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ITEM PROGRAM MANUAL

FORTRAN IV VERSION

By

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Greenbelt, Maryland

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A program of this nature must, of necessity, take a period of several years for its development; it is thus impossible to mention the names of all those who have contributed to its growth. It was originally conceived by S. Pines and H. Wolf at Republic Aviation Corporation under contract to NASA (NASW-109) beginning in 1959. This version is issued under contract to the Special Projects Branch, Theoretical Division of Goddard Space Flight Center (NAS 5-9085). Major contributors have been C. Bergren, C. Hipkins, L. Lefton, M. Wachman, F. Whitlock, and N. Levine.

Numerous additions and improvements have been made to the current version including reprogramming for the IBM 7090 and 7094 computers, and development is a continuing effort. This latest edition of the Program Manual covers the conversion of the original machine language program to FORTRAN IV.

The program has been and is available for general use to interested organizations.*

The authors express their appreciation to Mrs. Agnes Michalowski for her assistance in the final preparation of this manual.

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* Address: Head, Special Projects Branch, Theoretical Division, Goddard Space Flight Center, Greenbelt, Maryland

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I. INTRODUCTION

This report describes a general purpose Interplanetary Trajectory Encke Method (ITEM) Program, programmed in the FORTRAN IV language. The method employed is designed to give the maximum available accuracy without incurring prohibitive penalties in machine time. On the basis of research described in Reference 4, the Encke method was selected as best satisfying these requirements. However, the classical Encke method was modified to eliminate some of its objectionable features. This modified Encke method is described in Appendix A.

The perturbations included in this program are the gravitational attractions of the Earth, Moon, Sun, Jupiter, Venus and Mars considered as point masses. Additionally, the effects of the second, third and fourth zonal harmonics and the second and third tesseral harmonics of the Earth and Moon gravitational fields, as well as aerodynamic drag, small corrective thrusts, and radiation pressure including the shadow effect of the Earth, are considered. The input may be prepared in any one of several common systems and a great variety of output options are available.

II. NOTATION

Upper case - vectors; Hats - unit vectors; Lower case - magnitudes

Description	Symbol	Units
Cartesian coordinates of vehicle with respect to reference body	$x \; y \; z$	km
Velocity components in Cartesian coordinates	$\dot{x} \; \dot{y} \; \dot{z}$	km/sec
Time	t	hrs.
Longitude measured from Greenwich, + East (used in Section IV and Appendix H)	θ	degrees
Longitude of vernal equinox	θ_0	degrees
Speed	v	km/sec
Geodetic altitude*	h	km
Geodetic latitude	ϕ	degrees
Geodetic flight path angle	γ	degrees
Geodetic flight path azimuth	A	degrees
Acceleration parameter (defined in Appendix E)	u	
Right ascension	RA	degrees
Astronomical units	AU	
Earth radii	ER	
Earth mass	m_e	

*Note: The following 3 symbols with primes denote the corresponding geocentric quantities. Geocentric in this report refers to a spherical earth, i.e., $e^2 = 0$. In this case $\phi' = \delta =$ declination.

Description	Symbol
Vehicle position vector	R
Distance to vehicle	r
Perturbation displacement vector	ΔR
Perturbation displacement vector components	ξ, η, ζ
Perturbation acceleration	F
Coordinate functions and their time derivatives	$f, g, \dot{f}, \dot{g}$
Mass parameter	μ
Semi-major axis	a
Earth's eccentricity as used in Appendices H, I, L, S	e
Mean motion	n
Unit vectors for classical two-body orbit solution	$\hat{P}, \hat{Q}$
Eccentric anomaly as used in Appendix T	E
Elevation angle as used in Appendix I	E
$R_o \cdot \dot{R}_o$	d_o
Inclination of orbital plane	i
Right ascension of the ascending node	Ω

Description	Symbol
Argument of perigee	ω
Parameters which account for polar oblateness of the earth, defined in Appendix H	c, s
Right ascension of the station meridian	RA_s
Range measured from observation station	ρ
Direction cosines measured in a topocentric coordinate system	λ, μ, ν
Declination	δ

SUBSCRIPTS

Vehicle	v
ith perturbing body	i
Quantity obtained from Keplerian solution of two-body problem	k
Reference body as used in Appendix B	c
Station	s
$R_A - R_B$	R_{AB}
Value at rectification time	o
Corresponds to x, y, z components respectively	n = 1, 2, 3
Value at perigee	p

III. GENERAL PROCEDURE FOR USING PROGRAMS

Initial conditions, terminal conditions and print frequency, as well as other parameters controlling the flow of the program, are read as input. The computation of the trajectory then proceeds until one of the terminal conditions (e.g. maximum time) has been reached or an error is encountered. At this time the program prints the reason for its termination and proceeds to the next case. When an end of file is encountered on the input tape, control is transferred to the monitor.

IV. INITIAL CONDITIONS

The initial conditions necessary for the specification of a trajectory are:

1. Initial position of the vehicle relative to the reference body.

2. Initial velocity of the vehicle relative to the reference body.

3. Initial time of launch referenced to a base time.

For specification of the initial conditions, the reference systems and units shown below may be used.

A Cartesian Coordinates

The coordinate system is defined as follows:

1. The origin is at the center of the reference body.

2. The x axis is in the direction of the mean equinox of December 31.0 of the year of launch.

3. The xy plane is the mean equatorial plane of the Earth.

   Initial position is given by the x, y, z coordinates of the vehicle. Initial velocity is given by the $\dot{x}$, $\dot{y}$, $\dot{z}$ components of the vehicle. Initial time of launch from base time[(1)] (t) is also given. If the program is used in its standard form, the units[(2)] to be used for the above are:

---

(1) The base time is 0.0^h UT December 31 of the year previous to the year of launch.

(2) Scale factors are used to convert the input units to the units used internally (ER or AU and hrs). Any other set of units may be used by changing these scale factors with the appropriate ID card as described in Section VIII.

x, y, z - kilometers

$\dot{x}, \dot{y}, \dot{z}$ - kilometers/second

t - month, day, hours, minutes, and seconds from base time

The year of launch must also be given.

B Geodetic Polar Coordinates

Initial position of the vehicle is given by:

1. Geodetic latitude (φ)
2. Longitude[3] (θ), measured from Greenwich
3. Geodetic altitude (h)
4. Longitude of vernal equinox[3] at initial time (θ_o). This quantity may be computed by the program or may be loaded.

Initial velocity of the vehicle is given by:

1. Speed (v) with respect to the center of the Earth.
2. Flight path azimuth (A) measured clockwise from north in a plane normal to the geodetic altitude.
3. Flight path angle (γ) measured from a plane normal to the geodetic altitude.

---

(3) If the right ascension (RA) at initial time is known, it may be used in place of longitude (θ). The longitude of the vernal equinox (θ_o) is then set to zero.

Initial time of launch from base time[1] (t) must also be given.

The following units must be used with the above quantities:

1. φ, θ, and θ_o - degrees; h - kilometers

2. A and γ - degrees; v - kilometers/second

3. t - month, day, hours, minutes, and seconds

C Geocentric Polar Coordinates

Ordinarily an input given in polar coordinates will be interpreted as described in the preceding Paragraph B. However, if NOPT(1) = 1,[4] the program will interpret latitude as declination, height as distance above a spherical Earth of equatorial radius, and flight path angle and azimuth with reference to a plane normal to the radius vector.

D Osculating Element Input

The osculating elements to be input are:

Argument of perigee

Longitude of ascending node

Inclination

Semi-major axis (in Earth radii)

Eccentricity

Time of perigee, mean anomaly, eccentricity anomaly, or true anomaly (only one)

The program converts the above to Cartesian coordinates and then continues normally. (See Section IX, ID = 10.)

---

(4) See INPUT Section.

E Comments

1. The program computes in Cartesian coordinates. Units used internally in the computation are:

   a. position: Earth Radii (ER) - Astronomical Units (AU)

   b. velocity: ER/hour - AU/hour

   c. time: hours

   (Earth Radii units are used in the Earth or Moon reference. Astronomical Units are used in the Sun, Venus, Mars or Jupiter reference.)

2. Cartesian coordinates must be used when launching from any body other than Earth. Either polar or Cartesian coordinates may be used when launching from Earth.

3. Equations for converting the initial conditions from polar to Cartesian coordinates are shown in Appendix H.

V. TERMINATING CONDITIONS

The set of conditions which will terminate a trajectory may be summarized as:

1. Maximum time of flight - hours.

2. Maximum distance from any possible reference body considered in the solution. Last value in R-vector of integration and print block "ARRAY(1, n, J)."

3. Minimum distance from any possible reference body considered in the solution. First value in R-vector of integration and print block ARRAY(1, n, 1)*

Any of these conditions will terminate a trajectory. Loading a large number into any of the maxima and a zero into any of the minima will make the corresponding conditions inoperative. A proper choice of these numbers will permit complete computation of the desired trajectory, avoid extensive unnecessary computation and guard against faulty input.

---

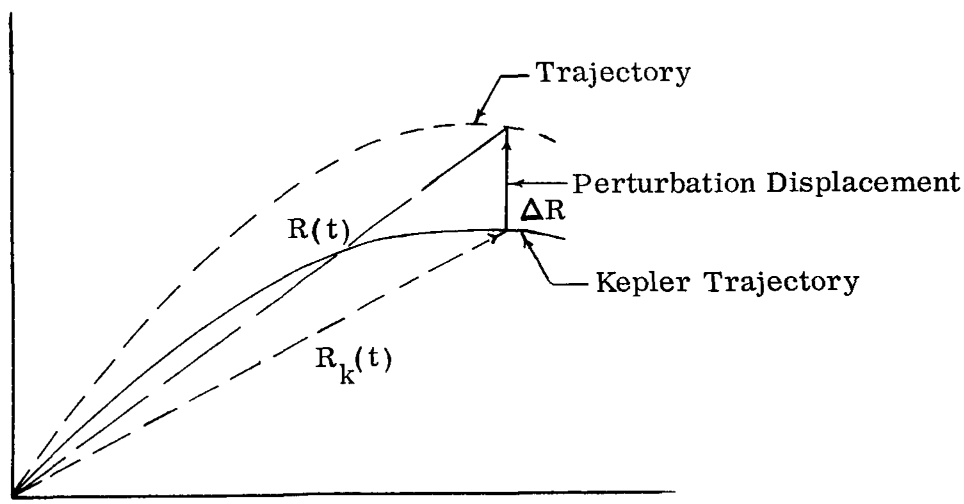
* n designates reference body.

n = 1	Earth
n = 2	Moon
n = 3	Sun
n = 4	Venus
n = 5	Mars
n = 6	Jupiter

VI. PERMISSIBLE PERTURBATIONS

The trajectory computation consists of two parts, the exact solution to the two-body problem and integrated additions to this solution for the effect of perturbations. The successful control of round-off errors in the modified Encke method depends on preventing the accumulated round-off error in the integrated perturbation displacement from affecting the computed position. This is achieved by always keeping the perturbation displacement small and rectifying whenever the perturbation exceeds specified limits. The constants mentioned below are used in determining the allowable limits as ratios of the perturbation position and velocity to the two-body position and velocity, respectively.

This ratio for the position vector is shown in the following sketch.



Encke Method

The recommended values for these ratios are as follows:

Position Ratio

POSRCS $\left(\frac{\Delta r}{r}\right)^2 \leq .0001$

Velocity Ratio

VELRCS $\left(\frac{\Delta \dot{r}}{\dot{r}}\right)^2 \leq .0001$

and these are incorporated into the program. Modifications may be made by altering the data subroutine.

VII. RADAR INFORMATION PROGRAM **

The program may be used to simulate radar data if desired. A maximum of 30 stations can be handled at one time. The following information is required for each station considered:

1. Station Name - for identification purposes

2. Position of Radar Station

   a. Longitude (θ) of the station from Greenwich - degrees, minutes and seconds* - positive eastward.

   b. Geodetic latitude (φ) of the station - degrees, minutes and seconds* - positive north.

   c. Altitude (h) of station above sea level - feet.

The simulated radar information consists of azimuth, elevation, topocentric right ascension and declination, slant range, and range rate. It is printed at every normal print time for which the elevation angle is positive. Refraction is not considered.

This section is coded as a subroutine and may be called at any time.

---

* Alternatively, these quantities may be given in degrees and decimals. Zero's must be loaded into the positions reserved for minutes and seconds.

The fractional parts will not appear in the printout reproducing the station coordinates. They will, however, be included in the computation.

** Not available in this version.

VIII. SUBROUTINE MODIF

Modifications to program constants, which normally remain unchanged during the running of a number of cases, may be made by using ID 12 of the INPUT in conjunction with a compilation of a subroutine called MODIF. This subroutine MODIF must contain the proper common blocks and may include data statements, ordinary FORTRAN statements, or read statements. The use of read statements is suggested to facilitate stacking of cases.

Modifications required more frequently may be accomplished through the use of other ID(I)'s, as described in the INPUT section (Section IX).

A Radiation Pressure

Radiation pressure may be included by loading a coefficient into

RACOE ____________**

The number to be loaded is:

$$\frac{K\,C_r\,A}{m}$$

** Floating point number

C_r is the radiation pressure in dynes/cm^2 at a distance of 1 AU from the Sun.

$$C_r = 4.6 \times 10^{-5} \frac{\text{dynes}}{\text{cm}^2} \qquad \text{(estimated value)}$$

A area in cm^2

m mass in grams

K scaler $3600^2 (23455.)^2 / 6378.165 \times 10^5 = .11178 \times 10^8$
seconds to hours, ER to AU, cm to ER

The radiation pressure will only be active if sunlight impinges on the vehicle. For correct results the radiation pressure should, therefore, be run only in conjunction with the optional shadow computation as described in Appendix O.

If, however, the expected trajectory may be safely assumed to be entirely out of the Earth's shadow, shadow testing may be avoided with a consequent saving in machine time. In this case, the following modification card must also be included:

SHDN = 1.**

B <u>Aerodynamic Drag</u>

If inclusion of the aerodynamic drag is desired, the drag parameter $1/2\ C_D\ A/m$ may be loaded into subroutine MODIF by means of the following card:

** Floating point number

COEFL = ________**

The units of $C_D A/m$ are the area in cm^2 and the mass in grams. A layered atmosphere rotating with the Earth is assumed. The density is obtained by a linear interpolation of the density-altitude table. The above may be incorporated into the DATA subroutine.

C Atmospheric Tables

Atmospheric tables for the drag computation are stored in core. They correspond to model #7, contained in Report #25 (Reference 2) of the Smithsonian Astrophysical Observatory, fitted to the ARDC Model Atmosphere of 1956 (Reference 3) at low altitudes. The units for the air density are grams/cm^3 and the height is given in ER from the center of the Earth. If it is desired to change this atmosphere, the following procedure has to be followed:

NTAR = ________*	The number to be entered is N - the number of entries in the density table.
RTBL(I) = ________**	I = 1, 2, - - -, N - the values of r for which densities are given, in ascending order (a maximum of 50).

** Floating point number

* Fixed point number

RHOT(I) = __________** I = 1, 2, - - -, N - the values for the air density in grams/cm^3 in respective order corresponding to the preceding r table.

If other units are used for the density table, the drag parameter described in Part B of this section must be read in with like units and the constant (-6378.165E5)** normally in DRSC has to be changed accordingly. The negative sign directs the drag force opposite to the velocity. This constant converts the drag from the units used for A, M and ρ to ER/$hour^2$.

DRSC = ______________** Conversion constant

D Printout+

The program provides a special printout described in the OUTPUT section (Section X-B-4) near the Earth, Moon, Sun, Venus, Mars and Jupiter. This printout occurs at every integration step and is useful for observing the behavior of these relevant quantities during ascent and re-entry. This feature is triggered by the following modification cards:

SERE(I) = __________** I = 1, 2, - - -, 6 Radial distance from indexed planet.

For printout near	Earth	use index 1	in	ER	units
	Moon	2		ER	
	Sun	3		AU	
	Venus	4		AU	
	Mars	5		AU	
	Jupiter	6		AU	

- -

** Floating point numbers

+ Not available in this version

The numbers used above are the radial distances within which the special printout is to be effective. The units are earth radii for the Earth and Moon references and astronomical units for the Sun, Venus, Mars and Jupiter references. A zero in any of the SERE(I) cards suppresses this feature.

E Ephemeris Time

The planetary coordinates are interpolated using ephemeris time.

$$ET = UT + \Delta TE$$

An approximate value of ΔTE (35 seconds) is used. To change this quantity, the following card, giving ΔTE in hours, is inserted:

ETMUT = ___________**	The value for ΔTE - hours.

To restore original quantity:

ETMUT = .009888888889**	ΔTE is 35 seconds.

** Floating point numbers

IX. INPUT

Input to the program is read in as follows:

Each set of input is preceded by an ID card which contains a fixed point number terminating in column 10. This card may also contain Hollerith information starting in column 11.

ID = 1 Permits one card of Hollerith information — usually used for case identification.

ID = 2 Permits one card containing a set of 72 fixed point 1's or 0's. Each flag (1 or 0) corresponds to the same numbered subroutine. A zero is used for normal operation and a one is used to print diagnostic information in the proper subroutine. A blank card after ID = 2 will be necessary if the system does not zero out core before load time and normal operation is desired. In the program, these flags are referred to as NC(I).

ID = 3 Performs similarly to ID = 2. It allows a card of 1's and 0's to be read into NOPT(I) (I=1 to 72). These flags permit the incorporation of various options into the program. Following are the currently available options:

NOPT(1) = 1 indicates polar geocentric coordinates

= 2 indicates geodetic coordinates when polar load is used.

NOPT (2-13) are used for print options.

= 0 indicates no print

= 1 indicates print

NOPT(2)	is associated with XR print
NOPT(3)	is associated with XRDT print
NOPT(4)	is associated with XVE print
NOPT(5)	is associated with XVM print
NOPT(6)	is associated with XME print
NOPT(7)	is associated with XVS print
NOPT(8)	is associated with XVVN print
NOPT(9)	is associated with XVMR print
NOPT(10)	is associated with XVJP print
NOPT(11)	is associated with XI print
NOPT(12)	is associated with XIDT print
NOPT(13)	is associated with D2XI print
NOPT(14) =1	prints data statement parameters
NOPT(15) =1	deletes regular print after rectification
NOPT(16) =1	deletes print in rectification
NOPT(17) =1	activates shadow routine
NOPT(67) =1	activates a backward integration at TIMEL (maximum time)
NOPT(68) =1	osculating print for solar engine
NOPT(69) =1	activates solar engine
NOPT(71) =1	activates Beta Integrator

ID = 4 Used to read in start time of flight in year, month, day, hours, minutes, and seconds; starting reference body; and target reference using the following format (5I5, F5.2, 2I5). The reference bodies are numbered as follows:

1 = Earth	4 = Venus
2 = Moon	5 = Mars
3 = Sun	6 = Jupiter

ID = 5 Used for polar load and reads in θ, φ, altitude, velocity, azimuth, γ, and θ_o. Format used is (7E10.0). The angles are in degrees. Altitude is in kilometers from the surface of the Earth, and the velocity is in kilometers per second. If θ_o is read in as 1000., the program will compute the proper θ_o.

ID = 6 Used for Cartesian input. x, y, z, $\dot{x}$, $\dot{y}$, and $\dot{z}$ are read in with the format (6E10.0). The program expects these coordinates to be equatorial in kilometers and kilometers per second, with the starting reference body as center.

ID = 7 This option generates initial conditions for a trajectory which is designed to get a spacecraft to the target within a specified number of days, without thrust. The input is: the Julian date of start time with the first three digits removed, the time of flight in days, option number, and desired radius of parking orbit, with a format of (4E10.0).
Option number 1 starts in Sun reference
Option number 2 starts in Earth reference.

ID = 8 This ID permits one to read in a vector of special print times. The first card after the ID card contains the number of such print times from 1 to 50, format (I10). The following card or cards contain the times, format (7E10.0). If this ID is used in conjunction with NOPT(69) (solar engine option), these times are used for starting and stopping the solar engine. Odd numbers start the engine, the even numbers shut the engine off.

ID = 9 This ID reads in the following: PRSP, TESP, TIMEL, STI, VI, CCNT, and ENPLAN using format (7E10.0).

PRSP	is print suppress time – normal print times will be suppressed until this time is reached.
TESP	is a trigger for calculating impact planes. A non-zero value activates this option.
TIMEL	is the maximum time of flight.
STI	is a trigger for iterating on moon trajectories. A non-zero value activates this option.
VI	is a trigger for activating subroutine VIT which may be altered for various iterations.
CCNT	triggers the nodal crossing print. It must be an integer n. Every n^{th} crossing is printed.
ENPLAN	is the number of bodies to be used in the calculations. This must be an integer from 1-6. If 1 is used, the ephemeris tape is not used.

ID = 10 This ID permits one to load the initial conditions as osculating elements of an ellipse. The following are read in: SOMEG, LOMEG, INC, A, ECC, ELOAD, ELTRIG with format (7E10.0).

SOMEG	is the argument of perigee
LOMEG	is the longitude of the ascending node
A	is the semi-major axis
ECC	is the eccentricity
ELOAD	depends on ELTRIG

If	ELTRIG = 1	ELOAD = time of perigee
	ELTRIG = 2	ELOAD = mean anomaly
	ELTRIG = 3	ELOAD = eccentric anomaly
	ELTRIG = 4	ELOAD = true anomaly

ID = 11 Permits one to alter the integration and print intervals of the various reference bodies. The number of cards to be read is a function of ENPLAN. If ENPLAN is 6, all reference bodies are expected and are read in with format (8E9.0) as follows:

Card 1 contains eight distances from Earth in ER
2 contains seven integration intervals in hrs.
3 contains seven print intervals in hrs.
4 contains eight distances from the Moon in ER
5 contains seven integration intervals in hrs.
6 contains seven print intervals in hrs.
7 contains eight distances from the Sun in AU
8 contains seven integration intervals in hrs.
9 contains seven print intervals in hrs.
10 contains eight distances from Venus in AU
11 contains seven integration intervals in hrs.
12 contains seven print intervals in hrs.
13 contains eight distances from Mars in AU
14 contains seven integration intervals in hrs.
15 contains seven print intervals in hrs.
16 contains eight distances from Jupiter in AU
17 contains seven integration intervals in hrs.
18 contains seven print intervals in hrs.

ID = 12 Permits one to make changes in the program's built-in data or to read in other-than-normal inputs. This can be done by using a subroutine called MODIF which must contain the proper block common.

ID = 13 Allows one to change input and output scale factors, using format (3E11.0). The card following the ID card contains TSCL, REKM, and XMDKM.

TSCL	is the time scale factor and sits in the program as 3600. It is used to change seconds to hours and hours to seconds.
REKM	sits in the program as 6378.165, the number of kilometers in one ER.
XMDKM	sits as 14.9599×10^7 and is the number of kilometers in one AU.

ID = 14 sets triggers for apogee, perigee, and nodal crossing prints. The following card reads in ICANT, ICPNT, and ICCNT, using format (3I10)

ICANT = n prints every n^{th} apogee

ICPNT = n prints every n^{th} perigee

ICCNT = n prints every n^{th} nodal crossing

ID = 15 Permits the reading in of the oblateness and tesseral harmonic coefficients, format (11E6.0). Six cards must follow the ID card as follows:

J 20(I), J 22(I), J 30(I), J 31(I), J 33(I), J 40(I), L22(I), L33(I), RADIUS(I), TESSTR(I).

I = 1-6 representing the reference body. TESSTR(I) is a trigger for the inclusion or deletion of tesseral harmonics for the reference body involved.

TESSTR(I) = 1 calculates tesseral harmonic coefficients

TESSTR(I) = 0 by-passes tesseral calculations.

ID = 16 Allows radar station data to be read in. The card following the ID card contains the number of stations to be read in with format (I10). The next two cards contain the name and coordinates of the first station with format (4A6/7E10.0). This last format is repeated until all stations have been accounted for.

ID = 17 is used for reading in solar engine information.

See Appendix

ID = 18 is used for the iterator.

This option is not available in this version.

ID = 20 Starts the program

ID = 4 must precede ID = 9 which also precedes ID = 11, since ID = 11 uses ENPLAN. Except for the preceding condition and ID = 20 which must be read in last, the ID's may be read in randomly. However, the user must bear in mind that the last ID read in prevails in a case where several conflicting ones are used. For example, if the Cartesian, polar and element load were in the input deck at the same time, the last one to be read in would take control of the program.

X. OUTPUT

A Program Outputs

The following information is printed as the output of the program.

1. Title
2. Case number and any identifying titles.
3. Launch time - days, hours, minutes, seconds.
4. Input - in the same units as they were entered into the program.
5. List of parameters used in run.
6. At each rectification the following data are printed:

___(b)___ RECTIFICATION PRINT ___(a)___ REFERENCE

PERT OVER UNPERT = ___(c)___ TIME = ___(d)___ DELTA T = ___(e)___

(a) Reference body

(b) and (c) indicate the reason for rectification

(c) If (c) = 0, rectification may be due either to switch of reference body or to change of integration interval.

If (c) $\neq$ 0, then the position, velocity, acceleration perturbations or the incremental eccentric anomaly have exceeded the permissible limits and (b) indicates which has been exceeded (see Section VI). These indications are given as:

PO - Position
VL - Velocity
TH - Incremental eccentric anomaly

(d) Time of rectification

(e) Integration interval

TIME IN DAYS, HRS, MINS, SECS, ____ ____ ____ ____ (a)

T = ________ (b)

XR ______ YR ______ ZR ______ RR ______ (c)

XRDT ______ YRDT ______ ZRDT ______ RRDT ______ (d)

RIGHT ASCENSION (DEG) = ______ DECL = ______ (e)

EARTH SUBSAT POINT LONG = ______ (f)

LAT = ______ (g)

HT = ______ (h)

GHA = ______ (i)

GEOCENTRIC AZIMUTH = ______ (j)

ELEVATION = ______ (k)

GEODETIC AZIMUTH = ______ (ℓ)

ELEVATION = ______ (m)

(a) Days, hours, minutes, seconds from time of launch

(b) Print time in hours from time of launch

(c) Position coordinates and magnitude of radius vector with respect to the reference body - kilometers

(d) Velocity components and magnitude of velocity vector with respect to the reference body - kilometers/second

(e) Right ascension and declination in Earth reference system - degrees

(f) Longitude or sub-satellite point - degrees

(g) Latitude (geodetic) - degrees

(h) Geodetic height above the Earth's surface - kilometers

(i) Greenwich hour angle - degrees

(j) Geocentric flight path azimuth - degrees

(k) Geocentric flight path angle - degrees

(ℓ) Geodetic flight path azimuth - degrees

(m) Geodetic flight path angle - degrees

MOON SUBSAT POINT	LONG	= ______	(a)
	LAT	= ______	(b)
	AZIM	= ______	(c)
	ELEV	= ______	(d)

OSCULATING ELEMENTS AT TIME T = ______

TRUE ANOMALY	= ______	(e)
SEM MAJ AXIS	= ______	(f)
ECCENT	= ______	(g)
PERICENT	= ______	(h)
APOCENT	= ______	(i)
INCLINATION	= ______	(j)

(a) Moon longitude - angle between the projection of the vector from the Moon to the vehicle onto the Moon's orbital plane and the Moon-Earth vector (Moon reference only) - degrees

(b) Moon latitude - angle between the radius vector connecting the Moon and the vehicle and its projection onto the orbital plane of the Moon about the Earth (Moon reference only) - degrees

(c) Selenocentric flight path azimuth - degrees

(d) Selenocentric flight path angle - degrees

(e) True anomaly - degrees

(f) Semi-major axis of trajectory - ER   + = ellipse
- = hyperbola

(g) Eccentricity of trajectory**

(h) Closest distance to the reference body (not necessarily the Earth)** - kilometers

(i) Farthest distance from the reference body (not necessarily the Earth)**(meaningful only for elliptic orbits) - kilometers

(j) Inclination of the orbital plane defined as the angle between the positive polar axis and the angular momentum vector** - degrees

---

** These are the osculating values and hence only constitute an estimate of the quantities described.

ARG PERIC	=	______	(a)
PERIOD	=	______	(b)
MEAN MOT	=	______	(c)
R A ASC NODE	=	______	(d)
M ANOMALY	=	______	(e)
E ANOMALY	=	______	(f)
T PERIC	=	______	(g)
UNIT PERICENTER POSITION VECTOR	=	______	(h)
UNIT ANGULAR MOMENTUM VECTOR	=	______	(i)

(a) Argument of pericenter - angle measured from the ascending node to the pericenter vector**- degrees.

Set to zero for circular orbits and poorly determined for near-circular orbits.

(b) Period**- hours

(c) Mean motion**- radians/hour

(d) Right ascension of the ascending node measured from the vernal equinox eastward along the equator**- degrees

(e) Mean anomaly** - radians

(f) Eccentric anomaly** - radians

(g) Time of nearest pericenter** - hours

(h) Components of the unit vector directed from reference toward pericenter**

(i) Components of the unit angular momentum vector

** Osculating values

B Optional Outputs

1. XVE	= _____	YVE	= _____	ZVE	= _____	RVE	= _____	(a)
XVM	= _____	YVM	= _____	ZVM	= _____	RVM	= _____	(b)
XME	= _____	YME	= _____	ZME	= _____	RME	= _____	(c)
XVS	= _____	YVS	= _____	ZVS	= _____	RVS	= _____	(d)
XVVN	= _____	YVVN	= _____	ZVVN	= _____	RVVN	= _____	(e)
XVMR	= _____	YVMR	= _____	ZVMR	= _____	RVMR	= _____	(f)
XVJP	= _____	YVJP	= _____	ZVJP	= _____	RVJP	= _____	(g)
XI	= _____	ETA	= _____	ZETA	= _____	PERT	= _____	(h)
XIDT	= _____	ETADT	= _____	ZETADT	= _____	VPERT	= _____	(i)
D2XI	= _____	D2ETA	= _____	D2ZETA	= _____	APERT	= _____	(j)

The above optional output appears between XRDT and RIGHT ASCENSION in the standard output. For instructions on how to obtain, see Section IX, ID = 3.

(a) Coordinates of vehicle with respect to the Earth - kilometers

(b) Coordinates of vehicle with respect to the Moon - kilometers

(c) Coordinates of the Moon with respect to the Earth - kilometers

(d) Coordinates of vehicle with respect to the Sun - kilometers

(e) Coordinates of vehicle with respect to Venus - kilometers

(f) Coordinates of vehicle with respect to Mars - kilometers

(g) Coordinates of vehicle with respect to Jupiter - kilometers

(h) Perturbation vector and magnitude of the perturbations with respect to the reference body - kilometers

(i) Perturbation velocity vector and magnitude - kilometers/second

(j) Perturbation acceleration vector and magnitude - kilometers/second^2

2. Shadow Print

$$\text{PASSAGE FROM} \left\{\begin{array}{l}\text{SHADOW}\\ \text{PENUMBRA}\\ \text{SUN}\\ \text{PENUMBRA}\end{array}\right\} \text{TO} \left\{\begin{array}{l}\text{PENUMBRA}\\ \text{SHADOW}\\ \text{PENUMBRA}\\ \text{SUN}\end{array}\right\}$$

$$\text{AT } \underline{\;(a)\;} \text{ TIME IN} \left\{\begin{array}{l}\text{SHADOW}\\ \text{PENUMBRA}\\ \text{SUN}\\ \text{PENUMBRA}\end{array}\right\} \underline{\;(b)\;} \text{ ACCUMULATED TIME } \underline{\;(c)\;}$$

The above optional output appears before TIME IN DAYS, HRS, MINS, SECS in the standard output. It is controlled through the INPUT subroutine [NOPT(17)] (see Section IX, ID = 3).

(a) Time at which vehicle traverses denoted shadow boundary - hours

(b) Total time the vehicle spends in denoted shadow region during current traverse - hours

(c) Total accumulated time spend in denoted shadow region since launch - hours

3. Radar Output

STATION	(a)
AZIMUTH	(b)
ELEVATION	(b)
TOPOC. R A	(c)
TOPOC. DECL.	(c)
SLT RNG	(d)
RANGE	(e)

This output appears at the tail end of a normal printout. An ID card in the INPUT subroutine will control this segment of the program (see Section IX, ID = 16).

(a) Station name (identification) for each station

(b) Azimuth and elevation with respect to each station - degrees

(c) Topocentric right ascension and declination with respect to each station - degrees

(d) The slant range to each station - kilometers

(e) Rate of change of slant range for each station - kilometers/second

If the elevation is negative (the vehicle is below the horizon), this print is suppressed for the station in question.

4. Reentry Output

REENTRY PRINT	TIME	INERTIAL SPEED (kilometers/second)

Right ascension, declination, Earth subsatellite points and flight path azimuth and angle as given above.

The above optional output appears between GEOD ELEV and MOON SUBSAT POINT in the standard output.

5. Trajectory Search Output

The output consists of the normal ITEM output for a nominal trajectory and the same trajectory output for each variation requested for each iteration. The output format used only for the trajectory search follows:

VARIATION IN INITIAL CONDITIONS	(a)	(b)	(c)	(d)	(e)	(f)	(g)

(a) Change in latitude - degress

(b) Change in longitude - degrees

(c) Change in altitude - kilometers

(d) Change in velocity - kilometers/second

(e) Change in azimuth - degrees

(f) Change in flight path angle - degrees

(g) Change in initial time - hours

5. Trajectory Search Output (cont.)

QUANTITY CODE		(a)	
DESIRED VALUES OF ABOVE QUANTITIES		(b)	
REQUIRED ACCURACY		(c)	
MATRIX OF PARTIAL DERIVATIVES		(d)	
RESIDUALS AND CHANGES IN INITIAL CONDITIONS	(e)	(f)	(g)

(a) Code indicating quantities (up to 7) to be searched for.

(b) Desired values of above quantities - degrees, kilometers, seconds.

(c) Tolerances allowed on above values - degrees, kilometers, seconds.

(d) Matrix with the dependent variables arranged by row. The independent by column.

(e) Residuals (desired-nominal) of quantities designated by the quantity code.

(f) Change required in initial conditions.

(g) Normalized changes in initial quantities in order of the variations.

The option associated with trajectory search routines is initiated by an ID card in the INPUT subroutine (see Section IX, ID = 18).

XI. INTERNAL PROCEDURES

A Units

The units used internally are Earth Radii and Earth Radii/hour in the Earth and Moon references, and Astronomical Units and Astronomical Units/hour in the Sun, Venus, Mars and Jupiter reference systems.

B Ephemeris Tape

The relative positions of the solar system bodies are obtained from a tape generated by the Jet Propulsion Laboratory. A separate program prepares a binary tape referred to the mean equinox of MID-FILE,* containing 16 days per record, in a form compatible with the main program. The main program searches the tape and reads in the proper file and record, keeping 32 days of tables in core storage at a time.

The first record on each file* consists of the year, number of records and number of files* in fixed decimal form. Each of the successive records contains the following information:

Word 1: Initial time of record in hours from base time. (0.0^h UT December 31 of year previous to launch)

---

* Pseudo file. Tape is prepared in overlapping two-year groups.

Equatorial coordinates of Mercury in two-day intervals follow (9 x values, 9 y values, 9 z values). Then 27 consecutive five-word blocks containing the equatorial coordinates, in four-day intervals, of

XVNE	YVNE	ZVNE	Venus with respect to the Sun
XSE	YSE	ZSE	Sun with respect to the Earth
XAS	YAS	ZAS	Mars with respect to the Sun
XJS	YJS	ZJS	Jupiter with respect to the Sun
XSAS	YSAS	ZSAS	Saturn with respect to the Sun
XUS	YUS	ZUS	Uranus with respect to the Sun
XNS	YNS	ZNS	Neptune with respect to the Sun
XPS	YPS	ZPS	Pluto with respect to the Sun
XBS	YBS	ZBS	Earth-Moon barycenter with respect to the Sun

are followed by three 32-word blocks containing the equatorial coordinates of the

XME	YME	ZME	Moon with respect to Earth

The Moon coordinates are stored in half-day intervals (0.0^h, $12^h.0$ UT) with distance measured in ER. All other tables are in AU.

The equatorial coordinates of the planets and of the Moon are followed by their velocities, in exactly the same order. Moon velocities are in ER/day. All other velocities are in AU/day.

At present, an ephemeris tape is available for 1968-1982, written in 15 two-year groups, each of which overlaps one year.

C Ephemeris in Core

The astronomical tables are stored in core in 96-hour intervals for the Sun and the planets, and 12-hour intervals for the Moon. There are always 32-days of tables available, arranged in such a way that the value of time for which the interpolation takes place is not near either end of the table.

In location TABLE(1), the time of the first entry from the initial time is stored. In TABLE(2) to TABLE(10) there are 9 x coordinates of the Sun with respect to the Earth. The following chart indicates the storage locations of the remaining astronomical data to be saved.

TABLE(11)	to	TABLE(19)	y coordinates of the Sun with respect to the Earth
TABLE(20)	to	TABLE(28)	z coordinates of the Sun with respect to the Earth
TABLE(29)	to	TABLE(55)	x, y, z coordinates of Jupiter with respect to the Sun
TABLE(56)	to	TABLE(82)	x, y, z coordinates of Mars with respect to the Sun
TABLE(83)	to	TABLE(109)	x, y, z coordinates of Venus with respect to the Sun
TABLE(299)	to	TABLE(363)	x coordinates of the Moon with respect to the Earth
TABLE(364)	to	TABLE(428)	y coordinates of the Moon with respect to the Earth
TABLE(429)	to	TABLE(493)	z coordinates of the Moon with respect to the Earth

These are followed by the velocities:

TABLE(494)	to	TABLE(520)	$\dot{x}$, $\dot{y}$, $\dot{z}$ coordinates of the Sun with respect to the Earth
TABLE(521)	to	TABLE(547)	$\dot{x}$, $\dot{y}$, $\dot{z}$ coordinates of Jupiter with respect to the Sun
TABLE(548)	to	TABLE(574)	$\dot{x}$, $\dot{y}$, $\dot{z}$ coordinates of Mars with respect to the Sun
TABLE(575)	to	TABLE(601)	$\dot{x}$, $\dot{y}$, $\dot{z}$ coordinates of Venus with respect to the Sun
TABLE(791)	to	TABLE(985)	$\dot{x}$, $\dot{y}$, $\dot{z}$ coordinates of the Moon with respect to Earth

D Perturbation Program

The perturbation program solves three differential equations for XI, ETA, ZETA. The differential equation for XI, with the various terms replaced by the storages containing them, is representative of all three equations and is given below:

```
D2XI = - GME  [VCOR(1)/VCOR(4)    - COMP(1)/COMP(4)]
       - GMVN [VCOR(19)/VCOR(22)  - COMP(19)/COMP(22)]
       - GMS  [VCOR(13)/VCOR(16)  - COMP(13)/COMP(16)]
       - GMMR [VCOR(25)/VCOR(28)  - COMP(25)/COMP(28)]
       - GMJP [VCOR(31)/VCOR(34)  - COMP(31)/COMP(34)]
       - GMM  [VCOR(7)/VCOR(10)   - COMP(7)/COMP(10)]
       + OTHER PERTURBATIONS
```

where, for example, in the first term GME = K^2 is the mass of the Earth, and VCOR(4) is the length cubed of the vector [VCOR(1), VCOR(2), VCOR(3)]. Similarly, in the other terms the denominator is the length cubed of the vector containing the corresponding numerator. In the case where the two terms within each of the brackets are nearly equal, they are computed by the special method described in Appendix E to avoid loss of accuracy.

The contents of the COMP storage at any time, t, depends upon the reference origin at that time.

CONTENTS OF COMP STORAGE

Earth Ref.	Moon Ref.	Sun Ref.	Venus Ref.	Mars Ref.	Jupiter Ref.	COMP(I)
XVE0	XME	XSE	XVNE	XMRE	XJPE	(1), (2), (3) = x, y, z (4), (5), (6) = R^3, R, R^2
XEM	XVM0	XSM	XVMM	XMRM	XJPM	(7), (8), (9)
XES	XMS	XVS0	XVNS	XMRS	XJPS	(13), (14), (15)
XEVN	XMVN	XSVN	XVVN0	XMRVN	XJPVN	(19), (20), (21)
XEMR	XMMR	XSMR	XVNMR	XVMR0	XJPMR	(25), (26), (27)
XEJP	XMJP	XSJP	XVNJP	XMRJP	XVJP0	(31), (32), (33)

Here XVE refers to the x component of the vehicle with respect to the Earth, with corresponding definitions for the other quantities. An additional subscript of 0 denotes quantity derived from the two-body problem.

CONTENTS OF VCOR STORAGE

All Refs.	VCOR(I)
XVE	(1), (2), (3), (4), (5), (6) = x, y, z, R^3, R, R^2
XVM	(7), (8), (9)
XVS	(13), (14), (15)
XVVN	(19), (20), (21)
XVMR	(25), (26), (27)
XVJP	(31), (32), (33)

XII. REFERENCES

1. Anon.; "Goddard Minimum Variance Orbit Determination Program," Report No. X-640-62-191, NASA Goddard Space Flight Center, Special Projects Branch, Theoretical Division, October 18, 1962.

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4. Pines, S., Payne, M. and Wolf, H.; "Comparison of Special Perturbation Methods in Celestial Mechanics," ARL TR 60-281, August 1960.

5. Morrison, J. and Pines, S.; "The Reduction from Geocentric to Geodetic Coordinates," A.J.66, No. 1, February 1961.

6. Campbell, J., Moore, W. E. and Wolf, H.; "MINMAX - A General Purpose Adaptive Iterator for Nonlinear Problems," Analytical Mechanics Associates, Inc. Report on Contract NAS 9-2527, June 30, 1964.

B. EQUATIONS OF MOTION

In a Newtonian system, the equations of motion of a particle in the gravitational field of n attracting bodies and subject to other perturbing accelerations such as thrust, drag, oblateness, radiation pressure, etc. are given by

$$\ddot{R}_v = -\sum_{i=1}^{n} \mu_i \frac{R_{vi}}{r_{vi}^3} + \sum_j F_j \tag{B. 1}$$

These equations are put into observable form by referring them to a reference body c. The equations of motion of the reference body are

$$\ddot{R}_c = -\sum_{\substack{i=1 \\ i \neq c}}^{n} \mu_i \frac{R_{ci}}{r_{ci}^3} \tag{B. 2}$$

Subtraction of Equation (B. 2) from Equation (B. 1) results in the equations of motion of the vehicle with respect to the reference body c.

$$\ddot{R}_{vc} = -(\mu_v + \mu_c) \frac{R_{vc}}{r_{vc}^3} - \sum_{\substack{i=1 \\ i \neq c}}^{n} \mu_i \left[\frac{R_{vi}}{r_{vi}^3} - \frac{R_{ci}}{r_{ci}^3} \right] + \sum_j F_j \tag{B. 3}$$

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Contract NAS 5-9085
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MATHEMATICAL APPENDIX

A. INTRODUCTION

The problem of orbit determination over long time periods requires a precise technique for integrating the equations of motion. Reference 4 contains an analysis of an integration procedure that yields the minimum loss of information due to the accumulation of numerical round-off errors. The Encke perturbation method has been shown to require minimum machine computation time for a minimum loss of numerical accuracy. The orbit prediction scheme presented herein uses a modified form of the Encke method with the initial position and velocity vectors replacing the conventional P and Q vectors of the Encke scheme. By avoiding reference to the position of perigee, it is possible to avoid numerical ambiguities arising from near-circular orbits and orbits of low inclination.

C. METHOD OF INTEGRATION

If Equation (B.3) is integrated directly by some numerical scheme, there results, after a number of step-by-step integrations, an accumulation of error which leads to inaccurate results. To avoid this loss in precision, it is convenient to write Equation (B.3) in the form

$$\ddot{R}_{vc} = \ddot{R}_k + \Delta\ddot{R} \tag{C.1}$$

The velocity and displacement vectors can be written as

$$\dot{R}_{vc} = \dot{R}_k + \Delta\dot{R} \tag{C.2}$$

$$R_{vc} = R_k + \Delta R \tag{C.3}$$

The reference body (the one in whose sphere of influence the vehicle travels) is chosen so as to minimize the perturbations.

In this method $\ddot{R}_k$ is taken as

$$\ddot{R}_k = -(\mu_v + \mu_c)\frac{R_k}{r_k^3} \tag{C.4}$$

and

$$\ddot{\xi} = -(\mu_v + \mu_c)\left[\frac{R_{vc}}{r_{vc}^3} - \frac{R_k}{r_k^3}\right] - \sum_{\substack{i=1 \\ i \neq c}}^{n} \mu_i \left[\frac{R_{vi}}{r_{vi}^3} - \frac{R_{ci}}{r_{ci}^3}\right] + \sum_j F_j \tag{C.5}$$

Equations (C.4) constitute the equations of motion of the Kepler problem and are solved as described in Appendix D.

Equations (C.5) are integrated numerically. The integration scheme employed by the ITEM program is a sixth order backward difference scheme, initiated by a Runge-Kutta scheme. The routine used is a Newton-Gregory integration scheme for second order difference equations written by S. Pines and J. Mohan of Analytical Mechanics Associates, Inc.

As derived in Appendix D, the solution of the Kepler problem may be represented by the vectors R_o, $\dot{R}_o$, the scalar a and the rectification time t_o.

The rectification process consists of moving R_{vc}, $\dot{R}_{vc}$ into the locations R_o and $\dot{R}_o$, t into t_o and the computation of a and n.

For computational convenience, the coefficients appearing in Equations (D.2) are also computed during rectification.

D. SOLUTION OF THE KEPLER TWO-BODY PROBLEM

The unified formulation of the two-body problem is used for both elliptic and hyperbolic cases.

$$\beta = \sqrt{|a|} \cdot \theta$$

$$\alpha = \beta^2 \left(\frac{1}{a}\right)$$

$$F_1(\alpha) = \frac{1}{6} - \frac{\alpha}{120} + \frac{\alpha^2}{5040} - - - - - = \sum_{i=0}^{\infty} \frac{(-\alpha)^i}{(2i+3)!}$$

$$F_2(\alpha) = \frac{1}{2} - \frac{\alpha}{24} + \frac{\alpha^2}{720} - - - - = \sum_{i=0}^{\infty} \frac{(-\alpha)^i}{(2i+2)!} \tag{D.1}$$

$$F_3(\alpha) = 1 - \alpha F_1$$

$$F_4(\alpha) = 1 - \alpha F_2$$

and

$$f = 1 - \frac{1}{r_o} \beta^2 F_2$$

$$g = \frac{r_o}{\sqrt{\mu}} \beta F_3 + \frac{d_o \beta^2 F_2}{\mu} \tag{D.2}$$

$$\dot{f} = - \frac{\sqrt{\mu}}{r_o r} \beta F_3$$

$$\dot{g} = 1 - \frac{1}{r} \beta^2 F_2$$

where

$$d_o = R_o \cdot \dot{R}_o$$

$$r = \beta^2 F_2 + r_o F_4 + \frac{d_o}{\sqrt{\mu}} \beta F_3$$

$$R = f R_o + g \dot{R}_o$$

$$\dot{R} = \dot{f} R_o + \dot{g} \dot{R}_o$$

$$a = \left(\frac{2}{r_o} - \frac{V_o^2}{\mu} \right)^{-1}$$

$$V_o^2 = \dot{R}_o \cdot \dot{R}_o$$

α is determined from the modified Kepler equation

$$\sqrt{\mu}\, \Delta t = \beta^3 F_1 + r_o \beta F_3 + \frac{d_o}{\sqrt{\mu}} \beta^2 F_2 \tag{D. 3}$$

See Figure 1 for the two-body orbit which results from the solution of Equation (C. 4) with the initial conditions:

$$\begin{aligned} R_k(t_o) &= R_{vc}(t_o) = R_o \\ \dot{R}_k(t_o) &= \dot{R}_{vc}(t_o) = \dot{R}_o \end{aligned} \tag{D. 4}$$

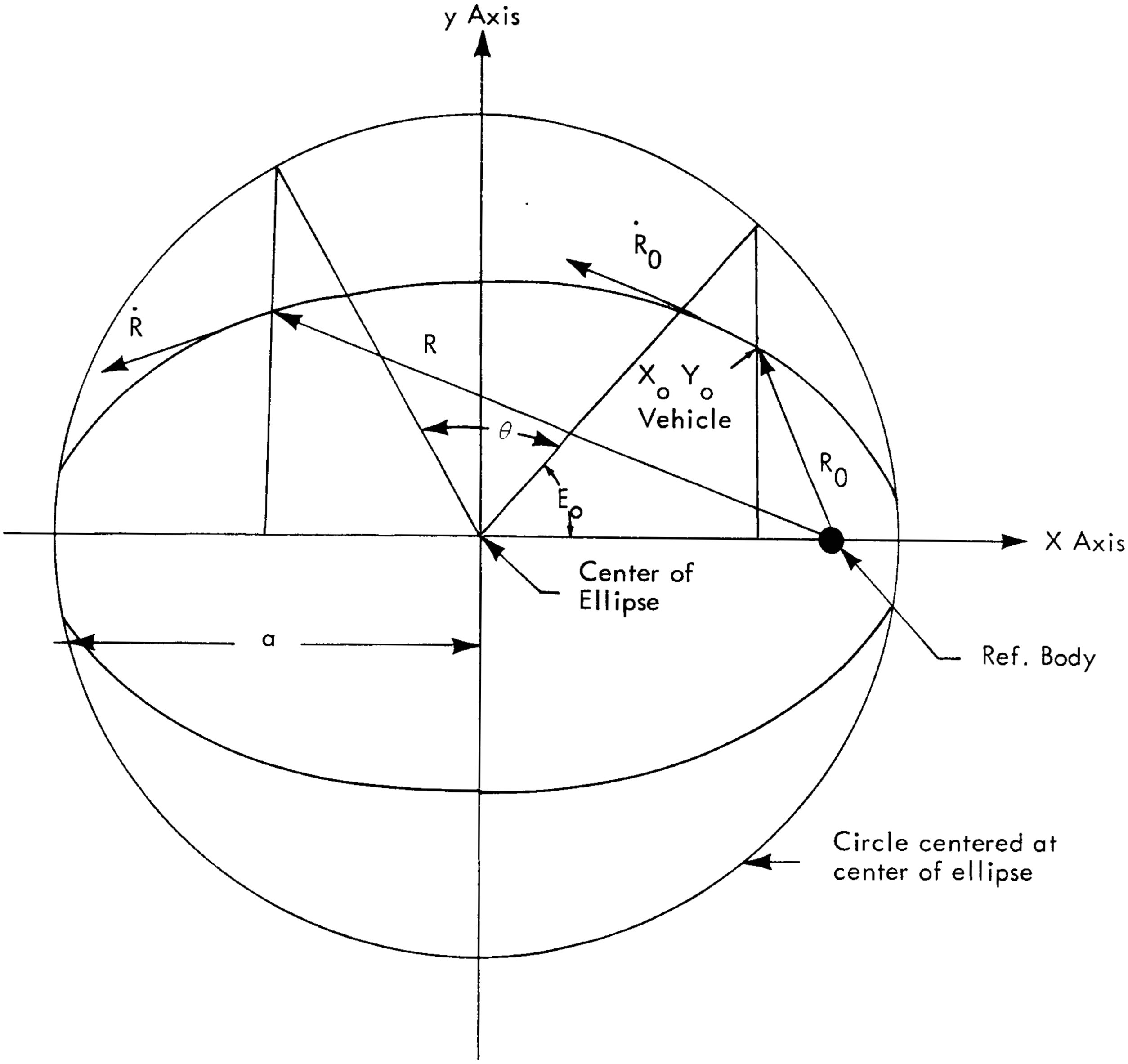

Figure 1 - Geometry of the Elliptic Two-Body Orbit

E. COMPUTATION OF PERTURBATION TERMS

The terms accounting for the Encke term and the planetary perturbations appearing on the right hand side of Equation (C.5) involve numerous terms of the form $\frac{R}{r^3} - \frac{R_o}{r_o^3}$ where R and R_o may differ only by small amounts. For the Encke term, for instance $R - R_o = \xi$ which is small, and for the planetary perturbations, the difference is R_{vc} which also often is small.

A computation scheme, which avoids loss of precision due to the subtraction of nearly equal terms and which also is correct when R_{vc} is not small, is employed. This scheme is described below: Find

$$\frac{R}{r^3} - \frac{R_o}{r_o^3}$$

$$u = \frac{2}{r_o^2}\left(R_o + \frac{1}{2}\Delta R\right) \cdot \Delta R \tag{E.1}$$

$$\frac{R}{r^3} - \frac{R_o}{r_o^3} = \frac{\Delta R}{r_o^3} + \frac{R(u^3 + 3u^2 + 3u)}{\left(1 + \frac{r^3}{r_o^3}\right)}$$

F. CONCLUSIONS

The method presented yields accurate trajectories using relatively little computer time. Summarizing some of the important features:

1. All significant solar system bodies may be included without undue complications.

2. Since the perturbations only are integrated, the allowable integration interval is fairly large over most of the path. Even in the vicinity of Earth or another planet a relatively large interval (compared to other schemes) may be used without limiting the stability and accuracy of the solutions.

3. The perturbations are kept small in two ways. First, the two-body orbit is rectified whenever the perturbations exceed a specified maximum value compared to the corresponding unperturbed values. This limits error build-up with respect to a particular reference body. Second, the reference body of the two-body problem is changed from Earth, to Sun, to planet accordingly, as that reference body would contribute the largest perturbing force otherwise.

4. This method will handle circular orbits, zero inclination, etc. The problem is defined in terms of parameters which have real physical significance (namely, the position and velocity vectors) which are directly relatable to measurable quantities.

G. OBLATENESS TERMS

The oblateness perturbation terms in Equations (C.5) are derived from the potential given by the following equation:

$$\varphi = \frac{\mu}{r}\left\{-\left(\frac{a_e}{r}\right)^2 J_{20}\left[\frac{3}{2}\left(\frac{z}{r}\right)^2 - \frac{1}{2}\right] - \left(\frac{a_e}{r}\right)^3 J_{30}\left[\frac{5}{2}\left(\frac{z}{r}\right)^3 - \frac{3}{2}\left(\frac{z}{r}\right)\right] - \left(\frac{a_e}{r}\right)^4 J_{40}\left[\frac{35}{8}\left(\frac{z}{r}\right)^4 - \frac{15}{4}\left(\frac{z}{r}\right)^2 - \frac{3}{8}\right]\right\} \tag{G.1}$$

where a_e is the equatorial radius of the earth.

$$F = \operatorname{grad} \varphi$$

This vector can be written in the form

$$F = \ell R + m\hat{k}$$

where $\hat{k}$ is a unit vector in the z-direction.

$$\ell = \frac{\mu}{r^3}\left\{\left(\frac{a_e}{r}\right)^2 J_{20}\left[\frac{15}{2}\left(\frac{z}{r}\right)^2 - \frac{3}{2}\right] + \left(\frac{a_e}{r}\right)^3 J_{30}\left[\frac{35}{2}\left(\frac{z}{r}\right)^3 - \frac{15}{2}\left(\frac{z}{r}\right)\right] + \left(\frac{a_e}{r}\right)^4 J_{40}\left[\frac{315}{8}\left(\frac{z}{r}\right)^4 - \frac{105}{4}\left(\frac{z}{r}\right)^2 - \frac{15}{8}\right]\right\} \tag{G.2}$$

$$m = -\frac{\mu}{r^2}\left\{\left(\frac{a_e}{r}\right)^2 J_{20}\left(3\frac{z}{r}\right) + \left(\frac{a_e}{r}\right)^3 J_{30}\left[\frac{15}{2}\left(\frac{z}{r}\right)^2 - \frac{3}{2}\right] + \left(\frac{a_e}{r}\right)^4 J_{40}\left[\frac{35}{2}\left(\frac{z}{r}\right)^3 - \frac{15}{2}\left(\frac{z}{r}\right)\right]\right\} \tag{G.3}$$

The perturbation acceleration due to tesseral harmonics are computed.

Constants: J_2^2, J_3^1, J_3^3, λ_2^2, λ_3^1, λ_3^3

$$J_2^2 = -1.9E-6 \qquad \lambda_2^2 = -21.^{\circ}$$

$$J_3^1 = -1.51E-6 \qquad \lambda_3^1 = 0 \tag{G.4}$$

$$J_3^3 = -\ .149E-6 \qquad \lambda_3^3 = 22.8^{\circ}$$

In initialization:

$$C_i^j = J_i^j \cos j\lambda_i^j$$

$$S_i^j = J_i^j \sin j\lambda_i^j \tag{G.5}$$

x, y, z are x', y', z'

F_{22} Term

$$A_{22} = \frac{5}{r^2}(x^2 - y^2)$$

$$B_{22} = -5\frac{xy}{r^2} \tag{G.6}$$

$$F_{22x'} = \frac{3\mu}{r^5}\left\{C_2^2(2x - x A_{22}) + 2 S_2^2(y - x B_{22})\right\}$$

$$F_{22y'} = \frac{3\mu}{r^5}\left\{C_2^2(-2y - y A_{22}) + 2 S_2^2(x - y B_{22})\right\} \quad \text{(G. 7)}$$

$$F_{22z'} = \frac{3\mu}{r^5}\left\{C_2^2(-z A_{22}) + 2 S_2^2(-z B_{22})\right\}$$

x, y, z are earth fixed coordinates. x, y in the equatorial plane, the x-axis toward the Greenwich meridian. Nutation is neglected.

$\underline{F_{31}\ \text{Term}}$

$$K_{31} = (4z^2 - x^2 - y^2)$$

$$A_{31} = \frac{7x}{r^2} K_{31} \quad \text{(G. 8)}$$

$$B_{31} = \frac{7y}{r^2} K_{31}$$

$$F_{31x'} = \frac{3}{2}\frac{\mu}{r^7}\left\{C_3^1[K_{31} - 2x^2 - x A_{31}] + S_3^1[-2xy - x B_{31}]\right\}$$

$$F_{31y'} = \frac{3}{2}\frac{\mu}{r^7}\left\{C_3^1[-2xy - y A_{31}] + S_3^1[K_{31} - 2y^2 - y B_{31}]\right\} \quad \text{(G. 9)}$$

$$F_{31z'} = \frac{3}{2}\frac{\mu}{r^7}\left\{C_3^1[8xz - z A_{31}] + S_3^1[8yz - z B_{31}]\right\}$$

$\underline{F_{33}\ \text{Term}}$

$$A_{33} = \frac{7x}{r^2}(x^2 - 3y^2)$$

$$B_{33} = \frac{7y}{r^2}(3x^2 - y^2) \tag{G.10}$$

$$F_{33x'} = \frac{15\mu}{r^7}\left\{C_3^3[3(x^2 - y^2) - xA_{33}] + S_3^3[6xy - xB_{33}]\right\}$$

$$F_{33y'} = \frac{15\mu}{r^7}\left\{C_3^3[-6xy - yA_{33}] + S_3^3[3(x^2 - y^2) - yB_{33}]\right\} \tag{G.11}$$

$$F_{33z'} = \frac{15\mu}{r^7}\left\{C_3^3[-zA_{33}] - S_3^3[zB_{33}]\right\}$$

$$F_{x'} = F_{22x'} + F_{31x'} + F_{33x'}$$

$$F_{y'} = F_{22y'} + F_{31y'} + F_{33y'} \tag{G.12}$$

$$F_{z'} = F_{22z'} + F_{31z'} + F_{33z'}$$

H. TRANSFORMATION EQUATIONS FROM GEODETIC POLAR COORDINATES TO CARTESIAN COORDINATES*

The geodetic polar coordinates in the program are referred to an ellipsoid of revolution. The equation of a cross section is given by

$$\frac{x^2}{a^2} + \frac{z^2}{b^2} = 1 \tag{H.1}$$

where

$$b^2 = a^2\left(1 - e^2\right)$$

The slope of the normal, along which h is measured is given by

$$\tan\phi = -\frac{1}{\frac{dz}{dx}} = \frac{a^2 z}{b^2 x} \qquad \textbf{(See Figure 2)} \tag{H.2}$$

and

$$\tan\phi' = \frac{z}{x} = \frac{b^2}{a^2}\tan\phi = \left(1 - e^2\right) \tan\phi$$

Eliminating x between equations (H.1) and (H.2) and solving for z results in:

$$z = \frac{a\left(1 - e^2\right) \sin\phi}{\left(1 - e^2 \sin^2\phi\right)^{1/2}}$$

*For geocentric (i.e. $e^2 = 0$) polar coordinates, c = s = 1. In this case the latitude input is interpreted as declination.

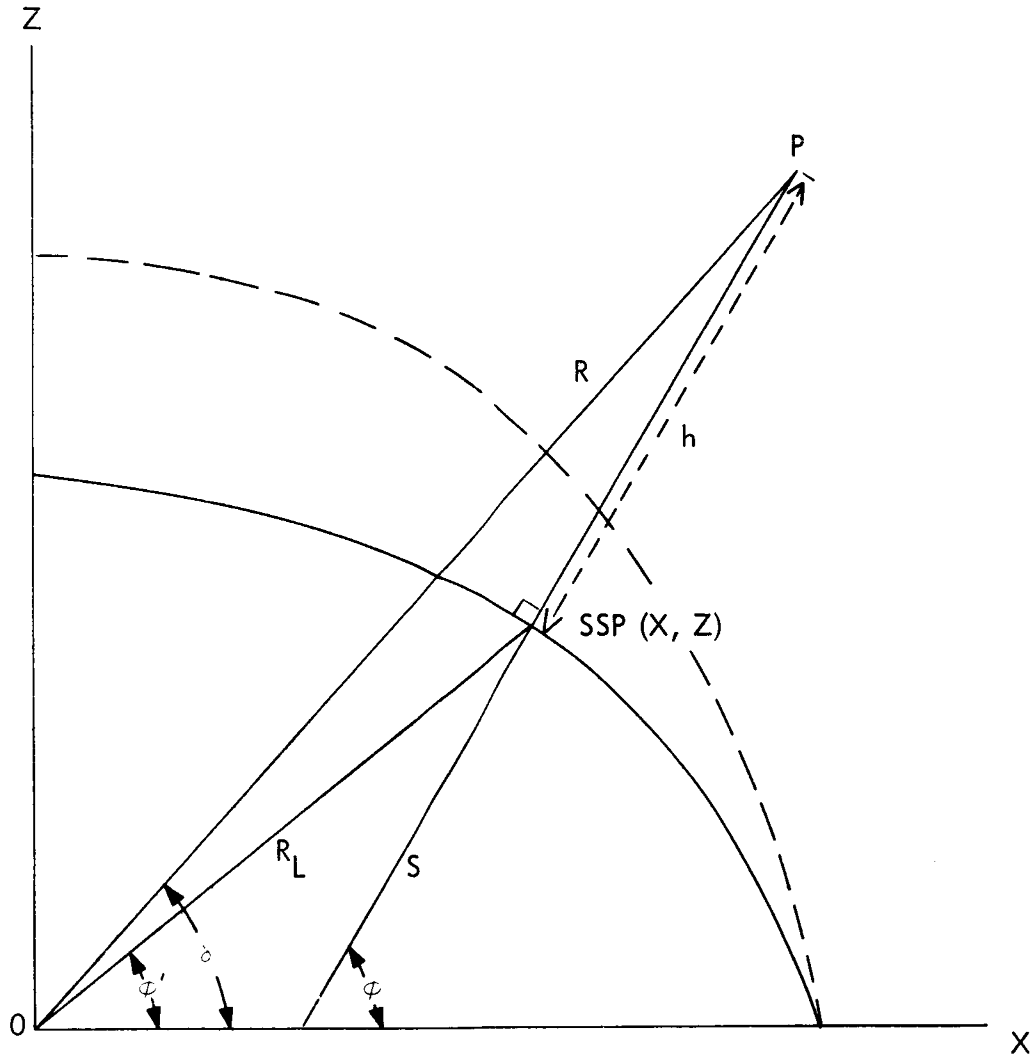

Figure 2 - Relation Between Declination, Geocentric and Geodetic Latitudes

and from equation (H.2) then

$$x = \frac{a \cos \phi}{\left(1 - e^2 \sin^2 \phi\right)^{1/2}}$$

In units of a_e, R and $\dot{R}$ are then given by equation (H.3)

$$
\begin{aligned}
c &= \left(1 - e^2 \sin^2 \phi\right)^{-1/2} \\
s &= \left(1 - e^2\right) c \\
x &= (c + h) \cos\phi \cos\left(\theta - \theta_0\right) \\
y &= (c + h) \cos\phi \sin\left(\theta - \theta_0\right) \\
z &= (s + h) \sin\phi \\
\dot{x} &= v \left\{ (\sin\gamma \cos\phi - \cos\gamma \cos A \sin\phi) \cos\left(\theta - \theta_0\right) \right. \\
&\qquad \left. - \cos\gamma \sin A \sin\left(\theta - \theta_0\right) \right\} \\
\dot{y} &= v \left\{ (\sin\gamma \cos\phi - \cos\gamma \cos A \sin\phi) \sin\left(\theta - \theta_0\right) \right. \\
&\qquad \left. + \cos\gamma \sin A \cos\left(\theta - \theta_0\right) \right\} \\
\dot{z} &= v \left\{ \sin\gamma \sin\phi + \cos\gamma \cos A \cos\phi \right\}
\end{aligned}
\tag{H.3}
$$

These equations include the effect of the rotation of the earth. The longitude of the vernal equinox $\left(\theta_0\right)$ at launch time is computed by the program from Newcomb's formula.

I. TRANSFORMATION EQUATIONS FOR RADAR SIMULATION

The program computes sight angles (in an azimuth-elevation system), slant range and range rate data for up to 30 radar stations. The vehicle coordinates are transformed from a system of geocentric cartesian coordinates (xyz), the x-axis in the direction of the vernal equinox and the x-y plane in the equatorial plane of the earth to the required topocentric azimuth elevation system. This is accomplished by a series of coordinate transformations as follows:

1. A rotation of the coordinate system about the z-axis through an angle RA_s so that x-y plane is in the meridian plane of the station.

$$x' = x \cos RA_s + y \sin RA_s$$

$$y' = -x \sin RA_s + y \cos RA_s \qquad (I.1)$$

$$z' = z$$

The velocity transformation must take the rotational velocity of the new coordinate system into account.

$$\dot{x}' = y' \omega_e + \dot{x} \cos RA_s + \dot{y} \sin RA_s$$

$$\dot{y}' = -x' \omega_e - \dot{x} \sin RA_s + \dot{y} \cos RA_s$$

$$\dot{z}' = \dot{z}$$

where x', y', z' are the rotated coordinates and RA_s is the right ascension of the station and ω_e is the sidereal rate of the earth's rotation. The G.H.A. necessary to obtain RA_s from the station longitude is computed by the program.

2. A translation of the origin of the coordinate system from the center of the earth to the station in question

$$x'' = x' - (c + h) \cos \phi$$

$$y'' = y'$$

$$z'' = z' - (s + h) \sin \phi \qquad (I.2)$$

where

$$c = \left(1 - e^2 \sin^2 \phi\right)^{-1/2}$$

$$s = \left(1 - e^2\right) c$$

$$\dot{x}'' = \dot{x}' \; ; \quad \dot{y}'' = \dot{y}' \; ; \quad \dot{z}'' = \dot{z}'$$

where x'', y'', z'' are the translated coordinates. ϕ is the geodetic latitude and h the height above sea level of the station in question.

3. A rotation of $(90 - \phi)$ about the y'' axis to place the (x'', z'')

plane into the horizon plane

$$
\begin{aligned}
x''' &= x'' \sin\phi + z'' \cos\phi \\
y''' &= y'' \\
z''' &= - x'' \cos\phi + z'' \sin\phi \\
\dot{x}''' &= \dot{x}'' \sin\phi + \dot{z}'' \cos\phi \\
\dot{y}''' &= \dot{y}'' \\
\dot{z}''' &= -\dot{x}'' \cos\phi + \dot{z}'' \sin\phi
\end{aligned}
\qquad (I.3)
$$

Now x''', y''', z''' are the coordinates of the vehicle in a topocentric azimuth elevation system, with z''' axis pointing to zenith and the x''' pointing south along the meridian. Range, range rate, azimuth and elevation are then given by

$$\rho = \left(x'''^2 + y'''^2 + z'''^2\right)^{1/2} = \text{Slant range} \qquad (I.4)$$

$$\dot{\rho} = \frac{x''' \dot{x}''' + y''' \dot{y}''' + z''' \dot{z}'''}{\rho} = \text{Range rate} \qquad (I.5)$$

$$E = \tan^{-1} \frac{z'''}{\left(x'''^2 + y'''^2\right)^{1/2}} = \text{Elevation} \qquad (I.6)$$

$$A' = \tan^{-1}\frac{y'''}{x'''}$$

$$A = \begin{cases} \pi - A' & A' < \pi \\ 3\pi - A' & A' > \pi \end{cases} \qquad (I.7)$$

J. TRIAXIAL MOON

Triaxial lunar potential constants (as used in the ITEM Program)

1. The values of the constants A, B and C for the perturbation accelerations due to the triaxial moon may be calculated using data from the NASA earth model meeting. These constants are currently being used in the ITEM Program.

2. The perturbation accelerations due to the triaxial moon are given by the partial derivatives of

$$\phi = \frac{C}{r^3}\left\{A\left(1 - \frac{3z^2}{r^2}\right) + B\left(1 - 3\frac{x^2}{r^2}\right)\right\} \tag{J.1}$$

where

$$C = \frac{\mu_m a_m^2}{3 a_e^2}\left(\frac{3 I_C}{2 m a_m^2}\right)$$

$$A = \frac{I_C - I_A}{I_C} \tag{J.2}$$

$$B = \frac{I_B - I_A}{I_C}$$

The form of the equations used in the ITEM Program are:

$$\frac{\partial \phi}{\partial x} = \frac{3xC}{r^5} F$$

$$\frac{\partial \phi}{\partial y} = \frac{3yC}{r^5} F - \frac{6CBy}{r^5} \qquad (J.3)$$

$$\frac{\partial \phi}{\partial z} = \frac{3zC}{r^5} F - \frac{6ACz}{r^5}$$

where

$$F = \left\{ A\left(\frac{5z^2}{r^2} - 1\right) + B\left(\frac{5y^2}{r^2} - 1\right) \right\}$$

Based on the NASA earth model meeting, the moments of inertia about the principal axes of the moon are:

$$I_A = .88746 \times 10^{35} \text{ kg meters}^2$$

$$I_B = .88764 \times 10^{35} \text{ kg meters}^2$$

$$I_C = .88801 \times 10^{35} \text{ kg meters}^2$$

(See Figure 3)

Other constants are:

$$\mu_e = 19.9094165 \times \frac{(ER)^3}{m^2}$$

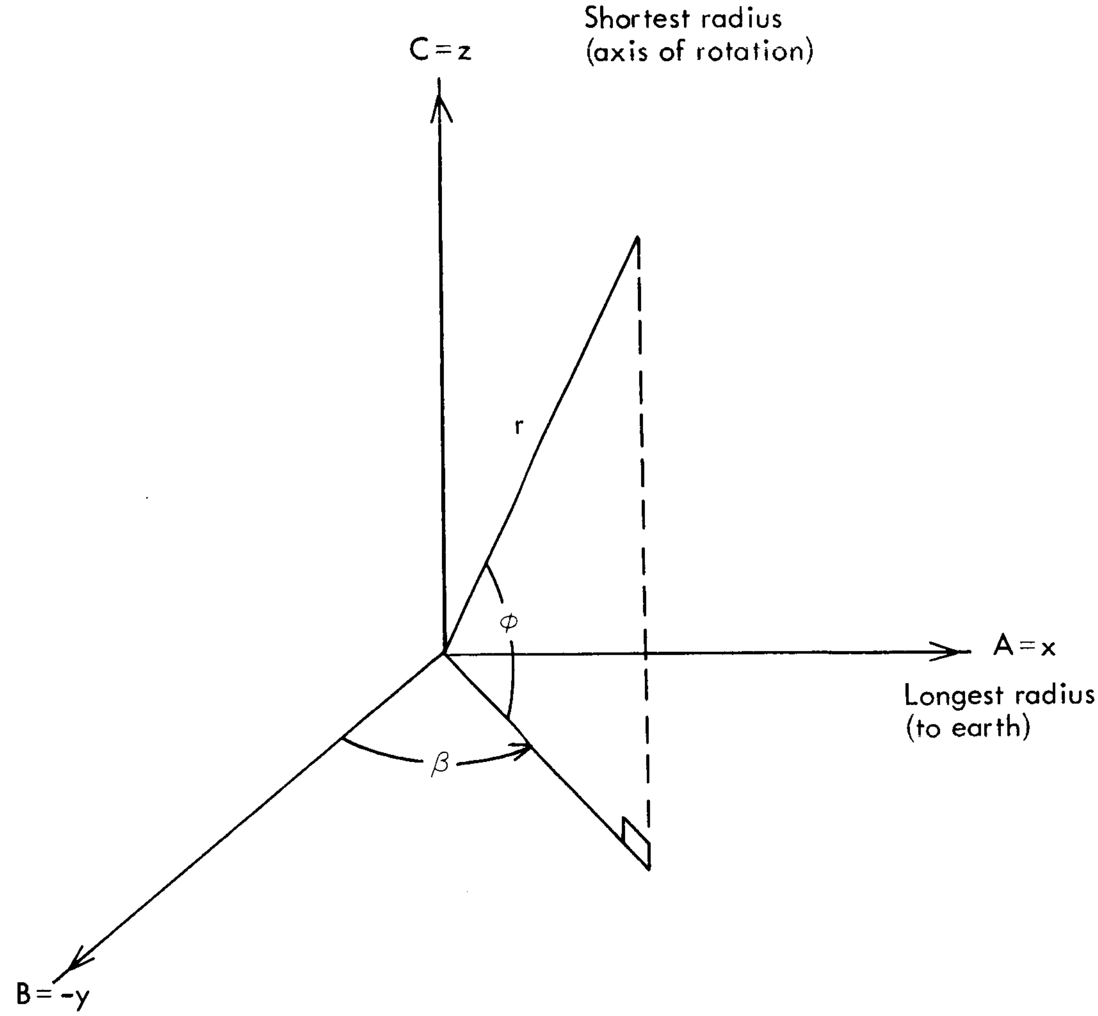

Figure 3 - Triaxial Moon

$$\mu_m = \frac{\mu_e}{81.335} = .2447829 \times \frac{(ER)^3}{m^2}$$

$$\frac{(a_m)^2}{(a_\epsilon)} = \frac{(1738.09)^2}{(6378.165)} = .0742595 \text{ (earth radii)}^2$$

$$m_e = 5.975 \times 10^{27} \text{ grams}$$

$$m = \frac{m_e}{81.335} = \frac{5.975 \times 10^{24}}{81.335} = 7.34616 \times 10^{22} \text{ kg}$$

$$a_m^2 = (1738.09)^2 = 3.0209568 \times 10^6 \text{ km}^2$$

The constants A, B and C may be calculated

$$A = \frac{I_C - I_A}{I_C} = \frac{(.88801-.88746)}{.88801} = 619.36 \times 10^{-6}$$

$$B = \frac{I_B - I_A}{I_C} = \frac{(.88764-.88746)}{.88801} = 202.70 \times 10^{-6}$$

$$C = \left(\frac{\mu_m a_m^2}{3 a_e^2}\right)\left(\frac{3 I_C}{2m a_m^2}\right)$$

For the ITEM Program the units of C are

$$\frac{(\text{earth radii})^5}{m^2}$$

therefore

$$C = \left\{\frac{\mu_m}{3}\left(\frac{a_M}{a_e}\right)^2\right\} \cdot \left(\frac{3 I_C}{2ma_m^2}\right)$$

$$= \left\{\frac{(.2447829)(.0742595)}{3}\right\}\left\{\frac{3(.88801) \times 10^{29}}{2(7.34616 \times 10^{22})(3.0209568 \times 10^8)}\right\}$$

$$C = 36.366998 \times 10^{-4} \text{ earth mass} \times (\text{earth radius})^2$$

In summary the constants used in the ITEM Program as based upon the NASA earth model meeting are:

$$A = 619.36 \times 10^{-6}$$

$$B = 202.70 \times 10^{-6}$$

$$C = 36.366998 \times 10^{-4} \text{ earth mass} \times (\text{earth radius})^2$$

K. DRAG COMPUTATION

The drag acceleration is computed, assuming a spherically symmetric atmosphere rotating with the earth. Thus:

$$D = -\frac{1}{2}\rho A \left|V_{eff}\right| V_{eff}$$

$$\Delta\ddot{R}_D = \frac{D}{m} \tag{K.1}$$

where

$$V_{eff} = \dot{R} - \omega \times R$$

ω is the sidereal rotation rate vector of the earth.

L. COMPUTATION OF SUBSATELLITE POINT

The geodetic coordinates of the subsatellite point are computed by the following method:

The geocentric latitude (declination) is obtained from

$$\sin \phi' = \frac{z}{r} \tag{L.1}$$

This latitude is then corrected to geodetic latitude by the formula

$$\phi = \phi' + a_2 \sin 2\phi' + a_4 \sin 4\phi' + a_6 \sin 6\phi' + a_8 \sin 8\phi' \tag{L.2}$$

where

$$a_2 = \frac{1}{1024r}\left\{512e^2 + 128e^4 + 60e^6 + 35e^8\right\}$$

$$+ \frac{1}{32r^2}\left\{e^6 + e^8\right\} - \frac{3}{256r^3}\left\{4e^6 + 3e^8\right\}$$

$$a_4 = -\frac{1}{1024r}\left\{64e^4 + 48e^6 + 35e^8\right\}$$

$$+ \frac{1}{16r^2}\left\{4e^4 + 2e^6 + e^8\right\} + \frac{15e^8}{256r^3} - \frac{e^8}{16r^4}$$

$$a_6 = \frac{3}{1024r}\left\{4e^6 + 5e^8\right\} - \frac{3}{32r^2}\left\{e^6 + e^8\right\}$$

$$+ \frac{35}{768r^3}\left\{4e^6 + 3e^8\right\} \tag{L.3}$$

$$a_8 = \frac{e^8}{2048}\left\{-\frac{5}{r}+\frac{64}{r^2}-\frac{252}{r^3}+\frac{320}{r^4}\right\}$$

e = the eccentricity of the earth

r = the distance from earth's center

See Reference 5.

The geodetic height is then given by

$$h = r\cos(\phi-\phi') - \sqrt{1-e^2\sin^2\phi} \tag{L.4}$$

The longitude is obtained by subtracting the sidereal time of Greenwich from the right ascension given by

$$\tan RA = \frac{y}{x} \tag{L.5}$$

M. POLAR COORDINATES REFERRED TO THE MOON

Moon longitude and latitude are defined in the coordinate system described in Appendix J. If the vehicle coordinates with respect to the center of the moon in this coordinate system are given by x, y, z, r, then

$$\theta = \text{longitude} = \tan^{-1} \frac{y}{x}$$

$$\varphi = \text{latitude} = \sin^{-1} \frac{z}{r}$$

N. SHADOW LOGIC

A coordinate system is set up in the plane defined by the centers of the light-emitting source, the shadowing body, and the probe. Both bodies are assumed to be spherical, and hence all testing can be carried out in this plane. The diagram in Figure 4 shows this plane.

The coordinates are defined by unit vectors i and j :

$$\underline{i} = \frac{\underline{R}_{c\ell}}{r_{c\ell}} \quad ; \quad \underline{i} \cdot \underline{i} = 1 \quad ; \quad \underline{i} \cdot \underline{j} = 0 \tag{N. 1}$$

$$\ell \underline{j} = - d \underline{i} + \underline{R}_{vc} \quad ; \quad \underline{j} \cdot \underline{j} = 1 \tag{N. 2}$$

where

$$d = \underline{R}_{vc} \cdot \underline{i}$$

Vehicle coordinates in this system are given by:

$$x_v = \underline{R}_{vc} \cdot \underline{i} = d \tag{N. 3}$$

$$y_v = \underline{R}_{vc} \cdot \underline{j} = [-d^2 + r_{vc}^{\ 2}]^{1/2} \tag{N. 4}$$

$$z_v = \underline{R}_{vc} \cdot \underline{K} = 0 \tag{N. 5}$$

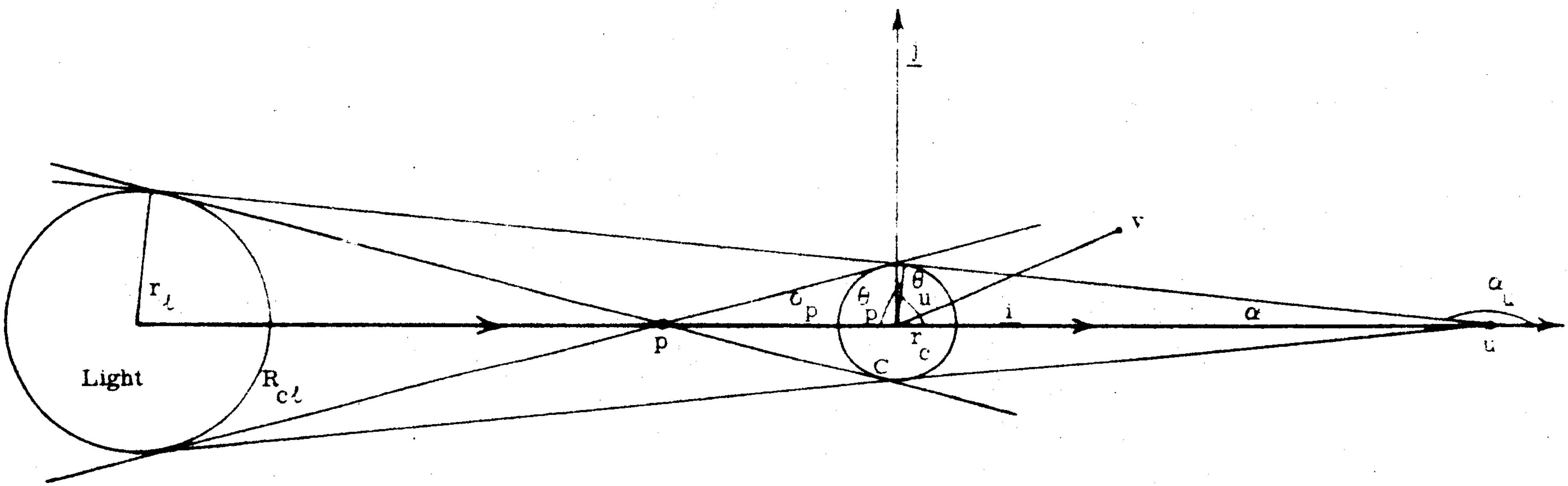

Figure 4 - Diagram for Shadow Logic

1. Shadow Parameters

a) The tips of the umbra and penumbra cones are:

$$d_u = \frac{r_{c\ell}}{\dfrac{r_\ell}{r_c} - 1} \quad , \quad d_p = - \frac{r_{c\ell}}{\dfrac{r_\ell}{r_c} + 1}$$

b) The slopes of the bounding lines are:

$$\sin \alpha_u = \cos \theta_u = \frac{r_c}{d_u}$$

$$- \cos \alpha_u = \sin \theta_u = \left[1 - \left(\frac{r_c}{d_u}\right)^2\right]^{\frac{1}{2}}$$

$$\tan \alpha_u = \frac{\sin \alpha_u}{\cos \alpha_u}$$

$$\tan \theta_u = \frac{\sin \theta_u}{\cos \theta_u}$$

$$\cos \theta_p = \sin \alpha_p = \frac{r_c}{d_p}$$

$$\sin \theta_p = \cos \alpha_p = \left[1 - \left(\frac{r_c}{d_p}\right)^2\right]^{\frac{1}{2}}$$

$$\tan \alpha_p = \frac{\sin \alpha_p}{\cos \alpha_p}$$

$$\tan \theta_p = \frac{\sin \theta_p}{\cos \theta_p}$$

c) Refraction Correction: (UMBRA)

$$\alpha'_u = \alpha_u - \epsilon \quad , \quad \theta'_u = \theta_u - \epsilon$$

$$\sin \alpha'_u = \sin \alpha_u \cos \epsilon - \cos \alpha_u \sin \epsilon$$

$$\tan \alpha'_u = \frac{\tan \alpha_u - \tan \epsilon}{1 + \tan \alpha_u \tan \epsilon}$$

$$\tan \theta'_u = \frac{\tan \theta_u - \tan \epsilon}{1 + \tan \theta_u \tan \epsilon}$$

$$d'_u = \frac{r_c}{\sin \alpha'_u}$$

d) Refraction Correction: (PENUMBRA)

Both $\epsilon < \alpha_p$ $\epsilon > \alpha_p$; $\alpha'_p = \alpha_p - \epsilon$

$$\sin \alpha'_p = | \sin \alpha_p \cos \epsilon - \cos \alpha_p \sin \epsilon |$$

$$\tan \alpha'_p = \frac{\tan \alpha_p - \tan \epsilon}{1 + \tan \alpha_p \tan \epsilon}$$

$$\tan \theta'_p = \frac{\tan \theta_p - \tan \epsilon}{1 + \tan \theta_p \tan \epsilon}$$

$$d'_p = - \operatorname{sign} (\tan \alpha'_p) \frac{r_c}{\sin \alpha'_p}$$

The equations of the bounding lines are given below.

2. <u>The Testing Procedure</u>

θ_p Line

$$Q_1 = \begin{cases} \dfrac{|y_v|}{\tan \theta'_p} - x_v \geq 0 & \text{Sunlight} \\[2ex] \dfrac{|y_v|}{\tan \theta'_p} - x_v < 0 & \text{Go to next test} \end{cases}$$

$$Q_2 = \begin{cases} |y_v| - (x_v - d'_p)\tan\alpha'_p > 0 & \text{Sunlight} \\ |y_v| - (x_v - d'_p)\tan\alpha'_p = 0 & \text{Sunlight penumbra boundary} \\ \qquad\qquad\qquad\qquad\quad < 0 & \text{Go to next test} \end{cases}$$

If $R_\ell = 0$, exit here.

$$Q_3 = \frac{|y_v|}{\tan\theta'_u} - x_v \quad \begin{cases} \geq 0 & \text{Penumbra} \\ < 0 & \text{Go to next test} \end{cases}$$

$$Q_4 = |y_v| - (x_v - d'_u)\tan\alpha'_u \quad \begin{cases} > 0 & \text{Penumbra} \\ = 0 & \text{Shadow penumbra boundary} \\ < 0 & \text{Shadow} \end{cases}$$

Q_2 and Q_4 are stored and saved. The crossing times are found by linearly interpolating for 0-values of Q_2 and Q_4 respectively, to guarantee that crossing from one region into another always occurs across these boundaries.

O. SOLAR RADIATION PRESSURE

The radiation pressure subroutine computes the force of solar radiation on the spacecraft if an appropriate pressure coefficient is used. The calculation relies on the shadow routine to set a trigger to multiply the pressure coefficient by 1.0, 0.5, or 0.0 for full sunlight, penumbra, and umbra, respectively. Therefore, the shadow subroutine must be used in conjunction with the radiation pressure routine for most cases. If the spacecraft is known to be continually in sunlight, the number 1.0 may be loaded into SHDN and thus elaborate shadow testing may be avoided.

$$P_{RP} = \frac{C_R A R_{VS}}{m r_{VS}^3} \qquad \text{(O. 1)}$$

(See Section VIII-A for definition of symbols.)

This radiation pressure subroutine has been found to be inexact for satellites of large area-to-mass ratio since it only controls the pressure to the nearest integration step. For such spacecraft (e.g., balloons), several degrees error in true anomaly may result after 100 days unless the integration is carried exactly to the boundaries. A modification to achieve this increased precision is available and will be included in future versions of the program.

P. ECLIPTIC COORDINATES

The ecliptic coordinates are an approximate set obtained by a simple rotation of the equatorial coordinates about the x-axis through a fixed angle i = 23°26'31" which is approximately the true obliquity for Jan 0.0, 1962. More exact coordinates may be obtained by changing NE, (unit normal to the ecliptic) as desired.

Q. MOON ROTATING AND FIXED COORDINATE SYSTEM

Geocentric coordinates of the vehicle based on the earth-moon plane are generated from the geocentric equatorial radius vector to the vehicle, R_{VE}, the geocentric unit vector in the direction of the moon $\hat{R}_{ME}$, and the vector in the direction of the moon's velocity, $\dot{R}_{ME}$.

Coordinates in the rotating system, XROT etc., are found by using the current values of these vectors at each time step in the relations

$$
\begin{aligned}
XROT &= R_{VE} \cdot \hat{R}_{ME} \\
YROT &= R_{VE} \cdot \left(\hat{H}_{ME} \times \hat{R}_{ME} \right) \qquad \text{(Q.1)} \\
ZROT &= R_{VE} \cdot \hat{H}_{ME}
\end{aligned}
$$

where

$$
\hat{H}_{ME} = \frac{\hat{R}_{ME} \times \dot{R}_{ME}}{\left| \hat{R}_{ME} \times \dot{R}_{ME} \right|}
$$

For the fixed axis system XINJ, etc., the initial vectors $\hat{R}_{ME}(t_0)$ and $\dot{R}_{ME}(t_0)$ at the time of injection are used with the current value of R_{VE}.

R. TRAJECTORY SEARCH

The program provides a search routine to obtain selected trajectories. The search is based on linear theory and varies the polar load input quantities (independent variables) to search for desired dependent variables. There are twelve possible dependent variables to select from, although a maximum of seven of the twelve may be used in any given search. The quantities, at present, are

$$i\,,\ \Omega\,,\ \omega\,,\ t_p\ \text{(pericenter time)},\ \ \text{and}$$

$$r_p\ \text{(pericenter radius)}$$

The above five variables at the moon and at the earth (in the case of earth return trajectories) constitute the first ten variables. (These quantities are normally referred to the equatorial plane. However, the earth-moon plane is also available.

In addition, the components of the impact parameter vector ($B \cdot T$, $B \cdot R$) may be selected. They are referred to the ecliptic plane for Mars and Venus trajectories and the moon's orbital plane for lunar trajectories. The number of independent variables must equal the number of dependent variables for this routine to operate.

This search routine is time consuming if the initial conditions are poorly approximated. Before using this routine, two things should be done.

1. A first guess of the initial conditions of the nominal trajectory should be obtained from a patched conic or a similar search program.

2. The number of variables should be kept to a minimum. It is planned to automate the iteration scheme to go from two-body, to patched conic, to full trajectory, and to increase the number of variables to be adjusted, in optimal fashion. Even in its present form, however, it is extremely useful.

The iterator uses a modified version of the MIN-MAX Principle (Reference 6).

A_{ij} is the matrix of partials

Δx_i is the vector of changes in the independent variables

λ_{ii} is a diagonal matrix of weights

y_i is a vector of residuals

The system to be solved is

$$(A_{ji} A_{ij} + \lambda_{ii}) \Delta x_i = A_{ji} y_i$$

$$A_{ji} A_{ij} + \lambda_{ii} = B_{ij}$$

$$A_{ji} y_i = z_i$$

Procedure

The system

$$B_{ij}\,\Delta x_i = z_i$$

is solved. If the value of Δx_i is greater than SIZER, some arbitrary amount, set

$$B_{ij} = B_{ij} + \lambda_{ii}$$

and solve the system again. Repeat these operations until Δx_i is less than or equal to SIZER. Now, run a new nominal trajectory with the new independent variable

$$x_i = x_i + \Delta x_i$$

a) If the new residuals y_i are greater than the previous ones, set

$$B_{ij} = B_{ij} + \lambda_{ii}$$

Solve the system again and continue solving until the new residuals are less than the old. Now the system is ready for a new iteration.

b) If the new residuals y_i are less than the old, set

$$B_{ij} = B_{ij} - \lambda_{ii}$$

Solve the system again and continue solving until the new residuals are greater or equal to the old.

The iteration continues until either the maximum number of iterations (input) is exceeded or the residuals are less than or equal to an input tolerance.

S. EQUATIONS FOR FLIGHT PATH AZIMUTH AND FLIGHT PATH ANGLE

A subroutine computes the flight path azimuth and flight path angle with the following equations:

1. Flight path angle

$$\gamma = \sin^{-1}\left[\frac{\dot{R}}{V} \cdot \hat{N}\right] \tag{S.1}$$

$\hat{N}$ is the vertical unit vector. In the geodetic system $\hat{N}$ is given by

$$\hat{N} = \left[\cos\phi \cos\left(\theta - \theta_0\right), \cos\phi \sin\left(\theta - \theta_0\right), \sin\phi\right]$$

In the geocentric system ϕ is replaced by ϕ'. Alternatively, in the latter system

$$\hat{N} = \frac{R}{r}$$

2. Flight path azimuth

$$A = \sin^{-1}\left[\frac{1}{\cos\gamma}\left\{\frac{\dot{y}}{v}\cos\left(\theta - \theta_0\right) - \frac{\dot{x}}{v}\sin\left(\theta - \theta_0\right)\right\}\right]$$

$$A = \cos^{-1}\left[\frac{1}{\cos\gamma\cos\phi}\left\{\frac{\dot{z}}{v} - \sin\gamma\sin\phi\right\}\right] \tag{S.2}$$

Both formulas are used to determine the proper quadrant of **A**. To obtain the geocentric output, $e^2 = 0$, ϕ is replaced by declination $\delta = \phi'$.

T. OSCULATING ELEMENTS

The osculating elements are obtained from the following equations:

$$a = \left(\frac{2}{r} - \frac{v^2}{\mu}\right)^{-1} \tag{T.1}$$

$$n = \mu^{1/2}|a|^{-3/2} \tag{T.2}$$

$$\left.\begin{matrix} e \cos E \\ e \cosh E \end{matrix}\right\} = 1 - \frac{r}{a} \tag{T.3}$$

$$\left.\begin{matrix} e \sin E \\ e \sinh E \end{matrix}\right\} = \frac{d}{\sqrt{|\mu a|}} \tag{T.4}$$

$$M = \begin{cases} E - e \sin E \\ e \sinh E - E \end{cases} \tag{T.5}$$

$$t_p = t - \frac{M}{n} \tag{T.6}$$

The angles Ω, ω, i are obtained from the vectors H and $\hat{P}$, where

$$H = R \times \dot{R} \tag{T.7}$$

$$eP = \left(\frac{1}{r} - \frac{1}{a}\right) R - \frac{d}{\mu}\dot{R} \tag{T.8}$$

In terms of these vectors:

$$\cos i = \frac{H_z}{h} \quad \text{in the first or fourth quadrant} \tag{T.9}$$

$$\sin \Omega = \frac{H_x}{h \sin i}$$

$$\cos \Omega = \frac{-H_y}{h \sin i} \tag{T.10}$$

$$\cos \omega = P_x \cos \Omega + P_y \sin \Omega$$

$$\sin \omega = \frac{P_z}{\sin i} \tag{T.11}$$

U. IMPACT PARAMETERS

The "impact parameters" are coordinates in the "impact" plane. This plane passes through the body (planet or the moon) and is normal to the incoming asymptote. The direction cosines of the asymptote are given by equations (U. 1, U. 2) in terms of unit vectors $\hat{P}$ (Appendix T) and

$$\hat{Q} = \frac{H}{h} \times \hat{P} \tag{U. 1}$$

$$\hat{S} = \frac{1}{e}\left[\hat{P} + \sqrt{(e^2 - 1)}\ \hat{Q}\right] \tag{U. 2}$$

In the plane defined by $\hat{S}$ as its normal, two unit vectors $\hat{T}_{IMP}$ and $\hat{R}_{IMP}$ are defined. $\hat{T}_{IMP}$ is parallel to the ecliptic plane for Mars and Venus impacts, and to the moon's orbital plane for moon impacts. Explicitly

$$\hat{T}_{IMP} = \frac{\hat{N} \times \hat{S}}{|\hat{N} \times \hat{S}|} \tag{U. 3}$$

where $\hat{N}$ is the unit normal to the ecliptic plane, or the moon's orbital plane. $\hat{R}_{IMP}$ is normal to both $\hat{S}$ and $\hat{T}_{IMP}$. B_{IMP} is the vector from the body to the vehicle as it crosses the impact plane. The data computed are the dot products

$$B_{IMP} \cdot \hat{T}_{IMP}$$

and

$$B_{IMP} \cdot \hat{R}_{IMP}$$

V. MOON'S ORBITAL PLANE INPUT AND OUTPUT

A polar coordinate system is available for input and output which uses as its reference plane the moon's orbital plane and the vector from moon to earth as unit vector. Polar coordinates in this system are defined analogous to geocentric polar coordinates. The cartesian coordinates in this system are computed by equations (H.3) with

$$c = s = r_B$$

and

$$\theta_0 = 0$$

Here r_B is the radius of the body of departure (earth or moon).

These coordinates are then transformed to equatorial coordinates by a matrix C computed as follows:

$$\hat{i} = \frac{R_{EM}}{r_{EM}}$$

$$\hat{k} = \frac{R_{EM} \times \dot{R}_{EM}}{\left| R_{EM} \times \dot{R}_{EM} \right|} \tag{V.1}$$

$$\hat{j} = \hat{k} \times \hat{i}$$

The transformation matrix C is then given by

$$C = \begin{pmatrix} i_x & j_x & k_x \\ i_y & j_y & k_y \\ i_z & j_z & k_z \end{pmatrix} \qquad (V.2)$$

and

$$\begin{aligned} R &= C R_{MOP} \\ \dot{R} &= C \dot{R}_{MOP} \end{aligned} \qquad (V.3)$$

The matrix C is unitary, and $C^{-1} = C^*$, permitting easy inversion of equations (V.2).

W. EQUATIONS FOR TRANSLUNAR PLANE INPUT

The translunar plane input is designed to permit easy visualization of the geometric relationships between initial conditions for circumlunar trajectories and the motion of the moon.

The initial conditions are given in a coordinate system referred to the translunar plane. This system has its x axis along the ascending node of the vehicle with respect to the moon's orbital plane, its y axis in the translunar plane at right angles to the ascending node, in the direction of motion. In this coordinate system, initial position and velocity vectors are given by

$$x_{TL} = (r_B + h) \cos \Psi$$
$$y_{TL} = (r_B + h) \sin \Psi \qquad (W.1)$$
$$z_{TL} = 0$$

Here r_B is the radius of the body of departure (earth or moon).

$$\dot{x}_{TL} = v \sin (\gamma - \Psi)$$
$$\dot{y}_{TL} = v \cos (\gamma - \Psi) \qquad (W.2)$$
$$\dot{z}_{TL} = 0$$

The translunar plane is positioned by giving its inclination i_{TL} with respect to the moon's orbital plane and the lunar lead angle φ, the angle between the moon's position at injection and the descending node. The vectors R_{TL} and $\dot{R}_{TL}$ may then be transformed into the equatorial system by the following series of rotations:

1. A rotation $-i_{TL}$ about the x_{TL} axis will rotate the translunar plane into the moon's orbital plane.

2. A rotation of $\pi - (\lambda_M + \phi)$ about the new z-axis will refer the moon's orbital plane coordinate system to the ascending node of the moon's orbital plane (with respect to the equator) as x-axis.

Here λ_M stands for the argument of latitude of the moon. These rotations are performed by multiplying R_{TL} and $\dot{R}_{TL}$ by the matrix:

$$A = \begin{pmatrix} -\cos(\lambda_M + \phi) & \sin(\lambda_M + \phi) & -\sin(\lambda_M + \phi)\sin i_{TL} \\ -\sin(\lambda_M + \phi) & -\cos(\lambda_M + \phi)\cos i_{TL} & \cos(\lambda_M + \phi)\sin i_{TL} \\ 0 & \sin i_{TL} & \cos i_{TL} \end{pmatrix} \qquad \text{(W-3)}$$

3. The moon's orbital plane (MOP) is rotated about its node through an angle $-i_M$ (the inclination of the MOP).

4. The ascending node is brought into coincidence with the vernal equinox by a rotation $-\Omega_M$. These two rotations are embodied in the matrix

$$B = \begin{pmatrix} \cos\Omega_M & -\sin\Omega_M \cos i_M & \sin\Omega_M \sin i_M \\ \sin\Omega_M & \cos\Omega_M \cos i_M & -\cos\Omega_M \sin i_M \\ 0 & \sin i_M & \cos i_M \end{pmatrix} \qquad \text{(W-4)}$$

and thus:

$$\mathbf{R} = (\mathrm{BA})\,\mathbf{R}_{TL}$$

$$\dot{\mathbf{R}} = (\mathrm{BA})\,\dot{\mathbf{R}}_{TL} \qquad (W.5)$$

Y. CHANGE OF INDEPENDENT VARIABLE - BETA MODE

According to the standard Encke method, we introduce a differential equation

$$\ddot{\rho} = -\mu \frac{\rho}{|\rho|^3} \tag{Y.1}$$

In the construction of the closed-form solution for (Y. 1), a parameter β arises. It is related to t by Kepler's equation,

$$t = t_o + \frac{f(\beta)}{\sqrt{\mu}} \tag{Y.2}$$

where f is a transcendental function of β and is obtained by summing several power series.

If t is taken as the independent variable, Equation (Y. 2) has to be solved for β by an iterative method, requiring numerous time-consuming evaluations of the function f for each integration step. Using β as the independent variable, however, only requires a single evaluation.

It remains, of course, to see what becomes of Equation (Y. 1) and

$$\begin{aligned} \ddot{\xi} &= \ddot{x} - \ddot{\rho} \\ &= -\mu\left(\frac{x}{|x|^3} - \frac{\rho}{|\rho|^3}\right) + F \end{aligned} \tag{Y.3}$$

if β is the independent variable. We have, from Kepler's equation, that

$$\frac{dt}{d\beta} = \frac{|\rho|}{\sqrt{\mu}} \tag{Y.4}$$

at any point along the solution of (Y.1). Thus

$$\dot{\rho} = \rho' \frac{\sqrt{\mu}}{|\rho|} \quad \text{and} \quad \rho' = \dot{\rho} \frac{|\rho|}{\sqrt{\mu}}$$

at any point along the solution of (Y.1) and the initial conditions become

$$\rho(\beta_o) = x_o \quad \text{and} \quad \rho'(\beta_o) = \frac{\dot{x}_o |x_o|}{\sqrt{\mu}} .$$

when

$$\beta_o = \beta(t_o) = 0$$

Now the solution for (Y.1), ρ and ρ', can be written in closed form for any β. As auxiliary quantities in this solution, we have $|\rho|$ and $D = \dfrac{\rho \cdot \rho'}{|\rho|}$. They are computed as functions of β before ρ and ρ' are known; that is, with accuracy at least as good as that of ρ and ρ'. Not only are they needed and easy to compute, but they also have the interesting property that

$$\frac{dt}{d\beta} = \frac{|\rho|}{\sqrt{\mu}}$$

and (Y.5)

$$\frac{d^2t}{d\beta^2} = \frac{D}{\sqrt{\mu}}$$

Thus Equation (Y.1) is solved more economically in terms of β than in terms of t.

Now we turn to Equation (Y.3). To treat it, we want to express ξ'' in terms of $\ddot{\xi}$. From (Y.5) we have that

$$\xi' = \dot{\xi}\frac{dt}{d\beta} = \dot{\xi}\frac{|\rho|}{\sqrt{\mu}} \tag{Y.6}$$

Differentiating with respect to β,

$$\begin{aligned}
\xi'' &= \frac{d\dot{\xi}}{d\beta}\frac{|\rho|}{\sqrt{\mu}} + \dot{\xi}\frac{d}{d\beta}\left(\frac{|\rho|}{\sqrt{\mu}}\right) \\
&= \ddot{\xi}\left(\frac{|\rho|}{\sqrt{\mu}}\right)^2 + \xi'\frac{\sqrt{\mu}}{|\rho|}\frac{D}{\sqrt{\mu}} \\
&= \ddot{\xi}\frac{|\rho|^2}{\mu} + \xi'\frac{D}{|\rho|} \\
&= -|\rho|^2\left(\frac{x}{|x|^3} - \frac{\rho}{|\rho|^3}\right) + \frac{|\rho|^2}{\mu}F + \xi'\frac{D}{|\rho|}
\end{aligned} \tag{Y.7}$$

Thus (Y.7) is the equation to be integrated numerically, instead of (Y.3). The coefficients $\frac{|\rho|^2}{\mu}$ and $\frac{D}{|\rho|}$ can be calculated with much more accuracy than the factors involving ξ, since they depend only on the two-body solution. For analysis of error propagation, we write (Y.7) as

$$\xi'' = -\frac{1}{|\rho|}\left[(\rho+\xi)\frac{|\rho|^3}{|\rho+\xi|^3} - \rho\right] + \frac{|\rho|^2}{\mu}F + \xi'\frac{D}{|\rho|} \tag{Y.8}$$

The mechanics of the procedure, then, are easy to enumerate. The initial conditions are x_o and $\dot{x}_o$. Let

$$\rho(t_o) = x_o$$

$$\rho'(t_o) = \frac{\dot{x}_o |x_o|}{\sqrt{\mu}} \tag{Y.9}$$

Using these initial conditions, evaluate $t, \frac{|\rho|^2}{\mu}, \frac{D}{|\rho|}, \rho, \rho'$ for each value of β to be considered.

Let $\xi_o = \xi'_o = 0$. Using these initial conditions, integrate Equation (Y.7) to get $\xi(\beta)$ and $\xi'(\beta)$. Note that the first two terms on the right-hand side of Equation (Y.7) are functions of x and possibly x'. These are obtained by

$$x(\beta) = \rho(\beta) + \xi(\beta)$$

$$x'(\beta) = \rho'(\beta) + \xi'(\beta) \tag{Y.10}$$

If at any point $\dot{x}$ is required, it can be found from

$$\dot{x}[t(\beta)] = x'(\beta) \frac{\sqrt{\mu}}{|\rho(\beta)|} \tag{Y.11}$$

Depending on the rectification control logic, there will be places where the solution to Equation (Y.1) must be started over. At this point, the values $t, x, \dot{x}$ become the new t_o, x_o, and $\dot{x}_o$, while β, ξ, and ξ' are reset to zero.

Comparison of Modes

a) It is immediately apparent that eliminating the necessity of iteratively solving Equation (Y. 2) will substantially increase the speed of computation.

b) An important advantage arises further from eliminating the sometimes ponderous logic which supplies initial guesses for the iterative process and guarantees convergence of the solution.

c) A third advantage of the β-method is not quite so apparent, but no less important. It is well known that the size of the integration time step can be increased as the distance from the center of attraction increases. This change of the integration interval requires a cumbersome restart procedure. An examination of Equation (Y. 5) shows that equal intervals of β correspond to time intervals of increasing lengths as the distance increases. The time interval thus automatically expands and contracts correctly without outside intervention.

d) Geometric stopping and printing conditions can usually be conveniently expressed in terms of β, whereas they often require iterative determinations of the time. This advantage, however, is slight.

e) If state vectors are required at fixed times, an iteration is necessary to find the corresponding value of β. In this case the β-method is no better, and no worse, than the standard methods. If such vectors are required at frequent, closely spaced, time points (as in orbit determination, for instance), the advantage of the β-method is marginal.