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VARIATION OF THE ANGULAR AND ENERGY DISTRIBUTION IN A CHARGED PARTICLE FLOW ACROSS A MAGNETIC FIELD

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SUMMARY

The influence is determined of the radiation retardation on the distribution by angles and energies in a flux of charged particles across a magnetic field layer of finite thickness. The results obtained may have a significance in astrophysics when considering the motion of fast electron fluxes through regions of stellar atmospheres and nebulae with intense magnetic fields.

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The influence of radiation retardation on the motion of a charged particle in a uniform magnetic field was considered in ref. [1]. We shall now determine how the radiation retardation influences the distribution by angles and energies in a flow of charged particles across a magnetic field layer of finite thickness.

We shall consider that the magnetic field is concentrated in a plane-parallel layer of thickness L, that it has inside the layer a constant intensity H and is directed along the normal to layer's surface. Assume that a flux of particles with a specific energy and a density $j_0(\theta_0)$, $0 \leqslant \theta_0 \leqslant \pi/2$, is incident upon it, θ_0 being the angle between the particle's velocity direction and the axis OZ of the coordinate system and the direction $\vec{H}$ coinciding with OZ. The character of the distribution by angles and energies of particles having emerged from the layer is easy to represent.

Let us denote by θ the angle between the direction of the velocity and the axis OZ at egress from the magnetic field layer. It is evident that at $\theta_0 = 0$ we shall also have $\theta = 0$. As θ_0 increases, so does θ. On the other hand, for $\theta_0 \approx \pi/2$ the particle will be moving in the layer a long time, losing nearly all of its tranverse velocity, so that $\theta \ll 1$. Therefore, one may expect,

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(*) IZMENENIYE UGLOVOGO I ENERGETICHESKOGO RASRREDELENIYA V POTOKE ZRYAZHENNYKH CHASTITS PRI PROKHOZHDENII CHEREZ MAGNITNOYE POLE.

and this is corroborated by direct calculation, that all egress angles θ are comprised within a cone $0 \leqslant \theta \leqslant \theta_m$, of which the aperture θ_m depends on the thickness of the layer and the intensity of the magnetic field. To every θ inside the cone correspond two values of θ_0 ; inasmuch as the radiation energy losses depend on θ_0 particles with two energy values move in the flow of particles having emerged from the layer along each direction θ.

Passing to the computation, we note that, according to [1], in the extreme relativistic case the transverse component $v_\perp$ decreases with time according to the law

$$v_\perp = v_\perp(0)/\mathrm{ch}\left(\frac{\delta t}{c} v_0 \sin\theta_0\right), \tag{1}$$

where

$$\delta = {}^2/_3 e^4 H^2 / m^3 c^5. \tag{2}$$

The longitudinal velocity $v_z = v_0 \cos\theta_0$ remains invariable. This is why the time $\underline{t}$ of motion through the layer may be expressed through v_z and the layer thickness L

$$t = L / v_z = L / v_0 \cos\theta_0. \tag{3}$$

Let us introduce $\mathrm{tg}\,\theta = v_\perp / v_z$ and denote

$$k = \frac{\delta L}{c} = {}^2/_3\, e^2/m^3c^5\, H^2 L; \tag{4}$$

then

$$\mathrm{tg}\,\theta = \mathrm{tg}\,\theta_0 \frac{1}{\mathrm{ch}\,(k\,\mathrm{tg}\,\theta_0)}. \tag{5}$$

It stems from this formula that to one value of θ correspond two values of θ_0 , provided θ does not exceed the value θ_m $(\theta_0' < \bar{\theta}_0\ , \theta_0'' > \bar{\theta}_0)$. The maximum value of θ_m is reached at $\theta_0 = \bar{\theta}_0$, where $k\,\mathrm{tg}\,\bar{\theta}_0 \approx 1{,}2$, and is determined by the relation $\mathrm{tg}\,\theta_m = 2/3k$.

The energy of the particle is determined by the formula

$$\frac{1}{E} - \frac{1}{E_0} = \frac{\sin\theta_0}{mc^2}\,\mathrm{th}\,(k\,\mathrm{tg}\,\theta_0). \tag{6}$$

which is obtained from formula (12) of ref.[1] at substitution of expression (3) for the time $\underline{t}$ by the angle θ_0 *.

Formulas (6) and (5) express the dependence of 1/E — 1/E on θ in a parametric form (by the angle θ_0). The graphs of this dependence are given in Fig.1 for several values of K. The flux of particles emerging from the layer through one element of solid angle,

$$dN = 2\pi j(\theta) \sin\theta\, d\theta,$$

is equal to the sum of fluxes of particles entering the layer at angles θ_0' and θ_0''.

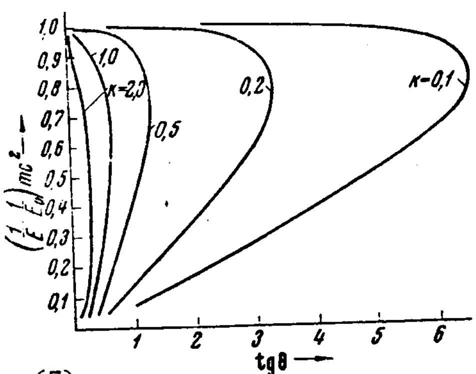

Fig.1

We have

$$j(\theta) = j_0(\theta_0')\, d\cos\theta / d\cos\theta_0' + j_0(\theta_0'')\, d\cos\theta / d\cos\theta_0''. \tag{7}$$

* see next page.

Utilizing (5), we may obtain

$$d\cos\theta / d\cos\theta_0 = \cos^3\theta_0 / \cos^3\theta \times \mathrm{ch}^2(k\,\mathrm{tg}\,\theta_0) \,/\, [1 - k\,\mathrm{tg}\theta_0\,\mathrm{th}\,(k\,\mathrm{tg}\,\theta_0)]. \qquad (8)$$

In conclusion we shall bring forth estimates for the values of the coefficient k for some of the cosmic objects. For the Sun, assuming $L \sim 10^{12}$ cм, $H \sim 1$ gāuss, we have $k \sim 5\cdot10^{-8}$. For a magnetic spot of the Sun, $L \sim 10^{10}$ cм, $H \sim 10^3$ gauss, $k \sim 5\cdot10^{-4}$. For some of the giant stars of later spectral classes [3] $L \sim 10^{13}$ cм, $H \sim 10^3$ gauss, $k \sim 0{,}5$.

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**** THE END ****

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* (from the preceding page) It follows from Eq. (6) the result, obtained by Pomeranchuk [2] as early as 1939, that as $E_0 \to \infty$, the final energy is approaching a constant limit, not dependent on E_0. However, contrary to [2], we do not replace here the true trajectory by a straight line.