

MEMORANDUM
RM-5242-NASA
JANUARY 1967

**THE RELATION BETWEEN THE DIAMETER
OF A LIGHTNING STREAMER AND ITS
RADIATED RADIO FREQUENCY SPECTRUM**

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BRIEF

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PURPOSE: To estimate the spectrum radiated by a current streamer whose cross-sectional dimensions are an appreciable fraction of a wavelength and in which the current varies discontinuously with time.

RELATED TO: RAND's research for NASA on the scientific utilization of meteorological satellite data and the implications of new techniques and measurements to meteorological satellite development.

DISCUSSION: Satellite measurements of the radio frequency radiation from thunderstorms can provide useful meteorological information about the distribution of thunderstorms and possibly about some properties of the lightning stroke. However, the only radiation that can reach a satellite is radiation above a few megacycles, because only at these frequencies will the signal penetrate the ionosphere. For this reason, the spectral distribution of radiated energy at frequencies above a few megacycles is of particular interest.

FINDINGS: If the current distribution is roughly uniform throughout the streamer and drops rapidly to zero at the edge, the amplitude of the radiated spectrum will decrease inversely as the frequency at frequencies below the one at which the projection of the mean diameter of the streamer in the direction of the observer is about 0.4λ . At higher frequencies, the spectrum decreases inversely as the $5/2$ power of the frequency. For streamers in which the current varies less abruptly or the edge of the streamer is not so well defined, the spectrum will decrease more rapidly than this at high frequencies. Experimental evidence suggests that the spectrum of lightning decreases inversely as the frequency up to a frequency of at least 500 Mc. If so, it is concluded that either the radiating streamer has an effective diameter of less than 24 cm (contrasting with visible diameters of 1 to 10 m) or that the streamer current is not the major source of radiation at frequencies above a few hundred megacycles.

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PREFACE

This Memorandum has been prepared as a part of RAND's work on satellite meteorology sponsored by NASA. It concerns the spectral distribution of radio frequency energy radiated by a stroke of lightning in the frequency band in which the radiated signal will penetrate the ionosphere and hence is detectable by satellites. Using the results obtained, experimental data on the radiated spectrum from lightning can be interpreted in terms of the size of the radiating streamer, and thus this Memorandum should be of interest to anyone who is concerned with the physical theory of lightning.

SUMMARY

An estimate is made of the spectrum radiated by a current streamer whose cross-sectional dimensions are an appreciable fraction of a wavelength and in which the current varies discontinuously with time. If the current distribution is roughly uniform throughout the streamer and drops rapidly to zero at the edge, it is found that the amplitude of the radiated spectrum will decrease inversely as the frequency at frequencies below the one at which the projection of the mean diameter of the streamer in the direction of the observer is about 0.4λ . At higher frequencies the spectrum decreases inversely as the $5/2$ power of the frequency. For streamers in which the current varies less abruptly or the edge of the streamer is not so well defined, the spectrum will decrease more rapidly than this at high frequencies. Experimental evidence suggests that the spectrum of lightning decreases inversely as the frequency up to a frequency of at least 500 Mc. If so, it is concluded that either the radiating streamer has an effective diameter of less than 24 cm (contrasting with visible diameters of one to ten meters), or that the streamer current is not the major source of radiation at frequencies above a few hundred megacycles.

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I. INTRODUCTION

Satellite measurements of the radio frequency radiation from thunderstorms can provide useful meteorological information about the distribution of thunderstorms and possibly about some properties of the lightning stroke. However, the only radiation that can reach a satellite is radiation at frequencies above a few megacycles, because only at these frequencies will the signal penetrate the ionosphere. For this reason the spectral distribution of radiated energy at frequencies above a few megacycles is of particular interest. Since most of the energy radiated by the main stroke of the lightning flash is in the frequency band below 100 Kc, it appears that the major source of radiation that penetrates the ionosphere may be the rapidly changing current in the leader strokes which precede the main stroke. Schonland⁽¹⁾ estimates that a stepped leader will have a diameter of one to ten meters, and it seems reasonable to assume that dart leaders in the same channel will have roughly the same diameter. If the wavelength of the radiation is comparable to this diameter, the radiation from different parts of the cross section of the leader will interfere destructively, thus significantly reducing the received signal strength. As a result, the radiated spectrum at frequencies of the order of 100 Mc or higher will decrease more rapidly than that from a simple impulse, and it is the object of this report to investigate this rate of decrease quantitatively.

II. FOURIER ANALYSIS OF THE FIELD RADIATED BY ONE SEGMENT
OF THE STEPPED LEADER

Since the actual path of a leader is very unpredictable, it will be convenient to consider first only a small segment of the leader, which can be assumed to be nearly straight and which is oriented vertically. The radiation from the entire streamer will then be estimated by combining the radiation from many of these segments of various locations, sizes, and orientations. The total current in one vertical segment of the leader will be assumed to depend on the altitude h and time t and will be denoted by $I(h, t)$. Current will be assumed to flow only in the vertical direction and to be distributed in the same way across all horizontal cross sections of the leader. If x and y are Cartesian coordinates in the horizontal plane, then the current density $J(x, y, h, t)$ can be written in the form

$$J(x, y, h, t) = I(h, t) f(x, y) \quad (1)$$

where $f(x, y)$ describes the distribution of the current across the leader. Since the total current I is the integral of J over any horizontal surface through the leader, it follows from Eq. (1) that

$$\int f(x, y) dx dy = 1 \quad (2)$$

where the integral is extended over the entire cross section of the leader. From this current distribution, the field radiated by one segment of the leader can be estimated.

Since the radiated field is usually detected by a receiver that is sensitive to only a narrow band of frequencies in the vicinity of some given center frequency, it is convenient to discuss only the Fourier components of the radiated field that lie in such a narrow band. These components are produced by the Fourier components of $J(x, y, h, t)$ in this particular band. If the Fourier component of $J(x, y, h, t)$ at the frequency of interest is $J^*(x, y, h, \omega)e^{-i\omega t}$, where J^* is in general a complex number and ω is 2π times the frequency, and if the corresponding

Fourier component of the total current $I(h, t)$ is $I^*(h, \omega)e^{-i\omega t}$, then from Eq. (1)

$$J^*(x, y, h, \omega) = I^*(h, \omega) f(x, y) \quad (3)$$

The Fourier transform of the field quantities can be computed directly from $J^*(x, y, h, \omega)$. If it is assumed that the distance r from the current element to the observer is so great that only terms proportional to $1/r$ need be considered and all terms proportional to higher powers of $1/r$ are neglected, then the Fourier transform of the magnetic field seen by an observer located at the point x', y', h' is given by⁽²⁾

$$H^*(x', y', h', \omega) = -\frac{i}{4\pi} \int J^*(x, y, h, \omega) \vec{u} \times \vec{k} \frac{e^{i\omega r/c}}{r} d\tau \quad (4)$$

where \vec{u} is the unit vector in the vertical direction, \vec{k} is the propagation vector whose magnitude is ω/c and whose direction is from the current element to the observer, and $d\tau$ is the element of volume at the current element under consideration. The notation is shown in Fig. 1.

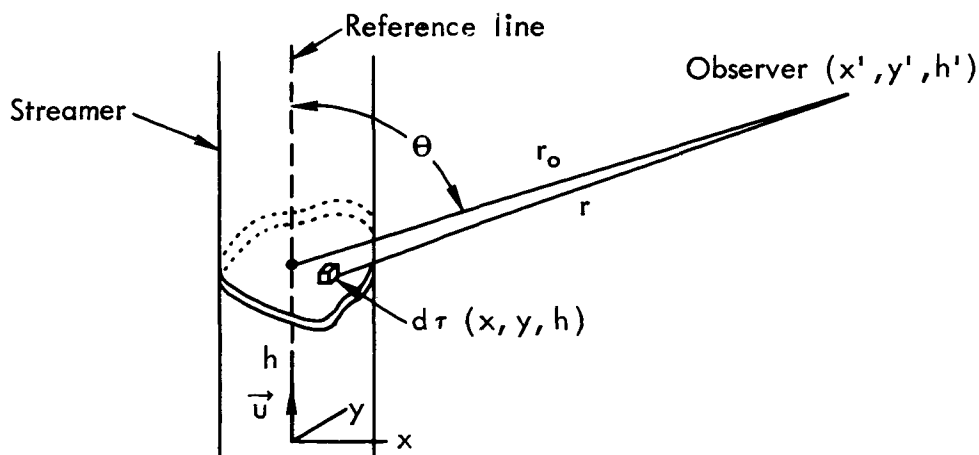


Fig. 1 -- Geometry of the streamer and the observer.

It will be convenient to introduce as a reference line (shown in Fig. 1) a vertical straight line within the streamer, and to measure the x and y axes from this line. The integral of Eq. (4) can then be evaluated by considering the thin horizontal section of the streamer shown in Fig. 1. Since the distance from the streamer to the observer is assumed to be far greater than the diameter of the streamer, the values of x and y will be much less than r. If the value of r when x = y = 0 is denoted by r₀, then the value of r at any point in the thin section under consideration can be approximated by

$$r = \sqrt{(x' - x)^2 + (y' - y)^2 + (h' - h)^2}$$

$$\approx r_0 - \frac{x'}{r_0} x - \frac{y'}{r_0} y$$

If the x and y axes are rotated so that the observer lies in the x, h plane, then y' = 0, and this equation becomes

$$r \approx r_0 - x \sin \theta \tag{5}$$

where θ is the angle between the h-axis and the line connecting the observer with the point where x = y = 0 in the section under consideration.

In the integral of Eq. (4) the value of J* can now be expressed by Eq. (3), the vector \vec{k} and the factor 1/r can be considered to be essentially independent of x and y since the diameter of the streamer is much smaller than its distance from the observer, the factor $e^{i\omega r/c}$ can be modified by replacing r with its value given in Eq. (5), and d τ can be replaced by dx dy dh, with the result that

$$H^*(x', y', h', \omega) = -\frac{i}{4\pi} \int I^*(h, \omega) \vec{u} \times \vec{k} \frac{e^{i\omega r_0/c}}{r_0} \left[\iint f(x, y) e^{-i(\omega/c)x \sin \theta} dx \right] dy \quad dh \tag{6}$$

For a vertical streamer segment that is very far from the observer, the angle θ is essentially independent of h , so the integral contained in brackets is a function of ω only and will be denoted by $F(\omega)$:

$$F(\omega) = \iint f(x, y) e^{-i(\omega/c)x \sin \theta} dx dy \quad (7)$$

where the integral is extended over the entire cross section of the streamer. Then Eq. (6) becomes

$$H^*(x', y', h', \omega) = -\frac{i}{4\pi} \int I^*(h, \omega) \vec{u} \times \vec{k} \frac{e^{-i\omega r_0/c}}{r_0} dh \cdot F(\omega) \quad (8)$$

By comparing the integral here with that of Eq. (4), it is seen that the field component H^* is equal to $F(\omega)$ times the field component that would have existed if all the actual current in the streamer had been concentrated on the vertical line through the origin of coordinates.

When H^* is known the corresponding electric field is perpendicular to H^* and related to it in magnitude by the characteristic impedance of free space, and hence is uniquely determined. Thus the entire effect of the nonvanishing cross-sectional area of the streamer on the radiation field is to multiply the field quantities by the factor $F(\omega)$.

If the current in one segment of the streamer were concentrated along a single line and rose from a small value to its maximum value almost instantaneously, as might be expected in such a discharge, its radiated frequency spectrum would vary inversely as the frequency, or as $1/\omega$. Thus the spectrum radiated from an actual stepped leader segment with a significantly large cross-sectional area will be proportional to $F(\omega)/\omega$. It remains to evaluate $F(\omega)$.

III. THE DETERMINATION OF F(ω)

Since the distribution of current across the streamer is not known, no exact determination of F(ω) is possible. However, a rough idea of the way in which F(ω) varies with ω can be gained by evaluating F(ω) for the simple case of a circularly cylindrical streamer with a uniform current distribution. In this case f(x, y) is a constant. If the radius of the streamer is R, then Eq. (2) requires that the actual value of f(x, y) must be 1/(πR²). If 2πc/(ω sin θ) is denoted by L, Eq. (7) becomes

$$F(\omega) = \frac{1}{\pi R^2} \iint e^{-i2\pi x/L} dx dy$$

Carrying out the integration over y in a circle of radius R, so that y runs from $-\sqrt{R^2 - x^2}$ to $\sqrt{R^2 - x^2}$, gives

$$F(\omega) = \frac{2}{\pi R^2} \int_{-R}^R \sqrt{R^2 - x^2} e^{-i2\pi x/L} dx$$

If u = x/R, this is

$$F(\omega) = \frac{2}{\pi} \int_{-1}^1 \sqrt{1 - u^2} e^{-i2\pi(R/L)u} du = \frac{J_1(2\pi R/L)}{\pi R/L} \quad (9)$$

where J₁ is the Bessel function of first order. Since the received signal strength depends only on the magnitude and not the sign of F(ω), the absolute magnitude of this value of F(ω) has been plotted as a function of 2πR/L in Fig. 2.

When 2πR/L is large, J₁(2πR/L) is given approximately by

$$J_1(2\pi R/L) \cong \frac{1}{\pi \sqrt{R/L}} \cos(2\pi R/L - \frac{3\pi}{4})$$

so that

$$F(\omega) \cong \frac{1}{\pi^2 (R/L)^{3/2}} \cos(2\pi R/L - 3\pi/4) \quad (10)$$

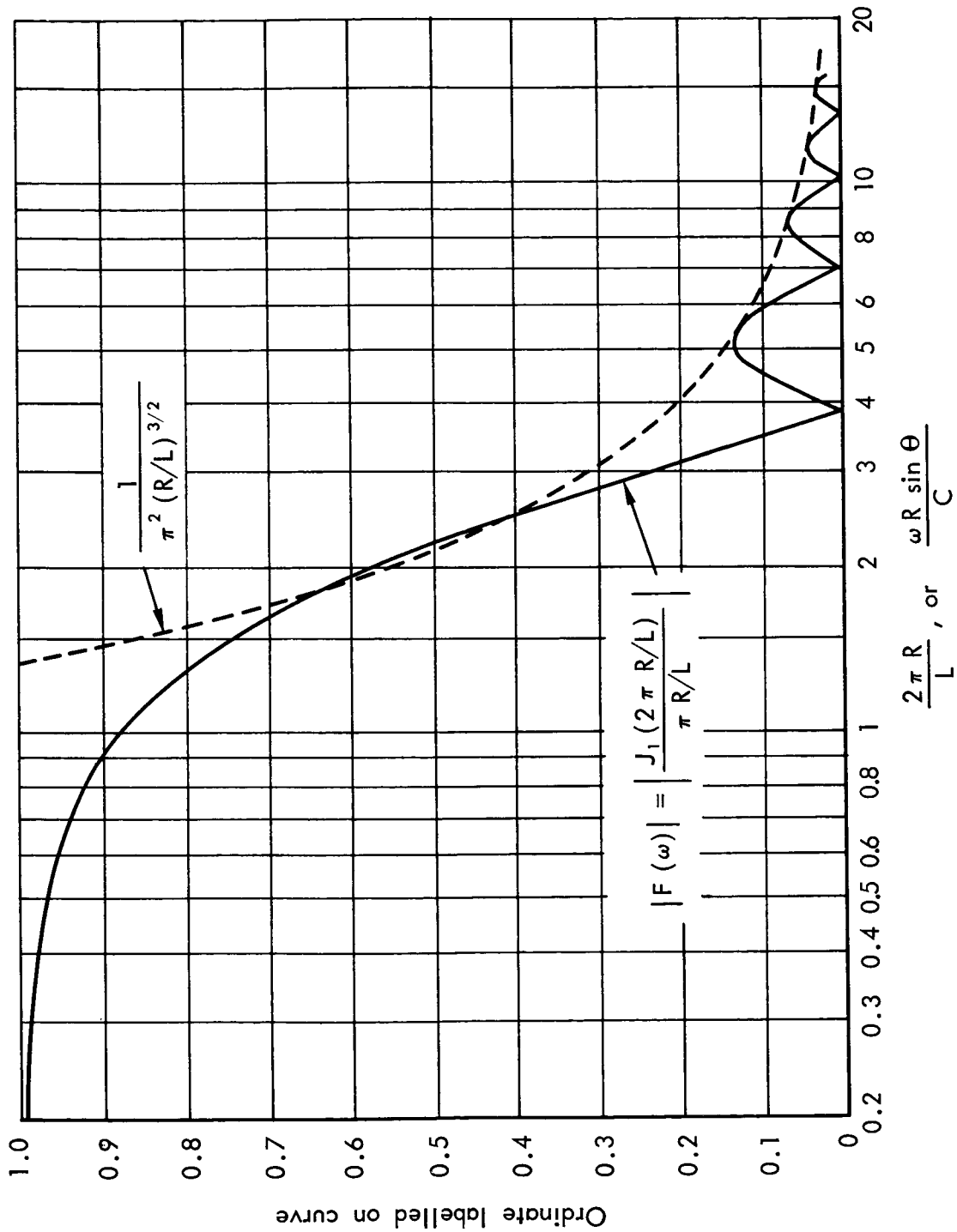


Fig. 2 -- Plot of $|F(\omega)|$ for a circularly cylindrical streamer with uniform current distribution as a function of $(\omega/c)R \sin \theta$.

Thus, to this approximation, $F(\omega)$ varies sinusoidally within an envelope that is shown by the dashed line in Fig. 2. It is apparent that Eq. (10) gives a very useful approximation for $F(\omega)$ whenever $2\pi R/L$ is greater than 3 or 4.

The envelope of the high-frequency end of the spectrum that is plotted in Fig. 2 has been derived for a circularly cylindrical streamer with a uniform current distribution. However, it can be generalized slightly to apply to any streamer that has a smoothly shaped boundary and whose current distribution varies only gradually within the boundary but drops rapidly to zero outside of it. To make this generalization, it is convenient to use a Fresnel-zone argument to evaluate the integral of Eq. (7).⁽³⁾ Let the cross section of an arbitrary streamer (shown in Fig. 3) be divided into Fresnel zones, each of which is of width $L/2$, where $L = 2\pi c/(\omega \sin \theta)$, as before.

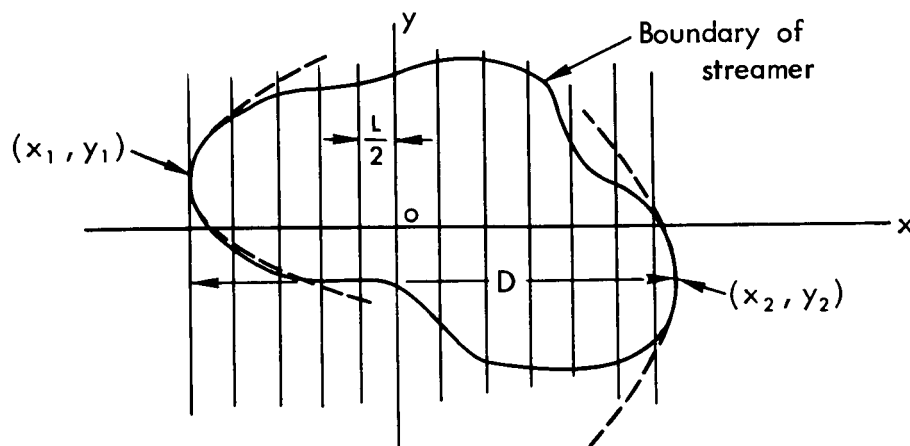


Fig. 3 -- Cross section of an arbitrary streamer.

If (x_1, y_1) and (x_2, y_2) are the points on the boundary that are farthest from the observer and closest to him, respectively, then the boundaries of the Fresnel zones are the vertical lines shown at the points where $x = x_1 + nL/2$, where n is an integer, and the integral of Eq. (7) can be written

$$F(\omega) = \iint f(x, y) e^{-i2\pi x/L} dx dy = \sum_{n=1}^N F_n \quad (11)$$

where F_n is the contribution of the n^{th} Fresnel zone to the integral, and N is the total number of zones. Then $F(\omega)$ can be written

$$F(\omega) = \frac{F_1}{2} + \left(\frac{F_1}{2} + F_2 + \frac{F_3}{2} \right) + \left(\frac{F_3}{2} + F_4 + \frac{F_5}{2} \right) + \dots \quad (12)$$

where the last term is $\frac{F_N}{2}$ if N is odd and $\frac{F_{N-1}}{2} + F_N$ if N is even. The exponential factors at corresponding points in the integrands of F_n and F_{n+1} differ only by the factor $e^{-i\pi}$, or -1 . The other factor in the integrand is $\int f(x, y) dy$, which depends on the range of integration of y and on the value of $f(x, y)$, both of which are assumed to vary only slowly from one zone to the next. Thus F_n and F_{n+1} will be of opposite sign and will differ only slightly in magnitude. It follows that F_n will be roughly the negative of the average of F_{n+1} and F_{n-1} , and the quantities in parentheses in Eq. (12) will tend to vanish as long as there are no strong periodicities of period L in $\int f(x, y) dy$. Thus, assuming that there are no such periodicities, the major contributions to $F(\omega)$ come from the first and last one or two Fresnel zones, and a good approximation to the integral in Eq. (11) can be obtained by replacing $\int f(x, y) dy$ by any quantity that closely approximates it in these zones and that varies in any convenient way in the other zones, provided only that it varies slowly from zone to zone.

With this observation, the integral of Eq. (11) can be evaluated directly. The boundary of the streamer can be approximated in the regions near the points (x_1, y_1) and (x_2, y_2) by two parabolas (shown dashed in Fig. 3) whose radii of curvature are the actual radii of curvature R_1 and R_2 of the boundary at these two points. The equations of these parabolas are

$$y - y_1 = \pm \sqrt{2R_1(x - x_1)}$$

and

$$y - y_2 = \pm \sqrt{2R_2(x_2 - x)}$$

respectively. Since $f(x, y)$ varies only gradually within the streamer, it can be approximated by the constant value f_1 in the region of the streamer near (x_1, y_1) and by the value f_2 in the region near (x_2, y_2) . Then in the regions near (x_1, y_1) and (x_2, y_2) we have approximately

$$\int f(x, y) dy = \begin{cases} 2f_1 \sqrt{2R_1(x - x_1)} & \text{near } (x_1, y_1) \\ 2f_2 \sqrt{2R_2(x_2 - x)} & \text{near } (x_2, y_2) \end{cases}$$

Between these two regions the value of $F(\omega)$ is insensitive to the value of $\int f(x, y) dy$ so long as $\int f(x, y) dy$ varies smoothly with x and has no marked periodicity with period L . Thus it is a good approximation to divide the entire streamer into two parts and use the first of these approximations throughout the left part and the second throughout the right part. The boundary between these two parts will be at a point $x = b$, where b is chosen so that the resulting approximation for $\int f(x, y) dy$ varies continuously across the boundary at $x = b$. From the expressions for $\int f(x, y) dy$ given above, it is easily seen that

$$b = \frac{x_1 f_1^2 R_1 + x_2 f_2^2 R_2}{f_1^2 R_1 + f_2^2 R_2} \quad (13)$$

and from Eq. (11) we have:

$$F(\omega) = \int_{x_1}^b 2f_1 \sqrt{2R_1(x - x_1)} e^{-i2\pi x/L} dx + \int_b^{x_2} 2f_2 \sqrt{2R_2(x_2 - x)} e^{-i2\pi x/L} dx$$

If we change the variables of integration by letting $u = 2\sqrt{\frac{x - x_1}{L}}$ in the first of these integrals and $u = 2\sqrt{\frac{x_2 - x}{L}}$ in the second, and employ the identical relation

$$\int u^2 e^{\pm i(\pi/2)u^2} du = \pm \frac{i}{\pi} u e^{\pm i(\pi/2)u^2} \pm \frac{i}{\pi} \int e^{\pm i(\pi/2)u^2} du$$

the result can be simplified by replacing b by the value given in Eq. (13) and becomes

$$F(\omega) = \frac{f_1 L}{i\pi} \sqrt{\frac{R_1 L}{2}} e^{-i2\pi x_1/L} \int_0^{\sqrt{\frac{b-x_1}{L}}} e^{-i(\pi/2)u^2} du$$

$$- \frac{f_2 L}{i\pi} \sqrt{\frac{R_2 L}{2}} e^{-i2\pi x_2/L} \int_0^{\sqrt{\frac{x_2-b}{L}}} e^{i(\pi/2)u^2} du$$

where the integrals are Fresnel's integrals. Since we are looking for an approximation for $F(\omega)$ at high frequencies, where L is small, we are primarily interested in these integrals when the upper limit of integration approaches infinity. If each of the two parts of the streamer, between x_1 and b and between b and x_2 , contains only one Fresnel zone, we will have $b - x_1 = L/2$ and $x_2 - b = L/2$, and the upper limit of each of the above integrals is $\sqrt{2}$. For an upper limit of $\sqrt{2}$ or more, the absolute magnitude of either integral is within about 30 percent of its value for an upper limit of infinity. Thus, for any streamer to which the Fresnel zone argument can reasonably be applied, a good first approximation to $F(\omega)$ can be obtained by letting the upper limits of both integrals be infinite. Then, since

$$\int_0^{\infty} e^{\pm i(\pi/2)u^2} du = \frac{1 \pm i}{2}$$

the expression for $F(\omega)$ becomes

$$F(\omega) = \frac{\sqrt{2} L^{3/2}}{4\pi} \frac{1-i}{i} e^{-i2\pi x_1/L} \left[f_1 \sqrt{R_1} - f_2 \sqrt{R_2} e^{-i2\pi \left(\frac{x_2 - x_1}{L} - \frac{1}{4} \right)} \right]$$

The amplitude of the received signal depends on the absolute magnitude of $F(\omega)$ which is given by

$$|F(\omega)| = \frac{L^{3/2}}{2\pi} \sqrt{f_1^2 R_1 + f_2^2 R_2 - 2f_1 f_2 \sqrt{R_1 R_2} \cos \left[2\pi \left(\frac{x_2 - x_1}{L} - \frac{1}{4} \right) \right]} \quad (14)$$

This is the general expression for $F(\omega)$ that was desired. For the particular case of a circularly cylindrical streamer with a radius R and with a uniform current distribution, we have $R_1 = R_2 = R$, $x_2 - x_1 = 2R$, and $f_1 = f_2 = 1/(\pi R^2)$, and Eq. (14) becomes Eq. (10).

The magnitude of $|F(\omega)|$ given by Eq. (14) is easily understood as the magnitude of the resultant of two vectors whose magnitudes are $\frac{1}{2\pi} L^{3/2} f_1 \sqrt{R_1}$ and $\frac{1}{2\pi} L^{3/2} f_2 \sqrt{R_2}$ and whose directions are separated by the angle $2\pi[(x_2 - x_1)/L + 1/4]$, as shown in Fig. 4.

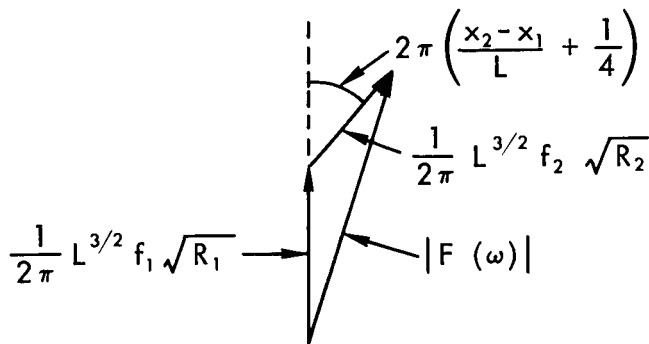


Fig. 4 -- Vector diagram for determining $|F(\omega)|$.

The frequency at which the observation is made enters the expression for $|F(\omega)|$ in Eq. (14) only through L , which varies inversely as the frequency. Thus as the frequency increases, the magnitudes of the two vectors decrease, and one of them rotates continuously relative to the other, with the result that $|F(\omega)|$ oscillates between the limits

$$|F(\omega)|_{\max} = \frac{1}{2\pi} L^{3/2} (f_1 \sqrt{R_1} + f_2 \sqrt{R_2}) \quad (15)$$

and

$$|F(\omega)|_{\min} = \frac{1}{2\pi} L^{3/2} |f_1 \sqrt{R_1} - f_2 \sqrt{R_2}| \quad (16)$$

Qualitatively, this variation is much like the one shown for high frequencies in Fig. 2 except that the minima will not be zero unless $f_1 \sqrt{R_1} = f_2 \sqrt{R_2}$. The variation of $|F(\omega)|$ at low frequencies for this more general type of streamer is also similar to that shown in Fig. 2, in that $F(\omega) \approx 1$ for any streamer as long as $x_2 - x_1$ is appreciably less than $L/2$. It is thus reasonable to conclude that an upper limit to the value of $|F(\omega)|$ is given by Eq. (15) at frequencies where Eq. (15) gives

a value of $|F(\omega)|$ less than one and is given by $|F(\omega)| = 1$ below this frequency, provided only that the streamer has a smooth, sharply defined boundary.

Furthermore, the analysis given above also suggests an upper limit to $|F(\omega)|$ that can be applied to streamers that lack a sharp boundary, that is, to streamers in which the current density [and hence $f(x, y)$] drops to zero only very gradually as the distance from the center of the streamer increases. In order that Eq. (2) can be satisfied by such a streamer, $f(x, y)$ must decrease more rapidly than $1/r^2$ for large distances r from the center of the streamer. Thus if we were to approximate the streamer by one that has the same current distribution within a large circle of radius R and that has no current outside of this circle, Eq. (15) would give an upper limit to the value of $|F(\omega)|$ at high frequencies that varies as $1/R^{3/2}$ and approaches zero as $R \rightarrow \infty$. Since in the limit as $R \rightarrow \infty$ the approximate streamer approaches the actual one, it is clear that the actual value of $|F(\omega)|$ at high frequencies is very small and is less than that predicted by Eq. (15) for any truncated streamer of the type considered above. Thus, in general, the effect of a gradual rather than abrupt drop in the current density at the edge of the streamer is to reduce appreciably the high-frequency radiation from the streamer. Since at low frequencies $|F(\omega)| \cong 1$ for any streamer, it is only the high-frequency radiation that is affected, and it can be concluded that the effect of a gradual rather than abrupt drop in the current density at the edge of the streamer is to make $|F(\omega)|$ decrease more rapidly than it otherwise would as the frequency increases.

These results give the following useful estimate of the value of $|F(\omega)|$ for almost any streamer that might occur in nature:

1. $|F(\omega)| \cong 1$ at low frequencies, where the maximum dimension of the streamer is much less than $L/2$.
2. $|F(\omega)|$ is given by Eq. (14) at frequencies so high that the maximum dimension of the streamer in the direction of the receiver is much greater than $L/2$, if the current distribution drops abruptly to zero at the edge of the streamer (making most of its variation in less than one Fresnel zone of width $L/2$).

3. If the current density at the edge of the streamer drops to zero more slowly than assumed above, $|F(\omega)|$ will be considerably less at high frequencies than the value given by Eq. (14) when f_1 and f_2 are evaluated on any large circle of radius R and $R_1 = R_2 = R$.
4. The case of a circularly cylindrical streamer with uniform current distribution (see Fig. 2) suggests that the high-frequency approximation is quite accurate down to the frequency at which it gives the value $|F(\omega)| = 1$, and the low-frequency approximation $|F(\omega)| \approx 1$ is reasonably good at frequencies below this one.

These results give an indication of the value of $|F(\omega)|$ for almost any leader stroke that might be encountered in nature. They would not apply when the current is concentrated almost entirely near the edge of the streamer and $f(x, y)$ becomes very large in this region, because $f(x, y)$ could not then be assumed to be roughly constant throughout the first and last Fresnel zones, and the integrals by which $|F(\omega)|$ has been determined would have to be re-evaluated. Similarly, if $f(x, y)$ is roughly constant but the edge of the channel is straight, so that either R_1 or R_2 becomes infinite, the conclusions above are not valid. In fact, if the channel is rectangular and is viewed perpendicular to one face, the envelope of the high-frequency radiation spectrum decreases as $1/\omega$ rather than as $1/\omega^{3/2}$. However, such situations appear to be rare and will be neglected here.

IV. THE RADIATED SPECTRUM FROM AN ACTUAL STREAMER

Because actual leader strokes vary widely in shape, any estimate of the radiation from a particular stroke must be somewhat qualitative. However, it is reasonable to assume that any stroke can be represented approximately by a sequence of segments, each of which has the form discussed above, but each of which is at a different location and is oriented in a slightly different direction from the others. The spectrum of an actual streamer is then the sum of the spectra of its segments, where the field components at any given frequency are added in their proper polarization and phase. Since the segments of the actual streamer tend to be oriented in roughly the same direction, it will be assumed that their fields all have the same polarization and thus can be added by adding their instantaneous amplitudes. Because the segments of the streamer are at different distances from the observer, it will be assumed that the components they radiate at any given frequency will arrive at the observer with randomly distributed phases.

Under these circumstances, the amplitude of the resultant field is the square root of the sum of the squares of the components received from the individual segments of the streamer. Each of these components has the form given by Eq. (8) and is assumed to vary with frequency proportional to $F_i(\omega)/\omega$, where $F_i(\omega)$ is the function $F(\omega)$ for the i^{th} segment of the streamer. It will be assumed that the factor of proportionality is roughly the same for each segment of the streamer, with the result that the frequency spectrum of the entire streamer can be written in the form of a constant factor times $|F_s(\omega)|/\omega$, where $|F_s(\omega)|$ is an effective value of $|F(\omega)|$ for the entire streamer composed of N segments. Then $|F_s(\omega)|$ is given by

$$|F_s(\omega)| = \sqrt{\sum_{i=1}^N |F_i(\omega)|^2} \quad (17)$$

At the low-frequency end of the spectrum $|F_i(\omega)| \approx 1$, and $|F_s(\omega)| \approx \sqrt{N}$. At the high-frequency end of the spectrum $|F_i(\omega)|$ has the form given by Eq. (14) and $|F_i(\omega)|^2$ is

$$\frac{L^3}{4\pi^2} \left\{ f_{1R_1}^2 + f_{2R_2}^2 - 2f_1 f_2 \sqrt{R_1 R_2} \cos \left[2\pi \left(\frac{x_2 - x_1}{L} - \frac{1}{4} \right) \right] \right\}$$

where each parameter is evaluated for the i^{th} segment of the streamer. It will be assumed that the parameters $x_2 - x_1$ and L vary enough from segment to segment so that the average value of $L^3 \cos \left[2\pi \left(\frac{x_2 - x_1}{L} \right) - \frac{1}{4} \right]$ over all segments is essentially zero. Furthermore, the averages over all segments of the quantities $L^3 f_{1R_1}^2$ and $L^3 f_{2R_2}^2$ will be roughly equal and will be denoted by $\overline{L^3 f^2 R}$, so that Eq. (17) gives

$$|F_s(\omega)| = \frac{1}{\pi} \sqrt{\frac{N L^3 f^2 R}{2}}$$

This expression depends on frequency only through the factor L , which is defined to be $2\pi c / (\omega \sin \theta)$, so $|F_s(\omega)|$ can be written

$$|F_s(\omega)| = 2 \sqrt{\frac{\pi N c^3}{\omega^3} \left(\frac{f^2 R}{\sin^3 \theta} \right)} \quad (18)$$

and it is clear that $|F_s(\omega)|$ varies as $1/\omega^{3/2}$. Since there is no reason to believe that the statistical distribution of the values of f at the edge of one segment of the streamer or of the radius of curvature R at that point is dependent on the orientation of that segment as specified by the angle θ , the average value of $f^2 R / \sin^3 \theta$ will be just the average of $f^2 R$ times the average of $1/\sin^3 \theta$. Defining an effective orientation angle θ_s between the general direction of the streamer and the direction of the observer by letting $1/\sin^3 \theta_s$ equal the average of $1/\sin^3 \theta$ over all segments of the streamer, we have

$$\overline{\left(\frac{f^2 R}{\sin^3 \theta} \right)} = \frac{1}{\sin^3 \theta_s} \overline{f^2 R}$$

The average value of $f^2 R$ depends on unknown parameters of the individual streamer segments. However, an estimate of this average can be obtained by assuming that the different segments of the streamer are statistically similar, so that the average of $f^2 R$ over the different segments is the same as the average of $f^2 R$ over all azimuthal directions for any one

segment of the streamer. If φ is an azimuth angle measured about the axis of one segment of the streamer between an arbitrary reference direction and the plane containing the axis and the observer, and if changing φ to $\varphi + d\varphi$ moves the point at which the perimeter of the streamer cross section is perpendicular to the direction of the observer by a distance ds along the perimeter, then the radius of curvature of the perimeter at that point is defined to be $R = |ds/d\varphi|$. If f is the value of $f(x, y)$ at that point, and if the perimeter of the streamer is assumed to be convex outward so that $ds/d\varphi$ is always positive, we can average $f^2 R$ over all values of φ from 0 to 2π to get

$$\overline{f^2 R} = \frac{1}{2\pi} \int_0^{2\pi} f^2 \frac{ds}{d\varphi} d\varphi = \frac{1}{2\pi} \int f^2 ds$$

Letting $\overline{f^2}$ be the average value of f^2 on the perimeter and letting P be the total length of the perimeter, we have

$$\overline{f^2 R} = \frac{P}{2\pi} \overline{f^2}$$

With these substitutions, Eq. (18) becomes

$$|F_s(\omega)| = \sqrt{\frac{2Nc^3 P \overline{f^2}}{\omega^3 \sin^3 \theta_s}} \quad (19)$$

Thus $|F_s(\omega)|$ is approximately \sqrt{N} at frequencies below the one where the right side of Eq. (19) equals \sqrt{N} , and it decreases as $1/\omega^{3/2}$ above that frequency. The frequency at which $|F_s(\omega)|$ changes slope is the one for which

$$\omega = \frac{c}{\sin \theta_s} \sqrt[3]{2P \overline{f^2}} \quad (20)$$

or for which the wavelength λ , is given by $\lambda = 2\pi c/\omega$, has the value

$$\lambda = \frac{2\pi \sin \theta_s}{\sqrt[3]{2P \overline{f^2}}} \quad (21)$$

This gives a reasonable estimate of $|F_s(\omega)|$ for a complete streamer when its perimeter and the average value of $f(x, y)$ on the perimeter are known. Actually, the only parameter of the streamer about which we have any information is its apparent diameter, and we would like to determine the frequency at which the slope of the spectrum changes as a function of this diameter. If it is assumed that the current distribution is roughly uniform throughout the streamer, then $f(x, y)$ is just the reciprocal of the cross sectional area of the streamer. If the effective streamer diameter D is defined so that this area is $\frac{\pi D^2}{4}$, then the perimeter P must be greater than πD , and the value of ω given by Eq. (20) will be greater than

$$\frac{2c}{D \sin \theta_s} \sqrt[3]{4/\pi}$$

and, from Eq. (21), λ at this point will be less than

$$\pi D \sin \theta_s \sqrt[3]{\pi/4}$$

If it is assumed that the streamer is roughly circular, these inequalities will be nearly equalities, and the frequency at which the slope of the radiated spectrum changes will be one for which $D \sin \theta_s$ is not more than about 0.4λ . Since the radiated spectrum varies as $(1/\omega)|F_s(\omega)|$, it is concluded that:

1. The amplitude spectrum will decrease as $1/\omega$ at frequencies below the one at which $D \sin \theta_s$ is about 0.4λ .
2. At frequencies higher than this one the amplitude spectrum decreases at least as rapidly as $1/\omega^{5/2}$.
3. If the rate of rise of the current in the streamer is not so rapid that the current can be considered to be a discontinuous function of time, or if the current distribution drops to zero only very gradually at the edge of the streamer, the amplitude spectrum at the higher frequencies will be less than the one indicated above.

It is now possible to compare these results with the frequency spectra that have been observed experimentally, as compiled by Kimpara. (4)

At any frequency between 100 Kc and 500 Mc, the measurements suggest that the amplitude spectrum decreases as $1/\omega$. Because of the spread of the experimental results, this conclusion cannot be regarded as certain; however, if it is accepted, the foregoing results show that the radiation cannot be considered to come from a single streamer with a roughly uniform current distribution unless $D \sin \theta_s$ is less than 0.4λ at 500 Mc, or 24 cm. If it is assumed that the measurements were made on streamers that were roughly perpendicular to the line of observation, this implies that the streamer diameter is less than 24 cm, arguing strongly against any theory in which the radiation is assumed to come primarily from a streamer 1 to 10 meters in diameter. To investigate the validity of this argument it would be very desirable to have more exact measurements of the frequency spectra radiated by individual lightning strokes, particularly at frequencies above 100 Mc, so that it could be determined whether the spectrum at these frequencies decreases as $1/\omega$ or as $1/\omega^{5/2}$, or even more rapidly.

V. CONCLUSIONS

It has been shown that the fact that the cross section of a lightning streamer is not infinitesimally small means that components of high-frequency radiation coming from different parts of the streamer cross section will not all add in phase; as a result, the radiated frequency spectrum will decrease more rapidly at high frequencies than it would if the streamer cross section were infinitesimally small. The resulting amplitude spectrum will decrease as $1/\omega$ at frequencies above about 100 Kc (where the return stroke can be ignored) until the frequency is so high that the diameter of the streamer is an appreciable part of a wavelength. Above this frequency the spectrum will decrease as $1/\omega^{5/2}$. For an observer who views the streamer from a direction roughly perpendicular to that of the streamer, this change in slope of the spectrum occurs when the diameter of the streamer is about four-tenths of the wavelength. If the current distribution in the streamer drops to zero only very slowly at the edge of the streamer, or if the current does not change in almost instantaneous jumps, the frequency spectrum will decrease even more rapidly than this at high frequencies. Experimental results suggest that the spectrum may decrease only as $1/\omega$ up to frequencies as high as 500 Mc. If this is true, then the main source of high-frequency radiation cannot be a single streamer of diameter greater than about 24 cm. Since visual diameters of stepped leaders are about 1 to 10 meters, and since dart leaders might be expected to have similar diameters, it appears that either the conducting core of the leader is much smaller than its visible diameter or the current in the leader is not the major source of high-frequency radiation.

REFERENCES

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