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PUBLICATION OF ORBITS DERIVED FROM PHOTOREDUCED BAKER-NUNN OBSERVATIONS

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BIOGRAPHICAL NOTE

Mr. Gaposchkin graduated in electrical engineering from Tufts University, Medford, Massachusetts, in 1957. He received a Degree of Numerical Analysis in 1959 from Cambridge University, England, and is now working toward a doctorate in geophysics at Harvard University.

Since joining the staff of the Smithsonian Astrophysical Observatory in 1959, Gaposchkin has held positions as programmer and Division Chief of the Computations Division, and is now a mathematician in the Research and Analysis Department. He has helped to develop the basic computer program used in all analyses of satellite motion.

His main interests include satellite geodesy and geophysics, and applied mathematics.

ABSTRACT

The precise orbital information published by the Smithsonian Astrophysical Observatory has been computed using various numerical values for the geophysical constants. This report details the numerical values of the constants used. The further reports of orbital data will include the same relevant information.

PUBLICATION OF ORBITS DERIVED FROM PHOTOREDUCED BAKER-NUNN OBSERVATIONS

E. M. Gaposchkin

The Smithsonian Astrophysical Observatory has published final orbital information (Staff of SAO, 1962, 1963, 1964a, and 1964b; referred to as E-1, E-2, E-3, E-4, respectively) that has been the by-product of certain investigations at the Observatory. Our purpose here is to detail information that will define these results more precisely (see Tables 1 to 5). In the future, we plan to supply such supporting information with each publication.

The tables were compiled by referring directly to the orbit files. In some cases, the information could not be retrieved (e. g., the station coordinates). Where the information is uncertain, it is appropriately designated. We now confine ourselves to a few remarks about these data.

A. The data published are the direct result of the DOI program. The distinction between mean and smoothed elements is based on the method of running the program. The details of the program, both theoretical and practical, are available elsewhere (Gaposchkin, 1964). We mention, without discussion, only aspects necessary for this summary.

a. Each orbital element as input to the DOI program is represented as a polynomial with trigonometric terms, such as

$$E(t) = E_0 + E_1 t + E_2 t^2 + \cdots + \sum_{i=1} A_i \sin (B_i + C_i t),$$

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where $t = T - T_0$; i. e., t is the time measured from the epoch of the orbit. In general, the E_j are determined by the DOI program, and the A_i, B_i, and C_i are determined theoretically. In this instance, the A_i, B_i, and C_i represent the long-period perturbation due to the earth's oblateness. Therefore, $B_i + C_i t \cong i\,\omega(t)$. The amplitudes A_i are evaluated from the theoretical development of Kozai (1964). Where used, they are given with the smoothed elements.

b. The smoothed elements are simply the values E_j, A_i, derived from orbital arcs of 1 to 2 weeks. The mean elements are the values of E(t) at the epoch (t = 0), given in the form of a table. They differ from the constant part of the polynomial only by the contribution of the long-period trigonometric terms. These mean elements are derived from shorter arcs (typically, 4 days), computed for epochs at 2-day intervals. Therefore, consecutive arcs are determined from overlapping intervals.

c. These elements, as contrasted with osculating elements, are "mean" elements in the sense that they are averages. In other words, they are the elements with the short-period perturbations subtracted.

d. The times of the epoch are given in Modified Julian Days (MJD), defined in terms of Julian Days (JD):

$$MJD = JD - 2\,400\,000.5 \quad .$$

e. The units of the orbital elements are degrees for angular quantities, megameters for linear quantities, and revolutions for the mean anomaly.

f. The values of the SAO elements are given for the argument of perigee ω, the right ascension of the ascending node Ω, and inclination I, the eccentricity e, and the mean anomaly M and its derivatives n, $\dot{n}/2$. They are given as functions of t measured in days. The number at the right of each constant represents the standard error for that element and refers to the last digits given.

g. The tabulation of "mean" elements gives the mean (anomalistic) motion n, the orbital acceleration $\dot{n}/2$, and the semimajor axis a or the geocentric distance of perigee q. In the last three columns, the one headed N indicates the number of observations used for the computation of a set of elements; the one headed D, the number of days used; and the one headed σ, the standard error relative to their assumed accuracy.

h. The semimajor axis a of the orbit is computed from the mean motion n by means of the following formula:

$$a = \left[\frac{GM}{n^2}\right]^{1/3} \left[1 - \frac{J_2 a_e^2}{2 p^2} (1 - e^2)^{1/2} (1 - \frac{3}{2} \sin^2 I)\right]$$

(see Kozai, 1959, equation (14)), where we put $A_2 = (3/2) J_2 a_e^2$. The value of J given in Table 2 is this A_2 expressed in M_m^2; n is the mean motion defined as the time derivative of the polynomial part of the mean anomaly; I is the inclination; and e is the eccentricity. The units for GM given in Table 2 are M_m^3 rev^2 $days^{-2}$. In reports E-1 to E-4 the mean motion was the total time derivative of the mean anomaly, and therefore included unwanted long-period effects.

i. In the DOI, the inclination and the argument of perigee are referred to the true equator of date, and the right ascension of the ascending node is measured from the mean equinox of 1950.0. To refer the orbit to the mean equinox of date, we should set the node equal to:

$$\Omega^\circ_{date} = \Omega^\circ_{DOI} + 3^\circ\!.508 \times 10^{-5} (MJD - 33281) \quad .$$

j. Two time systems are implicit in the DOI calculations. The first is the time used for computing the ephemeris; it should be a uniform system. The second, an empirically determined quantity, is used for computing the position of the earth (i. e., the station position). It should be UT1 because of the way we have defined the sidereal angle. The time given on a photo-reduced observation card is atomic time (A1) as published by the U. S. Naval Observatory; it is assumed to be a uniform time system. In reports E-1 to E-4 the DOI computed UT1 time from

$$UT1 = A1 + (-12.154 \times 10^{-6}) + (-0.015379 \times 10^{-6}) \times \tau \text{ days},$$

where $\tau = T - 37000$. This time was used in both the sidereal-time and the ephemeris calculations. Therefore, the orbits are referred to the UT1 time system as defined above. In E-5 and all subsequent reports the atomic time (as taken from the observation) is used in the ephemeris calculation, and the empirical UT1, given as the difference A1-UT1 on the observation card, is used in calculating the sidereal time. Therefore, these orbits are referred to the A1 time system.

B. The following comments apply to the tables.

a. In Table 1, under the columns headed "determination of $\dot{\omega}$ and $\dot{\Omega}$," a v indicates that the linear term in question was determined empirically by the DOI, and a c means that the term was held constant and determined by some other means. In the latter case, the linear terms were usually obtained by a separate least-squares program, which used the values of the element in question at successive epochs.

b. The constants referred to in Table 1 are given in Tables 2 to 5. The number given in Table 1 refers to the set of coefficients designated in the subsequent tables. Under time reference, 1 designates UT1 as defined above, and 2 designates A1.

c. In reports E-1 to E-4 only the short-period oblateness perturbations were included, while E-5 contains lunar perturbations and tesseral-harmonics perturbations. We detail the presence or absence of the perturbations and the values of the constants used. Unfortunately, a preliminary set of tesseral harmonics was used in E-5. The values are given here for reference, but the reader must be cautioned that they are not the set distributed by SAO.

Table 1. Summary of final orbital information published by the Smithsonian Astrophysical Observatory.

Satellite		Period of orbit	Mean or smoothed	Time ref.	Perturbations					Constants				Reference
					Determination of		Application of							
COSPAR code	Name				ω	Ω	Long-per.	Lunisol.	Tesser.	GM, J_2	Zonal	Tesser.	Sta. coor.	
5800402	1958 Delta 2 (Sputnik 3)	December 7-14, 1959	M	1	C	C	Y	N	N	1	1	N	1	
5900101	1959 Alpha 1 (Vanguard 2)	February 21-December 30, 1959	M	1	V	V	Y	N	N	1	1	N	1	
5900102	1959 Alpha 2 (Carrier rocket, Vanguard 2)	March 19-May 28, 1959	M	1	V	V	Y	N	N	1	1	N	1	
5900701	1959 Eta 1 (Vanguard 3)	September 23-December 30, 1959	M	1	V	V	Y	N	N	1	1	N	1	Staff of Smithsonian Astrophysical Observatory, 1962
6000902	1960 Iota 2 (Carrier rocket, Echo 1)	September 11, 1960-March 12, 1961	S	1			Y	N	N	1		N	1	
6000902	1960 Iota 2 (Carrier rocket, Echo 1)	September 11, 1960-March 12, 1961	M	1				N	N	1		N	1	
6001501	1960 Omicron 1 (Discoverer 17)	November 13-15, 1960	S	1			N	N	N	1		N	1	
6001801	1960 Sigma 1 (Discoverer 18)	December 8-10, 1960	S	1	V	V	N	N	N	1	N	N	1	
5900101	1959 Alpha 1 (Vanguard 2)	January 1, 1960-December 31, 1961	M	1	V	V	Y	N	N	1	1	N	2	
5900102	1959 Alpha 2 (Rocket, Vanguard 2)	April 6-August 26, 1960	M	1	V	V	Y	N	N	1	1	N	2	
5900701	1959 Eta 1 (Vanguard 3)	January 1, 1960-December 31, 1961	M	1	C&V	C&V	Y	N	N	1	1	N	2	Staff of Smithsonian Astrophysical Observatory, 1963
6000902	1960 Iota 2 (Rocket, Echo 1)	March 14-December 31, 1961	M	1	C	V	Y	N	N	1	1	N	2	
6100401	1961 Delta 1 (Explorer 19)	February 18-December 31, 1961	M	1	V	V	Y	N	N	1	1	N	2	
5900101	1959 Alpha 1 (Vanguard 2)	January 1-June 30, 1962	M	1	V	V	Y	N	N	1	1	N	2	
5900701	1959 Eta 1) (Vanguard 3)	January 1-June 30, 1962	M	1	V	V	Y	N	N	1	1	N	2	
6000201	1960 Beta 1 (Rocket body, Tiros 1)	April 12-May 26, 1960	M	1	C	C	Y	N	N	1	1	N	2	
6000202	1960 Beta 2 (Tiros 1)	April 12-September 15, 1960	M	1	C	C	Y	N	N	1	1	N	2	Staff of Smithsonian Astrophysical Observatory, 1964a
6000901	1960 Iota 1 (Echo 1)	August 14-30, 1960	M	1	V	V	N	N	N	1	N	N	2	
6000902	1960 Iota 2 (Rocket body, Echo 1)	January 1-June 30, 1962	M	1	V	V	Y	N	N	1	1	N	2	
6100401	1961 Delta 1 (Explorer 9)	January 1-June 30, 1962	M	1	V	V	Y	N	N	1	1	N	2	
6101501	1961 Omicron 1 (Transit 4A)	August 11, 1961-June 25, 1962	M	1	C	C	Y	N	N	1	1	N	2	
6101502	1961 Omicron 2 (Injun 3)	August 11, 1961-June 29, 1962	M	1	C	C	Y	N	N	1	1	N	2	

Table 1 (Cont.)

Satellite					Perturbations					Constants				
					Determination of		Application of							
COSPAR code	Name	Period of orbit	Mean or smoothed	Time ref.	$\dot{\omega}$	$\dot{\Omega}$	Long-per.	Lunisol.	Tesser.	GM, J_2	Zonal	Tesser.	Sta. coor.	Reference
5900101	1959 Alpha 1 (Vanguard 2)	July 1-December 31, 1962	M	1	V	V	Y	N	N	1	1	N	3	
5900701	1959 Eta 1 (Vanguard 3)	July 1-December 31, 1962	M	1	V	V	Y	N	N	1	1	N	3	
6000902	1960 Iota 2 (Rocket, Echo 1)	July 1-December 31, 1962	M	1	V	V	Y	N	N	1	1	N	3	
6100401	1961 Delta 1 (Explorer 9)	July 1-December 31, 1962	M	1	V	V	Y	N	N	1	1	N	3	
6101501	1961 Omicron 1 (Transit 4A)	July 19-December 31, 1962	M	1	C	V	Y	N	N	1	1	N	3	Staff of Smithsonian Astrophysical Observatory, 1964b
6101502	1961 Omicron 2 (Injun 3)	July 19-December 31, 1962	M	1	C	V	Y	N	N	1	1	N	3	
6102801	1961 Alpha Delta 1 (Midas 4)	March 13-December 31, 1962	S	1			Y	N	N	1	1	N	3	
6102801	1961 Alpha Delta 1 (Midas 4)	March 13-December 31, 1962	M	1	C	V	Y	N	N	1	1	N	3	
6202901	1962 Alpha Epsilon 1 (Telstar 1)	July 17-December 31, 1962	M	1	V	V	Y	N	N	1	1	N	3	
5900101	1959 Alpha 1 (Vanguard 2)	April 1-June 30, 1963	M	2	V	V	Y	Y	Y	2	2	1	4	
5900701	1959 Eta 1 (Vanguard 3)	April 1-June 30, 1963	M	2	V	V	Y	Y	Y	2	2	1	4	
6000902	1960 Iota 2 (Rocket, Echo 1)	April 1-June 30, 1963	M	2	V	V	Y	Y	Y	2	2	1	4	
6100401	1961 Delta 1 (Explorer 9)	April 1-June 30, 1963	M	2	V	V	Y	Y	Y	2	2	1	4	
6101501	1961 Omicron 1 (Transit 4A)	April 1-June 30, 1963	M	2	C	V	Y	Y	Y	2	2	1	4	Staff of Smithsonian Astrophysical Observatory, 1966
6101502	1961 Omicron 2 (Injun 3)	April 1-June 30, 1963	M	2	C	V	Y	Y	Y	2	2	1	4	
6102801	1961 Alpha Delta 1 (Midas 4)	April 1-June 30, 1963	M	2	C	V	N	Y	Y	2	2	1	4	
6202901	1962 Alpha Epsilon 1 (Telstar 1)	April 1-June 30, 1963	M	2	V	V	Y	Y	Y	2	2	1	4	
6206801	1962 Beta Upsilon 1 (A15 Relay)	April 1-June 30, 1963	M	2	C	C	Y	Y	Y	2	2	1	4	

Table 2. Values of the constants GM and J used in orbit calculation.

$\sqrt{GM}$	J	Set	Reference
274.54269	0.0660705456	1	
274.53910	0.0660546000	2	Gaposchkin, 1964; Kozai, 1962

Table 3. Zonal harmonics used in orbit calculation.

Harmonic	Value ($\times 10^{-6}$)	Set	Reference
J_2	1082.480	1	Kozai, 1962
J_3	-2.566		
J_4	-1.830		
J_5	-0.063		
J_6	0.390		
J_7	-0.469		
J_8	-0.020		
J_9	0.114		
J_2	1082.645	2	Kozai, 1964
J_3	-2.546		
J_4	-1.649		
J_5	-0.210		
J_6	0.646		
J_7	-0.333		
J_8	-0.270		
J_9	-0.053		
J_{10}	-0.054		
J_{11}	0.302		
J_{12}	-0.357		
J_{13}	-0.114		
J_{14}	0.179		

Table 4. Tesseral harmonics used in orbit calculation.

ℓ	m	$C_{\ell m}(\times 10^{-6})$	$S_{\ell m}(\times 10^{-6})$	Set	Reference
2	2	2.46	-1.30		
3	1	1.91	0.24		
3	2	0.77	-0.59		
3	3	0.39	1.54		
4	1	-0.54	-0.43		
4	2	0.37	0.65		
4	3	0.84	-0.20		
4	4	-0.04	0.33		
5	1	-0.10	-0.10		
5	2	0.61	-0.25		
5	3	-0.55	-0.09		
5	4	-0.22	0.06		
5	5	0.09	-0.57		
6	1	-0.04	-0.06		
6	2	0.09	-0.33	1	Gaposchkin and Izsak, 1965
6	3	-0.03	0.02		
6	4	-0.01	-0.46		
6	5	-0.28	-0.45		
8	1	0.00	0.10		
8	2	0.10	0.06		
8	4	-0.15	0.04		
9	1	0.11	0.05		
10	1	0.07	-0.12		
11	1	-0.09	0.02		
12	1	-0.10	-0.01		
13	12	0.01	-0.01		
13	13	-0.05	0.08		
15	12	-0.03	0.03		
15	13	-0.06	-0.06		
15	14	0.01	-0.02		

Table 5. Coordinates of the Baker-Nunn camera stations used in orbit calculation.

Station		Rectangular coordinates (in megameters)			Elliptical coordinates (in radians)		Set	Reference
COSPAR code	Name	X	Y	Z	ϕ	λ		
9001	New Mexico	-1. 535732	-5. 167226	3. 401154	0. 5659070	4. 4234972		
9002	South Africa	5. 056291	2. 716562	-2. 775723	-0. 4530812	0. 4930124		
9003	Australia	-3. 983755	3. 743360	-3. 275700	-0. 5428299	2. 3872948		
9004	Spain	5. 105755	-0. 555125	3. 769798	0. 6364140	6. 1748852		
9005	Japan	-3. 946681	3. 366587	3. 698891	0. 6226152	2. 4353495		
9006	India	1. 018286	5. 471276	3. 109564	0. 5124117	1. 3867868		
9007	Peru	1. 942731	-5. 804282	-1. 796795	-0. 2873626	5. 0353748	1	Veis, 1961
9008	Iran	3. 377040	4. 404061	3. 136326	0. 5172749	0. 9166290		
9009	Curaçao	2. 251817	-5. 817129	1. 327264	0. 2110292	5. 0817264		
9010	Florida	0. 976289	-5. 601610	2. 880356	0. 4716103	4. 8849429		
9011	Argentina	2. 280352	-4. 914876	-3. 355480	-0. 5575188	5. 1467990		
9012	Hawaii	-5. 466148	-2. 404268	2. 242482	0. 3614582	3. 5559712		
9001	New Mexico	-1. 535732	-5. 167225	3. 401154	0. 5659074	4. 4234971		
9002	South Africa	5. 056291	2. 716562	-2. 775723	-0. 4530812	0. 4930124		
9003	Australia	-3. 983757	3. 743372	-3. 275687	-0. 5428274	2. 3872937		
9004	Spain	5. 105791	-0. 555155	3. 769746	0. 6364040	6. 1748803		
9005	Japan	-3. 946681	3. 366587	3. 698895	0. 6226594	2. 4353495		
9006	India	1. 018303	5. 471246	3. 109595	0. 5124180	1. 3867828	2	Rolff, 1962. Internally distributed memorandum.
9007	Peru	1. 942732	-5. 804281	-1. 796792	-0. 2873622	5. 0353751		
9008	Iran	3. 377040	4. 404061	3. 136326	0. 5172751	0. 9166288		
9009	Curaçao	2. 251811	-5. 817127	1. 327282	0. 2110325	5. 0817256		
9010	Florida	0. 976289	-5. 601609	2. 880357	0. 4716108	4. 8849430		
9011	Argentina	2. 280741	-4. 914695	-3. 355481	-0. 5575189	5. 1468781		
9012	Hawaii	-5. 466148	-2. 404267	2. 242483	0. 3614585	3. 5559710		

Table 5 (Cont.)

Station		Rectangular coordinates (in megameters)			Elliptical coordinates (in radians)		Set	Reference
COSPAR code	Name	X	Y	Z	ϕ	λ		
9001	New Mexico	-1. 535702	-5. 167026	3. 401108	0. 5658951	4. 4234969	3	Gaposchkin, and Izsak, 1964. Internally distributed memorandum
9002	South Africa	5. 056123	2. 716523	-2. 775799	-0. 4530770	0. 4930260		
9003	Australia	-3. 983602	3. 743226	-3. 275656	-0. 5428274	2. 3872938		
9004	Spain	5. 105623	-0. 555194	3. 769670	0. 6363365	6. 1748626		
9005	Japan	-3. 946522	3. 366453	3. 698855	0. 6225545	2. 4354529		
9006	India	1. 018135	5. 471207	3. 109519	0. 5124156	1. 3863398		
9007	Peru	1. 942762	-5. 804082	-1. 796838	-0. 2874218	5. 0353814		
9008	Iran	3. 376872	4. 404022	3. 136250	0. 5173940	0. 9167524		
9009	Curaçao	2. 251841	-5. 816928	1. 327236	0. 2111383	5. 0817491		
9010	Florida	0. 976319	-5. 601410	2. 880311	0. 4716243	4. 8849901		
9011	Argentina	2. 280645	-4. 914512	-3. 355441	-0. 5573820	5. 1467738		
9012	Hawaii	-5. 466118	-2. 404068	2. 242437	0. 3614650	3. 5559335		
9001	New Mexico	-1. 535757	-5. 167003	3. 401057	0. 5658951	4. 4234969	4	Gaposchkin and Izsak, 1965
9002	South Africa	5. 056134	2. 716486	-2. 775825	-0. 4530770	0. 4930260		
9003	Australia	-3. 983753	3. 743116	-3. 275607	-0. 5428274	2. 3872938		
9004	Spain	5. 105603	-0. 555230	3. 769685	0. 6363365	6. 1748626		
9005	Japan	-3. 946699	3. 366295	3. 698854	0. 6225545	2. 4354529		
9006	India	1. 018201	5. 471105	3. 109619	0. 5124156	1. 3863398		
9007	Peru	1. 942769	-5. 804088	-1. 796967	-0. 2874218	5. 0353814		
9008	Iran	3. 376887	4. 403996	3. 136257	0. 5173940	0. 9167524		
9009	Curaçao	2. 251825	-5. 816922	1. 327165	0. 2111383	5. 0817491		
9010	Florida	0. 976290	-5. 601394	2. 880244	0. 4716243	4. 8849901		
9011	Argentina	2. 280568	-4. 914580	-3. 355468	-0. 5573820	5. 1467738		
9012	Hawaii	-5. 466064	-2. 404279	2. 242179	0. 3614650	3. 5559335		

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