_2} (K-v)^{-\frac{1}{2}}\, dz$$

where $z_1 < \zeta < z_2$. For a parabolic layer model, the expression for $v(z)$ is considered in the form

$$v = A + Q\,(z - z_*)^2 \; .$$

All integrals are elementary. The results for the several cases required are tabulated.

I. $Q > 0,\ K - A > 0,\ P = [(K - A)/Q]^{\frac{1}{2}}$

a: $z_* \le z_1 < z_2$

$$Z_1 = (z_1 - z_*)/P,\ Z_2 = (z_2 - z_*)/P$$

$$x_2 - x_1 = F(\zeta)\, Q^{-\frac{1}{2}}\, (\sin^{-1} Z_2 - \sin^{-1} Z_1)$$

b: $z_1 < z_2 \le z_*$

$$Z_1 = (z_* - z_1)/P,\ Z_2 = (z_* - z_2)/P$$

$$x_2 - x_1 = F\,(\zeta)\,(\sin^{-1} Z_1 - \sin^{-1} Z_2)Q^{-1/2}$$

II. $Q < 0,\ K\text{-}A > 0,\ P = [-(K - A)/Q]^{\frac{1}{2}}$

a: $z_* \le z_1 < z_2$

$$Z_1 = (z_1 - z_*)/P,\quad Z_2 = (z_2 - z_*)/P$$

$$x_2 - x_1 = F\,(\zeta)(-Q)^{-\frac{1}{2}} \log\left\{[Z_2 + (1+Z_2^2)^{\frac{1}{2}}]/[Z_1 + (1 + Z_1^2)^{\frac{1}{2}}]\right\}$$

b: $z_1 < z_2 \le z_*$

$$Z_1 = (z_* - z_1)/P,\quad Z_2 = (z_* - z_2)/P$$

$$x_2 - x_1 = F(\zeta)(-Q)^{-\frac{1}{2}} \log\left\{[Z_1 + (1+Z_1^2)^{\frac{1}{2}}]/[Z_2 + (1+Z_2^2)^{-\frac{1}{2}}]\right\}$$

III: $Q < 0$, $K - A < 0$, $P = [(K - A)/Q]^{\frac{1}{2}}$

a: $z_* \leq z_1 < z_2$

$$Z_1 = (z_1 - z_*)/P, \quad Z_2 = (z_2 - z_*)/P$$

$$x_2 - x_1 = F(\zeta)(-Q)^{-\frac{1}{2}} \log\left\{[Z_2 + (Z_2^2 - 1)^{\frac{1}{2}}]/[Z_1 + (Z_1^2 - 1)^{\frac{1}{2}}]\right\}$$

b: $z_1 < z_2 \leq z_*$

$$Z_1 = (z_* - z_1)/P, \quad Z_2 = (z_* - z_2)/P$$

$$x_2 - x_1 = F(\zeta)(-Q)^{-\frac{1}{2}} \log\left\{[Z_1 + (Z_1^2 - 1)^{\frac{1}{2}}]/[Z_2 + (Z_2^2 - 1)^{\frac{1}{2}}]\right\}$$

In the above, the maximum or minimum of $v(z)$ occurs at z_* and z_* is excluded from the interval z_1, z_2. In the event $z_1 < z_* < z_2$, a subdivision to place z_* at an end point may be made and one of the above forms may be used.

The linear cases with

$$v = A + B(z - z_1)$$

are handled in the expressions

IV: a: $B > 0$, $K - A > 0$, $z_* = z_1 + (K - A)/B$, $z_1 < z_2 \leq z_*$

$$P = (K - A)/B, \quad Q = (z_2 - z_1)/P$$

$$x_2 - x_1 = 2F(\zeta)(P/B)^{\frac{1}{2}}[1 - (1 - Q)^{\frac{1}{2}}]$$

b: $B < 0$, $K - A > 0$,

$$P = -(K - A)/B, \quad Q - (z_2 - z_1)/P$$

$$x_2 - x_1 = 2F(\zeta)(-P/B)^{\frac{1}{2}}[(1 + Q)^{\frac{1}{2}} - 1]$$

c: $B = 0,\ K > A$

$$x_2 - x_1 = F(\zeta)(z_2 - z_1)/(K-A)^{\frac{1}{2}}$$

The determination of $F(\zeta)$ can be accomplished only by an appeal to some sort of estimate of its basic variation with respect to altitude within the layer. If a linear variation is assumed, the integration is equivalent to that of

$$x_2 - x_1 = \int_{z_1}^{z_2} [F_1 + (F_2 - F_1)(z - z_1)/(z_2 - z_1)](K-v)^{-\frac{1}{2}}\, dz$$

with the results

I: a: $$F(\zeta) = F_1 + [(F_2 - F_1)/(Z_2 - Z_1)]\left\{[(1-Z_1^2)^{\frac{1}{2}} - (1-Z_2^2)^{\frac{1}{2}}]/(\sin^{-1} Z_2 - \sin^{-1} Z_1) - Z_1\right\}$$

b: $$F(\zeta) = F_1 + [(F_2 - F_1)/(Z_2 - Z_1)]\left\{[(1-Z_2^2)^{\frac{1}{2}} - (1-Z_1^2)^{\frac{1}{2}}]/(\sin^{-1} Z_1 - \sin^{-1} Z_2) - Z_1\right\}$$

II: a: $$F(\zeta) = F_1 + [(F_2 - F_1)/(Z_2 - Z_1)]\left\{[(1+Z_2^2)^{\frac{1}{2}} - (1+Z_1^2)^{\frac{1}{2}}]/\log\left\{[Z_2 + (1+Z_2^2)^{\frac{1}{2}}]/[Z_1 + (1+Z_1^2)^{\frac{1}{2}}]\right\} - Z_1\right\}$$

b: $$F(\zeta) = F_1 + [(F_2 - F_1)/Z_2 - Z_1)] \cdot \left\{[(1+Z_1^2)^{\frac{1}{2}} - (1+Z_2^2)^{\frac{1}{2}}]/\log\left\{[Z_1 + (1+Z_1^2)^{\frac{1}{2}}]/[Z_2 + (1+Z_2^2)^{\frac{1}{2}}]\right\} - Z_1\right\}$$

III: a: $$F(\zeta) = F_1 + [(F_2 - F_1)/(Z_2 - Z_1)] \cdot \left\{[(Z_2^2 - 1)^{\frac{1}{2}} - (Z_1^2 - 1)^{\frac{1}{2}}]/\log\left\{[Z_2 + (Z_2^2 - 1)^{\frac{1}{2}}]/[Z_1 + (Z_1^2 - 1)^{\frac{1}{2}}]\right\} - Z_1\right\}$$

b: $F(\zeta) = F_1 + [(F_2 - F_1)/(Z_2 - Z_1)]$.

$$\left\{(Z_1^2-1)^{\frac{1}{2}}-(Z_2^2-1)^{\frac{1}{2}}]/\log\left\{[Z_1+(Z_1^2-1)^{\frac{1}{2}}]/[Z_2+(Z_2^2-1)^{\frac{1}{2}}]\right\}-Z_1\right\}$$

IV: a: $F(\zeta) = F_1 + (F_2 - F_1)\left\{[1+Q-(1-Q)^{\frac{1}{2}}]/3Q\right\}$

b: $F(\zeta) = F_1 + (F_2 - F_1)\left\{[(1+Q)^{\frac{1}{2}}+Q-1]/3Q\right\}$

c: $F(\zeta) = (F_1 + F_2)/2$

where appropriate meaning for the symbols is listed in the previous tabulation of the several cases. For those cases for which $x_2 - x_1$ remains finite, it appears that for small layers the value $(F_2+F_1)/2$ is a reasonably good approximation. A better value would be obtained using the above. The situation would be still more exact if higher order approximations to $F(z)$ are used. This is a feasible procedure since the integrals involved are all elementary (but the calculations required become even more tedious than above).

4. Summary

The ray tracing method involves the evaluation of an improper integral (integrand becomes large without bound) in every instance in which a ray returns to earth. The integrand may be expressed in a form which isolates this particular factor of the denominator which becomes zero. The remaining terms may be lumped together in the numerator of the integrand. The numerator then becomes a slowly varying function of height. The assumptions about the variation of $v = u + c$ with altitude in the atmosphere strongly affect the results of the ray tracing method. The linear layer assumption (points of $v(z)$ joined by straight lines) has little virtue in that it may well fail to represent the atmosphere well, expecially if data are recorded at arbitrary times during ascent or heights (without reference to $v(z)$ itself). In such cases, a representation in terms of parabolic layers may be expected to describe the situation better. In addition, the formulation in terms of parabolic layers fits the physical assumptions behind the ray tracing method far better than the "linear layer" assumption (to which there are some significant objections). The formulation in the above terms permits evaluation of the integrals in elementary terms to a reasonably high degree of accuracy.

APPENDIX B

THEORETICAL TREATMENT OF VARIABILITY OF SOUND INTENSITY ESTIMATES

The application of information on the variability of wind, temperature, and vapor pressure (or relative humidity) to the problem of estimating sound intensity involves three distinct and different problems. First, knowing all about the atmosphere, it is necessary to be able to calculate the sound intensity. There is no clearly defined way to do this with a well-defined degree of accuracy. In the limit of high frequencies (or short wavelength), the ray tracing method provides some information, but the results must not be pushed too far because the method may be made to produce silly answers. On the other hand, this is all that is available in the present state of the art.

The second problem involves that of describing the state of the atmosphere. The most common description (which is adequate for many purposes) is that of the linear layer model. This model has the advantage that it permits the use of abbreviated methods in the construction of ray paths, but the very use of such a model and computing method violates the basic assumptions of ray acoustics. At least some of this difficulty may be ironed out by "rounding" some of the "corners" in the linear layer model. This process brings one back immediately to the exact description of this "rounding" process in terms of the real atmosphere. An adequately exact measurement of the radius of curvature of the "corners" would require fantastically complex meteorological observations.

The description of the variability of the atmosphere in time and space and its application to an ideally accurate sound intensity estimation procedure constitutes the third major problem. One approach, using a point-wise perturbation technique of the simplified linear layer model, is feasible but of doubtful accuracy in view of the limitations placed on the range of allowable parametric variation.

This method is discussed at some length in the following section. The somewhat more general approach to the perturbation of the ray differential equations does not appear to have a tractable solution.

1. Effect on the Focusing Factor

The errors in sound intensity estimates using the ray tracing method that are due to the way in which meteorological parameters affect the pattern of the returning rays may be discussed in terms of the focusing factor. From the basic relations

$$I = I_* f, \quad f = r/|dr/d\psi_0|\tan\psi_0 \quad , \tag{B.1}$$

it is seen at once that I_* is not subject to error from this source since it is the intensity that would have been present due to spherical spreading. The numerator of the expression for f (r, the distance from source to receiver) is also unaffected. One is concerned with the variations that may occur in the denominator terms, $|dr/d\psi_0|$ and $\tan\psi_0$, at a particular receiver point; i.e., at a fixed value of r and not in terms of the initial inclination angle, ψ_0. Put in different words, one is not at all concerned with the variations that may occur in the ray paths due to meteorological changes, but is concerned with the intensity of sound that arrives at a fixed point regardless of the path taken by the sound ray in reaching it.

Consider the source-receiver distance in terms of the initial inclination angle, ψ_0. Let the meteorological conditions be represented by the symbol a, where a is a many-component vector (or a continuous function in a more general sense). For a given set of conditions, a, there is an initial inclination angle, ψ_0, required that the ray reach the receiver. If the meteorological conditions are changed to $a + da$ and if the sound still may be traced to the receiver, it will come from an initial inclination angle $\psi_0 + d\psi_0$. The disturbed conditions may be expressed as a series expansion in terms of da and $d\psi_0$,

$$r(\psi_o+d\psi_o,\ a+da) = r(\psi_o,\ a) + (\partial r/\partial\psi_o)d\psi_o + (\partial r/\partial a)da + ---.$$

Since the rays arrive at the same place, one has, to terms of first order,

$$(\partial r/\partial\psi_o)d\Psi_o + (\partial r/\partial a)da = 0 \ . \tag{B.2}$$

The relative error in the focusing factor may be expressed as

$$\delta f/f = -\ \delta[\partial r/\partial\psi_o]/(\partial r/\partial\psi_o) - \sec^2\psi_o(\delta\psi_o)/\tan\psi_o \ . \tag{B.3}$$

Since ψ_o is considered as the independent variable,

$$\delta(\partial r/\partial\psi_o) = (\partial^2 r/\partial\psi_o^2)\delta\psi + (\partial^2 r/\partial\Psi\partial a)\delta a \quad ,$$

and we have from (B.1) and (B.3)

$$\delta f/f = -\ [(\partial^2 r/\partial\psi_o^2)/(\partial r/\partial\psi_o)]\delta\psi_o - [(\partial^2 r/\partial\psi_o\partial a)/(\partial r/\partial\psi_o)]\delta a -$$

$$(\sec^2\psi_o/\tan\psi_o)\delta\psi_o .$$

Using (B.2) in the above

$$\delta f/f = [(\partial^2 r/\partial\psi_o^2)/(\partial r/\partial\psi_o)^2 + \sec^2\psi_o/(\partial r/\partial\psi_o)\tan\psi_o](\partial r/\partial a)\delta a -$$

$$[(\partial^2 r/\partial\psi_o\partial a)/(\partial r/\partial\psi_o)]\delta a.$$

Since $\partial r/\partial\psi_o = r/f \ \tan\psi_o$,

$$\delta f/f = (f\cdot\tan\psi_o/r)[\left\{(\partial^2 r/\partial\psi_o^2)(f\cdot\tan\psi_o/r) + \sec^2\psi_o\right\}(\partial r/\partial a) - \partial^2 r/\partial\psi_o\partial a]\delta a \ . \tag{B.4}$$

Use of (B.4) to estimate the error of the focusing factor requires that one must estimate f, Ψ_o, $\partial^2 r/\partial\Psi_o^2$, $\partial r/\partial a$, and $\partial^2 r/\partial\Psi_o\partial a$ (r being given). One might think it was unnecessary to estimate the quantity $\partial r/\partial a$, the change of the distance of ray return with respect to meteorological parameter. This has crept in through the back door, so to speak. Though we could care less about which ray returns at the given distance, r, we need to know this derivative to estimate by how much the meteorological parameter change has changed the initial ray inclination angle.

The expression (B.4) for the relative change of the focusing factor is in the form of a sum of terms if the meteorological parameter, a, is representative of several items; a_1 , --- .

If we let

$$A = (\partial^2 r/\partial\Psi_o^2)\ (f\ \tan\Psi_o/r) + \sec^2\Psi_o\ ,$$

then

$$\delta f/f = (f\ \tan\Psi_o/r)[\{A(\partial r/\partial a_1) - \partial^2 r/\partial\Psi_o\partial a\}\delta a_1 + --- \\ +\{A(\partial r/\partial a_n) - \partial^2 r/\partial\Psi_o\partial a_n\}\delta a_n\ .$$

or

$$\delta f/f = (f\ \tan\Psi_o/r)[A\{(\partial r/\partial a_1)\delta a_1 + --- + (\partial r/\partial a_n)\delta a_n\} \\ -\{(\partial^2 r/\partial\Psi_o\partial a_1 + --- + (\partial^2 r/\partial\Psi_o\partial a_n)\delta a_n\}]$$

which may be written as

$$\delta f/f = (f\ \tan\Psi_o/r)[A(\delta F) - (\delta G)]\ , \qquad (B.5)$$

where

$$\delta F = (\partial r/\partial a_1)\delta a_1 + --- + (\partial r/\partial a_n)\delta a_n ,$$

$$\delta G = (\partial^2 r/\partial\psi_0\partial a_1)\delta a_1 + --- + (\partial^2 r/\partial\psi_0\partial a_n)\delta a_n . \tag{B.6}$$

2. Estimates on the Basis of an Elementary Model

To obtain a preliminary estimate of the effect of the change in meteorological parameter, δa, on the focusing factor, estimates of the quantities $\partial r/\partial a$ and $\partial^2 r/\partial\psi_0\partial a$ must be obtained. These may be approximated using the "circular arc" approximation in a simplified form. Then

$$1/R_i = - (\alpha_i \cos\psi_0 + \beta_i)/c_0 ,$$

and

$$x_i - x_{i-1} = -R_i(\sin\psi_i - \sin\psi_{i-1}) .$$

Consider a perturbation of the meteorological parameters c_k and u_k at the k'th level. Then

$$c_k' = c_k + \Delta c ,$$

$$u_k' = u_k + \Delta u ,$$

where c_k' , u_k' are the perturbed values and Δc, Δu, represent the amount of the perturbation. The parameters at the other levels are considered as unchanged. Then consider the ray path made up of the sum of terms in $x_i - x_{i-1}$. The values of α_i and β_i are given by

$$\alpha_i = (c_i - c_{i-1})/(z_i - z_{i-1}) ,$$

$$\beta_i = (u_i - u_{i-1})/(z_i - z_{i-1}) ,$$

so that changes of parameter at the k'th level change the values of R over the k'th and (k + 1)'th intervals and change the value of ψ . All other values in the sum remain unchanged. For a ray that reaches higher into the atmosphere than the (k + 1)'th level, the total horizontal distance traveled is given by

$$r = 2 \sum_{1}^{n} (x_i - x_{i-1})$$

in the unperturbed condition and is given by

$$r' = r + 2(x'_{k+1} - x_{k+1})$$

in the perturbed condition. In other words, the total perturbation of distance to the return point is due to the distance perturbation that takes place between levels k-1 and k+1 and is accounted for at the (k+1)'th level.

The analysis will be carried through with the approximation that $u = o$ so that only speed of sound changes need be considered. The wind is approximately accounted for by addition to the speed of sound. In this instance, $\varphi = \psi$ so that the quibble concerning ray tangents and phase normals is avoided.

Using the first order approximation

$$x'_{k+1} - x_{k+1} = (\partial x_{k+1}/\partial c_k)\Delta c_k .$$

Then since

$$x_{k+1} - x_{k-1} = -R_k(\sin\varphi_k - \sin\varphi_{k-1}) - R_{k+1}(\sin\varphi_{k+1} - \sin\varphi_k) ,$$

it follows that

$$\partial x_{k+1}/\partial c_k = (\partial R_k/\partial c_k)(\sin\varphi_k - \sin\varphi_{k-1}) + \partial R_{k+1}/\partial c_k)(\sin\varphi_{k+1} -$$

$$\sin\varphi_k) + (R_k - R_{k+1})\cos\varphi_k(\partial\varphi_k/\partial c_k)$$

since φ_{k-1} and φ_{k+1} are independent of c_k . Then

$$\partial R_k/\partial c_k = - R_k^2 \cos\varphi_0/[c_0(z_k - z_{k-1})],$$

$$\partial R_{k+1}/\partial c_k = + R_{k+1}^2 \cos\varphi_0/c_0(z_{k+1} - z_k) .$$

From Snell's Law one obtains

$$\partial\varphi_k/\partial c_k = - \cos\varphi_0/c_0 \sin\varphi_k .$$

Then

$$\partial x_{k+1}/\partial c_k = - (\cos\varphi_0/c_0)[R_k^2(\sin\varphi_k - \sin\varphi_{k-1})/(z_k - z_{k-1}) -$$

$$R_{k+1}^2(\sin\varphi_{k+1} - \sin\varphi_k)/(z_{k+1} - z_k) +$$

$$(R_k - R_{k+1})\cos\varphi_k/\sin\varphi_k] .$$

Since

$$z_k - z_{k-1} = - R_k (\cos\varphi_k - \cos\varphi_{k-1}),$$

$$z_{k+1} - z_k = - R_{k+1} (\cos\varphi_{k+1} - \cos\varphi_k) ,$$

we have

$$\partial x_{k+1}/\partial c_k = (\cos\varphi_0/c_0)[R_k \frac{\sin\varphi_k - \sin\varphi_{k-1}}{\cos\varphi_k - \cos\varphi_{k-1}} - R_{k+1} \frac{\sin\varphi_{k+1} - \sin\varphi_k}{\cos\varphi_{k+1} - \cos\varphi_k} -$$

$$(R_k - R_{k+1}) \cos\varphi_k/\sin\varphi_k] \quad .$$

Now

$$(\sin\alpha - \sin\beta)/(\cos\alpha - \cos\beta) = - \cot[(\alpha + \beta)/2] \quad ,$$

so that

$$\partial x_{k+1}/\partial c_k = (\cos\varphi_0/c_0)[R_k\{\cot[(\varphi_k + \varphi_{k-1})/2] - \cot\varphi_k\} -$$

$$R_{k+1}\{\cot[(\varphi_{k+1} + \varphi_k)/2] - \cot\varphi_k\}] \quad .$$

From the relation

$$\cot\alpha - \cot\beta = - \sin(\alpha-\beta)/(\sin\alpha \sin\beta),$$

it follows that

$$\partial x_{k+1}/\partial c_k = - \frac{\cos\varphi_0}{c_0 \sin\varphi_k} \left[R_k \frac{\sin[(\varphi_k-\varphi_{k-1})/2]}{\sin[(\varphi_k+\varphi_{k-1})/2]} + R_{k+1} \frac{\sin[(\varphi_{k+1}-\varphi_k)/2]}{\sin[(\varphi_{k+1}+\varphi_k)/2]}\right] .$$

The approximations

$$\varphi_{k+1}+\varphi_k \simeq 2\varphi_k \ , \quad \varphi_k + \varphi_{k-1} \simeq 2\varphi_k \ ,$$

and

$$\cos\varphi_k \simeq 1 \ , \quad \sin(\varphi_k/2) \simeq (\sin\varphi_k)/2 \ ,$$

are used to obtain

$$\partial x_{k+1}/\partial c_k \simeq - \cos\varphi_0 (x_{k+1} - x_{k-1})/(2c_0 \sin^2\varphi_k),$$

so that

$$\partial r/\partial c_k \simeq - \cos\varphi_0 (x_{k+1} - x_{k-1})/(c_0 \sin^2\varphi_k) . \qquad (B.7)$$

If the term $\partial^2 r/\partial\varphi_0\partial a$ is treated in the same manner, then one starts with

$$\partial r/\partial\varphi_0 = 2\tan\varphi_0[--- + R_k(1/\sin\varphi_k - 1/\sin\varphi_{k-1}) + R_{k+1}(1/\sin\varphi_{k+1} - 1/\sin\varphi_k) + ---] \ ,$$

so that

$$\partial^2_r/\partial\varphi_0\partial c_k = - 2\tan\varphi_0[(\partial R_k/\partial c_k)(1/\sin\varphi_k - 1/\sin\varphi_{k-1}) + (\partial R_{k+1}/\partial c_k)(1/\sin\varphi_{k+1} - 1/\sin\varphi_k) - (R_{k+1} - R_k)(\partial\varphi_k/\partial c_k)/\sin^2\varphi_k] \ .$$

When this is compared with the first expression for $\partial x_{k+1}/\partial c_k$, it is obvious that if we make the same approximations, one will eventually end up with

$$\partial^2 r/\partial\varphi\partial c_k \simeq (\tan\varphi_0/\sin^2\varphi_k)(\partial r/\partial c_k) \ . \qquad (B.8)$$

We are now in position to estimate the long parenthetical expressions of (B.6) on the basis of (B.7) and (B.8)

$$\delta F \simeq -(\cos\psi_0/c_0\sin^2\psi_0)[(\sin\psi_0/\sin\psi_1)^2(x_2'-x_0)(\delta c_1 + \delta u_1) + \cdots$$

$$+ (\sin\psi_0/\sin\psi_{h-2})^2(x_{n-1} - x_{n-3})(\delta c_{n-2} + \delta u_{n-2})] \;,$$

$$\delta G \simeq -(1/c_0\sin\psi_0\cos\psi_0)[(\sin\psi_0/\sin\psi_1)^4(x_2-x_0)(\delta c_1 + \delta u_1) + \cdots$$

$$+ (\sin\psi_0/\sin\psi_{n-2})^4(x_{n-1}-x_{n-3})(\delta c_{n-2}+ \delta u_{n-2})] \;,$$

where the values of Δc are now replaced by the part due to the error in speed of sound and wind separately.

It is seen that the error contribution at each level is proportional to the distance traveled in the adjacent layers and is weighted by a factor which is proportional to the inverse square of the sine of the ray inclination in one case and the inverse fourth power in the other. Consequently, the errors at the higher levels are much more strongly weighted than those of the lower levels.

If, in the above, we write

$$(z_{k+1} - z_{k-1})/(x_{k+1} - x_{k-1}) \simeq \tan\psi_k \simeq \sin\psi_k$$

and if $z_{k+1} - z_{k-1} = 2\Delta z$ where Δz = height interval between levels, then

$$\delta F \simeq -[2(\Delta z)\cos\psi_0/c_0\sin^3\psi_0]\sum_{k=1}^{n-2}(\sin\psi_0/\sin\psi_k)^3(\delta c_k + \delta u_k) \;,$$

$$\delta G \simeq -[2\Delta z)/\sin^3\psi_0 c_0][c_0 s\Psi]\sum_{k=1}^{n-2}(\sin\psi_0/\sin\psi_k)^5(\delta c_k + \delta u_k) \;.$$

The summation terminates at n-2 since the values at n-1 must remain as given by Snell's Law. The case for n-1, the level at the bottom of the layer in which the ray reaches its maximum, and that for n, the top of this layer, requires separate treatment. The same is true for the bottom, n = o.

a) Simplified Example

The calculation of the coefficients for δF and δG is dependent on the particular situation with respect to which the variation is computed. In a particularly simple case of a linear variation from the surface upward, the relative error of the focusing factor may be expressed approximately as

$$\delta f/f \simeq [1/2c_o(n-p)\sin^2\varphi_o] \sum_{k=1}^{n-2} [(n-p)/(n-p-k)]^{5/2} (\delta c_k + \delta u_k)$$

where n = the number of layers involved in the ray traced. If H is the maximum height of the ray and Δz the height interval between measurements of c and u, then $H/(\Delta z) = n-p$, where $0 < p \leq 1$; i.e., p is the measure in units of Δz by which the height H fails to reach the height of the top or n'th layer. The coefficients of the sum are given below for the particular case of n = 10, p = 0.5

k	$\left[\frac{n-p}{n-p-k}\right]^{5/2}$	k	$\left[\frac{n-p}{n-p-k}\right]^{5/2}$
1	1.32	5	6.89
2	1.82	6	12.2
3	2.55	7	28.3
4	3.49	8	101.

The last value for p = 0.1 and 0.9 would be 62.1 and 208., respectively. It is readily seen that the parameter value errors near the maximum height of the ray far outweigh the values at the lower levels.

b) Restraints on the Variation of Parameters

In the estimate of the change of r and $\partial r/\partial \psi_0$ due to the variation of the atmospheric parameters, there are some limits to the variation which should be kept in mind. These come under two categories.

1. The variations should be small enough that none of the assumptions of the ray method are violated in the perturbed sounding. (In a technical sense, this is impossible since the linear layer model violates these assumptions anyway. In another sense, the variations should be small since the method is basically a first order "differential method.")

2. What variations occur should not change the basic character of the curve of $c + u$ as a function of altitude. (Since $u = W\cos\theta$, W = wind speed, θ = azimuth, one might insist that this statement hold for any azimuth.)

The second point requires some explanation. By the preservation "character" of the curve, it is intended that the original and perturbed curves should exhibit maxima and minima at the same levels and that little secondary extremes are not introduced by the perturbations. This criterion may be expressed as

$$|\delta c_k| < \text{smaller of } \left\{ |c_{k+1} - c_k| \text{ , } |c_k - c_{k-1}| \right\}$$

with a similar expression for the wind component. It is evident that the allowable variation of δc_k is dependent on the thickness of the layers into which the atmosphere is divided since

$$|c_{k+1} - c_k| = |\alpha| (z_{k+1} - z_k) \text{ .}$$

c) The Top Two Levels

The contribution of the top two levels, n and n-1, to the error in the focusing factor require a modification of the previous treatment. It was previously assumed, in treating a perturbation at level k, that the ray entering at level $k-1$ made its exit at level $k+1$. The ray is assumed to bend earthward in the layer between levels $n-1$ and n so that the conditions are not fulfilled.

In the case of the top level, n, the perturbation affects only the radius of curvature in this layer which changes the distance traveled and height of the maximum. Let

$$x_n^* - x_{n-1} = - R_n \sin\varphi_{n-1}$$

be the distance traveled in the $(n-1,\ n)$ layer. We designate by $(x_n^*,\ z_n^*)$ the coordinates of the maximum (rather than using unmodified symbols since these have been associated with the level heights; $z_{n-1} < z_n^* < z_n$). The methodology is the same as before with the result that

$$\partial x_n^*/\partial c_n \simeq (R_n \cos\varphi_0/c_0)\left[\frac{-R_n}{z_n - z_{n-1}}\ \frac{z_n^* - z_{n-1}}{z_n - z_{n-1}}\right]^{\frac{1}{2}} .$$

The constraints on δc_n are stronger than before, since it must be assumed that the ray still returns earthward. There is a c_n^* such that $c_{n-1} < c_n^* < c_n$ for which it is required that

$$c_n^* - c_{n-1} < \delta c_n .$$

In the case of the n-1 level, the result is nearly the same as before

$$\partial x_n^* / \partial c_{n-1} \simeq (\cos\varphi_0 / c_0 \sin^2\varphi_{n-1})\Big\{(x_n^* - x_{n-2})/2 + [(x_n^* - x_{n-1})/2][1 - 4(z_n - z_n^*)/(z_n - z_{n-1})]\Big\}$$

with similar restraint on δc_{n-1} to assure that the ray concerned returns earthward in the level between z_{n-1} and z_n .

3. Variation Approach to the Meteorological Errors

It was seen in the previous section that at least a part of the variation of the focusing factor requires an estimate of the variation of the landing distance of a ray with respect to the variation of the meteorological parameter. To estimate this variation, the ray equations

$$dx/dt = c \cos\varphi + u$$

$$dz/dt = c \sin\varphi$$

and Snell's Law

$$c/\cos\varphi + u = c_o \cos\varphi_o + u_o$$

are subject to variation;

$$d(\delta x)/dt = \cos\varphi(\delta c) - c \sin\varphi(\delta\varphi) + (\delta u) ,$$

$$d(\delta z)/dt = \sin\varphi(\delta c) + c \cos\varphi(\delta\varphi)$$

$$\sec\varphi(\delta c) + c \sin\varphi \sec^2\varphi(\delta\varphi) + (\delta u) = 0 .$$

so that

$$\delta\varphi = -(\cos\varphi/c \sin\varphi)(\delta c + \delta u \cos\varphi) ;$$

$$d(\delta x)/dt = 2 \cos\varphi(\delta c) + (1 + \cos^2\varphi)(\delta u) ,$$

$$d(\delta z)/dt = (-1/\sin\varphi)[\cos 2\varphi(\delta c) + \cos^3\varphi(\delta u)] .$$

The variation δx , δz are obtained by integrating along the ray path

$$\delta x = \int_0^t \left[2\cos\varphi(\delta c) + (1 + \cos^2\varphi)(\delta u)\right]dt ,$$

$$\delta z = -\int_0^t (1/\sin\varphi)\left[\cos 2\varphi(\delta c) + \cos^3\varphi(\delta u)\right]dt .$$

The first integral may be approximated quite easily by the expression

$$\delta x = 2\int_0^t (\delta c + \delta u)dt$$

since in general the angle φ is small.

The second integral involves $\sin\varphi$ in the denominator so that the integral becomes large near $\varphi = 0$, or the crest of the ray. It is easily verified that the integral cannot be evaluated by the usual methods.

It would appear that the knowledge of δx , which was estimated easily, would be sufficient to estimate the variation of the landing point of the ray. This is not the case since the variation of the "range" is composed of two parts

$$\delta r = \delta x + (\delta z)\cot\psi_0$$

as shown in Fig. 17.

The variation of coordinates δx , δz transfers a point on the ray from the point P (original point of ray return) to the point B on the new ray. The point B may well not be the new location of ray return and, consequently, the new ray must be extended to the new return point P'.

This is connected with the fact that the limits of integration are over the time to traverse the ray originally so that $x + \delta x$, $z + \delta z$ is the point reached on the perturbed ray in the same time of travel that it took on the original ray to reach the ground. The perturbed return point is obtained by extending the perturbed ray to the ground along a straight line parallel to the original ray.

The situation is "doubly" bad since not only is the value δz given by an improper integral, but, in addition, the factor $\cot\psi_0$ is large, since the ray inclination angles are small.

a) Height as Independent Variable

The basic ray equation

$$dx/dz = (c \cos\varphi + u)/c \sin\varphi$$

and

$$c/\cos\varphi + u = c_0/\cos\varphi_0 + u_0$$

may be used to illustrate the problems in a direct approach to the estimate of errors by a perturbation method.

To start, note that the factor $\sin\varphi$ in the denominator of the expression for dx/dz is zero at the top of the ray (the ray is horizontal). The upper limit of integration is determined by this condition from Snell's Law:

$$c(z^*) + u(z^*) = c_0/\cos\varphi_0 + u_0 \ .$$

The integration may be performed, since, though improper, the integral converges at this upper limit. This is easily seen if $u \equiv 0$, $u_0 = 0$, in which case (for the linear layer model) one may write

$$\cos\varphi = 1 - (\alpha/c^*)(z^* - z)$$

so that

$$\sin\varphi \simeq (2\alpha/c^*)^{\frac{1}{2}}(z^* - z)^{\frac{1}{2}}$$

(the situation is not changed if $u \not\equiv 0$, but the algebra is more tedious.)

To estimate the perturbation in x from perturbations of c and u, the method of the preceding section leads to

$$d(\delta x)/dz = \left[(c \cos\varphi + u)/c^2 \sin^3\varphi\right](\delta u) +$$

$$\left[\left\{c(1+\sin^2\varphi) + u \cos^3\varphi\right\}/c^2 \sin^3\varphi\right](\delta c) \ .$$

In the integrated form

$$\delta x = \int_0^{z^*} (1/c \sin\varphi)\left[\delta u - (u/c)(\delta c)\right]dz + \left[(c \cos\varphi + u)/c \sin\varphi\right]_{z^*}(\delta z^*)$$

where the last term is the part due to the change in the upper limit of integration, δz^*, due to the changed atmosphere. This looks very fine, except that the last term is undefined, since $\varphi = 0$ at the upper limit.

The expression $\partial r/\partial\varphi_0$ appearing in the focusing factor is much more important in estimating the perturbation or errors of sound intensity. If one sets up the differential equation for this quantity, then

$$d(\partial x/\partial\varphi_0)/dz = -(c_0 \sin\varphi_0/\cos^2\varphi_0)\left[(c + u \cos\varphi)\cos^2\varphi/c^2 \sin^3\varphi\right].$$

This cannot be integrated using the linear layer model since at the upper limit the integral is improper like $(z^* - z)^{-3/2}$.

The difficulties in using differential methods to estimate δx and the improper expression for the integral of $\partial x/\partial\varphi_0$ are closely connected with the fact that the linear layer model of the atmosphere violates the second fundamental assumption of the ray method - smoothness.

APPENDIX C

CONSTRUCTION OF RANDOM NUMBERS HAVING GIVEN SECOND ORDER RELATIONS

1. Linear Transformation Method

Let u_i, $i = 1, ---, n$, be an ensemble of sequences of correlated random numbers such that $\overline{u_i^2} = \sigma_i^2$ and $\overline{u_i u_j} = \sigma_i \sigma_j r_{ij}$ are known. It is required to construct a particular realization or a set of realizations that have the required second order properties. Let the quantities u_i/σ_i be represented in terms of a linear transformation of some other set of numbers, x_i, $i = 1, ---, n$,

$$v_i = u_i/\sigma_i = \sum_{j=1}^{n} a_{ij}\, x_j\ , \quad i = 1\ , \ ---\ , \ n$$

where v_i has unit standard deviation and $\overline{v_i v_j} = r_{ij}$. In terms of the x_i's,

$$\overline{v_i v_j} = r_{ij} = \sum_{p=1}^{n}\sum_{q=1}^{n} a_{ip}\ a_{jq}(\overline{x_p x_q})\ .$$

It is convenient to take $\overline{x_p x_q} = 1$ if $p = q$ and $= 0$ if $p \neq q$. Such a set of numbers will be of unit variance (standard deviation equal to 1), independent, and are easily constructed. With these restrictions on the numbers x_i, the above reduces to

$$r_{ij} = \sum_{p=1}^{n} a_{ip}\ a_{jp} \qquad i, j = 1, ---, n$$

which is to be solved for the n^2 coefficients a_{ij} in terms of the $n(n+1)/2$ values of r_{ij} $(r_{ij} = r_{ji})$.

The last equation may be written in matrix form as

$$R = AA'$$

where $R = \{r_{ij}\}$, $A = \{a_{ij}\}$ and A' = transpose of A. The matrix R is symmetric and nonsingular so that it has an (upper triangular) square root matrix S, $S = \{s_{ij}\}$, $s_{ij} = 0$ if $i > j$, with the property

$$R = S'S$$

where S′ is the transpose of S. We simply identify A with the transpose of S

$$A = S'$$

so that A is simply the lower triangular square root of R. The algorithm for finding the elements of S (or of A) is straightforeward and easily carried out.

This simple method of computing A has not been noted in the literature. The usual procedure is to find the proper values and vectors of R by solving the matrix equation

$$RM = MD$$

where M is the matrix whose columns are the proper vectors of R (unit vectors since $MM' = I$) and D is the diagonal matrix of the proper values of R ($D = \{d_{ij}\}$, $d_{ij} = 0$ if $i \neq j$, the diagonal elements d_{ii}, are the proper values). Since R is symmetric and positive definite, then the proper values are all positive so that

$$R = MDM' = (MD^{\frac{1}{2}})(MD^{\frac{1}{2}})' \ .$$

Consequently, the value of A is obtained from

$$A = MD^{\frac{1}{2}}$$

where $D^{\frac{1}{2}}$ is the diagonal matrix with diagonal elements $\sqrt{d_{ii}}$.

The method for finding proper values and vectors involves much more computation than that of finding the triangular square root. All proper values and vectors are required, not just a few corresponding to the largest proper values.

It is not legitimate to equate the two values of A obtained by these methods. The first expression for A is a lower triangular matrix. The second expression for A does not necessarily have this property. The initial problem is under-determined and does not have a unique solution. A solution depends on the nature of the additional restrictions that are imposed.

There are trade-offs between the two methods for some purposes. The frequency function of u_i/σ_i has not been mentioned, nor has that of the independent numbers, x_i, with unit variance. If the values of x_i are normally distributed, then either method results in values of $v_i = u_i/\sigma_i$ that are normally distributed and properly correlated. If the values of x_i are not normally distributed, then the distribution of the $v_i = u_i/\sigma_i$ depends strongly on the method of constructing the matrix A.

In the first instance, in long form,

$$v_1 = u_1/\sigma_1 = s_{11}x_1 \ ,$$

$$v_2 = u_2/\sigma_2 = s_{12}x_1 + s_{22}x_2 \ ,$$

$$\text{---} \qquad \text{--------}$$

$$v_n = u_n/\sigma_n = s_{1n}x_1 + s_{2n}x_2 + \text{---} + s_{nn}x_n \ ,$$

so that $v_1 = u_1/\sigma_1$ has the same distribution function as x_1; $v_2 = u_2/\sigma_2$ will have a distribution depending on those of both x_1 and x_2; etc. In the second instance, each of the values of $v_i = u_i/\sigma_i$ depend linearly on all n values of x_i. In this case, if n is appreciable, then the distribution of v_i will be nearly normal ($n > 8$ is sufficient in many cases to bring about a nearly normal situation) regardless of the distribution of the x_i's. (For example, if x_i is chosen as +1, 0, -1, at random, then for reasonably large n the u_i's will be nearly normally distributed.) (See Law of Large Numbers in any standard text on probability.)

Fortunately, we are satisfied if the u_i's are normally distributed (or nearly so). If another specific distribution is required, the problem becomes very difficult

2. <u>Method of Averages</u>

The initial requirements for the method of averages are similar to those of Section 1 preceding. The linear averaging relation is given by

$$v_i = u_i/\sigma_i = \sum_{j=1}^{k} a_j x_{j+i} , \quad i = 1, \text{---}, n .$$

In this case, there are only k coefficients to be determined and each of the v_i's are determined using these same coefficients regardless of the value of i. There are $n + k$ values of the independent random numbers x_i to be provided in each instance. (This compares with the n^2 coefficients in the matrix A of the linear transformation method.)

The covariance of v_i and v_j (or correlation coefficient r_{ij}) is constructed from

$$r_{ij} = \overline{v_i v_j} = \sum_{p=1}^{k} \sum_{q=1}^{k} a_p a_q \left(\overline{x_{p+i} x_{q+j}}\right).$$

Since the x_i's are to be independent with unit variance, then $(x_{p+i}x_{q+j}) = 0$ if $p+i \neq q+j$ and $= 1$ if $p+i = q+j$. This requires that say $p = q + j - i$ whence

$$r_{ij} = \sum_{q=1}^{k} a_{q+j-i} \quad a_q \; .$$

A restriction is placed on the method because on the left the indices i, j appear only in the form $j - i$. Consequently, r_{ij} must be a function of only the difference in its indices, $j - i$. If the indices refer to height levels or time (or time lags), the spacing must be such that this condition on correlations is satisfied. This may be approximately the case when height or time spacings are equal. (It is certainly not true for equal spacings covering the large range of altitudes).

An additional convention is observed in the above summation: when the index $q+j-i$ is less than 1 or more than k, the value zero is assigned to a_{q+j-i} .

The method is sometimes formulated in symmetric form

$$v_i = u_i/\sigma_i = \sum_{j=-k}^{k} a_j x_{j+i}$$

with 2k+1 coefficient. The expressions differ primarily in the matter of indexing. Since there are an odd number of coefficients in the expression above, the case corresponds to an odd number of coefficients in the assymetric form.

Returning to the assymmetrical case, the equations to be solved, in any form, are (using $r_{j-i} = r_{ji} = r_{ij}$)

$$1 = r_o = a_1^2 + a_1^2 + --- + a_k^2 \; ,$$

$$r_1 = a_1 a_2 + a_2 a_3 + \text{---} + a_{k-1} a_k \;,$$

$$r_2 = a_1 a_3 + a_2 a_4 + \text{---} + a_{k-2} a_k \;,$$

$$r_{k-1} = a_1 a_k \;.$$

A <u>second restriction</u> is now placed on the method. For fixed k, the scheme can fit only $k-1$ given correlation coefficients. The remaining coefficient is used to "normalize" the solution in accordance with the first of the above relations.

It is now a question of solving the k simultaneous quadratic equations. A real solution is desired since all of the values of v_i are to be real. This imposes <u>additional restrictions</u> on the permissible values of r_i, $i = 0$, ---, $k - 1$.

Other methods, such as autoregressive schemes like

$$v_i = \sum_{j=1}^{k} a_j v_{i-j} + b\, x_i$$

where the x_i are independent random variates may also be used.

REFERENCES

1. Brekhovskikh, Leonid M.; Waves in a Layered Medea, (Translated by David Lieberman and Robert T. Beyer), Academic Press, New York, 1960, xi and 561 pp. (Russian edition, 1956).

2. Whipple, F. J. W.; The high temperature of the upper atmosphere as explanation of zones of audibility; Nature, Vol. III, p. 187.

3. Murgatroyd, R. J.; Wind and temperature to 50 km over England. Anomalous sound propagative experiments, 1944-45. Great Britain, Meteorological Office, Meteorological Memoirs, No. 95, 1955, 30 pp.

4. Reed, Jack W.; Weather determines blast predictions for atom tests, Weatherwise, Boston, Vol. 9, No. 6, pp. 202-204, Dec. 1956.

5. Burkhard, M. D., Karplus, H. B., and Sabine, H. J.; Sound Propagation near the Earth's Surface as Influenced by Weather Conditions, Wright Air Development Division, Air Research and Development Command, U. S. Air Force, Wright-Patterson Air Force Base, Ohio, WADC Technical Report 57-353, Part II, December, 1960, (AD No. 254670).

6. Sabine, H. J., Raelson, V. J., and Burkhard, M. D.; Sound Propagation near the Earth's Surface as Influenced by Weather Conditions, Wright Air Development Division, Air Research and Development Command, U. S. Air Force; Wright-Patterson Air Base, Ohio, WADC Technical Report No. 57-353, Part III, January 1961, (AD No. 254671).

7. Sabine, H. J.; Sound Propagation near the Earth's Surface as Influenced by Weather Conditions, Wright Air Development Division, Air Research and Development Command, U. S. Air Force, Wright-Patterson Air Force Base, Ohio, WADC Technical Report 57-353, Part IV, January 1961 (AD No. 254672).

7a. Brunt, David; Physical and Dynamical Meteorology, Cambridge, At the University Press, 1941, xxiv + 428 pp.

7b. Dedebant, G., and Wehrle, Ph.; "Mechanique Aleatoire," Portugaliae Physica, Vol. I, Fasc. 3, 1944, pp. 95-294.

8. Ratner, B.; Upper Wind Distribution Statistical Parameters Estimate, U. S. Weather Bureau Technical Paper No. 34, Washington, D. C., Nov. 1958, 59 pp.

9. Ratner, B.; Upper Air Climatology of the United States, Part 2 - Extremes and Standard Deviations of Average Heights and Temperatures, U. S. Weather Bureau Technical Paper No. 32, Washington, D. C., September, 1958, 140 pp.

10. Ratner, B.; Upper Wind Statistics Over the United States, Vol. 1, 1959, Vols. 2 and 3, 1960, (U. S. Weather Bureau -- Sandia Corp. Project in Climatology, Available from the U. S. Weather Bureau on microfilm.)

11. Cole, A. E. and Nee, P. F.; Correlations of Temperature, Pressure, and Density to 30 Kilometers, AFCRL-65-43, January 1965, Air Force Surveys in Geophysics No. 160, Air Force Cambridge Research Laboratories, L. Hanson Field, Bedford, Mass.

12. Nou, J. V.; Short-Period Variability of Upper Winds; The Pennsylvania State University, Graduate School, Department of Meteorology, (Thesis, Master of Science), January, 1961, v + 63 pp. (AD 251207).

13. Baginsky, W., Sissenwine, N., Davidson, B., and Lettau, H.; Review of Time and Space Wind Fluctuations Applicable to Conventional Ballistic Determinations, Air Force Surveys in Geophysics, No. 63, Air Force Cambridge Research Center, Cambridge, Mass., Dec. 1954, 38 pp. (AD 59394).

14. Durst, C. S.; Variation of Wind with Time and Distance, Geophysical Memoirs, No. 93, Meteorological Office, H. M. Stationery Office, London, 1954, 32 pp.

15. Bellucci, R. L.; Preliminary Estimates of Variability of Winds in the Lowest 500 Ft., U. S. Army Research and Development Laboratory, Ft. Monmouth, New Jersey, 15 June 1960, USAROL Tech. Rept. No. 2122, 16 pp.(AD 241176).

16. Buell, C. E.; Two-Point Variability of Wind, Vol. II; Kaman Nuclear, Colorado Springs, Colo., July 1962, Final Report, Contract AF 19(604)7282, Air Force Cambridge Research Laboratories, AFCRL-62-889(II).

17. Cox, Everett; Sound Propagation in Air; Encyclopedia of Physics, S. Flugge, Editor; Vol. XLVIII, Geophysics II, G. J. Bartels, Group Editor, Springer-Verlag, Berlin, 1957, pp. 456-478.

18. Godske, C. L.; Contribution to Statistical Meteorology, Geophysiske Publikasjoner, Vol. XXIV, Universitetsforlagt, Oslo, 1962(?), pp. 161-210 (Manuscript received Nov. 15, 1961).

19. Gossard, E. E.; Power Spectra of Temperature, Humidity, and Refractive Index from Aircraft and Tethered Balloon Measurements, I.R.E. Transactions on Antennas and Propagation, March 1960, pp. 186-201.

20. Crain, C. M., Deani, A. P., and Gerhardt, J. R.; Measurement of Troposphere Index of Refractive Fluctuations and Profiles, Proceedings of the I.R.E., Vol. 41, Feb. 1953, pp. 284-290.

21. Sawyer, J. S.; The Effect of Atmospheric Inhomogenieties on the Interpretation of Vertical Temperature Soundings, Nov. 18, 1952, "A paper of the Meteorological Research Committee, (London), M. R. P. No. 775 by Mr. J. S. Sawyer. A copy is available in the Library of the Meteorological Office."

22. Thompson, P. D.; Numerical Weather Analysis and Prediction, The MacMillan Co., N.Y., 1961, xiv + 170 pp.

23. Buell, C. E.; Likelihood of Dose Rate in Excess of Specified Amount under Various Wind Conditions, KN-183-9(R), Contract No. DA-49-146-XZ-048, for the Defense Atomic Support Agency, Feb. 1962.

24. Ingard, Uno; Acoustics, Handbook of Physics (Ed. by E. U. Condon and Hugh Odishaw), McGraw-Hill Co., Inc., New York, 1958, Chapter 8, pp. 3-112 to 3-133.

25. Blokhintsev, D. I.; Acoustics of a Nonhomogeneous Moving Medium. (Translated by S. Reiss from the 1946 Russian publication), National Advisory Committee for Aeronautics Technical Memorandum 1399, February, 1956. (Also translated by R. T. Beyer and D. Mintzer, Research, Analysis Group, Brown University, Providence 12, R.I., August, 1952. Now somewhat difficult to obtain: "out of print.")

26. Rayleigh, Lord; The Theory of Sound, Dover Publications, New York, 1945; Vol. I, xlii + 480 pp., Vol. II, xvi + 504 pp. (from the 1929 reprint of the second (1896) edition).

27. Emden, Richard; Beitrage Zur Thermodynamik der Atmosphare; Meteorologische Zeitschrift, Vol. 33, 1916, pp. 351-360; Vol. 35, 1918, pp. 13-20; pp. 74-81; pp. 114-123; Vol. 40, 1923, pp. 173-177.

28. Milne, E. A.; Sound Waves in the Atmosphere, Philosophical Magazine; Vol. 42, 1921, pp. 96-114.

29. Kornhauser, E. T.; Ray Theory for Moving Fluids, Journal of the Acoustical Society of America, Vol. 25, 1953, pp. 945-949.

30. Rothwell, P.; Calculation of Sound Rays in the Atmosphere, Journal of the Acoustical Society of America, Vol. 19, 1947, pp. 205-221.

31. Dorman, William J., and Brown, John A.; Meteorological Focusing of Sound and Blast Waves and its Prediction by Analogue Techniques, Ballistic Research Laboratories, Report No. 1014, April, 1957, 39 pp. (AD 139250).

32. Gutenberg, B.; Propagation of Sound Waves in the Atmosphere, Journal of the Acoustical Society of America, Vol. 13, 1942, pp. 151-155.

33. Morse, Philip M., and Ingard, K. Uno; Linear Acoustic Theory, Encyclopedia of Physics, S. Flugge, Editor; Vol. XI/1, Springer-Verlag, Berlin, 1961, pp. 1-128.

34. Kerr, Donald E.; Propagation of Short Radio Waves, MxGraw-Hill Book Co., Inc., New York, 1951, xvii + 728 pp.

35. Anderson, G., Gocht, R., and Serote, D.; Spreading Loss of Sound in an Inhomogeneous Medium; Journal of the Acoustical Society of America; Vol. 36, 1964, pp. 140-145.

35a. Kenney, J. F., and Keeping, E. S.; Mathematics of Statistics, Part Two, Second Edition, D. Van Nostrand Co., N.Y., 1951, xii + 429 pp. (p. 298).

36. Mackenzie, K. V.; Long-Range Shallow Water Transmission; Journal of the Acoustical Society of America, Vol. 33, 1961, pp. 1505-1514.

37. Pridmore-Brown, D. C., and Ingard, U.; Sound Propagation into the Shadow Zone in a Temperature-Stratified Atmosphere above a Plane Boundary, Journal of the Acoustical Society of America, Vol. 27, 1955, pp. 36-42.

38. Pridmore-Brown, D. C., and Ingard, Uno; Sound Propagation into the Shadow Zone in a Temperature-Stratified Atmosphere above a Plane Boundary, National Advisory Committee for Aeronautics, Washington, D. C., October 1955, Technical Note No. 3494, 57 pp.

39. Pridmore-Brown, D. C., and Ingard, Uno; Tentative Method for Calculation of the Sound Field about a Source over Ground Considering Diffraction and Scattering into Shadow Zones, National Advisory Committee for Aeronautics, Washington, D. C., 1956, Technical Note No. 3779. 33 pp.

40. Nyborg, W. L., and Mintzer, D.; Review of Sound Propagation in the Lower Atmosphere, Wright Air Development Center, Air Research and Development Command, U. S. Air Force, Wright-Patterson Air Force Base, Ohio, May 1955, WADC Technical Report 54-602, xv + 217 pp. (AD 67880).

41. Batchelor, G. K.; Wave Scattering Due to Turbulence; Symposium on Naval Hydrodynamics, National Academy of Sciences-National Research Council, Washington, D. C., Publication No. 515, 1957, pp. 409-430.

42. Batchelor, G. K.; The Theory Homogeneous Turbulence, Cambridge University Press, Cambridge, 1953, xi + 197 pp.

43. Kneser, H. O.; Interpretation of Anomalous Sound Absorption in Air and Oxygen in Terms of Molecular Collisions, Journal of the Acoustical Society of America; Vol. 5, 1933, p. 122 ff.

44. Haurwitz, Bernard; Dynamic Meteorology, McGraw-Hill Book Company, New York, 1941, x + 365 pp.

45. Byers, Horace R.; General Meteorology, McGraw-Hill Book Company, New York, 1944, x + 645 pp.

46. Chernov, Lev A.; Wave Propagation in a Random Medium, (Translated by R. A. Silverman); McGraw-Hill Book Company, Inc., New York, 1960, vii + 168 pp.

47. Tatarski, V. I.; Wave Propagation in a Turbulent Medium, (Translated by R. A. Silverman; McGraw-Hill Book Company, Inc., New York, 1961, xiv + 258 pp.

48. Stone, Robert G., and Mintzer, David; Range Dependence of Acoustic Fluctuations in a Randomly Inhomogeneous Medium, Journal of the Acoustical Society of America, Vol. 34, 1964, pp. 647-653.

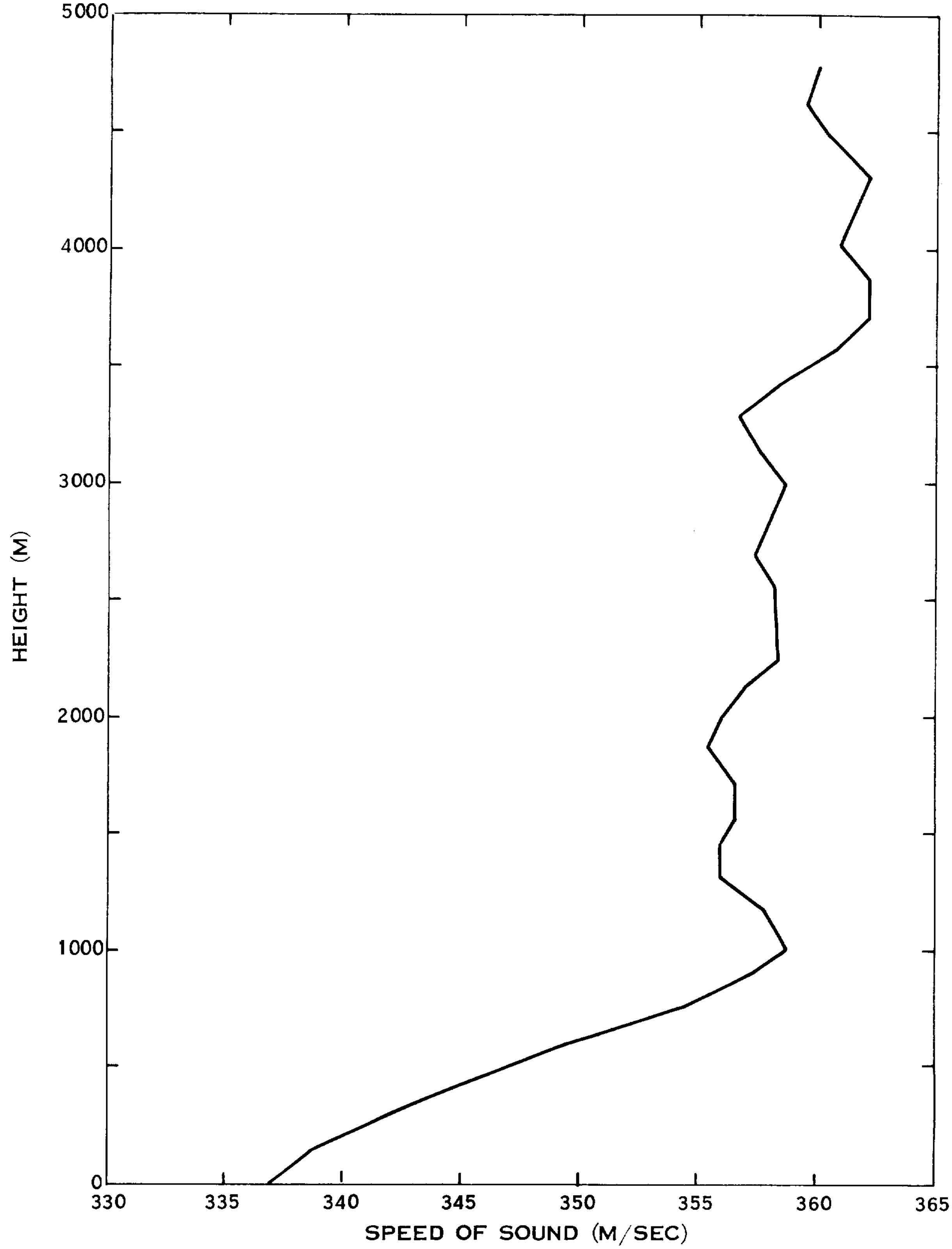


FIGURE 1a

VALUES OF c + u AS A FUNCTION OF HEIGHT, JANUARY 31, 1964, 1139 CST

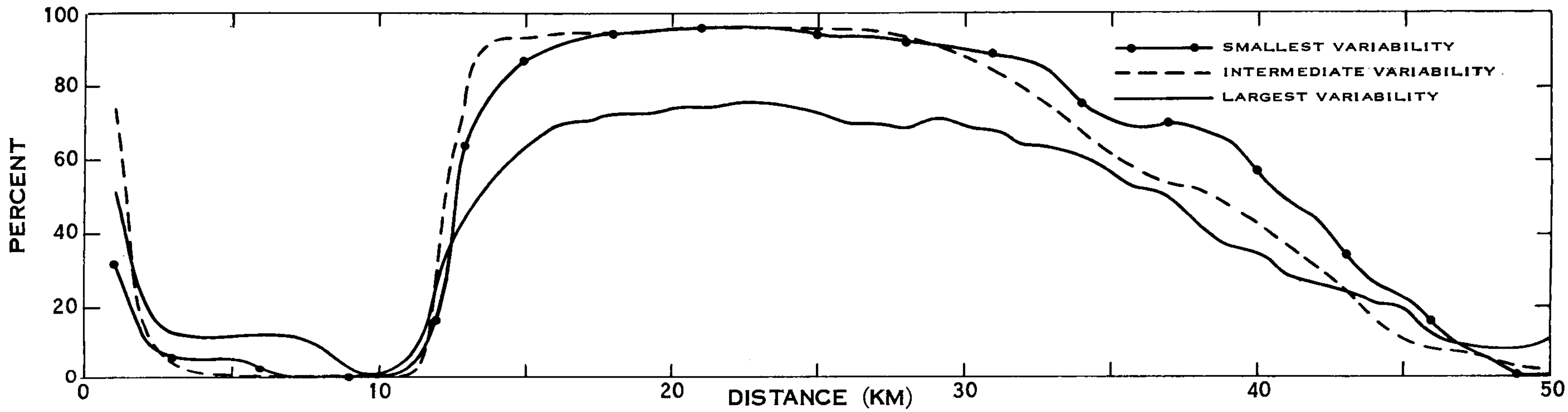


FIGURE 1b. PERCENT OF OCCASIONS WHEN NO SOUND RAYS RETURNED AT ASSIGNED DISTANCES FROM THE SOURCE; JANUARY 31, 1964, 1139 CST

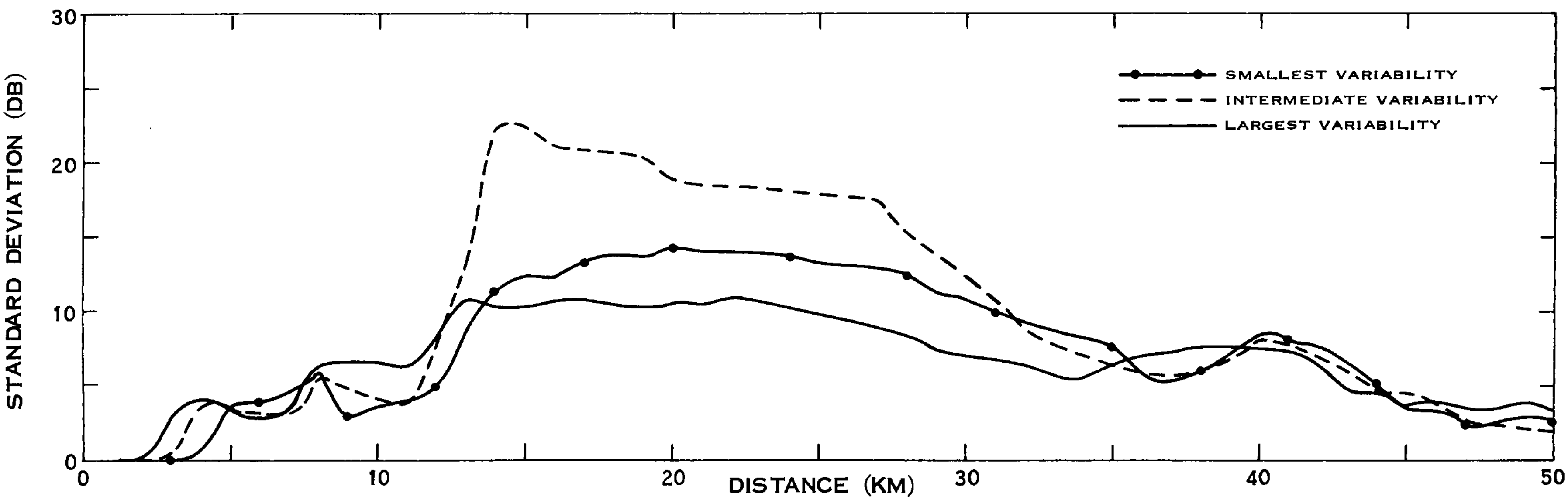


FIGURE 1c. STANDARD DEVIATION OF THE CALCULATED INTENSITY OF THE DIRECT SOUND RAYS FOR THOSE OCCASIONS WHEN RAYS RETURNED AT GIVEN DISTANCES FROM THE SOURCE; JANUARY 31, 1964, 1139 CST

152-1

152-2

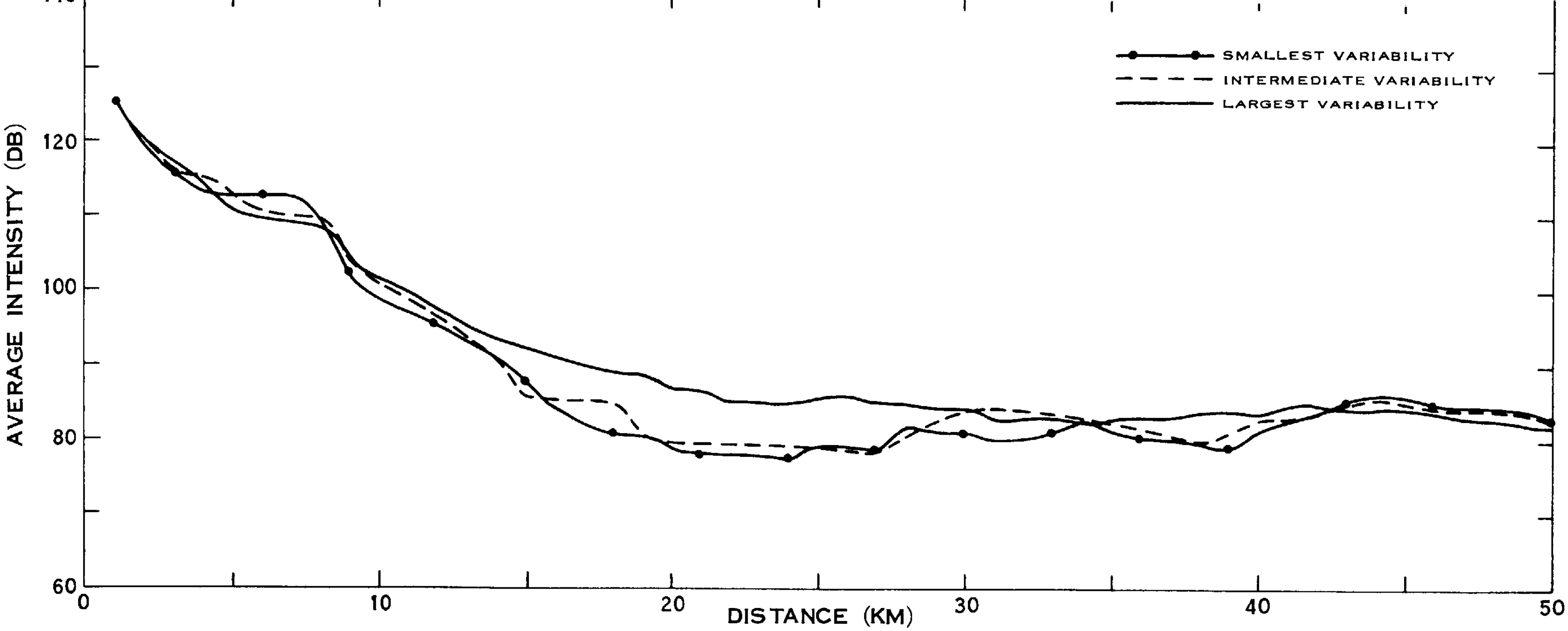


FIGURE 1d. AVERAGE INTENSITY OF THE CALCULATED INTENSITY OF DIRECT SOUND RAYS FOR THOSE OCCASIONS WHEN RAYS RETURNED AT GIVEN DISTANCES FROM THE SOURCE; JANUARY 31, 1964, 1139 CST

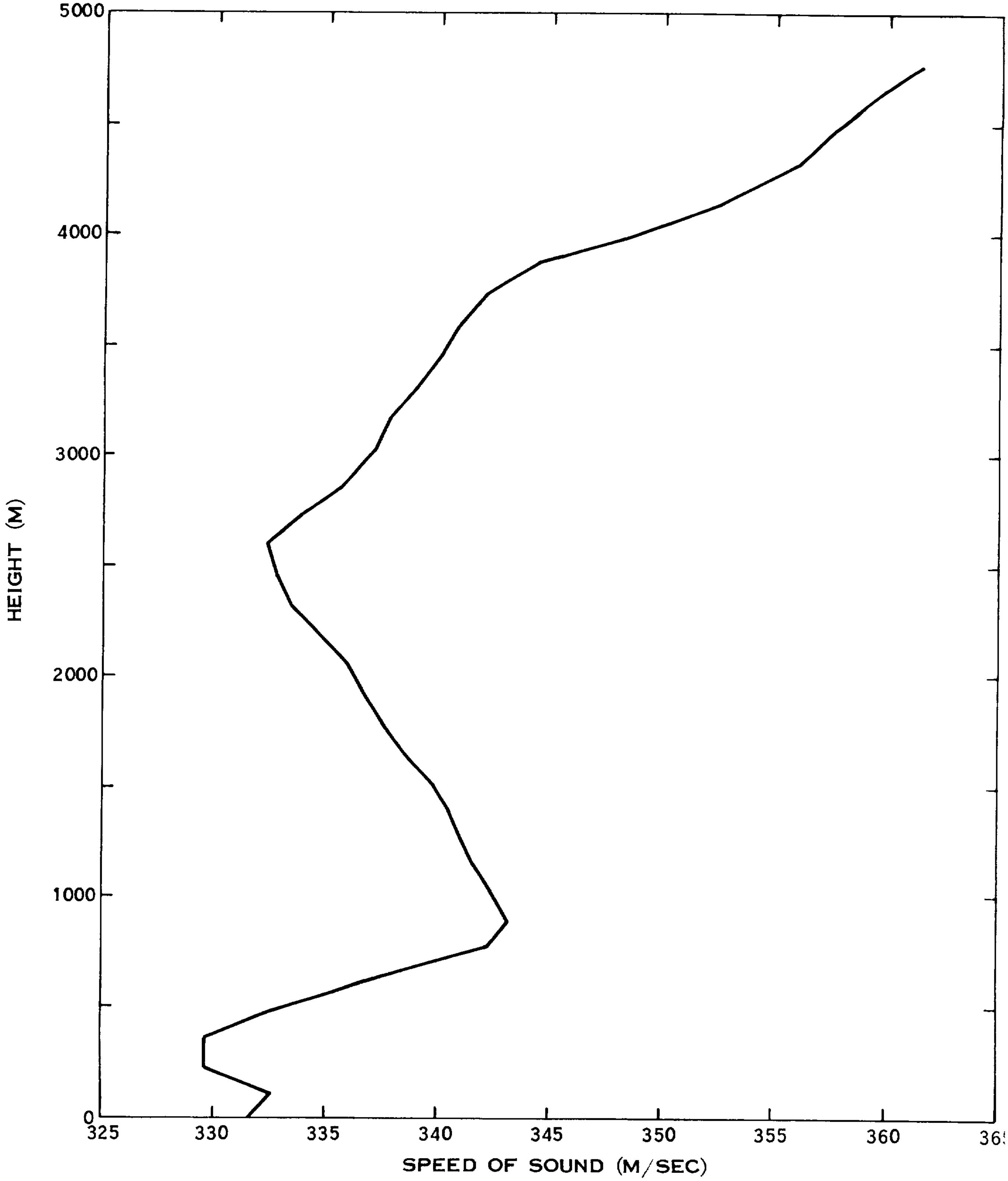


FIGURE 2a

VALUES OF c + u AS A FUNCTION OF HEIGHT, FEBRUARY 27, 1964, 1636 CST

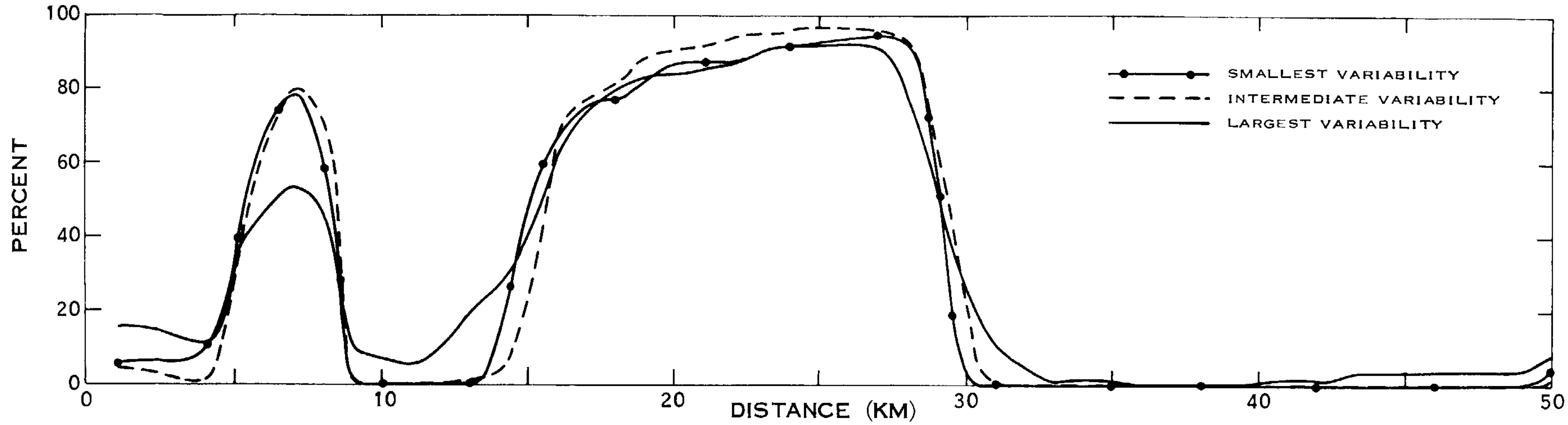


FIGURE 2b. PERCENT OF OCCASIONS WHEN NO SOUND RAYS RETURNED AT ASSIGNED DISTANCES FROM THE SOURCE; FEBRUARY 27, 1964, 1636 CST

154-1

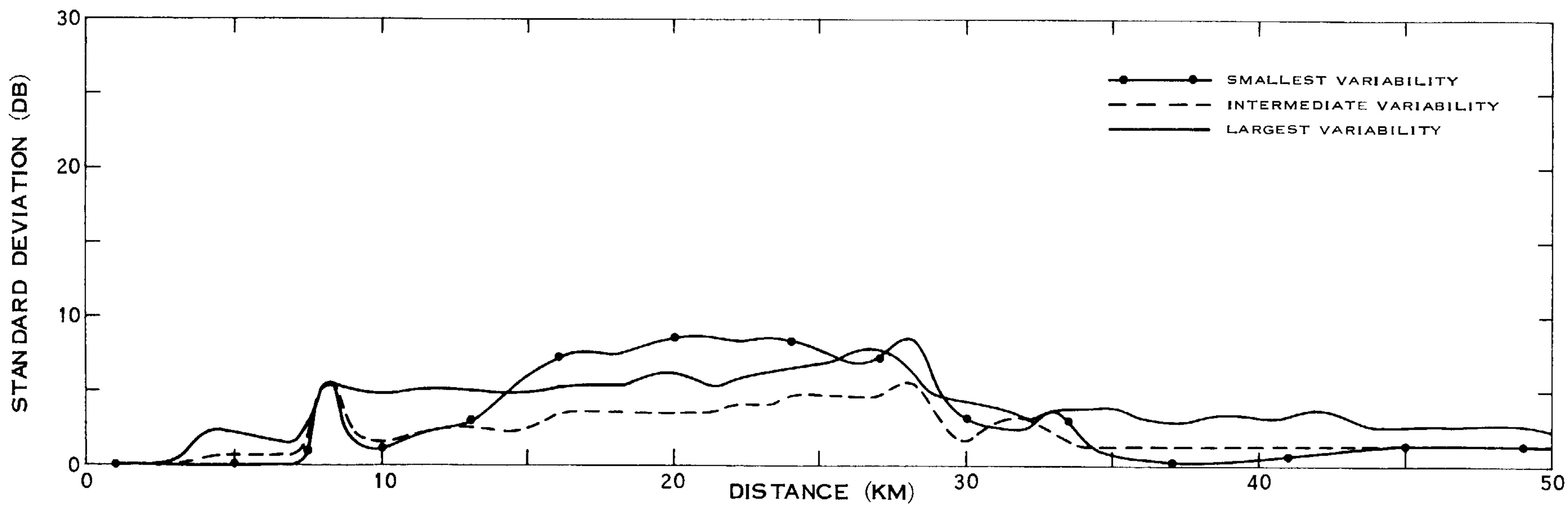
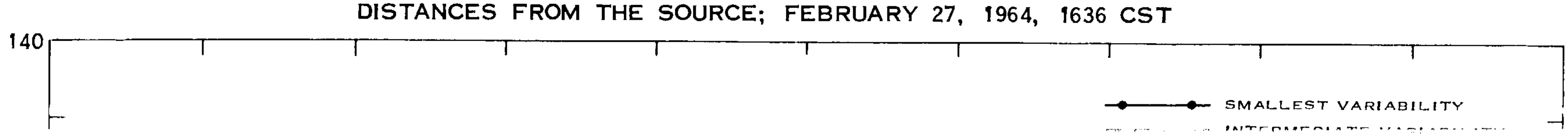


FIGURE 2c. STANDARD DEVIATION OF THE CALCULATED INTENSITY OF THE DIRECT SOUND RAYS FOR THOSE OCCASIONS WHEN RAYS RETURNED AT GIVEN DISTANCES FROM THE SOURCE; FEBRUARY 27, 1964, 1636 CST



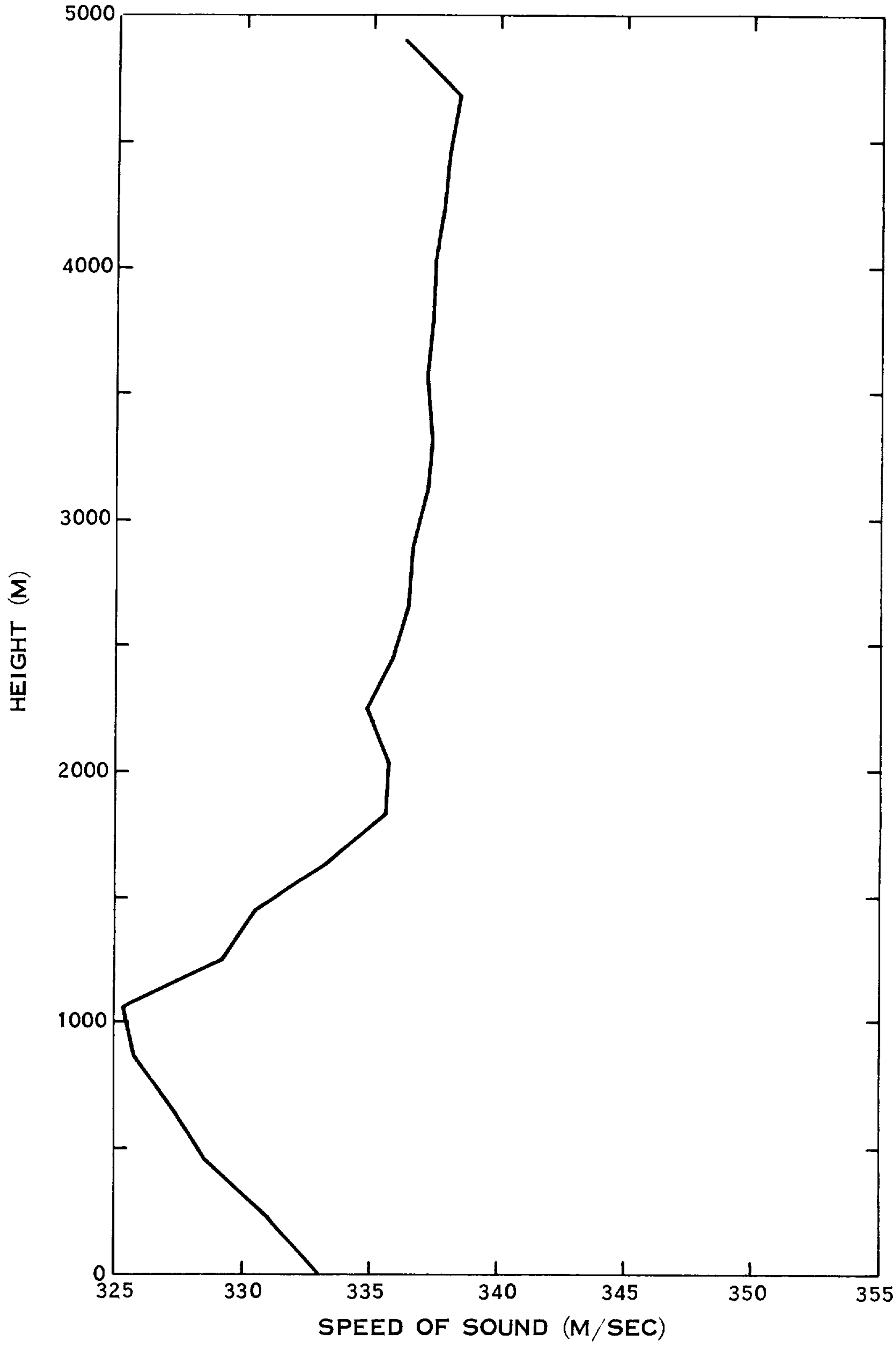


FIGURE 3a

VALUES OF c + u AS A FUNCTION OF HEIGHT, JANUARY 10, 1964, 1636 CST

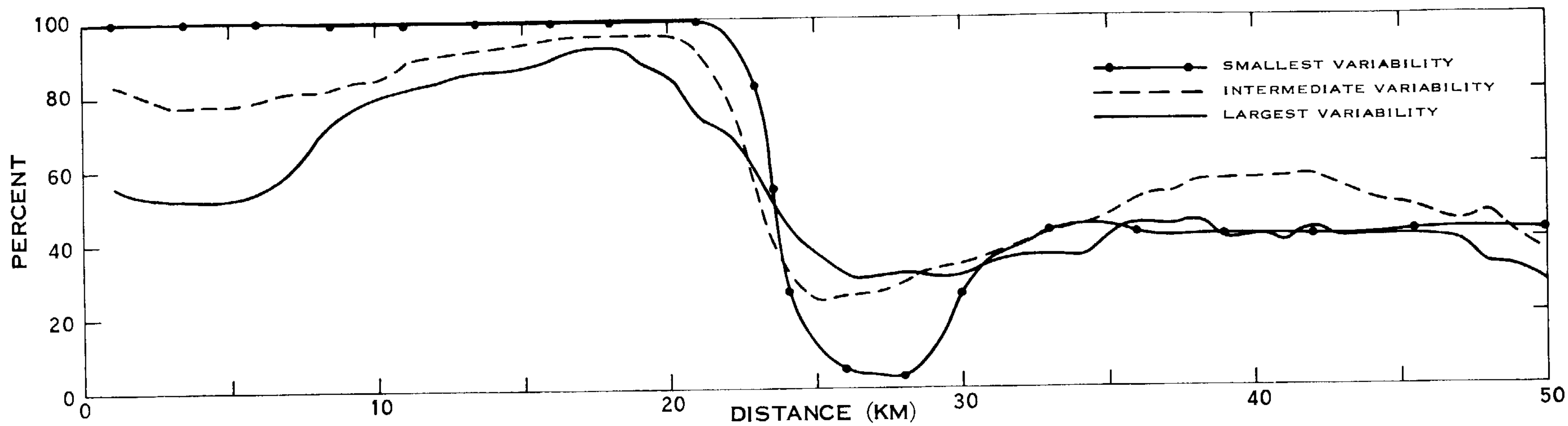


FIGURE 3b. PERCENT OF OCCASIONS WHEN NO SOUND RAYS RETURNED AT ASSIGNED DISTANCES FROM THE SOURCE; JANUARY 10, 1964, 1636 CST

156-1

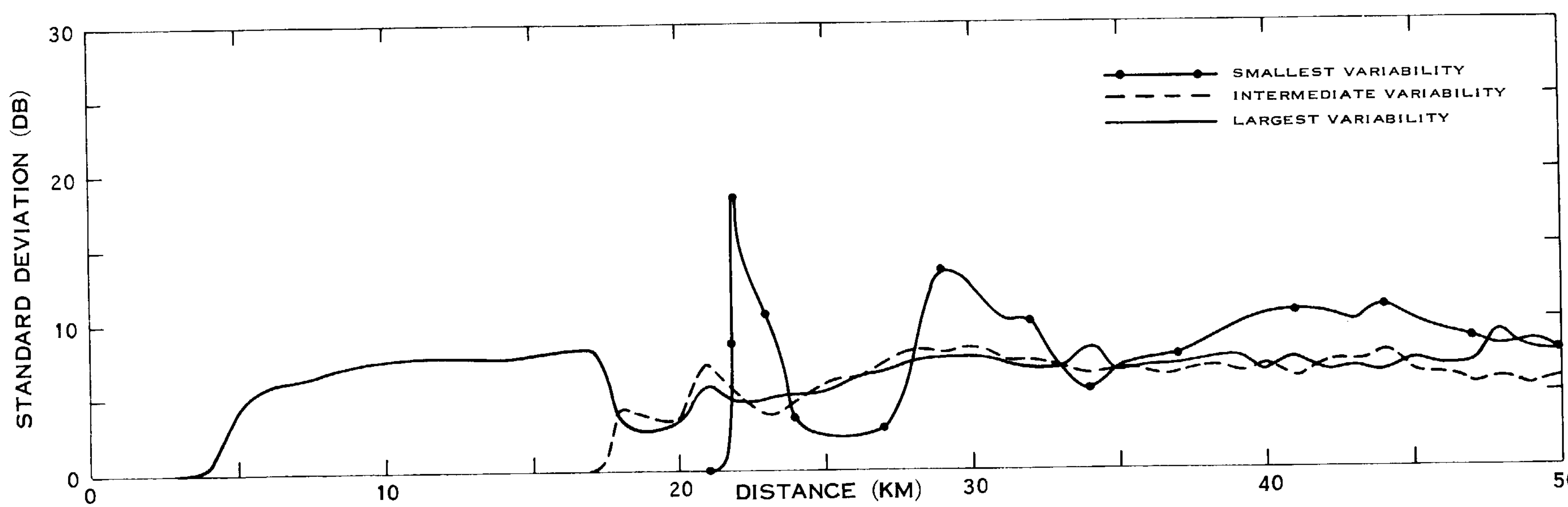
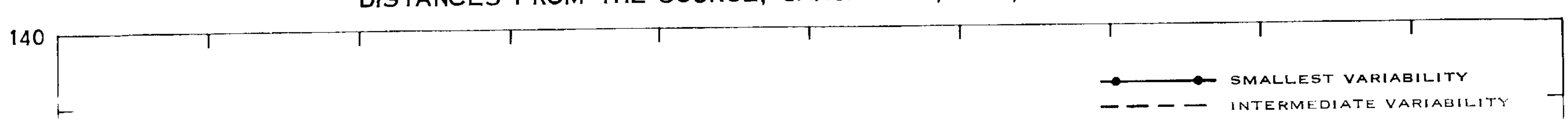


FIGURE 3c. STANDARD DEVIATION OF THE CALCULATED INTENSITY OF THE DIRECT SOUND RAYS FOR THOSE OCCASIONS WHEN RAYS RETURNED AT GIVEN DISTANCES FROM THE SOURCE; JANUARY 10, 1964, 1636 CST



156-2

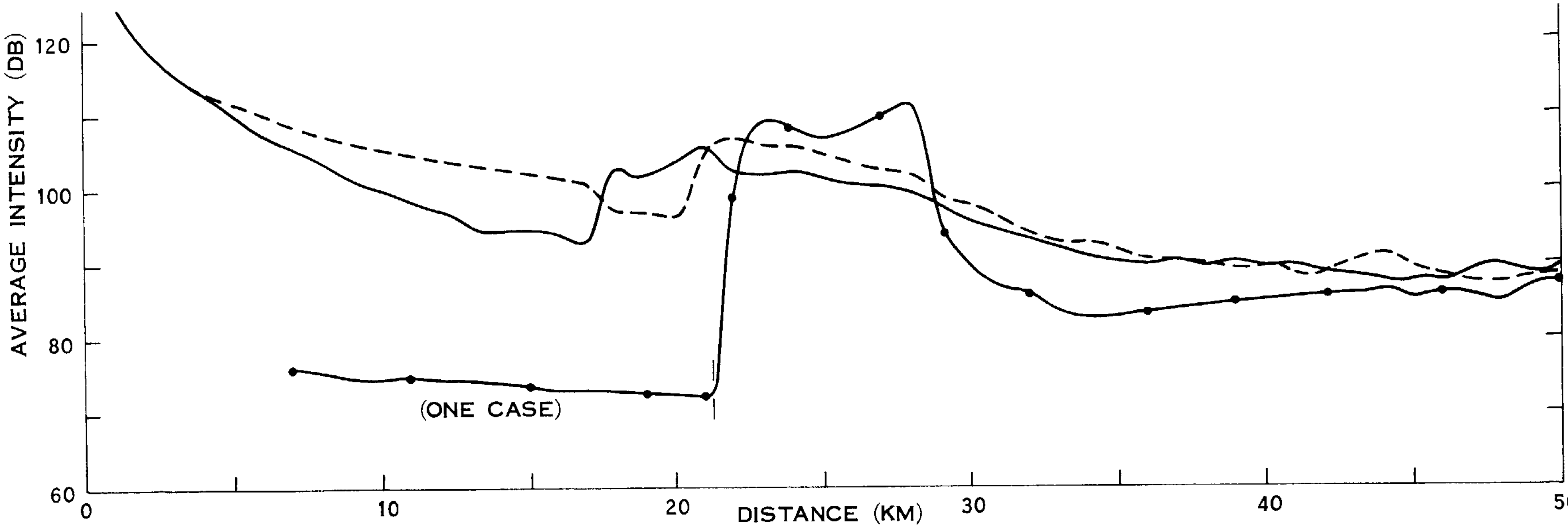


FIGURE 3d. AVERAGE INTENSITY OF THE CALCULATED INTENSITY OF DIRECT SOUND RAYS FOR THOSE OCCASIONS WHEN RAYS RETURNED AT GIVEN DISTANCES FROM THE SOURCE; JANUARY 10, 1964, 1636 CST

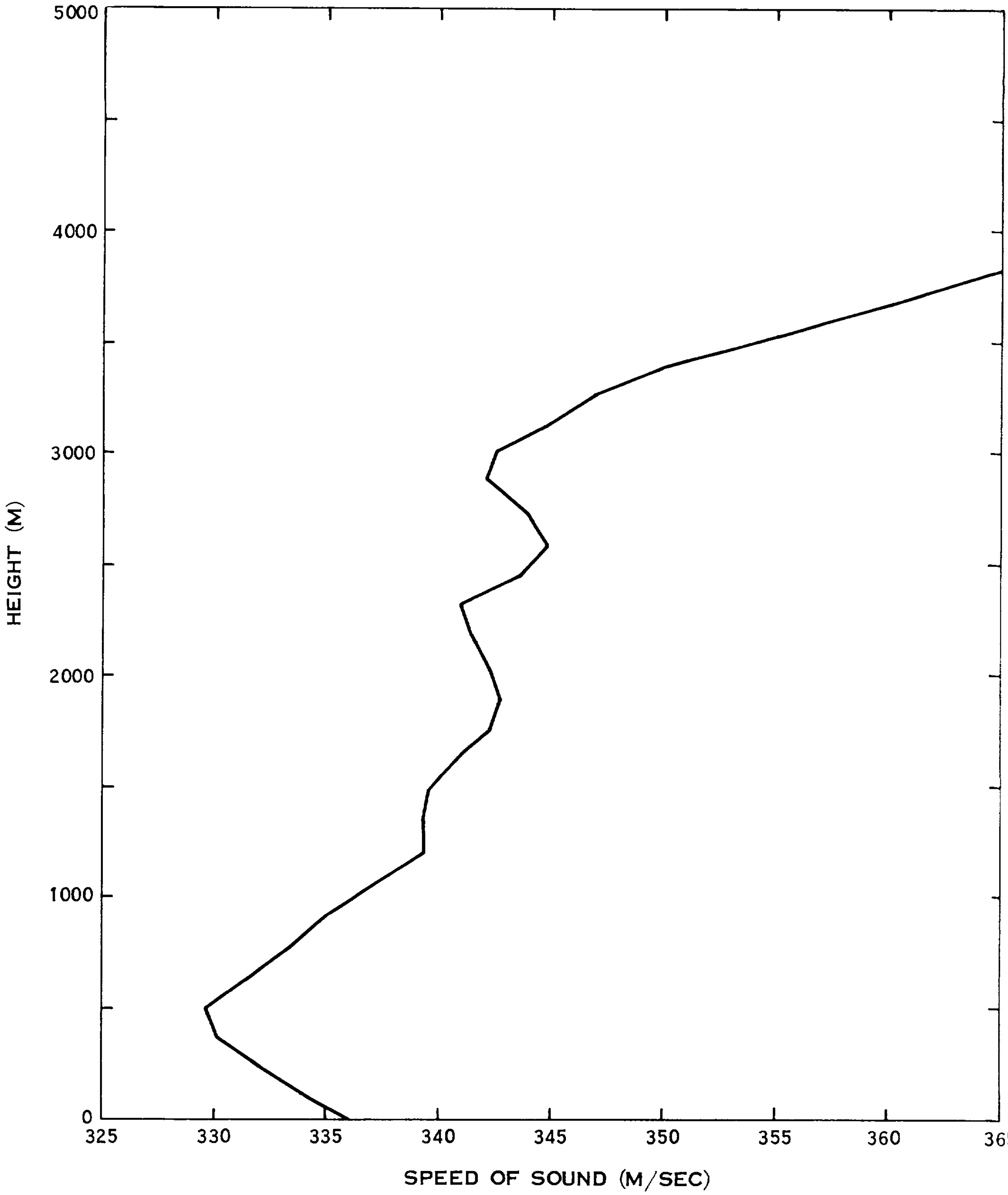


FIGURE 4a

VALUES OF c + u AS A FUNCTION OF HEIGHT, FEBRUARY 13, 1964, 1936 CST

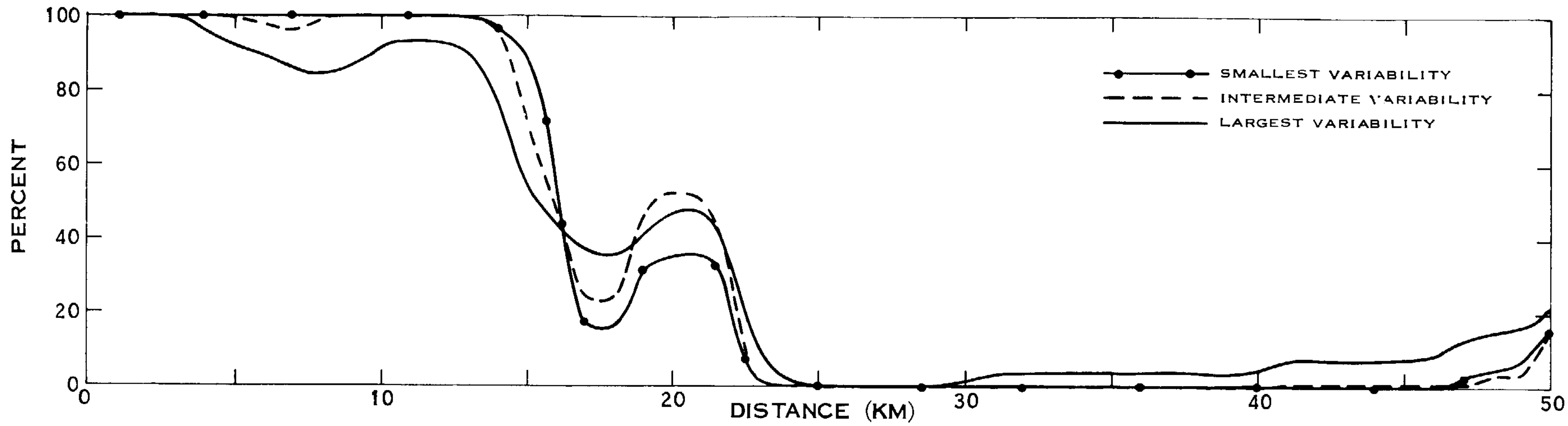


FIGURE 4b. PERCENT OF OCCASIONS WHEN NO SOUND RAYS RETURNED AT ASSIGNED DISTANCES FROM THE SOURCE; FEBRUARY 13, 1964, 1936 CST

1-851

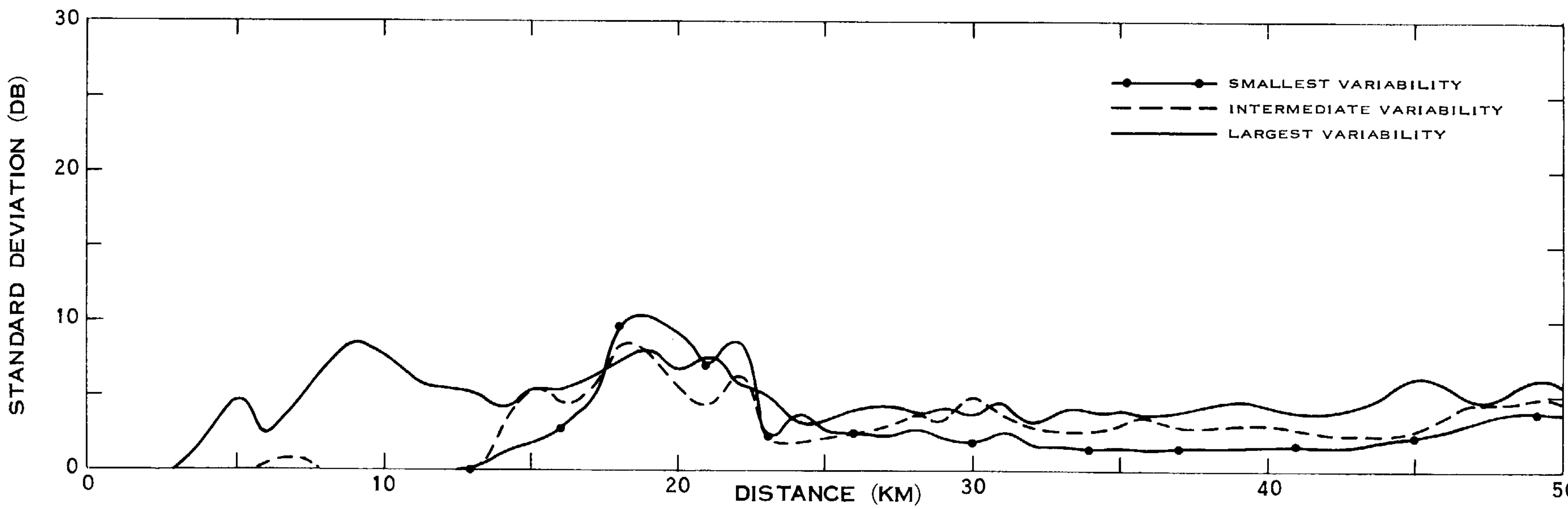
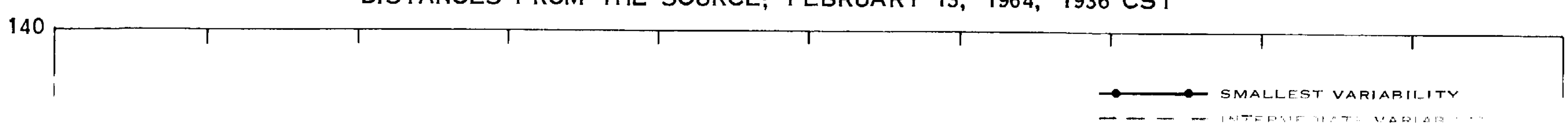


FIGURE 4c. STANDARD DEVIATION OF THE CALCULATED INTENSITY OF THE DIRECT SOUND RAYS FOR THOSE OCCASIONS WHEN RAYS RETURNED AT GIVEN DISTANCES FROM THE SOURCE; FEBRUARY 13, 1964, 1936 CST



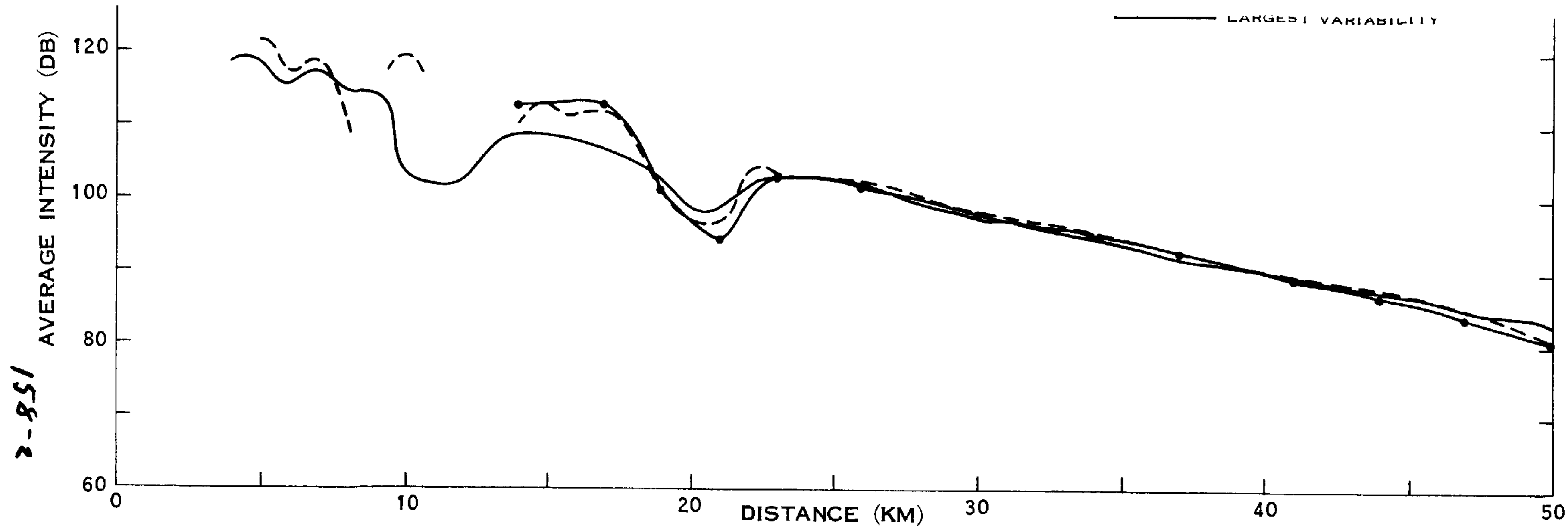


FIGURE 4d. AVERAGE INTENSITY OF THE CALCULATED INTENSITY OF DIRECT SOUND RAYS FOR THOSE OCCASIONS WHEN RAYS RETURNED AT GIVEN DISTANCES FROM THE SOURCE; FEBRUARY 13, 1964, 1936 CST

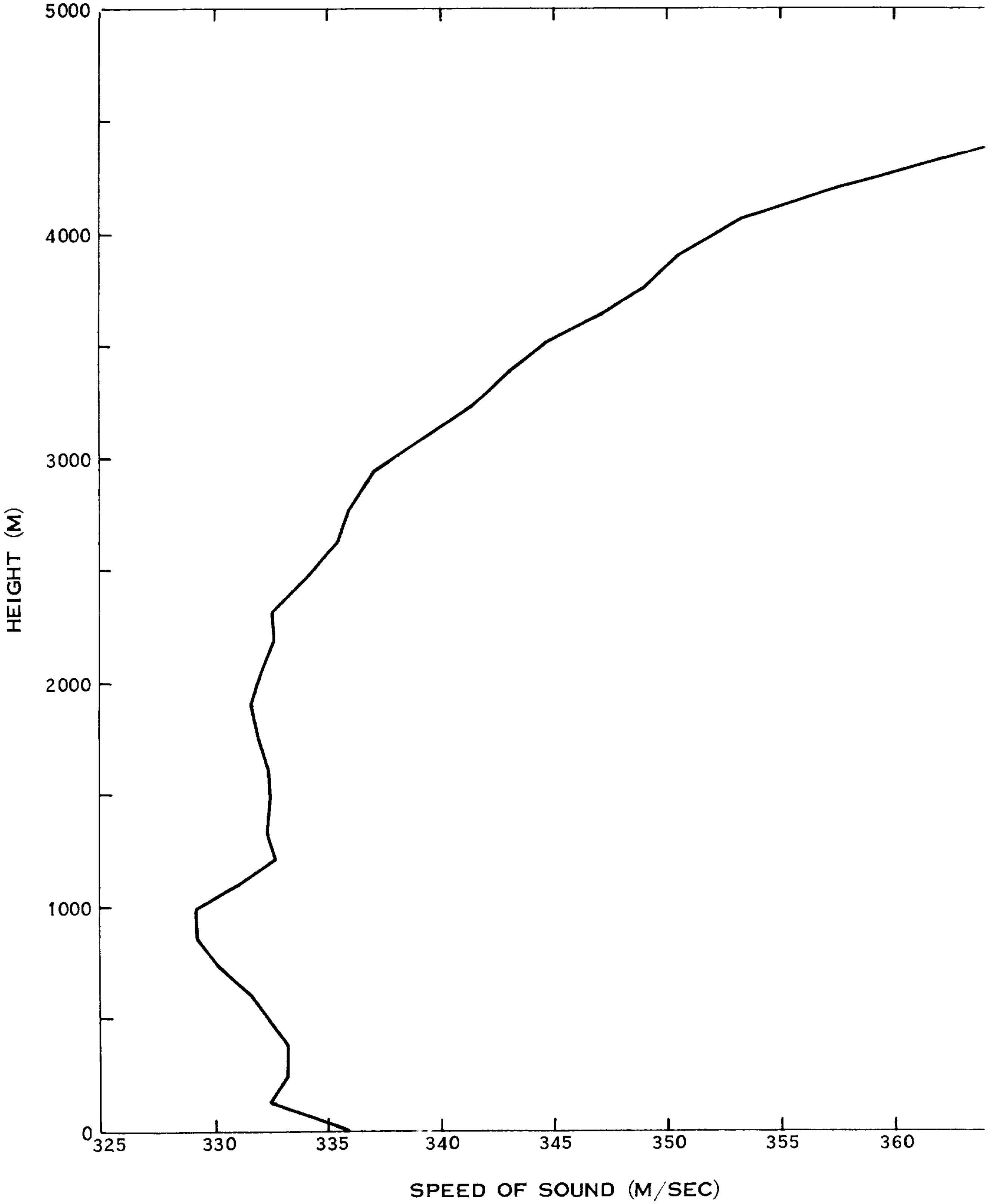


FIGURE 5a

VALUES OF c + u AS A FUNCTION OF HEIGHT, FEBRUARY 7, 1964, 1320 CS

1-091

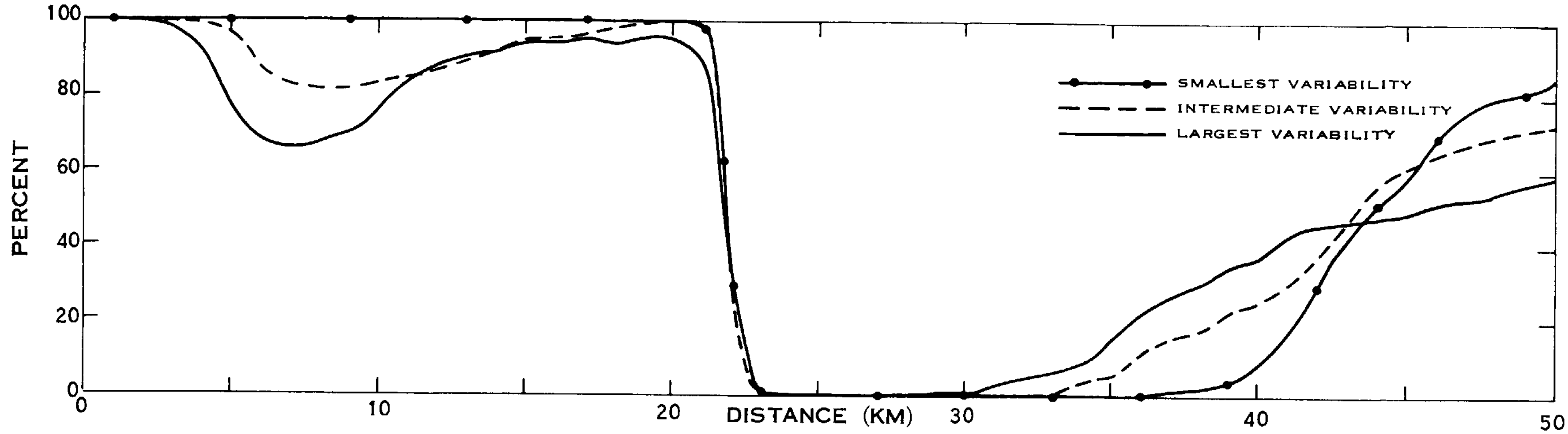


FIGURE 5b. PERCENT OF OCCASIONS WHEN NO SOUND RAYS RETURNED AT ASSIGNED DISTANCES FROM THE SOURCE; FEBRUARY 7, 1964, 1320 CST

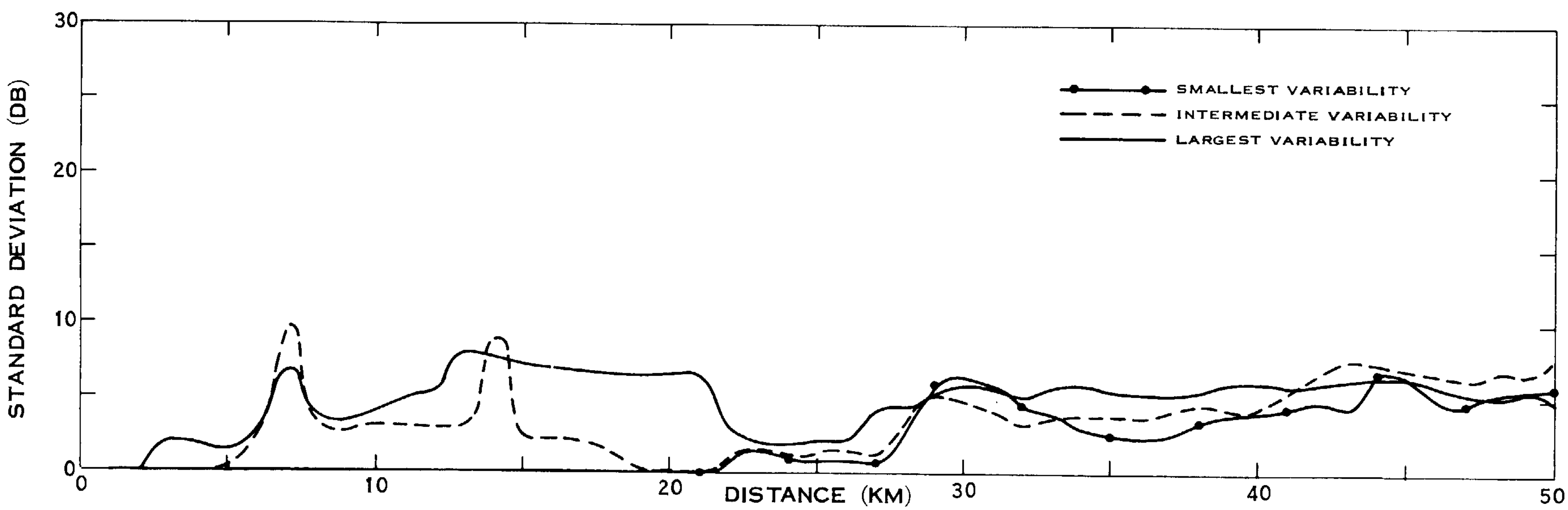
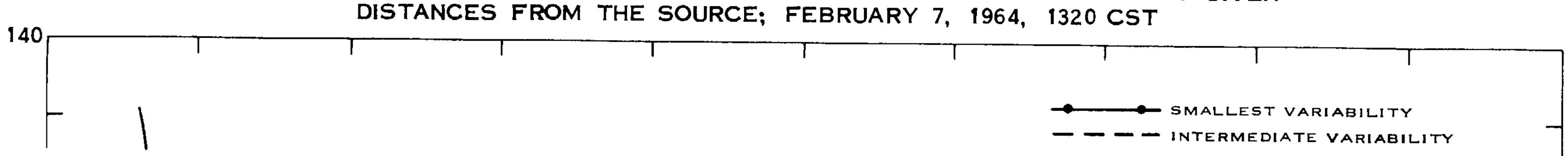


FIGURE 5c. STANDARD DEVIATION OF THE CALCULATED INTENSITY OF THE DIRECT SOUND RAYS FOR THOSE OCCASIONS WHEN RAYS RETURNED AT GIVEN DISTANCES FROM THE SOURCE; FEBRUARY 7, 1964, 1320 CST



160-2

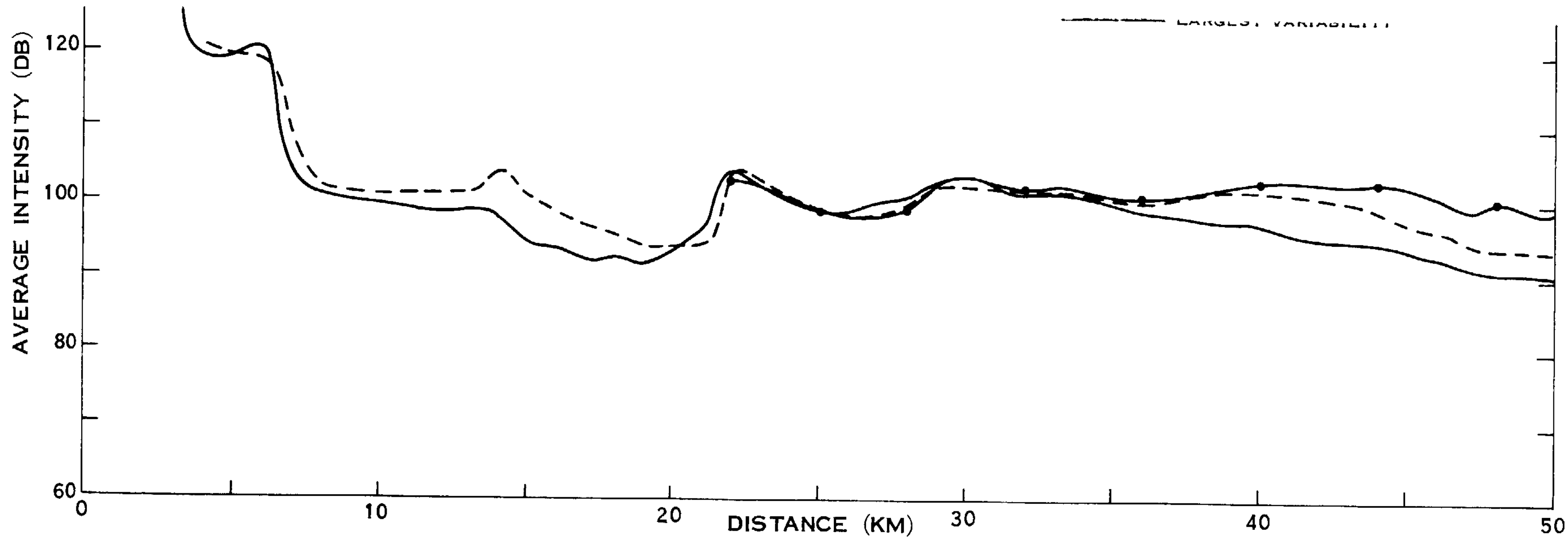


FIGURE 5d. AVERAGE INTENSITY OF THE CALCULATED INTENSITY OF DIRECT SOUND RAYS FOR THOSE OCCASIONS WHEN RAYS RETURNED AT GIVEN DISTANCES FROM THE SOURCE; FEBRUARY 7, 1964, 1320 CST

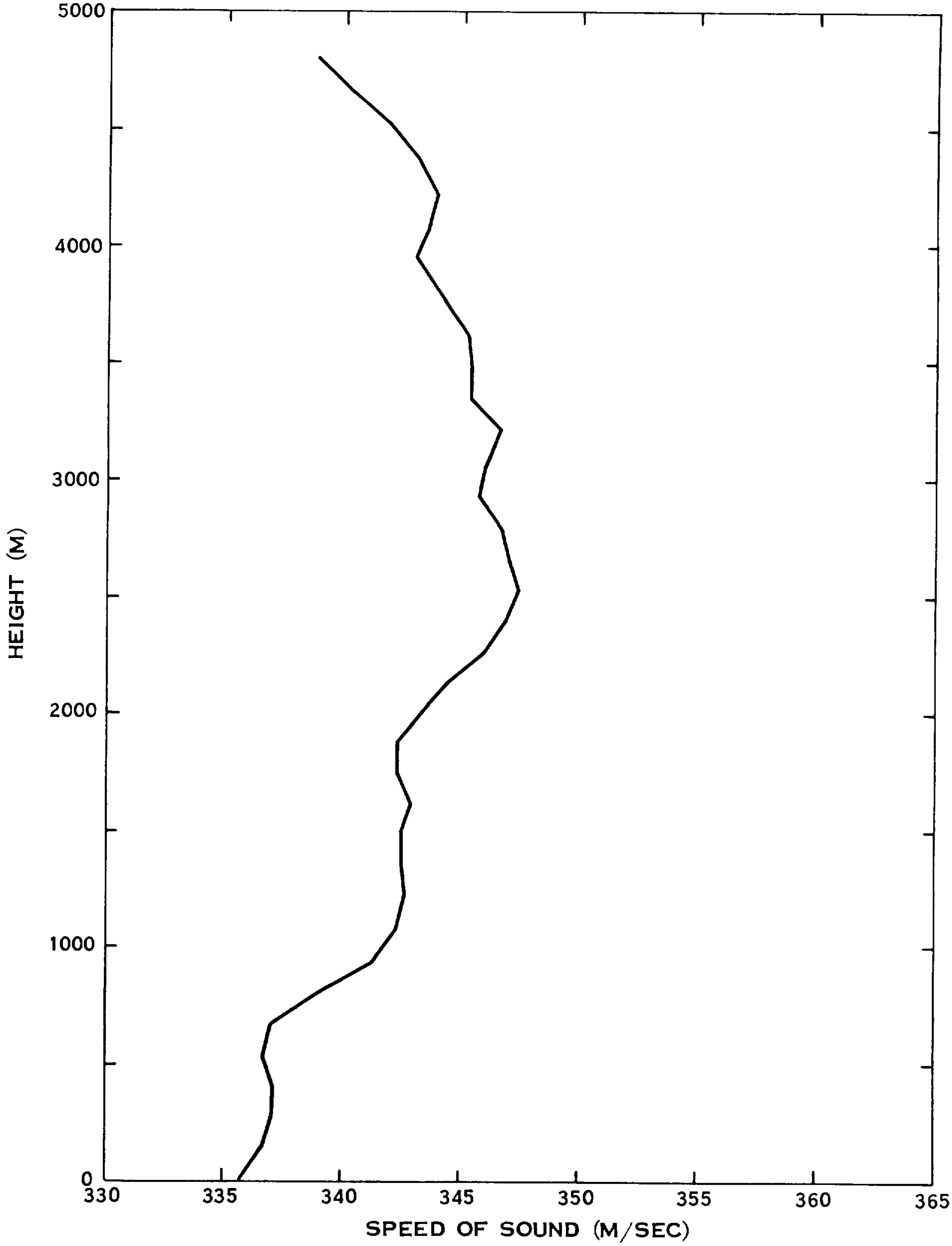


FIGURE 6a

VALUES OF c + u AS A FUNCTION OF HEIGHT, JANUARY 29, 1964, 1610 CST

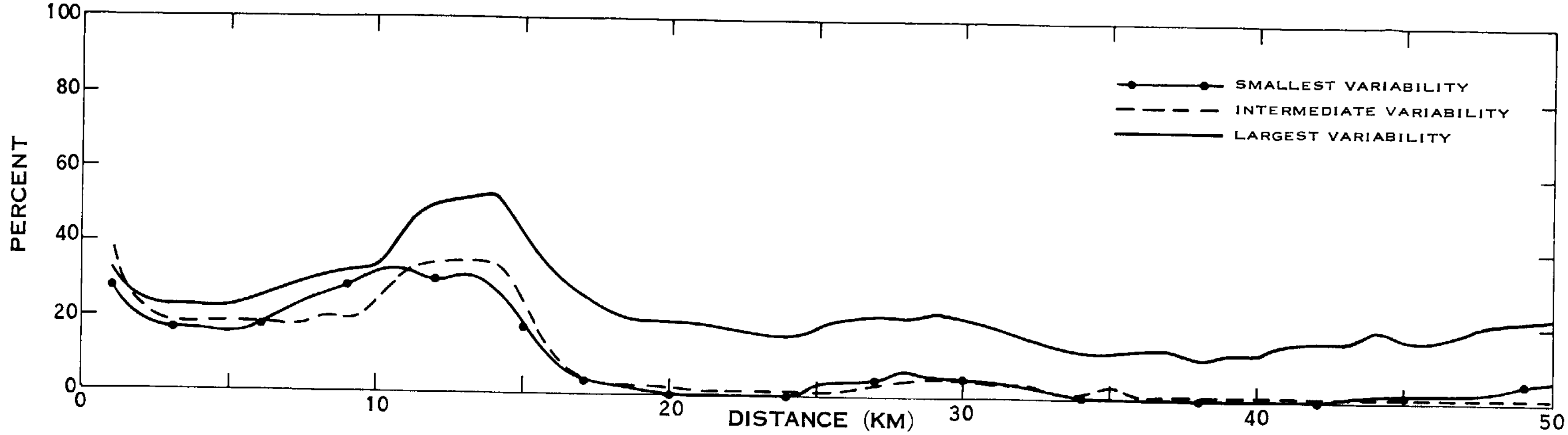


FIGURE 6b. PERCENT OF OCCASIONS WHEN NO SOUND RAYS RETURNED AT ASSIGNED DISTANCES FROM THE SOURCE; JANUARY 29, 1964, 1610 CST

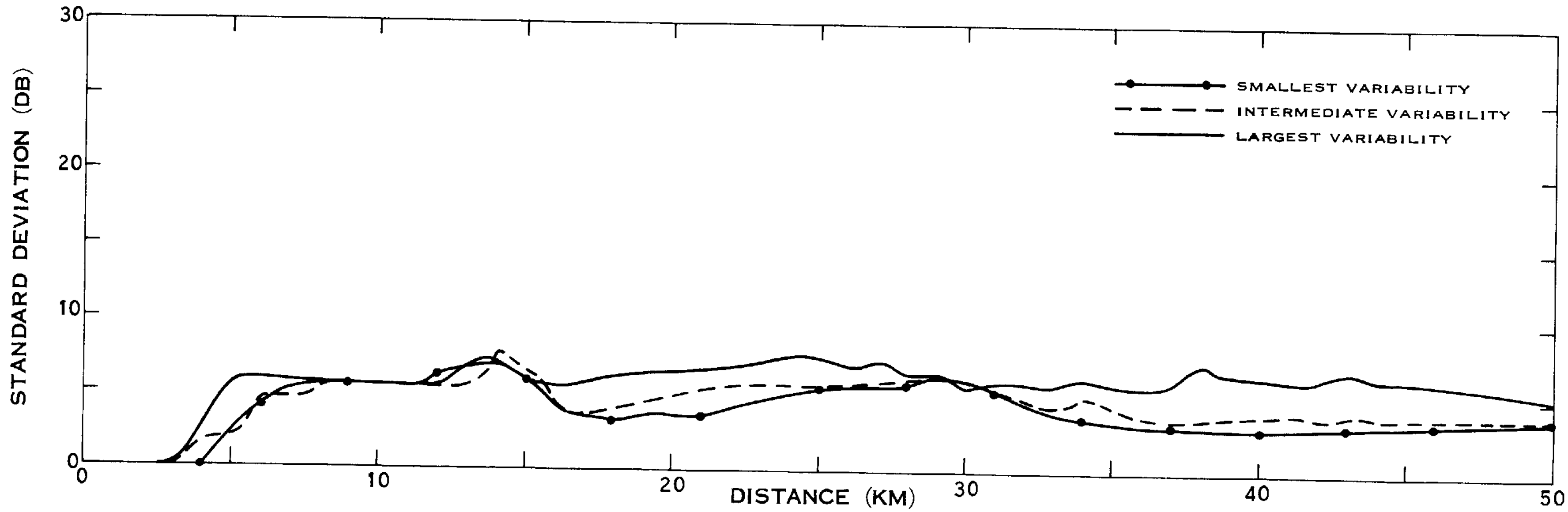
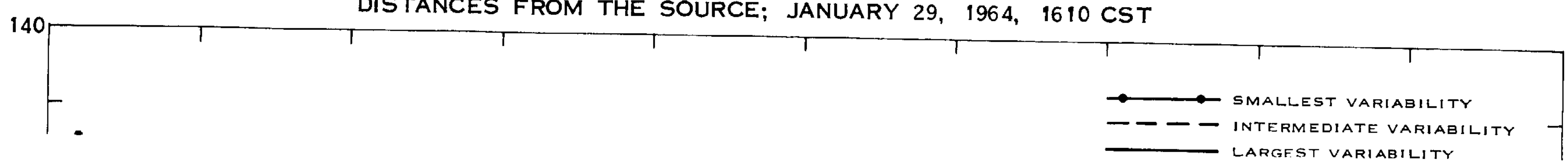


FIGURE 6c. STANDARD DEVIATION OF THE CALCULATED INTENSITY OF THE DIRECT SOUND RAYS FOR THOSE OCCASIONS WHEN RAYS RETURNED AT GIVEN DISTANCES FROM THE SOURCE; JANUARY 29, 1964, 1610 CST



7-191

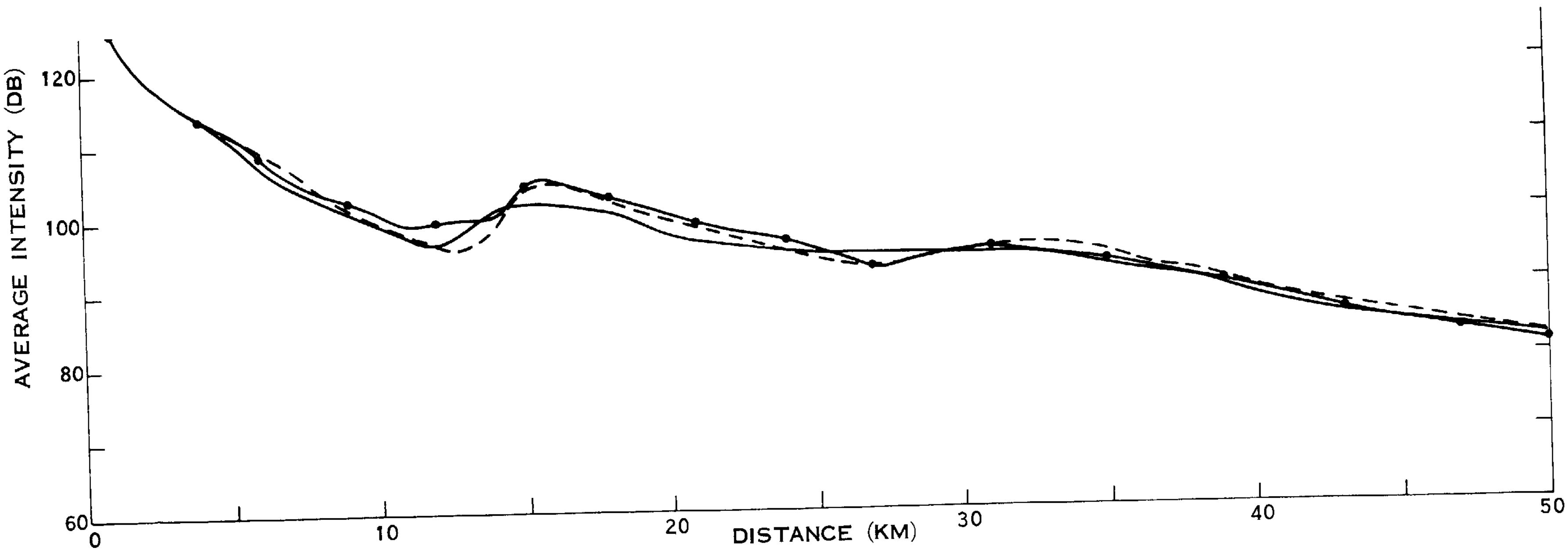


FIGURE 6d. AVERAGE INTENSITY OF THE CALCULATED INTENSITY OF DIRECT SOUND RAYS FOR THOSE OCCASIONS WHEN RAYS RETURNED AT GIVEN DISTANCES FROM THE SOURCE; JANUARY 29, 1964, 1610 CST

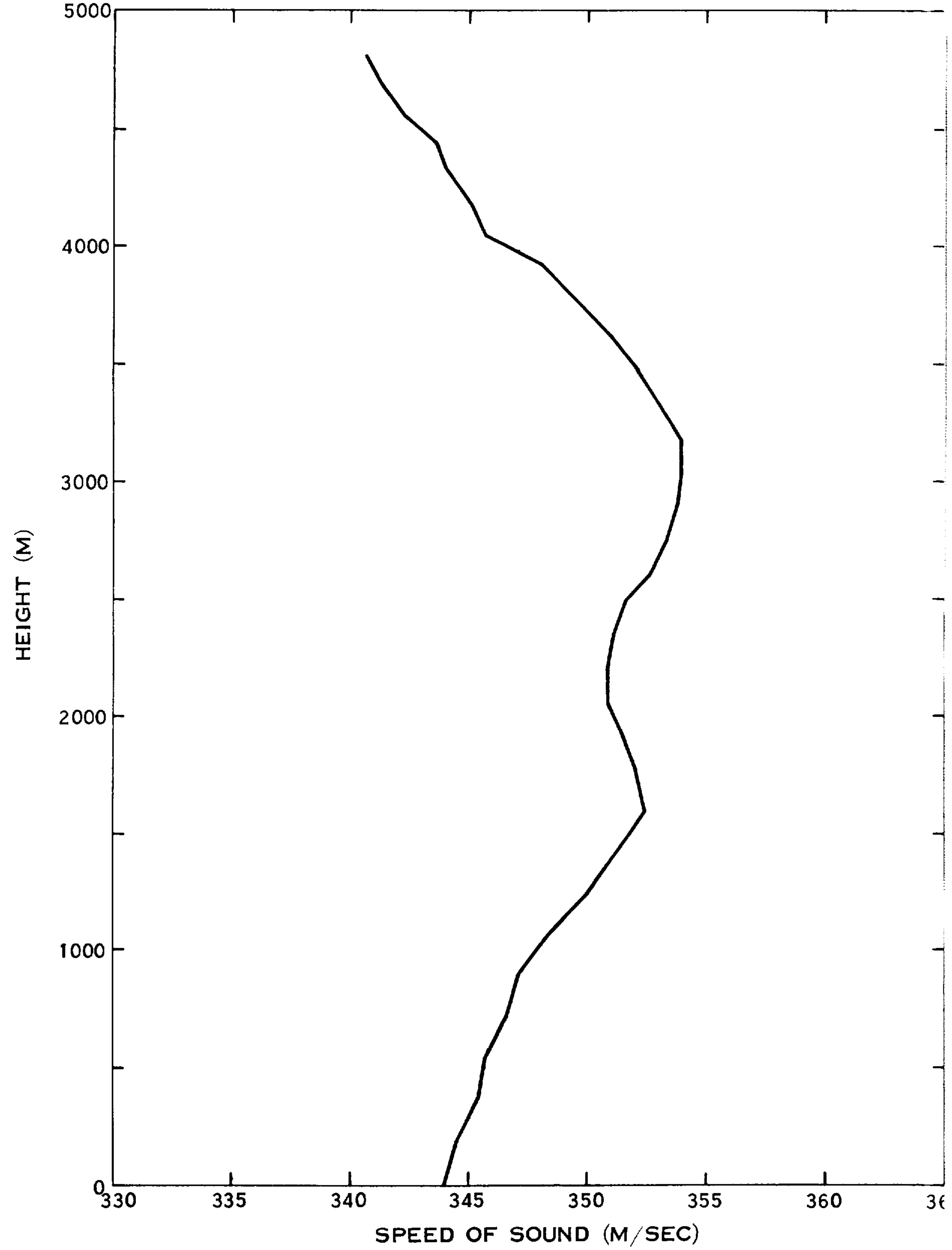
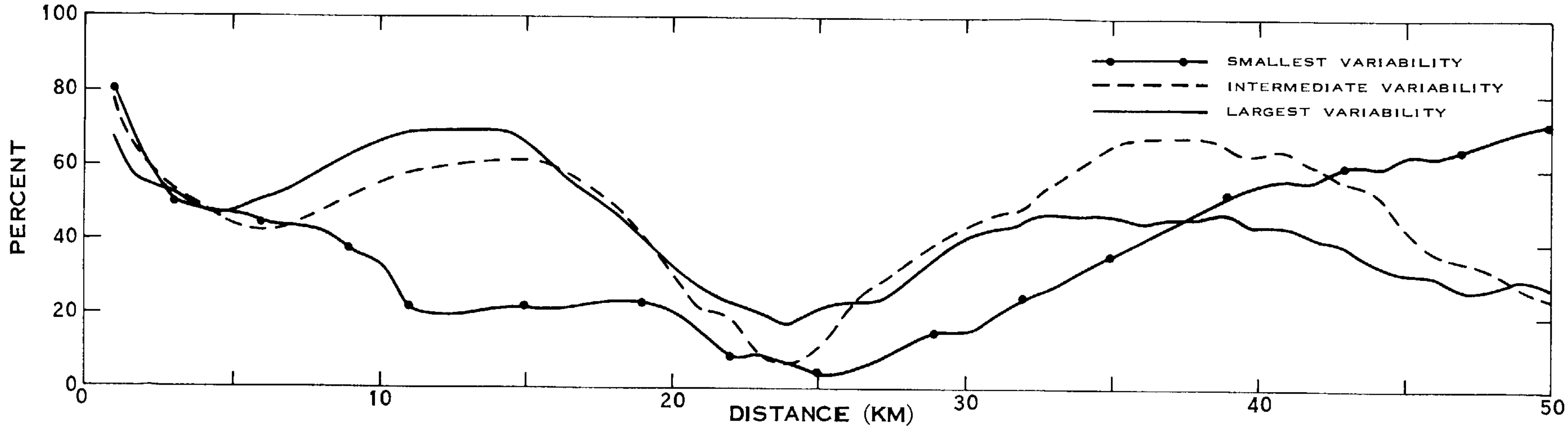


FIGURE 7a

VALUES OF c + u AS A FUNCTION OF HEIGHT, MARCH 13, 1964, 1633 CS



1641

FIGURE 7b. PERCENT OF OCCASIONS WHEN NO SOUND RAYS RETURNED AT ASSIGNED DISTANCES FROM THE SOURCE; MARCH 13, 1964, 1633 CST

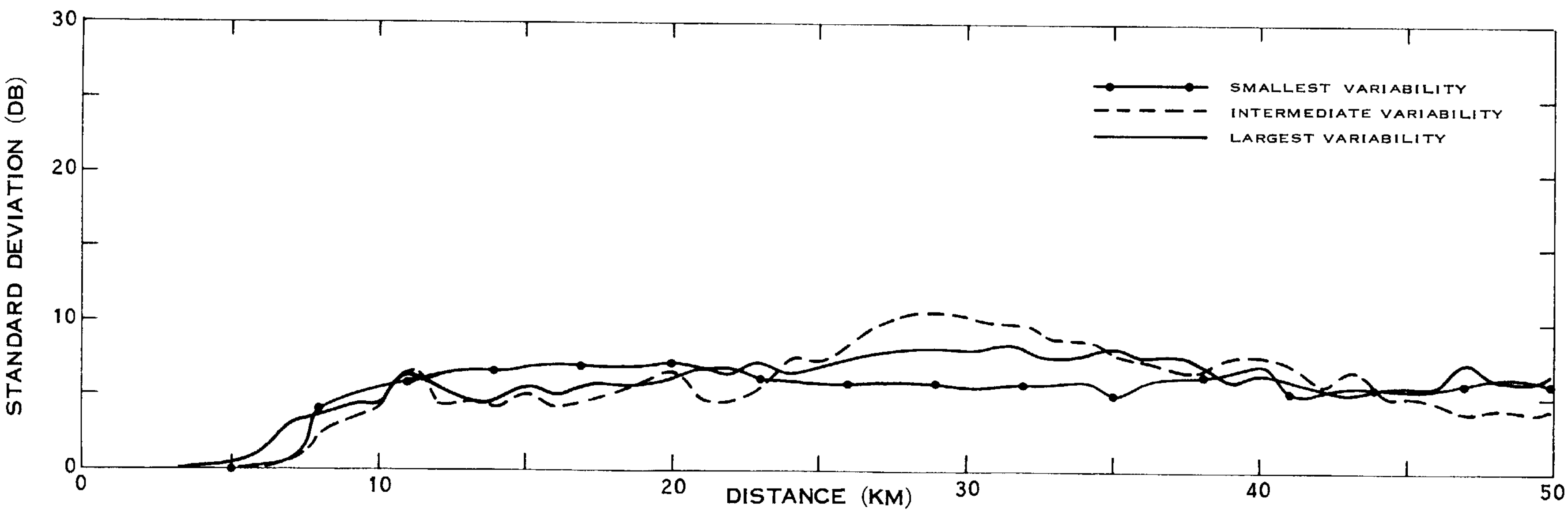
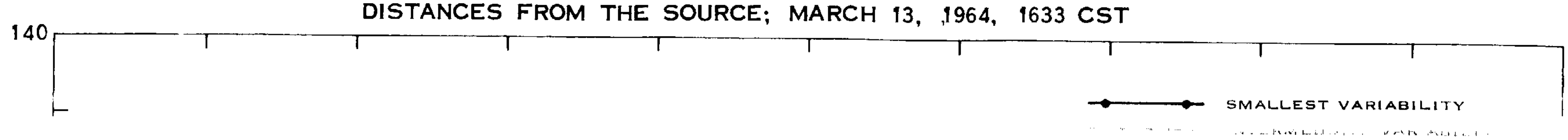


FIGURE 7c. STANDARD DEVIATION OF THE CALCULATED INTENSITY OF THE DIRECT SOUND RAYS FOR THOSE OCCASIONS WHEN RAYS RETURNED AT GIVEN DISTANCES FROM THE SOURCE; MARCH 13, 1964, 1633 CST



164-2

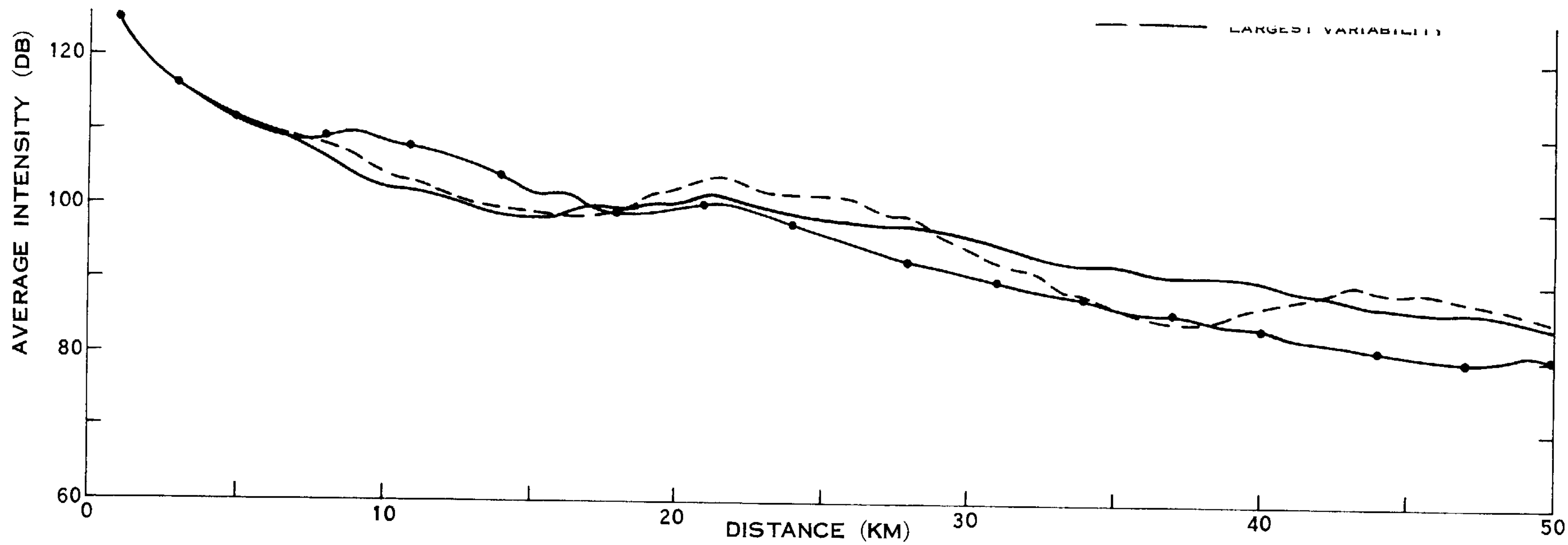


FIGURE 7d. AVERAGE INTENSITY OF THE CALCULATED INTENSITY OF DIRECT SOUND RAYS FOR THOSE OCCASIONS WHEN RAYS RETURNED AT GIVEN DISTANCES FROM THE SOURCE; MARCH 13, 1964, 1633 CST

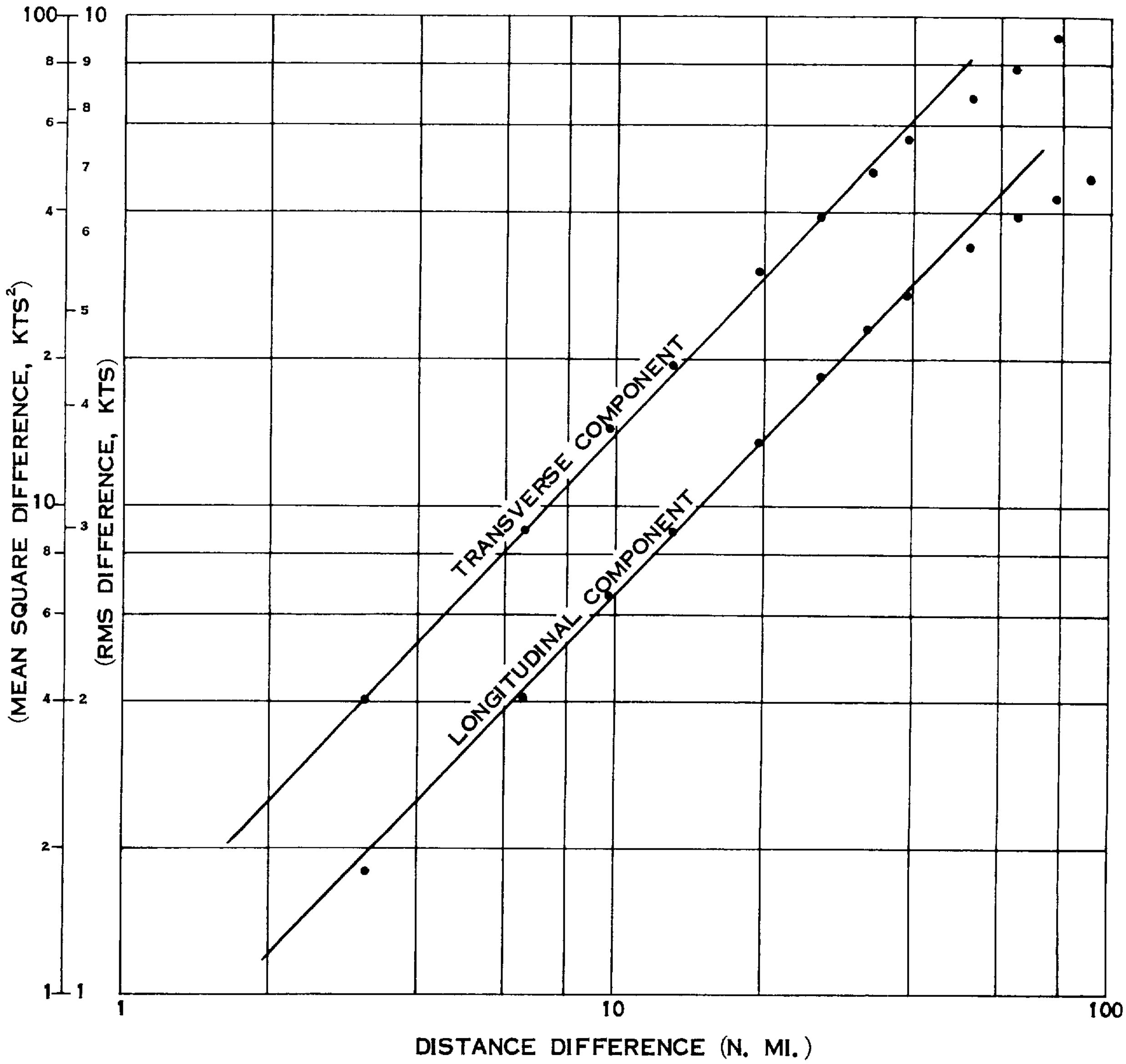


FIG. 8. WIND COMPONENT VARIABILITY AS A FUNCTION OF DISTANCE[41].

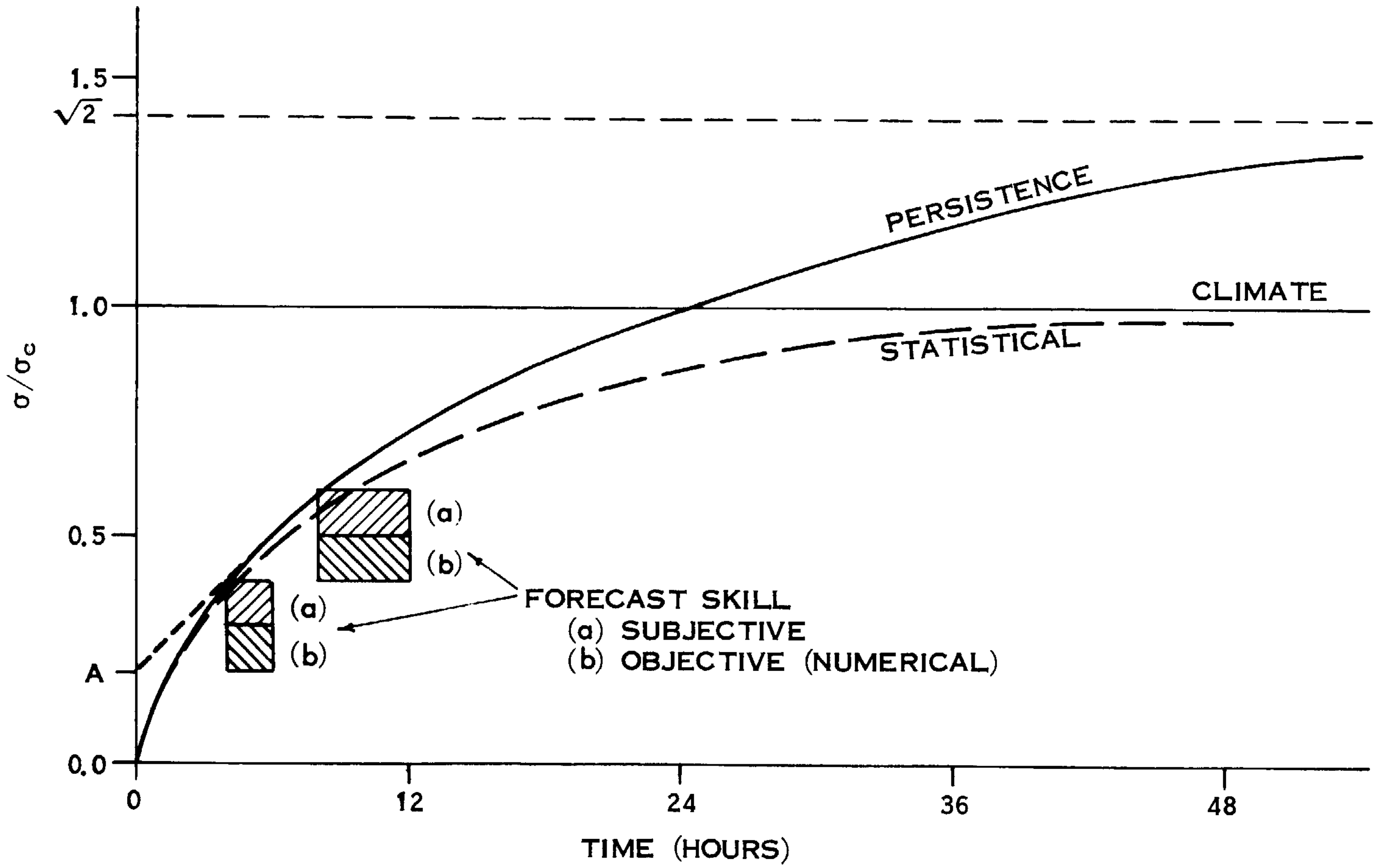


FIG. 9. RELATIVE ROOT MEAN SQUARE ERROR AS A FUNCTION OF TIME LAG FOR VARIOUS TYPES OF FORECASTS. SPECIFICALLY THE FIGURE APPLIES TO WIND ERRORS WITH σ_c AND σ REPRESENTING VECTOR STANDARD DEVIATIONS.

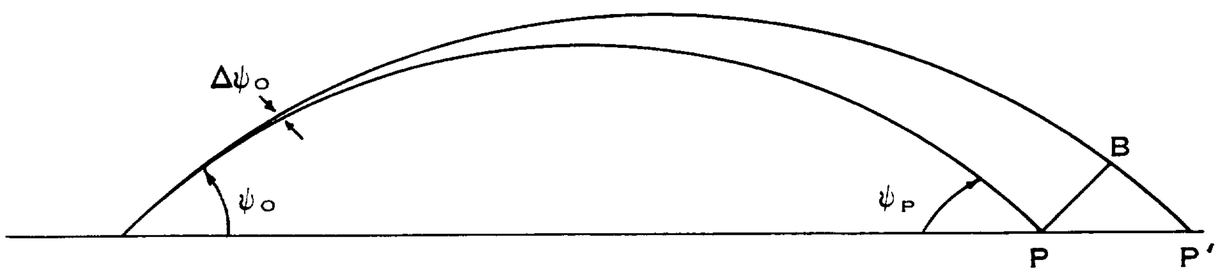


FIG. 10. CROSS SECTION OF A RETURNING RAY TUBE.

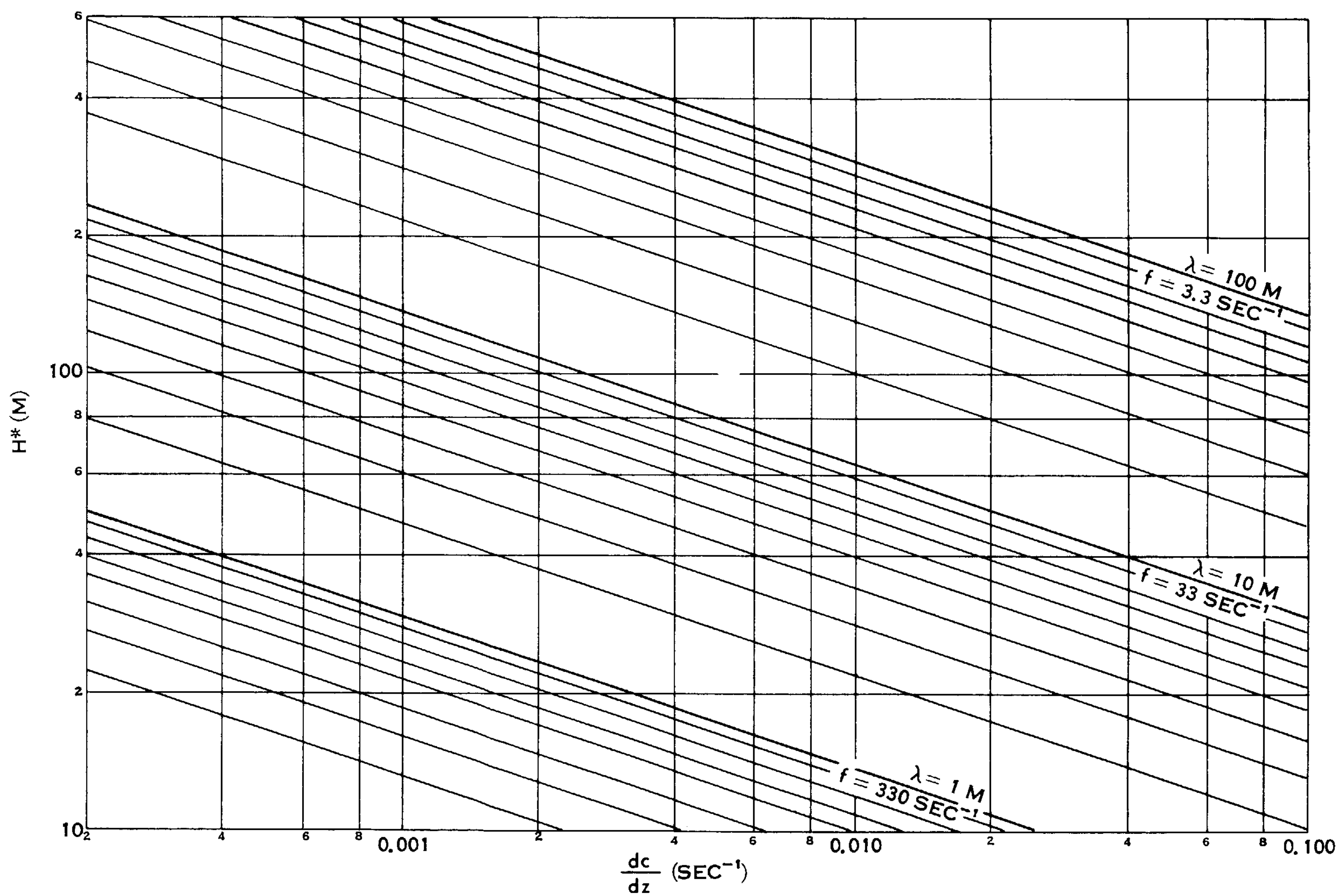


FIG. 11. GRAPH OF $H^* = \left[\lambda^2 c_o / 4\pi\,(dc/dz)\right]^{1/3}$. FOR RAY METHOD TO BE VALID, $H \gg H^*$, WHERE H = THE DEPTH OF THE UNIFORM LAYER, dc/dz = GRADIENT OF SPEED OF SOUND ABOVE THE UNIFORM LAYER, λ = WAVELENGTH.

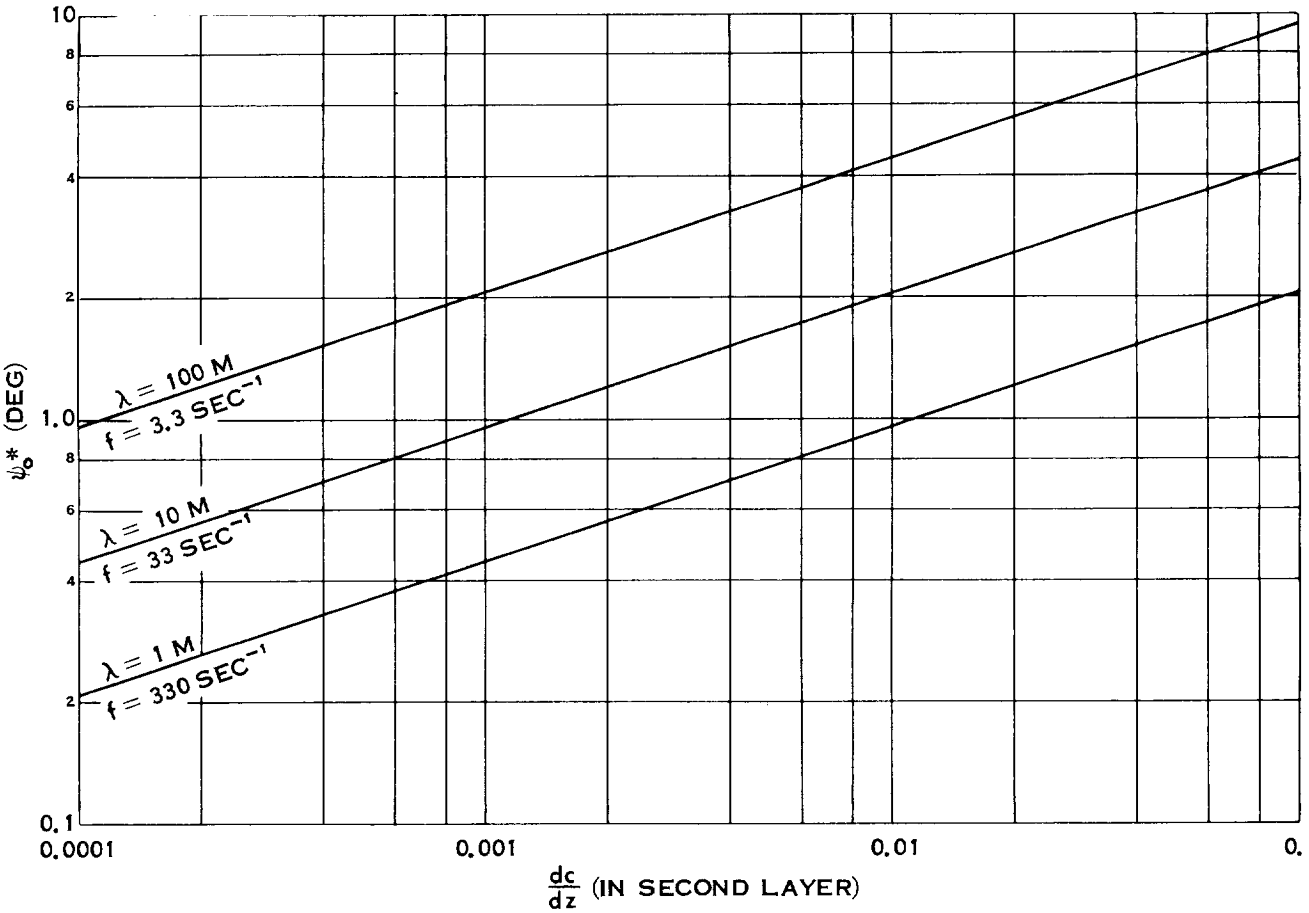


FIG. 12. GRAPH OF $\psi_o^* = [\lambda(dc/dz)/2\pi c_o]^{1/3}$. FOR RAY TRACING TO BE VALID, $\psi_o \gg \psi_o^*$ WHEN ψ_o = THE INITIAL ELEVATION ANGLE OF THE RAY. SEE FIG. 2.

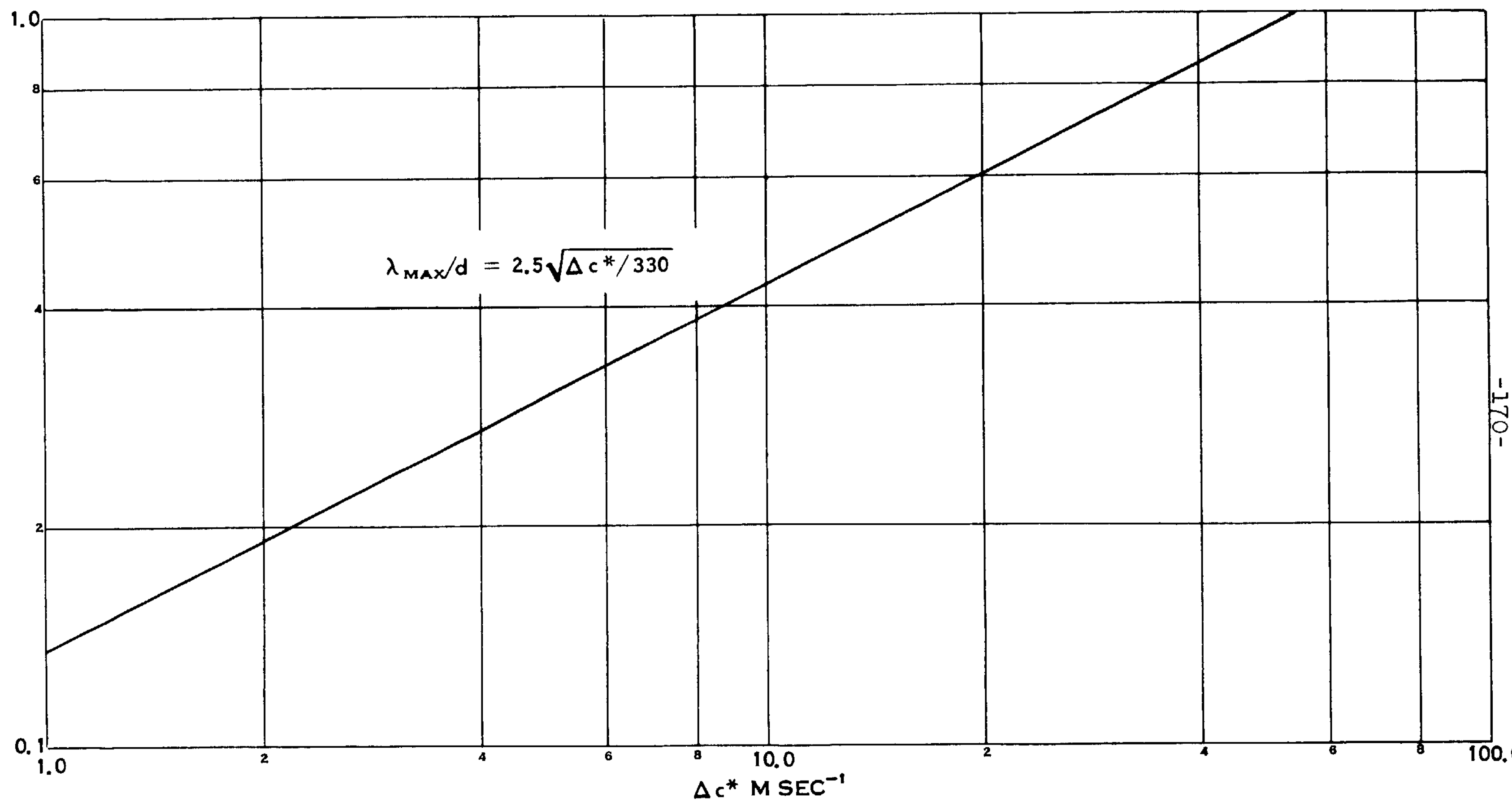


FIG. 13. MAXIMUM WAVELENGTH DUCTED, λ_{MAX}. ORDINATE IS λ_{MAX}/d WHERE d IS THE DEPTH OF THE DUCT. ABSCISSA IS Δc, THE INCREASE IN SPEED OF SOUND FROM GROUND TO TOP OF DUCT.

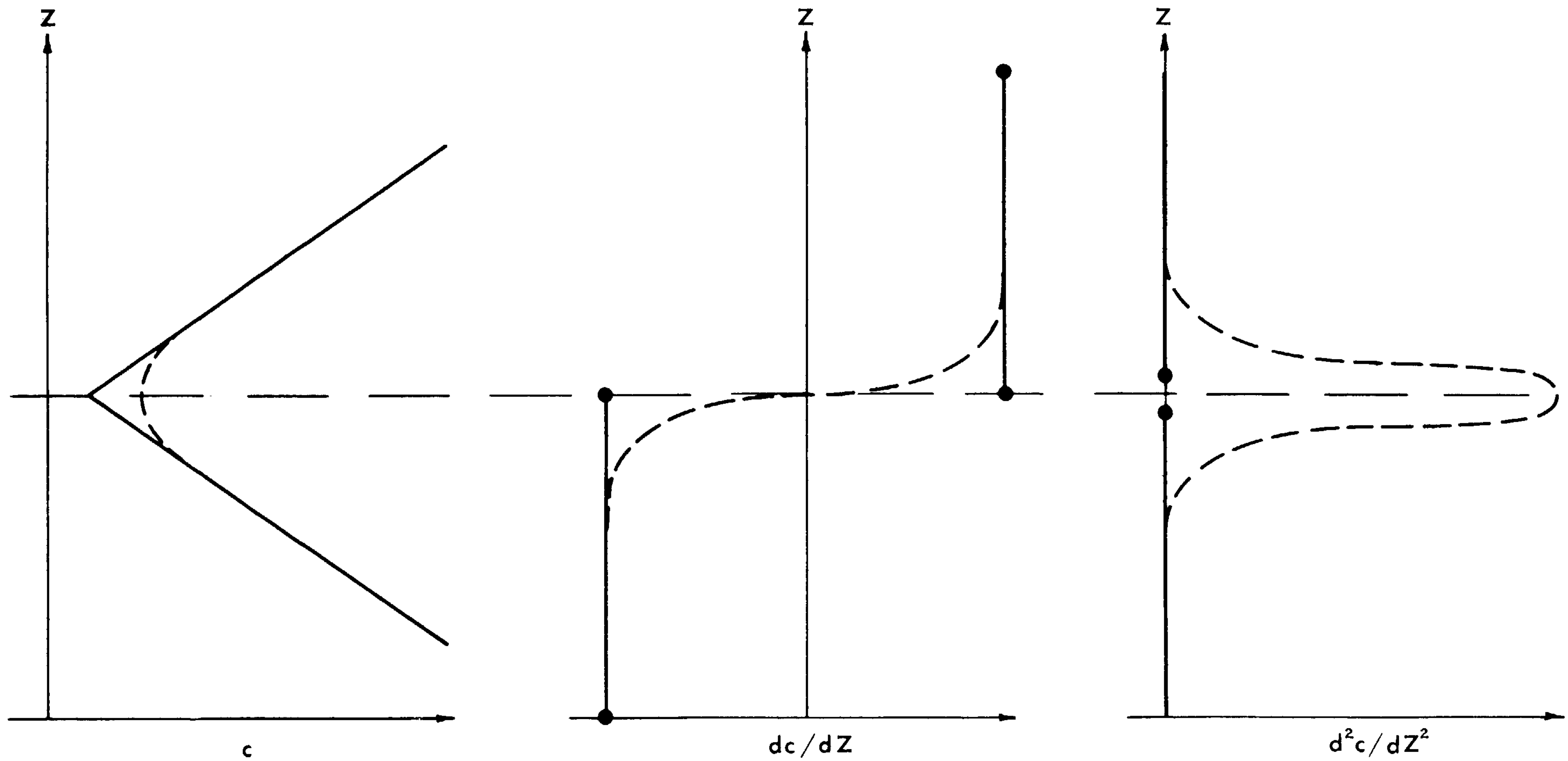


FIG. 14. THE CHANGES ACROSS A BOUNDARY BETWEEN LAYERS IN A "LINEAR LAYER" MODEL OF THE ATMOSPHERE (SOLID LINES) AND A "SMOOTHLY VARYING" ATMOSPHERE (DASHED CURVES).

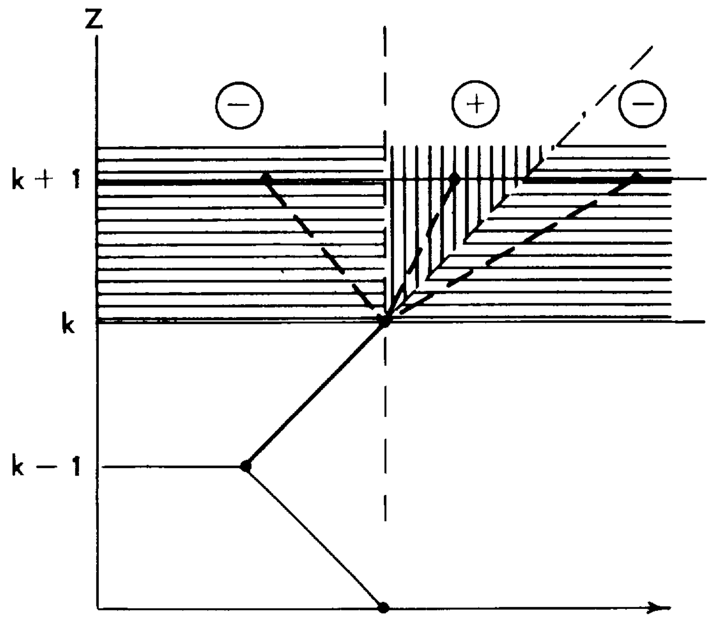


FIG. 15. THE SIGN OF THE DISCONTINUITY IN $dr/d\varphi$ AT A CHANGE IN THE SLOPE OF THE SPEED OF SOUND CURVE.

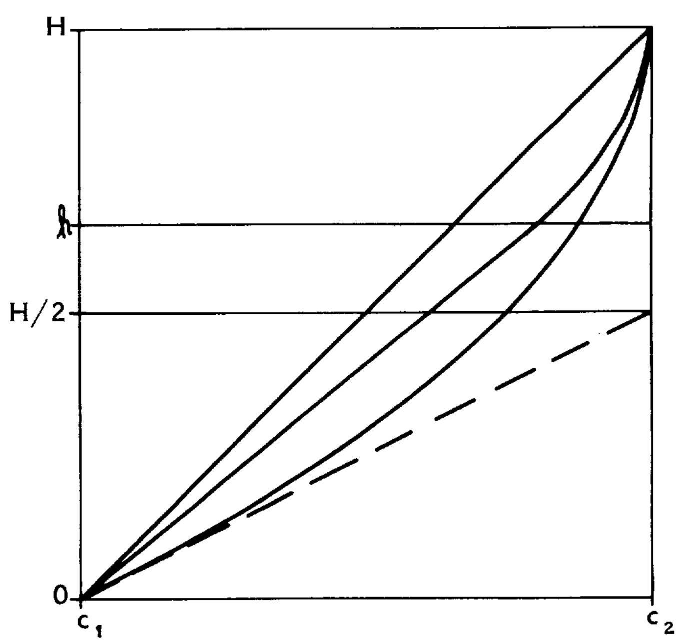


FIG. 16. LINEAR AND QUADRATIC LAYERS AND A COMBINATION OF LINEAR AND QUADRATIC DEPENDING ON THE PARAMETER h.

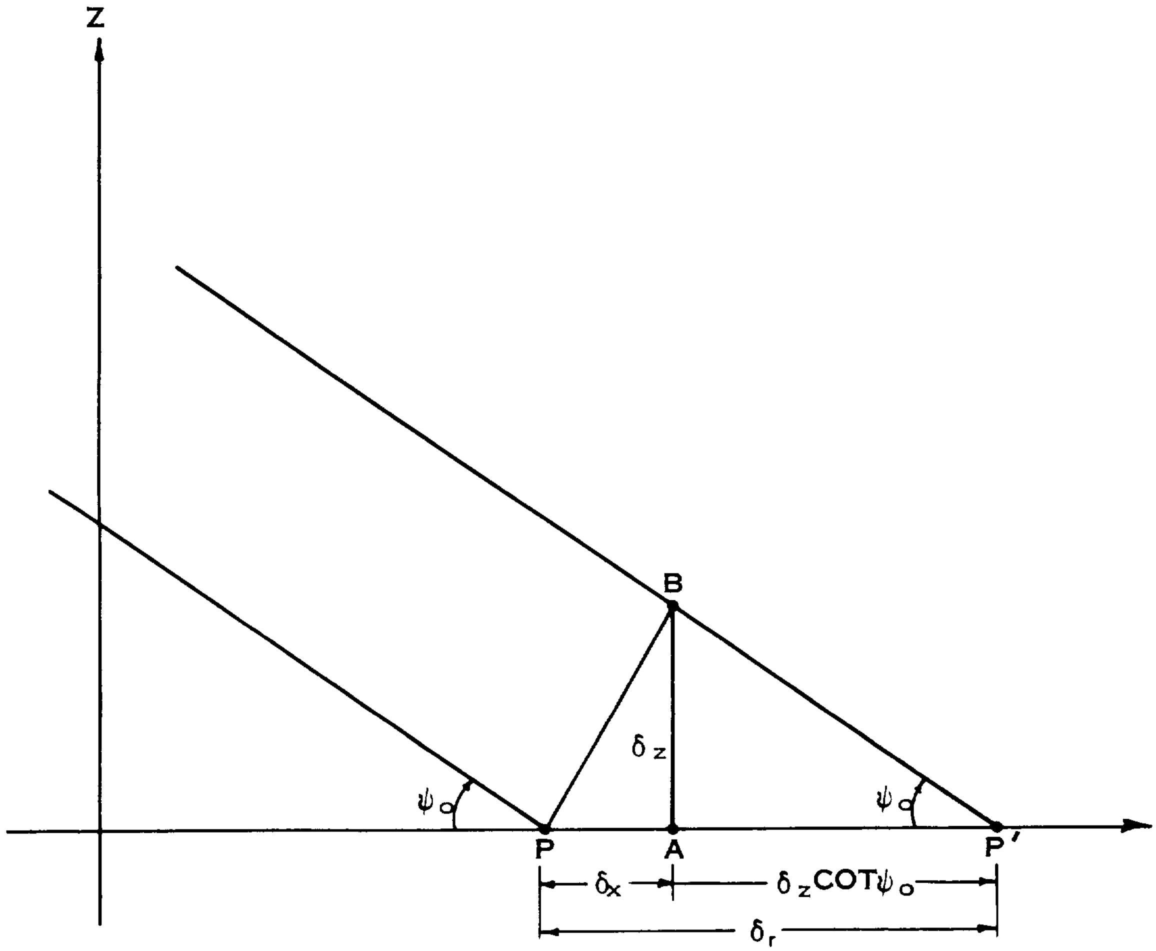


FIG. 17. VARIATION IN RANGE, δ_r, AS DEPENDENT ON VARIATION OF δ_x AND δ_z AND THE INCLINATION OF THE RAY ψ_o.

DISTRIBUTION

DEP-T
Dr. Rees

R-DIR
Mr. Weidner
Mr. Dannenberg

R-AS-DIR
Mr. Williams

R-EO-DIR
Dr. Johnson

MS-IP

MS-IPL (8)

MS-H

HME-P

CC-P

MS-T (8)

R-AERO-X
Mr. Thomae

R-AERO-DIR
Dr. Geissler
Mr. Jean
Mr. Clark

R-AERO-A
Mr. Dahm
Mr. Reed
Mr. Wilhold
Mr. J. Jones

R-AERO-D
Mr. Horn
Mr. Rheinfurth
Mr. Ryan
Mr. Deaton

R-AERO-F
Mr. Lindberg
Mr. Hill
Mr. Stone
Mr. Hagood

R-AERO-G
Mr. Baker
Mr. Blair
Mr. Hart
Mr. Schwaniger

R-AERO-P
Mr. McNair

R-AERO-T
Mr. Cummings
Dr. Heybey
Mr. Lavender
Dr. Liu
Mr. Murphree
Mr. J. Mabry
Mr. H. Krause

R-AERO-Y
Mr. Daniels
Mr. Dickey
Mr. Kaufman
Dr. Scoggins (2)
Mr. O. Smith (25)
Mr. O. Vaughan
Mr. W. Vaughan (2)
Mr. Brown
Mr. Sloan
Mr. Falls
Mr. Johnson
Mr. R. Turner
Mr. Camp
Mr. Susko
Mr. Hill
Mr. Dalton
Mr. R. Smith

R-ASTR-DIR
Dr. Haeussermann

DISTRIBUTION (CONT'D)

R-P&VE-DIR
Dr. Lucas
Mr. Hellebrand

R-P&VE-P
Mr. Paul

R-P&VE-S
Mr. Kroll
Mr. R. Hunt

R-P&VE-V
Mr. Aberg

R-QUAL-DIR
Mr. Grau

R-TEST-DIR
Mr. Heimburg

R-TEST-T
Dr. Reisig

R-TEST-I
Dr. Sieber
Mr. B. Blake
Mr. H. Rutledge

RSIC Library (2)

EXTERNAL

Director
Office of Manned Space Flight
NASA Headquarters
Washington, D. C. 20546
Attn: Dr. George Mueller, Director
Mr. Phil Bolger
Mr. John E. Greene
Mr. F. J. Demeritte

Office of Advanced Research and Technology
NASA Headquarters
Washington, D. C. 20546
Attn: Mr. Ames
Mr. R. Rhode
Mr. D. Gilstad
Mr. R. E. Littell
Dr. Jack Levine
Mr. Douglas Michell
Mr. Richard M. Bear
Dr. A. Kelley

Office of Space Science and Applications
NASA Headquarters
Washington, D. C. 20546
Attn: Dr. Homer Newell
Dr. John E. Naugle
Dr. M. Tepper
Mr. J. A. Salmanson
Mr. W. Spreen

NASA-Kennedy Space Flight Center
Cocoa Beach, Florida 32931
Attn: Dr. Bruns, INS-1
Dr. Knothe, TEC
Col. Petrone, PPR
Mr. A. Taiani, PPR-72
Technical Library
Lt. Col. R. L. Clark, SOP
Mr. J. Deese, EDV-23

NASA-Langley Research Center
Langley Field, Virginia 23365
Attn: Dr. F. L. Thompson, Director
Mr. R. Hord
Mr. T. Coleman
Mr. Homer Morgan
Mr. W. H. Reed, III
Mr. V. Alley
Mr. H. B. Tolefson
Mr. I. E. Garrick
Library (2)

DISTRIBUTION (CONT'D)

NASA-Goddard Space Flight Center
Greenbelt, Maryland 20771
Attn: Dr. W. Nordberg
Mr. W. Spencer
Library (2)

NASA-Flight Research Center
Edwards Air Force Base, California 93523
Attn: Mr. Tom Toll

Jet Propulsion Laboratory
California Institute of Technology
4800 Oak Grove Drive
Pasadena 3, California 91103
Attn: Technical Library

NASA-Lewis Research Center
21000 Brookpark Road
Cleveland, Ohio 44135
Attn: Mr. H. W. Plohr (Code: 9430)
Mr. C. Wentworth (Code: 9310)
Library

NASA-Ames Research Center
Moffett Field
Mountain View, California 94035
Attn: Dr. S. DeFrance, Director
Library

NASA-Manned Spacecraft Center
Houston, Texas 77001
Attn: Technical Library (2)
Mr. John P. Mayer
Mr. Robert F. Thompson

NASA-Mississippi Test Facility
Atmospheric Acoustic Facility
Bay St. Louis, Mississippi 39520
Attn: Mr. L. W. Nybo, I-MT-EM

Mr. William Elam (2)
Bellcomm, Inc.
1100 17th Street, NW
Washington, D. C. 20036

Commander
Headquarters, Air Weather Service
Scott Air Force Base, Illinois 6225
Attn: Dr. Robert D. Fletcher
Library (2)

Commander
Air Force Systems Command
Andrews Air Force Base
Washington, D. C. 20331

Mr. Kenneth Nagler
Environmental Science Services Administration
U. S. Weather Bureau
Washington, D. C. 20235

Office of Staff Meteorologist
AFSC (SCWTS)
Andrews Air Force Base
Washington, D. C. 20331

Air Force Systems Command (2)
Space Systems Division
Air Force Unit Post Office
Los Angeles 45, California 90045

Meteorological & Geoastrophysical Abstracts
P. O. Box 1736
Washington 13, D. C. 20013

Dr. Walter Boccius (2)
Lockheed Missiles & Space Corporation
Sunnyvale, California 94000

Office of Scientific and Technical Information (25)
Attn: NASA Representative (S-AK/RKT)
P. O. Box 33
College Park, Maryland 20740

DISTRIBUTION (CONT'D)

Air Force Cambridge Research Laboratories
Bedford, Massachusetts 01730
Attn: CRFL, Dr. J. S. Salisbury
Technical Library (5)

Michoud Operations, George C. Marshall Space Flight Center
P. O. Box 26078
I-MICH-D
New Orleans, Louisiana 70126

Sandia Corporation
Box 5800
Albuquerque, New Mexico 87115
Attn: Mr. Floyd Forsythe, 9323

Member, MWG
Inter-Range Instrumentation Group
Attn: Major Vick
APGC (PGGW)
Eglin Air Force Base, Florida 32542

Member, MWG
Inter-Range Instrumentation Group
Attn: Lt. Col. H. R. Montague (3)
AFETR
Patrick Air Force Base, Florida 32925

Dr. O. Essenwanger
AMSMI-RRA, Bldg. 5429
U. S. Army Missile Command
Redstone Arsenal, Alabama

Member, MWG
Inter-Range Instrumentation Group
Attn: Mr. Orville Daniel
PAWA/GMRD, AFMTC, MU-235,
Technical Library
Patrick Air Force Base, Florida 32925

Member, MWG
Inter-Range Instrumentation Group
Attn: Lt. Col. I. Van Blunt
AFFTC (FTZBW)
Edwards Air Force Base, California 93523

Member, MWG
Inter-Range Instrumentation Group
Attn: Mr. Royal C. Gould
Code 3069
Naval Ordnance Test Station
China Lake, California 93557

Member, MWG
Inter-Range Instrumentation Group
Attn: Mr. Tom Carr
Code 3150 (2)
Comdr. Summerville,
Code 3250, Box 22 (2)
Pacific Missile Range
Point Mugu, California 93041

Member, MWG
Inter-Range Instrumentation Group
Attn: Mr. Willis L. Webb (3)
SELWS-M, USAERDA
White Sands Missile Range,
New Mexico 88002

Associated Member, MWG
Inter-Range Instrumentation Group
Attn: Mr. Robert Leviton
AFCRL (CRER), OAR
1065 Main Street
Waltham, Massachusetts 02154

DISTRIBUTION (CONT'D)

Associated Member, MWG
Inter-Range Instrumentation Group
Attn: Mr. V. S. Hardin
AWSSS/TRSD
Air Weather Service
Scott Air Force Base, Illinois

Associated Member, MWG
Inter-Range Instrumentation Group
Attn: Mr. Kenneth Steelmen
AMSEL-RD-SM
USAERDL
Fort Monmouth, New Jersey

Associate Member, MWG
Inter-Range Instrumentation Group
Attn: Mr. A. Lewis Miller
Meteorological Management Division
Bureau of Naval Weapons
Washington 25, D. C.

Associate Member, MWG
Inter-Range Instrumentation Group
Attn: Mr. John F. Spurling
NASA-Wallops Station
Wallops Island, Virginia 23337

Associated Member, MWG
Inter-Range Instrumentation Group
Attn: Mr. Clifford A. Olson
Code 7261
Sandia Corporation
Sandia Base
Albuquerque, New Mexico 87110

Dr. Jack L. Hough
Department of Meteorology and
Oceanography
University of Michigan
Ann Arbor, Michigan 48105

Commander
Air Force Flight Dynamics Laboratory
Air Force Systems Command
Wright-Patterson AFB, Ohio 45433
Attn: Mr. Thomas D. Lemley
Mr. George Muller

Aerospace Corporation
P. O. Box 95081
El Segundo, California 90245
Attn: Dr. Henry G. Maier

Dr. Edwin F. Danielsen
The Pennsylvania State University
Department of Meteorology
College of Mineral Industries
University Park, Pennsylvania 16802

Mr. H. C. S. Thom
Environmental Data Service
Gramax Building
8060 13th Street, N. W.
Silver Springs, Maryland 20910

Dr. Harold L. Crutcher
National Weather Records Center
Federal Building
Asheville, North Carolina 28801

Dr. A. C. Cohen, Jr.
Department of Statistics
University of Georgia
Athens, Georgia 30601

Dr. C. E. Buell (25)
Kaman Nuclear
Garden of the Gods Road
Colorado Springs, Colorado 80907

Dr. Thomas Gleeson
Professor of Meteorology
Department of Meteorology
Florida State University
Tallahassee, Florida 32306

Professor Noel LaSuer
Department of Meteorology
Florida State University
Tallahassee, Florida 32306

DISTRIBUTION (CONT'D)

Professor C. Jordan
Department of Meteorology
Florida State University
Tallahassee, Florida 32306

Dr. Gerald L. Barger
Environmental Data Service
Gramax Building, Room 730
8060 13th Street, N. W.
Silver Springs, Maryland 20910

The Boeing Company
New Orleans, Louisiana
Attn: Mr. Wayne R. Covington

Dr. Arnold Court
17168 Septo Street
Northridge, California 91324

Dr. Sidney Teweles
U. S. Weather Bureau
Washington, D. C. 20331

Mr. Norman Sissenwine (2)
Air Force Cambridge Research Laboratories
Code: CREW
L. G. Hanscom Field
Bedford, Massachusetts

Mr. Jack Reed
Sandia Corporation
Albuquerque, New Mexico

Mr. Edmund Blond
Manager, Special Studies
Aerospace Corporation
2350 E. El Segundo Blvd.
El Segundo, California 90245

Mr. Thomas Y. Palmer
Research Specialist
Aerospace Division
Boeing Company MS 23-81 656-PS88
P. O. Box 3707
Seattle, Washington

Professor Elmar R. Reiter
Department of Atmospheric Science
Colorado State University
Fort Collins, Colorado 80521

Mr. Marvin Diamond
U. S. Army Electronics R&D Activity
Attn: Environmental Sciences Department
White Sands Missile Range,
New Mexico 88002

Director
Meteorological Division
U. S. Army Signal R&D Laboratories
Fort Monmouth, New Jersey

Mr. John C. Mester
U. S. Army Ballistics Research Laboratories
Attn: AMXBR-B
Aberdeen Proving Ground, Maryland 21005

U. S. Army Electronics Research and Development Activity
Fort Hauachuca, Arizona 86513
Attn: Mr. A. V. Carlson, SELHU-M (2)
Meteorology Department
Library (2)

Commanding Officer (2)
United States Army
Frankford Arsenal
Attn: Mr. David Askin, Q6200
Building 230
Philadelphia, Pennsylvania 19137

Commanding Officer
U. S. Army Security Agency Processing Station
Vint Hill Farms Station
Attn: IAV APC-R&D
Captain T. H. Tyvand
Warrenton, Virginia 22186

Martin-Marietta Corp.
P. O. Box 179
Denver 1, Colorado
Attn: C. R. Gunnison, A-175

AVCO Corporation
Research and Advanced Development Division
201 Lowell Street
Wilmington, Mass. 01887

C.O. B.R.L.
Aberdeen Proving Ground, Maryland
Attn: A. A. Thompson, T. B. L.

Lockheed Missiles & Space Co.
Huntsville Research & Engineering Center
Huntsville Research Park
4800 Bradford Drive
Huntsville, Alabama
Attn: Mr. John Woo