THE PROBABILITY OF MISSION SUCCESS (POMS) PROGRAM

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Introduction

The selection of mission trajectory parameters has usually been made on the basis of the resulting deterministic trajectory. Error analyses have sometimes been carried out subsequently to determine the statistical effects of random disturbances on the trajectory. It is thus possible to estimate the probability that the pre-planned mission will be successful.

Techniques have been developed, and a computer program prepared and tested, for choosing trajectory parameters to maximize the probability of success of a mission directly. The main idea is the recognition that the statistical distribution of perturbed trajectories, as well as the nominal trajectory itself, depends on the nominal trajectory parameters. The mission probability of success is thus completely determined by the input statistics, assumed specified, and the nominal trajectory parameters.

The principal advantage of the technique developed is that it is not necessary to do Monte Carlo simulations to calculate the probability of success. This immediately foretells an order of magnitude reduction in computing time, at least, over procedures requiring large numbers of simulations. In order to optimize the trajectory parameters, furthermore, it is necessary to use a successive improvement scheme, changing the parameters each iteration to produce a modest, if not small, increase in the success probability. An efficient scheme for computing the success probability thus becomes essential.

The gradient program which has been prepared allows optimization of a particular translunar mission, although all the structure and most of the equations are included for any open-loop space mission. The program is also suitable for the addition of orbit determination and feedback control system equations. Although these naturally represent major additions, it appears that existing programs could be added largely intact.

General Problem Formulation

The general problem considered assumes there are a number of mission trajectory parameters, β, which are free to be chosen. These parameters determine the nominal trajectory. Statistics of random disturbances are given so that it is possible to calculate the covariance of trajectory perturbations. By assuming that the perturbations from the nominal flight path are always small enough so that their governing differential equations may be linearized, and assuming that all input disturbances are Gaussian random variables, the probability distribution of the state may be (approximately) calculated. The optimization problem is choosing β to maximize the probability that the terminal state (position and velocity) variables are within certain bounds. The number of bounds may be as large, in principle, as the number of state variables, although the computational strain increases if more than two are imposed.

The state of the system is denoted by x, where

$$x = \begin{bmatrix} R \\ \dot{R} \end{bmatrix} \tag{1}$$

R is the position vector

$(\dot{\ }) = \frac{d}{dt}(\ \ )$

t = time

The nominal value of $x(t)$ is denoted $\bar{x}(t)$. Perturbations from $\bar{x}$ are denoted δx so that

$$x = \bar{x} + \delta x \tag{2}$$

For concreteness, a trajectory will be considered successful if two functions of the terminal state, $\varphi_1(x_f)$ and $\varphi_2(x_f)$ are between a_1, b_1 and a_2, b_2 respectively.* With the small perturbation assumption

$$\begin{aligned}\varphi_i(x_f) &= \varphi_i(\bar{x}_f) + \delta\varphi_i \\ &= \varphi_i(\bar{x}_f) + \frac{\partial\varphi_i}{\partial x}\delta x_f \qquad i = 1, 2\end{aligned} \tag{3}$$

Equation (3) assumes that the terminal time t_f is fixed. The theory for the extension to variable terminal time has been carried out in other analyses and could easily be included.

With linear equations for $\delta\dot{x}$ and Gaussian random inputs, the perturbation variables δx will each be Gaussian random variables. Then $\delta\varphi_i$, being a linear combination of Gaussian random variables, is also a Gaussian random variable. The mean of $\delta\varphi_i$ is zero with the assumption of zero mean disturbances. The variances are given by

$$\sigma_i^2 = \mathcal{E}\left[(\delta\varphi_i)^2\right] = \mathrm{tr}\left[\left(\frac{\partial\varphi_i}{\partial x}\right)^T \frac{\partial\varphi_i}{\partial x} X\right]_f \tag{4}$$

where

$X = \mathcal{E}[\delta x \ \delta x^T]$

$\mathcal{E}(\ ) =$ ensemble average of ( )

$(\ )^T =$ ( ) transposed

$\mathrm{tr}(\ ) =$ trace ( )

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* $(\ )_o =$ ( ) evaluated at the initial point

$(\ )_f =$ ( ) evaluated at the terminal point

The cross correlation is

$$\mathcal{E}\left[\delta\varphi_1 \; \delta\varphi_2\right] = \rho\,\sigma_1\,\sigma_2$$

$$= \mathrm{tr}\left[\left(\frac{\partial\varphi_1}{\partial x}\right)^T \frac{\partial\varphi_2}{\partial x}\, X\right]_f \tag{5}$$

Equation (5) defines ρ, the correlation coefficient.

The joint probability distribution of $\delta\varphi_1$, $\delta\varphi_2$ is the bivariate Gaussian

$$\mathrm{pr}(\delta\varphi_1, \delta\varphi_2) = \frac{1}{2\pi\,\sigma_1\,\sigma_2\sqrt{1-\rho^2}}\exp\left\{-\frac{1}{2}[\delta\varphi_1, \delta\varphi_2]\begin{bmatrix}\sigma_1^2 & \rho\,\sigma_1\,\sigma_2\\ \rho\,\sigma_1\,\sigma_2 & \sigma_2^2\end{bmatrix}^{-1}\begin{bmatrix}\delta\varphi_1\\ \delta\varphi_2\end{bmatrix}\right\} \tag{6}$$

A trajectory is successful if $\overline{\varphi}_1 + \delta\varphi_1$ and $\overline{\varphi}_2 + \delta\varphi_2$ are between a_1, b_1 and a_2, b_2 respectively.* The probability of success is thus

$$J = \int_{a_1-\overline{\varphi}_1}^{b_1-\overline{\varphi}_1} d(\delta\varphi_1) \int_{a_2-\overline{\varphi}_2}^{b_2-\overline{\varphi}_2} d(\delta\varphi_2)\;\mathrm{pr}(\delta\varphi_1, \delta\varphi_2) \tag{7}$$

J is seen to depend only on σ_i, ρ, $\frac{\partial\varphi_i}{\partial x}$ and $\overline{\varphi}_i$. These quantities in turn are all determined from $\overline{x}(t_f)$ and $X(t_f)$. By showing how $\overline{x}$ and X depend on the β parameters, J may be reduced to a function only of β and various specified quantities.

Almost by definition, the nominal trajectory itself depends on the control parameters. The parameters are either initial conditions, jump conditions, or constants that appear in the equations of motion:

$$\dot{\overline{x}} = f(\overline{x}, \beta, t) \tag{8}$$

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* $\overline{\varphi}_i = \varphi_i(\overline{x}_f)$

Integrating (8) from t_o to t_f gives $\overline{x}(t_f)$ and consequently $\varphi_i(\overline{x}_f)$. The integration method is a modified Encke in which the major portion of the solution is obtained analytically, the remaining portion from numerical integration.

The covariance matrix, X, may be closely approximated by using only the analytic functions from the nominal trajectory solution. In this way $X(t)$ may be obtained from a series of algebraic equations between "rectification" times. The rectification times are determined, in the course of obtaining the nominal trajectory, on the basis of keeping the major portion of $\overline{x}(t)$ piecewise analytic. The entire approximation for $X(t)$ is piecewise analytic. Since the analytic functions are taken directly from the nominal trajectory, the dependence of $X(t)$ on β is immediately determined.

The mathematical problem is the choosing of β to maximize J. A global maximum would be desirable, but it appears that any currently feasible technique will only be able to guarantee a relative maximum.

Equations for the Nominal and Covariance Matrix Histories

Inertial Cartesian coordinates are used to describe the position and velocity of the vehicle center of mass. The position is a three-vector R, the velocity $\dot{R}$. The launch site position, R_o, is input, with the earth center $R = 0$. The launch time, t_o, in this case is one of the control parameters, although any parameter could easily be made an input quantity instead. Conversely, most constants in the program could be made control parameters with minor modifications. A simplified boost model is used. The parking orbit altitude, the earth angle traversed during boost, and the duration of the boost are input. At t_1, the time of injection into parking orbit

$$R_1 = r_1 [\hat{R}_o \cos(\beta - \gamma) + \hat{\dot{R}}_o \sin(\beta - \gamma)] \tag{9}$$

$$\dot{R}_1 = v_1 [-\hat{R}_o \sin(\beta - \gamma) + \hat{\dot{R}}_o \cos(\beta - \gamma)] \tag{10}$$

where r is the magnitude of R, v is the magnitude of $\dot{R}$, β is the earth angle traversed during boost, $r_1 - r_o$ is the parking orbit altitude, γ is the flight path angle at t_1, $(\hat{\ })$ is a unit vector in the direction of ( ),

$$\hat{\dot{R}}_o = \hat{N} \cos \psi_o + \hat{E} \sin \psi_o \tag{11}$$

$$\hat{E} = \frac{\hat{k} \times \hat{R}_o}{|\hat{k} \times \hat{R}_o|} \tag{12}$$

$$\hat{N} = \hat{R}_o \times \hat{E} \tag{13}$$

$\hat{N}$, $\hat{E}$ are unit vectors in the north and east directions at launch, $\hat{k}$ is a unit vector in the z inertial direction, ψ_o is the launch azimuth. $t_1 - t_o$, $r_1 - r_o$, v_1, β, γ are each input, ψ_o is one of the control parameters.

The vehicle coasts from t_1 until t_2, the beginning of the second burn. The interval $t_2 - t_o$ is one of the control parameters. Modified Encke method coast equations previously used by Analytical Mechanics Associates, Inc., Ref. 1, are used to obtain R_2, $\dot{R}_2$. The thrust magnitude starting at t_2 is input, the thrust direction is given in terms of pitch and yaw angles p and y:

$$\frac{T}{k} = \hat{\dot{R}} \cos p \cos y + (\dot{R} \times \hat{H}) \sin p \cos y + \hat{H} \sin y \tag{14}$$

T is the thrust, k is the thrust magnitude, H is the angular momentum. The pitch and yaw angles are control parameters.

Instead of integrating $\ddot{R}$ directly, the substitution, from Ref. 2,

$$R = S + P \tag{15}$$

is used. S is a relatively simple analytic function chosen to approximate R, so that P is essentially a small correction term. S and $\dot{S}$ are written as functions of t and t_r, where t_r is the last rectification time. At each rectification time, S is set equal to R and P is set equal to zero: $S(t_r+) = R(t_r)$, $P(t_r+) = 0$. The rectification times are chosen when P exceeds certain limits.

The length of the second burn, $t_3 - t_2$, is input. From t_3 to t_4 the coast equations are again used. This interval is the midcourse region, a coast out to the vicinity of the moon in this program. During this time, it is assumed that a switch from earth reference to moon reference (for calculating S) is made. The total flight time up to retrofire, $t_4 - t_o$, is a control parameter, but the duration of the retrofire burn is an input number. The alignment (inertial) of the vehicle axis at t_4 is just as it was at t_3. The thrust is in the opposite direction. Retrofire burnout time is t_5.

At the same time the nominal trajectory is calculated, the covariance matrix approximation is obtained. In this problem $X(t_1)$ is input. $X(t_2)$ is obtained from

$$X(t_2) = \Phi(t_2, t_1) X(t_1) \Phi^T(t_2, t_1) \tag{16}$$

where

$$\Phi(t, t_1) = \Phi(t, t_r+) \Phi(t_r-, t_1) \tag{17}$$

if $t > t_r$, where t_r is the last rectification time. The equations for $\Phi(t, t_r+)$ for coasting periods are given in Ref. 1.

For the thrusting interval $t_2 \leq t \leq t_3$, $X(t)$ is calculated from

$$X(t) = \Phi(t, t_r+) X(t_r) \Phi^T(t, t_r+) + U(t, t_r+) Y(t_r) U^T(t, t_r+) \tag{18}$$

The equations for $\Phi(t, t_r+)$ and $U(t, t_r+)$ are given in Ref. 2. The Y matrix is the covariance of thrust execution errors. It is derived assuming that the thrust magnitude has a Gaussian random error and that the thrust direction has a Gaussian random error. This leads to

$$Y = \left|\frac{T}{m_r}\right|^2 \left\{\left[\left(\frac{\delta k}{k}\right)^2 - \frac{\tan^2 \alpha}{2}\right]\left[\frac{T}{k}\right]\left[\frac{T}{k}\right]^T + \frac{\tan^2 \alpha}{2} I\right\} \tag{19}$$

m is the total vehicle mass, $\frac{\delta k}{k}$ is the root mean square relative error in thrust magnitude, α is the root mean square error in thrust direction relative to the nominal direction. These errors are input quantities.

The propagation of X during subsequent coasting and thrusting periods is just as given above. X is calculated along with S. Numerical integrations are not required for S and X; they obey difference equations between rectification times.

Gradient Method of Optimization

The control parameters may be arranged as a vector

$$\beta = \begin{bmatrix} t_o \\ \psi_o \\ t_2 - t_o \\ p \\ y \\ t_4 - t_o \end{bmatrix} \tag{20}$$

A gradient procedure starts with an initial β vector and successively makes changes, $d\beta$, so as to increase J according to the linearized approximation

$$dJ = \frac{\partial J}{\partial \beta} d\beta \tag{21}$$

The specific gradient procedure followed here is to select $d\beta$ so as to maximize dJ subject to specified dP, where

$$(dP)^2 = d\beta^T W \, d\beta \tag{22}$$

W is the metric of the β space; it is an input diagonal matrix for the program. The value of dP is determined automatically by the program on each iteration after the first.

If $\frac{\partial J}{\partial \beta}$, the vector of partial derivatives of J with respect to the components of β, is available, the optimal $d\beta$ with constraint (22) is

$$d\beta = \frac{dP}{G} W^{-1} \left(\frac{\partial J}{\partial \beta}\right)^T \tag{23}$$

where

$$G^2 = \frac{\partial J}{\partial \beta} W^{-1} \left(\frac{\partial J}{\partial \beta}\right)^T \tag{24}$$

The heart of this procedure is the gradient vector $\frac{\partial J}{\partial \beta}$. Through the use of adjoint theory, it is possible to construct a method for calculating $\frac{\partial J}{\partial \beta}$ exactly (within the limits of numerical integration accuracy). The concepts of this approach are given in Ref. 3, which gives details for a continuous control optimization. Adapting this analysis to the control parameter problem of this report is straightforward. Because this approach leads to a much larger programming effort, it was decided, for the present program, to calculate the gradient numerically with the approximation

$$\frac{\partial J}{\partial \beta_i} \cong \frac{\Delta_i J}{\Delta \beta_i} \tag{25}$$

where $\Delta_i J$ is the change in J produced if only component β_i is changed by an amount $\Delta\beta_i$. There is a problem in selecting the $\Delta\beta_i$. On the one hand, if $\Delta\beta_i$ is selected too large, the secant approximation of (25) will not be accurate. On the other hand, $\Delta_i J$ produced by a too small $\Delta\beta_i$ may be inaccurate because it is the difference of two nearly equal numbers, $J(\beta_i + \Delta\beta_i)$ and $J(\beta_i)$.

The program operates automatically to produce each $\Delta\beta_i$ yielding $|\Delta_i J|$ between .0001 and .001. If $|\Delta_i J|$ is outside this range, the next $\Delta\beta_i$ is adjusted accordingly. As J is accurate to six decimal places, $\frac{\Delta_i J}{\Delta\beta_i}$ consequently retains three significant figures, which should be comparable to the closeness of approximating $\frac{\partial J}{\partial \beta_i}$.

Another decision with the gradient method is the choice of the matrix W. In this problem W is chosen so that for each i, $\frac{\Delta_i J}{\Delta \beta_i} d\beta_i$ will be the same order. In words, each $d\beta_i$ will give roughly the same predicted increase in J. This is a somewhat conservative approach in that it "holds back" the more influential parameters, but it does provide steady, easily controlled, convergence.

Denoting the i^{th} diagonal element of W as w_i, W is chosen according to

$$\frac{1}{w_i} = (\Delta\beta_{i_{new}})^2 \tag{26}$$

where $\Delta\beta_{i_{new}}$ is the value of $\Delta\beta_i$ to be used on the next estimation of $\frac{\partial J}{\partial \beta_i}$. This leads to

$$d\beta_i = \frac{dP}{G}\left(\frac{\Delta_i J}{\Delta\beta_i}\right)\Delta\beta_{i_{new}} \tag{27}$$

This, in turn, leads to a predicted increase in J of

$$dJ_{pred} = dP\left[\sum_i \left(\frac{\Delta_i J}{\Delta\beta_i}\right)^2 \left(\Delta\beta_{i_{new}}\right)^2\right]^{1/2} \tag{28}$$

The remaining choice in the gradient procedure is the selection of the step size dP. The main criterion is that the predicted dJ be "reasonably" close to the actual dJ that is obtained. Closeness is measured according to the value of

$$\zeta = \frac{dJ_{act} - dJ_{pred}}{dJ_{pred}} \tag{29}$$

If ζ is between −.05 and .15, dP is increased by 30% in order to accelerate convergence. If ζ is over .15 or between −.15 and −.05, dP is kept the same. If ζ is less than −.15, dP is reduced by 20-60%, depending on the ζ value. The reduction factors can be noted from the program. These factors could be changed if desired, but it is believed that the present program values are quite workable and that it would not be fruitful to readjust them unless a more sophisticated dP were to be used.

The Anchored IMP Problem, with Example

The goal of the particular translunar mission that the POMS program designs is achieving a lunar orbit with long term stability. Such stability is considered achieved if the apocynthion and pericynthion of the lunar orbit calculated using the two-body approximation are within specified limits. Hence, the calculations end following the retrofire near the moon. The probability of success is calculated based on retrofire burnout conditions, both nominal and statistical.

In the nomenclature of the general problem formulation, the functions $\varphi_i(x_f)$ are the apocynthion and pericynthion calculated at t_5. Hence,

$$\varphi_1(x_f) = [a(1+e)]_{t=t_5} \tag{30}$$

$$\varphi_2(x_f) = [a(1-e)]_{t=t_5} \tag{31}$$

where

$$a = \left[\frac{2}{r} - \frac{v^2}{\mu}\right]^{-1} \tag{32}$$

$$e = \left[\left(\frac{rv^2}{\mu} - 1\right)^2 + \frac{(R \cdot \dot{R})^2}{\mu a}\right]^{1/2} \tag{33}$$

A complete sequence of iterations was made in testing the program. The upper bound on apocynthion used was 38,000 km; the lower bound on pericynthion 3346 km. The initial nominal trajectory used is described in detail in the last section. The probability of success, J, was .05 for that trial trajectory. A sequence of forty-two iterations on β resulted in J of .95. The most significant numbers for each iteration are given in Table 1.

Table 1

Test Case Results

Iteration	J (%)	dP	ΔJ_{pred} (%)	ΔJ_{act} (%)	φ_1 (km)	σ_1 (km)	φ_2 (km)	σ_2 (km)	ρ	t_o (hr)	ψ_o (deg)	$t_2 - t_o$ (hr)	p (deg)	y (deg)	$t_4 - t_o$ (hr)
0	5.00				113900	127100	18530	20390	-.9517	0	89.40	.3110	0	0	73.00
1	8.40	10.0	3.90	3.41	75580	79700	15520	19170	-.9421	-.0537	89.20	.3108	.0009	- .0004	73.21
2	11.58	10.0	2.85	3.17	64520	61520	15060	16900	-.9250	-.0663	88.99	.3115	.0144	- .0165	73.31
3	15.29	13.0	3.39	3.71	56340	48700	14790	15180	-.9030	-.0805	88.80	.3125	.0287	- .0415	73.51
4	19.64	16.9	4.02	4.35	49450	39590	14370	13830	-.8849	-.0944	88.65	.3139	.0582	- .0766	73.94
5	24.49	22.0	4.69	4.85	45830	31380	15010	12860	-.8241	-.1071	88.53	.3159	.1100	- .1344	74.36
6	26.10	28.6	6.18	1.61	38730	35740	11250	9720	-.9444	-.1209	88.44	.3141	.2004	- .2027	74.85
7	30.25	22.8	6.09	4.15	38930	27890	12920	11450	-.8572	-.1164	88.40	.3161	.2381	- .2976	74.81
8	33.76	18.3	3.33	3.52	36800	25420	12660	10850	-.8513	-.1243	88.36	.3168	.3154	- .3866	75.10
9	38.31	23.8	4.39	4.54	34680	22700	12570	10320	-.8419	-.1346	88.34	.3179	.4223	- .5236	75.49
10	44.35	30.9	5.98	6.05	32710	19020	13080	10690	-.8072	-.1475	88.30	.3193	.5728	- .7318	75.78
11	52.00	40.2	7.43	7.65	29870	16420	13040	10380	-.8207	-.1703	88.25	.3202	.6890	-1.069	76.04
12	59.69	52.2	9.75	7.69	28420	11930	14490	12830	-.7122	-.1866	88.17	.3224	.8380	-1.370	76.38
13	63.13	41.8	12.26	3.44	26210	14200	13240	9390	-.8728	-.1983	88.14	.3213	.9773	-1.218	76.51
14	66.03	33.4	5.32	2.90	25810	11680	13250	11110	-.8005	-.1894	88.10	.3222	1.020	-1.351	76.65
15	69.46	26.7	4.11	3.43	24710	12800	12530	8276	-.8708	-.1989	88.08	.3216	1.060	-1.403	76.84
16	72.52	21.4	3.10	3.06	24120	11990	12020	7732	-.8574	-.1955	88.06	.3218	1.088	-1.484	77.01
17	76.45	27.8	3.92	3.93	23520	12660	11520	5413	-.8734	-.1980	88.03	.3212	1.128	-1.563	77.15
18	78.91	36.1	6.22	2.46	22850	10510	10540	6044	-.8070	-.1812	88.01	.3216	1.158	-1.705	77.31
19	81.45	20.2	8.05	2.54	23110	12780	10220	2523	-.7090	-.1841	88.00	.3203	1.171	-1.697	77.34
20	83.54	16.2	5.87	2.10	22530	11060	10140	4086	-.7919	-.1819	88.00	.3209	1.175	-1.809	77.34
21	85.48	13.0	2.79	1.94	22460	11410	9996	3009	-.7410	-.1825	87.99	.3206	1.188	-1.845	77.40
22	86.88	10.4	1.47	1.40	22200	10930	9848	3060	-.7309	-.1810	87.99	.3207	1.200	-1.902	77.44
23	88.48	13.5	1.83	1.60	21980	10740	9665	2554	-.6607	-.1800	87.98	.3206	1.218	-1.962	77.51
24	89.73	13.5	1.88	1.25	21620	10000	9513	2997	-.6953	-.1780	87.97	.3208	1.232	-2.041	77.55
25	90.80	10.8	1.64	1.07	21520	10090	9432	2419	-.6063	-.1785	87.96	.3206	1.246	-2.062	77.59
26	91.59	8.62	.96	.79	21280	9704	9331	2576	-.6329	-.1773	87.95	.3207	1.256	-2.118	77.60
27	92.22	6.90	.70	.62	21170	9609	9253	2382	-.5905	-.1767	87.95	.3206	1.266	-2.148	77.63
28	92.82	6.90	.64	.61	21000	9368	9114	2381	-.5762	-.1748	87.95	.3206	1.276	-2.187	77.64
29	93.03	6.90	.74	.20	20960	9502	9027	1914	-.4152	-.1743	87.94	.3204	1.286	-2.213	77.66
30	93.58	5.52	.97	.55	20810	9089	8810	2271	-.5130	-.1700	87.94	.3205	1.289	-2.238	77.67
31	93.76	4.41	.56	.18	20800	9223	8866	1944	-.4040	-.1719	87.94	.3204	1.294	-2.248	77.69
32	94.06	3.53	.45	.31	20700	8979	8740	2149	-.4667	-.1694	87.94	.3204	1.298	-2.266	77.69
33	94.23	2.82	.22	.17	20680	9017	8757	1985	-.4069	-.1703	87.93	.3204	1.301	-2.277	77.71
34	94.42	2.26	.20	.19	20620	8895	8695	2058	-.4278	-.1691	87.93	.3204	1.304	-2.291	77.71
35	94.69	2.26	.32	.27	20520	8818	8680	2001	-.4033	-.1697	87.93	.3204	1.308	-2.303	77.72
36	94.79	1.81	.13	.10	20490	8816	8696	1934	-.3756	-.1703	87.92	.3204	1.311	-2.314	77.73
37	94.91	1.45	.12	.12	20440	8692	8613	2035	-.4077	-.1686	87.92	.3204	1.313	-2.322	77.73
38	94.91	1.88	.13	.00	20460	8776	8656	1901	-.3501	-.1698	87.92	.3204	1.313	-2.325	77.73
39	94.99	.74	.07	.08	20430	8704	8611	1963	-.3745	-.1698	87.92	.3204	1.314	-2.329	77.74
40	95.01	.74	.06	.02	20430	8730	8622	1901	-.3446	-.1692	87.92	.3204	1.315	-2.333	77.74
41	95.07	.59	.05	.06	20410	8683	8592	1936	-.3583	-.1687	87.92	.3204	1.316	-2.336	77.74
42	95.08	.59	.04	.01	20400	8709	8618	1886	-.3359	-.1693	87.92	.3204	1.317	-2.340	77.74

The initial trajectory had φ_1 (apocynthion) of 113,900 km, φ_2 (pericynthion) of 18,500 km. The standard deviations σ_1 and σ_2 were 127,100 km and 20,400 km respectively. The major change in the first ten iterations was the reduction of φ_1 to 32,700, σ_1 to 19,000, and σ_2 to 10,700 km. At this point J was .44. In the next seven iterations φ_1 and φ_2 changed only slightly, but σ_1 and σ_2 were reduced to 12,700 and 5,400 km respectively, with J increasing to .76. The remaining iterations achieved much slower progress with J = .90 obtained on the twenty-fifth iteration and, finally, J = .95 on the forty-second.

Description of the Computer Program

POMS (FORTRAN II) is an automatic parameter optimization program which is presently designed for a lunar capture mission. The program includes a simple approximation for the booster ascent phase which yields the injection conditions for the earth parking orbit. The covariance matrix of the state associated with these injection conditions is assumed to be a diagonal matrix with terms

$$d_{1-3} = (\overline{\Delta R})^2 = \left(\frac{\bar{R}\,10^{-4}}{2}\right)^2$$

$$d_{4-6} = (\overline{\Delta \dot{R}})^2 = \left(\frac{\bar{\dot{R}}\,10^{-3}}{2}\right)^2$$

The basic structure of the program assumes that, after injection into earth parking orbit, the vehicle coasts for a prescribed length of time. Then a thrusting period occurs, followed by a coast and another thrust. The durations of the above phases are given as input. Different Encke methods are used to solve the equations of motion during the thrust and coast stages. The covariance matrix of the state is propagated to the end of the last thrust. During thrust, errors due to the uncertainties associated with the thrust magnitude and direction contribute to the state covariance matrix.

It is not until the final thrust is over that the program is concerned with particular application to a lunar capture mission. The covariance matrix of the state is used to compute the probability that the apocynthion and pericynthion lie within the bounds given as input. Two FORTRAN II SHARE routines are used to compute this probability; ERR1 (SDN #1322) and BVN (SDN #1323). These write-ups describe the probability as having an error of one unit in the sixth decimal place.

If a different mission objective were desired (rather than prescribing limits on apocynthion and pericynthion), it would be a simple matter to rewrite this portion of the program. If the number of mission objectives is increased, however, it becomes numerically more difficult to compute the probability of success.

The program uses an automated gradient procedure to yield a nominal mission plan which has the largest probability of success. It accomplishes this by choosing optimal values of the following six parameters:

1) launch time
2) azimuth of the launch plane
3) time from launch to first thrust
4) pitch orientation of thrust axis
5) yaw orientation of thrust axis
6) time from launch to second thrust

The automated gradient procedure numerically computes the change in the probability of success as each one of the control parameters is subsequently varied by small amounts. Based upon these results, new values of the control parameters are chosen, each change giving a predicted increase in the probability. The automated part of this scheme is choosing the small variations and the size of the expected increase so that an increase is indeed attained. If a decrease does occur, the last set of variations of the control parameters are reduced until an increase in the probability occurs. (The sample test case encountered only 3 decreases in 45 attempts to improve the probability.)

The program requires that a nominal mission plan achieving lunar capture be used to start the iteration procedure. Since such a plan may be difficult to find, the program includes a feature which helps the analyst to find one. It allows the second thrust to be triggered not on time from launch but on time after entering the moon's gravitational sphere of influence. So, if the analyst stacks cases of a promising nominal where only this time parameter is changed in an arbitrary way, one run on the computer will provide a key to a satisfactory set of control parameters.

The integration routine used to solve the perturbation equation is the scheme proposed by Samuel Pines. The disturbing functions are the gravitational forces for up to six planets and thrust. The program is designed to use less computer time when only earth and moon gravitational forces are included. It takes .95 min. on the 7094 Mod 1 for a complete lunar trajectory with a transit time of 75 hours.

POMS Input

Each piece of input is defined as one of four categories: integer, fixed point, floating point, or alphanumeric with the notation I, FX, FL, or A, respectively. The quantity in the description column is entered on the specified card in the appropriate columns. The name given is the name used for the quantity internally in the program. An asterisk before a name means that the dimension is determined by KLM.

Card	Cols.	Name	Type	Description
1	1-5	NUMSTA	I	Number of stations
	6-10	KLM	I	0 - Input in ER and ER/hr 1 - Input in km and km/sec
	11-15	MREF	I	Reference body (1-6)
	16-20	KOND	I	Print indicator for trajectory parameters
	21-25	IFLAG	I	Print indicator for statistics
	26-30	ITMAX	I	Maximum number of iterations
	31-35	IPR	I	Print indicator for iterations
2	12 cols. per value	VAR(1)	FL	Launch time (hrs) from nominal
		VAR(2)	FL	Inertial launch azimuth (deg)
		VAR(3)	FL	Length of time from launch to first thrust (hrs)
		VAR(4)	FL	Pitch of thrust vector (deg)
		VAR(5)	FL	Yaw of thrust vector (deg)
		VAR(6)	FL	Length of time from launch to second thrust (hrs)
3	12 cols. per value	PVAR(1-6)	FL	Changes made to the above parameters to compute the partials of the probability numerically (same dimensions as the parameters)
4	12 cols. per value	WTD(1)	FL	Weight flow of first thrust (#/sec)
		WTD(2)	FL	Weight flow of second thrust (#/sec)

Card	Cols.	Name	Type	Description
4	12 cols. per value	DTB(1)	FL	Length of first thrust (sec)
		DTB(2)	FL	Length of second thrust (sec)
		D(1)	FL	Thrust of first burn (# force)
		D(2)	FL	Thrust of second burn (# force)
5	12 cols. per value	DTBOS	FL	Time for booster ascent (hrs)
		DWBOS	FL	Excess weight dropped after first thrust (#)
		WTIN	FL	Weight at injection into earth parking orbit (#)
		DKOK	FL	Relative error of thrust magnitude
		ALP	FL	Thrust direction error (deg)
		GAM	FL	Geocentric angle measured in the azimuth plane from launch to the earth parking orbit injection point (deg)
6	12 cols. per value	*RMAGE	FL	Altitude of earth parking orbit
		*RAL	FL	Lower limit on apocynthion
		*RAU	FL	Upper limit on apocynthion
		*RPL	FL	Lower limit on pericynthion
		*RPU	FL	Upper limit on pericynthion
		TAMR	FL	Time (hrs), after entering moon's sphere of influence, to thrust
7	12 cols. per value	SWGHT	FL	Not being used at present
		DP	FL	Gradient step size (assumed to be 10 if no value is input)
		FPA	FL	Flight path angle at injection into earth parking orbit (deg)
		VELO	FL	Earth parking orbit injection velocity (km/sec)
8				Nominal launch date
	1-6	NYEARP	I	Year
	7-12	DAYS	FX	Day
	13-18	HRS	FX	Hour

Card	Cols.	Name	Type	Description
8	19-24	HMIN	FX	Minutes
	25-30	SEC	FX	Seconds
9				Launch site
	1-2	K	I	Station number
	3-14	STANM	A	Station name
	15-26	SLON	FX	Longitude - degrees
	27-29	SLONM	FX	- minutes
	30-35	SLONS	FX	- seconds
	36-47	SLAT	FX	Geodetic latitude - degrees
	48-49	SLATM	FX	- minutes
	50-55	SLATS	FX	- seconds
	56-67	SALT	FX	Geodetic altitude (feet)
10	1-6	BMU	FX	Six indicator for planets: 1 - planet included 0 - planet excluded
	7-18	DTC	FL	Integration interval for coast
	19-30	DTPR	FL	Print interval for coast
	31-42	DTT	FL	Integration interval for thrust
	43-54	DTPRTH	FL	Print interval for thrust

Some of the input quantities require further explanation.

MREF is chosen from the table below:

MREF	Reference
1	Earth
2	Sun
3	Moon
4	Venus
5	Mars
6	Jupiter

The BMU indicators represent the planets according to the preceding table.

KOND governs the printing of the osculating elements and is the frequency in minutes that print-out is desired.

IFLAG governs the printing of the covariance matrix, S matrix and Ω matrix, and is the frequency in minutes that print-out is desired.

IPR governs all print-out for an iteration except the results of the iteration.

1: Print-out on first and last iterations

2: Every iteration

3: Every iteration and every variation trajectory

TAMR is a quantity that is only used to obtain an acceptable nominal trajectory. Its value is the number of hours after entering moon reference that the second thrust should occur. Many cases may be stacked where only the value of TAMR is changed, thus automating the choice of a nominal trajectory captured by the moon after the second thrust.

DTC is the integration interval for the translunar leg of the mission. This value is multiplied by 1/16 to obtain the near-earth integration interval and by 1/8 to obtain the near-moon interval.

Input Values for the Initial Nominal of the Test Case

The test case chosen was for a mission scheduled for launch from Cape Kennedy at 16 hours 12 minutes of the 182 day of 1966. The coordinates used for the Cape were:

Longitude :	-80° 34' 35.45"
Geodetic latitude :	28° 28' 54.34"
Geodetic altitude :	44.78 feet

The nominal azimuth of the launch plane is 89.4°. The booster ascent carries a payload of 806 pounds downrange in the launch plane through a geocentric angle of 22° in .15075 hours to an altitude of 146.16 km with a velocity of 8.37565226 km/sec.

The first thrust occurs .311 hours after launch with a 6188.4 pound thrust for 22.6 seconds and a weight flow of 22.460177 pounds per second. After thrust, 86.4 pounds of excess weight is dropped.

The second thrust occurs 73 hours after launch with an 845.945945 pound thrust for 22.2 seconds and a weight flow of 3.0765765 pounds per second.

The RMS value of the relative error of the magnitude is .003319. The RMS value of the half cone angle for the thrust direction error is 1.462°. The limits for apocynthion and pericynthion are 3346 km and 38000 km. This nominal input produced a 5% probability of success for the above mission plan. The program was able to attain an apparent maximum probability of 95% after 42 iterations by choosing better values of the control parameters.

References

1. Pines, S., Wolf, H. and Mohan, J.; Final Report for Minimum Variance Precision Tracking and Orbit Prediction Program, Contract NAS 5-2535, prepared by Analytical Mechanics Associates, Inc., May 1963.

2. Pines, S.; Precision Trajectories for Thrusting Vehicles and the Solution of the Variational Equations, prepared by Analytical Mechanics Associates, Inc., April 1964.

3. Denham, W.F.; Choosing a Nominal Path to Maximize the Probability of Reaching a Given Region of State Space, prepared by Analytical Mechanics Associates, Inc., October 1964.