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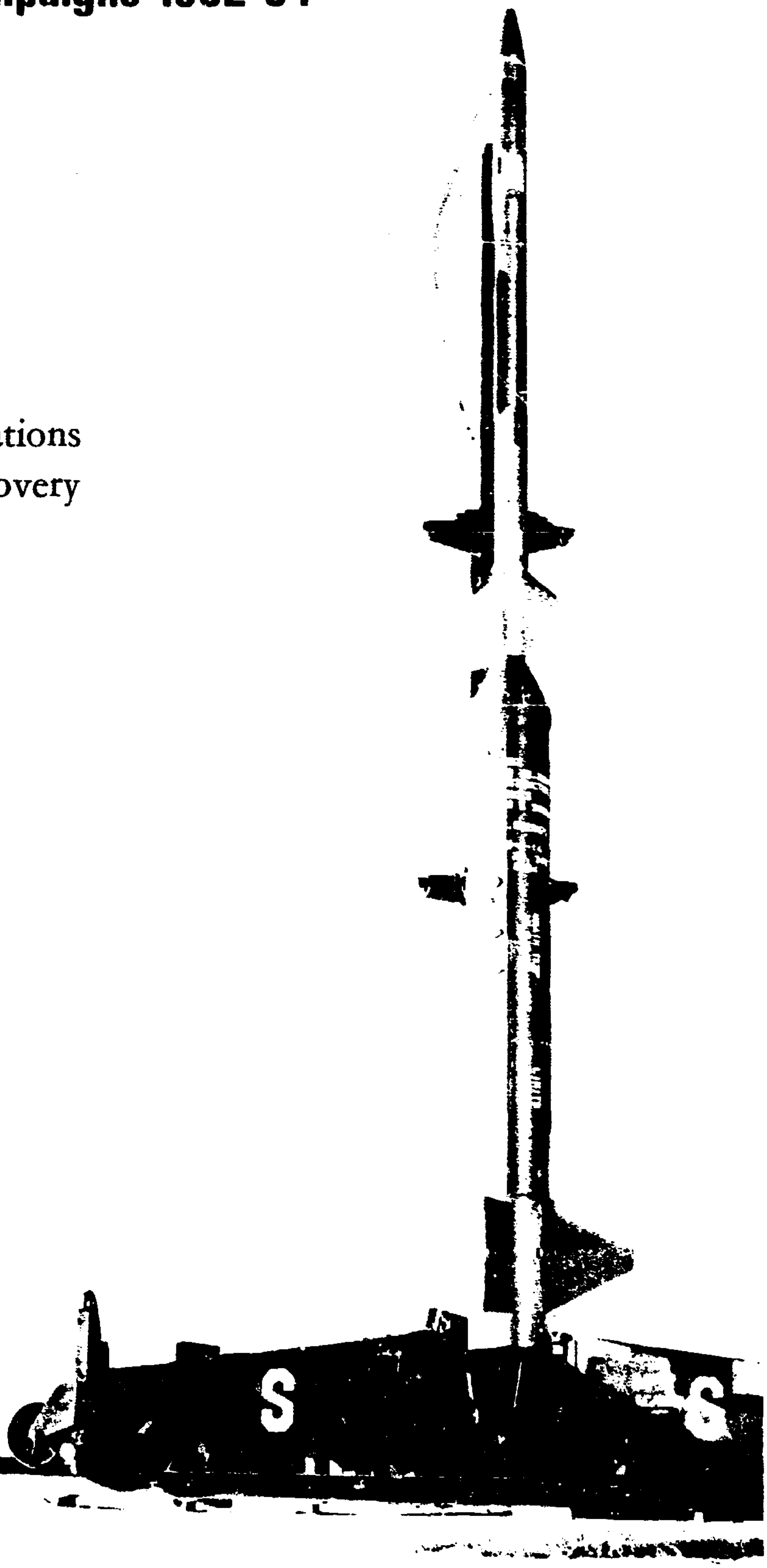

No 67-K2

Sound ranging computations for sounding rocket recovery

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SOUND RANGING COMPUTATIONS FOR SOUNDING ROCKET RECOVERY

Johan Martin-Löf

ABSTRACT

An acoustic method to locate an object that enters into the earth's atmosphere at supersonic speed is presented. The direction to the object is computed from shock wave arrival times observed with the aid of infra-sonic microphones. A numerical method for the computations is described. This method gives a least squares estimate of the direction of the normal to an idealized plane shock wave. It has been successfully used in a computer program to reduce data collected during the sounding rocket experiments in northern Sweden in 1964.

FOREWORD

The Kronogård reports

During the summer of 1962, 1963 and 1964 a series of sounding rocket experiments were performed at Kronogård in northern Sweden under a cooperative agreement between the US National Aeronautics and Space Administration (NASA) and the Swedish Space Research Committee. The main experimenter on the Swedish side was the Institute of Meteorology, University of Stockholm and on the US side groups from USAF Cambridge Research Laboratories (AFCRL) and NASA Goddard Space Flight Center.

The Swedish Space Research Committee set up a technical group to take care of the technical and operational parts of the experiments. While the scientific results from the experiments have been and will be published by the experimenters, this group is preparing a special series of reports covering its activities during the campaigns.

The group is since the 1st of July 1965 a division of TUAB, Teleutredningar AB under the name of Space Technology Group.

CONTENTS

1. INTRODUCTION

In many sounding rocket experiments it is necessary to recover the instrumentation after the flight for examination. Usually the payload is separated from the rocket and is slowed down by a parachute which gives a moderate descent rate. The impact can be located by means of radar or, if radar is not available, by some active homing system in the payload. In case of separation or power failure in the rocket, these systems might give no data, however, and a back-up system is desirable.

Sound ranging is very well suited for this purpose. During reentry a sounding rocket creates one or more shock waves which propagate through the atmosphere. They can easily be detected by means of low-frequency microphone systems located in surveyed positions on the ground. From the observed arrival times, it is possible to estimate the direction to the origin of the shock waves from each microphone system and thus by extrapolation to get a fix on the impact point. The minimum number of microphones in each system is three, but it is preferable to have more in order to get redundancy and an estimate of errors in the result.

This report deals with the numerical problem of computing the direction of incidence of a plane sound wave from observed arrival times to a microphone system. It will be shown that it is a linear least squares problem with a quadratic constraint. The problem is often ill-conditioned especially for low elevation angles of the wave normal. Additional difficulties arise because the problem degenerates when the microphones lie in a common plane. A method will be presented which gives a unique solution in all practical cases, also where previous methods (1, 2) have failed. In the presentation will be used a geometric interpretation to visualize the numerical problem.

2. THE MATHEMATICAL PROBLEM

The aim is to obtain an estimate of the direction of the normal and the time of arrival of a plane sound wave from observations of the arrival to a system of microphones.

The speed of sound is assumed constant in the lowest part of atmosphere over the microphones. Wind influence on the speed of sound is neglected. These idealizations are tolerable as will be discussed below in paragraph 3.

The number of microphones is m. Their positions are given relative to an arbitrary cartesian coordinate system by the column vectors:

$$\underline{r}_i = \begin{bmatrix} r_{i1} \\ r_{i2} \\ r_{i3} \end{bmatrix} \qquad i = 1, \ldots m$$

In order to obtain a solution it is necessary that $m \geq 3$.

$\underline{n}$ is the unit wave normal pointing in the direction from which the wave is coming.

c is the speed of sound.

$t_1, \ldots t_m$ are the observed arrival times at the microphones.

In practice it is possible to determine the microphone position coordinates with an accuracy that is one order of magnitude better than the accuracy in the determination of the arrival times. This is due to the fact that the sound recordings are made with the aid of low-frequency microphones which are disturbed by the general noise in the atmosphere. We will thus here neglect the errors in the microphone positions $\underline{r}_i$.

The arrival time at the origin of the coordinate system can be calculated from each observation by

$$t_i' = t_i + (\underline{r}_i^T \underline{n})/c \qquad i = 1, \ldots m$$

The observations are given equal weight and the average of these quantities, t_o, is taken as estimate of the time of arrival at the origin.

Introducing the average values

$$\bar{t} = \frac{1}{m} \sum_{i=1}^{m} t_i \qquad \bar{\underline{r}} = \frac{1}{m} \sum_{i=1}^{m} \underline{r}_i$$

we get

$$t_o = \frac{1}{m} \sum_{i=1}^{m} t_i' = \bar{t} + (\bar{\underline{r}}^T \underline{n})/c$$

We choose our estimate of $\underline{n}$ so that the mean square deviation from t_o is minimized.

Thus we seek the minimum of the error function

$$F(\underline{n}) = \sum_{i=1}^{m} e_i^2 = \sum_{i=1}^{m} (t_i' - t_o)^2 =$$

$$= \sum_{i=1}^{m} \left\{(t_1 - \bar{t}) + (\underline{r}_i^T - \bar{\underline{r}}^T)\underline{n}/c\right\}^2$$

with the subsidiary condition $|\underline{n}| = 1$
introduce new coordinates

$$u_i = t_i - \bar{t}$$

$$i = 1, \ldots m$$

$$\underline{s}_i = (\underline{r}_i - \bar{\underline{r}})/c.$$

and the matrix

$$S = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ \cdot & & \\ \cdot & & \\ s_{m1} & s_{m2} & s_{m3} \end{bmatrix}$$

Our problem is equivalent to obtaining the least squares solution of the system of linear equations

$$S\underline{n} = - \underline{u}$$

with the quadratic constraint

$$|\underline{n}|^2 = \underline{n}^T \underline{n} = 1$$

We have two cases depending on the rank of the matrix S.
If the vectors

$$\underline{s}_1 \dots \underline{s}_m$$

span the whole 3-dimensional space the rank is 3, otherwise it is 2 and we have a degenerate problem.
As the coordinates $\underline{s}_i$ have the property

$$\sum_{i=1}^{m} \underline{s}_i = 0$$

the vectors span the whole space only if $m \geq 4$ and if the microphones are not located in a common plane. For numerical reasons it is also necessary that the microphones are not even very close to a common plane.

2.1. Non-degenerate case

We can write the error function

$$F(\underline{n}) = |\underline{u} + S\underline{n}|^2 = (\underline{u} + S\underline{n})^T (\underline{u} + S\underline{n})$$

The symmetric matrix $S^T S$ is positive definite and can thus be inverted. The function F can then be written:

$$F(\underline{n}) = (\underline{n} + (S^T S)^{-1} S^T \underline{u})^T S^T S(\underline{n} + (S^T S)^{-1} S^T \underline{u}) +$$

$$+ \underline{u}^T (I - S(S^T S)^{-1} S^T)\underline{u}$$

The second term is constant.
As $S^T S$ is positive definite the set of surfaces

$$F(\underline{n}) = \text{const.}$$

is a set of ellipsoids with their common center in

$$\underline{C} = -(S^T S)^{-1} S^T \underline{u}$$

Our problem is equivalent to finding the tangent point between the unit sphere

$$\underline{n}^T \underline{n} = 1$$

and the smallest ellipsoid in the set that touches this sphere. In this point the normal to the ellipsoid is also normal to the sphere. The normal to an ellipsoid is parallel to the gradient

$$\text{grad } F = 2S^T(\underline{u} + S\underline{n})$$

Thus tangent point is the solution of the non-linear equation

$$\underline{n} = \pm \frac{\text{grad } F}{|\text{grad } F|} = \pm \frac{S^T\underline{u} + S^T S\underline{n}}{|S^T\underline{u} + S^T S\underline{n}|}$$

The plus sign is used if the center $\underline{C}$ of the ellipsoids is outside the unit sphere and the minus sign if it is inside. In the special case that it is exactly on the surface of the sphere we have the immediate solution

$$\underline{n} = \underline{C}$$

which is the well known solution to the problem without the constraint.

Generally it is necessary to make an iterative solution to the problem and we will here first discuss the previous methods that have been used and why they often fail in recovery applications.

Our microphone system is for practical reasons placed on the ground so that the vertical separation between microphones is much smaller than the horisontal. This leads to poor accuracy in elevation and consequently the error ellipsoids are long and cigarshaped with their long axis roughly along the vertical. For high elevations the equi-error contours on the unit sphere are essentially circular with the minimum point in the middle. For lower elevations the equi-error contours get more and more oval and the minimum is located in a "valley" that is very sharp in the azimuth direction and shallow in the elevation direction.

In previous reports (1, 2) dealing with this problem the following iterative method has been used to find the minimum. Starting from an approximate point on the unit sphere the minimum point in the tangent plane is located. This latter point is no longer on the unit sphere, but normalizing its position vector gives a new and better approximation of the minimum on the sphere. The method has been successfully used by the author of this report in connection with the rocket grenade experiment, where the elevation is around 80°.

When trying to use this method for recovery computations, however, it was frequently found that no convergence could be obtained due to the fact that the minimum is in a narrow valley.

Therefore a quite different method has been devised. Let us study the locus of all points on the error ellipsoids from which the normal passes through the origin. Introducing a parameter k, the points on this locus are the solutions to the equation

$$\text{grad } F(\underline{n}) + k\underline{n} = 0$$

or

$$(S^T\underline{u} + S^TS\underline{n}) + k\underline{n} = 0$$

or

$$\underline{n}(k) = -(S^TS + kI)^{-1}S^T\underline{u}$$

The function $\underline{n}(k)$ is a continuous function of k and traces all points on the locus as k passes from $+\infty$ to $-\infty$. Let us study its general shape (see fig. 1). For k = 0 we have

$\underline{n}(0) = \underline{C}$

That is the center of the ellipsoids. As k increases towards $+\infty$ the trajectory approaches the origin. As k decreases towards $-\lambda_3$ where λ_3 is the smallest of the (positive) eigenvalues of S^TS the trajectory goes towards infinity along the direction of the corresponding eigenvector. As k passes $-\lambda_3$ the trajectory comes back from infinity along the opposite direction. As k further passes $-\lambda_2$ and $-\lambda_1$ where λ_2 and λ_1 are the next largest and the largest eigenvalue of S^TS, the trajectory shows a similar behaviour with respect to the corresponding eigenvectors. Finally, as k approaches $-\infty$ the trajectory approaches the origin again.

This trajectory can have 2, 4 or 6 intersections with the unit sphere and thus there exists a corresponding number of points from which a unit normal can be drawn from an ellipsoid to the origin. In half of these points the error function F is smaller, in the others it is greater than in a neighbourhood on the unit sphere.

The k-values corresponding to these points are the solutions of the equation

$$|\underline{n}(k)|^2 = 1$$

which is of order 6.

To simplify the equation let us rotate the coordinate system with an orthogonal matrix Q that diagonalizes S^TS.

$$Q^TS^TSQ = D \qquad\qquad Q^T = Q^{-1}$$

The columns of Q are the eigenvectors of S^TS. The diagonal elements of D are the corresponding eigenvalues λ_1, λ_2 and λ_3. Suppose they are ordered: $\lambda_1 > \lambda_2 > \lambda_3$. Multiplying the expressions for $\underline{n}(k)$ from the left with Q^T we get:

$$Q^T\underline{n}(k) = -Q^T(S^TS + kI)^{-1}QQ^TS^T\underline{u} =$$

SENSITIVITY ANALYSIS

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ABSTRACT

The problem of calculation of the sensitivity coefficients can be resolved efficiently if an analytical expression for the network function is available. The flowgraph technique provides a partial answer to this problem, since a symbolic representation of the network function can be obtained directly from the flowgraph. A brief discussion of flowgraph techniques is presented; the discussion is based upon the dichotomy of the equivalent network. Sensitivity formulas are also developed from the basic topology equation of the flowgraph.

A computer program (NASAP) has been written which utilizes an efficient algorithm to solve the topology equation, H = 0. The network is encoded using a code for each element which completely characterizes the network. Dependent and independent sources can be handled and the method is being extended to non-linear networks. Examples of sensitivity calculations are presented for both passive and active networks. The equivalent circuit, network coding, and computer printout are included for each example.

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INTRODUCTION

Tagging procedures and dichotomous techniques can be used to obtain the sensitivity function symbolically.[1,2] There are three distinct ways to obtain the sensitivity function. These are (a) perturbational, (b) matrix inversion, and (c) symbolic.

The symbolic method provides insight into the factors contributing to the sensitivity function and also gives an exact solution. An illustrative example of this is the investigation of how sensitive the overall gain is to changes in h_{fe}, in the transistor. This is useful information since the designer constructs a circuit to operate for off-the-shelf transistors.

DICHOTOMOUS TECHNIQUES

Dichotomous techniques imply a graph theoretical transformation from the equivalent circuit to topologically represent the corresponding equations of the associated linear system. The method is based upon the seperation of network elements into controlled current and voltage sources, this is the origin of the term network dichotomy. This procedure is equivalent to choosing an appropriate tree for the network, which in turn defines uniquely a set of independent variables. There exists a unique solution since all the voltage generators are included in the tree (branches) and all the current generators are placed in the cotree (links).

The equivalent circuit must specify a necessary and sufficient statement of the problem. This requirement is met by coding the equivalent circuit. The coded equivalent circuit contains the necessary information for obtaining a fundamental circuit and cut-set matrix. The dependence of the sources is also clearly established in the coding scheme for problem statement. Once the equivalent circuit has been coded the associated flowgraph can be constructed, Figure 1.

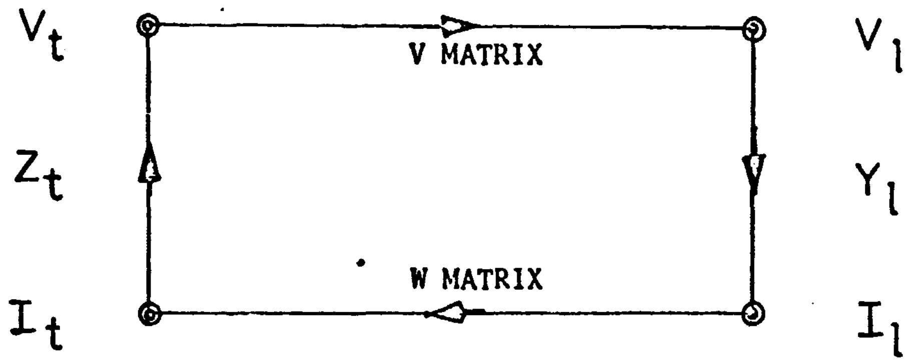

Figure 1 Scheme for constructing flowgraph

Figure 1 is termed an open flowgraph since it contains strictly dependent or strictly independent nodes. To facilitate a computer-based evaluation the flowgraph is closed by placing a "dummy" transmittance between two nodes. This transmittance has a value associated with it which gives the functional relationship between the nodes it connects. Using this artifice one can evaluate the topology equation (H) for the desired "dummy" transmittance.

The topology equation is

$H = S(N)(-1)^N L(N)$[4], where L(N) denotes the sum of all Nth order loops. The summation is denoted by S(N) and includes loops of all orders in the flowgraph. For a closed flowgraph H = 0, this constraint results in the determination of the "dummy" transmittance which is the functional relationship desired.

The procedure is implemented on a digital computer by appropriate tagging techniques. The sensitivity coefficients will now be related to the loops.

SENSITIVITY COEFFICIENTS

In the discussion which follows there will be need for the following sensitivity criteria relating two parameters P (system performance criteria) and Q (system parameter).

These are denoted by:

$$G_Q^P = P/Q$$

$$B_Q^P = dP/dQ, \text{ and}$$

$$S_Q^P = d(\ln P)/d(\ln Q).$$

G_Q^P is a "large-signal" sensitivity coefficient. For example, let P be the voltage at the output of an amplifier and Q be the input voltage, to same. The G_Q^P would give a measure of how sensitive the output voltage is to the input voltage - commonly called "gain".

B_Q^P is a "small-signal" sensitivity criterion. For example, let P be the small-signal voltage from the base to the emitter of a transistor and Q be the collector to emitter small-signal voltage. Then B_Q^P would represent the reverse voltage amplification factor h_{re}.

S_Q^P is a "system-variation" sensitivity coefficient. This is the reciprocal of the classical (or Bode) sensitivity. This coefficient relates a relative change in parameter variation to the relative change in system performance. There are also other sensitivity criteria, one which is of interest is called "zero-sensitivity", which is approximately the change in a zero location in the s plane to the fractional change in a parameter.

A derivation of how G_Q^P, B_Q^P and S_Q^P are functional related to the feedback loops follows.

Let H represent the topology equation for a flowgraph closed by the "dummy" transmittance P, Q is assumed to be present in the open flowgraph. Then H = 0, for the closed flowgraph.

Now $H(P)$ consists of two mutually disjoint parts,

$$H(P) = H(\overline{P}) + PH(P'),$$

where $H(\overline{P})$ are those feedback loops devoid of P and $PH(P')$ are those feedback loops which contain P. Similarly,

$$H(Q) = H(\overline{Q}) + QH(Q').$$

Therefore since $H(P) = 0$ and $H(Q) = 0$, then

$$G_Q^P = P/Q = \frac{H(\overline{P})H(Q')}{H(\overline{Q})H(P')}$$

To determine the relationship of feedback loops to the small-signal sensitivity coefficient, dP/dQ needs to be determined. Let us proceed by finding the total differential of $H(P,Q)$. Now,

$dH(P,Q) = [H(P',\bar{Q}) + QH(P',Q')]\ dP + [H(\bar{P},Q') + PH(P',Q')]\ dQ$, but

$dH(P,Q) = 0$ and $H(P') = H(P',\bar{Q}) + QH(P',Q')$, $H(Q') = H(\bar{P},Q') + PH(P',Q')$.

Therefore, $dP/dQ = -H(Q')/H(P') = B_Q^P$

To determine the classical sensitivity, notice that

$S_Q^P = d(\ln P)/d(\ln Q) = B_Q^P/G_Q^P$, therefore,

$$S_Q^P = -H(\bar{Q})/H(\bar{P}).$$

Equations for higher order system sensitivities are formulated in terms of the loops, in much the same manner as the above.[5]

EXAMPLES

To illustrate the manner in which the sensitivity coefficients are obtained consider the loaded twin-T filter of Figure 2a. The object of the analysis is to determine the sensitivity of the network's voltage transfer function (P) to variations in the value of resistor R1 (Q). The first step in the solution involves the preparation of the equivalent circuit, Figure 2b. Since the parameter of interest is the voltage transfer function, a controlled voltage generator, element E1, represented by a double arrow in the assumed direction of current flow is placed across the equivalent circuit's input terminals. This generator is made dependent on the output voltage across element 8, which represents resistor R4. The addition of element 1, relating the input and output voltages, closes the flowgraph and allows the previously developed algorithms for sensitivity to be utilized. In order to determine a unique solution, a tree is selected for the equivalent circuit consisting of E1, D3, E6, and E8, all of which are controlled voltage generators. The remaining elements E2, E4, E5, and E7, form the co-tree and are thus represented as controlled current sources.

The equivalent circuit is now encoded for computer input as indicated in Table 1. The rows of this table represent the elements in the equivalent circuit as enumerated in column E. Columns A and B contain the nodal interconnection data and therefore define the circuit topology with the assumed direction of positive current flow from A to B. Column G designates the circuit dichotomy by encoding the controlled voltage generators (tree branches) as zeros and the controlled current generators (co-tree links) as ones. Columns C and D denote the control, as voltage (C=0) or current (C=1), and the controlling variable, respectively. Thus E1 is encoded as a voltage generator (G=0) controlled by the voltage (C=0) across E8 (D=8). For passive elements, which are represented as tree impedances by current controlled voltage generators or as link admittances by voltage controlled current generators, column C contains the binary complement of G while columns D and E are identical. Column F reveals the frequency dependence of the element transmittance as the function s^F. The single unit entry in column H (E1) tags the "dummy" element utilized to close the flowgraph and thus identifies the system performance criterion P while the unit entry in column K tags the system parameter Q.

From the coded equivalent circuit the computer develops the closed flowgraph of Figure 3. This appears in the output encoded in the form of the cut-set, circuit, and transmittance matrices. Each node of the flowgraph has been labeled with the corresponding equivalent circuit element number. The computer enumerates the feedback loops and develops the voltage transfer function,

$$G(S) = \frac{S^3 + .417E{+}3\ S^2 + .173E{+}6\ S + .722E{+}8}{S^3 + .248E{+}4\ S^2 + .120E{+}7\ S + .141E{+}9}$$

The computer now examines the loops for the tagged elements E1, and E3, computes $H(\overline{P}) = H(\overline{G(S)}) = H(\overline{E1})$ and $H(\overline{Q}) = H(\overline{R1}) = H(\overline{E3})$, and lists the system variation sensitivity as determined by S_Q^P,

$$S_{R1}^G = \frac{.833E{+}3\ S^5 + .135E{+}7\ S^4 + .642E{+}9\ S^3 + .118E{+}12\ S^2 + .120E{+}15\ S}{S^6 + .289E{+}4\ S^5 + .241E{+}7\ S^4 + .114E{+}10\ S^3 + .446E{+}12\ S^2 + .111E{+}15\ S + .100E{+}7}$$

This function reveals that R1 has negligible influence on the transfer function at both very low and very high frequencies. This observation could be expected due to the behavior of capacitors C2 and C3 under these conditions.

Since $H(\overline{P})$ is independent of Q, the transfer function sensitivity may now be determined with respect to any remaining circuit element i by computing the corresponding value of $H(\overline{i})$. The sensitivity matrix can be generated by evaluating $H(\overline{i})$ with respect to each of the circuit components and parameters. This column matrix is defined

$$\left[S_i^P\right] = \left[-H(\overline{i})\right] / H(\overline{P})$$

The sensitivity matrix enables the circuit designer or reliability analyst to predict, for any desired frequency range, those components or parameters which will have the most significant influence on the circuit performance. This array would then pinpoint the areas where additional component accuracy or stability may be warranted.

The sensitivity analysis procedure is readily applicable to active networks containing dependent sources. In the example that follows the voltage gain sensitivity of the RC-coupled common-source amplifier stage of Figure 4a is to be determined with respect to variations in the transconductance g_{fs} of the field effect transistor. The AC equivalent circuit of the amplifier stage is illustrated in Figure 4b, in which the transistor has been characterized by a small signal linear model. The input and output impedances of the device are assumed large in comparison to resistors R_G and R_D, respectively, and therefore these parameters are not included in the model. Except for the dependent elements, the equivalent symbolic circuit of Figure 4c is constructed following the methods of the previous example. Since the system performance criterion is voltage gain, the flowgraph is again closed by means of a "dummy" voltage generator (E1) at the circuit input and dependent upon the circuit output voltage across element E8. The dependent current source of the field effect transistor model is controlled by the gate to source voltage. This current generator is simulated by two voltage controlled current generators (E4 and E5) with transconductance g_{fs}. In Figure 4c the numbers placed within the arrows of these links denote the controlling branch voltages. Hence, E4 is controlled by the gate to ground voltage appearing across element E3, while E5 is controlled by the source to ground voltage appearing across E6. Since these current generators are connected in parallel but of opposite polarity, the net drain current, I_D, from node 5 to node 4 is

$$I_D = I_{E4} - I_{E5} = (V_G - V_{gnd}) \quad - (V_S - V_{gnd}) \quad = V_{GS}$$

The equivalent circuit is encoded as indicated in Table II. From this the program develops the flowgraph of Figure 5 and performs the sensitivity analysis as indicated in the first example. Literal expressions may be determined for both the voltage gain and sensitivity function by correlating the loops designated in the solution listing with the elements of the flowgraph. In this manner the voltage gain is found to be

$$A_V(S) = \frac{(R_G R_L R_s C_i C_s g_{fs})\ S^2 + R_G R_L C_i g_{fs}(1-2R_s g_{fs})\ S}{(R_G R_s C_i C_s)\ S^2 + (R_L C_i + R_s C_s + R_L R_s C_i g_{fs})\ S^1 + (1+R_s g_{fs})}$$

and the sensitivity of A_V with respect to g_{fs} is

$$S^{A_V}_{g_{fs}} = \frac{(R_G R_s C_i C_s)\ S^2 + (R_G C_i + R_s C_s)\ S^1 + 1}{(R_G R_s C_i C_s)\ S^2 + (R_L C_i + R_s C_s + R_L R_s C_i g_{fs})\ S^1 + (1+R_s g_{fs})}$$

or numerically

$$S^{A_V}_{g_{fs}} = \frac{S^2 + .934E+2\ S + .834E+3}{S^2 + .109E+4\ S + .108E+5}$$

It may be observed that the sensitivity is considerably less at lower frequencies. This is due primarily to the fact that for these frequencies the source resistor R_S is not completely by-passed and the resulting negative feedback diminishes the influence of the transconductance on the voltage gain.

The sensitivity matrix was evaluated for this example and appears in Table III.

CONCLUSIONS

Dichotomous techniques provide an effective means of obtaining both numeric and symbolic expression for system performance. The implementation of these concepts, as in the NASAP circuit analysis program, provides the designer with a convenient means of appraising circuit component accuracy and stability requirements. This program is available from NASA/COSMIC and also from cooperating universities participating in the development of NASAP.

REFERENCES

1. W. W. Happ, "Flowgraph Techniques for Closed Systems", IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-2, No. 3, pp. 252-264, May, 1966.

2. W. W. Happ and D. E. Moody, "Topological Techniques for Sensitivity Analysis", IEEE Transactions on Aerospace and Navigational Electronics, Vol. ANE-11, No. 4, pp. 249-254, December, 1964.

3. W. W. Happ, "Flowgraphs as a Teaching Aid", IEEE Transactions on Education, Vol. E-9, No.2, pp. 69-80, June, 1966.

4. C. S. Lorens, Flowgraphs. New York: McGraw-Hill, 1964.

5. R. M. Carpenter, "Computer-Oriented Sensitivity and Tolerance Techniques", to be published.

A	B	C	D	E	F	G	H	K	NUMERIC
2	1	0	8	1	0	0	1	0	.100E+1
2	3	0	2	2	1	1	0	0	.500E-7
3	1	1	3	3	0	0	0	1	.240E+5
3	5	0	4	4	1	1	0	0	.500E-7
2	4	0	5	5	0	1	0	0	.208E-4
4	1	1	6	6	-1	0	0	0	.100E+8
4	5	0	7	7	0	1	0	0	.208E-4
5	1	1	8	8	0	0	0	0	.100E+6

Table I - Input data for twin-T filter

A	B	C	D	E	F	G	H	K	NUMERIC
1	2	0	8	1	0	0	1	0	.100E+1
2	3	0	2	2	1	1	0	0	.100E-7
3	1	1	3	3	0	0	0	0	.100E+8
5	4	0	3	4	1	1	0	1	.100E-2
4	5	0	6	5	1	1	0	1	.100E-2
4	1	1	6	6	0	0	0	0	.120E+5
4	1	0	7	7	1	1	0	0	.100E-5
5	1	1	8	8	0	0	0	0	.100E+6

Table II - Input data for FET-amplifier

ELEMENT	S^2	S^1	S^0
C_i	0.0	-.100E+1	-.108E+4
R_g	0.0	-.100E+1	-.108E+4
g_{fs}	-1.0	-.934E+2	-.834E+3
R_s	0.0	.100E+5	.100E+6
	0.0	.120E+0	.100E+2
C_s	-.120E+3	-.120E+4	0.0
	0.0	.120E+0	.100E+2
R_L	-1.0	-.109E+4	-.108E+4

$H(\bar{P})$ $\quad S^2 \quad + \quad .109E+4\ S \quad + \quad 1.08E+4$

Table III - Sensitivity matrix for FET amplifier

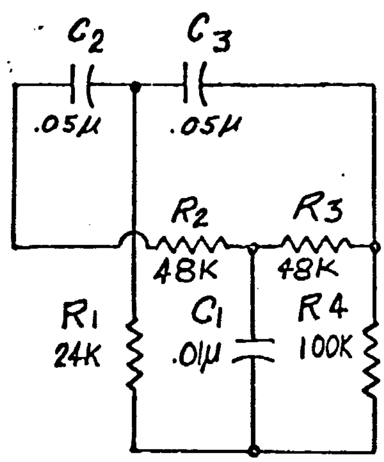

Figure 2(a)

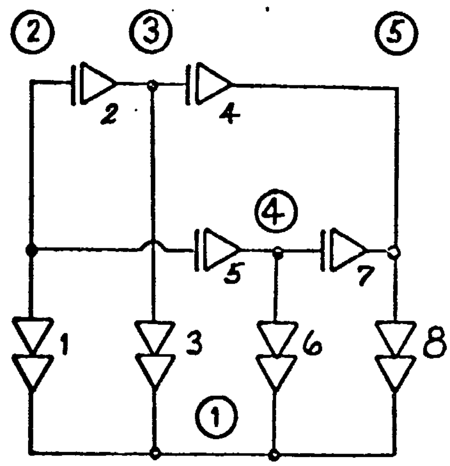

Figure 2(b)

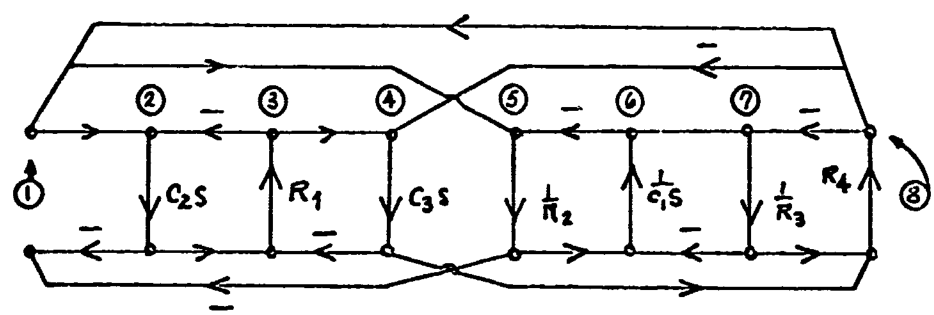

Figure 3

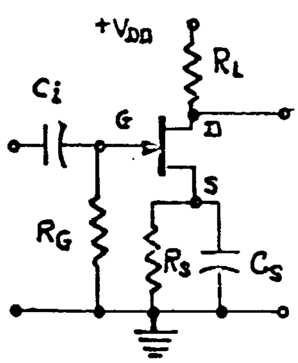

Figure 4(a)

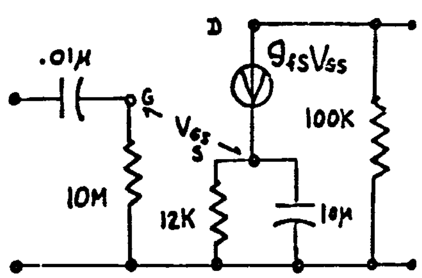

Figure 4(b)

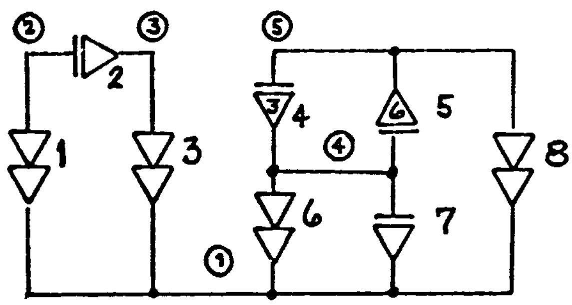

Figure 4(c)

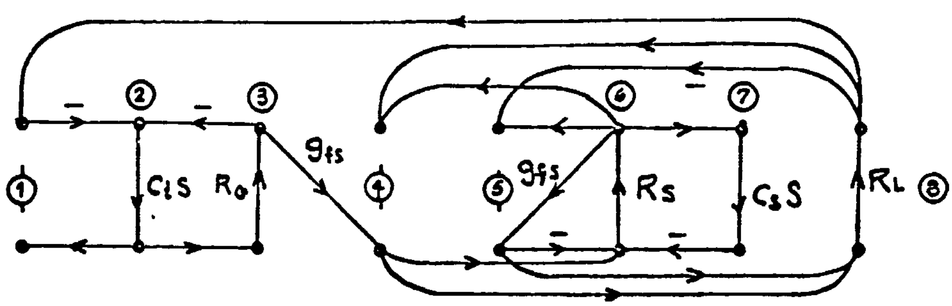

Figure 5