https://ntrs.nasa.gov/search.jsp?R=19670013516 2020-03-16T18:34:18+00:00Z

General Disclaimer

One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.

- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.

- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.

- This document is paginated as submitted by the original source.

- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

Produced by the NASA Center for Aerospace Information (CASI)

10-047-009-1

SNAP-8

DIVISION

TM 345163-1-106
DATE 22 April 1963
W.O. 0743-05-2000
CR-72213

TECHNICAL MEMORANDUM

PREPARED BY: E. F. Perez, F. R. Cleveland,
L. Rico

ABSTRACT

SNAP-8 THIRD LOOP OPTIMIZATION

This report presents a summary of the investigations conducted to define optimum operating parameters for the SNAP-8 Third Loop. Eutectic NaK and OS-124 were considered as coolant fluids for this loop. A comparison is made between the fluids on the basis of optimum equivalent weights. Also included are analyses developed for optimizing design parameters for the heat exchanger, condenser and the flat tube and fin radiator.

FACILITY FORM 602
N67-22845 (ACCESSION NUMBER)
(THRU) 1
66 (PAGES)
(CODE) 1
CR-72212 (NASA CR OR TMX OR AD NUMBER)
22 (CATEGORY)

LIBRARY COPY
APR 14 1967
LEWIS LIBRARY, NASA
CLEVELAND, OHIO

APPROVED:

DEPARTMENT HEAD P. I. Wood   DIVISION MANAGER C. C. Ross

NOTE: This document is considered preliminary and is subject to revision as analysis progresses and additional data are acquired. The general reader may encounter internal reference not available to him.

SNAP-8 THIRD LOOP OPTIMIZATION

I. INTRODUCTION

This analysis was conducted to determine optimum third loop parameters and to make weight comparisons for systems using OS-124 and eutectic NaK as third loop fluids. The result of this analysis was a consideration in selecting NaK as the third loop fluid. The analysis was performed on a reference system which included a mercury-to-coolant fluid heat exchanger condenser and a flat tube-and-fin space radiator.

From a preliminary cycle analysis it was determined that an optimum weight system could be obtained by having the mercury condensing temperature at a lower level than would be practical from mercury pump NPSH requirements. Therefore, condensing temperature and mercury subcooling were selected to satisfy the mercury pump requirements and were held constant throughout these investigations.

II. DISCUSSION

A. THIRD LOOP ANALYSIS

The third loop analysis was a parametric study in which operating parameters and component design parameters were optimized in order to determine an optimum weight system for each of the fluids. The analysis was made by varying the coolant flow rate and the radiator inlet temperature for a specified total heat rejection load of 330 kw. The condenser and radiator were optimized for each design point. In addition, pumping power equivalent weights were evaluated and included in assessing component and system weights. The power equivalent weight was based on an estimate of 200 lb of system weight per kilowatt of electrical power.

Certain criteria were used in making this analysis. These criteria are reviewed in the following discussion in order to qualify the analytical results.

B. RADIATOR ANALYSIS

The radiator was considered to be a flat tube-and-fin configuration for both OS-124 and NaK. Because the radiator comprises a high percentage of the overall system weight, a method was developed for optimizing radiator parameters in order to have a minimum weight radiator for a given set of operating conditions. The radiator optimization analysis is included in Appendix A of this report. The actual radiator weight calculations were made with an approximate analysis which differed slightly from that described in Appendix A. An error analysis was made to determine the difference in results which could be expected if the more exact analysis were used. The results showed that the maximum difference in radiator weight could be approximately 2.6% and that the maximum difference in projected area would be approximately 5.5%. These differences, however, do not have a significant effect when making a relative comparison between two fluids since the same analysis is used for both fluids.

Some of the significant radiator criteria which were applied in this analysis are as follows:

1. Heat rejection load was 330 kw thermal
2. Optimum weight tapered fins were designed using the data of D. B. Mackay and C. P. Bacha (Reference 1)
3. Incident heat flux from solar and planetary sources were evaluated on the basis of a 500 mile earth orbit
4. Emissivity and absorbtivity of .85 and .60, respectively, were used
5. Armor thickness for micrometeorite protection was evaluated from a preliminary analysis using the Bjork penetration model and micrometeorite data from Whipple. An armor thickness of .320 in. was calculated. This armor thickness was kept constant throughout these analyses
6. Radiator parameters were optimized in each case including tube diameters and manifold diameters, lengths of tubes and manifolds, number of tubes and fin dimensions.

C. CONDENSER ANALYSIS

1. OS-124 Compact Fin and Plate Condenser

In the case of the organic OS-124 coolant, the coolant film coefficient is the controlling resistance to heat transfer. Because of the low organic film coefficients it was decided to consider the use of a compact fin and plate type heat exchanger in order to obtain as large a heat transfer area as possible. Subsequently, an analysis was made on a concentric tube heat exchanger for one design point and the results of this analysis showed that comparable equivalent weights could be obtained with a heat exchanger of this type. An analysis of the compact plate and fin type heat exchanger is covered in Appendix B. The condensers were optimized for each design point. A typical condenser optimization is shown in Figure 1 which shows the component weight and pumping power equivalent weight as a function of coolant Reynolds number.

In evaluating NaK as a third loop coolant fluid, a tube and shell type heat exchanger condenser was considered. The analysis used for evaluating the NaK condensers is described in Appendix C. The NaK condenser was also optimized for each design point. Figure 2 shows a typical optimization of the condenser as a function of NaK film coefficient and flow rate.

III. RESULTS OF THIRD LOOP OPTIMIZATION ANALYSIS

In evaluating optimum operating and component design parameters, a number of cases were computed. These cases consisted of a series of design points in which the third loop flow rate and the radiator inlet temperature were varied. First in order to determine the near optimum tube diameter for the radiator tubes, some preliminary cases were computed in which the tube diameter was varied. The results of these cases is shown on Figures 3 and 4 for OS-124 and NaK respectively. The optimum diameters were used in subsequent cases in which the flow rate and radiator inlet temperature were varied.

The results of the OS-124 and NaK optimization are summarized in Figures 4 and 5 respectively. These figures show the variation of equivalent weight (including weight of radiator, condenser, subcooler, and equivalent weight of pumping power) as a function of radiator inlet temperature and coolant flow rate. It was concluded from this analysis that the third loop fluid could not be selected on the basis of system weight since the system equivalent weights are comparable. NaK was finally selected as the working fluid on the basis of other considerations such as degradation of heat transfer properties and possible decompositions of OS-124. These considerations are discussed in Reference 2.

A summary of the optimum NaK condenser and radiator data is as follows:

Fin and Tube Condenser

Tube length	2.97 ft
Number of tubes	78
Tube inlet diameter	.348 in.
Tube outlet diameter	.313 in.
Shell diameter	3.65 in.
Tube and shell weight	25 lb
Hg inventory	5 lb
NaK inventory	21.5 lb

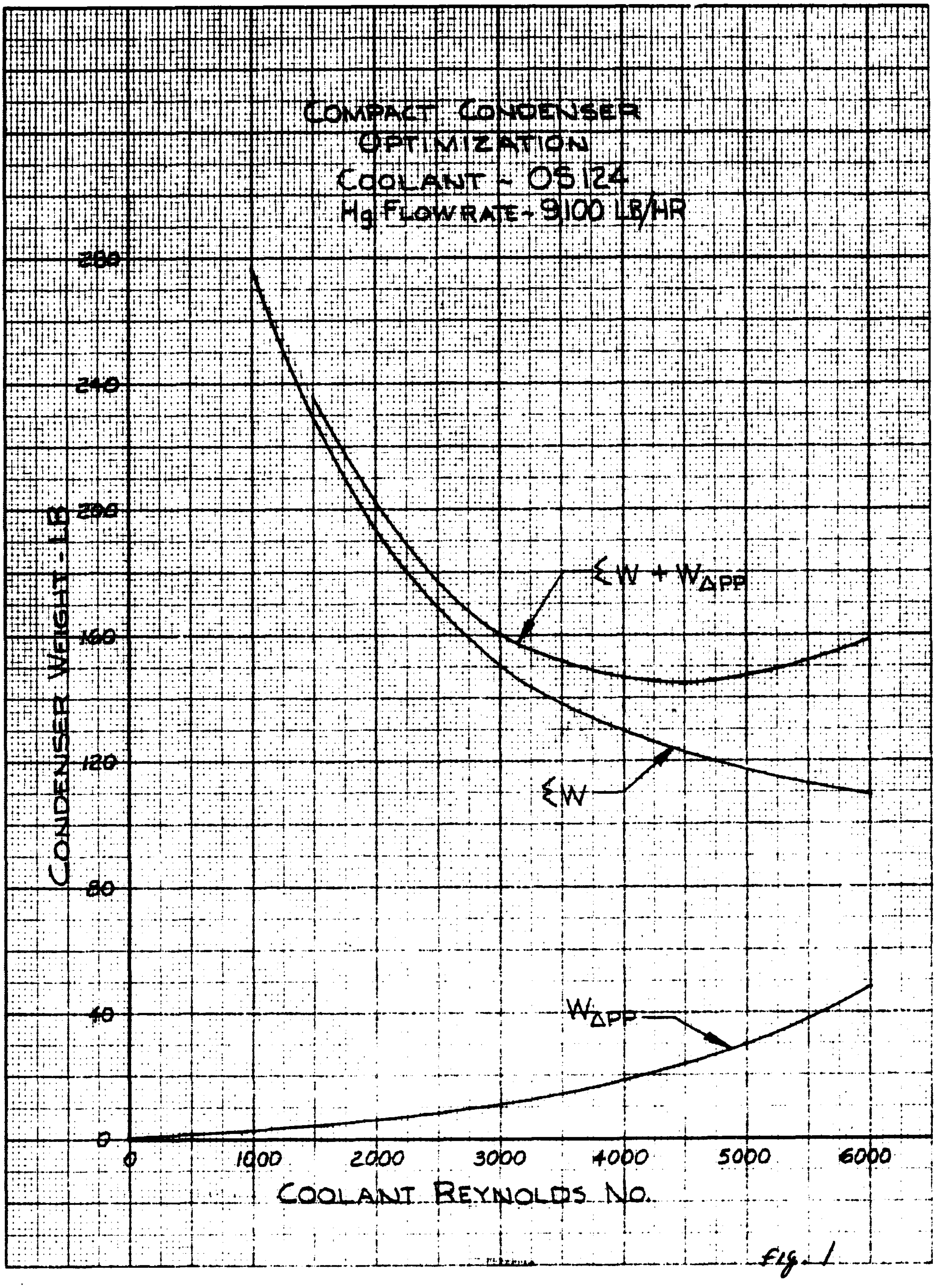
Tube and Fin Radiator

Total tube and fin length	815.9 ft
Number of tubes	38
Armor Thickness	.320 in.
Tube inside diameter	.200 in.
Inlet manifold ID	1.14 in.
Outlet manifold	.816 in.
Manifold length	15.8 ft
Fin half width	4.55 in.
Fin thickness at root	.064 in.
Fin thickness at tip	.016 in.
Weight of radiator including manifold	1030 lb

REFERENCES

1 D. B. Mackay, C. P. Bacha, Space Radiation Design and Analysis Part I, ASD Technical Report 61-30 dated October 1961.

2 Comparison of NaK and Organics as SNAP-8 Fluids, AGC Report 2413.



COMPACT CONDENSER
OPTIMIZATION
COOLANT ~ OS124
Hg FLOWRATE ~ 9100 LB/HR
CONDENSER WEIGHT - LB
280
240
200
160
120
80
40
0
0
1000
2000
3000
4000
5000
6000
COOLANT REYNOLDS NO.
ΣW + W_ΔPP
ΣW
W_ΔPP

Fig. 1

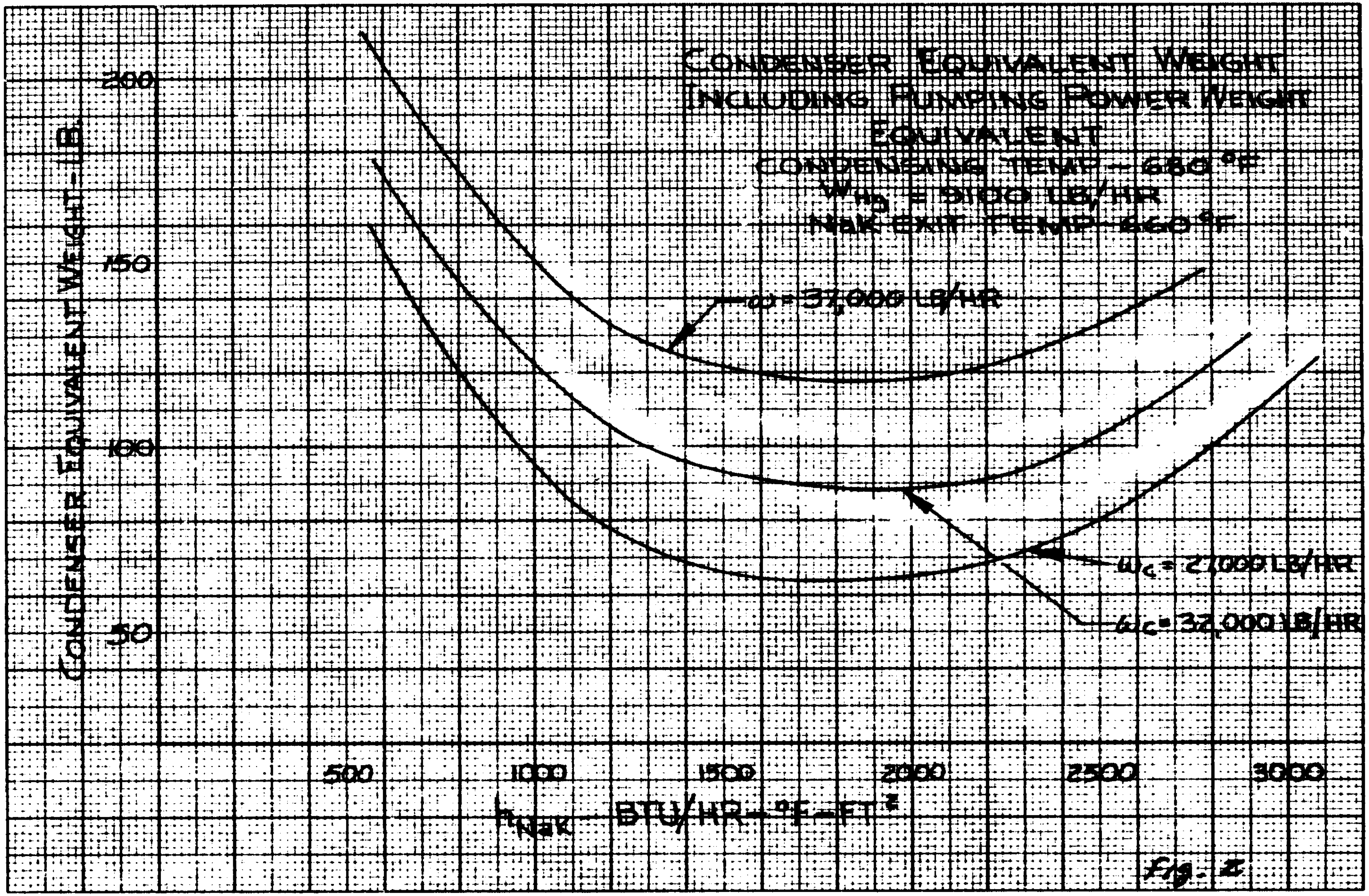

CONDENSER EQUIVALENT WEIGHT
INCLUDING PUMPING POWER WEIGHT
EQUIVALENT
CONDENSING TEMP – 680 °F
W_Hg = 9100 LB/HR
NaK EXIT TEMP 460 °F
ω = 37,000 LB/HR
ω_c = 27,000 LB/HR
ω_c = 32,000 LB/HR
200
150
100
50
CONDENSER EQUIVALENT WEIGHT – LB
500
1000
1500
2000
2500
3000
h_NaK BTU/HR – °F – FT²

Fig. II

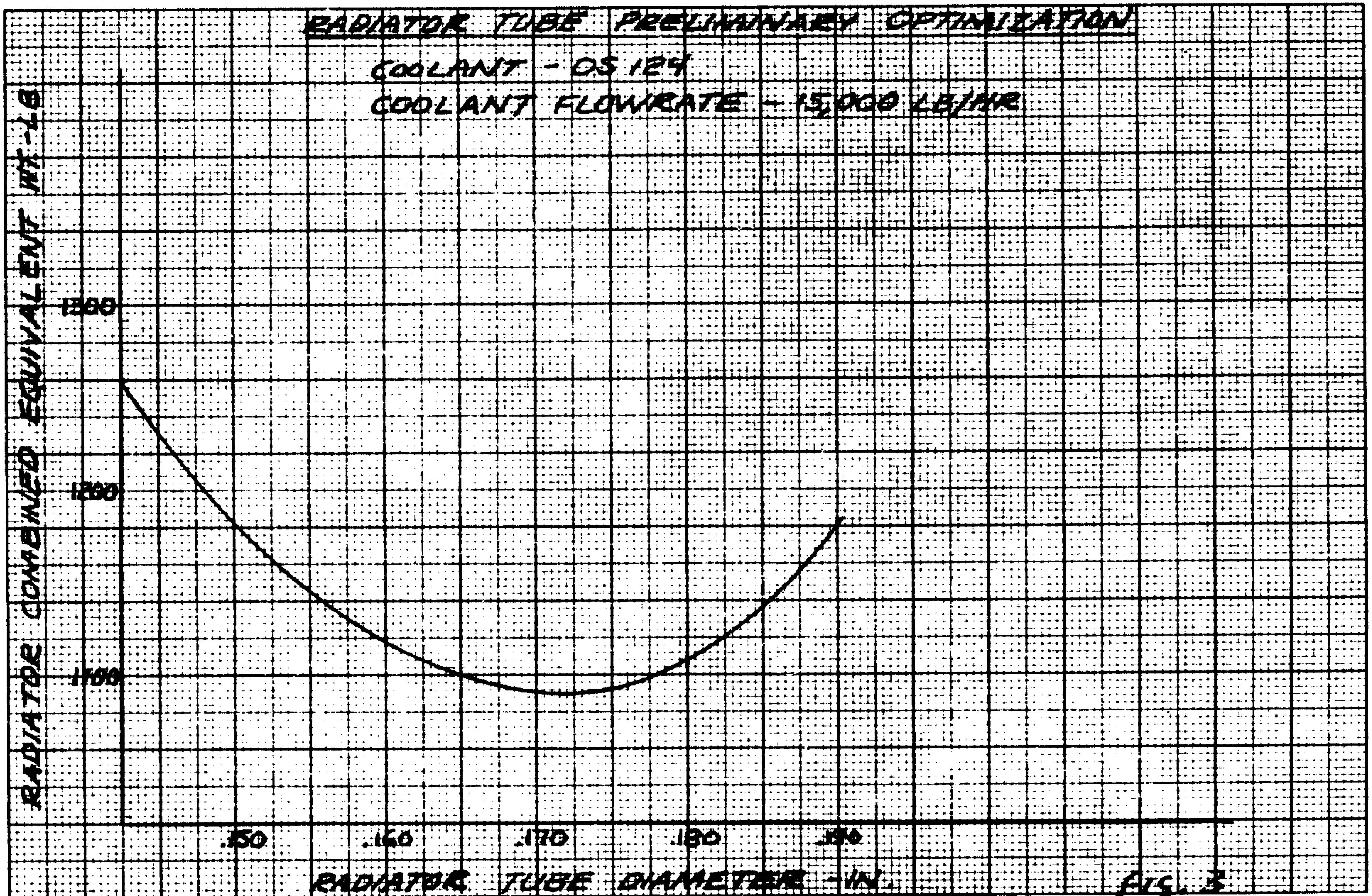

RADIATOR TUBE PRELIMINARY OPTIMIZATION
COOLANT - OS 124
COOLANT FLOWRATE - 15,000 LB/HR
RADIATOR COMBINED EQUIVALENT WT-LB
1300
1200
1100
.150
.160
.170
.180
.190
RADIATOR TUBE DIAMETER -IN.

FIG. 3

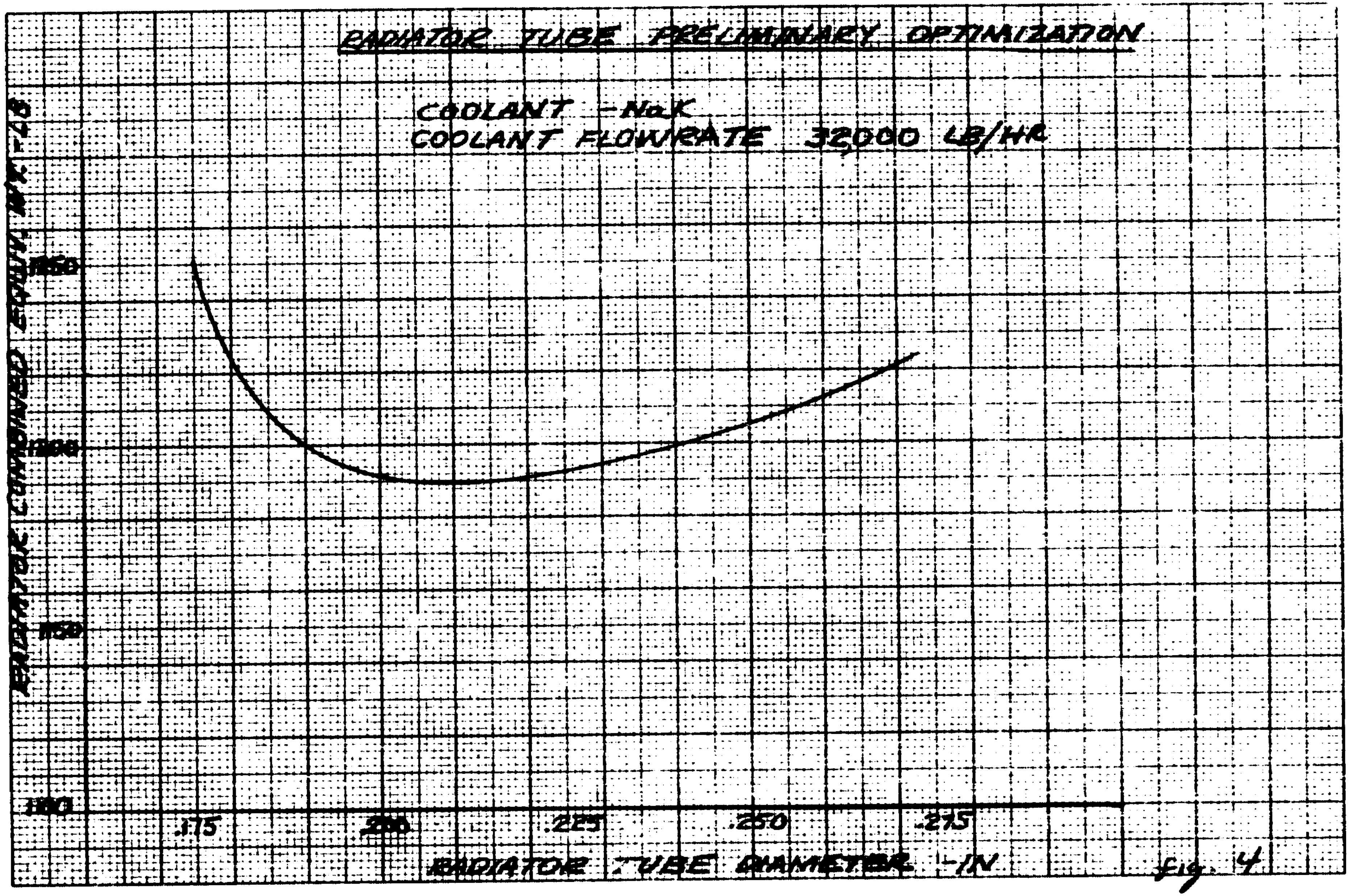

RADIATOR TUBE PRELIMINARY OPTIMIZATION
COOLANT - NaK
COOLANT FLOWRATE 32,000 LB/HR
RADIATOR COMBINED EQUIV. WT - LB
1250
1200
1150
1100
.175
.200
.225
.250
.275
RADIATOR TUBE DIAMETER - IN

Fig. 4

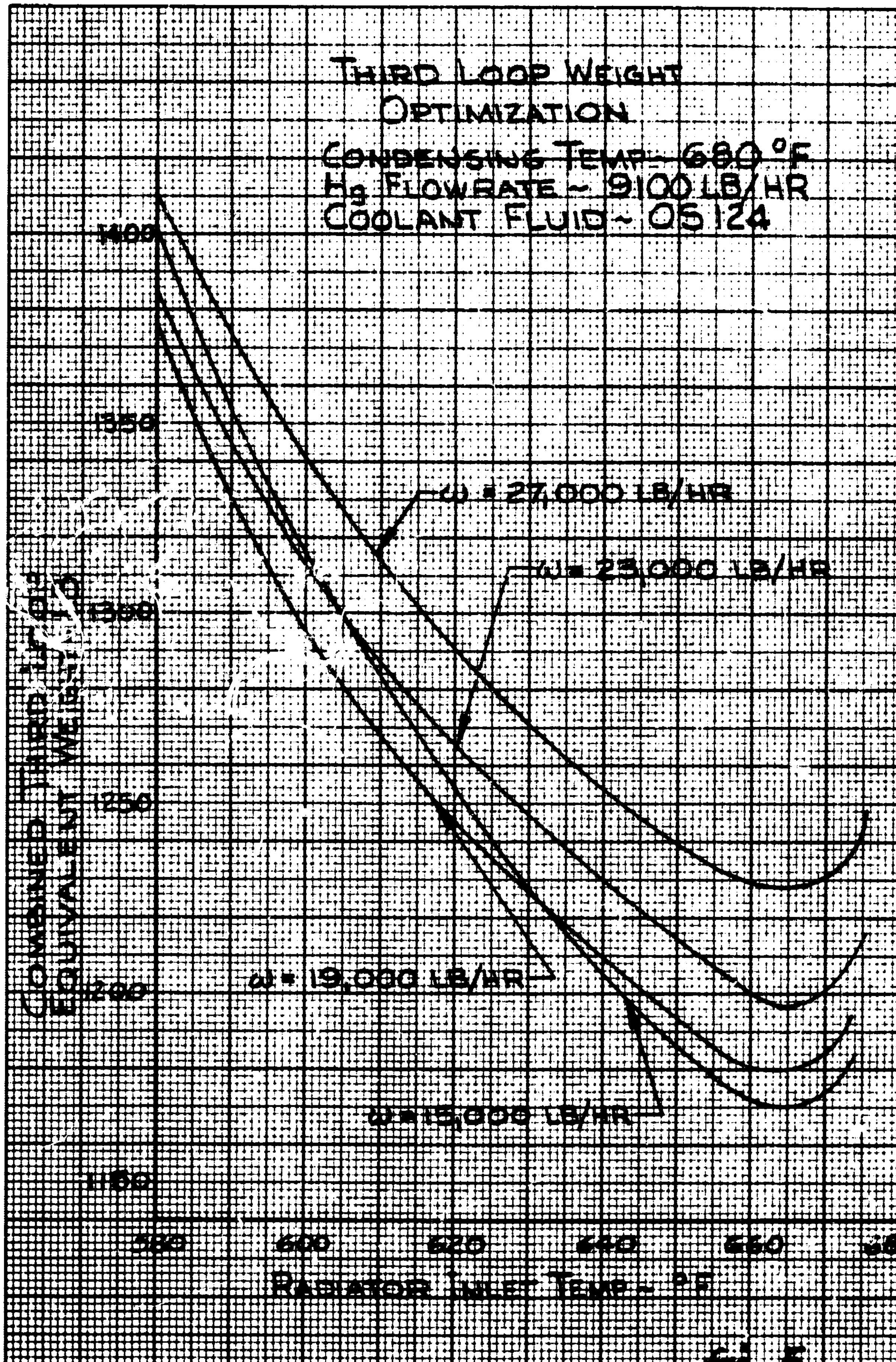
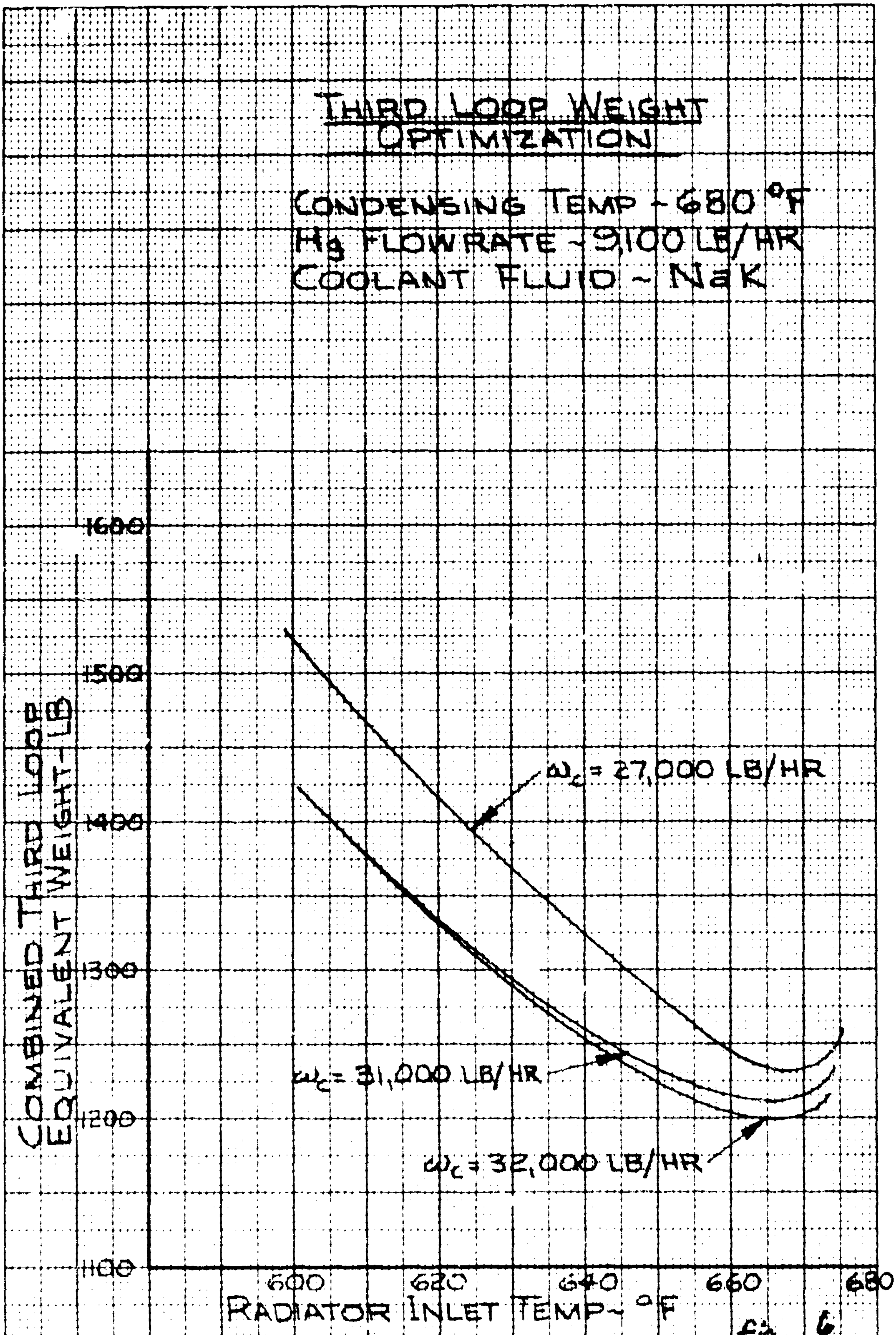

THIRD LOOP WEIGHT
OPTIMIZATION
CONDENSING TEMP ~ 680 °F
Hg FLOWRATE ~ 9100 LB/HR
COOLANT FLUID ~ OS124
1400
1350
1300
1250
1200
1150
ω = 27,000 LB/HR
ω = 23,000 LB/HR
ω = 19,000 LB/HR
ω = 15,000 LB/HR
COMBINED THIRD LOOP
EQUIVALENT WEIGHT
580
600
620
640
660
680
RADIATOR INLET TEMP ~ °F

Fig. 5



THIRD LOOP WEIGHT OPTIMIZATION
CONDENSING TEMP ~ 680 °F
Hg FLOWRATE ~ 9,100 LB/HR
COOLANT FLUID ~ NaK
ω_c = 27,000 LB/HR
ω_c = 31,000 LB/HR
ω_c = 32,000 LB/HR
1600
1500
1400
1300
1200
1100
600
620
640
660
680
COMBINED THIRD LOOP EQUIVALENT WEIGHT - LB
RADIATOR INLET TEMP ~ °F

Fig. 6

APPENDIX A

DETERMINATION OF METHOD FOR WEIGHT OPTIMIZATION OF RADIATOR PARAMETERS

THE OBJECT OF THIS ANALYSIS IS TO PROVIDE A METHOD FOR SELECTING RADIATOR PARAMETERS WHICH WILL RESULT IN A MINIMUM WEIGHT FOR A SPECIFIED HEAT REJECTION RATE AND TEMPERATURE LEVEL.

THE FIRST PART OF THIS ANALYSIS IS CONCERNED WITH THE DESIGN OF FINS AND ARMORED TUBES. THE SECOND PART OF THE ANALYSIS IS CONCERNED WITH THE DETERMINATION OF THE NUMBER OF TUBES AND SELECTION OF MANIFOLD PARAMETERS FOR MINIMUM WEIGHT.

APPLICATION OF THIS ANALYSIS WILL RESULT IN AN IDEAL RADIATOR CONFIGURATION. IT IS RECOGNIZED THAT FROM PRACTICAL CONSIDERATIONS, SUCH AS STRESS, AND PHYSICAL ARRANGEMENTS, THE IDEAL PARAMETERS WILL HAVE TO BE MODIFIED TO SOME EXTENT. HOWEVER; THE IDEAL OPTIMUM CONFIGURATION CAN BE USED AS A REFERENCE FOR, 1) EVALUATION OF OTHER MORE PRACTICAL RADIATOR CONFIGURATIONS, AND 2) EVALUATING OPTIMUM OPERATING PARAMETERS FOR THE SNAP-8 EGS.

THE DATA OF MACKAY AND BACHA REF. 1 ARE USE FOR EVALUATING HEAT REJECTION FROM THE FINS.

REF. 1 D.B MACKAY, C.P. BACHA, "SPACE RADIATOR DESIGN & ANALYSIS PART I ASD TECHNICAL REPORT 61-30 OCT. 1961

NOMENCLATURE AND SYMBOLS

A RADIATOR PROJECTED AREA

A_x FLOW CROSS SECTIONAL AREA AT x ON MANIFOLD

A_t TUBE FLOW CROSS SECTIONAL AREA

A_1, A_2, A_3 ETC SHORT HAND NOTATIONS IN WEIGHT EQUATIONS

a, b, c, e SHORT HAND NOTATIONS DEFINED IN P. 5

C_1 CONSTANT USED IN REF. 1 $= \sigma(\epsilon_a + \epsilon_b)$ WHERE ϵ_a & ϵ_b ARE THE EMMISIVITIES ON EITHE SIDE OF THE RADIATOR

C_2 CONSTANT USED IN REF. 1 WHICH IS EQUIVALENT TO THE ABSORBED EXTERNAL HEAT FLUX

D_m INLET TUBE DIAMETER AT INLET MANIFOLD

D_o DIAMETER AT OUTER SURFACE OF TUBE ARMOR

D_t TUBE INSIDE DIAMETER

E_p ENVIRONMENTAL PARAMETER IN REF 1 $= \frac{C_2}{C_1 T_h^4}$

f TUBE FRICTION FACTOR

K_{al} THERMAL CONDUCTIVITY OF ALUMINUM

K_H CONSTANT RELATING HYDRAULIC PUMPING POWER TO SYSTEM EQUIVALENT WEIGHT

K_1, K_2 ---- SHORT HAND NOTATIONS IN WEIGHT EQUATIONS

L_h FIN HALF WIDTH

L_W LENGTH OF TUBE AND FIN

L_m LENGTH OF MANIFOLD

L SYMBOL DENOTING LENGTH

N_1 NUMBER OF TUBES IN ONE HALF OF ON PANEL

N_T TOTAL NUMBER OF TUBES IN RADIATOR

q TOTAL HEAT REJECTED BY TUBES & FINS

q_F HEAT REJECTED FROM FINS

q_T HEAT REJECTED FROM TUBE

P_{PM} HYDRAULIC POWER FOR CIRCULATION THROUGH THE MANIFOLDS

P_{PT} HYDRAULIC POWER FOR CIRCULATION THROUGH THE TUBES

S SPAN BETWEEN TUBES

T_h TEMPERATURE AT OUTER SURFACE OF ARMOR & AT THE BASE OF THE FINS

t_t THICKNESS OF STAINLESS STEEL TUBE

W_R TOTAL WEIGHT OF TUBE AND FINS

W_T WEIGHT PER FOOT OF TUBE AND ARMOR.

W_{m_1} WEIGHT OF INLET MANIFOLD

W_{m_2} WEIGHT OF OUTLET MANIFOLD

W_{ppm_1} EQUIVALENT WEIGHT FOR PUMPING POWER IN INLET MANIFOLD

W_{ppm_2} EQUIVALENT WEIGHT FOR PUMPING POWER IN RETURN MANIFOLD.

W_{ppT} EQUIVALENT WEIGHT FOR PUMPING POWER IN TUBES

W_{ppL} EQUIVALENT WEIGHT FOR MANIFOLD-TUBE INLET & OUTLET LOSSES

W_{RE} TOTAL RADIATOR EQUIVALENT WEIGHT.

W_c TOTAL FLUID FLOWRATE

W_o FLUID FLOWRATE IN ONE HALF OF RADIATOR

W_x FLOW IN MANIFOLD AS A FUNCTION OF DISTANCE

W_t COOLANT FLOWRATE PER TUBE

ΔP_m PRESSURE DROP ACROSS MANIFOLD

ΔP_A PRESSURE DROP IN INLET MANIFOLD BETWEEN TUBES

ΔP_B PRESSURE DROP IN RETURN MANIFOLD BETWEEN TUBES

$\Delta P_c = \Delta P_A + \Delta P_B$

ϵ_a EMISSIVITY OF SURFACE FACING SUN

ϵ_b EMISSIVITY OF SURFACE FACING AWAY FROM SUN

δ_h THICKNESS OF FIN AT THE ROOT

δ_c THICKNESS OF FIN AT THE COLD EDGE

ξ_p PROFILE NUMBER AS DEFINED IN REF. 1

ρ_a DENSITY OF ALUMINUM

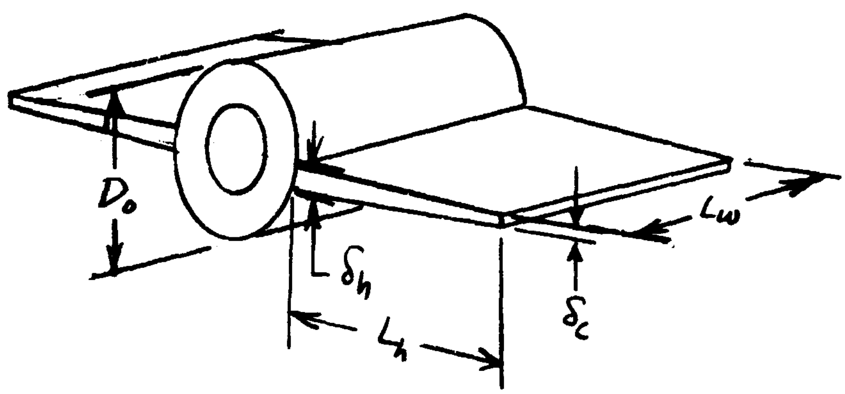
ρ_c DENSITY OF COOLANT FLUID

ρ_t DENSITY OF STAINLESS STEEL TUBE

Ω FIN EFFECTIVNESS – RATIO OF ACTUAL HEAT RADIATED FROM FIN TO HEAT WHICH WOULD BE REJECTED IF FIN WAS AT CONSTANT TEMPERATURE

TUBE AND FIN OPTIMIZATION

THE RADIATOR TUBE AND FIN ARE ANALYZED BY LOOKING AT SEVERAL EQUAL HEAT REJECTION NODES IN WHICH THE TEMPERATURE DROPS AS A FUNCTION OF THE HEAT REJECTED FROM EACH NODE. THE TUBE AND FIN MODEL ARE SHOWN ON THE FOLLOWING DIAGRAM



HEAT REJECTION FROM THE TUBE AND FIN

$$q = q_F + q_T$$

(1) $$q = \left(\frac{C_1}{2} T_h^4 \pi D_o - D_o\right) L_w + 2C_1 \Omega L_h L_w T_h^4$$

WHERE
$C_1 = \sigma(\epsilon_a + \epsilon_b)$
C_2 = EXTERNAL ABSORBED HEAT FLUX
σ = STEFAN BOLTZMAN CONSTANT
Ω IS THE FIN EFFECTIVENESS WHICH IS A FUNCTION OF SEVERAL PARAMETERS INCLUDING THE EXTERNAL HEAT FLUX AND IS EVALUATED USING THE DATA OF MACKAY & BACHA REF. 1

WEIGHT OF FIN AND TUBE

$$W = w_T L_w + \rho_1 L_h L_w \delta_h \left(1 + \frac{\delta_h}{\delta_h}\right) \qquad (2)$$

FROM REF 1.

$$\delta_h = \frac{C_1 T_h^3 L_h^2}{K \xi_p}$$

SUBSTITUTING INTO EQN (2)

$$W = w_T L_w + \frac{\rho_1 C_1 T_h^3 L_h^3 L_w}{K_{al} \xi_p} \left(1 + \frac{\delta_o}{\delta_h}\right) \qquad (2a)$$

WHERE: W_T = WEIGHT PER FOOT OF TUBE AND ARMOR

EQNS (1) and (2a) CAN BE SIMPLIFIED TO THE FORM

$$q = a L_w + b L_w L_h \qquad (3)$$

$$w = c L_w + e L_h^3 L_w \qquad (4)$$

SPECIFYING: q (CONST), $\frac{\delta_c}{\delta_h}$, T_h, Ω, ξ_o, E_p, C_2

WE DIFFERENTIATE EQN (3) WRT L_h

$$0 = (a + bL_h)\frac{dL_w}{dL_h} + bL_w$$

$$\frac{dL_w}{dL_h} = \frac{bL_w}{(a + bL_h)} \qquad (5)$$

DIFFERENTIATING (4) WRT TO L_h

$$\frac{dW}{dL_h} = c\frac{dL_w}{dL_h} + 3eL_h^2 L_w + eL_h^3\frac{dL_w}{dL_h}$$

$$\frac{dW}{dL_h} = (c + eL_h^3)\frac{dW}{dL_h} + 3eL_h^2 L_w \qquad (6)$$

SUBSTITUTE EQN (5) INTO (6) AND FOR MINIMUM WEIGHT SET EQN (6) TO ZERO

$$\frac{dW}{dL_h} = bL_w\left(\frac{c + eL_h^3}{a + bL_h}\right) + 3eL_h^2 L_w = 0$$

FROM WHICH $L_h^3 + \frac{3}{2}\frac{a}{b}L_h^2 - \frac{c}{2e} = 0$ (7)

THIS EQ'N. DEFINES AN OPTIMUM FIN WIDTH

FROM EQ'N (3)

$$L_w = \frac{q}{a + bL_h}$$ OPTIM. LENGTH (8)

RE ARRANGING EQN (7) AND COMBINING WITH EQN (4) IT CAN BE SHOWN THAT THE OPTIMUM COMBINED TUBE AND FIN WEIGHT IS

$$W_R = \frac{3}{2} W_T L_\omega \left[\frac{bL_h + a}{bL_h + \frac{3}{2}a}\right]$$ (9)

WHERE:

$$a = \left(\frac{C_1}{2} T_h^4 \pi D_0 - C_2 D_0\right)$$

$$b = 2C_1 \Omega$$

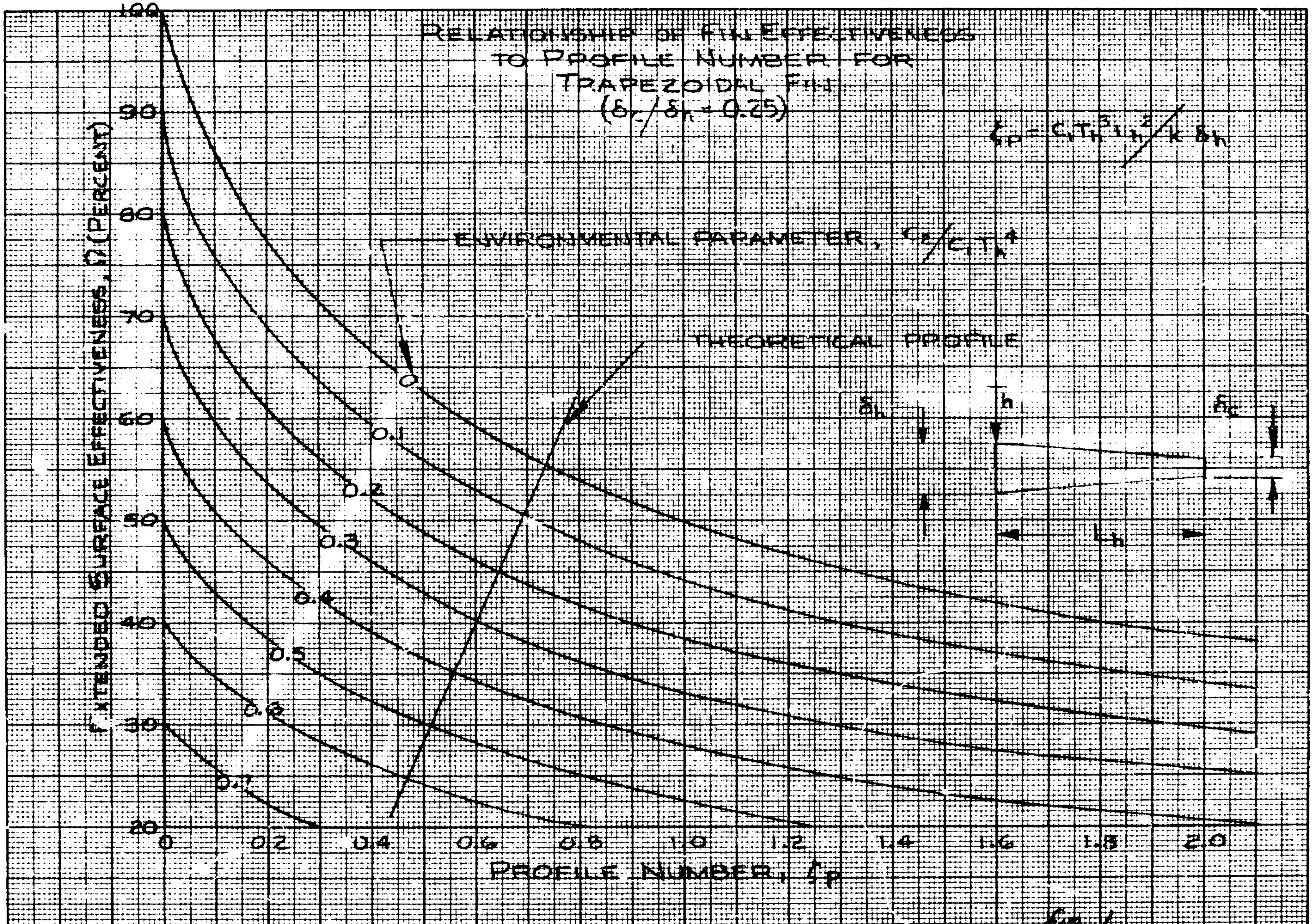
$$c = W_T$$

$$e = \frac{\rho_{al} C_1 T_h^3 \left(1 + \frac{\delta_c}{\delta_h}\right)}{K_{al} \xi_f}$$

PROJECTED AREA OF FIN & TUBE

$$A = D_0 L_\omega + 2 L_\omega L_h$$

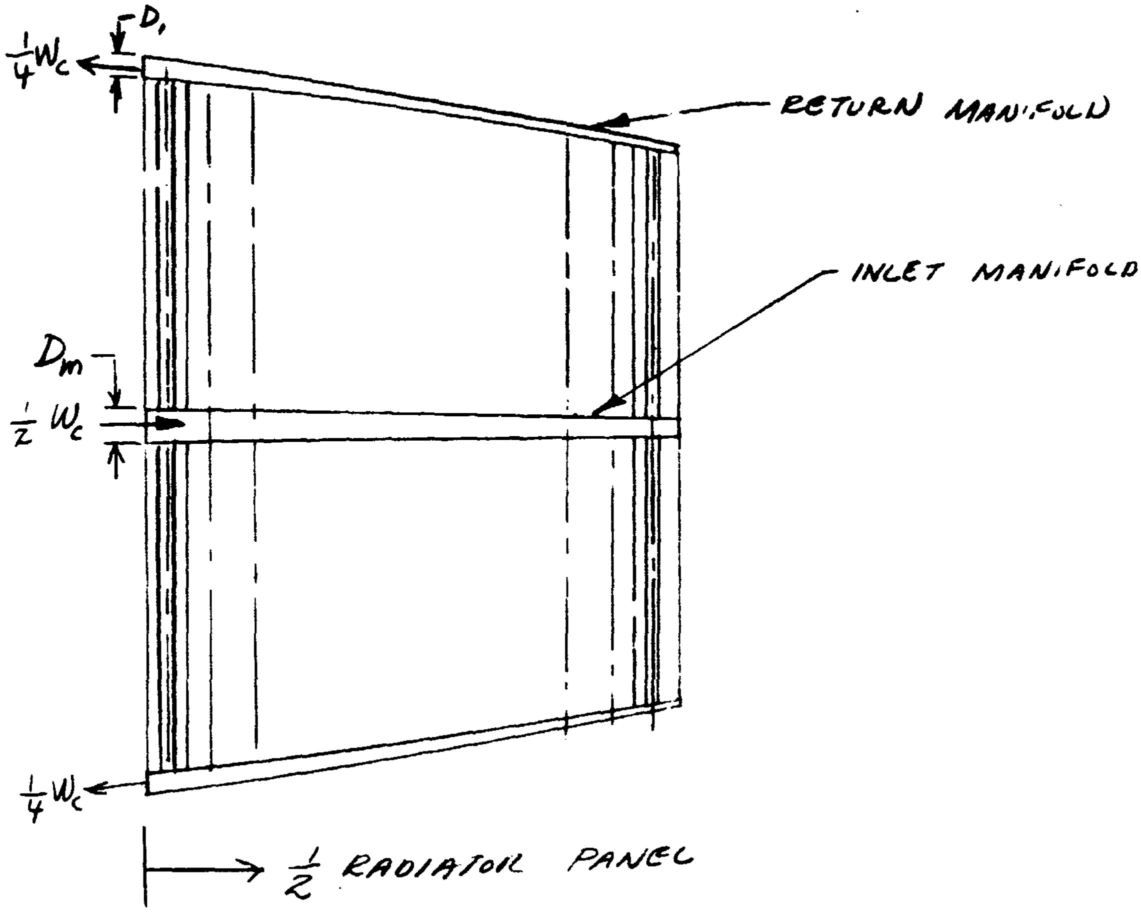
THE DATA OF MACKAY & BACHA IS ILLUSTRATED IN FIGURE 1 WHICH SHOWS FIN EFFECTIVNESS AS A FUNCTION OF SEVERAL PARAMETERS, THE THEORETICAL PROFILE LINE IS DEFINED AS A LOCI OF POINTS WHICH DESCRIBES A MINIMUM WEIGHT FIN FOR A GIVEN ENVIRONMENTAL PARAMETER. DATA ALONG THE THEORETICAL PROFILE LINE ARE USED IN OPTIMIZING TUBE AND FIN DIMENSIONS IN THE PRECEDING ANALYSIS.



RELATIONSHIP OF FIN EFFECTIVENESS
TO PROFILE NUMBER FOR
TRAPEZOIDAL FIN
$(\delta_c/\delta_h = 0.25)$
$\xi_p = C_1 T_h^3 L_h^2 / k\,\delta_h$
ENVIRONMENTAL PARAMETER, $C_2/C_1T_h^4$
THEORETICAL PROFILE
δ_h
h
δ_c
L_h
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
EXTENDED SURFACE EFFECTIVENESS, η (PERCENT)
100
90
80
70
60
50
40
30
20
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
PROFILE NUMBER, ξ_p
Fig. 1

RADIATOR MANIFOLD OPTIMIZATION

IN OPTIMIZING THE RADIATOR A BASIC CONFIGURATION MUST BE ASSUMED. IN THIS ANALYSIS THE RADIATOR IS ASSUMED TO BE IN A FLAT CONFIGURATION WITH THE TUBES AND MANIFOLDS ARRANGED AS SHOWN BELOW.

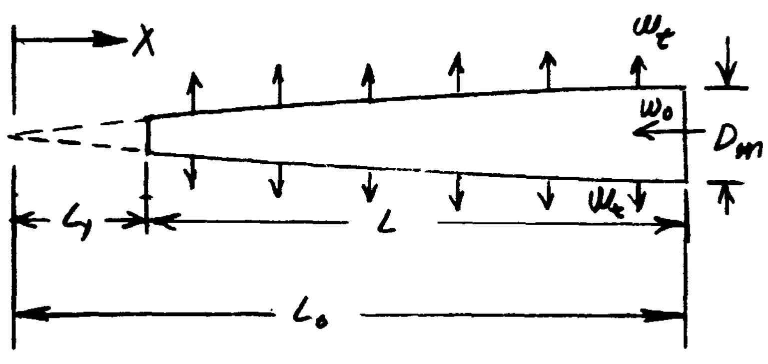


IN OPTIMIZING THE RADIATOR FOR MINIMUM WEIGHT CONSIDERATION IS GIVEN TO THE WEIGHT EQUIVALENT OF PUMPING POWER REQUIRED TO CIRCULATE THE COOLANT FLUID IN THE RADIATOR. THE MANIFOLDS ARE ASSUMED TO HAVE A VARYING FLOW CROSS SECTION ALONG THEIR LENGTH, AND THAT THE PRESSURE DROP

PER UNIT LENGTH REMAINS CONSTANT ALONG THE MANIFOLD.

IN THE ANALYSIS WHICH FOLLOWS FIRST THE INLET AND RETURN MANIFOLD DIAMETERS ARE OPTIMIZED FOR MINIMUM WEIGHT THEN THE NUMBER OF TUBES IS OPTIMIZED

INLET MANIFOLD



$$w_o = \tfrac{1}{2} W_C \ \#/sec$$

ASSUME

$$\frac{\Delta P}{\Delta X} = CONST$$

$$\frac{\Delta P}{\Delta X} = \frac{f}{D_X} \frac{V_X^2}{2g} \qquad f = \text{FRICTION FACTOR}$$

$$V_X = \frac{W_X}{A_X \rho_C} = \frac{4\, W_X}{\pi D_X^2 \rho_C}$$

THEREFORE:

$$\frac{\Delta P}{\Delta X} = \left(\frac{8\, f}{g \pi^2 \rho_C^2}\right)\left(\frac{W_X^2}{D_X^5}\right) \qquad (10)$$

IF WE HAVE EQUAL FLOW DISTRIBUTION FOR ALL TUBES

$$W_x = K_1 x$$

THEREFORE

$$D_x = K_2 x^{0.4}$$

MANIFOLD WEIGHT INCLUDING INVENTORY

FLUID TUBE

$$W_{m_1} = \frac{\pi}{4}\int_{L_1}^{L_0} D_x^2 \rho_c\, dx + \pi\int_{L_1}^{L_0}(D_x + t_t)t_t \rho_t\, dx + \pi\int_{L_1}^{L_0}(D_x + 2t + t_a)\rho_a t\, dx$$

$$W_{m_1} = \frac{\pi}{4}\rho_c \int_{L_1}^{L_0} K_2^2 x^{0.8} dx + \pi(\rho_t t_t + \rho_a t_a)\int_{L_1}^{L_0} K_2 x^{0.4} dx + \pi\left[t_t^2 \rho_t + (2t_t + t_a)\rho_a t_a\right] x \Big]_{L_1}^{L_0}$$

BUT $D_M = K_2 L_0^{0.4}$

IF W_t = FLOW/TUBE AND WE LET

$$\frac{W_t}{W_0} = \frac{L_1}{L_0}$$

IT CAN BE SHOWN THAT

$$W_{m_1} = \frac{\pi}{4}\rho_c \frac{D_m^2 L}{1.8}\left[\frac{1-\left(\frac{w_t}{w_o}\right)^{.8}}{1-\frac{w_t}{w_o}}\right] + \pi(\rho_t t_t + \rho_a t_a)\frac{D_m L}{1.4}\left[\frac{1-\left(\frac{w_t}{w_o}\right)^{1.4}}{1-\frac{w_t}{w_o}}\right]$$
$$+ \pi\left[t_t^2\rho_t + (2t_t + t_a)\rho_a t_a\right] L$$

THIS EXPRESSION CAN BE SIMPLIFIED BY MAKING SOME APPROXIMATIONS

$$\lim_{\frac{w_1}{w_o}\to 1} \frac{1-\left(\frac{w_t}{w_o}\right)^{1.4}}{1-\frac{w_t}{w_o}} = \lim_{\frac{w_1}{w_o}\to 1} \frac{-1.4\frac{w_1}{w_o}^{0.4}}{-1} = 1.4$$

$$\lim_{\frac{w_1}{w_o}\to 0} \frac{1-\left(\frac{w_t}{w_o}\right)^{1.4}}{1-\frac{w_t}{w_o}} = 1.0$$

if $\frac{w_t}{w_o} << \frac{1}{30}$ $\quad 1 < \frac{1-\left(\frac{w_t}{w_o}\right)^{1.4}}{1-\frac{w_t}{w_o}} < \frac{1-.00086}{1-.0333} = 1.024$

if $\frac{w_t}{w_o} << \frac{1}{30}$ $\quad 1 < \frac{1-\left(\frac{w_t}{w_o}\right)^{1.8}}{1-\frac{w_t}{w_o}} < \frac{1-.00022}{1-.0333} = 1.034$

THEREFORE

$$W_{m_1} \cong \frac{1.03\pi L \rho_t D_m^2}{1.8 \times 4} + \frac{1.02\pi}{1.4} L(\rho_t t_t + \rho_a t_a) D_m$$
$$+ \pi\left[t_t^2\rho_t + (2t + t_a)t_a\rho_a\right]L$$

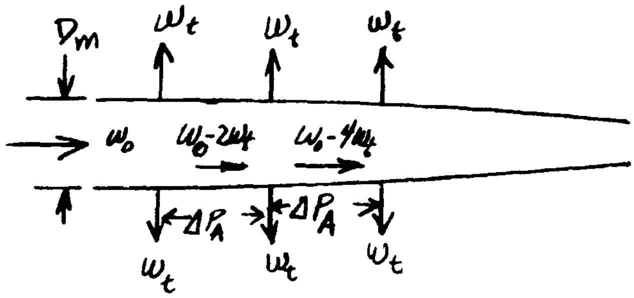
$$W_{m_1} \cong K_{10} D_m^2 L + K_{11} D_m L + K_{12} L \qquad (11)$$

MANIFOLD PUMPING POWER WEIGHT EQUIVALENT

FROM EQN (10) THE INLET MANIFOLD PRESSURE DROP IS

$$\Delta P_M = \left(\frac{8f}{g\pi^2\rho_c^2}\right)\left(\frac{w_0}{D_M^5}\right) L_M$$

THE MANIFOLD PUMPING POWER MAY BE EVALUATED BY CONSIDERING THE FLOW DISTRIBUTION IN THE MANIFOLD



$$P_{PM} = w_0 \Delta P_1 + (w_0 - 2w_t)\Delta P_2 + (w_0 - 4w_t)\Delta P_3 \cdots (w_0 - 2N w_t)\Delta P_N$$

WHERE:

$N_1 = \frac{1}{2}$ NUMBER OF TUBES PER MANIFOLD

$\Delta P_1 = \Delta P_2 = \Delta P_N = \Delta P_A$ PRESSURE DROP IN MANIFOLD BETWEEN TUBES

$$P_{PM} = w_0 \Delta P_m - 2 w_1 \Delta P_A \left[1 + 2 + 3 + \cdots N_1\right]$$

$$= w_0 \Delta P_m - 2 w_1 \Delta P_A \frac{N_1(N_1+1)}{2}$$

01-071-002

QUADRILLE WORK SHEET

AEROJET-GENERAL CORPORATION
AZUSA, CALIFORNIA

DATE

SUBJECT BY WORK ORDER

BUT $\Delta P_A \; N_1 = \Delta P_m$

$$2 w_1 N_1 = w_0$$

$$P_{pm} = w_0 \Delta P_m - \frac{w_0}{2N} \Delta P_m \, (N+1)$$

$$= \frac{1}{2} w_0 \Delta P_m \left(1 - \frac{1}{N_1}\right)$$

IF $N_1 > 15$ $\quad .934 < 1 - \frac{1}{N_1} < 1$

THEREFORE APPROXIMATE

$$W_{ppm} = \frac{.934}{2} w_0 \Delta P_m K_H$$

WHERE $K_H = \dfrac{LB \text{ (SYSTEM WEIGHT)}}{\frac{FT-LB}{HR} \text{ (HYDRAULIC POWER)}}$

IF WE LET $K_3 = \left(\dfrac{8 f w_0^3}{g \pi^2 \rho_c^2}\right)\left(\dfrac{.934}{2} K_H\right)$

THEN

$$W_{ppm} = \frac{K_3 \, L_m}{D_m^5}$$

INLET MANIFOLD EQUIVALENT WEIGHT

$$W_{ME} = W_{m_1} + W_{ppm} = K_{10} D_m^2 L + K_{11} D_m L + K_{12} L + K_3 L D_m^{-5}$$

DIFFERENTIATING WRT D_m WITH L CONSTANT AND EQUATING TO ZERO

$$\left.\frac{\partial W_{MS}}{\partial D_0}\right|_{L=CONST} = 2K_{10}D_m L + K_{11}L - 5K_3 L D_m^{-6} = 0$$

$$D_m^7 + \frac{K_{11}}{2K_{10}} D_m^6 + \frac{5K_3}{2K_{10}} = 0 \qquad (12)$$

EQ'N 12 CAN BE SOLVED TO DETERMINE THE OPTIMUM MANIFOLD INLET DIAMETER.

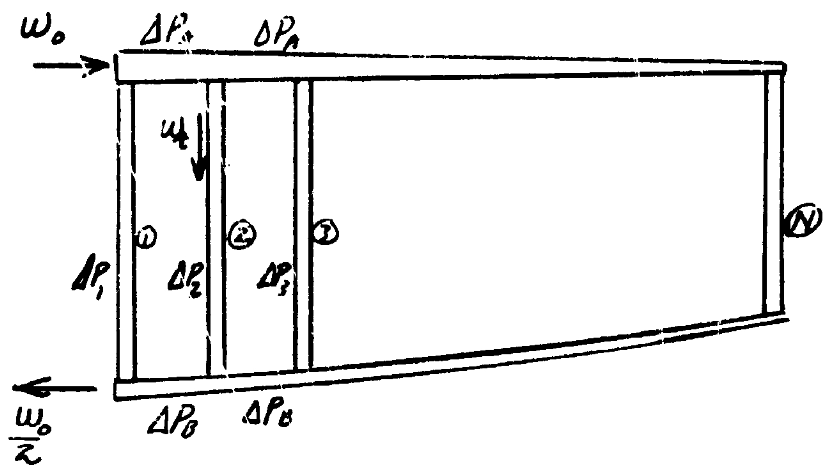
IN A SIMILAR FASHION IT CAN BE SHOWN THAT THE RETURN MANIFOLD OPTIMUM OUTLET DIAMETER CAN BE EVALUATE BY THE EQUATION

$$D_1^7 + \frac{1}{2}\frac{K_{11}}{K_{10}} D_1^6 - \frac{5}{16}\frac{K_3}{K_{10}} = 0 \qquad (13)$$

RETURN MANIFOLD EQUIVALENT WEIGHT

$$W_2 = K_{10} D_1^2 L + K_{11} D_1 L + K_{12} + \frac{K_3}{8} D_1^{-5} \qquad (14)$$

OPTIMIZATION OF NUMBER OF TUBES



IF WE DEFINE;

$$\Delta P_A + \Delta P_B = \Delta P_C$$

ΔP_1 PRESSURE DROP IN FIRST TUBE
ΔP_2 " " " 2nd TUBE
ΔP_N " " " Nth TUB

$$\Delta P_2 = \Delta P_1 - \Delta P_C \qquad \Delta P_3 = \Delta P_2 - \Delta P_C$$

THEREFORE THE PUMPING POWER FOR THE TUBES IS

$$P_{PT} = W_T(\Delta P_1 - \tfrac{1}{2}\Delta P_C) + W_t(\Delta P_1 - 1\tfrac{1}{2}\Delta P_C) + W_t(\Delta P_1 - 2\tfrac{1}{2}\Delta P_C)$$

$$+ \cdots \qquad W_t\left[\Delta P_1 - (N - \tfrac{1}{2})\Delta P_C\right]$$

WHICH CAN BE REDUCED TO THE FORM

$$P_{Pt} = W_t N\left[\Delta P_1 - \frac{\Delta P_C}{2}N\right] = W_o\left[\Delta P_1 - \frac{\Delta P_{M_1} + \Delta P_{M_0}}{2}\right]$$

$$P_{pt} = w_o \Delta P_{AVG}$$

WHERE ΔP_{AVE} IS THE AVERAGE PRESSURE DROP ACROSS THE TUBES.

THE EQUIVALENT WEIGHT FOR PUMPING POWER REQUIRED TO CIRCULATE FLUID THROUGH THE TUBES

$$W_{PPT} = K_H w_o \Delta P_{AVE} = K_H w_o f \frac{L_A}{D_t} \frac{V_t^2}{2g}$$

BUT L_A = AVE TUBE LENGTH = $\frac{L_w}{N_T}$

N_T = TOTAL NUMBER OF TUBES IN RADIATOR

D_t = TUBE I.D.

A_t = TUBE CROSS SECTIONAL AREA

f = FRICTION FACTOR

$$W_{PPT} = \left(\frac{K_H f}{D_t A_t^2 \rho^2 2g}\right) w_t^2 \frac{w_o L_w}{N_t}$$

$$= \left(\frac{K_H f w_o^3 L_w}{D_t A_t^2 \rho^2 2g}\right) \frac{1}{N_t^3} = A_5 N_t^{-3} \qquad (15)$$

THE EQUIVALENT WEIGHT FOR TUBE INLET LOSSES

$$W_{PP_L} = 1.4 K \frac{V_t^2}{2g} w_t = \frac{0.7 K w_o^3}{g A_t^2 \rho^2} \cdot \frac{1}{N_t^3} = A_6 N_t^{-3} \qquad (16)$$

THE TOTAL RADIATOR EQUIVALENT WEIGHT

$$W_{RE} = W_R + W_{m_1} + 2W_{m_2} + W_{ppm_1} + 2W_{ppm_2} + W_{ppT} + W_{ppL}$$

IF S = SPAN ACROSS ONE TUBE AND FIN

$$S = D_0 + 2L_n$$

$$L_m = \frac{S N_T}{2} \quad \text{MANIFOLD LENGTH ONE PANEL}$$

$$W_R = \frac{3}{2} W_t L_w \left[\frac{b L_n + a}{b L_n + \frac{1}{2}a}\right] = A_0$$

$$W_{m_1} = \left[K_{10} D_m^2 + K_{11} D_m + K_{12}\right] L_m = \frac{A_2 S N_T}{2}$$

$$W_{m_2} = \left[K_{10} D_1^2 + K_{11} D_1 + K_{12}\right] L_m = \frac{A_3 S N_T}{2}$$

$$W_{ppm_1} = \frac{K_3}{D_m^5} L_m = \frac{A_3 S N_T}{2} \quad \text{(INLET MANIFOLD)}$$

$$W_{ppm_2} = \frac{K_3}{8 D_1^5} L_m = \frac{A_4 S N_T}{2} \quad \text{(RETURN MANIFOLD)}$$

$$W_{ppT} = \left(\frac{K_H f W_0^3 L_w}{D_t A_t^2 \rho^2 2g}\right) \frac{1}{N_T^3} = A_5 N_T^{-3}$$

$$W_{ppL} = \left(\frac{0.7 K_H W_0^3}{g A_t^2 \rho^2}\right) \frac{1}{N_T^{-3}} = A_6 N_T^3$$

THEREFORE FOR OPTIMUM N_T

$$W_{RE} = W_R + \left[\frac{A_1 S}{2} + A_2 S + \frac{A_3 S}{2} + A_4 S\right] N + \left[A_5 + A_6\right] N_T^{-3}$$

$$\left.\frac{\partial W_{RE}}{\partial N_T}\right]_{W_R \, CONST} = \left(\frac{A_1}{2} + A_2 + \frac{A_3}{2} + A_4\right) S - 3(A_5 + A_6) N_T^{-4} = 0$$

$$N_{T\,OPT} = \left[\frac{\left(\frac{A_1}{2} + A_2 + \frac{A_3}{2} + A_4\right) S}{3(A_5 + A_6)}\right]^{-1/4} \qquad (17)$$

APPENDIX B

INTRODUCTION

This report presents the method of analysis used to determine the optimum weight of a compact mercury condenser for the SNAP-8 system. This method of analysis was used to produce curves such as the one shown in Figure 1. The detailed calculations, results and conclusions of the analysis are not a part of this report.

DISCUSSION

The condenser analyzed was a compact counterflow type. The coolant passages have a constant height and width along the condenser. The mercury passages have a constant height but the width is gradually reduced toward the exit, resulting in a tapered passage. The mercury condensing film coefficient considered in the analysis was so high compared to the coolant coefficient that fins were not used on the mercury passages. The core geometry and condenser configuration are shown on Figure 2. The following given data was kept constant for ~~all the~~ each case~~s~~ analyzed:

a. Condenser heat load

b. Coolant flow rate

c. Coolant exit temperature

d. Mercury inlet and exit temperature

e. Mercury inlet quality

f. Allowable mercury pressure drop

g. No subcooling of the condensate

h. Coolant - OS-124

In general, the optimization procedure consisted of the following steps:

a. Based on the given data determine the required heat transfer overall conductance (UA).

b. Assume 3 different Reynolds numbers for the coolant. For each one determine:

    1. Coolant flow area. The mercury inlet flow area was taken as equal to the coolant flow area. For a counterflow type heat exchanger the two added together define the condenser frontal area. At the mercury exit the area was fixed by the tapering of the mercury passage. The coolant flow area was constant. From these the exit total area was calculated.
    2. Overall unit conductance (U).
    3. Heat transfer area.
    4. Condenser dimensions.
    5. Weight of the condenser, coolant and mercury inventory and manifolds. Let these added together be equal to $\sum W$.
    6. Pressure drop of coolant through the condenser and the associated pumping power.
    7. Weight penalty due to pumping power. ($W_{\Delta PP}$).

c. Plot $\sum W$ and $W_{\Delta PP}$ versus coolant Reynolds number. A typical plot is shown in Figure 1. By adding $\sum W$ and $W_{\Delta PP}$ at various Reynolds numbers the curve of total weight penalty (W_T) is obtained. As can be seen in the figure, this curve has a minimum value, which represents the condenser of minimum weight. It also has a corresponding coolant optimum Reynolds number. The size of the optimum condenser can be determined by repeating the calculation using the optimum Reynolds number or by interpolating the results already obtained.

The detailed description of the method of analysis is divided in two parts, (A and B). Part A covers the complete analysis as described above. Part B is an extrapolation of the results obtained in Part A to correct for a higher condensing temperature and heat load.

SYMBOLS AND NOMENCLATURE

Symbols	Description	Units
$A_{blockage}$	Condenser frontal area blocked by vertical plates	IN^2
$A_{frontal}$	Condenser frontal area	IN^2
$(A_f)_c$	Coolant flow area	IN^2
$(A_f)_{Hg}$	Mercury flow area at mercury inlet	IN^2
$(A_{HT})_c$	Heat transfer area based on coolant side	FT^2
b	Plate spacing in coolant passages	IN
b'	Plate spacing in mercury passage at mercury inlet	IN
C_1	Group of parameters	IN^2
C_2	Proportionality constant	--
$(C_p)_c$	Specific heat of coolant	BTU/Lb-°F
$(C_p)_{Hg}$	Specific heat of mercury	BTU/Lb-°F
C_c	Total heat capacity of coolant	BTU/Hr-°F
C_{Hg}	Total heat capacity of mercury	BTU/Hr-°F
C_{min}	Value of lower heat capacity	BTU/Hr-°F
C_{max}	Value of higher heat capacity	BTU/Hr-°F
D	Dimension in coolant manifold	IN
D_H	Hydraulic diameter	FT
f	Coolant friction factor	Dimensionless
h_c	Coolant heat transfer film coefficient	BTU/Hr-°F-FT^2
K	Thermal conductivity of coolant	BTU/Hr-°F-FT
K_{ss}	Thermal conductivity of stainless steel	BTU/Hr-°F-FT

Symbols	Description	Units
ℓ	Length of fins	FT
m_c	Fin parameter	FT^{-1}
N	Total number of mercury and coolant passages	--
N + 1	Total number of vertical divider plates between passages	--
$N_F)_c$	Fin effectiveness, coolant side	--
$N_F)_{Hg}$	Fin effectiveness, mercury side	--
$N_o)_c$	Surface effectiveness, coolant side	--
$N_o)_{Hg}$	Surface effectiveness, mercury side	--
NTU	Number of transfer units	Dimensionless
N_{PR}	Prandtl Number, coolant	Dimensionless
N_{RE}	Reynolds Number, coolant	Dimensionless
ΔP_c	Coolant pressure drop	PSI
Q_c	Condenser heat load	BTU/HR
ΔQ	Increment in condenser heat load	BTU/HR
r	Dimension in mercury manifold	IN
s	Dimension in mercury manifold	IN
T_{c_1}	Coolant inlet temperature	^oF
T_{c_2}	Coolant exit temperature	^oF
T_{Hg_1}	Mercury inlet temperature	^oF
T_{Hg_2}	Mercury exit temperature	^oF
ΔT_{log}	Log mean temperature difference	^oF
t_1	Plate spacing in mercury passages at mercury exit	IN

Symbols	Description	Units
U_c	Overall unit conductance	BTU/HR-^oF-FT^2
$(UA)_{req}$	Overall conductance	BTU/HR-^oF
V_c	Coolant volume	FT^3
$\dot{W}_c$	Coolant flow rate	LB/HR
$\dot{W}_{Hg}$	Mercury flow rate	LB/HR
W_{Hg_1}	Weight of mercury in mercury exit manifold	LBS
W_{Hg_2}	Weight of mercury in coolant inlet manifold	LBS
W_c	Weight of coolant in condenser	LBS
$(W_c)_{manif.}$	Weight of coolant in two coolant manifolds	LBS
$(W_{manif})_c$	Total weight of coolant manifolds	LBS
$(W_{manif})_{Hg}$	Total weight of mercury manifolds	LBS
W_{cond}	Condenser weight (empty)	LBS
W_{plates}	Weight of condenser plates	LBS
W_{fins}	Weight of condenser fins	LBS
$W_{\Delta PP}$	Weight penalty due to pumping power	LBS
W_T	Total weight penalty	LBS
X	Mercury inlet quality	--
x	Width of condenser at mercury inlet	IN
x_o	Width of condenser at mercury exit	IN
y	Height of condenser	IN
Z	Length of condenser	IN
ΔZ	Increment in condenser length	IN

Greek Symbols	Description	Units
α	Angle in mercury manifold	Degrees
β	$\frac{\text{Condenser heat transfer area}}{\text{Condenser volume between the plates}}$	$\frac{FT^2}{FT^3}$
δ	Fin thickness	IN
0	$\frac{\text{Total fin area}}{\text{Total heat transfer area}}$	--
ψ	Group of parameters	--
μ_c	Coolant dynamic viscosity	LB/FT-HR
ρ_c	Density of coolant	LB/FT^3
ρ_{ss}	Density of stainless steel	LB/FT^3
ϵ	Heat exchanger effectiveness	--

METHOD OF ANALYSIS

PART A

1. Given Data:

Q_c = condenser heat load

$\dot{W}_c$ = coolant flow rate

$\dot{W}_{Hg}$ = mercury flow rate

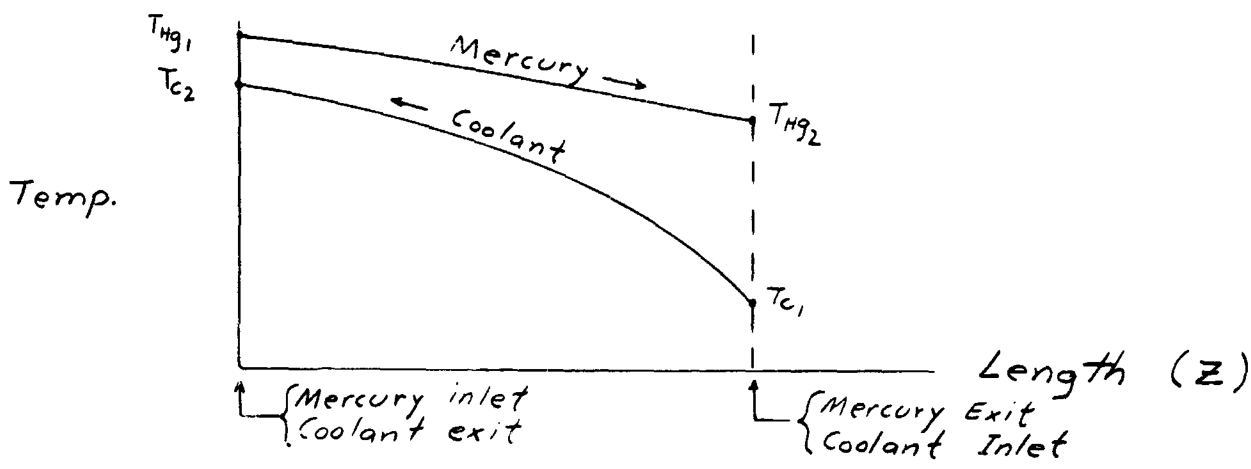
X = mercury inlet quality

T_{Hg_1} = mercury inlet temperature

T_{Hg_2} = mercury temperature at end of condensing process

T_{c_2} = coolant exit temperature

2. Thermo Cycle - Assume a counterflow heat exchanger



For the cases analyzed $T_{Hg_1} - T_{Hg_2} = 20^\circ F$

3. Coolant Inlet Temperature (T_{c_1})

$$T_{c_2} - T_{c_1} = \frac{Q_{cond}}{C_{p_c} \hat{W}_c}$$

Compute $T_{c_1} = T_{c_2} - \dfrac{Q_{cond}}{C_{p_c} \hat{W}_c}$

C_{p_c} must be at average of T_{c_2} and T_{c_1} and will require some trial and error calculations to use the proper value.

4. Heat Exchanger Effectiveness (ϵ)

Let $C_{Hg} = W_{Hg} C_{p_{Hg}}$

Let $C_c = W_c C_{p_c}$

For a condensing process $C_{p_{Hg}} = \infty$. However, there is a drop in temperature from T_{Hg_1} to T_{Hg_2} due to the mercury pressure drop and we will define a fictitious C'_{Hg} as follows:

$$C'_{Hg} (T_{Hg_1} - T_{Hg_2}) = C_c (T_{c_2} - T_{c_1})$$

$$\frac{C_c}{C'_{Hg}} = \frac{T_{Hg_1} - T_{Hg_2}}{T_{c_2} - T_{c_1}} = \frac{20^{\circ}F}{T_{c_2} - T_{c_1}}$$

$\dfrac{C_c}{C'_{Hg}}$ is the equivalent of $\dfrac{C_{min}}{C_{max}}$ used by Compact Heat Exchangers Book, Reference (1).

Let $\frac{C_c}{C_{Hg}} = \frac{C_{min}}{C_{max}}$

Compute $\frac{C_{min}}{C_{max}}$

Compute $\epsilon = \frac{C_c (T_{c_2} - T_{c_1})}{C_{min} (T_{Hg_1} - T_{c_1})}$

$$= \frac{T_{c_2} - T_{c_1}}{T_{Hg_1} - T_{c_1}}$$

Read NTU (Number of Transfer Units) from Figure 2 of Reference (1) as a function of ϵ and $\frac{C_{min}}{C_{max}}$

5. Overall Heat Transfer Conductance (UA_{req})

Compute $UA_{req} = (NTU)(\dot{W}_c \, C_{p_c}) = BTU/HR\text{-}^{o}F$

6. Heat Exchanger Core

Assume plain plate surface type 11.1 for coolant core (no fins on mercury side). The following data for that type of surface obtained from Reference (1).

D_H	:	Hydraulic diameter	.01012 FT
δ	:	Fin thickness	.006 IN.
β	:	$\frac{\text{Heat Transfer Area}}{\text{Volume Between Plates}}$	367 $\frac{FT^2}{FT^3}$
ϕ	:	$\frac{\text{Fin Area}}{\text{Total Area}}$	.756
b	:	Plate Spacing	.25 IN.

It is also assumed that the plates are made of stainless steel 316 and are .035 in. thick.

7. Coolant Properties

Get the following properties at the average of T_{c_1} and T_{c_2}

μ_c

C_{p_c}

K_c

$$N_{PR_c} = \frac{\mu_c \, C_{P_c}}{K_c}$$

8. Coolant Flow Area Required $(A_f)_c$

Assume a Reynolds number for the coolant (N_{RE})

$$\text{Compute } (A_f)_c = \frac{D_H \, \dot{W}_c}{\mu_c \, N_{RE}} = FT^2$$

9. Coolant Film Coefficient (h_c)

For N_{RE} assumed get Ψ from Figure 63 of Reference (1).

$$\text{Where } \Psi = \left[\frac{h_c \, (A_f)_c}{\dot{W}_c \, (C_p)_c}\right] \left[N_{PR}\right]^{2/3}$$

$$\text{Compute } h_c = \frac{\Psi \, \dot{W}_c \, C_{p_c}}{(N_{PR})^{2/3} \, (A_f)_c} = \frac{BTU}{HR\text{-}^{\circ}F\text{-}FT^2}$$

10. <u>Fin Effectiveness - Coolant Side</u> $(N_f)_c$

Compute $m_c = \sqrt{\dfrac{2\, h_c}{K_{ss}\, \delta}} = (FT)^{-1}$

(δ is defined in Section 6)

Compute $\ell = \dfrac{b}{(12)(2)} = FT$

(b is defined in Section 6)

Compute $m_c \ell$

Get $\operatorname{Tan h} (m_c \ell)$

Compute $(N_f)_c = \dfrac{\operatorname{Tan h} (m_c \ell)}{m_c \ell}$

11. <u>Fin Effectiveness - Mercury Side</u> $(N_f)_{Hg}$

A condensing film coefficient of 10,000 BTU/HR-^{o}F-FT^2 was assumed on the mercury side, as suggested by Reference (2). This will make $(N_f)_{Hg}$ so low that it can be neglected. In other words, the thermal resistance due to the mercury film will be neglected.

12. <u>Surface Effectiveness - Coolant Side</u> $(N_o)_c$

$(N_o)_c = 1 - \phi \left[1 - (N_f)_c \right]$

where ϕ is defined in Section 6

13. Overall Unit Conductance (U_c)
(Based on coolant side)

(from Reference (1))

$$\frac{1}{U_c} = \frac{1}{(N_o)_c\, h_c} + \frac{1}{\frac{(A_{HT})_{Hg}}{(A_{HT})_c}\,(N_o)_{Hg}\,(h_{Hg})}$$

Neglecting the right term because of the high value of h_{Hg} we get:

$$U_c = (N_o)_c\, h_c = BTU/HR\text{-}^oF\text{-}FT^2$$

14. Heat Transfer Area Required $(A_{HT})_c$
(Based on coolant side)

$$(A_{HT})_c = \frac{UA_{req}\ \text{(from Section 5)}}{U_c\ \text{(from section 13)}} = FT^2$$

15. Coolant Volume (V_c)

Compute $V_c = \dfrac{(A_{HT})_c\ \text{(from Section 14)}}{\beta\ \text{(from Section 6)}} = FT^3$

16. Length of Condenser Z

Compute $Z = \dfrac{V_c\ \text{(from Section 15)}}{(A_f)_c\ \text{(from Section 8)}} = FT$

17. Condenser Dimensions on Mercury Inlet

Let $(A_f)_c$ = Coolant flow area; same at mercury inlet and mercury exit (in^2)

Let $(A_f)_{Hg}$ = Mercury flow area at inlet (in^2)

Let N = Total number of coolant and mercury passages

Let b = Coolant gap (in)

Let b' = Mercury gap (in)

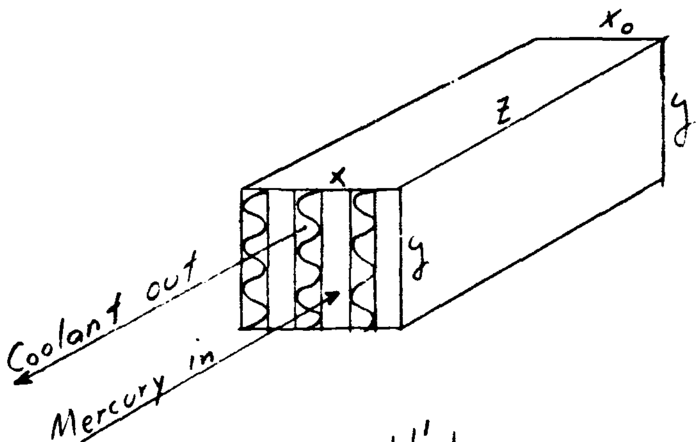
Let x = Width of condenser at mercury inlet (in)

Let y = Height of condenser at mercury inlet (in)

Based on experimental data on condensing mercury pressure drop presented in Reference (2), a proportion was established between the condenser of Reference (2) and the condenser being analyzed. In order to maintain a pressure drop of 4 psi on the mercury, the proportion resulted in the following expression for the mercury passage gap on the mercury inlet (b').

$$b' = 2.29 \left[\frac{Z}{(y\ N)^2} \right]^{1/3} \tag{1}$$

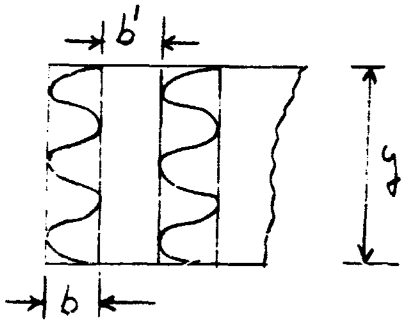
(b' = inches, Z = FT, y = inches)



Let $x = y$

$x > x_o$

Plate thickness = .035 in.



From the above figure we have:

$$(A_f)_c = y\, b\, \frac{N}{2} \qquad (2)$$

$$\therefore\ y\, N = \frac{2\,(A_f)_c}{b}$$

Substituting this value of $(y\ N)$ in Equation (1), we get:

$$b' = 2.29 \left[\frac{Z\, b^2}{4\, (A_f)_c^{\,2}} \right]^{1/3} = \text{inches}$$

The mercury flow area is given by:

$$(A_f)_{Hg} = b'\, y\, \frac{N}{2} = \frac{b'}{2}\, y\, N \qquad (3)$$

Substituting $(y\ N)$ from Equation (2) into (3), we get:

$$(A_f)_{Hg} = \frac{b'}{2} \left[\frac{2 (A_f)_c}{b} \right]$$

Let $x = y$

$$A_{frontal} = (A_f)_c + (A_f)_{Hg} + A_{blockage}$$

$$= (A_f)_c + (A_f)_{Hg} + .035(N+1)y$$

Where $N + 1$ = total number of plates

$$A_{frontal} = (A_f)_c + (A_f)_{Hg} + .035y + .035Ny$$

Substitute $N = \left[\frac{2(A_f)_c}{yb} \right]$ from Equation (2)

$$\therefore A_{frontal} = (A_f)_c + (A_f)_{Hg} + .035y + .035 \left[\frac{2(A_f)_c}{yb} \right] y$$

$$= (A_f)_c + (A_f)_{Hg} + \frac{.07(A_f)_c}{b} + .035\,y$$

$$= C_1 + .035y \qquad (4)$$

Where $C_1 = (A_f)_c + (A_f)_{Hg} + \frac{.07(A_f)_c}{b}$

Also $A_{frontal} = x\,y = y^2$ (by letting $x = y$) (5)

Setting (4) = (5) we get:

$$y^2 = C_1 + .035y$$

$$y^2 - .035y - C_1 = 0$$

$$y = \tfrac{1}{2} \left[.035 + \sqrt{(.035)^2 - 4\,(-C_1)} \right]$$

$$= \tfrac{1}{2} \left[.035 + \sqrt{.001225 + 4\,C_1} \right]$$

Since x = y, we also have solved for value of x

The number of passes (N) is obtained from Equation (2) as follows:

$$N = \left[\frac{2(A_f)_c}{yb} \right]$$

18. Condenser Dimensions on Mercury Exit Side

The condenser height (y) is the same at the mercury inlet and exit.

Let x_o = condenser width at mercury exit (inches)

Let t_1 = gap between mercury plates at exit

Let N = N (same as in the mercury inlet) = total number of mercury and coolant passages

Using the same approach described under Section 17 based on mercury pressure drop data from Reference (2), we get for t_1:

$$t_1 = .59 \left[\frac{Z}{(y\ N)^2}\right]^{1/3}$$

(Z = FT, y = inches, t_1 = inches)

We can then compute X_o as follows:

$$X_o = \frac{Nb}{2} + \frac{N}{2} t_1 + (N + 1)(.035)$$

19. Coolant Pressure Drop ($\triangle P_c$)

Disregarding the inlet and exit pressure loss:

$$\triangle P_c = \frac{1.08 \times 10^{-4}}{\rho_c} \left[\frac{\dot{W}_c}{3600(A_f)_c}\right]^2 (f) \left[\frac{(A_{HT})_c}{(A_f)_c}\right]$$

$\triangle P_c$ = psi

ρ_c = coolant average density, LB/FT^3

$\dot{W}_c$ = coolant flow rate, LB/HR

$(A_f)_c$ = coolant area, FT^2

$(A_{HT})_c$ = coolant h... transfer area, FT^2

f = friction factor, function of N_{RE}, obtained from Figure 63 of Reference (1) for plain fin surface 11.1

20. Weight Penalty Due to Pumping Power ($W_{\Delta PP}$)

$$W_{\Delta PP} = \left[\frac{\Delta P_c \, \dot{W}_c \times 144}{\rho_c}\right] \left[\text{(Penalty Factor)} \right]$$

Where Penalty Factor $= \dfrac{1}{1.328 \times 10^4}$

$W_{\Delta PP}$ = Lbs

ΔP_c = psi

$\dot{W}_c$ = Lb/Hr

ρ_c = Lb/FT^3

21. Weight of Condenser (W_{cond})

$W_{cond} = W_{plates} + W_{fin}$ (Lbs)

$$W_{plates} = (N + 1)(y\ z)\left(\frac{.035}{12}\right)(\rho_{ss})$$

y, z = ft

ρ_{ss} = 500 Lb/FT^3 (stainless steel)

$\therefore \; W_{plates} = 1.46\ (N + 1)(y\ z)$

$$W_{fins} = \left[\frac{A_{fin}}{(A_{HT})_c}\right] \left[(A_{HT})_c\right] \left[\frac{\delta}{12}\right] \left[\rho_{ss}\right]$$

For surface 11.1 :

$$\frac{A_{fin}}{(A_{HT})_c} = .756$$

δ = .006 in

ρ_{ss} = 500 Lb/FT^3

$\therefore W_{fins} = .189\ (A_{HT})_c$

22. <u>Weight of Coolant</u>

$W_c = V_c\ \rho_c$

If ρ_c is taken as 61 Lb/FT^3

$W_c = 61\ V_c$

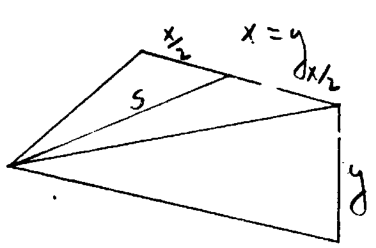
(V_c from Section 15)

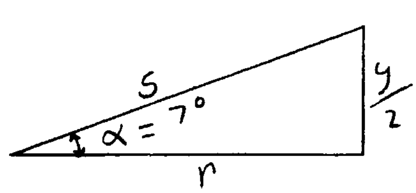
23. <u>Weight of Mercury</u>

The weight of mercury between the inlet and the vapor-liquid interface was assumed to be negligible.

24. <u>Weight of Mercury Manifold</u> $(W_{manif})_{Hg}$

Assume a pyromidal shape with an angle $\alpha = 7^o$ (inlet and outlet equal)





$r = \frac{y}{2\ \text{Tan}\ 7^o} = \frac{y}{2(.1225)} = \frac{y}{.245}$

$s = \frac{y}{2\ \text{Sin}\ 7^o} = \frac{y}{.244}$

Surface Area = $\frac{1}{2}$ (Perimeter of base) (S)

$= \frac{1}{2}\ (2\ x + 2\ y)\ (s)$

$= \frac{1}{2}\ (4\ y)\ (S) = 2y\ (\frac{y}{.244})$

$= 8.2\ y^2 = in^2$

Weight of 2 manifolds = (Surface Area)(Plate thickness)(ρ_{ss})(2)

$$= 8.2y^2 \,(.035)\left(\frac{500}{1728}\right)(2)$$

$$= .164\, y^2 = \text{Lbs}$$

$$= (W_{manif})_{Hg}$$

(y = in)

25. <u>Weight of Liquid Mercury in Mercury Exit Manifold</u> (W_{Hg_1})

$$W_{Hg_1} = (Vol)(\rho_{Hg}) = 1/3\,(z)(xy)(\rho_{Hg})$$

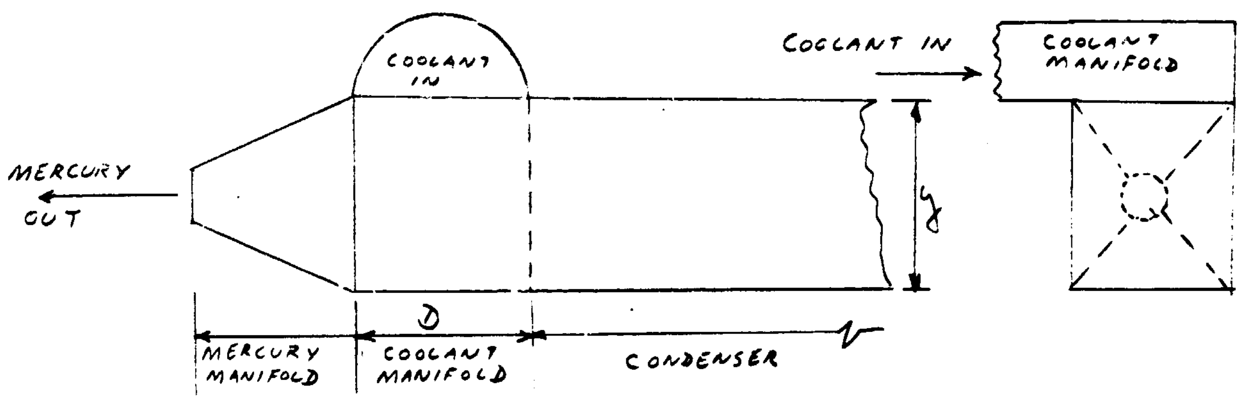
(refer to sketch in Section 24)

Expressing x and z in terms of y and substituting the mercury density:

$$W_{Hg_1} = .66\, y^3 \quad (y = \text{inches})$$

20% of the weight computed from the above equation was used as a realistic mercury inventory. It is expected that a manifold of the requisite internal volume can be produced.

26. <u>Coolant Manifold</u>



Let area of top half cylinder be equal to coolant flow area $(A_f)_c$

Let $\rho_{ss} = \frac{500}{1728}$ Lbs/in^3 Plate thickness = .035 in.

a) Area of half cylinder = $(A_f)_c = \left[\frac{\pi}{4} D^2\right]\left[\frac{1}{2}\right]$

Compute $D = 1.6 \sqrt{(A_f)_c}$

D = in, $(A_f)_c$ = in^2

b) W of half cylinder $= y \left(\frac{\pi D}{2}\right)(.035)\left(\frac{500}{1728}\right)$

$= .016\ yD$ (lbs) (y in inches)

c) W of plates $= (N + 1)(y)(D)(.035)\left(\frac{500}{1728}\right)$

$= (N + 1)(yD)(.0105)$

d) W of end plate $= \frac{x\ y}{2}\ (.035)\left(\frac{500}{1728}\right)$

$= .0053\ y^2\ (x = y)$

e) W of top and bottom $= \left(y\ D + y\ \frac{D}{2}\right)(.035)\left(\frac{500}{1728}\right)$

$= .008\ y\ D$

f) The total weight for 2 manifolds will be $\left(W'_{manif}\right)$

$= 2\left[.016\ y\ D + .0105(N+1)(yD) + .0053\ y^2 + .008\ y\ D\right]$

$= 2\ y\left[.024D + .0105(N+1) + .0053y\right]$

g) Weight of coolant in 2 manifolds:

$W_{c_{manif}} = 2\left[x\ y\ D\ \rho_c\right]\left[\frac{1}{2}\right]$

$= .035\ y^2\ D$

(Assuming x=y, $\rho_c = \frac{61}{1728}$ Lb/in^3)

50% of the weight computed from the above equation was used as a realistic mercury inventory. It is expected that a manifold of the requisite internal volume can be produced.

h) Weight of liquid mercury in coolant manifold at mercury exit end $(W_{Hg})_2$

$$W_{Hg_2} = \frac{x\ y\ D\ \rho_{Hg}}{2} = \frac{y^2\ D\ (840)}{2\ (1728)}$$

$$= .244\ y^2\ D \quad (y \text{ in inches}) \quad (D \text{ in inches})$$

14% of the weight computed from the above equation was used as a realistic mercury inventory. It is expected that a manifold of the requisite internal volume can be produced.

i) Total weight of coolant manifold

$$(W_{manif})_c = W'_{manif} + W_{c_{manif}} + W_{Hg_2}$$

27. Total Weight of Mercury and Coolant Manifolds

$$W_{manif} = (W_{manif})_c + (W_{manif})_{Hg} + W_{Hg_1}$$

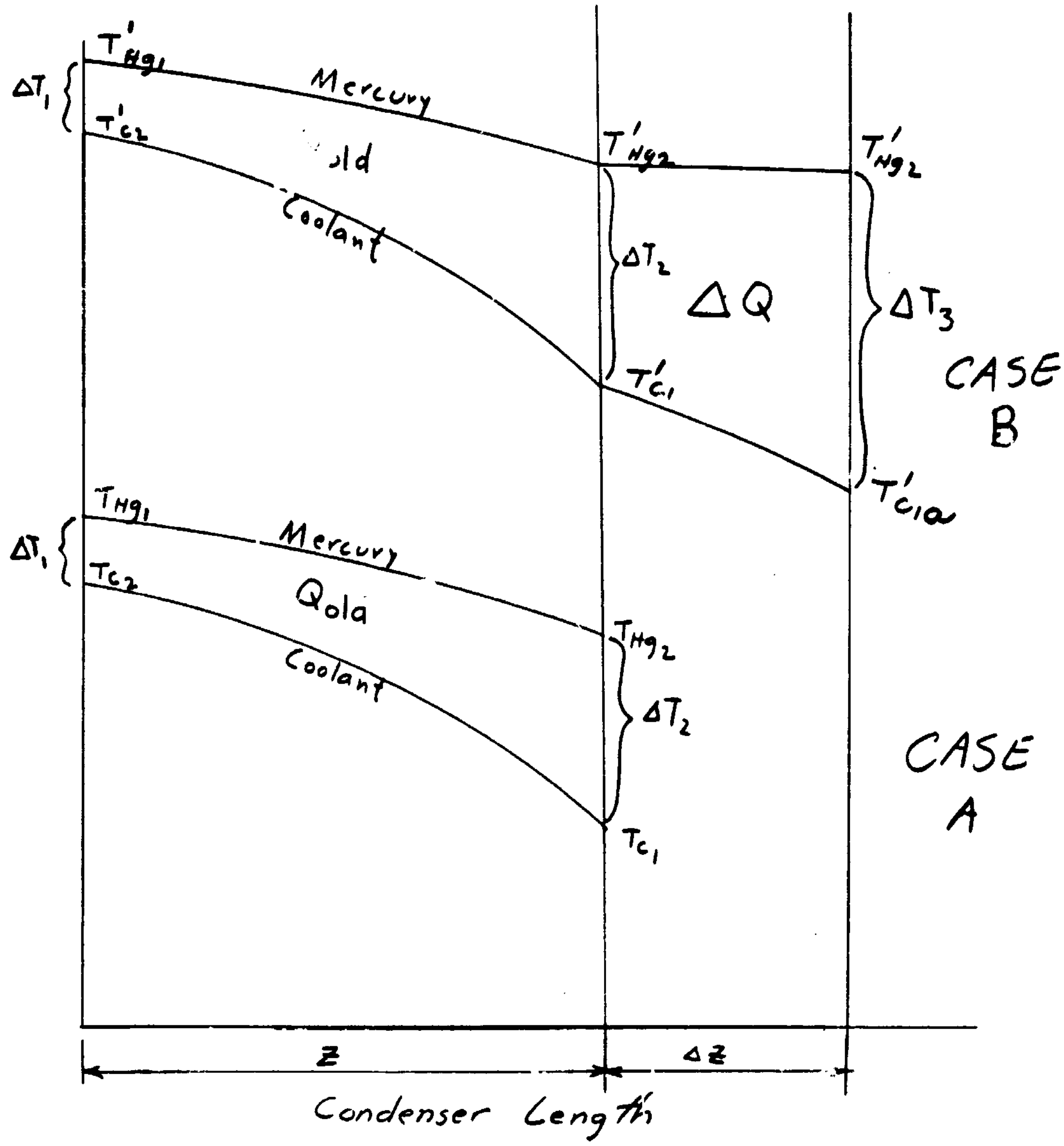
Three cases were solved and a curve of W_{manif} vs $(A_f)_c$ was plotted to eliminate calculations.

PART B

Extrapolation to a Higher Condensing Temperature and Higher Load

28. Thermo Cycle

The coolant outlet and mercury inlet and outlet temperatures will be increased 20°F. The "old" heat load will be increased by an amount ΔQ. Such a thermo cycle can be represented as follows:



Given:

$T'_{Hg_1} = T_{Hg_1} + 20°F$

$T'_{Hg_2} = T_{Hg_2} + 20°F$

$T'_{c_2} = T_{c_2} + 20°F$

$T_{Hg_1} - T_{Hg_2} = 20°F$

$T'_{Hg_1} - T'_{Hg_2} = 20°F$

Temp.

Refer to figure above.

$$Q_{old} = UA_1 \Delta T_{log\ old} = U\ (C_2\ Z)\ \Delta\ T_{log\ old}$$

$$\Delta\ Q = UA_2 \Delta T_{log\ new} = U\ (C_2\quad Z)\ \Delta\ T_{log\ new}$$

Where C_2 is a proportionality constant. If we divide one equation by the other we get:

$$\frac{Q_{old}}{\Delta Q} = \frac{Z \cdot \Delta T_{log\ old}}{\Delta Z \Delta T_{log\ new}}$$

$$\therefore \quad \Delta Z = Z \left[\frac{\Delta Q}{Q_{old}} \quad \frac{\Delta T_{log\ old}}{\Delta T_{log\ new}} \right]$$

ΔZ = increase in condenser length to handle additional load. Based on the "old" x, y, and U derived in Part A we can establish the following proportions:

$$W_{cond\ new} = (W_{cond\ old}) \left(\frac{Z_{new}}{Z_{old}} \right)$$

where $Z_{new} = Z_{old} + \Delta Z$

$$W_{cool\ new} = (W_{cool\ old}) \left(\frac{Z_{new}}{Z_{old}} \right)$$

$$W_{\Delta PP\ new} = (W_{\Delta PP\ old}) \left(\frac{Z_{new}}{Z_{old}} \right)$$

$$W_{manif.\ new} = W_{manif.\ old}$$

(See Section 29 on how to compute $\Delta T_{log\ old}$, $\Delta T_{log\ new}$, etc.)

29. Coolant Temperatures

Given $Q_{old} = 1.082 \times 10^6$ BTU/Hr

Given $Q_{new} = 1.147 \times 10^6$ BTU/Hr

Given $T_{Hg_1} = 680^\circ F$

Given $T_{Hg_2} = 660^\circ F$

T_{c_2} = variable input data

$\Delta Q = Q_{new} - Q_{old} = 65{,}000$ BTU/Hr

Compute $T'_{Hg_1} = T_{Hg_1} + 20^\circ F = 700^\circ F$

(prime refers to new; no prime means old)

Compute $T'_{Hg_2} = T_{Hg_2} + 20^\circ F = 680^\circ F$

Compute $T'_{c_2} = T_{c_2} + 20^\circ F$

Compute $T'_{c_2} - T'_{c_1} = \dfrac{Q_{old}}{\hat{W}_c \, C_{p_c} \text{ ave}}$

$T_{c_1} - T_{c_2} = T'_{c_1} - T'_{c_2}$

Compute $T_{c_1} = T_{c_2} + (T'_{c_2} - T'_{c_1})$

Compute $T'_{c_1} = T_{c_1} + 20^\circ F$

Compute $\Delta T_1 = T'_{Hg_1} - T'_{c_2}$

Compute $\Delta T_2 = T'_{Hg_2} - T'_{c_1}$

Compute $\Delta T_{leg\ old} = \dfrac{\Delta T_2 - \Delta T_1}{\ln \left[\dfrac{\Delta T_2}{\Delta T_1}\right]}$

Compute $T_{c_1}' - T_{c_{1_a}}' = \dfrac{\Delta Q}{\hat{W}_c \, C_{p_c} \text{ ave}}$

Compute $T_{c_{1_a}}' = T_{c_1}' - (T_{c_1}' - T_{c_{1_a}}')$

Compute $\Delta T_3 = T'_{Hg_2} - T'_{c_{1_a}}$

Compute $\Delta T_{log\ new} = \dfrac{\Delta T_3 - \Delta T_2}{\ln \left[\dfrac{\Delta T_3}{\Delta T_2}\right]}$

Compute $\Delta Z = Z_{old} \left[\dfrac{\Delta Q}{Q_{old}} \quad \dfrac{\Delta T_{log\ old}}{\Delta T_{log\ new}}\right]$

Compute $Z_{new} = Z_{old} + \Delta Z$

Compute $W_{cond\ new}$)
" $W_{cool\ new}$) From relations derived on
" $W_{\Delta PP\ new}$) previous section

LIST OF REFERENCES

1. Kays, W. and London, A. L., Compact Heat Exchangers, The National Press, Palo Alto, California, 1955.

2. SNAP-8 Radiator Topical Report, AGC Nucleonics Division, San Ramon, California, June 1962.

APPENDIX C

OPTIMIZATION OF NaK TUBE AND SHELL CONDENSER

1. The purpose of the SNAP-8 Condenser Optimization Study is to select the length, average diameter, and number of condenser tubes so as to minimize the total weight of the condenser. The total weight includes the weights of tubes, shell, mercury, coolant, and pump. The configuration examined is a counter-flow tube-in-shell condenser. Coolant is a eutectic mixture of NaK. Initial conditions are as follows:

Mercury flow rate	9100 lb/hr
Condensing temperature	680°F
Subcooling temperature	520°F

2. A total of 36 cases were computed. The variable parameters and their values are listed below:

Coolant heat transfer coefficient	625, 1250, 2500 Btu/ft^{2}hr$^{\circ}$F
Coolant flow rate	27K, 32K, 37K lb/hr
Coolant outlet temperature	600, 620, 640, 660°F

3. The following paragraphs present a brief explanation of the methods and formulas employed in the study.

a. First, the product of tube length, average diameter, and number of tubes is computed from a consideration of the overall heat transfer coefficient in the condensing region.

$$(L)(N)(D) = \frac{UA}{w}\left[\frac{1}{h_{Hg}} + \frac{1}{K_w(1 + \delta/2D)} + \frac{1}{h_c(1 + \delta/D)}\right]$$

where: $UA = Q_{cond} / \Delta T_{log}$

$$Q_{cond} = \dot{W}_{Hg} H_u$$

$$T_{log} = (T_{RI} - T_c) / \ln\left[(T_{cond} - T_c) / (T_{cond} - T_{RI})\right]$$

$$T_c = T_{RI} - Q_{cond} / \dot{W}_c C_c$$

The following assumptions are made:

$h_{Hg} = 1250\ Btu/ft^2hr^oF$ (Ref. a)

$k_w = 10\ Btu/ft\ hr\ ^oF$

$\delta/D = 0.10$

$H_v = 125.7\ Btu/lb$

$C_c = 0.214\ Btu/lb^oF$

b. Second, the product of the average tube diameter and number of tubes is computed from a consideration of the pressure drop in the condensing region. Reference (a) presents a curve of the ratio of actual pressure drop in the condensing region to pressure drop if vapor only were present versus the parameter $(Re_{Hg})_{in}\ (\frac{v_{Hg}}{10})^{1.25}$

The following point is chosen for the analysis: $\phi = 1.8$ at

$(Re_{Hg})_{in}\ (\frac{v_{Hg}}{10})^{1.25} = 10,000.$

$ND_{in} = 4\ \dot{W}_{Hg}\ / \pi \mu_{Hg}\ (Re_{Hg})_{in}$

where: $D_{in} = D/0.95$ (assumed)

$\mu_{Hg} = 0.148\ lb/ft\ hr$

$(Re_{Hg})_{in} = 10,000\ (\frac{v_{Hg}}{10})^{1.25}$

$v_{Hg} = 3.85\ ft^3/lb$

c. Third, the parameter $D^5N^2/L = D^3(ND)^3\ /\ (LND)$ is computed from the same pressure drop criterion. The single-phase vapor pressure drop is given by:

$$\Delta P_v = f\ (L/D) \rho_{Hg} (V_{Hg}{}^2/2g)$$

$$= f\ (L/D) (\rho_{Hg}/2g)\ (4\ \dot{W}_{Hg}\ /N\ \pi\ D^2 \rho_{Hg})^2$$

Rearranging, this becomes:

$$D^5\ N^2/L = 8f\ \dot{W}_{Hg}{}^2/ \pi^2 g\ \rho_{Hg}\ \Delta P_v$$

where: $f = 0.0225$ (Ref. b)

$$\rho_{Hg} = 0.26 \text{ lb/ft}^3$$

$$\Delta P_v = \Delta P_{tp} / \phi$$

$$\Delta P_{tp} = 4 \text{ psi} \quad \text{(assumed)}$$

d. Fourth, from the three independent relations between tube length, average diameter, and number of tubes, unique values for these quantities may be computed. The computed length for the condensing section is then increased by the ratio of subcooling heat load to condensing heat load.

$$L' = L(1 + Q_{sub} / Q_{cond})$$

where: $Q_{sub} = \dot{W}_{Hg} C_{Hg} (T_{cond} - T_{sub})$

$C_{Hg} = 0.0325 \text{ Btu/lb}^\circ\text{F}$

e. Fifth, the weight of the tubes is computed.

$$W_t = \pi L' N D \delta \rho_t$$

where: $\delta = 0.20$ in.

$\rho_t = 0.28 \text{ lb/in.}^3$

f. Sixth, the weight of the shell is computed assuming a hexagonal packing of the tubes with an average spacing of 0.050 in. between tubes. It is further assumed that the manifolds attached to the shell weigh the same as five disks the size of the cross-section of the shell.

$$W_s = \pi D_s t L' \rho_s + \frac{5\pi}{4} D_s^2 t \rho_s$$

where: $t = 0.063$ in.

$\rho_s = 0.280 \text{ lb/in.}^3$

g. Seventh, the weight of the liquid mercury in the subcooling portion is computed.

$$W_{Hg} = \frac{\pi}{4} D^2 (L' - L) \rho_{HgL} N$$

where: $\rho_{HgL} = 800 \text{ lb/ft}^3$

h. Eighth, the weight of the coolant is computed.

$$W_c = \frac{\pi}{4} (D_s^2 - ND^2) L' \rho_c$$

where: $\rho_c = 50.9 \text{ lb/ft}^3$

i. Ninth, the equivalent weight of the pumping power is computed making use of the Nusselt-Reynolds-Prandtl number correlation given in Reference (c):

$$Nu = 61.2 \left[(A_f^2 / A_H^2) \, Pr \, Re \right]^{3/5}$$

$$W_p = 0.271 \, (\dot{W}_c \, \Delta P_c / \rho_c)$$

where: $\Delta P_c = f \, V_c^2 \, L' \rho_c / 2g \, D_H$

$f = \psi(Re)$ (Ref. b)

$V_c = Re \, \mu_c / D_H \, \rho_c$

$Re = (Nu/61.2)^{5/3} \, A_H^2 / A_f^2 \, Pr$

$Nu = h_c \, D_H / k_c$

$D_H = 4 \frac{\pi}{4} (D_s^2 - ND^2) / \pi (D_s + ND)$

$A_H = \pi L' ND$

$A_f = \frac{\pi}{4} (D_s^2 - ND^2)$

The following assumptions are made:

$k_c = 14.74$ Btu/ft hr °F

$Pr = 0.00972$

$\mu_c = 0.670$ lb/ft hr

j. Finally, the total weight is computed by summing the five component weights computed above.

4. References:

a. "Government-Industry Confere ·e on Mercury Condensing" NASA TN D-1188, February 1962

b. B. Gebhart, "Heat Transfer," McGraw-Hill Book Co., 1961

c. "Sodium-NaK Supplement, Liquid Metals Handbook," TID 5277, AEC and Department of Navy, July 1955.

5. Nomenclature:

A_f	Coolant flow area
A_H	Heat transfer area
c	Specific heat
D	Diameter
D_H	Hydraulic diameter
f	Fanning friction factor
g	Conversion factor (32.2 lb/slug)
h	Heat transfer coefficient
H_v	Heat of vaporization
k	Thermal conductivity
L	Condensing length
L'	Total length
N	Number of tubes
Nu	Nusselt number
Pr	Prandtl number
Q	Heat transfer rate
Re	Reynolds number
t	Shell thickness
T	Temperature
UA	Overall heat transfer coefficient