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CALCULATION OF CERTAIN CHARACTERISTICS OF EXTENDED AIR SHOWERS IN THE LOWER PART OF THE ATMOSPHERE FOR LARGE FLUCTUATIONS OF THE ELEMENTARY ACT

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SUMMARY

Certain characteristics of extended air showers (EAS) are calculated in the assumption that the inelasticity coefficient and the multiplicity of particles produced in the interactions of protons with the nuclei of air atoms vary considerably from case to case.

The role of interactions with a large inelasticity coefficient in the development of EAS is considered. Results are given of calculations of energy distribution of primary protons generating EAS with a given number of particles, and of energy flux in the electron-photon and the nuclear-interacting components of EAS.

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The large number of recently obtained experimental data point to the fact that numerous characteristics of extended air showers (EAS) differ strongly from case to case. Referred to here is the broad distribution of showers by the age parameter S [1, 2], a great spread in the energy ratio of the nucleoactive and the electron-photon component of the shower [2, 3], the intensity fluctuations of the Cerenkov radiation attending the EAS [4], and so forth. All these data show that fluctuations play an important role in the development of a shower. Up to the present time several were published, in which the fluctuations' role in either kind of shower development is considered. However, most of these works consider only the fluctuations at places of interaction of the "leading" particle for an invariable elementary act [5, 6].

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The experimental data obtained by us during the study of interaction of particles with energy $10^{12}—5 \cdot 10^{13}$ ev [7,8] have shown that there exist, at least in that energy range, interactions characterized by an anomalously high value of the inelasticity coefficient ($\alpha > 0.8$). In connection with this it seems to us that accounting for the fluctuations of the inelasticity coefficient is required in all calculations of EAS development. In particular, it is apparently difficult to explain the experimentally observed broad spread of the energy ratio of the nucleoactive to electron-photon components of the shower without taking into account the fluctuations of inelasticity coefficient in the first stages of the shower.

We considered as early as in 1958 a simplified model of EAS development in the assumption that at interaction of high and ultra-high energy particles the inelasticity coefficient may vary within broad limits [9]. It was shown at the same time that if the showers develop only from interactions with a large inelasticity coefficient, they must have identical average characteristics as those experimentally observed. However, in order to simplify the calculations, we neglected in [9] the interactions with small values of the inelasticity coefficient. In the present paper, which constitutes in essence the subsequent development of the model described in [9], are presented the results of calculation of some characteristics of EAS with a more correct accounting of fluctuations of the inelasticity coefficient, and also taking into account the fluctuations of places of particle interaction.

1. FUNDAMENTAL HYPOTHESES

The experimental data obtained during the study of interaction of particles with energy $1 \cdot 10^{12}—5 \cdot 10^{13}$ev with light nuclei by method of controlled photoemulsions [7] and during the study of young air showers [8], have shown that there exist alongside with the usual intractions characterized by comparatively small inelasticity factor ($\alpha \approx 0.3—0.4$) and great multiplicity of generating particles (which we shall call in the following the pionization process) interactions, in which the inelasticity coefficient (or factor) $\alpha \gtrsim 0.8$, and the share of energy α_{π^0}, then transmitted by π^0-mesons, is $\alpha_{\pi^0} \gtrsim 0.6—0.7$. The peculiarity of these interactions consists in that the multiplicity of generated π^0-mesons [2 - 4] is substantially less than in the pionization process. According to our estimates the probability of such events is ~10 percent.

Starting from our experimental data we assumed that particle interactions, included those with energies > $5 \cdot 10^{13}$ ev, may in the first approximation be subdivided into two types: interactions with inelasticity coefficient equal to the unity ($\alpha_1 = 1$), which we shall call "catastrophic" for then the particle loses all its energy and does not participate in the further development of the shower and the usual (standard) interactions, i. e., the pionization process. We have assumed further that alongside with the interactions, when nearly all the energy of the primary particle is transferred by π^0-mesons with a probability twice greater (20%), analogous events are materialized with energy transfer by charged π-mesons. Therefore the "catastrophic" interactions are realized with the probability $w_1 = 0.3$, and the pionization process with the probability $w_2 = 0.7$.

For the mean value of the inelasticity coefficient to be 0.5 it is necessary to consider that in the pionization process it is $\alpha_2 = 0.29$. Since the absorption L_π and interaction $L_{\text{r}3}$ ranges of nucleoactive particles in the air are interrelated by means of the average value of the inelasticity coefficient and the index γ of the energy spectrum of particles, by assuming L_π = 120 g/cm and γ = 1.7, we shall obtain for the interaction range in the air the value $L_{\text{B}3} = 83$ g/cm .

Inasmuch as in interactions characterized by great values of the inelasticity coefficient the effective multiplicity of produced particles is small, we considered for the sake of definiteness, that in "catastrophic" interactions 70% of the energy of primary particle is transferred to three π-mesons, these being in one third of the cases only π^0-mesons, and in the remaining cases -- only the charged ones. The remaining energy is transferred to a large number of mesons in the same fashion as in the pionization processes, for which we considered that the energy lost is equally distributed among $10^{-2}\cdot E_0^{1/4}$ π-mesons.

On the basis of the above assumptions we computed the electronic-nuclear cascade curves for the pionization process. At the same time we assumed that in the course of the pionization process π^0-mesons constitute one third of produced particles. We considered the inelasticity coefficient for $\pi^\pm$-mesons to be the unity. The computation of the nuclear cascade was conducted by the method of consecutive generations taking into account the decay of $\pi^\pm$mesons [10]. Since the number of generations is then determined by the critical energy of $\pi^\pm$-mesons (that is, energy at which their decay range is compared with the range for interaction), which depends in its turn on the height of the observation level, we constructed the family of electronic-nuclear cascade curves for the pionization process $N_\pi(\varepsilon;\, t)$ for two concrete levels — the atmosphere depth 1030 g/cm^2 (sea level) and 690 g/cm^2 (3.2 km altitude above the sea level); ε is the energy transferred to all mesons during the interaction.

The electronic-nuclear cascade curves for catastrophic interactions with transfer of the greater part of energy to π^0-mesons $N_{\pi^0}(\varepsilon;\, t)$ were obtained by summation of six electron-photon curves and the electronic-nuclear curve for the pionization process for the corresponding energies. The cascade curves for catastrophic interactions with energy transfer to charged π-mesons $N_{\pi^\pm}(\varepsilon;\, t)$ were constructed analogously. Therefore, for each of the two observation levels three families of cascade curves (N_π; N_{π^0} and $N_{\pi^\pm}$) were constructed for the region of cascade energy ε variation from $6\cdot10^{11}$ to $3\cdot10^{16}$ ev.

2. METHOD OF CALCULATION

During the calculations we took into account the places of catastrophic interactions and, for the sake of simplification, we considered that the interactions preceding them, with $\alpha_2 = 0.29$, are uniformly distributed in the overlying atmosphere layer.

Let us consider the case when the catastrophic interaction is the k-th by count $(1 < k < \infty)$. The probability of this is $-u(k) = w_1\cdot w_2^{k-1}$. Assume that this interaction takes place in the atmosphere, at the depth t, which, in principle, may be less or more than the depth of observation t_H. It is natural that in the first case the catastrophic interaction will contribute to the shower observed at the depth t_H, and in the second case the development of the shower to the observation level will be determined only by pionization processes. Both these cases are schematically represented in Fig.1 (a and b).

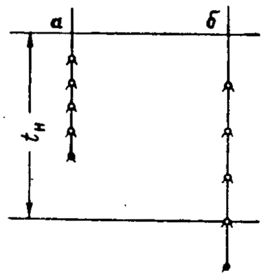

Fig.1. Schematic representation of two possible cases of EAS development. The white circles indicate the interactions with small inelasticity coefficient; the black circles show the catastrophic interactions.

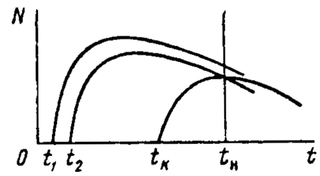

Fig.2. Illustration for the calculation of the number of particles of EAS. The atmosphere depth is in the abscissa and the number of showers' particles is in ordinates.

The probability that the k-th interaction (by count) takes place at the depth from t to t + dt [p(k; t)dt] is equal to the probability that in a layer of depth t k — 1 interactions took place [P(k — 1) is the Poisson function], multiplied by the probability of interaction in the layer dt, i. e., $dt/L_{вз}$. Therefore,

$$p(k;t)\,dt = P(k-1)\frac{dt}{L_{вз}} = e^{-t/L_{вз}}\left(\frac{t}{L_{вз}}\right)^{k-1}\frac{1}{(k-1)!}\frac{dt}{L_{вз}}, \qquad (1)$$

and the probability that the catastrophic interaction was the k-th by count and took place at depth from t to t + dt will be u(k)p(k; t)dt.

The places of interactions preceding the catastrophic one are uniformly distributed at the depth t. This is why the m-th interaction by count (pionization process) takes place at the depth

$$t_m = \frac{(2m-1)}{2}\frac{t}{k}. \qquad (2)$$

Let us consider the concrete case when the catastrophic interaction with transfer of 70% of energy from the primary particle to three π^0-mesons takes place above the observation level, that is $t < t_H$ (Fig.2). The number of shower particles at the level t_H from the m-th (by count) interaction with $\alpha_2 = 0.29$ is $N_\pi(\varepsilon_m;\, t_H - t_m)$. Inasmuch as all the preceding interaction also had 0.29 for the inelasticity coefficient, we have

$$\varepsilon_m = (1-\alpha_2)^{m-1}E_0, \qquad (3)$$

where E_0 is the energy of the primary particle creating the shower. The number of shower particles at level $t_н$ from all interactions preceding the catastrophic is equal to

$$\sum_{m=1}^{k-1} N_\pi(\varepsilon_m; t_н - t_m).$$

The number of particles at the level $t_н$ from the catastrophic interaction is $N_{\pi^0}(\varepsilon_k; t_н - t)$. (It is evident that $\varepsilon_k = (1-\alpha_2)^{k-1}E_0$.)This is why the total number of particles in EAS at the $t_н$ level is in the considered case

$$N_{\pi^0}(t_н) = \sum_{m=1}^{k-1} N_\pi(\varepsilon_m; t_н - t_m) + N_{\pi^0}(\varepsilon_k; t_н - t), \tag{4}$$

where t_m and ε_m are determined by formulas (2) and (3). An analogous expression is also obtained for the case when at catastrophic interaction nearly the entire energy is obtained by the charged π- mesons.

In the case when the catastrophic interaction takes place below the observaion level (Fig.1, b), the number of particles in the shower is

$$N_\pi(t_н) = \sum_{m=1}^{m^*} N_\pi(\varepsilon_m; t_н - t_m), \tag{5}$$

where m* is the ordinal number of the last interaction taking place above the observation level. At the same time, all the observed shower particles are produced only in pionization processes.

Relations (4) and (5) provide the possibility of determining the number of particles in the shower created by them for any values of k and t, so long as the energy of the primary photon is known. However, most of the available experimental data refer to EAS with known number of particles (and not with the given energy of the primary particle). This is why we required that in the case considered (catastrophic interaction with energy transfer to π^0 -mesons is the k-th by count and takes place at depth t to t + dt) the number of particles in the shower be included in the interval from N to B + dN. For the fulfillement of this requirement it was necessary that the energy of the primary particle having induced the shower be included in the interval from $E_{\pi^0}(N; k; t)$ to $E_{\pi^0}(N; k; t) + dE_{\pi^0}(N; dN; k; t)$, where $E_{\pi^0}(N; k; t)$ denotes such an energy of the particle that the shower induced by it at the level $t_н$ consist of N particles. If the spectrum of particles at atmosphere boundary has the form

$$F(E)\,dE = AE^{-(\nu+1)}dE,$$

the number of such particles will be

$$AE_{\pi^0}^{-(\nu+1)}(N; k; t)\,dE_{\pi^0}(N; dN; k; t). \tag{6}$$

The total number of showers with a number of particles from N to N + dN in the considered case will be equal to the number of particles of correspon- ding energy at the boundary of the atmosphere; it will be given by expression (6) multiplied by the probability that the catastrophic interaction is k-th by count and takes place at the depth of between d and d + dt in the atmosphere, i. e.,

$$n_{\pi^0}(N; dN; t_{\text{н}}; k; t) = \frac{A}{3} u(k) p(k; t) dt E_{\pi^0}^{-(\nu+1)} (N; k; t) dE_{\pi^0}(N; dN; k; t). \tag{7}$$

Inasmuch as the k-th interaction by count may, in principle, take place at any atmosphere depth from 0 to ∞, the number of all showers, in the course of whose development the catastrophic interaction was k-th by count, and contributed to the registered shower, is

$$n_{\pi^0}(N; dN; t_{\text{н}}; k) = \frac{A}{3} u(k) \times \int_0^t p(k; t) E_{\pi^0}^{-(\nu+1)} (N; k; t) dE_{\pi^0}(N; dN; k; t) dt. \tag{8}$$

Finally, since the catastrophic interaction may be of any ordinal number by count, the total number of EAS, in whose development catastrophic interactions have contributed with energy transfer to π^0-mesons, is

$$n_{\pi^0}(N; dN; t_{\text{н}}) = \frac{A}{3} \sum_{k=1}^{\infty} u(k) \int_0^{t_{\text{н}}} p(k; t) E_{\pi^0}^{-(\nu+1)} (N; k; t) dE_{\pi^0}(N; dN; k; t) dt =$$

$$= A \sum_{k=1}^{\infty} I_{\pi^0}(N; dN; t_{\text{н}}; k).$$

Analogously, for the total number of showers, into whose development con- ribution originated from catastrophic interactions with energy transfer to char- ged π-mesons, we shall have

$$n_{\pi^\pm}(N; dN; t_{\text{н}}) = \frac{2A}{3} \sum_{k=1}^{\infty} u(k) \times$$

$$\times \int_0^{t_{\text{н}}} p(k; t) E_{\pi^\pm}^{-(\nu+1)} (N; k; t) dE_{\pi^\pm}(N; dN; k; t) dt = A \sum_{k=1}^{\infty} I_{\pi^\pm}(N; dN; t_{\text{н}}; k). \tag{10}$$

Utilizing formulas (9) and (10), it is easy to obtain the number of showers that develop only from catastrophic interactions. Postulating k = 1 (for k > 2 the catastrophic interaction will be preceded by pionization processes), we shall obtain

$$n_{\text{кат}}(N; dN; t_{\text{н}}) = A [I_{\pi^0}(N; dN; t_{\text{н}}; 1) + I_{\pi^\pm}(N; dN; t_{\text{н}}; 1)]. \tag{11}$$

The matter is somewhat different with the number of showers developing only from standard interactions (pionization processes). As may be seen from Fig.1 b, in this case the catastrophic interaction must take place below the observation level, i. e., $t \geqslant t_{\text{н}}$. Moreover, in this case the catastrophic inter- action can not be first by count, that is, k > 2. This is why the total number of showers developing only from pionization processes is

$$n_\pi(N;\,dN;\,t_н) = A\sum_{k=2}^{\infty} u(k)\int_{t_н}^{\infty} p(k;\,t)\,E_\pi^{-(\gamma+1)}(N;\,k;\,t)\,dE_\pi(N;\,dN;\,k;\,t)\,dt =$$

$$= A\sum_{k=2}^{\infty} I_\pi(N;\,dN;\,t_н;\,k). \qquad (12)$$

The above formulas are initial for subsequent calculations. All these subsequent calculations were performed by the numerical method. The quantities E(N; k; t) and dE(N; dN; k; t) were determined by way of assortment (trial-and error solution), utilizing formulas (4) and (5). Utilized also at the same time were the computed electronic-nuclear cascade curves N_π; N_{π^0}; $N_{\pi^\pm}$ (see Sec.1). For the simplification of the assortment during the construction of families of cascade curves, energies ε_m were so chosen that $\varepsilon_m = (1-\alpha_2)\varepsilon_{m-1}$. For a given number k (ordinal number of catastrophic interaction), the quantities E and dE were determined for several values of t. For the intermediate values of t they were determined by interpolation. After computations of integrands the values of integrals of I(N; dN; t_H; k) were determined. It was admitted during calculations that the index of the differential energy spectrum of primary particles $\gamma + 1 = 2.7$. The energy E was expressed in ev. The results of calculations (values of energy I) for atmosphere depth of 1030 g/cm^2 at N = 10^4, 10^5, 10^6 and for atmosphere depth of 690 g/cm^2 at N = 10^5 are compiled in Table 1 below. In all cases dN = 1 is the particle.

TABLE 1

k	$I(10^4;1;1030\ г/см^2;k)\cdot10^{31}$			$I(10^5;1;1030\ г/см^2;k)\cdot10^{34}$			$I(10^6;1;1030\ г/см^2;k)\cdot10^{36}$			$I(10^5;1;680\ г/см^2;k)\cdot10^{33}$		
	π^0	$\pi\pm$	π	π^0	$\pi\pm$	π	π^0	$\pi\pm$	π	π^0	$\pi\pm$	π
1	65	96	—	288	374	—	147	135	—	525	695	—
2	111	152	0,1	394	539	0,5	159	184	0,2	433	587	1,6
3	127	172	0,5	386	573	1,8	136	171	0,6	281	423	4,9
4	117	160	1,4	324	524	4.7	101	145	1,6	144	242	11,4
5	88	120	3,2	236	365	10,2	68	107	3,2	87	148	22,4
6	56	83	6,0	165	248	18,2	47	77	5,5	52	90	32,2
7	36	53	8,9	95	164	26,4	29	52	7,7	28	54	38,5
8	25	35	11,5	57	98	33,4	17	33	9,4	16	31	43,4
9	12	19	12,9	28	54	36,4	9,7	19	10,3	7,3	14	40,5
10	5,9	10	12,8	18	32	36,5	5,3	10	10,1	3,5	6,9	31,8
11	2,5	4,3	11,4	9,4	18	34,4	3,0	5,5	9,5	1,7	3,3	26,3
12	0,9	1,7	10,0	4,6	9,0	29,6	1,7	3,1	8,4	0,8	1,6	18,8
13	0,3	0,7	7,8	2,2	4,2	23,9	0,9	1,7	6,7	0,4	0,8	12.5
14	0,1	0,3	5,8	0,9	1,9	18,9	0,5	0,9	5,4	0,2	0,4	10,6
⩾15	0.1	0,3	13,5	0,8	1,5	66,5	0,4	0,2	19,5	0,2	0,4	60.0
Sum	647	907	106	2009	3006	331	726	945	98,1	1580	2297	355

3. ROLE OF INTERACTIONS WITH GREAT INELASTICITY COEFFICIENT IN THE DEVELOPMENT OF E.A.S

As already noted above, one of the fundamental problems standing ahead of the current work, is the clarification of the role of interactions with great inelasticity coefficient in the development of showers, that is, to determine which part of shower particles originates as a result of catastrophic interactions. If such shower particles constitute an insignificant part of the total number of particles of EAS, the fluctuations of the inelasticity coefficient have apparently little effect on the development of showers, and we may neglect them in the first approximation. If, to the contrary, they constitute the bulk among all shower particles, the fluctuations of the inelasticity coefficient cannot be ignored. Moreover, it should then be considered that interactions with great inelasticity coefficient are determinant in the development of showers and, consequently, the experimentally measured characteristics of EAS fundamentally reflect the characteristics of interactions with great inelasticity coefficient.

In order to estimate the role of catastrophic interactions in the development of showers we have determined the quantity N_k/N, where N is the total number of particles in a shower at observation level, N_k is the number of shower particles having emerged as a result of catastrophic interactions. It is obvious that in showers developing only from catastrophic interactions (k = 1), $N_k/N = 1$. In showers developing only from pionization processes, the quantity $N_k/N = 0$. The number of showers with $N_K/N = 1$, equal to the unity or to zero, is determined respectively by formulas (11) and (12), and their percentage among all showers is compiled in Table 2.

TABLE 2

$t_{н}$ (г/см)²	N	$w(N_K/N=1)$	$w(N_K/N=0)$	$w(N_K/N\geqslant 0{,}5)$	$w(N_K/N\geqslant 0{,}7)$	$\langle N_K/N\rangle$
1030	10^4	9	6	86	74	0,75
	10^5	12	6	82	70	0,72
	10^6	16	5	80	67	0,72
680	10^5	28	8	72	58	0,66

As follows from Table 2, the fundamental part among registered showers is constituted by those, in which pionization processes (no more than 4 - 5, as a rule) precede the catastrophic interaction. The total number of particles in such showers is determined by expression (4), where the first addend gives the number of shower particles having emerged in pionization processes, and the second addend indicates the number of particles from catastrophic interaction. This is why , for the case when the catastrophic interaction with energy transfer to π^0-mesons was k-th by count and took place at the depth <u>t</u>

$$\frac{N_K}{N}=\frac{N_{\pi^0}(\varepsilon_k;\,t_н-t)}{\sum\limits_{m=1}^{k-1}N_\pi(\varepsilon_m;\,t_н-t_m)+N_{\pi^0}(\varepsilon_k;\,t_н-t)} \qquad (13)$$

where $\varepsilon_m = (1-\alpha_2)^{m-1}E_{\pi^0}(N;\, k;\, t)$, and t_m is determined by formula (2). An analogous expression is also obtained for interactions with energy transfer to charged mesons.

Determining the values of N_k/N for various combinations of k and t, and taking into account the corresponding statistical weight

$$u(k)p(k;\, t)E^{-(\gamma+1)}(N;\, k;\, t)dE(N;\, dN;\, k;\, t),$$

we constructed the distribution of the quantity N_k/N. These distributions for showers with $N = 10^5$ at two observation levels (1030 and 680 g/cm^2) are plotted in Fig.3, where the values of N_k/N are in abscissa and the percentage of showers in the corresponding interval of values of N_k/N are in ordinates.

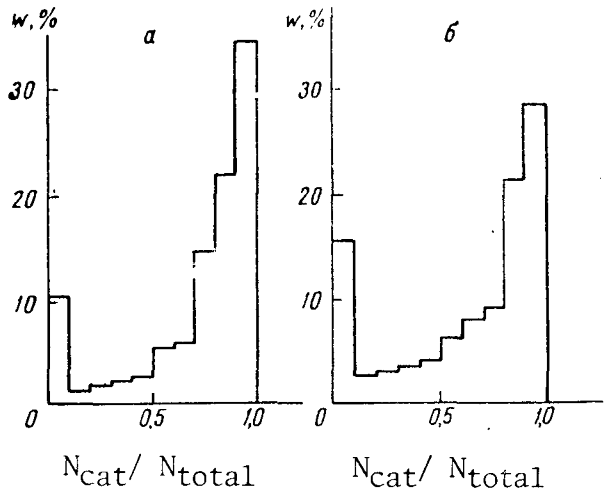

Fig.3. Distribution of showers with $N = 10^5$ by the value of N_k/N: a) at sea level; b) at atmosphere depth of 680 g/cm^2

As may be seen from Fig.3, particles produced in catastrophic interactions constitute in the overwhelming part of showers more than one half of all shower particles. The catastrophic interactions are determinant in the development of these showers. The percentage of EAS in which shower particles stemming from catastrophic interactions constitute $\geqslant 50\%$ and $\geqslant 70\%$ of all shower particles is compiled in Table 2.

Utilizing the data plotted in Fig.3, it is possible to determine the average value of $\overline{N_k/N}$. It is also included in Table 2, equalling 0.72 for sea level and 0.66 for the altitude of 3.2 km. Thus, under the assumptions made by us, interactions with great inelasticity coefficient determine also the development of showers in the average.

Let us note still one more circumstance. Under our initial assumptions the probability ratio of catastrophic interactions with transfer of 70% of energy to neutral and charged π-mesons was found to be 0.5. As to the ratio of the number of showers having emerged from catastrophic interactions with energy transfer to neutral and charged mesons at sea level, it is 0.7. This means that interactions with great inelasticity coefficient and energy transfer to π^0-mesons are the most effective for the generation of extended air showers.

4. ENERGY DISTRIBUTION OF PRIMARY PROTONS GENERATING E.A.S.

As already indicated above, in order to compute the number of showers $n(N;\, dN;\, t_H)$, we determined the energy of primary proton $E(N;\, k;\, t)$ for various values of k and t. If we now take these energy values with corresponding statistical weight, we shall obtain the distribution of primary protons inducing the showers with given N, by energy.

The distribution by energy of primary protons for showers with N = 10^4, 10^5 and 10^6 particles at sea level, and for particles with N = 10^5 at 3.2 km height is plotted in Fig.4. The energy of primary particles is plotted in logarithmic scale along the abscissa axis, and the number of particles (in percent), comprised in the corresponding energy range — in ordinates. As may be seen from Fig.4, the distribution of primary protons by energy becomes narrower with number of particle increase in the shower, as well as with altitude increase of the observation level, which was noted by us in the work [9]. In connection with this one may expect that as the power of sorted showers or altitude of observation level increase, the experimentally observed fluctuations of the various characteristics of E.A.S. must decrease.

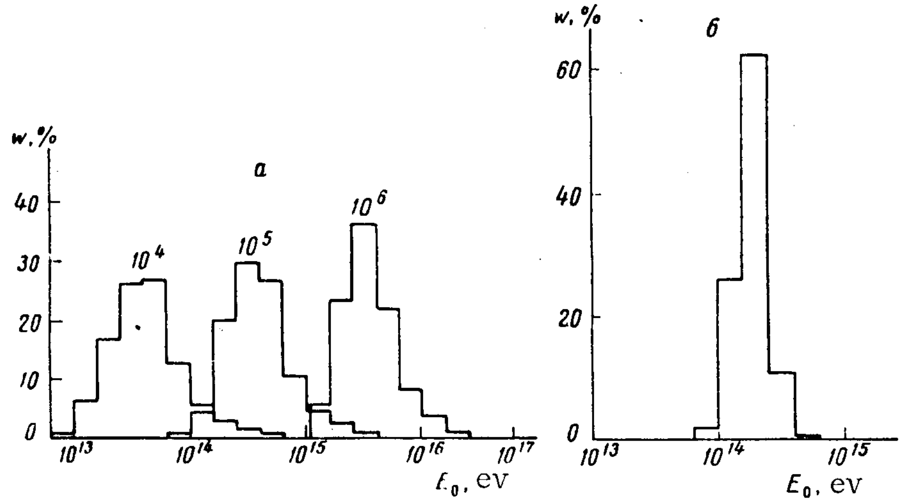

Fig.4. Distribution of primary protons by energy: a) for showers with N = 10^4, 10^5, 10^6 at sea level; b) — with N = 10^5 at 3.2 km.

At sea level, the most probable energy of primary protons for showers with numbers of particles 10^4, 10^5, 10^6 constitutes respectively $4.0 \cdot 10^{13}$ev, $3.6 \cdot 10^{14}$ ev and $3.1 \cdot 10^{15}$ ev. These values are 20% lower than those obtained in the work [6]. Had we accounted for the fluctuations at places of interactions preceding the catastrophic ones, the discrepancy would have been still greater. This is why we should consider that the spectrum of primary particles, reduced from the spectrum of EAS by the number of particles and brought out in ref.[6], where fluctuations of the inelasticity coefficient were not taken into account, gives a somewhat overrated flux of high-energy particles.

From the distributions illustated in Fig.4, we may determine the mean value of energy of particles creating EAS with given N. For showers with N = 10^5 registered at sea level, it is equal to $5.0 \cdot 10^{14}$ ev. For the same showers at 3.2 km altitude it is $1.9 \cdot 10^{14}$ ev, i. e., it is 2.75 times lower. The mean energy ratio of particles inducing showers with N = 10^5 at sea level and at 3860 m altitude (Pamir) is ~ 3. At the same time, in [6], where only the fluctuations of places of interactions were taken into account, the mean energies of primary protons creating showers with N = 10^5 at sea level and in Pamir, differ by a factor of 3.2.

It is well known that the mean value of Cerenkov radiation attending the showers with a given number of particles, is proportional to the mean energy of primary particles. According to available experimental data the Čerenkov radiation in showers with $N \geqslant 10^5$ is 7 to 10 times less in Pamir than in Moscow [11]. Therefore, when accounting for the fluctuations of the inelasticiy coefficient, the ratio of mean values of primary protons inducing showers with identical number of particles in Moscow and in Pamir, is obtained closer to experimental data than in the work [6], though this ratio is, one way or another, less by two-three times than the experimental. In this connection the following should be noted. The highest ratio of energies of primary protons for showers with identical number of particles in Moscow and in Pamir is obtained in the assumption that the shower develops generally without any kind of fluctuations. However, even in this, knowingly unrealizable case, the ratio of mean energies will in all be 4.7, which is 1.5 to 2 times less than the experimental result.

5. ENERGY FLUXES OF ELECTRON-PHOTON AND NUCLEOACTIVE COMPONENTS OF E.A.S.

On the basis of the considered model of E.A.S. development, certain energetic characteristics of showers were also computed. To that effect fluxes of electron-photon and nucleoactive energy components were sought for beforehand at different levels of development of electronic-nuclear cascades of the pionization process (see Sec.1).

Solution of equations of pionization nucleocascade process by method of consecutive generations [10] gives for the energy flux of nucleoactive particles the expression

$$S_{\text{n. a.}}(t) = \frac{2}{3} E_0 e^{-t/L_{\text{B3}}} \sum_{i=1}^{l} \frac{1}{(i-1)!} \left(\frac{2}{3} \frac{t}{L_{\text{B3}}} \right)^{i-1}, \qquad (14)$$

where l is the number of the last generation of nucleoactive particles.* Calculation by formula (14) was performed for generations of π-mesons with energy $>10^{11}$ ev.

For the energy flux of the electron-photon component in the same cascade the following equation may be written:

$$S_{\text{el-ph}}(t) = E_{\pi^0}(t) - E_{\text{ion}}(t), \qquad (15)$$

where $E_{\pi^0}(t)$ is the energy transferred by π^0 -mesons to the electron-nuclear cascade having covered the path t; $E_{\text{ion}}(t)$ is the energy expended by shower particles for the ionization over the same path.

For the first term of the right-hand part of Eq.(15) we may obtain according to [11]

$$E_{\pi^0}(t) = \frac{2}{3} E_0 \left\{ \left[1 - \left(\frac{2}{3} \right)^m \right] - e^{-t/L_{\text{B3}}} \sum_{i=1}^{m} \frac{1}{(i-1)!} \left(\frac{t}{L_{\text{B3}}} \right)^{i-1} \left[\left(\frac{2}{3} \right)^{i-1} - \left(\frac{2}{3} \right)^m \right] \right\}, \qquad (16)$$

---

* $S_{\text{n.a}}$ stands for $S_{\text{nucleoactive}}$.

where m is the number of the last generation of nucleoactive particles, this time determined by the critical energy of π-mesons (see Sec.1). For the second term we have

$$E_{\text{иoн}}(t) = \beta \int_0^t N_\pi(E_0;\, t)\,dt, \quad (*) \qquad (17)$$

where β denotes the ionization losses of shower particles. Function $E_{ion}(t)$ was found by numerical integration of computed electron-nuclear cascade curves for the pionization process.

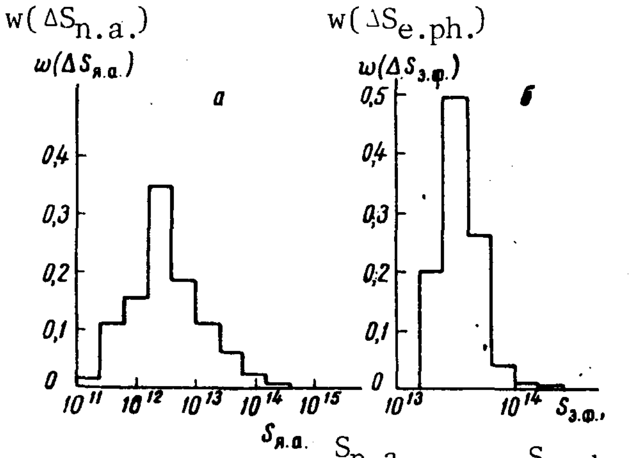

Fig.5. Distribution of energy fluxes in showers with N = 10^5 at sea level: a) nucleoactive component; b) electron-photon component. Abscissa) energy of the respective component; ordinates) fraction of cases comprised within the corresponding energy range

On the basis of formulas (14)-(17) we computed by numerical method the fluxes of nucleoactive and electron-photon components at first for cascade curves of the pionization process (N_π), and then, with the aid of the method described in Sec.2. for the corresponding cascade curves from processes of catastrophic interactions (N_{π^0}, $N_{\pi^\pm}$).

After these preparatory computations we may determine the energy fluxes of the nucleoactive and electron-photon components of the shower. Besides, we know the statistical weights for the corresponding combinations of k and t. Consequently, we may also compute the distribution of energy fluxes of both components.

The result of computation of the distribution of energy fluxes of the high-energy nucleoactive component ($E_\pi \geqslant 10^{11}$ ev) in showers with N = 10^5 particles, registered at sea level is plotted in Fig.5 a. As may be seen from that figure, this distribution is rather broad: the energy of the nucleoactive component of the EAS is comprised within the 10^{11} to 10^{14} energy range. The energy of the nucleoactive component in showers, in the course of development of which catastrophic interactions were manifest with transfer of the basic part of energy to π^0-mesons, constitutes by order of magnitude from 10^{11} to 10^{12} ev. In showers from catastrophic interactions with energy transfer to charged π-mesons it is $\sim 10^{12}$ -- 10^{13} ev. Finally, in showers, developing only from pionization processes, the energy carried by the nucleoactive component is $\gtrsim 10^{13}$ ev.

The mean computed value of energy flux of the nucleoactive component in showers with N = 10^5 particles registered at sea level, constitutes $1 \cdot 10^{13}$ ev. For comparison we shall indicate that the experiment of identical showers gives the value $(1.0 \pm 0.3) \cdot 10^{13}$ ev. [1].

(*) $E_{\text{иoн}}(t)$ stands for $E_{ion}(t)$

The following should be noted in connection with this. Starting from the cascade curves [12], it may be shown that when the energy of the primary γ-quantum $E_0 \gtrsim 10^{10}$ ev in electromagnetic cascades, the energy of the shower can not be less than $2.3 \cdot 10^8$ N with age parameter S = 1.2 (which is close to the average value S for showers registered at sea level). When the primary energy is 10^{13} ev, the energy flux of the electron-photon component constitutes $3 \cdot 10^8$ N. The accounting for the "booster" to electromagnetic cascade by nucleoactive particles may only lead to energy increase of shower's electron-photon component. This is why it seems to us that neither of the lately considered models of EAS development (including the present) can assure such an energy of the electron-photon component as the one observed in the experiment.

Finally, we present in Fig.6 the computed distribution of the ratio of energy fluxes of the nucleoactive to the electron-photon components in showers with $N = 10^5$ at sea level. In such showers the mean energy ratio of the nucleoactive to the electron-photon components is equal to 0.21 and, consequently, the energy of the nucleoactive component constitutes 17 percent of the energy of the whole shower.

It seems to us that further refining of experimental data on energy fluxes of the electron-photon and, most particularly of the nucleoactive components of showers, on their mean values and distributions may help substantially in the choice of either mode of development of extended air showers, and in the ascertaining of the main characteristics of interactions leading to the formation of showers.

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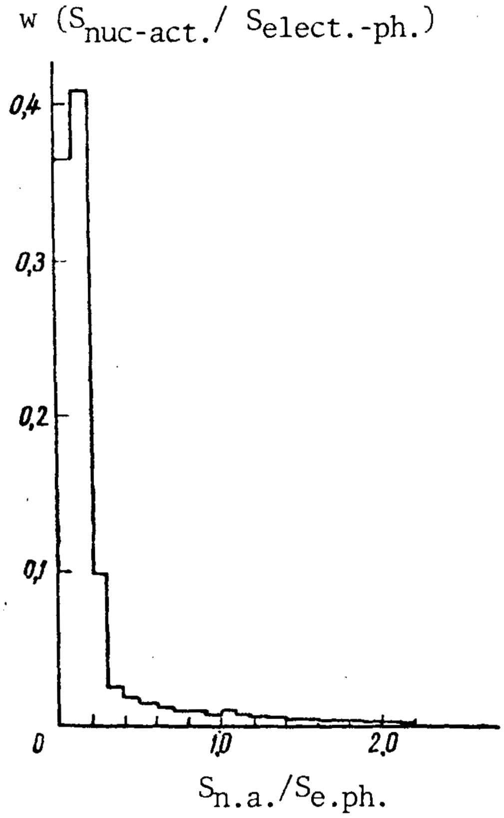

Fig.6. Distribution of the ratio of energy fluxes of the nucleoactive and electron-photon components in showers with $N = 10^5$ at sea level. The ratio is in the abscissa and the fraction of cases comprised in the corresponding intervals is in ordinates

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