AN EXPERIMENTAL STUDY OF IMPERFECTLY CONDUCTING DIPOLES

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AN EXPERIMENTAL STUDY OF IMPERFECTLY CONDUCTING DIPOLES

Division of Engineering and Applied Physics
Harvard University   Cambridge, Massachusetts

ABSTRACT

Input admittances of dipole antennas with moderately high internal impedance were measured in the UHF range with the antenna lengths varying from 1/10 - wavelength to such a value that the antenna behaved as if infinitely long. The measured results are compared with the three-term theory of King and Wu [1] and with the theoretical values obtained by Shen and Wu [3] for an infinitely long antenna.

1. Motivations

In a recent paper by King and Wu [1], a combination of three trigonometric functions were used to represent the current distribution I(z) on a dipole antenna made of a cylinder with an internal impedance per unit length z^i:

$$I(z) = I_V \sin k(h-|z|) + I_U (\cos kz - \cos kh) + I_D \left(\cos \frac{k_o z}{2} - \cos \frac{k_o h}{2}\right), \quad (1)$$

where z is the axial coordinate measured from the driving point, h is the half-length of the antenna, k_o is the free-space wave number, and I_V, I_U, and I_D are constant that can be determined from Hallen's well-known integral equation. Furthermore, the value of k in (1) is related to the internal impedance z^i by the following equation:

$$k^2 = k_o^2 (1 - j2\lambda z^i / \zeta_o \Psi_{dR}), \quad (2)$$

where λ is the wavelength, ζ_o is the intrinsic impedance of free space, and Ψ_{dR} is a constant defined by equation (54b) in [1].

The three trigonometric functions in (1) were obtained following a careful analysis of the behavior of the integral in the integral equation. This three-term theory gives accurate values of input admittances, current distributions, as well as field patterns as compared with the experiment when the internal impedance is small. It is certainly true that other theories such as the King-Middleton iterative procedure or the numerical method can also yield satisfactory results, but the simplicity in the representation of the current distribution enjoyed by the three-term theory is unique among the existing accurate theories. Numerical results based on the three-term theory are available in [2].

The three-term theory, however, has been verified by experiment only when the antenna is made of metal and a/λ is of the order 0.007 (a is the radius of the antenna) so that the parameter $\Phi_i = 2\lambda z^i/\zeta_0$ is quite small (of the order 0.01). In such cases, the complex wave number k defined by (2) is essentially real and the conductivity of the antenna can be thought to be infinite if the antenna is not many wavelengths long. In many applications, on the other hand, the ratio a/λ is smaller than 0.007 and the effect of the imperfect conductivity can not be ignored. This calls for a critical experimental confirmation of the three-term theory when applied to antennas with moderately high internal impedances.

Furthermore, the three-term theory is known to be applicable for antenna lengths limited to about 5/8 - wavelength. Shen and Wu [3] studied the problem of an infinitely long resistive antenna hoping to acquire some understanding of the behavior of long but finite antennas. The theoretical input admittance of an infinitely long antenna has been obtained but has not been verified experimentally. An experimental verification is possible since when z^i is moderately high, the antenna under test actually need not be very long in order to resemble an infinitely long antenna.

2. Descriptions of the Experiment

In this experiment tubular conductors with conducting layers thinner than the skin depth were used. Solid metallic conductors were not used since it is inconvenient to handle antennas when the ratio a/λ is as small as is required to get sufficiently high values of z^i. The z^i of a tubular conductor with a conducting layer thinner than the skin depth is equal to the d-c resistance of the conductor. This is a quantity that is readily and accurately measureable. The use of thin tubular conductors made it possible to carry out the experiment in the UHF range. It is to be noted that, due to the nature of the approximations involved in the three-term theory, should the theory be confirmed experimentally for a given value of a/λ it would be possible to predict even better accuracy for smaller values of a/λ .

The tubular conductor was constructed by spraying resistive paint onto a dielectric cylinder. The resistive paint, a mixture of carbon powder and adhesives, had a conductivity equal to about 1.5×10^3 mho per meter. The corresponding skin depths are 0.043 cm at 900 MHz and 0.061 cm at 450 MHz. The thickness of the coating was 0.0035 cm for z^i equal to 850 ohms per meter. Since the thickness of the coating is an order of magnitude smaller than the skin depth, z^i is well approximated by the d-c resistance of the coating. A detailed description of the resistive coating is given in [4].

Input admittances of antennas with $a = 0.32$ cm and z^i equal to either 700 or 1400 ohms per meter were measured in the frequency range 500 to 900 MHz, with antenna heights ranging from 6.1 cm to a value for which the antenna behaved as if infinitely long. The values of z^i were chosen so that the parameter Φ_i was in the range of 1.24 to 4.46 which, as can be seen from Figs. 8 and 9 of [2] , should be of the greatest interest to the present investigation.

The results of the measurement are shown in Figs. 1, 2, and 3, and the experimental data are also tabulated in Tables 1 and 2. The

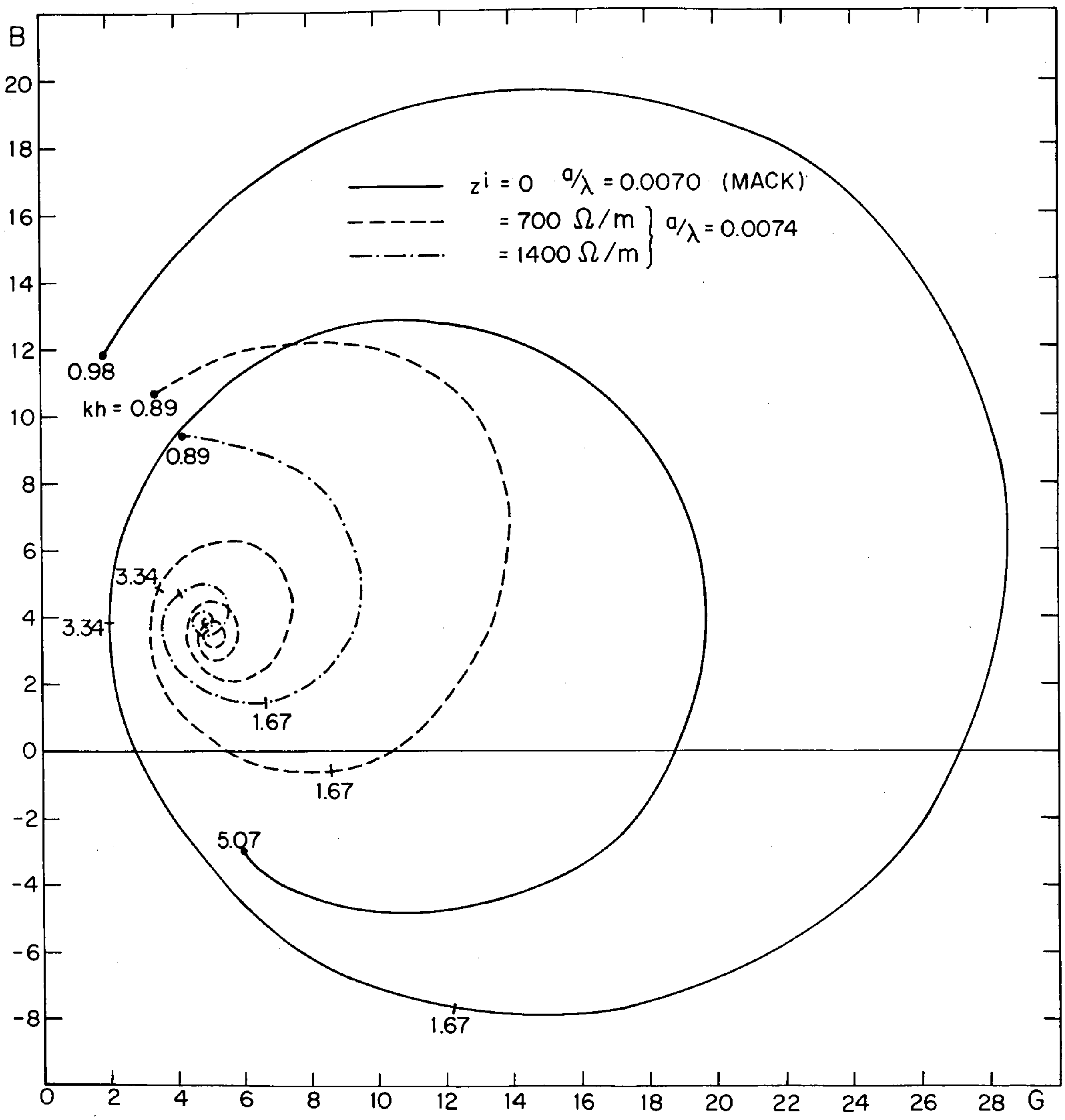
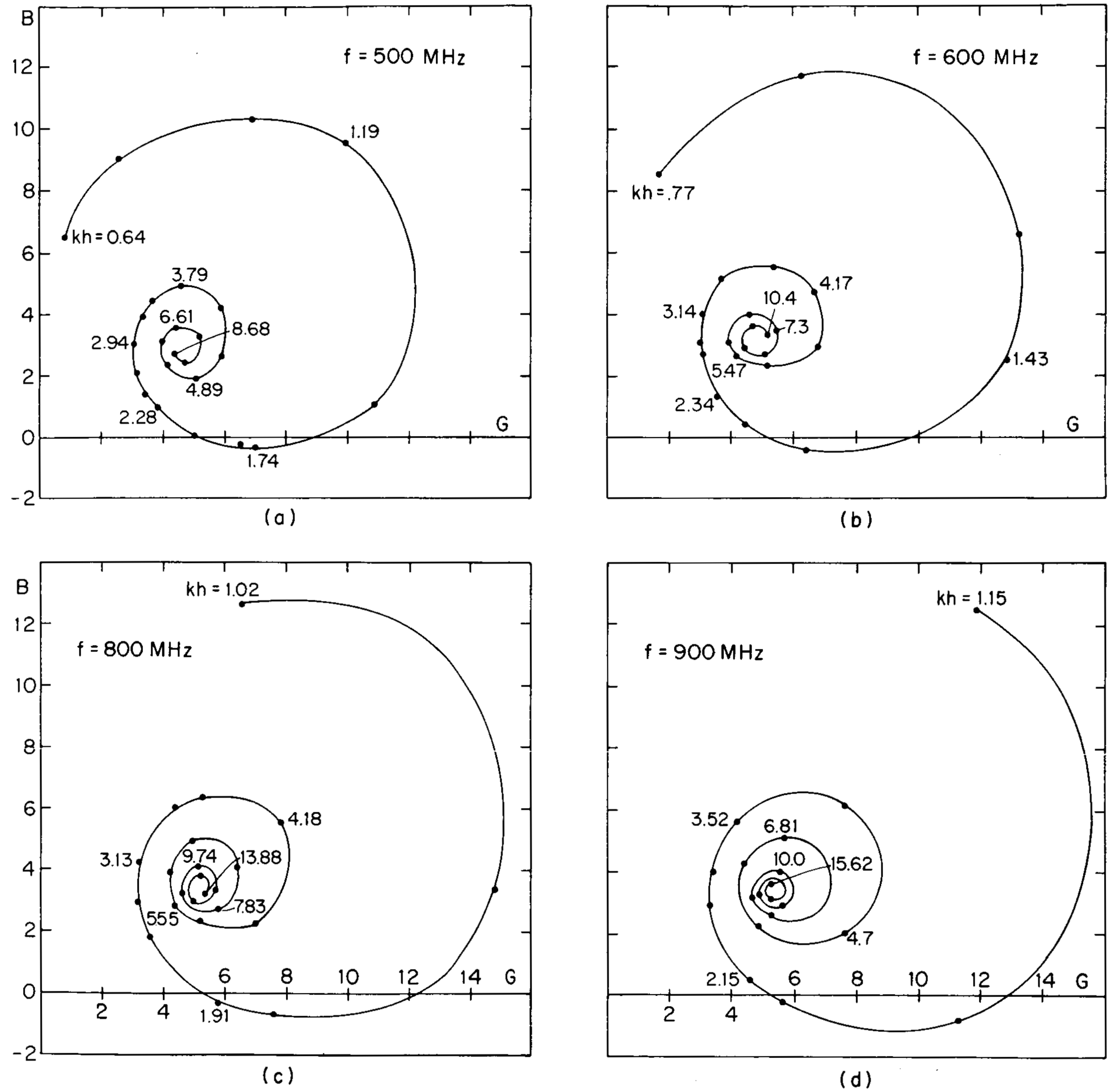


FIG. 1 MEASURED INPUT ADMITTANCES, a = 0.32 cm



(a) (b)

(c) (d)

FIG. 2 MEASURED INPUT ADMITTANCES $z^i = 700$ Ω/METER, $a = 0.32$ cm

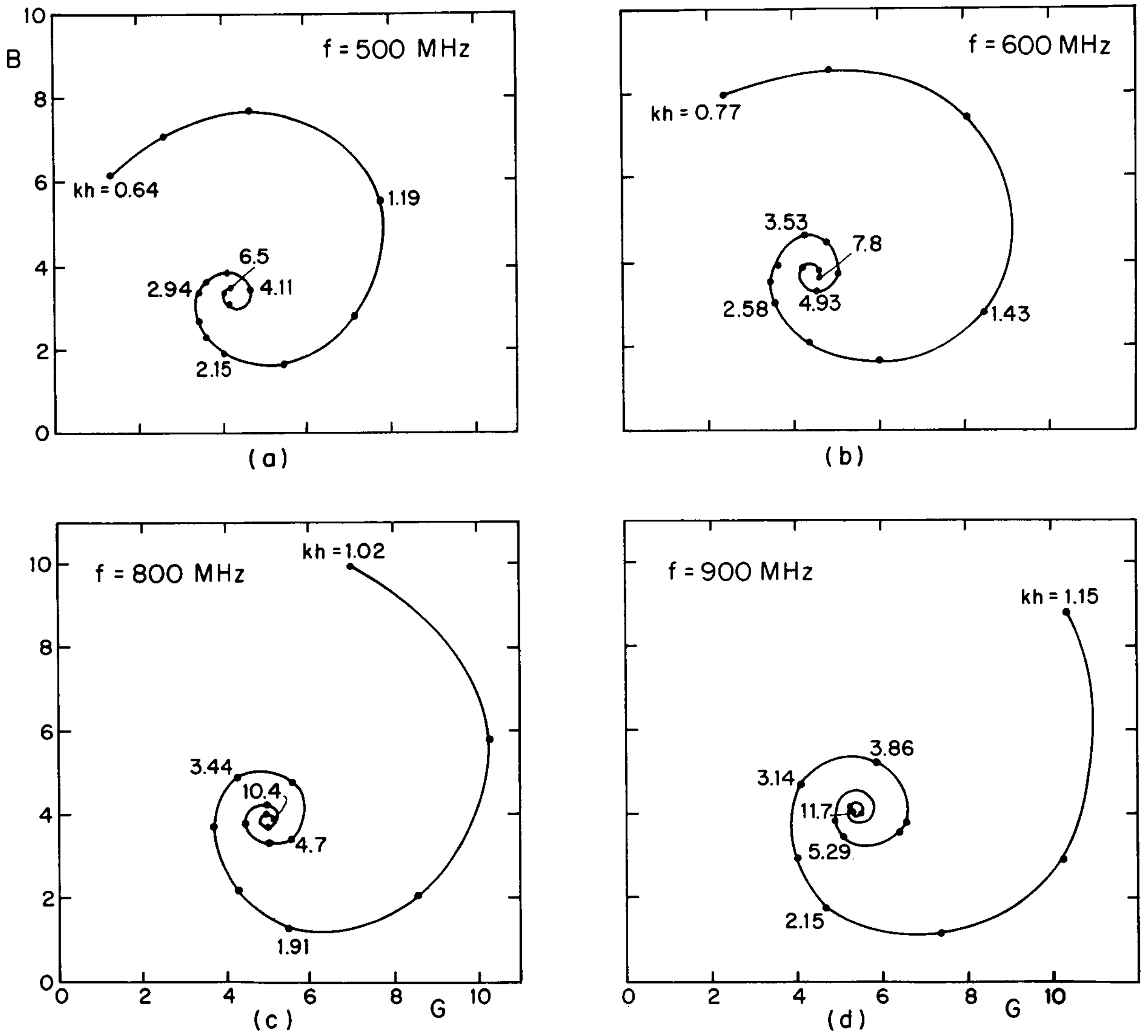


FIG. 3 MEASURED INPUT ADMITTANCE z^i = 1400 Ω / METER, a = 0.32 cm

admittances are apparent admittances measured from a coaxial line with outer and inner radii equal to 0.64 and 0.32 cm respectively. The coaxial line was terminated in the resistive antenna at the ground plane. Thus, the measured data for the monopole should be compared with twice the theoretical value for a dipole antenna.

3. Discussions

Several interesting observations can be made from Figs. 1, 2, and 3. As the length of the antenna increases, the input admittance traces on the admittance plane a clockwise spiral that converges to the point representing the input admittance of the infinitely long antenna. This spiral is similar to that obtained experimentally by Mack [5] for brass dipoles which is shown in Fig. 1 for comparison. It can be seen that the rate of convergence of the admittance spiral for the resistive antenna is much faster than that for a brass antenna. This is quite understandable. The spiral moves upward on the admittance plane as z^i is increased. The result is that when Φ_i is of the order of 1, the spiral stays almost entirely on the upper admittance plane so that the susceptance is always capacitive.

In Fig. 4 the measured input admittances are compared with the theoretical values. The conductance is seen to agree fairly well with the theory while the theoretical susceptance is generally less than the measured result. The same discrepancy in theoretical and experimental susceptances was also observed in Mack's experiment [5] where z^i was very small. For comparison, the difference between the theoretical susceptance and the experimental value is listed in Table 3 for three

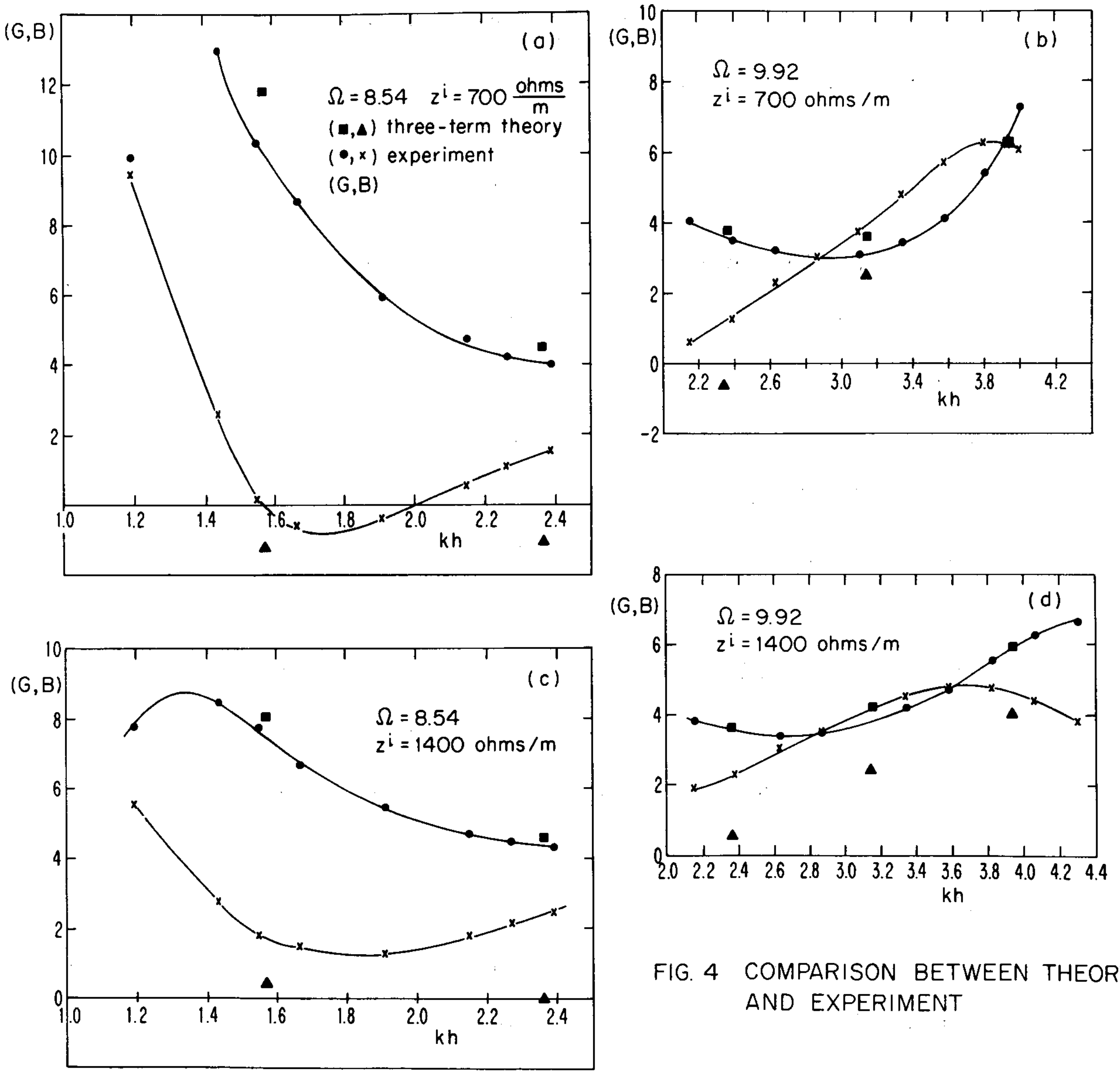


FIG. 4 COMPARISON BETWEEN THEORY AND EXPERIMENT

Table 1 Measured Input Admittances

z^i = 700 ohms/meter

f	$2\lambda z^i/\zeta_o$	ka	kh	G	B
MHz				millimho	millimho
450	2.48	0.0299	2.149	4.028	0.597
500	2.23	0.0332	0.639	0.886	6.441
			0.869	2.589	9.014
			1.084	6.947	10.266
			1.194	9.979	9.512
			1.508	10.898	1.036
			1.744	7.076	-0.411
			1.953	5.075	-0.038
			2.288	3.889	0.924
			2.388	3.478	1.278
			2.613	3.212	2.036
			2.937	3.081	2.956
			3.252	3.398	3.817
			3.471	3.734	4.305
			3.786	4.646	4.832
			4.121	5.925	4.116
			4.555	5.984	2.537
			4.890	5.154	1.854
			5.529	4.183	2.321
			6.084	4.034	3.053
			6.613	4.483	3.449
			7.168	5.215	3.202
			8.037	4.758	2.378
			8.676	4.434	2.661
550	2.03	0.0366	2.626	3.222	2.303
600	1.86	0.0399	0.767	1.715	8.542
			1.043	6.301	11.793
			1.301	13.289	6.640

Table 1 (Cont'd)

			1.433	12.948	2.600
			1.810	6.498	-0.334
			2.092	4.584	0.462
			2.344	3.609	1.397
			2.733	3.165	2.736
			2.865	3.051	3.086
			3.135	3.164	4.021
			3.525	3.747	5.197
			3.902	5.456	5.541
			4.166	6.735	4.777
			4.543	6.876	2.980
			4.945	5.259	2.380
			5.466	4.264	2.667
			5.868	3.994	3.131
			6.635	4.678	4.005
			7.301	5.571	3.542
			7.936	5.159	2.752
			8.602	4.517	2.967
			9.645	4.810	3.683
			10.411	5.251	3.362
650	1.71	0.0432	1.552	10.386	0.189
			3.104	3.121	3.743
700	1.59	0.0465	0.894	3.312	10.662
			1.217	12.826	10.255
			1.517	11.170	0.572
			1.671	8.652	-0.595
			2.111	4.576	0.696
			2.441	3.580	1.755
			2.734	3.234	2.878
			3.189	3.346	4.338
			3.343	3.444	4.821
			3.658	4.428	5.946

Table 1 (Cont'd)

			4.112	6.895	5.749
			4.552	7.157	3.216
			4.860	5.945	2.161
			5.300	4.776	2.459
			5.769	4.244	3.480
			6.377	4.453	4.332
			6.847	5.273	4.420
			7.741	5.753	2.946
			8.518	4.709	2.912
			9.258	4.662	3.695
			10.035	5.302	3.872
			11.252	5.114	3.054
			12.146	4.703	3.492
750	1.49	0.0499	3.581	4.120	5.711
800	1.39	0.0532	1.022	6.644	12.603
			1.391	14.897	3.388
			1.734	7.752	-0.721
			1.910	5.947	-0.377
			2.413	3.733	1.773
			2.790	3.336	2.930
			3.125	3.365	4.177
			3.644	4.513	5.974
			3.820	5.399	6.256
			4.180	7.946	5.517
			4.700	7.150	2.206
			5.202	5.353	2.315
			5.554	4.487	2.741
			6.057	4.362	3.892
			6.593	5.123	4.892
			7.288	6.535	4.021
			7.825	5.963	2.669
			8.847	4.728	3.205
			9.735	5.297	4.041

Table 1 (Cont'd)

			10.581	5.869	3.302
			11.469	5.091	2.960
			12.860	5.394	3.744
			13.882	5.533	3.181
850	1.31	0.0565	4.059	7.260	6.078
900	1.24	0.0598	1.150	11.961	12.561
			1.565	11.441	-0.763
			1.951	5.833	-0.160
			2.149	4.782	0.544
			2.714	3.464	2.915
			3.138	3.495	4.221
			3.515	4.320	5.692
			4.100	7.744	6.165
			4.703	7.782	2.029
			5.287	5.066	2.297
			5.853	4.685	3.613
			6.249	4.593	4.325
			6.814	5.886	5.164
			7.417	7.316	3.888
			8.200	5.456	2.714
			8.803	4.820	3.208
			9.953	5.717	4.032
			10.952	5.794	2.947
			11.903	5.071	3.311
			12.903	5.419	3.892
			14.467	5.415	3.152
			15.617	5.403	3.661
950	1.29	0.0632	2.268	4.226	1.114
1000	1.11	0.0665	2.388	4.025	1.565

Table 2 Measured Input Admittances

$z^i = 1400$ ohms/meter

f	$2\lambda z^i/\zeta_o$	ka	kh	G	B
MHz				millimho	millimho
450	4.95	0.0299	2.149	3.814	1.932
500	4.46	0.0332	0.639	1.343	6.193
			0.775	2.623	7.055
			0.953	4.705	7.700
			1.194	7.748	5.522
			1.414	7.131	2.814
			1.744	5.448	1.630
			2.147	4.008	1.900
			2.388	3.576	2.289
			2.519	3.421	2.671
			2.937	3.445	3.378
			3.157	3.572	3.624
			3.560	4.108	3.822
			4.110	4.706	3.411
			5.084	4.187	3.073
			5.859	4.047	3.340
			6.498	4.204	3.437
550	4.06	0.0366	2.626	3.427	3.032
600	3.72	0.0399	0.767	2.400	7.963
			0.930	4.861	8.554
			1.144	8.092	7.396
			1.433	8.469	2.780
			1.696	5.986	1.612
			2.092	4.363	2.076
			2.576	3.561	3.022
			2.865	3.487	3.530
			3.022	3.659	3.903
			3.525	4.273	4.616
			3.789	4.771	4.471
			4.273	5.036	3.716
			4.932	4.563	3.307
			6.101	4.260	3.888
			7.031	4.621	3.804
			7.797	4.626	3.632
650	3.42	0.0432	1.552	7.718	1.785

Table 2 (Cont'd)

700	3.18	0.0465	0.894	4.263	9.405
			1.085	7.885	8.252
			1.334	9.551	4.459
			1.671	6.691	1.437
			1.979	4.853	1.706
			2.441	3.755	2.879
			3.005	3.608	4.083
			3.343	4.178	4.535
			3.526	4.507	4.866
			4.112	5.637	4.409
			4.420	5.466	3.752
			4.985	4.734	3.418
			5.754	4.443	3.872
			7.118	5.049	3.980
			8.203	4.755	3.780
			9.097	4.678	3.950
750	2.98	0.0499	3.581	4.678	4.823
800	2.78	0.0532	1.022	7.017	9.933
			1.240	10.250	5.802
			1.525	8.543	2.081
			1.910	5.452	1.282
			2.262	4.287	2.191
			2.790	3.697	3.716
			3.435	4.304	4.913
			3.820	5.554	4.771
			4.030	5.793	4.562
			4.700	5.546	3.385
			5.052	5.051	3.312
			5.697	4.479	3.802
			6.576	4.991	4.230
			8.135	5.037	3.726
			9.375	4.998	4.041
			10.397	5.227	3.901
850	2.62	0.0565	4.059	6.237	4.395
900	2.48	0.0598	1.150	10.374	8.779
			1.395	10.636	2.895
			1.715	7.336	1.207

Table 2 (Cont'd)

			2.149	4.673	1.782
			2.545	4.062	2.940
			3.138	4.107	4.691
			3.864	5.851	5.262
			4.298	6.623	3.796
			4.533	6.394	3.588
			5.287	5.103	3.471
			5.683	4.903	3.857
			6.409	5.056	4.434
			7.398	5.777	3.958
			9.151	5.259	4.137
			10.546	5.518	4.025
			11.696	5.297	4.041
950	2.58	0.0632	2.268	4.487	2.131
1000	2.22	0.0665	2.388	4.326	2.458

Table 3 Theoretical and Experimental Susceptances

Ω	h/λ	$2\lambda z^i/\zeta_o$	$B_{exp.}$	B_{theory}	ΔB(millimho)
8.54	0.25	0	-7.5	-8.9	1.4
		1.69	0.0	-1.2	1.2
		3.38	1.7	0.4	1.3
9.92	0.50	0	2.8	2.0	0.8
		1.69	3.9	2.5	1.4
		3.38	4.1	2.4	1.7

different Φ_i's . It is seen that the difference is roughly a constant for constant a/λ when the input susceptance is near a minimum (that is, h/λ near 0.25). Minimum conductances do not occur at the same ratio of h/λ (Fig. 1) for different z^i hence a meaningful comparison is difficult although the sign of the difference is always found to be positive, that is, the theoretical susceptance is always smaller than the experimental value. In the measurement of the current distribution on a brass antenna [6], it was observed that the imaginary part of the current increases very rapidly near the driving point. The smoothly varying trigonometric functions that represent the current in the three-term theory just fail to follow the rapid change near the driving point. This observation explains the discrepancy found in the susceptances just mentioned.

The theoretical input admittance of the infinitely long antenna is shown in Fig. 5, together with the experimental result. The theoretical curve is obtained according to the formulas given in [3]. The experimental values are obtained by extrapolating the admittance spirals shown in Figs. 1, 2, and 3. The agreement between theory and experiment is seen to be good. The theoretical admittance is obtained through a different approach from that used for the three-term theory, and the discrepancy in the theoretical and the experimental susceptance no longer appears.

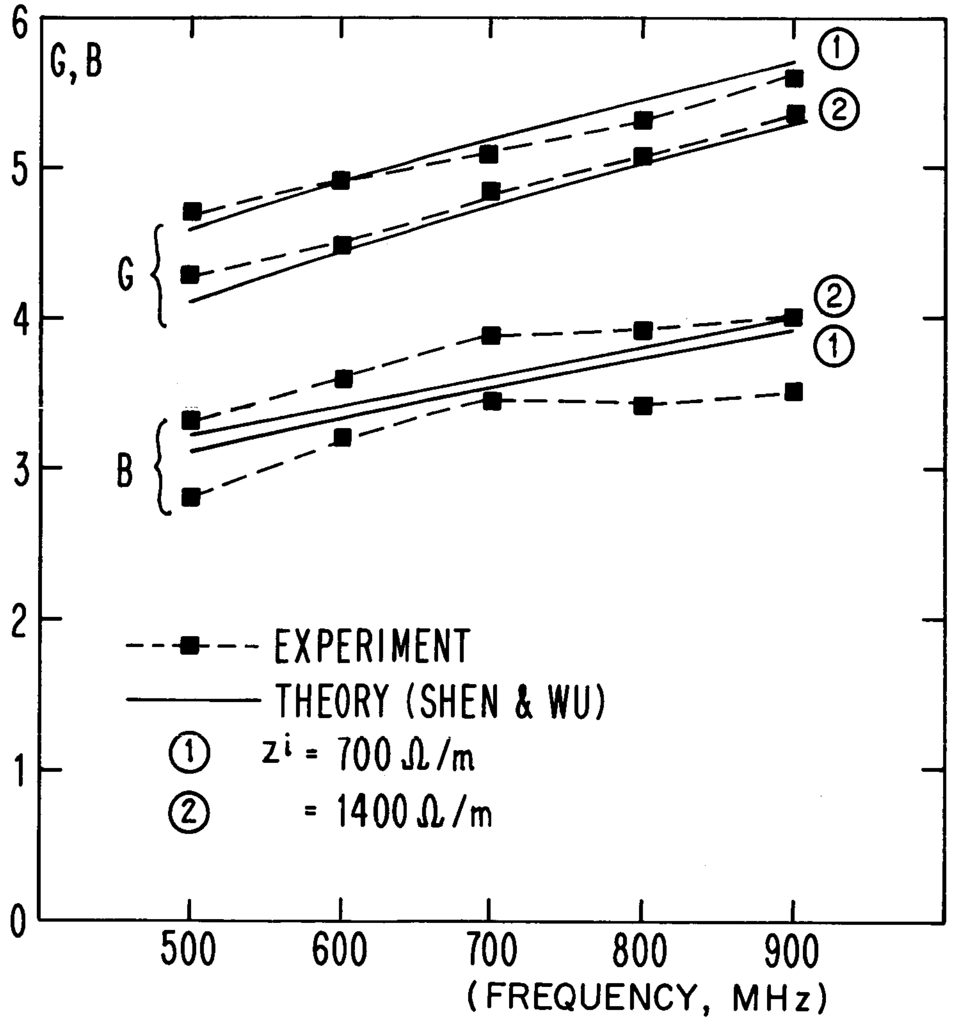


FIG. 5 INPUT ADMITTANCES OF INFINITELY LONG ANTENNAS

4. Conclusions

An experimental study of the resistive antenna has been completed. It provides a critical check on the three-term theory for moderate values of the impedance parameter Φ_i . It also confirms the theoretical work of Shen and Wu on infinitely long resistive antennas.

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13. ABSTRACT

Input admittances of dipole antennas with moderately high internal impedance were measured in the UHF range with the antenna lengths varying from 1/10- wavelength to such a value that the antenna behaved as if infinitely long. The measured results are compared with the three-term theory of King and Wu [1] and with the theoretical values obtained by Shen and Wu [3] for an infinitely long antenna.

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