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INTERNAL NOTE NO.

SINGLE PARAMETER TESTING

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RELIABILITY AND QUALITY ASSURANCE LABORATORY

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SINGLE PARAMETER TESTING

SUMMARY

The objective of the single parameter testing program was to investigate techniques for solving a variation of the identification problem. The general identification problem consists in determining the input-output relationships of a black box by experimental means. The form of the identification problem dealt with in this program was in determining deviations of the parameters of a component from their nominal values. Two techniques have been investigated for solving this problem: (1) the use of growing exponentials as the input probing signal and (2) the use of transient response sampling.

Both linear and nonlinear component models were studied to establish the range over which accurate parameter predictions could be made. The final phase of the program involved applying the developed techniques to AC and DC amplifiers with the objective of measuring the transfer function parameters and, therefore, the frequency response characteristic. Also investigated was a technique for measuring amplifier linearity.

The study shows that single parameter testing techniques can be applied to some components. Technical and economic considerations for the selection of a component to be tested when using these techniques are described in this report.

SECTION I. INTRODUCTION

The objective of the single parameter testing program, NAS8-11715, Part III, was to investigate techniques for simultaneously testing several parameters of a component with one testing signal, thereby achieving faster checkout time. Two techniques were studied during the program, the use of growing exponential probing signals and the use of transient response sampling. The program to investigate single parameter testing techniques was divided into five phases:

1. Phase A—A survey was conducted of several techniques for performing single parameter testing with emphasis on the use of growing exponential signals as the input testing signal. The use of this technique to test linear first and second order systems was evaluated by studying analog computer models (Ref. 1).

2. Phase B—A further investigation of the growing exponential probing signal technique was made. Linear active networks were also considered during this phase (Ref. 2).

3. Phase C—Investigation was made of testing implementation by studying the pen position control system of an X-Y plotter with the growing exponential probing signal technique. A linear model (a sixth order transfer function) of the S-IB thrust vector control system was studied with both the growing exponential probing signal technique and the transient response sampling technique (Ref. 3 and 4).

4. Phase C Extension—A nonlinear model of the S-IB thrust vector control system was tested using transient response sampling (Ref. 5).

5. Phase D—A survey was made of the components which the Electrical Test and Analysis Branch of the Reliability and Quality Assurance Laboratory is responsible for testing. The phase report gives the components that can be considered for single parameter testing and the particular component parameters that can be measured (Ref. 6).

6. Phase E—The transient response sampling technique was applied to an AC and a DC amplifier both of which are used as signal processors between the space vehicle transducers and the telemetry equipment. The amplifier characteristics to be measured are the frequency response and the amplifier linearity (Ref. 7).

The complete results of Phases A, B, and C are presented in References 1 through 5. Section II of this report contains a review of these results and conclusions. Section III is a summary of the Phase D results previously reported in Reference 6. Sections IV and V contain the testing results obtained with the AC and the DC amplifiers, respectively. Section VI gives the technical and economic considerations used in determining if a component should be selected for single parameter testing. The program conclusions are given in Section VII.

SECTION II. REVIEW OF THE PHASE A, B, AND C RESULTS

A. GENERAL

The objective of the single parameter testing program was to investigate techniques for solving a variation of the identification problem. The general identification problem is in determining the input-output relationship of a black box by experimental means. The form of the identification problem with which this study was concerned was in determining component parameter deviations from the nominal values.

Two techniques were investigated for solving this problem. A technique of using growing exponentials as the input probing signal and a technique of using transient response sampling. The initial evaluation of these techniques was performed during Phases A, B, and C of the program. The complete results of these phases are reported in References 1 through 5 and will be reviewed in this section.

B. GROWING EXPONENTIALS AS A PROBING SIGNAL

The theory of this technique of component identification is presented by S. Litman and W.H. Huggins in Reference 8 and was reviewed in previous phase reports of this program, specifically References 1 and 4. A brief review of the implementation of this technique will be presented in this section.

A block diagram of the implementation is shown in Figure 1. Assuming an electrical component under test, the nominal model can be built of passive components, or an analog computer model, or an actual nominal component may be used. The orthogonal filter bank is used both to initially generate the probing signal and then analyze the error waveform in performing the identification process. The impulse response of the filter bank produces a decaying exponential signal at the output of each filter stage. By recording these signals with a tape recorder and then reversing the tape end-for-end, the orthogonal growing exponential signals are obtained. The probing signal is then selected as a combination of these signals. Again assuming an electrical output signal, the subtraction process, orthogonal filter bank, estimator, and the sampler can be constructed on an analog computing device.

C. TRANSIENT RESPONSE SAMPLING TECHNIQUE

The basic theory of this technique for system identification is presented in Reference 9. This method was extended by the inclusion of second order effects as described in a previous phase report (Ref. 7). Because this was the technique chosen for the Phase E testing of the AC and DC amplifiers, the theory will again be reviewed.

The implementation of the technique is shown in Figure 2. The systems which can be tested with this method are describable in terms of transfer functions, that is, an expression of the gain and phase shift of the system as a function of frequency. The parameters of the system that are measured are, then, the time constants, natural frequencies, damping ratios, and the gain associated with the circuit. The test signal to be used must have its energy content in the region of the system frequency response where

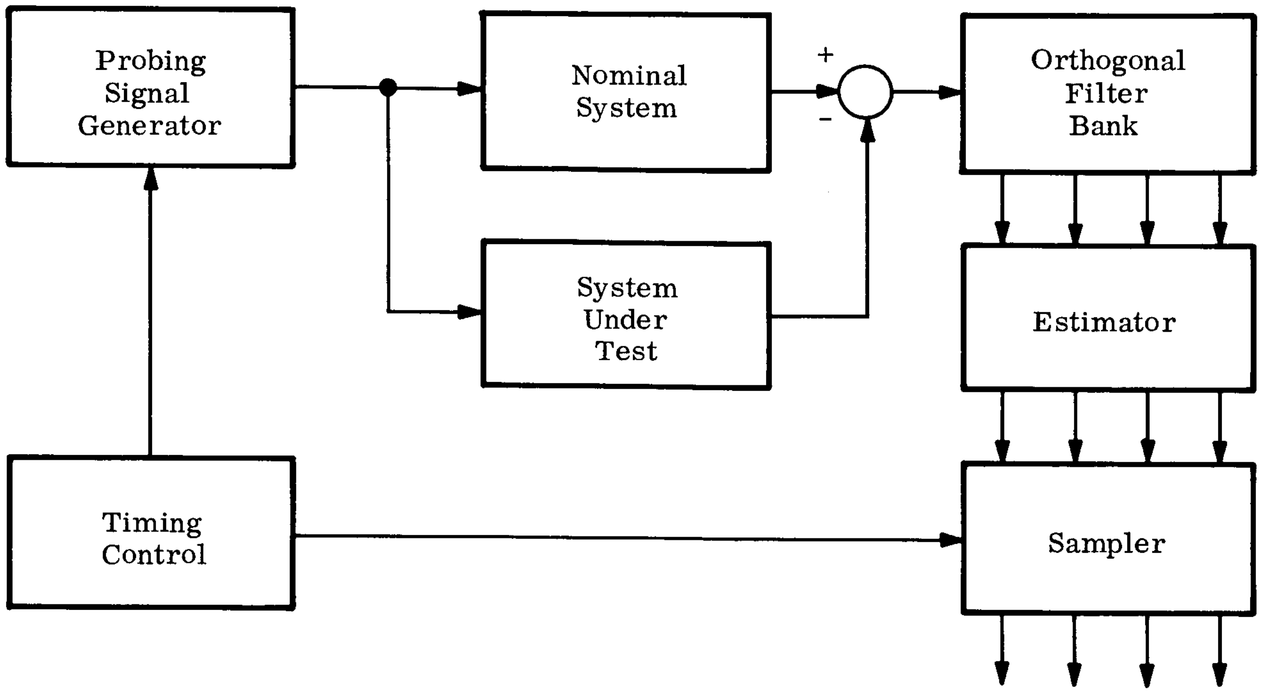


Figure 1. Single Parameter Testing Using Growing Exponential Signals

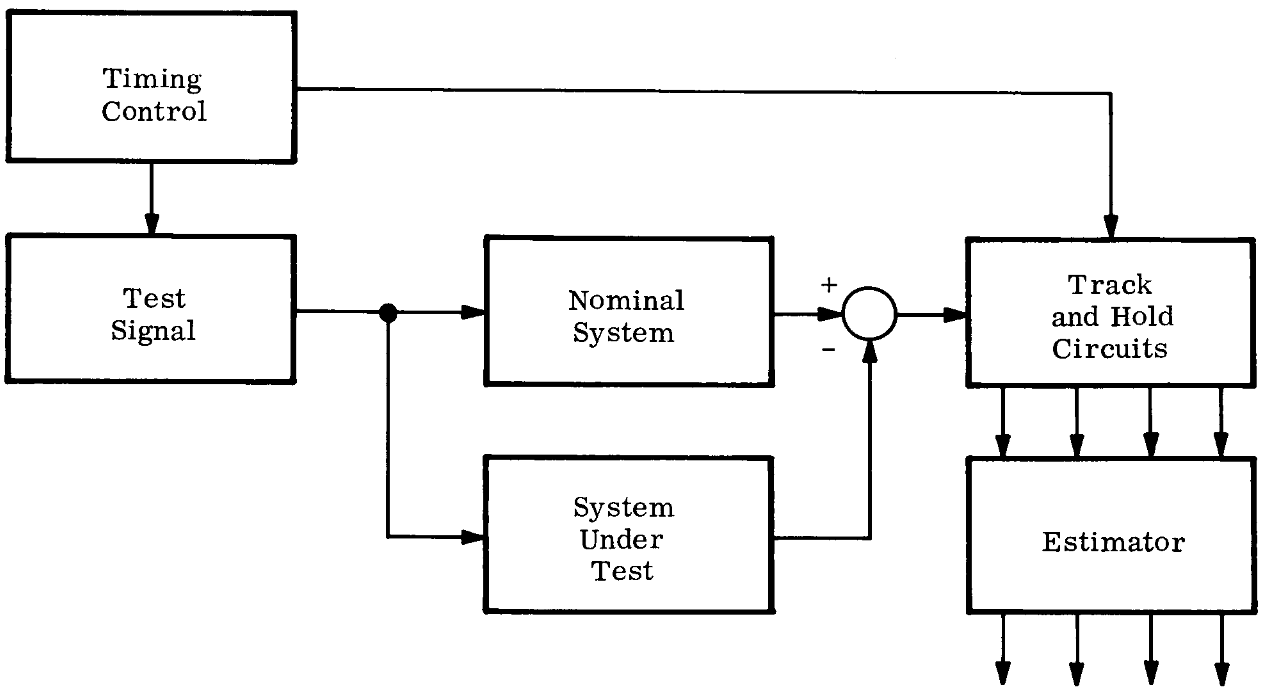


Figure 2. Implementation of the Transient Response Sampling Technique

the poles and zeros to be measured are located. Several test signals can be considered such as a pulse, a ramp, one cycle of a square wave, a triangular wave, etc. The spectrums of these signals are given in Reference 7. In particular, it was found that a pulse or one cycle of a square wave gave good single parameter testing results.

To determine if all the parameters of a transfer function can be independently measured, the partial derivatives of the transfer function (G) with respect to each parameter (P) are calculated and substituted into

$$A_1 \frac{\partial G}{\partial P_1} + A_2 \frac{\partial G}{\partial P_2} + \cdots A_n \frac{\partial G}{\partial P_n} = 0 \tag{1}$$

If the only values of A for which Equation 1 is satisfied are

$$A_1 = A_2 = \cdots A_n = 0$$

then the partial systems are linearly independent and the parameters can be measured.

The error waveform at the output of the difference circuit can be expressed

$$\begin{aligned} V(t) = {} & a_1(t)\Delta P_1 + b_1(t)\Delta P_1^2 + a_2(t)\Delta P_2 + b_2(t)\Delta P_2^2 + \cdots \\ & + a_n(t)\Delta P_n + b_n(t)\Delta P_n^2 + E \end{aligned} \tag{2}$$

where V (t) is the difference waveform as a function of time

$$\Delta P = 10(P - P_n)/P_n$$

and

a = Linear coefficients (a function of time).
b = Second order coefficients (a function of time).
E = An error term to take into account the higher order terms.

Note that ΔP has been defined for convenience, so that it is equal to unity for a 10 percent change in the parameter P. The coefficients can be determined experimentally by varying one parameter at a time and measuring the error waveform value. Because only one parameter is varied at a time, Equation 2 reduces to

$$V(t) = a(t)\Delta P + b(t)\Delta P^2 \tag{3}$$

if ΔP is chosen small so that E can be considered negligible. For a particular sampling time t_1 two runs are made, one with a parameter change of +20 percent ($\Delta P = 2$) and the other for a change of -20 percent ($\Delta P = -2$). If the experimentally determined sampled values of the difference waveform are X and Y, respectively, then

$$\left.\begin{aligned} V(t_1) &= a(t_1)\,\Delta P + b(t_1)\,\Delta P^2 \\ X &= a(t_1)\,(2) + b(t_1)\,(2)^2 \\ Y &= a(t_1)(-2) + b(t_1)(-2)^2 \end{aligned}\right\} \quad (4)$$

and, therefore

$$a(t_1) = \frac{X - Y}{4} \text{ and } b(t_1) = \frac{X + Y}{8} \quad (5)$$

This coefficient determination process must be carried out for all parameters of interest and for each time sample point. The number of time sample points must equal twice the number of parameters to be measured. After the coefficients have been determined the following set of simultaneous equations can be written.

$$\left.\begin{aligned} V(t_1) &= a_1(t_1)\Delta P_1 + b_1(t_1)\Delta P_1^2 + \cdots + b_n(t_1)\Delta P_n^2 \\ &\vdots \\ V(t_{2n}) &= a_1(t_{2n})\Delta P_1 + b_1(t_{2n})\Delta P_n^2 + \cdots + b_n(t_{2n})\Delta P_n^2 \end{aligned}\right\} \quad (6)$$

or using matrix form

$$\begin{bmatrix} V(t_1) \\ \\ \vdots \\ V(t_{2n}) \end{bmatrix} = \begin{bmatrix} a_1(t_1) & b_1(t_1) & \cdots & b_n(t_1) \\ & & & \\ \vdots & \vdots & & \vdots \\ a_1(t_{2n}) & b_1(t_{2n}) & \cdots & b_n(t_{2n}) \end{bmatrix} \begin{bmatrix} \Delta P_1 \\ \Delta P_1^2 \\ \vdots \\ \Delta P_n^2 \end{bmatrix} \quad (7)$$

This equation can be represented by $V = AP$ and the parameter vector solved for using

$$P = A^{-1} V \quad (8)$$

The implementation of this equation involves a bank of track and hold circuits to hold the error waveform values corresponding to the sample times of interest and a set of potentiometers and amplifiers to implement the matrix inverse. This matrix inverse can be easily calculated using digital programs for this purpose. Only the linear terms (ΔP) and not the second order terms (ΔP^2) need be solved in the implementation.

As the theory has been presented, we have approximated the error waveform (Equation 2) with first and second order terms neglecting higher order effects and cross order effects. Another approach is to use only the linear terms, again keeping the number of sample times equal to the number of terms in the approximation equation. Still, another way is to represent some parameter changes with just linear terms and other parameters with first and second order terms. There is a tradeoff to be made in this choice. If the second order effects are used, the representation of the error waveform is more accurate and good single parameter testing results can be obtained for a wider range of parameter variations. However, the problem is not quite as easy as this. For each additional term used, the order of the A matrix is increased by one. If each row of the A matrix is considered as a vector in n dimensional space, then the A matrix can be written as a column matrix

$$A = \begin{bmatrix} \underline{a}_1 \\ \underline{a}_2 \\ \underline{a}_n \end{bmatrix} \qquad (9)$$

If each of these "$\underline{a}$" vectors were orthogonal to the other vectors then the size of the matrix would not affect the single parameter testing results. But, except for a very simple situation, it is impossible to select the time sample points so that the vectors are orthogonal. The vector $\underline{a}$ can be written

$$\underline{a}_n = (a_{n1}, a_{n2}, \cdots a_{nn}) \qquad (10)$$

and the angle (θ) between the i^{th} vector and the j^{th} vector is calculated from

$$\cos^2 \theta_{ij} = \left[\sum_{k=1}^{n} a_{ik} a_{jk} \right]^2 \div \left[\sum_{k=1}^{n} a_{ik}^2 \right] \left[\sum_{k=1}^{n} a_{jk}^2 \right] \qquad (11)$$

If any of these angles are small, it is possible to have numerical stability problems. In terms of the estimator in the single parameter testing implementation, poor parameter prediction will result. The more components in each $\underline{a}$ vector the more difficult it will be to pick "n" time sample points so as to have large angles between the "n" vectors.

Because of this type of problem, it is best to use only the linear terms in the approximation equation (Equation 2) when the number of parameters to be measured is large (four or more). A good method to use in selecting the time sample points is to select more points than are needed and then, using Equation 11, calculate the angles, and eliminate those vectors that lie along about the same direction as other $\underline{a}$ vectors.

D. TESTING RESULTS

In order to establish the accuracy and range of single parameter testing parameter predictions, several test situations were modeled on the analog computer. A summary of some of these results obtained for simple transfer functions is given in Table 1. The data presented in Table 1 was obtained using the transient response sampling

Table 1. Parameter Prediction Data

Transfer Function	Run No.	Testing Signal	Number of Terms		Parameters Measured	Average Accuracy* %
			Linear	Second Order		
$\frac{K}{TS+1}$	1 2	Square Wave	2 2	0 1	K, T	1.4 1.1
$\frac{K}{(T_1 S+1)(T_2 S+1)}$ $T_1 = 4T_2$	3 4	Square Wave	3 3	0 2	K, T_1, T_2	1.9 1.5
$\frac{K}{S^2 + 2\zeta \omega S + \omega^2}$ $\zeta = 1$	5	Square Wave	3	0	K, ζ, ω	1.3
$\frac{K}{S^2 + 2\zeta \omega S + \omega^2}$ $\zeta = 0.1$	6 7	Square Wave	3 3	0 1	K, ζ, ω	4.0 1.3
$\frac{KS}{S^2 + 2\zeta \omega S + \omega^2}$ $\zeta = 0.1$	8	Square Wave	3	1	K, ζ, ω	1.3
$\frac{KS}{TS+1}$	9 10	Pulse	2 2	0 1	K, T	2.0 1.1

*Parameter prediction average accuracy of about 30 data points with one or more parameters varying in the range of ±20 percent.

single parameter testing technique. Data with comparable accuracy would be obtained with the growing exponential test signal technique.

The average accuracy for each run is the average parameter prediction accuracy of about 30 data points with one or more parameters varying in the range of ±20 percent at each point. Note that the inclusion of second order effects improved the prediction accuracy. With the transfer function associated with Run 5, the two nominal pole locations are the same. In this case, it is impossible to separate effects which are caused by the individual poles. Parameters that can be measured are

$$\omega = \sqrt{P_1 P_2} \quad , \quad \zeta = \frac{P_1 + P_2}{2\sqrt{P_1 P_2}} \tag{12}$$

where P_1 and P_2 are the two poles.

As a result of the model testing performed, it has been concluded that components that can be single parameter tested have transfer functions that are in one of the classes shown in Table 2.

Table 2. Component Types for Which Single Parameter Testing is Applicable

Number of Parameters to be Measured	Example Transfer Function	Parameters	Parameter Variation Range	Parameter Prediction Accuracy
2	$\frac{K}{TS + 1}$	K, T	±40%	±3%
3	$\frac{K}{(T_1 S + 1)(T_2 S + 1)}$	K, T_1, T_2	±25%	±3%
4	$\frac{K(S + a)}{S^2 + 2\zeta\omega S + \omega^2}$	K, a, ζ, ω	±15%	±3%
5	$\frac{KS(S + a)}{(S^2 + 2\zeta\omega S + \omega^2)(S + b)}$	K, a, b, ζ, ω	±10%	±3%

Components with more complicated transfer functions can be considered if the extra poles are far away from the poles of interest such that they do not affect the response, or if it can be assumed that the extra poles cannot vary and, therefore, do not need to be tested. More complicated systems can also be handled by considering them as subsystems, each of which is simple enough to be tested by itself, and providing a separate testing input and output connection for each subsystem. This reduces the complex system into subsystems which have transfer functions that can be single parameter tested.

If a component has a nonlinearity which must also be measured, then this reduces the number of other parameters that can be simultaneously tested. It is possible to measure a nonlinear characteristic simultaneously with the transfer function parameters. This is discussed further in paragraph F of Section II and also in Reference 7.

E. TESTING AN X-Y PLOTTER SERVO SYSTEM

The technical approach of single parameter testing with growing exponential signals was applied to the servo-loop controlling pen position on an X-Y plotter. The primary purpose of the test was to establish the test procedure for a physical system and gain insight into practical implementation problems. The total transfer function of the system obtained by a detailed analysis and from measured data is

$$G(S) = \frac{K(S + 275)}{(S^2 + 15.08\,S + 184.4)(S + 81.5)(S + 279)} \tag{13}$$

No specific quantitative single parameter testing results could be obtained with the X-Y plotter. This was due to a combination of reasons. The conclusions that have been drawn are:

1. Signal levels between the testing equipment and the system under evaluation must be compatible. This was a problem with the X-Y plotter that operates with millivolt level signals and with the analog computer that operates from 10 millivolts to 100 volts. It would have been desirable to test the X-Y plotter with a transistorized testing system design to operate with low signal levels.

2. It may be difficult to develop an accurate nominal model of some components. To obtain the necessary accuracy of matching the nominal plotter control system model to the actual system, it was necessary to use a second X-Y plotter as the model.

3. The instrumentation for implementing the method must be compatible with input/output impedance relations of the component under evaluation.

F. THRUST VECTOR CONTROL SYSTEM TESTING

During Phase C the transfer function model of the Saturn IB thrust vector control system was selected and tested using both the growing exponential probing signal technique and also the transient response sampling technique. The thrust vector control system uses a Moog's Model 16-120A dynamic pressure feedback servo-value and a Moog's Model 17-150 actuator. The linearized closed loop transfer function of the system derived from empirical data is

$$G(S) = K\left[\left(S + \omega_1\right)\left(S + \omega_3\right)\left(S^2 + 2\zeta_2\omega_2 S + \omega_2^2\right)\left(S^2 + 2\zeta_4\omega_4 S + \omega_4^2\right)\right]^{-1} \tag{14}$$

where

$$\omega_1 = 21.02 \text{ rad/sec} \qquad \omega_4 = 262.7 \text{ rad/sec}$$

$$\omega_2 = 49.52 \text{ rad/sec} \qquad \zeta_2 = 0.202$$

$$\omega_3 = 302.5 \text{ rad/sec} \qquad \zeta_4 = 0.528$$

The study of this control system and derivation of the transfer function is found in Reference 10. Also in this reference, is a specification of the permissible amplitude ratio and phase lag as a function of frequency. The range of changes in each parameter studied was determined from these specifications to be

$$\omega_1 \pm 10\% \qquad \omega_3 \pm 10\%$$

$$\omega_2 \pm 5\% \qquad \zeta_2 \pm 5\%$$

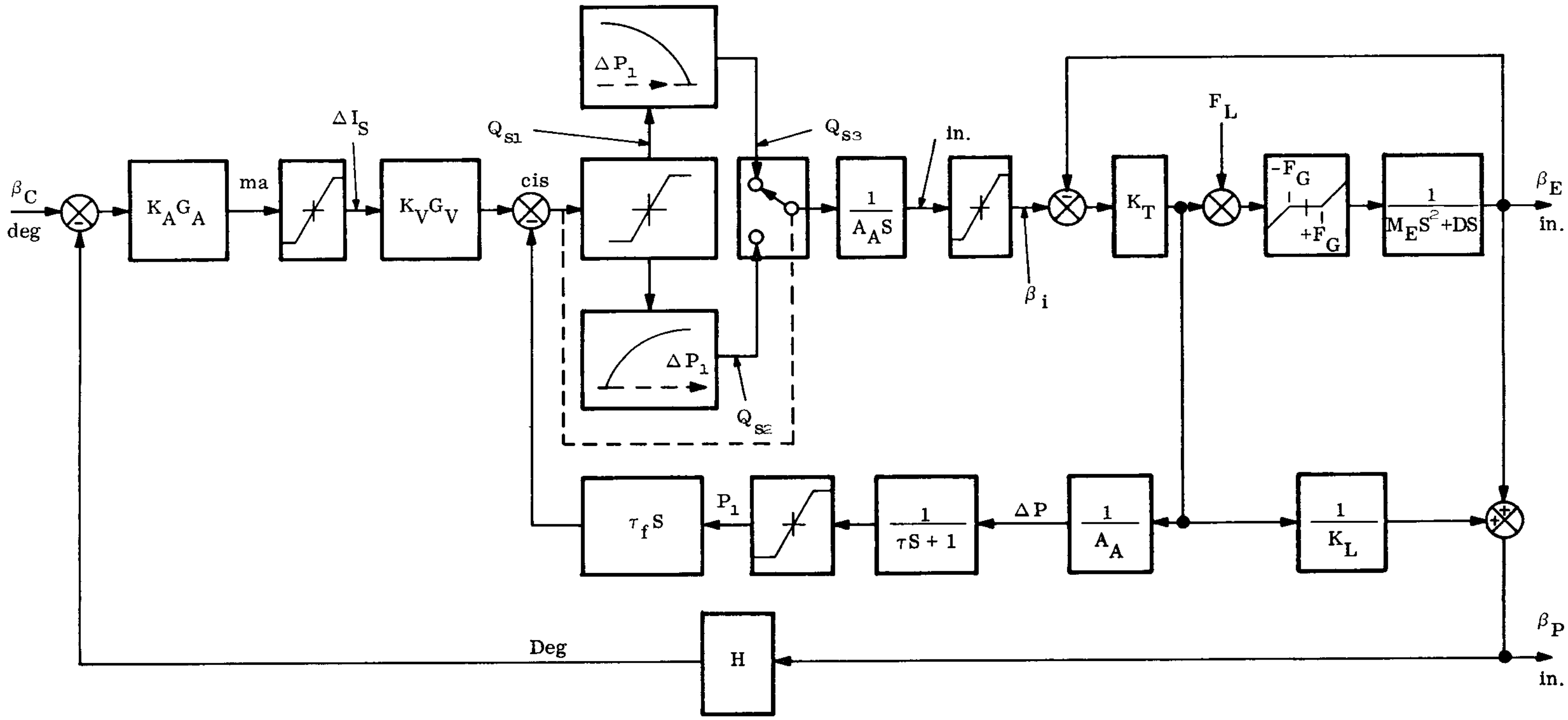
Using the technique of growing exponential signals, as it had been used to test the simpler transfer functions, led to the subtraction of large signals in the estimator. This resulted in errors in the prediction of the parameter changes. Therefore, a modification was used which allowed the inclusion of second order effects. An experimental design plan was conducted to express the sampled output of each orthogonal filter as a function of changes in the parameters of interest; that is, an equation of the form

$$\Phi = a_o + \sum_{n=1}^{4} \left(a_n \Delta P_n + b_n \Delta P_n^{2} \right) + E \tag{15}$$

where ΔP represents the parameter changes of interest. The a and b coefficients are determined from the experimental design data and E is an error term which expresses the result of the neglected higher order terms and cross product terms. The experimental design plan which was performed is discussed in the Phase C report (Ref. 4). The single parameter testing results obtained showed that changes in all four parameters could be estimated with an average error of about 3 percent when the magnitude of a parameter error (or errors) was less than 10 percent. When parameter errors were greater than 10 percent, the prediction accuracy decreased.

The single parameter testing technique of using transient response sampling was also applied to the linear sixth order model of the thrust vector control system. The test signal for this technique was formed by recording the impulse response of the nominal system on tape and then reversing the tape end-for-end. The parameter prediction results obtained were very close to the previous results obtained with the growing exponential signals. The changes in all four parameters could be estimated with an average error of about 4 percent when the magnitude of a parameter error (or errors) was less than 10 percent. When a parameter error was greater than 10 percent, the prediction accuracy decreased.

The time sampling single parameter testing technique was also used to test the complex nonlinear model of the thrust vector control system shown in Figure 3. The test signal was formed by recording the impulse response of the nominal system on tape



$K_A = 8$ ma/deg

$G_A = \dfrac{1}{\left[\dfrac{S^2}{(2\pi 43)^2} + \dfrac{2(0.7)S}{2\pi 43} + 1\right]}$

$K_V = 5.45$ cis/ma

$G_V = \dfrac{1}{\left(\dfrac{S}{2\pi 50} + 1\right)}$

$A_A = 5$ in.^{2}

$K_T = 57{,}910$ lb/in.

$K_L = 66{,}800$ lb/in.

$M_E = 16.1 \dfrac{\text{lb/sec}^2}{\text{in.}}$

$D = 34.25 \dfrac{\text{lb/sec}}{\text{in.}}$

$\tau_f = 6.37 \times 10^{-4}$ sec

$\tau = \dfrac{1}{2\pi(3)}$

$H = 2.092$ deg/in.

$P_S = 3{,}000$ psi (nominal)

$(\Delta I_S)_{max} = \pm 16$ ma

$(Q_{s1})_{max} = \pm 65$ cis

$(\beta_i)_{max} = \pm 3.82$ in.

$\dfrac{Q_{s2}}{Q_{s1}} = \left(\dfrac{P_S + \Delta P}{P_S}\right)^{\frac{1}{2}}$

$\dfrac{Q_{s3}}{Q_{s1}} = \left(\dfrac{P_S - \Delta P}{P_S}\right)^{\frac{1}{2}}$

$F_G = 1{,}800$ lb

$F_L = 7{,}700 + 15{,}000 (\sin 500)$

$-P_S < \Delta P_1 < +P_S$

$-1{,}750 < P_1 < +1{,}750$ psi

Figure 3. Thrust Vector Control System Nonlinear Block Diagram

and then reversing the tape end-for-end. The parameters which were measured with the testing setup are shown in Table 3. The results obtained with the test setup established that single parameter testing techniques can be used on nonlinear components and that at

Table 3. Parameters to be Measured

Parameter	Nominal Value
Servo-valve frequency (ω)	50 Hz
Servo-valve gain (K_V)	5.45 cis/ma
Load (M_E)	16.1 lb sec^2/in.
Nominal pressure (P_S)	3000 psi
Flow rate limit (Q_S)	±65 cis

least some nonlinearities can be measured. Certain nonlinearities such as the deadband and the 500 rad/sec sine wave in the load part of the nonlinear block diagram act like a gain change in the loop and, therefore, could not be distinguished from a gain change. However, the limiting action on the flow rate (+65 cis) and the current limitation of ±16 ma could be measured.

The range over which accurate parameter predictions can be made was approximately the same for the testing of both the linear and nonlinear model of the thrust vector control system. The accuracy of the parameter prediction was less for the nonlinear system. The reason for this was the complexity of the system model, however, and not the fact that the system contained nonlinearities. The system complexity led to problems in matching the nominal system to the actual system under zero parameter error conditions and problems in data repeatability.

SECTION III. REVIEW OF THE PHASE D SURVEY RESULTS

The objective of Phase D was to show the applicability of single parameter testing techniques to the components which the Electrical Test and Analysis Branch of the Reliability and Quality Assurance Laboratory is responsible for testing. The four groups within the branch are the Measuring, RF, Telemetry, and Networks Group. The complete results of this phase of the program are reported in Reference 6.

Many of the components that the Measuring Group tests can be considered for single parameter testing. Among these components are:

1. Accelerometers.
2. DC amplifiers.
3. Emitter followers
4. Pressure transducers
5. Rate gyroscopes.
6. Turbine tachometer.
7. First motion and cutoff module.
8. Sync buffer unit.
9. Frame rate and frequency to DC converter.
10. Voltage inverter.
11. Frequency to DC converter.

The information presented in Reference 6 for each category of components includes a general description, acceptance level testing, final checkout, and calibration level testing, tests that single parameter testing techniques can perform, how these tests are now performed, and a list of the documents which were examined.

In general, the information that could be obtained using single parameter testing techniques for the preceding components would be the transfer function parameters from which the frequency response can be determined, plus the added data of what part of the component is in an off-nominal condition. This information would be obtained using just one input testing signal as opposed to 15-to-20 sequentially applied sinusoidal test signals. Using a growing exponential signal as suggested in Reference 11, information concerning the linearity of the component could be obtained with just one testing signal.

With the time and information available during Phase D, a complete study of the selected components could not be performed and the technical and economic considerations to be given in Section VI need to be checked before finally selecting a component for single parameter testing.

Single parameter testing techniques are also applicable to several other classes of equipment such as hydraulic, pneumatic, and other electrical components as well as systems involving these components. Examples of hydraulic components that can be considered as valves, pumps, and motors. Pneumatic components that can be evaluated for single parameter testing are valves, rotary actuators, and controllers. Some examples of electrical components are motors, generators, amplifiers, rotating amplifiers, and filters.

SECTION IV. AC AMPLIFIER TESTING

A. GENERAL

During Phase E, the transient response sampling technique was applied to the testing of an AC amplifier. This type of amplifier is used as a signal processor between the space vehicle transducers and the telemetry equipment. The objective was to measure the amplifier frequency response and the amplifier linearity. The AC amplifier passband is between 50 and 3,000 Hz.

B. MODELING THE AC AMPLIFIER

The first task in setting up a single parameter test for a component is determining an accurate nominal model. The first approach that was used for the AC amplifier was by taking frequency response data of the AC amplifier that was available. The phase shift and amplitude gain versus frequency data was then used as input to the transfer function determination program which was described in the Phase E Quarterly Report (Ref. 7). The output of this program is the transfer function that gives the best fit to the input data using a least squares criterion.

The second approach was a theoretical circuit analysis using schematic drawing No. 50M04426, Revision A, dated 14 October 1965. The circuit was divided into stages that could be more easily analyzed to obtain transfer functions. In order to obtain numerical values in the overall transfer function, several assumptions were necessary. These assumptions were:

1. The transistors can be represented by either an ideal voltage source equal to $\beta R_E I$ in the case of an emitter-follower or an ideal current source equal to βI_{in} for the case of a grounded emitter amplifier.

2. The transistor forward current gain (β) was assumed to be 150 for all transistors over the operating range.

3. The nominal component values were used in the calculations. Some of the component tolerances are ±10 percent.

4. The two coil resistances in the third stage were assumed to be 1000 ohms.

5. Because of the way the amplifier has been divided into stages some small loading effects have been neglected.

6. The compensating resistor used in the third and fourth stage calculations as measured on the amplifier was 320 ohms.

Because of the assumptions necessary to obtain numerical values, it is estimated that the calculated pole and zero locations are accurate within 5 to 10 percent. The amplifier circuit analysis was divided into six stages with each division occurring at a transistor. This individual stage analysis is found in Reference 7. The composite transfer function is the product of each of the stage transfer functions and is equal to

$$G(S) = \frac{KS^4 (S + 35.1)(S + 285.4)(S^2 + 149.5S + 2.27 \times 10^4) H(S)}{(S + 0.123)(S + 1.25)(S + 1.77)(S + 1.96)(S + 50.1)(S + 185.4)}$$

$$\times \frac{1}{(S + 3.65 \times 10^4)(S^2 + 245.7S + 5.1 \times 10^4)} \tag{16}$$

$$\times \frac{1}{(S^2 + 3.11 \times 10^4 S + 896 \times 10^6)(S^2 + 6.02 \times 10^4 S + 7.27 \times 10^9)}$$

where K is the overall amplifier midband gain and H(S) is the transfer function contributed by the sixth stage. For stage six not enough information was known about the transformer to substitute circuit parameter values into the general stage transfer function. The similar frequency response of the theoretical transfer function and the frequency response determined in the experimental analysis indicate that the assumptions and simplifications in determining the transfer function are valid within the region of interest.

C. TEST IMPLEMENTATION

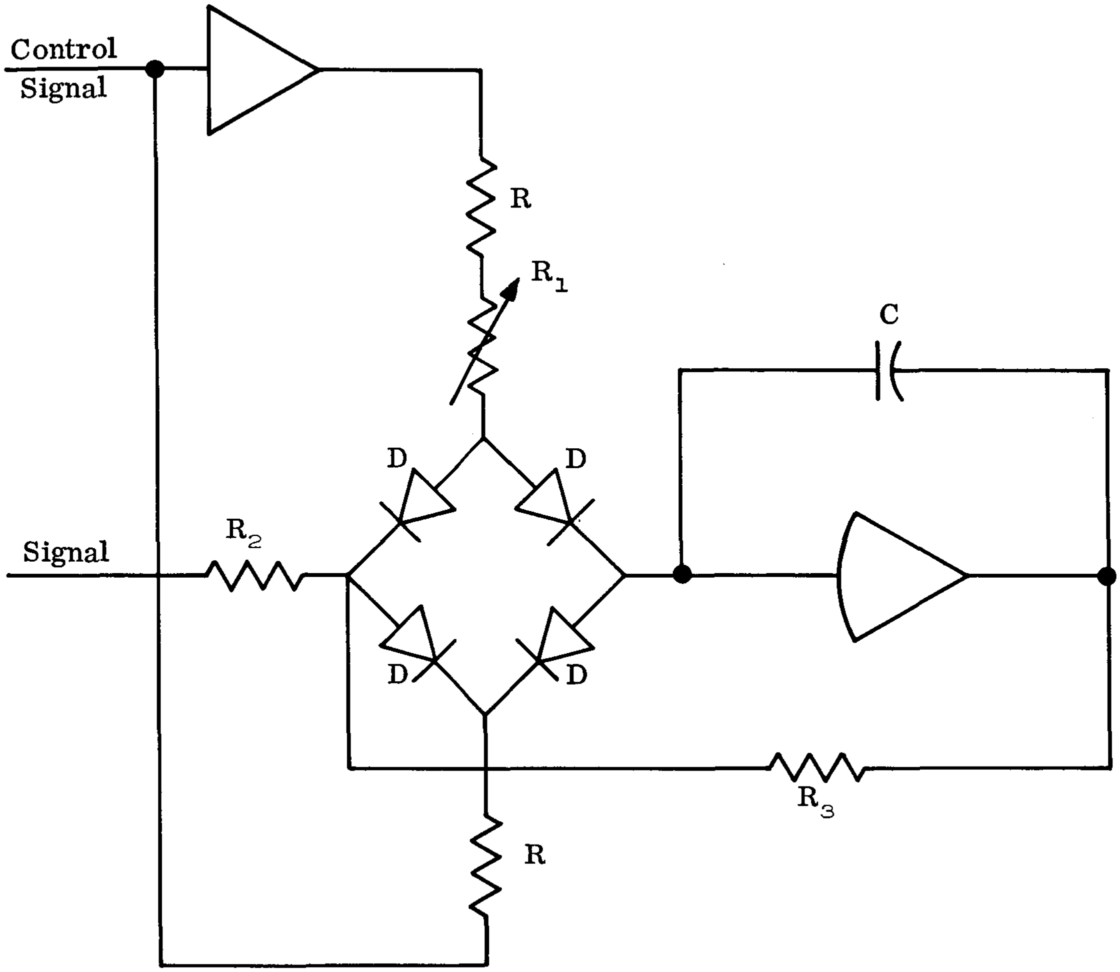
The transient response sampling technique was implemented with the AC amplifier by using an analog computing device and digital logic cards. The test signal used, which was generated on the digital logic cards, was one cycle of a square wave lasting 100 microseconds. The timing control for the test was also generated with the logic cards. One timing control step function was generated for each of the eight track and hold circuits. These circuits, as shown in Figure 4, were constructed using an operational amplifier and diode switching to perform the amplifier mode control between operate and hold.

The AC amplifier model was implemented using passive R, L, C components. The component values were selected to correspond to the poles and zeros in Equation 16. The variable passive component values were then adjusted to optimize the match with the AC amplifier. The estimator coefficients were determined by varying the passive component parameter known amounts and recording the error waveform values at each sample time. The coefficients were calculated from this data using the technique shown in paragraph C of Section I. These estimator coefficients were then programmed on an analog computing device.

Thus, the implementation consists basically of the digital logic cards and the analog computing device. In addition, an oscillator, power supply, and oscilloscope, all of which are normally available, are required.

D. LINEAR TESTING RESULTS

Several attempts were made but the AC amplifier transfer function parameters could not be predicted using the test implementation. To confirm these results the amplifier transfer function was studied on the analog computer using a low frequency model. To simplify the mathematical transfer function further, this function was divided into a lowpass filter containing five poles, which represent the upper end of the passband, and a highpass filter containing three poles and three zeros, which represent the lower end of the passband. A single parameter test was then developed for each model. The results obtained for each of these test cases confirmed the AC amplifier parameter prediction problem.



Track Time Constant = R_3C	Control Signal ± 10 volts	R_1 = 1K
Gain = R_3/R_2	R = 4.7K	D = 1N459

Figure 4. Track and Hold Circuit

The problem is that there are too many poles and zeros and that these are too close together to be separable. At the upper end of the passband there are five poles within an octave of frequency. At the lower end of the passband there are three poles and three zeros all within an octave of frequency. This problem has been discussed with several people who have been involved with the identification problem and it would seem that any identification technique would have a great deal of trouble separating the effects of the particular poles and zeros associated with the AC amplifier.

E. NONLINEAR TESTING RESULTS

The implementation setup to perform the nonlinearity testing is shown in Figure 5. The growing exponential signal which is generated by the circuit shown in Figure 6

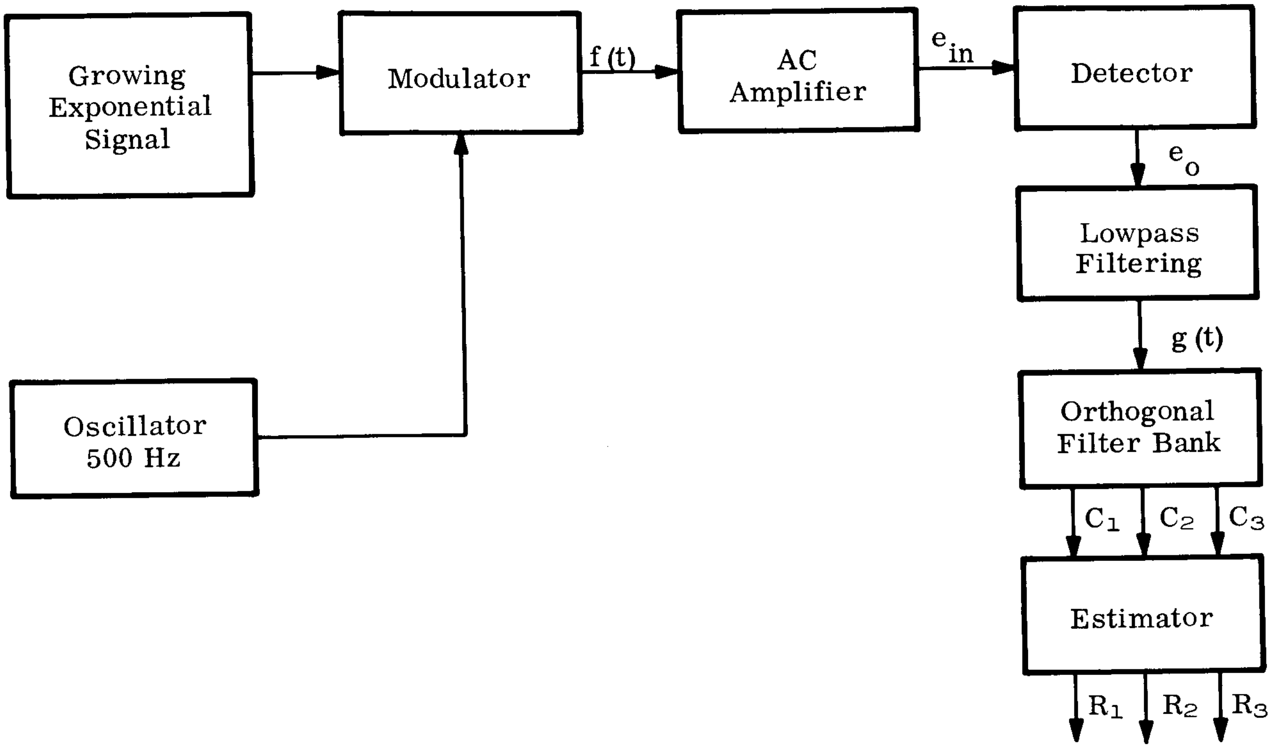


Figure 5. Linearity Testing of the AC Amplifier

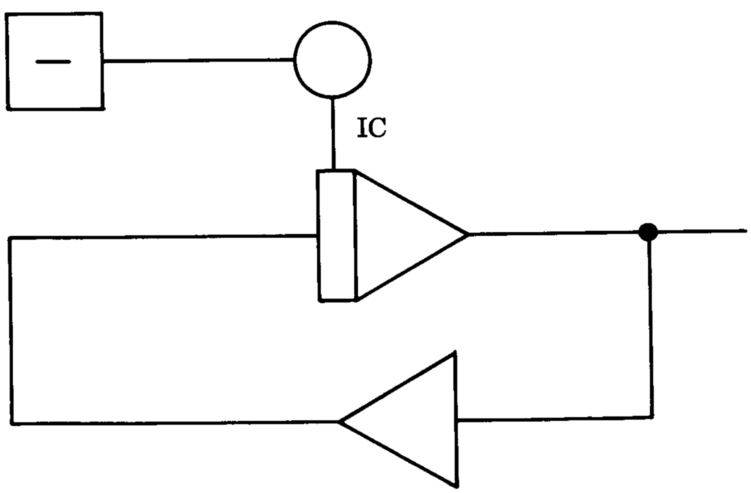


Figure 6. Growing Exponential Signal Generator

is modulated onto a 500-Hz sine wave to form the input testing signal. Then, this signal can be expressed

$$
\begin{aligned}
f(t) &= A \exp(t - 6) \sin(1000\pi) t, \qquad 0 \le t \le 6 \text{ sec} \\
f(t) &= 0 \qquad\qquad\qquad\qquad\qquad\quad 6 \le t
\end{aligned}
\tag{17}
$$

Thus, the linearity of the amplifier is being measured at just the frequency of 500 Hz and the amplifier highpass and lowpass characteristics will not affect the demodulated growing exponential test signal. The detector output is

$$
\begin{aligned}
e_o &= e_{in}, \qquad e_{in} > 0 \\
e_o &= 0 \qquad\quad e_{in} \le 0
\end{aligned}
\tag{18}
$$

or

$$
\begin{aligned}
e_o &= 0, \qquad e_{in} > 0 \\
e_o &= e_{in} \qquad e_{in} \le 0
\end{aligned}
\tag{19}
$$

depending upon the part of the linearity characteristic that it is desired to measure. The lowpass filtering was selected as two RC filter stages each having a 100-rad/second bandwidth to eliminate the carrier frequency. The orthogonal filter bank consists of three filters with the following transfer functions

$$
\begin{aligned}
\frac{C_1(S)}{g(S)} &= \frac{\sqrt{2}}{S + 1} \\
\frac{C_2(S)}{C_1(S)} &= \sqrt{2}\,\frac{S - 1}{S + 2} \\
\frac{C_3(S)}{C_2(S)} &= \sqrt{1.5}\,\frac{S - 2}{S + 3}
\end{aligned}
\tag{20}
$$

The equations which the estimator solves are

$$
\begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} = \begin{bmatrix} \sqrt{2} & -4 & 3\sqrt{6} \\ 0 & 6 & -12\sqrt{6} \\ 0 & 0 & 10\sqrt{6} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}
\tag{21}
$$

Further discussion of the theory of this technique and example results can be found in References 7 and 11. Example testing results of the linearity of the AC amplifier are given in Figures 7 and 8. The data given in each figure is for the positive side of the AC amplifier transfer characteristic, that is, the positive part of the signal is detected at the amplifier output. The arm of the X-Y plotter is driven by the growing exponential signal for both figures. The linear term (R_1), second order term (R_2), and third order term (R_3) are plotted for different ranges of amplifier input values in the two figures. Note the high degree of linearity shown in Figure 7, and where the limiting starts at about an input signal level of 0.15 volts in Figure 8. The amplifier output can be expressed as the sum of the three signals R_1, R_2, and R_3 where these three signals are such that this weighted mean-square error between this sum and the output is minimized. Thus, this technique can be used to obtain a measure of the linearity of the amplifier transfer characteristic at a particular frequency.

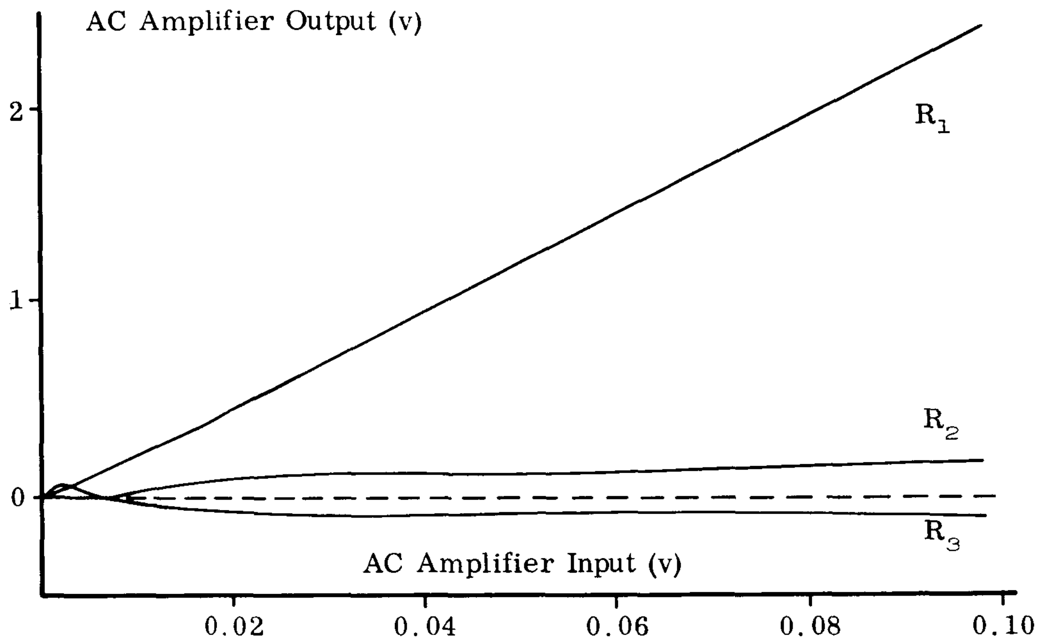


Figure 7. AC Amplifier Linearity

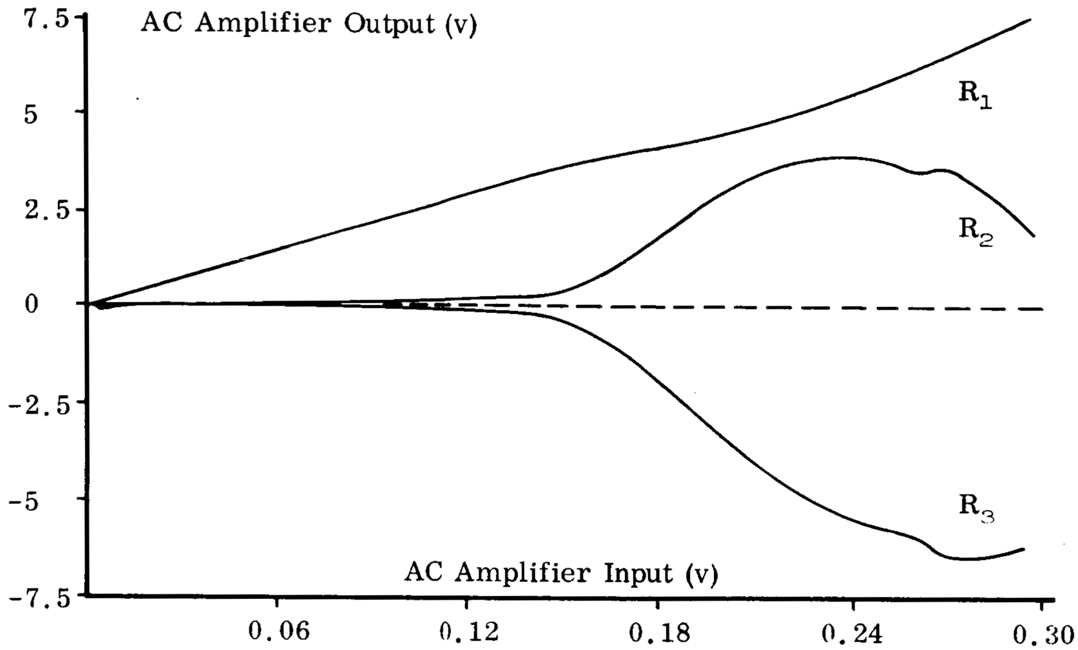


Figure 8. AC Amplifier Linearity

SECTION V. DC AMPLIFIER TESTING

A. GENERAL

During Phase E, the transient response sampling technique was applied to the testing of a DC amplifier. This type of amplifier is used as a signal processor between the space vehicle transducers and the telemetry equipment. The amplifier characteristics to be measured are the frequency response and the amplifier linearity. The DC amplifier passband is between 0 and 40 Hz.

B. MODELING THE DC AMPLIFIER

The approach used to determine the DC amplifier model was to take frequency response data on the DC amplifier. This phase shift and amplitude gain versus frequency data was then used as input to the transfer function determination program which was described in the Phase E Quarterly Report (Ref. 7). The output of this program is the transfer function that gives the best fit to the input data using a least squares criterion. The results indicated a second order transfer function of

$$G(S) = \frac{K}{1 + \frac{2\zeta}{\omega_o} S + \left(\frac{S}{\omega_o}\right)^2} = \frac{5}{1 + \frac{2(0.695)}{228} S + \left(\frac{S}{228}\right)^2} . \qquad (22)$$

C. TEST IMPLEMENTATION

The transient response sampling technique was implemented with the DC amplifier using an analog computer. The test signal generation and track and hold circuits were built using the computer comparators. The test signal used was an eight-millisecond pulse. The amplifier model could be adjusted to obtain 40-db cancellation with the DC amplifier.

The estimator coefficients were determined by varying the model parameters known amounts and recording the error waveform values at each sample time. Then, the coefficients were calculated from this data using the technique shown in paragraph C of Section I. These estimator coefficients were programmed on the analog computing device.

D. LINEAR TESTING RESULTS

In order to determine the range and accuracy of the estimator measurements, parameter variations were made on the amplifier model. Thus, the DC amplifier actually served the role of the nominal component while the amplifier model was the component under test.

Table 4 shows sample results obtained of parameter predictions. The first three columns are the actual variations of the three parameters (K, ζ, ω) as programmed on the computer model. The final three columns are the parameter prediction outputs of the estimator. The results show that the parameter prediction accuracy is quite good for any combination of the parameters varying up to changes of ±20 percent.

Table 4. DC Amplifier Testing Results

Actual Parameter Variation %			Parameter Prediction %		
K	ζ	ω	K	ζ	ω
0	0	0	0.1	0	-0.4
0	0	0	0	-0.3	-0.5
10	0	0	10	0	0
20	0	0	20	0	0
40	0	0	40	0	0
-10	0	0	-10	0	0
-20	0	0	-20	1	1
-40	0	0	-40	2	2
0	10	0	0	9	0
0	20	0	0	16	0
0	40	0	-3	26	0
0	-10	0	0	-11	0
0	-20	0	-1	-25	-1
0	-40	0	-6	-68	-5
0	0	10	0	0	10
0	0	20	0	3	22
0	0	40	-1	17	49
0	0	-10	0	1	-9
0	0	-20	0	5	-16
0	0	-40	-5	18	-24
0	-20	-20	1	-17	-23
0	-20	20	-4	-19	28
0	20	20	0	18	18
0	20	-20	-3	20	-11
-20	0	-20	-20	6	-11
-20	0	20	-20	3	18
20	0	-20	19	5	-20
20	0	20	19	6	28
20	20	0	18	18	0
20	-20	0	18	-30	0
-20	-20	0	-20	-20	0
-20	20	0	-20	14	1
-20	20	20	-19	16	15
-20	20	-20	-23	20	-5
-20	-20	-20	-18	-13	-17
-20	-20	20	-23	-16	22
20	-20	20	15	-20	36
20	-20	-20	22	-22	-28
20	20	-20	15	22	-15
20	20	20	19	22	21
10	10	10	10	10	10
10	10	-10	9	11	-9
10	-10	-10	10	-10	-12
10	-10	10	9	-10	13
-10	-10	10	-10	-8	11
-10	-10	-10	-9	-8	-9
-10	10	-10	-10	10	-6
-10	10	10	-9	9	9

Another approach to verify the DC amplifier single parameter test was to use the developed estimator to predict the parameters of five DC amplifiers that were available. The amplifier parameter predictions are given in Table 5. Figures 9 through 13 show the predicted transfer function plotted as a smooth line for each amplifier. To verify that these predicted transfer functions do represent the particular amplifiers, experimental frequency response data was taken and is plotted on each figure as circles. The close correlation between the experimental data and the predicted transfer function shows that the single parameter test can be used to predict the DC amplifier frequency response.

Table 5. DC Amplifier Predicted Parameter Changes

DC Amplifier Number	Predicted Parameter Change		
	K	ζ	ω
1	2	-15	-18
2	1	2	9
3	1	2	5
4	1	-4	-2
5	0	0	0

E. NONLINEAR TESTING RESULTS

The implementation of the nonlinear test for the DC amplifier was identical to that used for the AC amplifier (see Figure 5), except that the modulation process was not necessary; that is, the growing exponential signal was fed directly into the DC amplifier and the amplifier output was fed directly to the orthogonal filter bank.

Sample testing results of the linearity of the DC amplifier are given in Figures 14 and 15. Figure 14 is for positive inputs and Figure 15 is for the negative inputs to the amplifier. The arm of the X-Y plotter is driven by the growing exponential signal for both figures. The linear term (R_1), second order term (R_2), and third order term (R_3) are plotted as a function of the amplifier input value in the two figures. Note the high degree of linearity shown in Figure 14, up to where the limiting starts at about an input signal level of 1.36 volts. In Figure 15 the amplifier output is linear down to input levels of -0.15 volts and then the limiting starts.

The amplifier output can be expressed as the sum of the three signals R_1, R_2, and R_3 where these three signals are such that the weighted mean-square error between this sum and the output is minimized. Thus, this technique can be used to obtain a measure of the linearity of the amplifier transfer characteristic.

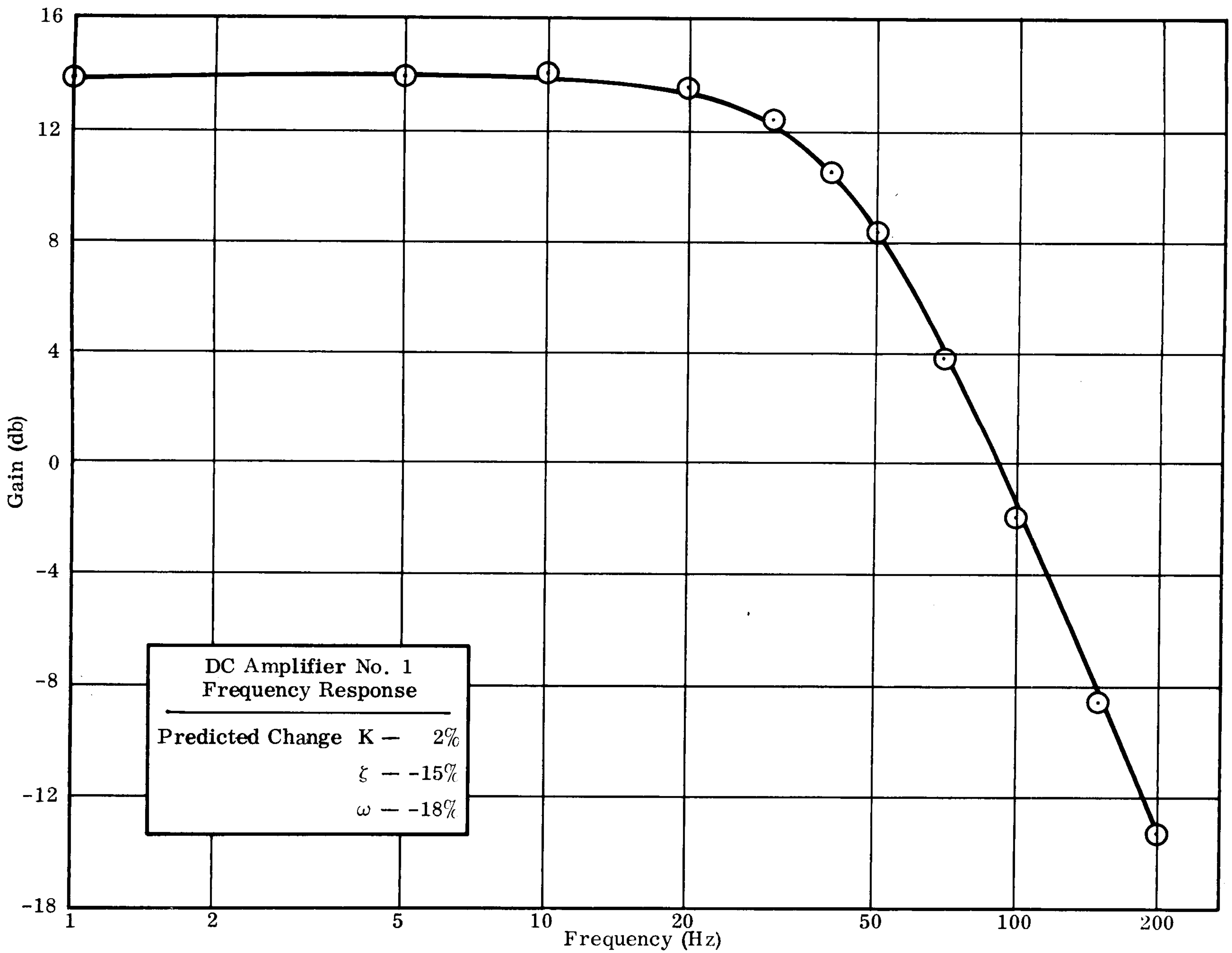


Figure 9. DC Amplifier No. 1

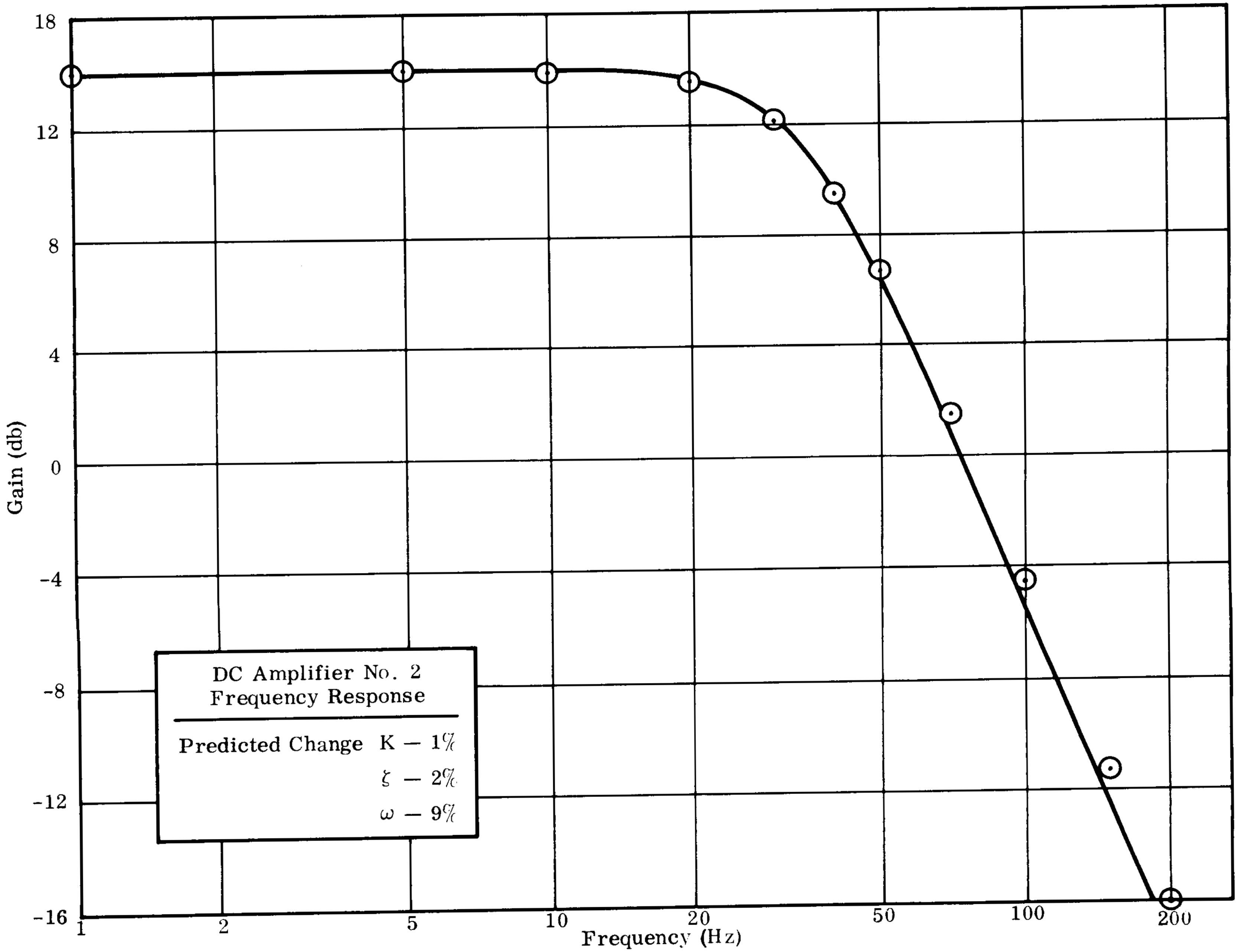


Figure 10. DC Amplifier No. 2

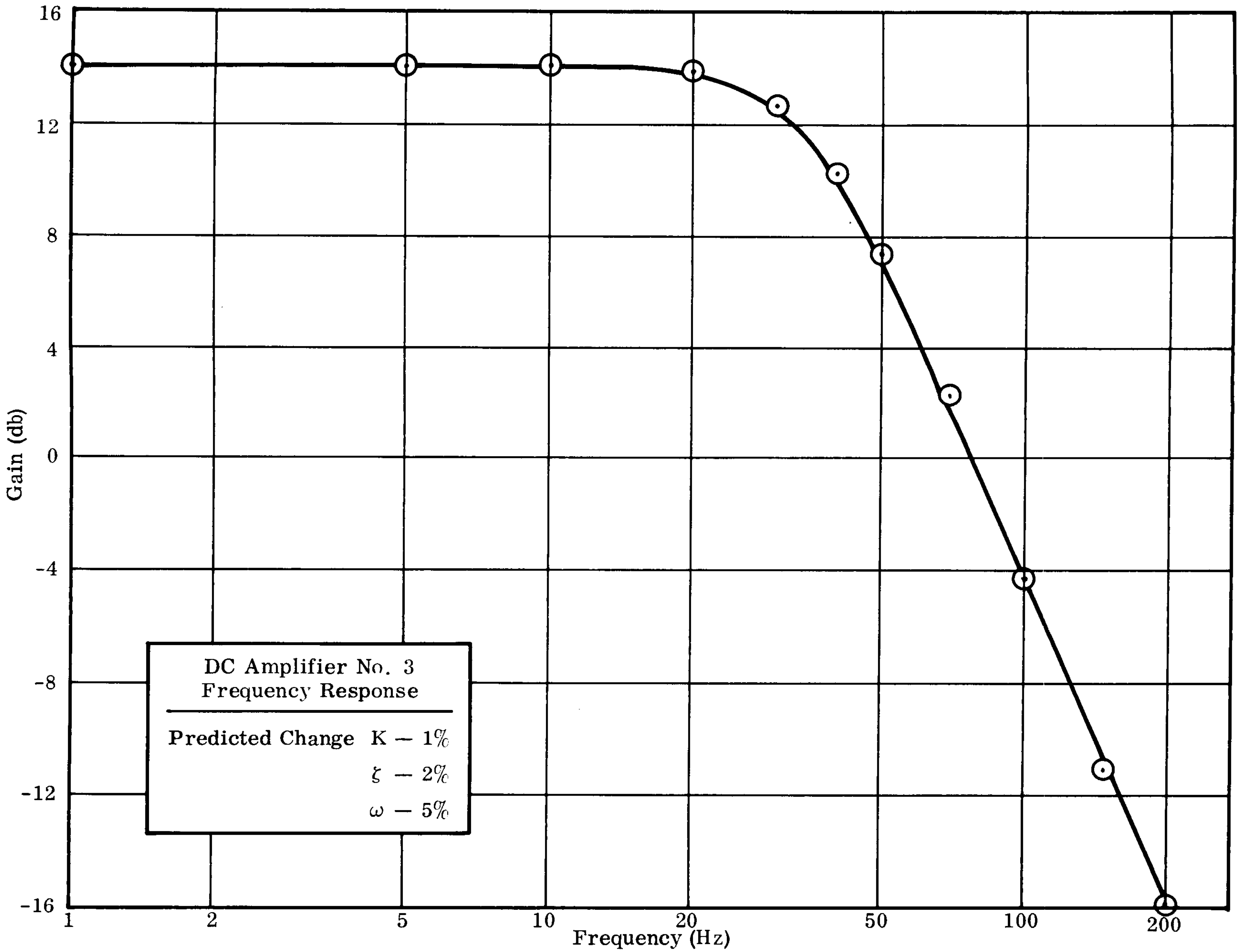


Figure 11. DC Amplifier No. 3

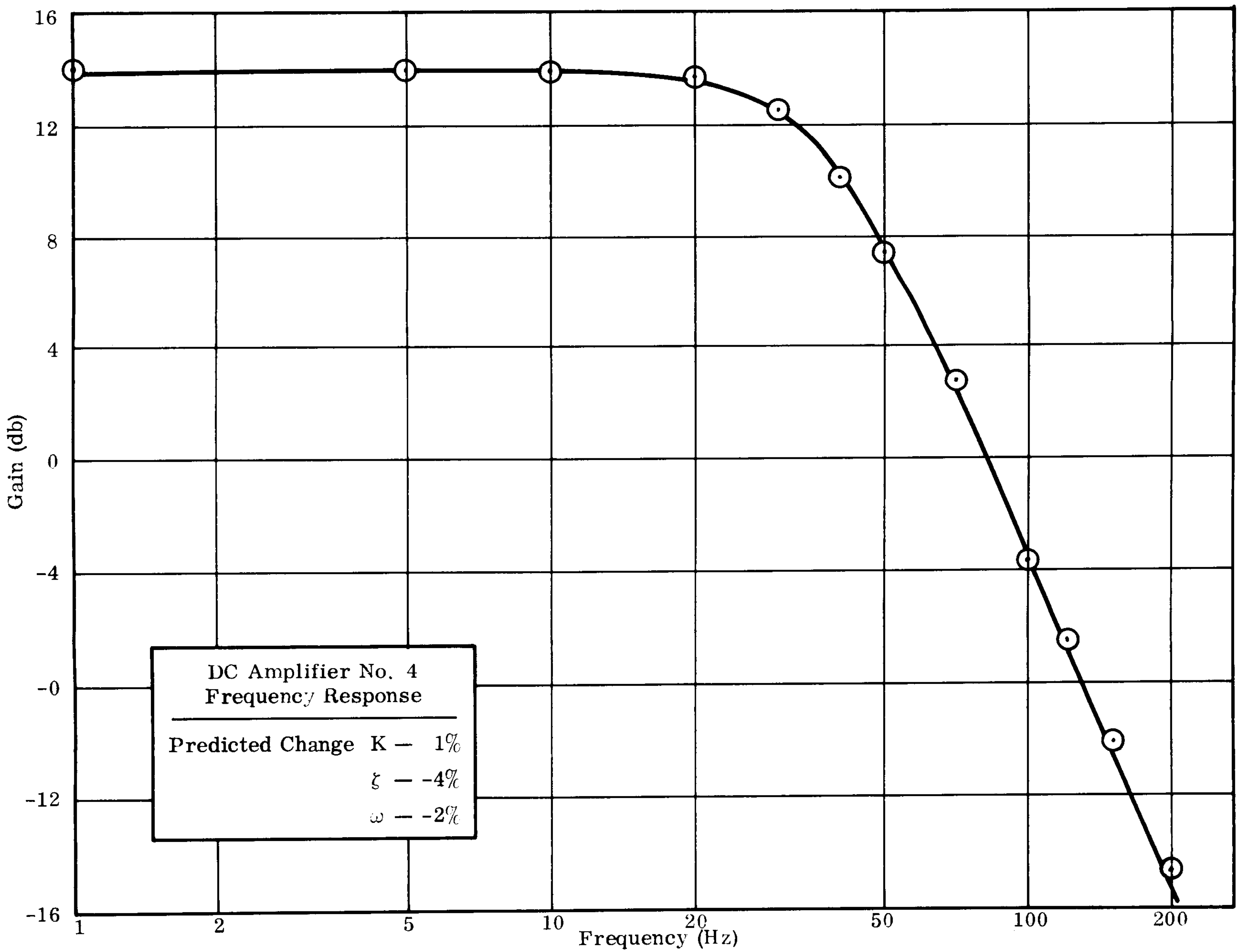


Figure 12. DC Amplifier No. 4

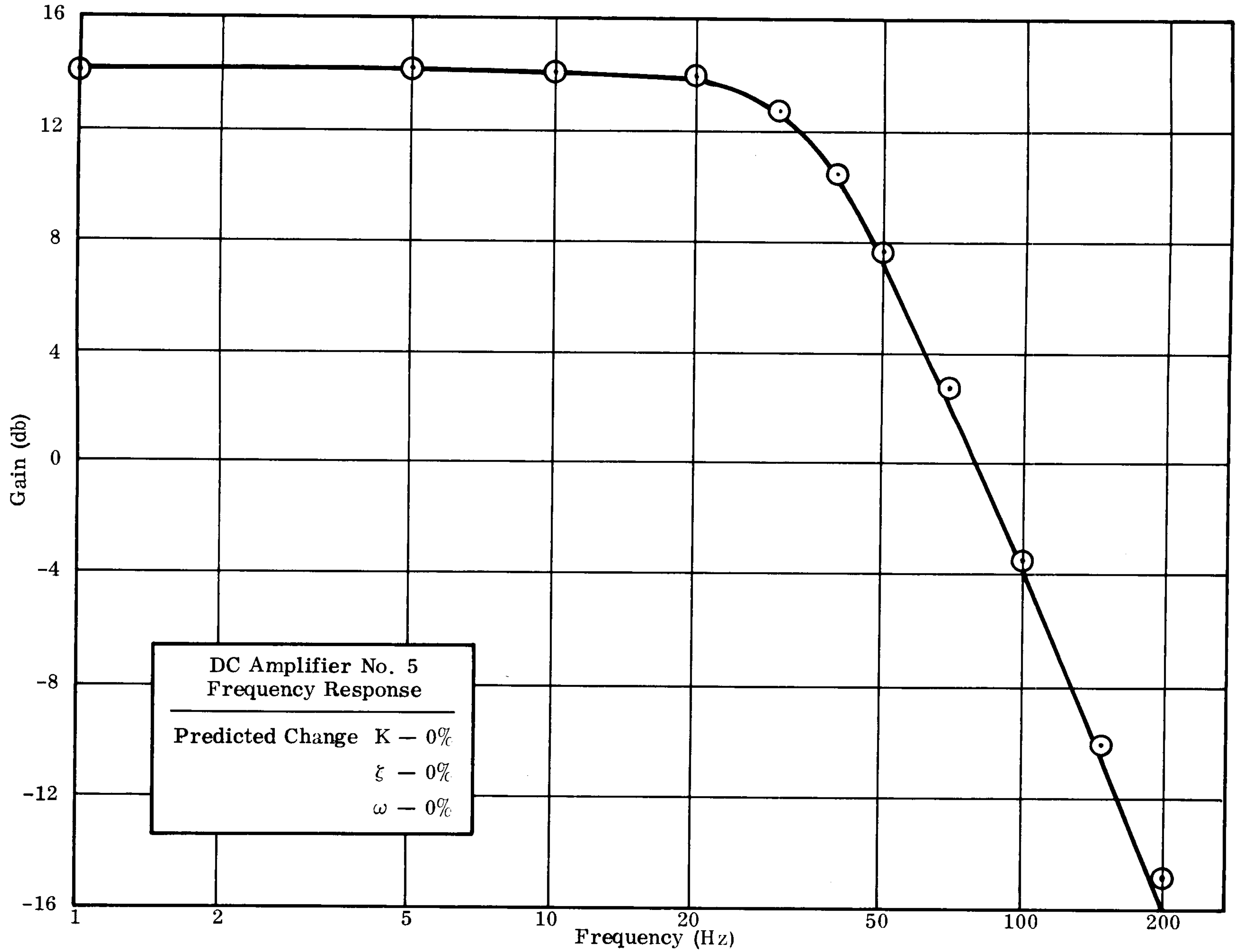


Figure 13. DC Amplifier No. 5

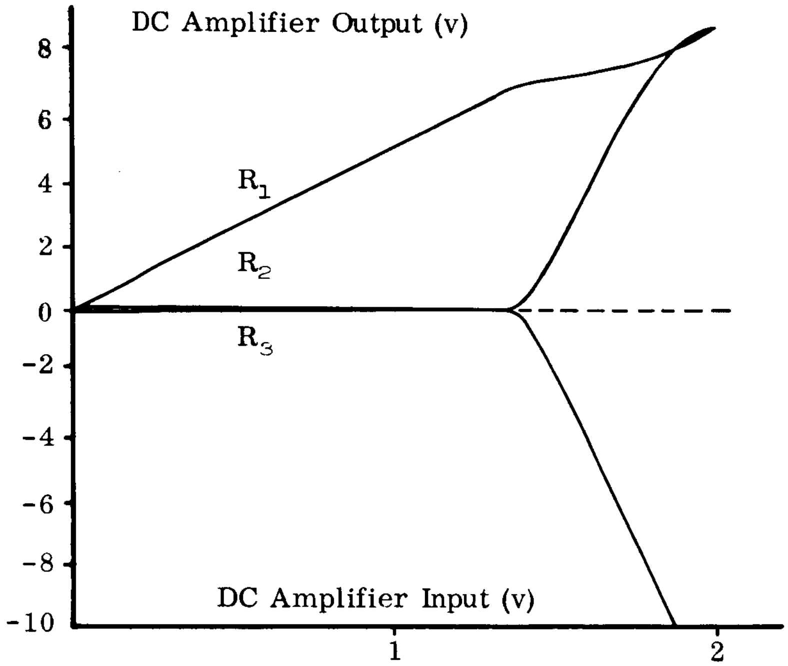


Figure 14. DC Amplifier Linearity

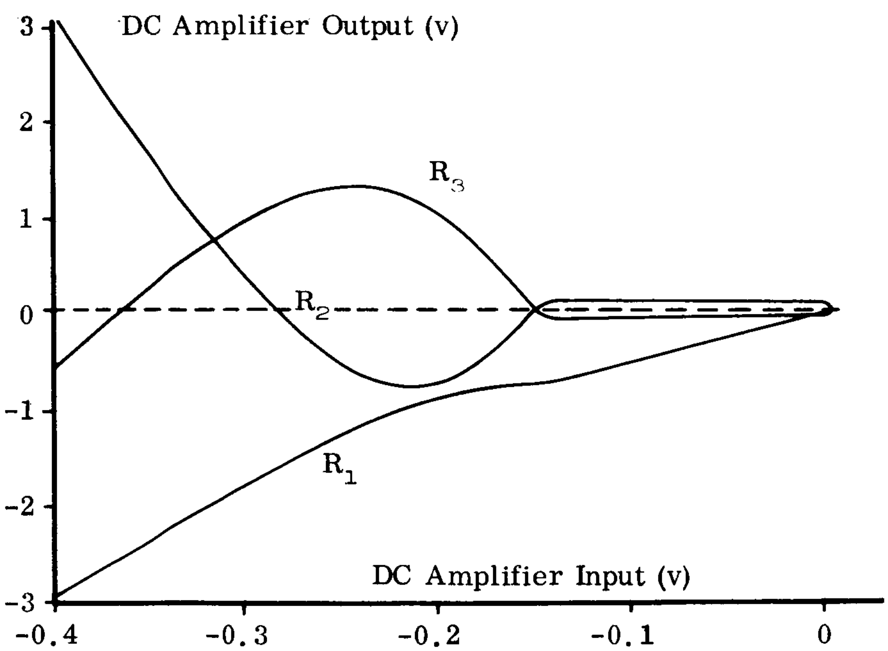


Figure 15. DC Amplifier Linearity

SECTION VI. DETERMINING THE APPLICABILITY OF SINGLE PARAMETER TESTING FOR A COMPONENT

A. GENERAL

In this section a summary will be made of factors to be considered for selecting components for single parameter testing and a description of the equipment needed for implementation. The steps needed to implement the single parameter testing technique using transient response sampling were discussed in Section II. They consist basically of developing an accurate component model, selecting sample times, taking data to determine an estimator, and checking out the final implementation. How these steps were carried out for the AC and DC amplifiers is presented in Sections IV and V.

B. TECHNICAL REQUIREMENTS

As a result of this study the following technical requirements have been found for selecting a component for single parameter testing:

1. The component transfer function is one of the classes shown in Table 2. Components with more complicated transfer functions can be considered if the extra poles are far away from the poles of interest such that they do not affect the response, or if it can be assumed that the extra poles cannot vary and, therefore, do not need to be tested. More complicated systems can also be handled by considering them as subsystems, each of which is simple enough to be tested by itself, and providing a separate testing input and output connection for each subsystem. This reduces the complex system into subsystems that have transfer functions that can be single parameter tested.

2. Table 2 defines the range over which the parameters can vary and the parameter prediction accuracy that can be obtained.

3. If a component has a nonlinearity which must also be measured, then this reduces the number of other parameters that can be simultaneously tested. It may be possible to select a test signal to separately test the linear and nonlinear regions of the component. This is true in the case of an amplifier that exhibits limiting when the input signal amplitude exceeds some value.

4. The component to be tested must have a dynamic range of at least 34 db.

5. The testing equipment must be compatible with the component input/output signals in amplitude and frequency content.

6. It must be possible to obtain an accurate nominal model of the component. This may be difficult for some components, in particular for some hydraulic and pneumatic components.

C. COST EFFECTIVENESS CONSIDERATIONS

The cost of developing a single parameter test for a given component will depend upon a number of the following considerations:

1. Testing level—the cost effectiveness of the single parameter test will be a function of the component information desired and the testing level at which it is applied. Examples of testing levels are:

   a. Design and development testing.

   b. Qualification testing.

   c. Reproduction acceptance testing.

   d. Final checkout and calibration testing.

   e. Operational checkout testing.

   f. Troubleshooting and maintenance testing.

Information about individual parameters may not be required at all testing levels. Once a single parameter test technique is developed for application at one testing level, it could then be used at a subsequent level to obtain increased information.

2. Number of identical components to be tested.

3. Number of data points needed to determine a parameter. For example, a frequency response normally may require data to be taken at many test points. Thus, a single parameter test technique may achieve a large time savings.

4. Test development effort.

   a. Is an engineer available to develop the component single parameter test?

   b. Is an accurate nominal component model available or can it be determined?

   c. Will extra equipment be needed to single parameter test this component?

5. Free information—a single parameter testing technique may give additional parameter information with no increase in testing time.

The preceding items are representative of some of the factors that need to be considered in doing a cost effectiveness evaluation of the development of a single parameter test for a component. Basically, the choice is an evaluation of the testing time savings and increased parameter information obtained versus the test development and test equipment cost.

D. IMPLEMENTATION OF SINGLE PARAMETER TESTING

The implementation of a single parameter test will be illustrated by describing the equipment needed to test a component which has the following characteristics:

1. Electrical input and output.

2. Frequency range, 0 to 40 Kc.

3. Amplitude range, ±10 volts.

The implementation for a nonelectrical component would differ only in the equipment needed to develop the test signal and the sensor needed to convert the component output into an electrical signal. The block diagram of the test setup is shown in Figure 2. The test signal can be generated using digital logic cards. These cards can also be used to perform the timing control function. The digital logic circuits recommended would be:

1. Twelve flip-flop circuits.

2. Twelve adjustable delay multivibrator circuits.

3. Twelve power amplifier circuits.

4. Four NAND gate circuits

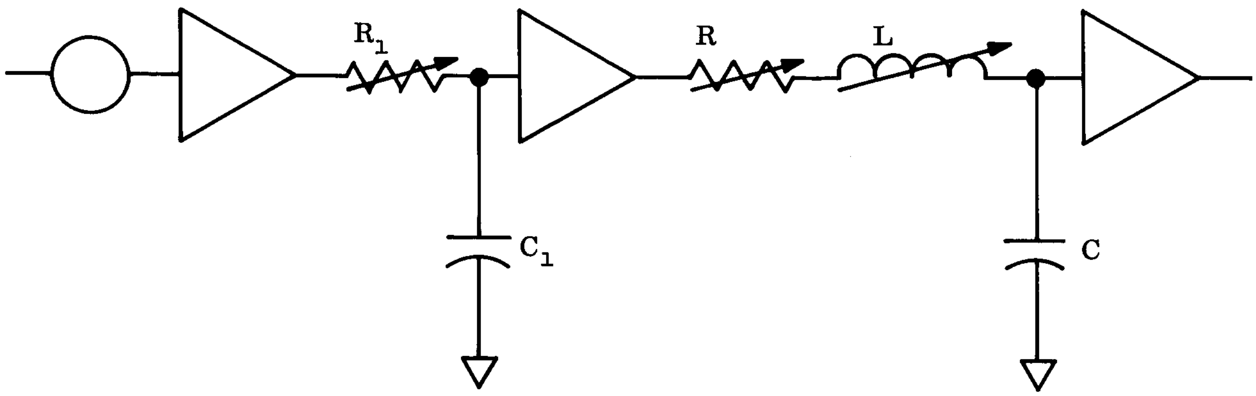
5. Ten variable capacitors.

The digital equipment including a rack to hold the cards and power supply would cost about $3,500.

The nominal system could be an actual system or a model of the system built with passive components such as the example shown in Figure 16. The passive components cost relatively little and the operational amplifiers would be part of the analog computing device to be discussed.

The track and hold circuits can each be constructed using an operational amplifier and diode switching to perform mode control. The cost of the diodes and other passive components to perform the track and hold function would be less than $100. The amplifiers are part of an analog computing device which also has programmed on it the estimator. This analog computing device consists of 40 operational amplifiers and 40 potentiometers that would cost less than $25,000 including power supply.

Basically, the implementation consists of the digital logic cards and the small analog computing device. In addition, an oscillator, power supply, and oscilloscope which are normally available would be required. Further details on the implementation are contained in Section IV where the AC amplifier testing is discussed.



$$H(S) = \frac{K}{(S + \omega_1)(S^2 + 2\zeta\omega S + \omega^2)}$$

$$\omega_1 = \frac{1}{R_1 C_1}, \qquad \omega_2^2 = \frac{1}{LC}, \qquad \zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$$

Figure 16. Example Component Model

SECTION VII. CONCLUSIONS

The objective of the single parameter testing program has been to investigate techniques for determining deviations from the nominal values of the parameters of a component. These techniques have been evaluated using linear and nonlinear models to establish the range over which accurate parameter predictions can be made. The transient response sampling technique was applied successfully to the testing of a DC amplifier during the final phase of the program.

The basic conclusion is that single parameter testing techniques can be applied successfully to some components when using the technical and economic considerations described for the selection of a component to be tested.

REFERENCES

1. Berger, Eugene L. and Jackson, James C., Single Parameter Testing, Phase A Report, ASD, General Electric Company, Daytona Beach, Florida, November 1964.

2. Berger, Eugene L. and Jackson, James C., Single Parameter Testing, Phase B Report, ASD, General Electric Company, Daytona Beach, Florida, January 1965.

3. Berger, Eugene L. and Jackson, James C., Single Parameter Testing, Phase C Quarterly Report, ASD, General Electric Company, Daytona Beach, Florida, April 1965.

4. Berger, Eugene L., Jackson, James C. and Sterling, John T., Single Parameter Testing, Final Report for Phases A, B, and C, ASD, General Electric Company, Daytona Beach, Florida, August 1965.

5. Sterling, John T., Single Parameter Testing, Final Report Addendum, ASD, General Electric Company, Daytona Beach, Florida, September 1965.

6. Berger, Eugene L., Grunden, Charles A. and Sterling, John T., Single Parameter Testing, Phase D Report, ASD, General Electric Company, Daytona Beach, Florida, April 1966.

7. Berger, Eugene L., Grunden, Charles A. and Sterling, John T., Single Parameter Testing, Phase E Quarterly Report, ASD, General Electric Company, Daytona Beach, Florida, July 1966.

8. Litman S. and Huggins, W.H., Growing Exponentials as a Probing Signal for System Identification, Proceedings of the IEEE, pp. 917-923, June 1963.

9. Chorzel, J., Thompson, J. and Myers, R., System Parameter Measurement Using Transient Response Sampling, Symposium Proceedings of the Automatic Support Systems for Advanced Maintainability Symposium, St. Louis, 1965.

10. Moore, F.B., SAT-IB Engine Positioning Control Systems Data, MSFC R-ASTR-NFM-128-65, March 1965.

11. Lory, H.J., Lai, D.C. and Huggins, W.H., On the Use of Growing Harmonic Exponentials to Identify Static Nonlinear Operators, IRE Transactions on Automatic Control, November, 1959.

INTERNAL NOTE NO.

SINGLE PARAMETER TESTING

By

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J.T. Sterling

ABSTRACT

This report presents the results obtained in evaluating two testing techniques for determining off-nominal parameter values of a component. These two techniques are the use of growing exponential probing signals with an orthogonal filter bank and the use of transient response sampling. The results obtained in testing linear and some nonlinear component models are described and the conclusions reached when applying the transient response sampling technique to an AC amplifier and a DC amplifier are given.

This report shows that single parameter testing techniques can be applied successfully to some components when using the technical and economic considerations recommended for the selection of a component to be tested.