TECHNICAL NOTE R-34

AN EVALUATION OF POTENTIAL LOCATIONS FOR AROD GROUND STATIONS

Prepared By

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ABSTRACT

Transponder stations are selected for a 105° bearing launch to a 105 NM circular orbit. The selection is based on minimizing Geometric Dilution of Precision and the provision of continuous coverage through the launch phase by a minimum of three stations with a total of five stations.

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The purpose of this technical note is to evaluate the available range stations in terms of minimum Geometric Dilution of Precision for typical Saturn trajectories on a bearing angle of 105° from true North.

Co-ordinates of typical stations in each area are evaluated in terms of a minimum elevation angle (5 degrees) for each trajectory position. This determines which stations can establish line-of-sight communications with the vehicle for each trajectory position. All combinations of such stations are then taken three at a time and root-mean-square error volumes are calculated. The groups of three are then arranged in order of least error volume. The group consistent with continuous tracking having the least error is selected.

For all stations satisfying the line of sight communication criteria, line-of-sight range, range rate, rate of change of range rate (doppler rate), azimuth angle, azimuth angle rate, elevation angle, elevation angle rate, and aspect angle (look angle) with respect to the body axis can be calculated for each trajectory position, but were not included in this note due to the limited time available and the lack of sufficient trajectory information.

All stations were positioned relative to a spherical earth; however, local earth radius may be used and local site altitude accounted for by reading these in as station co-ordinates. The trajectory used was based on a spherical, fixed earth model.

Errors in all station co-ordinates were assumed to be seven (7) meters*. Where the accuracy of survey is known for particular station, these, too, may be used in the program. Errors in range were calculated as a function of range from the results of the AROD Feasibility Study by:

$$\Delta R = 1.37 + 9.1 \times 10^{-7} R \quad \text{(meters)}$$

No effort was made to calculate range rate error or a velocity error volume for different station groupings in this technical note.

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*After an estimate made in reference 1 based on best survey accuracy currently available.

ANALYSIS

Ground Stations Considered

The required inputs for these calculations are the co-ordinates of possible ground stations, the accuracy of such co-ordinates, any station preference based on available facilities and instrumentation, and sufficient trajectory data with which to determine vehicle position relative to space-fixed co-ordinates and vehicle vector velocity as a function of position or time.

The choice of stations resulting from these calculations is based on three trajectory points due to the lack of trajectory information at this time. Subsequent calculations will attempt to check this choice as a function of other trajectory positions. The three trajectory points chosen are those for the beginning and the end of the S-IV (second stage) portion of the trajectory and the range midpoint of that portion.

The ground stations considered are required to be within line-of-sight of one of the trajectory points for a minimum elevation angle of 5 degrees. Where it is possible, stations under NASA control are considered. The positions considered are taken to be representative of an immediate area and an attempt was made to represent most areas within range of a significant portion of the trajectory. Table I gives a list of stations considered.

TABLE I

LIST OF GROUND STATIONS CONSIDERED FOR AROD TRANSPONDERS BASED ON A 105 NM CIRCULAR ORBIT LAUNCH PHASE TRAJECTORY, INITIAL BEARING ANGLE = 105°

Station	Location Lat.	Location Long.	Location Control or Equipment	Trajectory Point in Range 1	2	3
Cape Canaveral	28°27.6'N	80°33.6'W	NASA	x	x	
Grand Bahama	26°36.9'N	78°20.9'W	NASA	x	x	
Fort Myers, Fla.	26°36.9'N	81°51.9'W	NASA	x	x	
Eleuthera	25°16.1'N	76°18.8'W	SCR-584	x	x	
Charleston, S. C.	32°48.0'N	88°00.0'W		x	x	
Jupiter	27°01.2'N	80°06.8'W	NASA	x	x	
Mayaguez	18°10.5'N	67°05.3'W	SCR-584		x	x
Great Inague	20°54.0'N	73°42.0'W			x	x
San Salvador	24°04.0'N	74°32.2'W	SCR-584		x	x
Bermuda	32°21.0'N	64°39.4'W	NASA		x	x
Grand Turk	21°26.0'N	71°08.7'W	NASA		x	x
Santa Lucia	13°48.0'N	61°06.0'W				x
Antigua	17°08.6'N	61°46.7'W	NASA			x

Trajectory Point 1 - Altitude 61 km, Range 30 NM

2 - Altitude 195 km, Range 480 NM

3 - Altitude 195 km, Range 990 NM

Table I also notes those stations within range of trajectory point 1, at a ground range of 30 NM and an altitude of 61 kilometers. Note that the number of possible combinations of n stations taken c at a time is given by:

$$N = \frac{n!}{c! \ (n - c)!}$$

For trajectory point one, we must consider six (6) stations three (3) at a time for a total of 20 combinations. Also note those stations within range of trajectory point 2 (mid-range). There are 165 combinations of these 11 stations taken three at a time. Those stations within range of trajectory point 3 (separation) provide 35 combinations of 7 stations taken three at a time.

Vehicle position rms error volumes are calculated for each combination of ground stations within range of the trajectory points.

Method of Calculating Volume Error

The co-variance matrix of position error, E_p, is given by:

$$E_p = \begin{vmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{vmatrix}$$

where σ_{xx} = variance of x position error, $\sigma_{xy} = \sigma_{yx}$ = co-variance of x and y position errors. The information contained in E_p describes an error volume in space within which the vehicle lies with a specified probability.

A spherical volume with center at the computed position of the vehicle is related to E_p by:

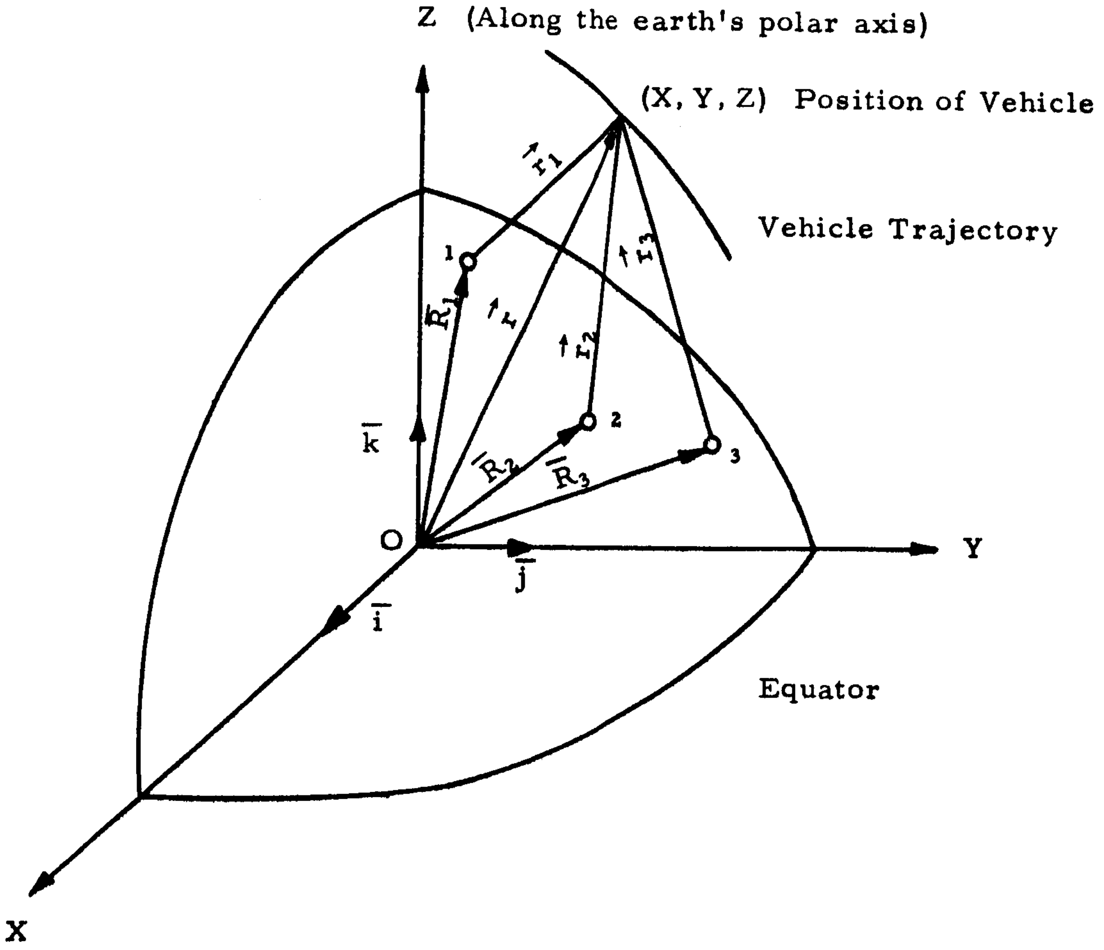
$$P_p = K \left| E_p \right|^{1/6}$$

where P_p is the radius of the sphere. The probability that the vehicle is actually in the sphere is related to the proportionality factor, K, by a chi-square distribution function.

The quantity P_p/K is used as an overall estimate of position error.

A computer program was written to calculate the elements of E_p and the value of P_p/K. Inputs to the program are three co-ordinates

Figure 1
Co-ordinate System Used in Error Analysis



X, Y, Z	Geocentric Inertial Frame
$\bar{R}_1, \bar{R}_2, \bar{R}_3$	Location vectors for the three stations, 1, 2, 3
$\vec{r}_1, \vec{r}_2, \vec{r}_3$	Range vectors from stations to vehicle
$\bar{r}$	Position vector of vehicle
(X_i, Y_i, Z_i) $i = 1, 2, 3$	Co-ordinates of the station complex
$\bar{i}, \bar{j}, \bar{k}$	Unit Vectors along X, Y, Z respectively

of vehicle position, nine co-ordinates for the locations of the three stations, the RMS errors in range measurement, and the RMS errors in the nine station co-ordinates.

P_p/K is computed as follows:

From Figure 1, the range of the vehicle relative to the three stations is given by:

$$r_i^2 = (x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 \qquad (1)$$

From equation (1) the errors in vehicle position Δx, Δy, Δz, are related to range errors Δr_1, Δr_2, Δr_3, and errors in the location of the three stations, $\left[\Delta x_i, \Delta y_i, \Delta z_i,\right] i = 1, 2, 3$, by:

$$\begin{aligned} r_1 \Delta r_1 &= (x - x_1)(\Delta x - \Delta x_1) + (y - y_1)(\Delta y - \Delta y_1) \\ &\quad + (z - z_1)(\Delta z - \Delta z_1) \end{aligned} \qquad (2)$$

$$\begin{aligned} r_2 \Delta r_2 &= (x - x_2)(\Delta x - \Delta x_2) + (y - y_2)(\Delta y - \Delta y_2) \\ &\quad + (z - z_2)(\Delta z - \Delta z_2) \end{aligned} \qquad (3)$$

$$\begin{aligned} r_3 \Delta r_3 &= (x - x_3)(\Delta x - \Delta x_3) + (y - y_3)(\Delta y - \Delta y_3) \\ &\quad + (z - z_3)(\Delta z - \Delta z_3) \end{aligned} \qquad (4)$$

The elements of E_p are determined in the following manner. After dividing equations (2), (3), and (4) by r_1, r_2, and r_3 respectively and rearranging, they become:

$$\Delta r_1 + a_{11}\,\Delta x_1 + a_{12}\,\Delta y_1 + a_{13}\,\Delta z_1 = a_{11}\,\Delta x + a_{12}\,\Delta y + a_{13}\,\Delta z \qquad (4)$$

$$\Delta r_2 + a_{21}\,\Delta x_2 + a_{22}\,\Delta y_2 + a_{23}\,\Delta z_2 = a_{21}\,\Delta x + a_{22}\,\Delta y + a_{23}\,\Delta z \qquad (5)$$

$$\Delta r_3 + a_{31}\,\Delta x_3 + a_{32}\,\Delta y_3 + a_{33}\,\Delta z_3 = a_{31}\,\Delta x + a_{32}\,\Delta y + a_{33}\,\Delta z \qquad (6)$$

where the a's are the elements of a 3 x 3 matrix M.

$$M = \begin{vmatrix} \frac{x - x_1}{r_1} & \frac{y - y_1}{r_1} & \frac{z - z_1}{r_1} \\ \frac{x - x_2}{r_2} & \frac{y - y_2}{r_2} & \frac{z - z_2}{r_2} \\ \frac{x - x_3}{r_3} & \frac{y - y_3}{r_3} & \frac{z - z_3}{r_3} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Equations (4), (5), and (6) can be written using matrix notation as:

$$M \begin{vmatrix} \Delta x \\ \Delta y \\ \Delta z \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} \Delta r_1 \\ \Delta r_2 \\ \Delta r_3 \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} \Delta x_1 \\ \Delta y_1 \\ \Delta z_1 \end{vmatrix}$$

$$+ \begin{vmatrix} 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} \Delta x_2 \\ \Delta y_2 \\ \Delta z_2 \end{vmatrix} + \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \begin{vmatrix} \Delta x_3 \\ \Delta y_3 \\ \Delta z_3 \end{vmatrix}$$

$$\text{Let } \Delta P = \begin{vmatrix} \Delta x \\ \Delta y \\ \Delta z \end{vmatrix} ; \quad M_1 = \begin{vmatrix} a_{11} & a_{12} & a_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} ; \quad \Delta S_1 = \begin{vmatrix} \Delta x_1 \\ \Delta y_1 \\ \Delta z_1 \end{vmatrix}$$

$$M_2 = \begin{vmatrix} 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 0 \end{vmatrix} ; \quad \Delta S_2 = \begin{vmatrix} \Delta x_2 \\ \Delta x_2 \\ \Delta x_2 \end{vmatrix} ;$$

$$M_3 = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} \end{vmatrix} ; \quad \Delta S_3 = \begin{vmatrix} \Delta x_3 \\ \Delta y_3 \\ \Delta z_3 \end{vmatrix} ; \quad \Delta r = \begin{vmatrix} \Delta r_1 \\ \Delta r_2 \\ \Delta r_3 \end{vmatrix}$$

Now ΔP the matrix representing error in vehicle position may be written as:

$$\Delta P = M^{-1} (\Delta r + M_1 \Delta S_1 + M_2 \Delta S_2 + M_3 \Delta S_3) \qquad (7)$$

E_p can be obtained by right multiplying both sides of equation (7) by their respective transposes and statistically averaging both sides of the resulting matrix equation.

$$E_p = \Delta P \Delta P^t = \begin{vmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{vmatrix} \qquad \text{left side of (7)}$$

To obtain the right side of (7) multiplied by its transpose, the matrix identities $(A + B)^t = A^t + B^t$, and $(AB)^t = B^t A^t$ are used to give:

$$(\Delta r + M_1\Delta S_1 + M_2\Delta S_2 + M_3\Delta S_3)(\Delta r^t + \Delta S_1{}^t M_1{}^t + \Delta S_2 M_2{}^t + \Delta S_3{}^t M_3{}^t)$$

$$= \Delta r \Delta r^t + \Delta r \Delta S_1{}^t M_1{}^t + \Delta r \Delta S_2{}^t M_2{}^t + \Delta r \Delta S_3{}^t M_3{}^t + M_1 \Delta S_1 \Delta r^t$$

$$+ M_1 \Delta S_1 \Delta S_1{}^t M_1{}^t + M_1 \Delta S_1 \Delta S_2{}^t M_2{}^t + M_1 \Delta S_1 \Delta S_3{}^t M_3{}^t + M_2 \Delta S_2 \Delta r^t$$

$$+ M_2 \Delta S_2 \Delta S_1{}^t M_1{}^t + M_2 \Delta S_2 \Delta S_2{}^t M_2{}^t + M_2 \Delta S_2 \Delta S_3{}^t M_3{}^t + M_3 \Delta S_3 \Delta r^t$$

$$+ M_3 \Delta S_3 \Delta S_1{}^t M_1{}^t + M_2 \Delta S_3 \Delta S_2{}^t M_2{}^t + M_3 \Delta S_3 \Delta S_3{}^t M_3{}^t$$

In this analysis, errors in range and in station location are assumed to be independent; therefore, all cross terms such as $M_2 \Delta S_2 \Delta S_1{}^t M_1{}^t$ vanish when statistically averaged.

E_p is now given by:

$$E_p = M^{-1} (\Delta r \Delta r^t + M_1 \Delta S_1{}^t M_1{}^t + M_2 \Delta S_2 \Delta S_2{}^t M_2{}^t + M_3 \Delta S_3 \Delta S_3{}^t M_3{}^t)(M^{-1})^t$$

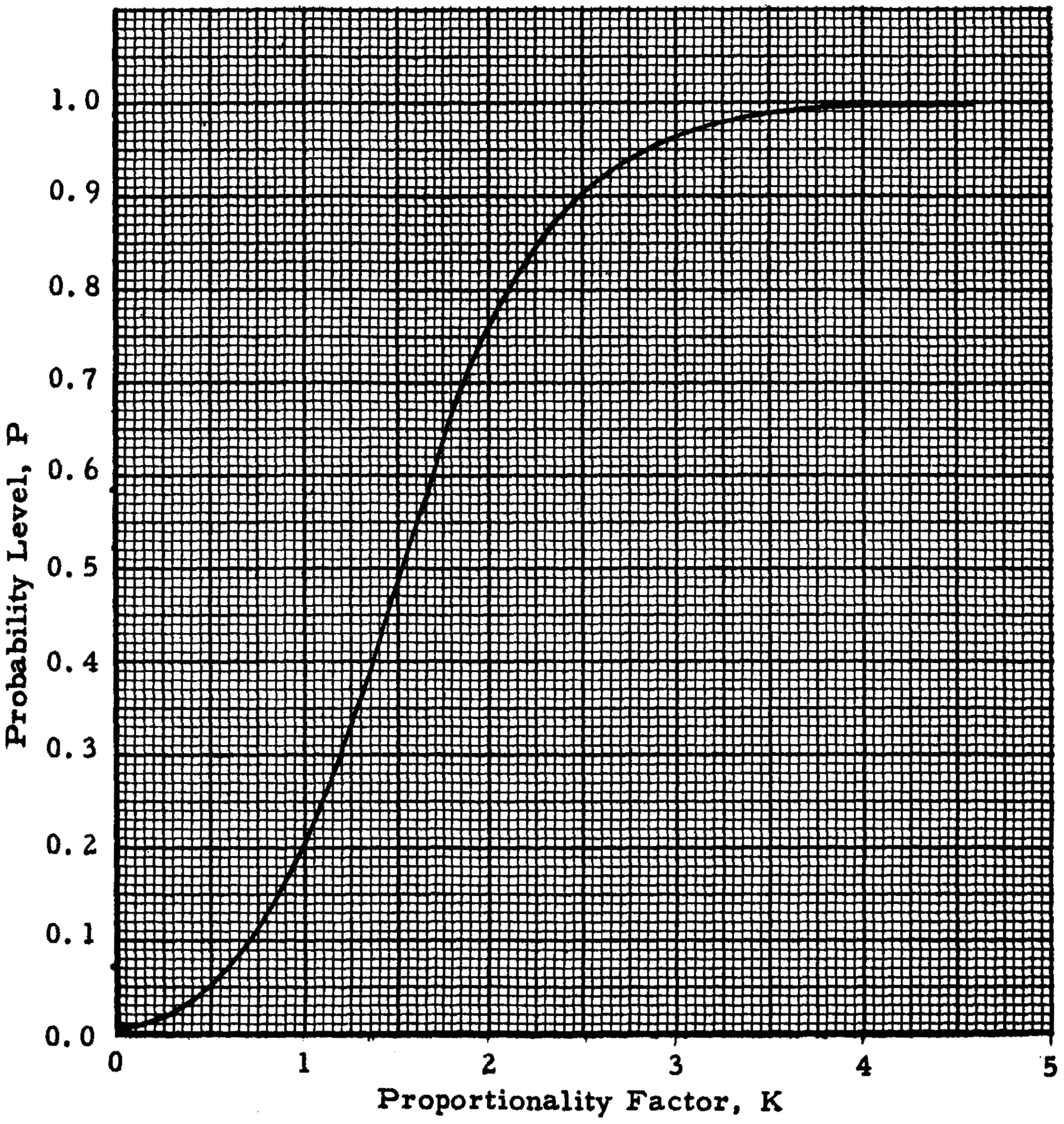
With the elements of E_p determined P_p/K can be computed by:

$$P_p/K = \left| E_p \right|^{1/6}$$

The computer program is written so that for each set of input data, P_p/K is printed out. P_p/K is related to position error volume radii for any probability level P by Figure 2. For example, with a minimum value of $P_p/K = 15$ meters, the vehicle is estimated to be within a sphere centered at that particular position and having a radius of 15 meters. The approximate probability level P is given as a function of K in Figure 2. Thus, for K = 1,

Figure 2

Chi-Square Distribution
for Three Degrees of Freedom



the probability of the vehicle being within the sphere is 20%; for K = 2, P = .75 and a sphere of radius 30 meters corresponds to a 75% probability level.

DISCUSSION OF RESULTS

Error volumes are determined from computer readouts of P_p/K for each combination of possible transponder locations. The values of P_p/K for each trajectory point are listed in the order of increasing error in Tables II, III, and IV.

For convenience in discussing the choices made from these lists based on a five station limit for continuous coverage of the Saturn S-IVB stage they are listed together in Table V. Pairs common to other trajectory points are indicated. Note that Antigua appears in the three minimum error combinations for trajectory point 3. For this trajectory portion Antigua is chosen above St. Lucia. Bermuda is common to all combinations for points 2 and 3 and is our second choice. The combination of Great Inague (GI), Bermuda and Antigua is eliminated since it has no pair of stations occurring in the minimum error group of combinations from trajectory point 2 (mid-stage). Note that there is no pair of stations common to any combinations from point 1 and point 2. This indicates that we cannot assure an overlap of ground transponder coverage between these two points with only five stations for the tracking mission. The best we can do is to select a station within range of both trajectory points 2 and 3. From trajectory point 2 we have a large choice. From trajectory point 3 our choice is limited to Bermuda, Grand Turk and San Salvador.

TABLE II

ERRORS ASSOCIATED WITH DIFFERENT COMBINATIONS OF STATIONS WITHIN RANGE OF TRAJECTORY POINT 1

Stations	P_p/K
Cape, Ft. Myers, Grand Bahama	7.735
Cape, Ft. Myers, Eleuthera	7.807
Cape, Charleston, Eleuthera	8.024
Cape, Grand Bahama, Charleston	8.038
Cape, Jupiter, Ft. Myers	8.796
Cape, Jupiter, Eleuthera	8.887
Cape, Jupiter, Grand Bahama	9.084
Ft. Myers, Grand Bahama, Charleston	9.310
Ft. Myers, Charleston, Eleuthera	9.551

Total Combinations = 20
Highest Error, $P_p/K = 22.31$
Cut-Off for Consideration, $P_p/K = 9.39$, Rank 9
Range = 30 NM Altitude = 61 km
Time = 155 sec

TABLE III

ERRORS ASSOCIATED WITH DIFFERENT COMBINATIONS OF STATIONS WITHIN RANGE OF TRAJECTORY POINT 2

Stations	P_p/K
Grand Bahama, Bermuda, Grand Turk	8.087
Eleuthera, Mayaguez, Bermuda	8.131
Cape, Bermuda, Grand Turk	8.142
Eleuthera, Bermuda, Grand Turk	8.145
Jupiter, Bermuda, Grand Turk	8.167
Mayaguez, Bermuda, San Salvador	8.261
Ft. Myers, Bermuda, Grand Turk	8.276
Grand Bahama, Mayaguez, Bermuda	8.336
Charleston, Bermuda, Grand Turk	8.357

Total Combinations = 165
Highest Error, $P_p/K = 73.770$
Cut-Off for Consideration, $P_p/K \leq 9.39$, Rank 26
Range = 990 NM Altitude = 195 km
Time = 624.5 sec

TABLE IV

ERRORS ASSOCIATED WITH DIFFERENT COMBINATIONS OF STATIONS WITHIN RANGE OF TRAJECTORY POINT 3

Stations	P_p/K
Bermuda, Grand Turk, Antigua	8.896
Great Inague Island, Bermuda, Antigua	8.914
Bermuda, San Salvador, Antiqua	9.051
Bermuda, Grand Turk, St. Lucia	9.229
Great Inague Island, Bermuda, St. Lucia	9.252
Bermuda, San Salvador, St. Lucia	9.402

Total Combinations = 35
Highest Error, $P_p/K = 34.27$
Cut-Off for Consideration, $P_p \leq 9.39$, Rank 6
Range = 990 NM Altitude = 195 km
Time = 624.5 sec

TABLE V. CHOICES OF COMBINATIONS AVAILABLE FOR CONTINUOUS COVERAGE

Trajectory Pt. 1	P_p/K	Trajectory Pt. 2		P_p/K	Trajectory Pt. 3		P_p/K
Cape Ft. Myers GBI	7.74	GBI 3 | Bermuda 3 | Grand Turk		8.09	2 | Bermuda 2 | Grand Turk Antigua		8.90
Cape Ft. Myers Eleuthera	7.8	Eleuthera ~~Mayaguez~~ ~~Bermuda~~	No pair from Pt. 3	8.13	GI ~~Bermuda~~ ~~Antigua~~	No pair from Pt. 2	8.91
Cape Charleston Eleuthera	8.02	Cape 3 | Bermuda 3 | Grand Turk		8.13	2 | Bermuda 2 | San Salvador Antigua		9.05
Cape GBI Charleston	8.04	Eleuthera 3 | Bermuda 3 | Grand Turk		8.14	Bermuda ~~Grand Turk~~ ~~St. Lucia~~		9.23
Cape Jupiter Ft. Myers	8.80	Jupiter 3 | Bermuda 3 | Grand Turk		8.14	GI ~~Bermuda~~ ~~St. Lucia~~	Antigua Best	9.25
Cape Jupiter Eleuthera	8.89	Mayaguez 3 | Bermuda 3 | San Salvador		8.26	Bermuda ~~San Salvador~~ ~~St. Lucia~~		9.40
Cape Jupiter GBI	9.08	Ft. Myers 3 | Bermuda 3 | Grand Turk		8.28			
Ft. Myers GBI Charleston	9.31	GBI ~~Mayaguez~~ ~~Bermuda~~	No pair from Pt. 3	8.34			

Bermuda is within range at t = 400 seconds, ground range = 360 NM, and goes out of range at t = 657 seconds, ground range = 1040 NM. Grand Turk comes within range at 185 NM, t = 282 seconds and goes out of range at 1040 NM, t = 657 seconds. This gives coverage at burnout and separation. San Salvador comes within range at 70 NM, t = 615 seconds but goes out of range at 955 NM, t = 615 seconds and cannot "see" the vehicle at separation (988 NM at t = 624.5 seconds).

From this analysis it is evident that the five best stations for continuous coverage with minimum error should include San Salvador for earliest acquisition but must include Grand Turk if it is necessary to "see" the separation point with three stations.

If San Salvador is selected we must go beyond the list of minimum error combination for trajectory point 2 given in Table V to find a pair common to point 3 and point 2 containing San Salvador. The first such combination is Charleston, Bermuda, San Salvador with $P_p/K = 8.38$. The next is the Cape, Bermuda, San Salvador with $P_p/K = 8.71$. The next is Jupiter, Bermuda, San Salvador with $P_p/K = 9.05$. Unless there is some over-riding reason for using Cape Canaveral as a location, the selection should be Charleston or use a combination in the initial portion of the trajectory involving both. If the line of sight from the vehicle is limited by the vehicle antenna pattern to within 70° of the trajectory path (see Figure 3), we must eliminate any consideration of Charleston because it will be outside the antenna pattern until the path angle is nearly 90°. Station Jupiter is within the pattern and we will presume that the Cape transponder station will be located down-range so that it too will be within the antenna pattern. Our selection of five stations for early track is then determined to be San Salvador, Jupiter, Cape Canaveral, Bermuda and Antigua.

If Grand Turk is selected we have several choices of pairs from trajectory point 2. Bermuda, Antigua and Grand Turk are already determined as three of the stations from trajectory point 3 criteria. Our alternatives at point 2 from line-of-sight 5° above horizon considerations with minimum error volume are Grand Bahama, Cape Canaveral, Eleuthera, Jupiter, Ft. Myers and Charleston in order of minimum error for point 2 tracking. The antenna pattern limit of 70° from trajectory eliminates Ft. Myers and Charleston. Table VI lists error volumes calculated for these combinations as seen from the point of initial track by Grand Turk. For minimum error, the logical choice is Grand Turk, Jupiter, Cape Canaveral.

TABLE VI

ERROR VOLUMES (P_p/K) ASSOCIATED WITH STATION COMBINATIONS AS SEEN FROM THE TRAJECTORY POINT INITIAL TRACK FROM GRAND TURK

Station Combinations	P_p/K
Grand Turk, Jupiter, Cape Canaveral	12.64
Grand Turk, Grand Bahama, Cape Canaveral	14.77
Grand Turk, Grand Bahama, Jupiter	15.55

Antenna beam limitations were considered on the basis of mounting and pattern problems to be -20° from the local horizon perpendicular to the trajectory. Of this, -15° was due to the necessity for mounting the receiving and transmitting antennas on opposite sides of the vehicle 15° away from that point on the vehicle which is nominally closest to the earth and -5° was due to the ± 5° roll specification on the vehicle. Assuming an antenna pattern that extends 90° on each side of its center-line, the requirement for a transponder to be simultaneously in contact with the transmitter and receiver gives a limit of 70° on the effective beam half-angle measured from the plane of the trajectory. The coverage limit due to this criteria is shown in Figure 3.

To take full advantage of the extent of coverage afforded by the requirement for a minimum elevation angle from the transponder site of 5° above the horizon, an effective beam half-angle of 75°10' is required for an altitude of 105 nautical miles. If it is possible to communicate between the vehicle and transponder on the horizon (zero degrees elevation), an effective beam half-angle of 76°01' may be utilized for a 105 nautical mile orbit. For lower altitudes, the effective beam half-angle must be larger to include the horizon. For higher altitude orbits, the effective beam half-angle may be less for full horizon-to-horizon coverage.

The bars along the trajectory line in Figure 3 indicate the zones of coverage of the indicated stations for the given trajectory. This was based on altitude versus ground range and a minimum elevation angle of 5° above the horizon. For ground ranges in excess of 900 nautical miles, this altitude is the orbital altitude of 105 nautical miles and the effective beam half-angle required is 75°10'. (See Figure 4).

SUMMARY OF CONCLUSIONS

The choices based on the criteria given may be summarized. Bermuda, Antigua, Cape Canaveral, and Jupiter are the logical choices for four of the stations. If early track is desired, the fifth station should be San Salvador. If burnout and separation must be tracked, then the fifth station must be Grand Turk.

Figure 3 shows limits imposed by the vehicle antenna location and describes the continuity of coverage by these three stations.

Figure 4 gives maximum ground range (surface distance) on a spherical earth from which the vehicle is in view at 5° above the horizon as a function of ground range from launch. Figure 5 gives the altitude of the vehicle as a function of time from launch and ground range from launch.

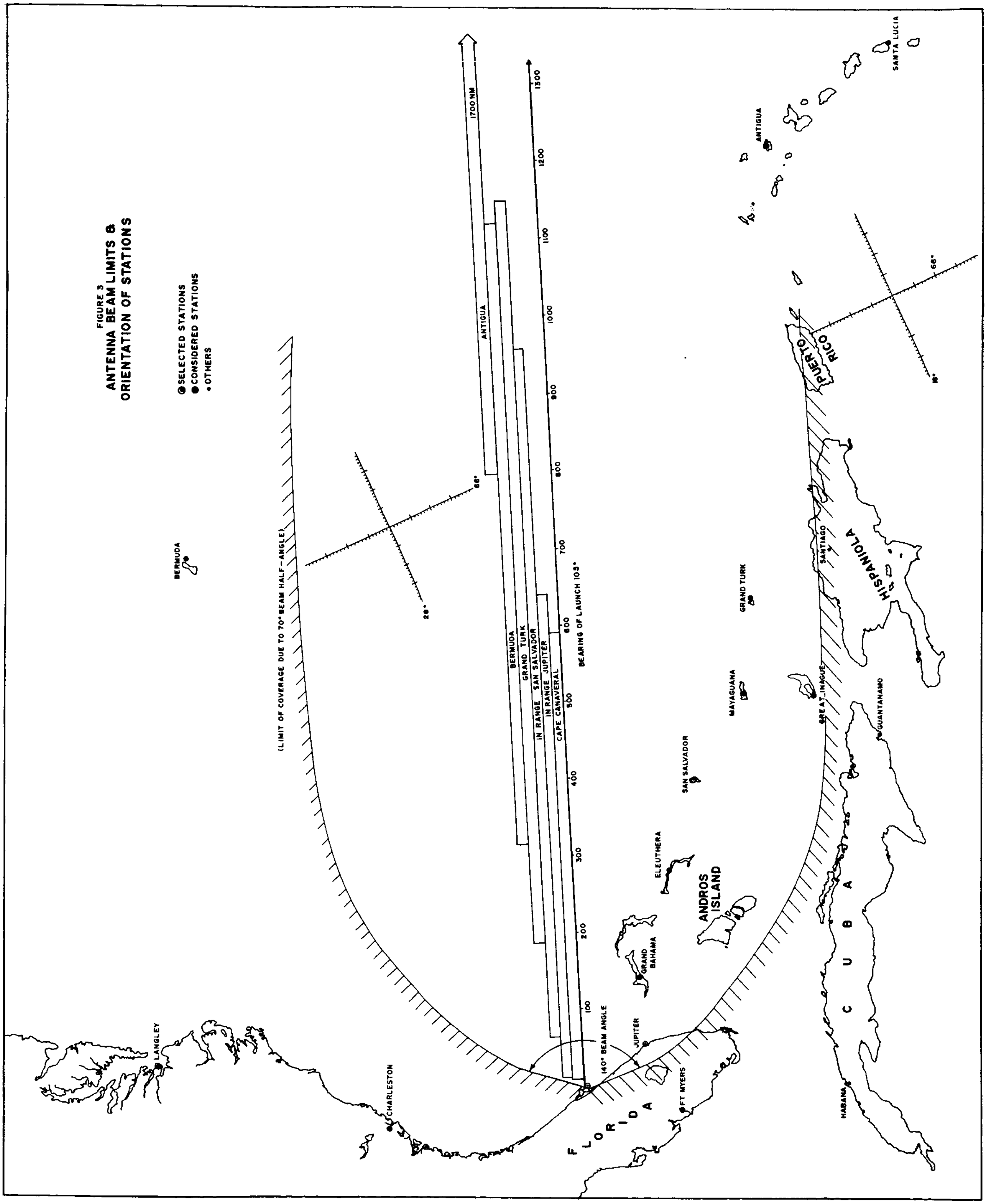
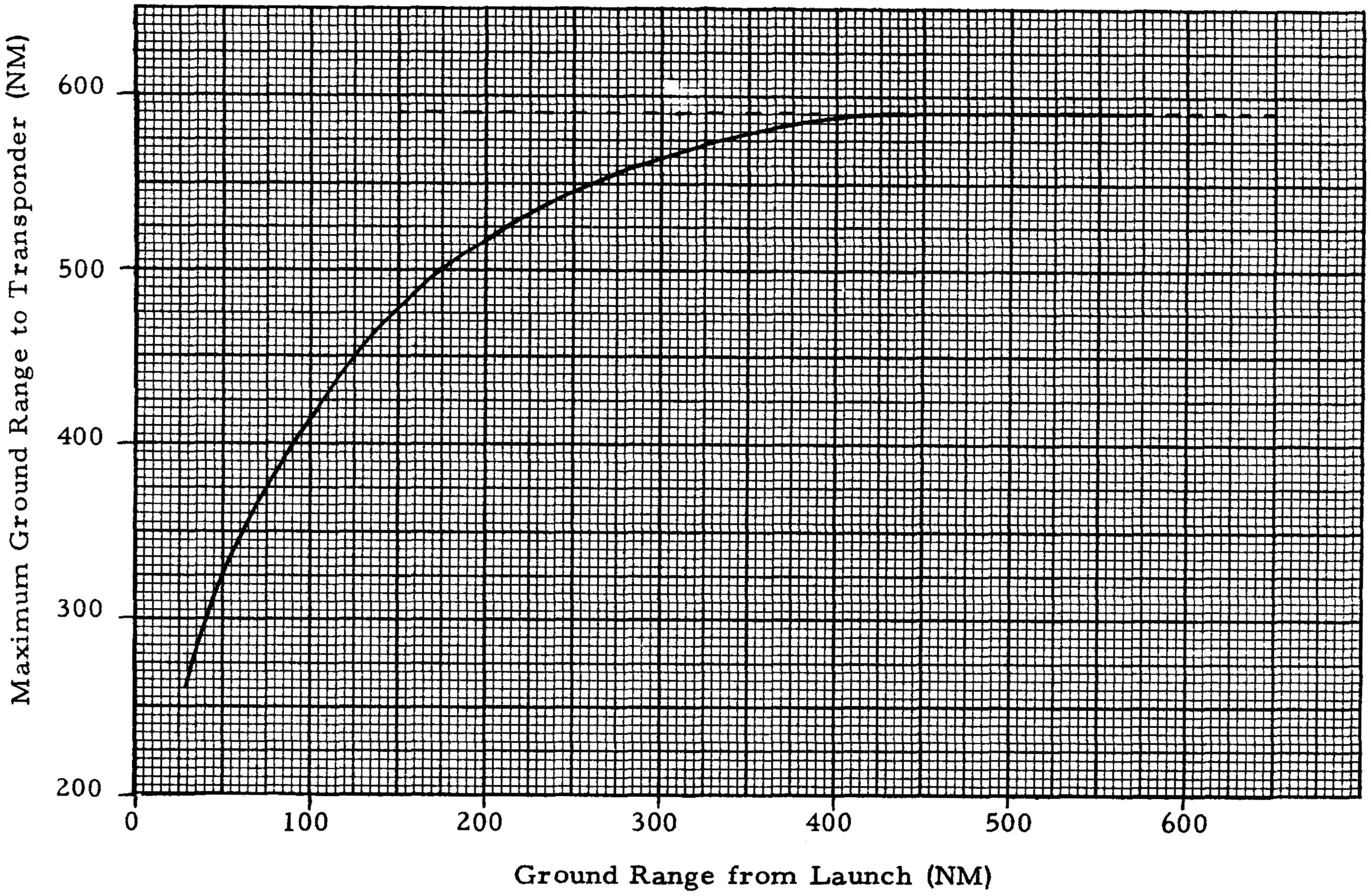


FIGURE 3
ANTENNA BEAM LIMITS & ORIENTATION OF STATIONS

Figure 4

Ground Range from Launch vs Maximum Transponder Ground Range
for the Second Stage of 105 NM Circular Orbit Mission
Based on a Minimum Elevation Angle of 5° Above the Horizon



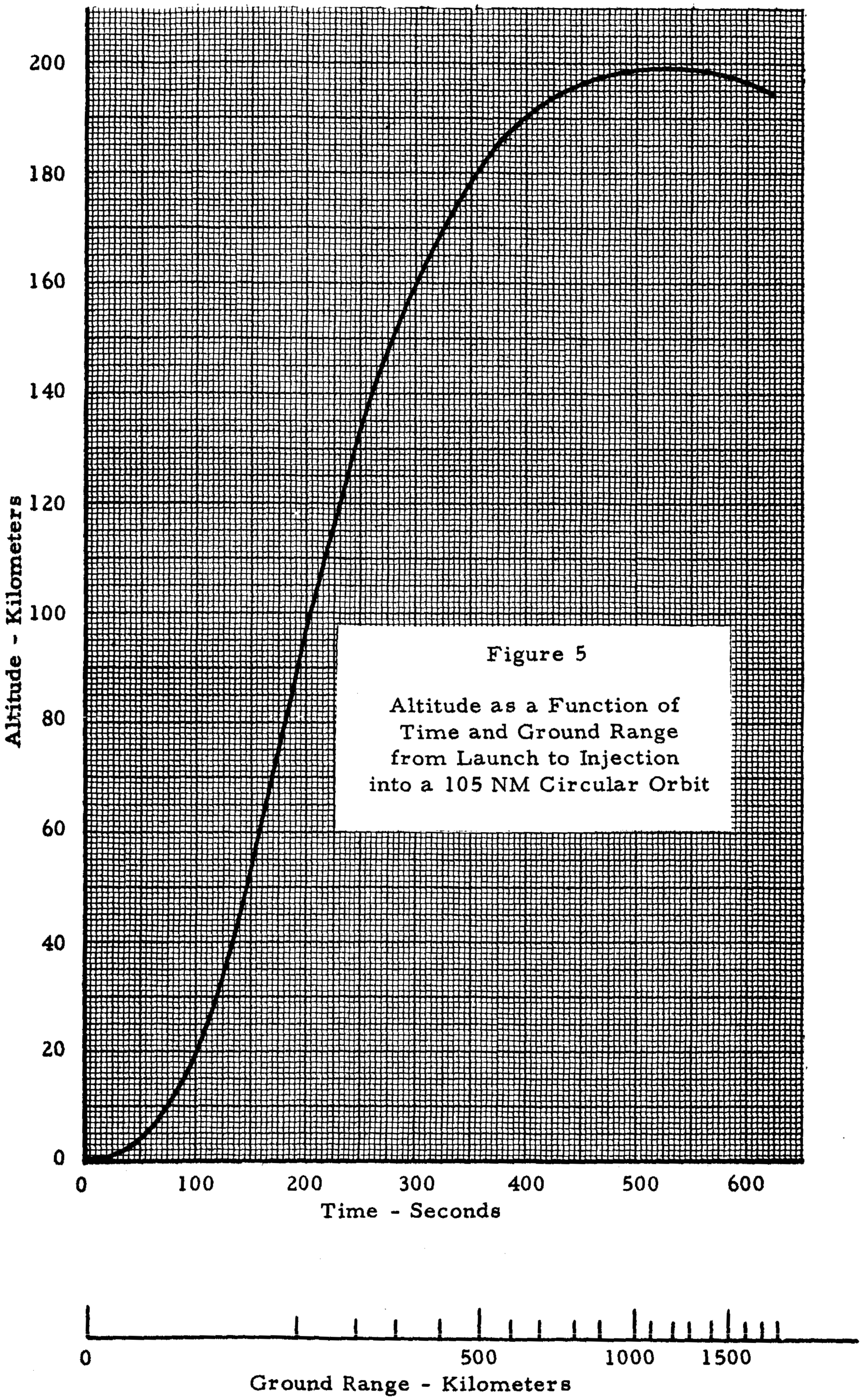


Figure 5

Altitude as a Function of Time and Ground Range from Launch to Injection into a 105 NM Circular Orbit

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SUPPLEMENT TO TECHNICAL NOTE R-34,
3 ACCURACY OF THE AROD SYSTEM IN THE EARLY MINUTES OF FLIGHT 6

N67-20409

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From Technical Memorandum R-9-63-2 by R. Holmes

In order to demonstrate the accuracy of the AROD tracking system in the early minutes of flight, error calculations were made for "close-in" trajectory points along a launch trajectory with initial bearing angle of 105 degrees. For these calculations, points were chosen along the trajectory at twenty second intervals from time equal to 157 seconds through 397 seconds. The first point is approximately 50 kilometers from launch and yet high enough, 61 kilometers, for most stations to "see"; the last is as far down range as San Salvador.

Combinations of eleven ground-based tracking stations that can cover the early part of the trajectory were used. The stations considered are Andros Central, Bermuda, Cape Canaveral, Eleuthera, Grand Bahama, Grand Turk, Jupiter, Mayaguana, New Smyrna Beach, Page Field, and San Salvador.

After the calculations for each point had been made for all combinations of stations taken three at a time, the radius of the equivalent spherical error volume for position was used as a convenient and accurate method for comparing the results (1). For each trajectory point the ten best combinations of stations were listed in the order of their equivalent radius. The selection of the best combinations of stations was based on a minimum spherical volume error with continuous coverage to obtain the AROD test objectives.

Since no combination of stations is common to the lists for all points, no one group of stations can be used to provide coverage over the early part of the trajectory. However, two combinations can provide adequate coverage. From the initial point to time equal to 257 seconds, the combination of stations that give the best overall coverage is Cape Canaveral, Grand Bahama, Page Field. At time equal to 277 seconds, Andros Central, New Smyrna Beach, San Salvador become the best choice for the rest of the points.

If New Smyrna and Page Field are to be used, additional antennas must be mounted on the missile, since they are outside the wave pattern for an antenna mounted to point down range with half-angle of 70°. There is no loss of acquisition problem. Once the antenna pattern has been aligned to include a station, the station will remain in the pattern until the spacecraft goes over the horizon.

The listings of the ten best combinations for each point with the radius of their equivalent spherical error volume are included. Also the dimensions of the semi-axes of the geometric and velocity error ellipsoids, and the angles that determine their projections into a geocentric system are listed for the suggested combinations of stations (2), (3).

Again the dimensions given correspond to a probability level of 20%. To obtain the dimensions for any probability level, multiply the dimensions by the proportionality factor K obtained from the chi-square distribution curve (see Figure 2).

The error assumptions made in these calculations were seven meters in station location, 0.5 meters/sec error in range rate determination, and range error = $1.37 + 9.1 \times 10^{-7} R$ (range in meters). Any improvement in system accuracy will, of course, yield better results, but these assumptions, according to the "AROD Feasibility Report", are realistic.

TABLE I

Combinations of Stations Giving Best Coverage

Time 157 sec		Stations	Equivalent Radius (meters)
1	*	Cape Canaveral, Grand Bahama, Page Field	7.99
2		Cape Canaveral, Andros Central, Page Field	8.39
3		Grand Bahama, New Smyrna Beach, Page Field	8.41
4		Andros Central, New Smyrna Beach, Page Field	9.13
5		Cape Canaveral, Jupiter, Page Field	9.19
6		Cape Canaveral, Grand Bahama, Jupiter	9.36
7		Jupiter, New Smyrna Beach, Page Field	9.48
8		Grand Bahama, Jupiter, New Smyrna Beach	9.76
9		Cape Canaveral, New Smyrna Beach, Page Field	10.67
10		Andros Central, Cape Canaveral, Jupiter	11.51
Time 177 sec			
1	*	Cape Canaveral, Grand Bahama, Page Field	8.19
2		Cape Canaveral, Eleuthera, Page Field	8.33
3		Grand Bahama, New Smyrna Beach, Page Field	8.36
4		Eleuthera, New Smyrna Beach, Page Field	8.63
5		Andros Central, Cape Canaveral, Page Field	8.71
6		Andros Central, New Smyrna Beach, Page Field	9.18
7		Cape Canaveral, Eleuthera, Jupiter	9.29
8		Cape Canaveral, Grand Bahama, Jupiter	9.44
9		Eleuthera, Jupiter, New Smyrna Beach	9.45
10		Grand Bahama, Jupiter, New Smyrna Beach	9.53
Time 197 sec			
1		Grand Bahama, New Smyrna Beach, Page Field	8.51
2	*	Cape Canaveral, Grand Bahama, Page Field	8.53
3		Cape Canaveral, Eleuthera, Page Field	8.74
4		Cape Canaveral, Page Field, San Salvador	8.84
5		Eleuthera, New Smyrna Beach, Page Field	8.86
6		New Smyrna Beach, Page Field, San Salvador	8.99
7		Andros Central, Cape Canaveral, Page Field	9.19
8		Andros Central, New Smyrna Beach, Page Field	9.49
9		San Salvador, Jupiter, New Smyrna Beach	9.61
10		Eleuthera, Jupiter, New Smyrna Beach	9.66

* Suggested Combination of Stations

Time 217 sec		Stations	Equivalent Radius (meters)
1		Grand Bahama, New Smyrna, Page Field	8.73
2	*	Cape Canaveral, Grand Bahama, Page Field	8.87
3		Eleuthera, New Smyrna Beach, Page Field	9.08
4		Cape Canaveral, Eleuthera, Page Field	9.08
5		Cape Canaveral, Page Field, San Salvador	9.24
6		New Smyrna Beach, Page Field, San Salvador	9.27
7		Andros Central, Cape Canaveral, Page Field	9.65
8		Andros Central, New Smyrna Beach, Page Field	9.82
9		Eleuthera, Jupiter, New Smyrna Beach	9.90
10		Grand Bahama, Jupiter, New Smyrna Beach	9.93

Time 237 sec			
1		Grand Bahama, New Smyrna Beach, Page Field	9.07
2	*	Cape Canaveral, Grand Bahama, Page Field	9.31
3		Eleuthera, New Smyrna Beach, Page Field	9.36
4		Cape Canaveral, Eleuthera, Page Field	9.45
5		New Smyrna Beach, Page Field, San Salvador	9.63
6		Cape Canaveral, Page Field, San Salvador	9.68
7		Andros Central, Cape Canaveral, Page Field	10.18
8		Andros Central, New Smyrna, Page Field	10.26
9		Eleuthera, Jupiter, New Smyrna Beach	10.30
10		Jupiter, New Smyrna Beach, San Salvador	10.42

Time 257 sec			
1		Grand Bahama, New Smyrna, Page Field	9.55
2		Eleuthera, New Smyrna Beach, Page Field	9.57
3		Cape Canaveral, Eleuthera, Page Field	9.73
4	*	Cap Canaveral, Grand Bahama, Page Field	9.87
5		New Smyrna, Page Field, San Salvador	9.88
6		Cape Canaveral, Page Field, San Salvador	10.00
7		Mayaguana, New Smyrna Beach, Page Field	10.15
8		Cape Canaveral, Mayaguana, Page Field	10.23
9		Andros Central, Cape Canaveral, San Salvador	10.59
10		Andros Central, New Smyrna Beach, San Salvador	10.61

* Suggested Combination of Stations

Time 277 sec		Stations	Equivalent Radius (meters)
1		Eleuthera, New Smyrna Beach, Page Field	9.79
2		Cape Canaveral, Eleuthera, Page Field	10.00
3		New Smyrna Beach, Page Field, San Salvador	10.13
4		Grand Bahama, New Smyrna Beach, Page Field	10.18
5		Cape Canaveral, Page Field, San Salvador	10.30
6		Mayaguana, New Smyrna, Page Field	10.44
7		Andros Central, New Smyrna Beach, San Salvador	10.53
8	*	Andros Central, Cape Canaveral, San Salvador	10.55
9		Andros Central, Eleuthera, New Smyrna Beach	10.57
10		Cape Canaveral, Grand Bahama, Page Field	10.58
Time 297 sec			
1		Eleuthera, New Smyrna Beach, Page Field	10.33
2		New Smyrna Beach, Page Field, San Salvador	10.62
3		Cape Canaveral, Eleuthera, Page Field	10.63
4		Andros Central, New Smyrna Beach, San Salvador	10.70
5		Andros Central, Grand Turk, New Smyrna Beach	10.77
6	*	Andros Central, Cape Canaveral, San Salvador	10.81
7		Andros Central, Eleuthera, New Smyrna Beach	10.85
8		Cape Canaveral, Page Field, San Salvador	10.88
9		Andros Central, Cape Canaveral, Grand Turk	10.88
10		Andros Central, Cape Canaveral, Eleuthera	10.94
Time 317 sec			
1		Andros Central, New Smyrna Beach, San Salvador	10.55
2		Eleuthera, New Smyrna Beach, Page Field	10.55
3	*	Andros Central, Cape Canaveral, San Salvador	10.68
4		Andros Central, Eleuthera, New Smyrna Beach	10.71
5		Andros Central, Grand Turk, New Smyrna	10.72
6		Andros Central, Cape Canaveral, Eleuthera	10.82
7		New Smyrna Beach, Page Field, San Salvador	10.83
8		Andros Central, Cape Canaveral, Grand Turk	10.86
9		Cape Canaveral, Eleuthera, Page Field	10.88
10		Andros Central, Grand Bahama, Grand Turk	11.08

* Suggested Combination of Stations

Time 337 sec		Stations	Equivalent Radius (meters)
1		Andros Central, New Smyrna Beach, San Salvador	10.78
2	*	Andros Central, Cape Canaveral, San Salvador	10.95
3		Andros Central, Grand Turk, New Smyrna Beach	10.96
4		Andros Central, Cape Canaveral, Grand Turk	11.14
5		Eleuthera, New Smyrna Beach, Page Field	11.19
6		Andros Central, Eleuthera, New Smyrna Beach	11.20
7		New Smyrna Beach, Page Field, San Salvador	11.24
8		Andros Central, Cape Canaveral, Eleuthera	11.35
9		Andros Central, Mayaguana, New Smyrna Beach	11.37
10		Andros Central, Grand Bahama, Grand Turk	11.50

Time 357 sec			
1		Andros Central, New Smyrna Beach, San Salvador	11.11
2		Andros Central, Grand Turk, New Smyrna Beach	11.25
3	*	Andros Central, Cape Canaveral, San Salvador	11.32
4		Andros Central, Cape Canaveral, Grand Turk	11.47
5		New Smyrna Beach, Page Field, San Salvador	11.68
6		Andros Central, Mayaguana, New Smyrna Beach	11.68
7		Andros Central, Cape Canaveral, Mayaguana	11.92
8		Andros Central, Eleuthera, New Smyrna Beach	11.93
9		Andros Central, Grand Bahama, Grand Turk	11.99
10		Eleuthera, New Smyrna Beach, Page Field	12.00

Time 377 sec			
1		Andros Central, New Smyrna Beach, San Salvador	11.08
2	*	Andros Central, Cape Canaveral, San Salvador	11.30
3		Andros Central, Grand Turk, New Smyrna Beach	11.31
4		Andros Central, Cape Canaveral, Grand Turk	11.55
5		Andros Central, Mayaguana, New Smyrna Beach	11.70
6		New Smyrna Beach, Page Field, San Salvador	11.82
7		Andros Central, Cape Canaveral, Mayaguana	11.96
8		Andros Central, Grand Bahama, Grand Turk	12.05
9		Andros Central, Grand Bahama, San Salvador	12.12
10		Cape Canaveral, Page Field, San Salvador	12.24

* Suggested Combination of Stations

Time 397 sec		Stations	Equivalent Radius (meters)
1		Andros Central, Grand Turk, New Smyrna Beach	11.75
2		Andros Central, New Smyrna Beach, San Salvador	11.76
3	*	Andros Central, Cape Canaveral, San Salvador	12.02
4		Andros Central, Cape Canaveral, Grand Turk	12.03
5		Andros Central, Mayaguana, New Smyrna Beach	12.21
6		Andros Central, Cape Canaveral, New Smyrna Beach	12.51
7		New Smyrna Beach, Page Field, San Salvador	12.52
8		Mayaguana, New Smyrna Beach, Page Field	12.52
9		Andros Central, Grand Bahama, Grand Turk	12.77
10		Grand Turk, New Smyrna Beach, Page Field	12.99

* Suggested Combination of Stations

TABLE II

Velocity and Position Data for Suggested Combinations of Stations

Time 157 sec

Missile LAT. = 28.4°;LONG. = 80.0°; Alt. = 61.0 km
Stations: Cape Canaveral, Grand Bahama, Page Field

EX = 13.3 m/sec	AX = -16.8°	ELX = 51.5°	
EY = 5.73	AY = 76.0	ELY = 28.9	GDOP
EZ = 6.72	AZ = 64.0	ELZ = 60.6	
EX = 1.07 m/sec	AX = -40.5°	ELX = 23.3°	
EY = .411	AY = 46.1	ELY = 7.91	VDOP
EZ = .472	AZ = -61.3	ELZ = 65.3	

Time 177 sec

Missile LAT. = 28.3°;LONG. = 79.7°; Alt. = 79.5 km
Stations: Cape Canaveral, Grand Bahama, Page Field

EX = 14.5 m/sec	AX = -14.3°	ELX = 1.57°	
EY = 5.63	AY = 76.4	ELY = 24.0	GDOP
EZ = 6.75	AZ = 72.2	ELZ = 65.9	
EX = 1.11 m/sec	AX = -41.5°	ELX = 23.4°	
EY = .403	AY = 44.5	ELY = 9.17	VDOP
EZ = .474	AZ = -65.3	ELZ = 64.7	

Time 197 sec

Missile LAT. = 28.2°; LONG. = 79.3°; Alt. = 96.3 km
Stations: Cape Canaveral, Grand Bahama, Page Field

EX = 16.6 m/sec	AX = -11.8°	ELX = 1.76°	
EY = 5.51	AY = 77.3	ELY = 26.2	GDOP
EZ = 6.80	AZ = 81.8	ELZ = 63.7	
EX =	AX =	ELX =	
EY =	AY =	ELY =	VDOP
EZ =	AZ =	ELZ =	

Time 217 sec

Missile LAT. = 28.1°; LONG. = 78.9°; Alt. = 111.6 km
Stations: Cape Canaveral, Grand Bahama, Page Field

EX = 18.6 m/sec	AX = -10.7°	ELX = 4.60°	
EY = 5.35	AY = 76.5	ELY = 31.1	GDOP
EZ = 7.01	AZ = 86.8	ELZ = 58.5	
EX = 1.37 m/sec	AX = -45.8°	ELX = 19.9°	
EY = .388	AY = 47.4	ELY = 8.82	VDOP
EZ = .499	AZ = -19.9	ELZ = 68.2	

Time 237 sec

Missile LAT. = 27.9°; LONG. = 78.4°; Alt. = 125.4 km
Stations: Cape Canaveral, Grand Bahama, Page Field

EX = 20.8 m/sec	AX = -10.3°	ELX = 7.44°	
EY = 5.10	AY = 75.2	ELY = 31.0	GDOP
EZ = 7.60	AZ = -88.3	ELZ = 57.9	
EX = 1.51 m/sec	AX = -48.6°	ELX = 19.2°	
EY = .376	AY = 47.8	ELY = 17.7	VDOP
EZ = .550	AZ = -2.64	ELZ = 63.6	

Time 257 sec

Missile LAT. = 27.8°; LONG. = 78.0; Alt. = 137.8 km
Stations: Cape Canaveral, Grand Bahama, Page Field

EX = 23.1 m/sec	AX = -10.3°	ELX = 8.17°	
EY = 4.85	AY = 75.0	ELY = 29.5	GDOP
EZ = 8.58	AZ = -86.4	ELZ = 59.2	
EX = 1.67 m/sec	AX = -51.3°	ELX = 19.3°	
EY = .363	AY = -47.4	ELY = 23.3	VDOP
EZ = .629	AZ = 3.15	ELZ = 59.0	

Time 277 sec

Missile LAT. 27.7°; LONG. 77.6°; Alt. 148.8 km
Stations: Andros Central, Cape Canaveral, San Salvador

EX = 31.9 m/sec	AX = -13.9°	ELX = 2.74°	
EY = 5.24	AY = 80.3	ELY = 56.8	GDOP
EZ = 7.03	AZ = 74.3	ELZ = 33.0	
EX = 2.32 m/sec	AX = -50.6°	ELX = 44.1°	
EY = .364	AY = -55.9	ELY = 45.8	VDOP
EZ = .532	AZ = 36.8	ELZ = 2.64	

Time 297 sec

Missile LAT. = 27.7°; LONG. = 77.0; Alt. = 158.5 km
Stations: Andros Central, Cape Canaveral, San Salvador

EX = 35.2 m/sec	AX = -10.5°	ELX = .377°	
EY = 5.44	AY = 80.5	ELY = 68.3	GDOP
EZ = 6.59	AZ = 79.4	ELZ = 21.7	
EX = 2.59 m/sec	AX = -50.2°	ELX = 41.4°	
EY = .379	AY = -60.2	ELY = 48.1	VDOP
EZ = .504	AZ = 35.5	ELZ = 4.93	

Time 317 sec

Missile LAT. = 27.4°; LONG. = 76.5°; Alt. = 166.9 km
Stations: Andros Central, Cape Canaveral, San Salvador

EX = 34.2 m/sec	AX = -9.34°	ELX = 10.9°	
EY = 5.59	AY = 86.0	ELY = 78.4	GDOP
EZ = 6.36	AZ = 80.4	ELZ = 11.5	
EX = 2.49 m/sec	AX = -43.2	ELX = 39.2	
EY = .393	AY = -50.5	ELY = 50.6	VDOP
EZ = .489	AZ = 43.8	ELZ = 3.61	

Time 337 sec

Missile LAT. = 27.3°; LONG. = 75.9°; Alt. = 174.1 km
Stations: Andros Central, Cape Canaveral, San Salvador

EX = 37.0 m/sec   AX = -7.40°   ELX = 30.7°
EY = 5.63   AY = 81.6   ELY = 73.5   GDOP
EZ = 6.29   AZ = 82.7   ELZ = 16.5

EX = 2.72 m/sec   AX = -48.8°   ELX = 36.9°
EY = .415   AY = -55.5   ELY = 52.9   VDOP
EZ = .476   AZ = 38.8   ELZ = 32.1

Time 357 sec

EX = 40.6 m/sec   AX = -5.91°   ELX = .396°
EY = 5.48   AY = 84.6   ELY = 54.1   GDOP
EZ = 6.52   AZ = 83.8   ELZ = 35.9

EX = 3.02 m/sec   AX = -50.0°   ELX = 34.5°
EY = .443   AY = -60.2   ELY = 55.0   VDOP
EZ = .469   AZ = 36.8   ELZ = 47.0

Time 377 sec

Missile LAT. = 26.8°; LONG. = 74.8°; Alt. = 185.3 km
Stations: Andros Central, Cape Canaveral, San Salvador

EX = 39.9 m/sec   AX = -5.84°   ELX = 1.45°
EY = 5.36   AY = 82.9   ELY = 42.1   GDOP
EZ = 6.74   AZ = 85.8   ELZ = 47.9

EX = 2.93 m/sec   AX = -50.3°   ELX = 32.4°
EY = .454   AY = 34.3   ELY = 83.7   VDOP
EZ = .486   AZ = -68.8   ELZ = 56.4

Time 397 sec

Missile LAT. = 26.7°; LONG. = 74.1°; Alt. = 189.4 km
Stations: Andros Central, Cape Canaveral, San Salvador

EX = 46.2 m/sec	AX = -4.56°	ELX = .126°	
EY = 5.10	AY = 85.5	ELY = 36.2	GDOP
EZ = 7.38	AZ = 85.3	ELZ = 53.8	

EX = 3.48	AX = -53.1	ELX = 31.0	
EY = .446	AY = 40.0	ELY = 5.21	VDOP
EZ = .553	AZ = -41.4	ELZ = 58.4	

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