National Aeronautics and Space Administration
Goddard Space Flight Center
Contract No.NAS-5-12487

ST-PF-GM-10569

ON THE MOTION OF A CHARGED PARTICLE IN THE UNIDIMENSIONAL MODEL OF MAGNETOSPHERE'S TRANSITIONAL LAYER

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23 FEBRUARY 1967

ST PF GM 10569

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Kosmicheskiye Issledovaniya
Tom 5, vyp.1, 157-159
Izdatel'stvo "NAUKA", 1967

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SUMMARY

The trajectory of a charged particle in a unidimensional model of the transitional layer of the magnetosphere is discussed in the light of specific hypotheses concerning the physical conditions in that layer, and as a function of particle's angle of incidence upon it.

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Measurements of the magnetic field on AES Explorer-12 [1] and Explorer-18 [2] (IMP.1) lead to the conclusion that there exists between the shock wave forming as a consequence of sub-Alfvén solar wind flow past the geomagnetic field and the outer boundary of the magnetosphere a region of irregular magnetic field. This region is called the transition region of the magnetosphere and is characterized by a chaotic variation of the magnetic field intensity vector in direction, as well as in magnitude.

It should be noted that a transitional region with such magnetic field properties exists always, even during quiet geomagnetic periods. A rough estimate according to data of Explorer-12 [1] gives for the characteristic amplitude of oscillations 10γ and for the characteristic macroscopic length — $2 \cdot 10^3$ km. The question on the physical nature of this transitional region is still obscure at present. The irregular field variations are conditioned either by turbulence, or are the consequence of contraction of the inhomogenous interplanetary medium in front of such an obstacle as the geomagnetic cavity is [3].

Digressing from the effects conditioned by the chaoticity of field structure, and taking into account only the effects linked with field variation in space, we shall neglect in the proposed model of magnetosphere's transitional layer the temporal variations of the magnetic field, and we shall substitute

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* [O DVIZHENII ZARYAZHENNOY CHASTITSY V ODNOMERNOY MODEL PEREKHODNOGO SLOYA MAGNITOSFERY]

for the true pattern of sharp spatial variations in magnitude and time of the magnetic field a unidimensional sinusoidal field with a characteristic amplitude of 10γ and wavelength $\lambda = 2 \cdot 10^3$km. The plane $z = 0$ is the boundary of the transitional layer and the magnetic field B is directed tangentially (along the axis x)

$$B = B_x = B_0 + B_1 \cos kz \quad (k = 2\pi/\lambda). \tag{1}$$

Let a charged particle enter the considered region at the time $t = 0$ under the following initial conditions:

$$y(0) = z(0) = v_y(0) = 0, \; v_z(0) = v_0.$$

The equations of motion

$$m\frac{dv_y}{dt} = \frac{q}{c} v_z B(z), \qquad m\frac{dv_z}{dt} = -\frac{q}{c} v_y B(z) \tag{2}$$

are easy to integrate. As a result we obtain

$$t = \int_0^z \frac{dz}{\sqrt{v_0^2 - \omega^2\left(z + \frac{b}{k}\sin kz\right)^2}}. \tag{3}$$

Here $\omega = -qB_0/mc$, $b = B_1/B_0$.

For the sake of simplicity we may assume that the constant component B_0 is small by comparison with the amplitude B_1, which is justified, inasmuch as in real transitional region of the magnetosphere the field varies sharply in direction. Neglecting the constant field component, we obtain from (3)

$$kv_0 t = F\left(\frac{\omega_1}{kv_0}, kz\right) \quad \text{for} \left|\frac{\omega_1}{kv_0}\right| < 1, \tag{4}$$

$$\omega_1 t = F\left(\frac{kv_0}{\omega_1}, \arcsin\left(\frac{\omega_1}{kv_0}\sin kz\right)\right) \quad \text{for} \left|\frac{\omega_1}{kv_0}\right| > 1. \tag{5}$$

Here $\omega_1 = -qB_1/mc$, and $F(k,\phi)$ is an elliptical integral.

The particle trajectory equation of first kind is of interest; it has the following form

$$ky = \operatorname{arsh}\frac{\varkappa\cos kz}{\sqrt{1-\varkappa^2}} - \operatorname{arsh}\frac{\varkappa}{\sqrt{1-\varkappa^2}} \quad \text{for} \quad |\varkappa| < 1, \tag{6}$$

$$ky = \operatorname{arsh}\frac{\cos\{\arcsin(\varkappa\sin kz)\}}{\varkappa\sqrt{1-1/\varkappa^2}} - \operatorname{arsh}\frac{1}{\varkappa\sqrt{1-1/\varkappa^2}} \quad \text{for} \quad |\varkappa| > 1, \tag{7}$$

where

If $|\varkappa| \gg 1$, the trajectory (7) becomes the Larmor circumference.

Expression (4) describes an infinite motion that takes place for

$$v > |\omega_1|/k.$$

In the opposite case ($v_0 < |\omega_1| /k$), the motion of the particle is finite. Therefore, the sinusoidal magnetic field may retain particles with sufficiently small impulse. The explanation consists in that a particle with small impulse has a Larmor radius smaller than the characteristic scale of field variation, so that during its motion the particle does not "perceive" the variation of the field in space.

For the assumed values of B_1 and k, electrons, moving with the velocity of the solar wind, have finite trajectories (they are Larmor circumferences, since $\kappa \sim 10^3$). As to ions, they may move along either closed or open trajectories. For protons the critical velocity $v_{0\,cr}$ constitutes 311 km/sec. For $v_0 > v_{0cr}$ the proton permeates the transitional layer of the magnetosphere with such parameters, while for $v_0 < v_{0cr}$ it describes a closed trajectory in the region of the first magnetic field maximum. Plotted in the Fig.1 below are the trajectory of proton motion at v_0 = 400 km/sec (the curve 1 is an open trajectory) and v_0 = 300 km/sec (curve 2, closed trajectory).

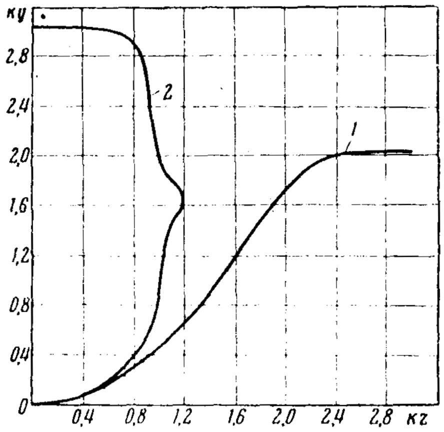

Fig.1

If the particle incidence upon the transitional layer of the magnetosphere takes place at a certain angle to the axis z (oblique incidence), the condition of finiteness of the trajectory (4) changes in its form. Indeed, assume $v_z(0) = v_0 \cos\alpha$, $v_y(0) = v_0 \sin\alpha$, where $-\pi/2 \leqslant \alpha \leqslant \pi/2$. Considering that the constant component $B_0 = 0$ and integrating the equations of motion (2), taking into account the boundary conditions, we shall obtain:

$$\frac{dz}{dt} = \sqrt{v_0^2 \cos^2 \alpha - \frac{\omega_1^2}{k^2} \sin^2 kz + \frac{2\omega_1 v_0 \sin \alpha}{k} \sin kz}. \qquad (8)$$

It is clear that the trajectory of the particle is finite when the radicand in the right-hand part of (8) becomes zero. Hence it follows that the condition for trajectory finiteness has the form

$$\frac{kv_0}{|\omega_1|}(1 - |\sin \alpha|) \leqslant 1.$$

At $\alpha = \pm\pi/2$, the trajectory of the charged particle is always finite, no matter what its energy (at relativistic velocities the quiescent mass m must be replaced in the equations of motion (2) by $m / \sqrt{1 - v_0^2/c^2}$).

*** THE END ***

Received on 15 Nov.1965

R E F E R E N C E S

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CONTRACT No.NAS-5-12487
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Translated by ANDRE L. BRICHANT

on 23 February 1967

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