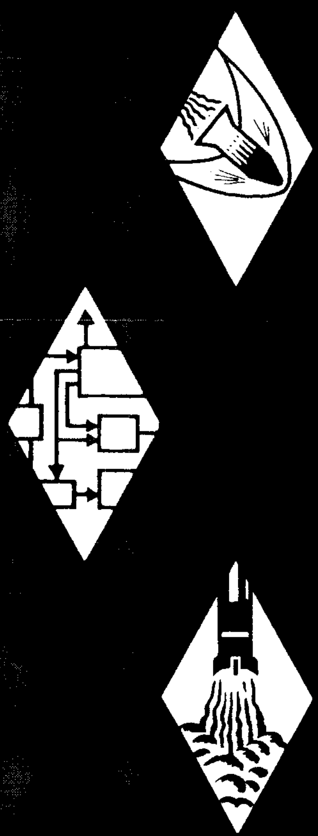
AEROSPACE RESEARCH • AERODYNAMICS • PROPULSION • STRUCTURAL DYNAMICS • ELECTRONIC SYSTEMS AND INSTRUMENTS • COMPUTER MODULES

NY



RESEARCH
ENGINEERING
PRODUCTION

RADIATION FROM CARBON IN A ROCKET PLUME MIXING REGION WITH COUPLED CONVECTIVE & RADIATIVE ENERGY FLUXES & GENERAL OPTICAL THICKNESS

TECHNICAL REPORT NO. 628

FACILITY FORM

N67-31152	
(ACCESSION NUMBER)	(THRU)
42	1
(PAGES)	(CODE)
CR-85835	28
(NASA CR OR TMX OR AD NUMBER)	(CATEGORY)

GPO PRICE $ ____________

CFSTI PRICE(S) $ ____________

Hard copy (HC) 3.00

Microfiche (MF) .65

ff 653 July 65

S67-43710

GENERAL APPLIED SCIENCE LABORATORIES, INC.
MERRICK and STEWART AVENUES, WESTBURY, L.I., N.Y. · (516) ED 3-6960

Job No. 8043
Contract No. NAS8-20298

Copy (24) of (30)
Total No. Pages i & 34

RADIATION FROM CARBON IN A ROCKET PLUME MIXING REGION WITH COUPLED CONVECTIVE & RADIATIVE ENERGY FLUXES & GENERAL OPTICAL THICKNESS

TECHNICAL REPORT NO. 628

by: S. Slutsky
J. D. Melnick

Prepared for

NATIONAL AERONAUTICS & SPACE ADMINISTRATION
GEORGE C. MARSHALL SPACE FLIGHT CENTER
HUNTSVILLE, ALABAMA

Prepared by

General Applied Science Laboratories, Inc.
Merrick and Stewart Avenues
Westbury, L.I., New York

September 1966

Approved by: Antonio Ferri /SS
Antonio Ferri
President

ABSTRACT

An analysis is presented of the coupled aerodynamic and radiation field in a plume mixing region, including the effects of viscosity, diffusion, conductive, convective and normal radiative heat transfer as well as radiative heat losses from the flow. The formulation of the chemical heat release is based on the use of the finite rate reactions of the relevant chemical species or of the "overall reaction" simplifications useful for treatment of the higher hydrocarbons. The radiative transport equation is treated in the optically "intermediate" case assuming carbon as the principal emitter and thereby justifying a "gray gas" treatment. Scattering is neglected because of the large wavelength to particle size encountered. The radiation transport equation is treated on the basis of both the plane stratified model and of the curved stratified model and the area of applicability of each is discussed.

Table of Contents

LIST OF SYMBOLS

λ	wavelength
$\bar{\varkappa}$	average absorption coefficient
σ	Stefan-Boltzman constant
T	temperature
B	black body intensity in energy per unit time, area, and solid angle
j	emission coefficient
y	boundary layer coordinate in local normal direction
I	specific intensity in energy per unit time, area and solid angle
μ	cosine of the angle between the directions of y and I
ρ_c	carbon particle density
k_a	constant of proportionality in absorption coefficient
δ	solid density of carbon
P	ρ_c/δ
α	exponent of wavelength in absorption coefficient
$F(\lambda)$	extinction equation defined in Appendix
n_1, n_2	real and imagninary parts, respectively, of the index of refraction of carbon spheres.

λ_{max}	wavelength used to normalize the average absorption coefficient
h	Planck's constant
c	speed of light
k	Boltzmann constant
ξ	optical depth
q	flux over all solid angles of I
y_2	coordinate of the top of the mixing layer
y_1	coordinate of the bottom of the mixing layer
ξ_L	optical depth corresponding to y_2-y_1
E_1	$\int_0^1 e^{-z/\mu} \frac{d\mu}{\mu}$
x	boundary layer coordinate in streamwise direction
ρ	total fluid density
u,v	velocity components in the x and y direction respectively
α_i	mass fraction of i^{th} species
w_i	chemical production rate of i^{th} species
μ_t	turbulent viscosity coefficient
K	turbulent thermal conductivity
$\bar{c}_p$	mixture specific heat at constant pressure

L	turbulent Lewis number, $\frac{\bar{D}c_p}{K}$
Pr	turbulent Prandtl number, $\frac{\bar{c}_p \mu_t}{K}$
$\frac{dp}{dx}$	pressure gradient
c_{pi}	specific heat at constant pressure of the i^{th} species
R	universal gas constant
M_i	molecular weight of the i^{th} species
h_i	enthalpy per unit mass of the i^{th} species
h	total enthalpy per unit mass of the mixture
x, Ψ	stream coordinates defined by Eq. 21
F, a	any general functions of x and Ψ in Eq. 29
$a_{n, m\pm\frac{1}{2}}$	defined by Eq. 30
n, m	x and Ψ grid points
z	plume centerline coordinate
φ	angle of the tangent cone
$R(z), z$	plume outer surface coordinates
θ	angle between the normal to a point on the base and the vector to a point on the plume
D	length of the vector from a point on the base to a point on the plume surface
ψ	polar angle in (R, z, ψ) coordinates
ω	$2\pi c/\lambda$
$\theta(\omega)$, $\varphi(\omega)$	defined in Appendix

s	path length
T	optical layer thickness (Sect. V)
R_c	local radius of curvature
t_o	$R_c\,(1 - \sqrt{1-\mu^2}\,)$

RADIATION FROM CARBON IN A ROCKET PLUME MIXING REGION WITH COUPLED CONVECTIVE AND RADIATIVE ENERGY FLUXES AND GENERAL OPTICAL THICKNESS

by S. Slutsky
J. D. Melnick

I. INTRODUCTION

A problem of importance in the design of larger rocket vehicles is the estimation of the radiant heat transfer from the exhaust plume to the base of the vehicle. In one of the configurations of interest the exhaust gases result from the combustion of the fuel-rich RP1-LOX system. Consequently, combustible components, including solid carbon, are present. The products mix and react with the ambient air along the plume boundary producing a high temperature radiating layer. The most important radiating species will be carbon, which closely resembles a black body emitter. By contrast, the emissivities of H_2O and CO_2, the next most important radiation emitters, will be an order of magnitude lower[5,6]. If, however, the concentration of carbon particles is sufficiently low, the rotation-vibration bands of CO, CO_2, and particularly H_2O[7], will become the significant radiation sources.

In the present investigation it is assumed that the carbon cloud is the principal emitter and is composed of particles having effective diameters ranging from 100 to 1000 angstroms[1], which have the local temperature and velocity of the local gas phase flow. The carbon particles are assumed to have temperature levels of 2500° K or less.

Since the carbon particles are roughly spherical of less than 0.1 μ diameter, the density number of the carbon particle "gas" is about 6×10^{14} per gram-mole. Defining a mean free path for collisions of the carbon particles leads to a value of 1 mm., smaller than any physically relevant dimension of the flow. Consequently the carbon gas is assumed to obey the continuum gas-dynamic and radiative transfer equations. On the other hand, the interaction of the carbon with the radiation field can be treated by classical electromagnetic theory due to the large particle size relative to molecular dimensions. In consequence, the wavelength dependence of the absorption coefficient is continuous, i.e., the permissible energy levels of the system are continuous, as contrasted with the discrete rotation-vibration energy values of molecules which are governed by the Schrodinger equation. From the black body intensity curve at $2250^{\circ}K.$, the range of wavelength λ of interest is 1.0 to 10 microns, which is far greater than the particle size. This has the very important consequence of eliminating scattering as a significant phenomenon in carbon clouds(1).

The principle assumptions in the present formulation of the radiative transfer equation are:

1. the gray gas approximation is appropriate
2. the system is in local thermodynamic equilibrium
3. within the plume the radiative flux components parallel to the layer are negligible compared to the flux component normal to the layer.

Under the gray gas approximation the average is taken of the absorption coefficient over the relevant range of wavelengths, thereby eliminating the need to integrate the wavelength-dependent intensity to obtain the total intensity. To obtain $\bar{\varkappa}$, the wavelength-dependent absorption coefficient is averaged over the wavelength range of 1.0 μ to 10 μ.

The local thermodynamic equilibrium assumption is that the emitted radiation intensity is determined by the local thermal state of the medium at the temperature T. Thus the radiation source function is $j = \bar{\varkappa} B$ with $B = \sigma T^4$, where σ is the Stefan-Boltzmann constant.

Siddall and McGrath[2] give the form of $\rho_c \varkappa_\lambda$ at $T = 2250^\circ K$ for amorphous carbon. Huffaker[8] indicates possible experimental discrepancies from this theoretically predicted absorption coefficient at the expected temperatures. For the present investigation, the Siddall and McGrath equation, which is based on the D.C. conductivity of small absorbing spheres of baked electrode carbon is used but can be replaced when more appropriate formulas are available. This prescription for the absorption coefficient is given by:

$$\rho_c \varkappa_\lambda = 36\pi P F(\lambda) \lambda^{-1} = k_a \lambda^{-\alpha} \quad , \qquad (1)$$

where, for $1\mu < \lambda < 10\mu$,

$\alpha = 0.906 + 0.283 \ln \lambda$.

$k_a = 36\pi P F(\lambda = 1\mu)$.

P = average volume of particles per unit volume of gas,

$= \rho_c / \delta$.

ρ_c = gas density of carbon species.

δ = solid density of carbon particles = 2 gm/cm^3 .

$F(\lambda) = f(n_1, n_2)$.

n_1 and n_2 are obtained from the work of Stull and Plass[1] from a set of two equations which give n_1 and n_2, respectively the real and imaginary parts of the index of refraction, as functions of λ. $F(\lambda)$, derived for carbon spheres by Hawksley[3] is defined in the Appendix.

Thus for the gray gas assumption, an average absorptivity over the wavelength range of interest may be defined as follows:

$$\bar{\varkappa} = \frac{36\pi F(\lambda=1\mu)}{\lambda_{max}\ \delta} \left[\int_{1\mu}^{10\mu} \frac{d\lambda}{\lambda^{\alpha}} \right] \tag{2}$$

where

$$\lambda_{max} = 0.2014 \ \frac{hc}{kT} \tag{3}$$

λ_{max} is the wavelength of maximum emission given by Wien's Law.

h = Planck's constant.

c = speed of light.

k = Boltzman constant.

The third principal assumption regarding the direction of the radiative flux vector is dependent on the boundary layer nature of the flow and is crucial for the validity of the mathematical and numerical technique employed for the solution of the coupled system of gas radiation and flow. According to this assumption, the free carbon, which is the principal luminous emitter, is injected into the F-1 engine plume along with other combustible gases such as CO, free kerosene and free H_2, at the dividing streamline between the main nozzle flow and the external free airstream. When the oxygen and the fuels diffuse across the dividing streamline, combustion results and the free carbon becomes incandescent.

It is assumed in the present study that the mixing mechanism can be treated by boundary layer theory to approximate the interaction between the engine jet exhaust and the external free airstream; and that the basic assumptions of boundary layer theory, i.e., that the flow field gradients in the streamwise and circumferential directions are of lower order than the gradients normal to the stream surfaces, are applicable. These circumstances define conditions appropriate to the problem of thermal radiation in a stratified atmosphere, in which the local absorption coefficient and temperature are functions of the normal coordinate alone. This formulation permits expression of the transfer equation as a differential equation in one independent variable.

The use of an explicit finite difference formulation of the conservation equations (including radiation) in the parabolic boundary layer approximation further simplifies the system. Thus it becomes possible in the numerical solution scheme to march forward, one station at a time and to treat the radiation source term as a completely known function. The intensity distribution is then found by solving an ordinary first order differential equation containing a known source function thereby determining the emergent radiative energy flux. The divergence of the radiative flux is then obtained and is added to the energy equation in order to compute the flow conditions at the next station.

II. THE RADIATION FIELD WITHIN THE BOUNDARY LAYER

The radiation transfer equation along a ray travelling through an atmosphere in local thermodynamic equilibrium is

$$\frac{1}{\rho_c \varkappa_\lambda} \frac{dI_\lambda(s)}{ds} + I_\lambda(s) = B_\lambda(T), \tag{4}$$

where ds is the element of path length in the direction of the ray, λ is the wavelength of the radiation, ρ_c is the mass density of the carbon "gas," $\varkappa_\lambda$ is the absorption coefficient of the carbon at the wavelength λ and $B_\lambda(T)$ is the source strength at wavelength λ and temperature T. Since locally, $T = T(s)$, we can write implicitly $B_\lambda = B_\lambda(s)$.

The terms in the above equation (4) can be integrated over the whole wavelength range of importance, which for the temperature range of interest is the infrared region between 1μ and 10μ; thus,

$$\int_{1\mu}^{10\mu} I_\lambda \, d\lambda = I,$$

$$\int_{1\mu}^{10\mu} B_\lambda \, d\lambda \cong B = \frac{\sigma T^4}{\pi} .$$

Then the Eq. (4) can be written

$$\frac{1}{\rho_c \bar{\varkappa}} \frac{dI(s)}{ds} + I(s) = B(s), \tag{5}$$

provided a suitable mean value $\bar{\varkappa}$ can be found. One such mean value is given by

$$\bar{\varkappa} = \frac{1}{\lambda_{max}} \int_{\lambda_1}^{\lambda_2} \varkappa_\lambda \, d_\lambda$$

where λ_{max} is the wavelength characterizing the Wien Law emission peak and λ_1 and λ_2 are wavelengths respectively less and greater than λ_{max} which suitably encompass the range of significant luminosity.

Substitution of the Siddall and McGrath formula for $\bar{\varkappa}$ results in the expression of Eq. (1).

We now use the boundary layer approximation to relate the properties ρ_c and $\bar{\varkappa}$ to the thickness coordinate of the mixing region. In the determination of the radiation flux we further use the idealization of local planar stratification. However, in determining the radiation intensity to the base, this planar model is inadequate as will be shown later. For the planar model the geometry is defined in terms of a single variable, y, normal to the flow direction. Then

$$ds = \frac{dy}{\mu}$$

where μ is the direction cosine of the light ray relative to the normal. Because of the planar geometry μ is constant along the ray. Then Eq. (5) becomes

$$\frac{\mu}{\rho_c \bar{\varkappa}} \frac{dI(\mu,y)}{dy} + I(\mu,y) = B(y) \quad . \tag{6}$$

The boundary conditions appropriate are that at the outer surface $y = y_2$, the incoming intensity ($\mu < 0$) is zero. The outgoing radiation ($\mu > 0$) is related to the incoming radiation at the inner boundary $y = y_1$ by the equality $I(y_1,\mu) = I(y_1,-\mu)$.

Defining

$$d\xi = \rho_c \bar{\varkappa}\, dy; \quad \xi = \int_{y_1}^{y} \rho_c \bar{\varkappa}\, dy \quad . \tag{7}$$

Equation (4) becomes

$$\mu \frac{dI}{d\xi} + I = B . \tag{8}$$

The flux over all solid angles can be written

$$q(\xi) = 2\pi \int_{-1}^{1} I(\xi,\mu)\ \mu d\mu \tag{9}$$

since I is assumed locally independent of aximuthal angle.

The solution of (8) is:

$$I^{+}(\xi,\mu) = e^{-\xi/\mu}\left[\int_{0}^{\xi} B(\zeta)e^{-\zeta/\mu}\ \frac{d\zeta}{\mu} + \int_{0}^{\xi} B(\zeta)e^{-\zeta/\mu}\ \frac{d\zeta}{\mu}\right], \quad \mu > 0.$$

$$I^{-}(\xi,\mu) = -e^{-\xi/\mu}\int_{\xi}^{\infty} B(\zeta)e^{\zeta/\mu}\ \frac{d\zeta}{\mu}, \quad \mu < 0 . \tag{10}$$

The integration is cut off at $y = y_2$ instead of going out to ∞ since ρ_c and hence $d\xi/dy$ is zero for $y > y_2$ and ξ cannot exceed the upper limit, ξ_L . The flux is thus given by:

$$q(\xi) = 2\pi \int_0^1 \left[\int_0^{\xi_L} B(\zeta) e^{-(\xi+\zeta)/\mu} \, d\zeta \right.$$

$$\left. + \int_0^{\xi} B(\zeta) e^{-(\xi-\zeta)/\mu} \, d\zeta - \int_{\xi}^{\xi_L} B(\zeta) e^{(\xi-\zeta)/\mu} \, d\zeta \right] d\mu \, , \tag{11}$$

and the divergence of the radiation flux to be added to the right side of the boundary layer energy equation is given by

$$\frac{-\partial q}{\partial y} = 2\pi \rho_c(y) \bar{\varkappa} \left[\int_0^{\xi_L} B(\zeta) E_1 \, (\xi+\zeta) d\zeta + \int_0^{\xi} B(\zeta) E_1 \, (\xi-\zeta) \, d\zeta \right.$$

$$\left. + \int_{\xi}^{\xi_L} B(\zeta) E_1 \, (\zeta-\xi) d\zeta - 2B(\xi) \right] , \tag{12}$$

where

$$E_1(z) = \int_0^1 e^{-z/\mu} \, \frac{d\mu}{\mu} \, . \tag{13}$$

The incorporation of the effect of the radiation transfer (assuming the plane stratified model) into the flow field is described next.

III. BOUNDARY LAYER FLOW FIELD

The mixing and radiating region of the plume field is assumed to be initially confined for several diameters downstream of the nozzle lip to a relatively narrow annular region straddling the dividing streamline. It is expedient to treat this mixing region as a locally two-dimensional flow.

The governing gas-dynamic equations for the viscous layer including the radiation energy flux then become:

Global continuity:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \tag{14}$$

Species continuity:

$$\rho u \frac{\partial \alpha_i}{\partial x} + \rho v \frac{\partial \alpha_i}{\partial y} = \frac{\partial}{\partial y}\left[\frac{L}{Pr}\,\mu\,\frac{\partial \alpha_i}{\partial y}\right] + \rho w_i \,. \tag{15}$$

Momentum:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{-dp}{dx} + \frac{\partial}{\partial y}\left[\mu_t \frac{\partial u}{\partial y}\right] . \tag{16}$$

Energy:

$$\bar{c}_p \rho u \frac{\partial T}{\partial x} + \bar{c}_p \rho v \frac{\partial T}{\partial x} = u \frac{dp}{dx} + \mu_t \left(\frac{\partial u}{\partial y}\right)^2$$

$$- \rho \sum_i w_i h_i + \frac{\partial}{\partial y}\left[\frac{\bar{c}_p}{Pr}\,\mu_t \frac{\partial T}{\partial y}\right] + \frac{\mu_t L}{Pr}\,\frac{\partial T}{\partial y}\,\sum_i c_{pi}\,\frac{\partial \alpha_i}{\partial y} - \frac{\partial q}{\partial y} \,.$$

(17)

Equation of state:

$$p = \rho RT \sum_i \frac{\alpha_i}{M_i} \quad . \tag{18}$$

In the above equations α_i is the mass fraction of the i^{th} species whose production rate is w_i and whose molecular weight is M_i. The mixture enthalpy is

$$h = \Sigma\, \alpha_i h_i\ (T) \quad . \tag{19}$$

The mixture specific heat is

$$\bar{c}_p = \Sigma\, \alpha_i c_{pi} \ , \tag{20}$$

and q is the radiation flux in the y-direction.

The equations are transformed to stream coordinates (x, Ψ) defined by

$$\frac{\partial \Psi}{\partial y} = \rho u \tag{21}$$

$$\frac{\partial \Psi}{\partial x} = - \rho v \tag{21}$$

Thus,

$$\left(\frac{\partial}{\partial y}\right)_x = \rho u \left(\frac{\partial}{\partial \Psi}\right)_x \ , \tag{22}$$

$$\left(\frac{\partial}{\partial x}\right)_y = \left(\frac{\partial}{\partial x}\right)_\Psi - \rho v \left(\frac{\partial}{\partial \Psi}\right)_x \ .$$

The describing equations become:

Species Continuity:

$$\frac{\partial \alpha_i}{\partial x} = \frac{\partial}{\partial \Psi}\left[\frac{L\mu_t \rho u}{Pr}\left(\frac{\partial \alpha_i}{\partial \Psi}\right)\right] + \frac{w_i}{u} \quad . \tag{23}$$

Momentum:

$$\frac{\partial u}{\partial x} = - \frac{1}{\rho u}\left(\frac{dp}{dx}\right)_y + \frac{\partial}{\partial \Psi}\left[\mu_t \rho u \left(\frac{\partial u}{\partial \Psi}\right)\right] . \tag{24}$$

Energy:

$$\bar{c}_p \frac{\partial T}{\partial x} = \frac{1}{\rho}\left(\frac{dp}{dx}\right)_y + \frac{\partial}{\partial \Psi}\left[\frac{\bar{c}_p \mu_t \rho u}{Pr}\left(\frac{\partial T}{\partial \Psi}\right)\right]$$

$$+ \mu_t \rho u \left(\frac{\partial u}{\partial \Psi}\right)^2 - \frac{1}{u}\sum_i h_i w_i - \frac{\partial q}{\partial \Psi} \tag{25}$$

$$+ \frac{\mu_t L \rho u}{Pr}\left(\frac{\partial T}{\partial \Psi}\right)\sum_i c_{pi}\left(\frac{\partial \alpha_i}{\partial \Psi}\right) \quad .$$

The radiation energy flux in (x, Ψ) coordinates is

$$- \frac{\partial q}{\partial \Psi} = \frac{2\sigma\rho_c(\Psi)\bar{\varkappa}}{\rho u}\left[\int_0^{\xi_L} T^4(\zeta)E_1(\xi+\zeta)d\zeta + \int_0^{\xi} T^4(\zeta)E_1(\xi-\zeta)d\zeta \right.$$

$$\left. + \int_{\xi}^{\xi_L} T^4(\zeta)E_1(\zeta-\xi)d\zeta - 2T^4(\xi)\right] . \tag{26}$$

$$\xi = \int_{y_1}^{y} \bar{\chi}\, \rho_c(y')dy' = \int_{\Psi_1}^{\Psi} \frac{\bar{\chi}\, \rho_c(\Psi')d\Psi'}{\rho u} \tag{27}$$

$$\xi_L = \int_{\Psi_1}^{\Psi_2} \frac{\bar{\chi}\, \rho_c d\Psi'}{\rho u} \quad .$$

where (28)

$$d\zeta = \frac{\bar{\chi}\, \rho_c d\Psi}{\rho u} \quad .$$

In the rectangular (x, Ψ) grid, if n refers to x position and m to Ψ position, the following explicit finite difference scheme is used for the above parabolic differential equations:

$$\left(\frac{\partial F}{\partial x}\right)_{n+1,m} = \frac{F_{n+1,m} - F_{n,m}}{\Delta x}$$

$$\left(\frac{\partial F}{\partial \Psi}\right)_{n,m} = \frac{F_{n,m+1} - F_{n,m-1}}{2\Delta \Psi} \tag{29}$$

$$\left[\frac{\partial}{\Psi}\left(a\frac{\partial F}{\partial \Psi}\right)\right]_{n,m} = \frac{a_{n,m+\frac{1}{2}}(F_{n,m+1} - F_{n,m}) - a_{n,m-\frac{1}{2}}(F_{n,m} - F_{n,m-1})}{(\Delta \Psi)^2}$$

where

$$\Delta\Psi = \Psi_{n+1} - \Psi_n, \quad \Delta x = x_{n+1} - x_n, \quad a_{n,m\pm\frac{1}{2}} = \frac{1}{2}\left[a_{n,m} + a_{n,m\pm 1}\right] . \tag{30}$$

Defining

$$a \equiv \mu_t \rho u ,$$

the finite difference equations become:

$$\frac{\alpha^i_{n+1,m} - \alpha^i_{n,m}}{\Delta x} = \frac{1}{(\Delta\Psi)^2}\left(\frac{La}{Pr}\right)_{n,m+\frac{1}{2}}\left[\alpha^i_{n,m+1} - \alpha^i_{n,m}\right] - \frac{1}{(\Delta\Psi)^2}\left(\frac{La}{Pr}\right)_{n,m-\frac{1}{2}}\left[\alpha^i_{n,m} - \alpha^i_{n,m-1}\right] + \left(\frac{\dot{w}^i}{u}\right)_{n,m} \tag{31}$$

$$\frac{u_{n+1,m} - u_{n,m}}{\Delta x} = -\frac{1}{(\rho u)_{n,m}}\left(\frac{dp}{dx}\right)_{n+1} + \frac{1}{(\Delta\Psi)^2} a_{n,m+\frac{1}{2}}\left[u_{n,m+1} - u_{n,m}\right] - \frac{1}{(\Delta\Psi)^2} a_{n,m-\frac{1}{2}}\left[u_{n,m} - u_{n,m-1}\right] . \tag{32}$$

$$\frac{T_{n+1,m}-T_{n,m}}{\Delta x} = \frac{1}{(\bar{c}_p\rho)_{n,m}}\left(\frac{dp}{dx}\right)_{n+1}$$

$$+\frac{1}{(\Delta\Psi)^2}\frac{1}{(\bar{c}_p)_{n,m}}\left(\frac{\bar{c}_p a}{Pr}\right)_{n,m+\frac{1}{2}}\left[T_{n,m+1}-T_{n,m}\right]$$

$$-\frac{1}{(\Delta\Psi)^2}\frac{1}{(\bar{c}_p)_{n,m}}\left(\frac{\bar{c}_p a}{Pr}\right)_{n,m+\frac{1}{2}}\left[T_{n,m}-T_{n,m-1}\right]$$

$$+\frac{a_{n,m}}{\bar{c}_{p_{n,m}}}\frac{(u_{n,m+1}-u_{n,m-1})^2}{4(\Delta\Psi)^2}-\frac{1}{(u\bar{c}_p)_{n,m}}\sum_i h^i_{n,m}w^i_{n,m}$$

$$+\frac{A_{nm}}{(\bar{c}_p)_{nm}}+\left(\frac{aL}{\bar{c}_p Pr}\right)_{n,m}\left[\frac{T_{n,m+1}-T_{n,m-1}}{2\Delta\Psi}\right]x$$

$$\sum_i \frac{c_{p_{i_{n,m}}}\left[\alpha^i_{n,m+1}-\alpha^i_{n,m-1}\right]}{2\Delta\Psi} \quad . \tag{33}$$

where A_{nm} is the function on the right side of Eq. (26) evaluated at (n,m).

Solving for $\alpha_{n+1,m}$, $u_{n+1,m}$, $T_{n+1,m}$, algebraic equations are obtained for the $(n+1)^{th}$ line in terms of quantities known along the n^{th} line.

IV. RADIATION TO THE BASE REGION

In the foregoing section the temperature distribution within the plume was computed taking into account the effects of mixing and chemical reaction as well as that of radiative losses.

The computation of the radiation field at the base is next determined by evaluating a surface integral over the plume, rather than a volume integral. Thus, the radiant energy flux from a point Q of the plume surface to a point P of the gas is given by the integral over the solid angle.

$$q(P) = \int I(QP) \cos\theta \, d\Omega \qquad (34)$$

where θ is the angle between the vector QP and the normal to the base, and I(QP) is the intensity of the radiation in the direction QP. The element of solid angle dw can be written

$$d\Omega = \frac{\mu dA}{D^2} \qquad (35)$$

where μ is the cosine of the angle between the normal to the plume surface element of area dA and the vector QP.

In order to specify the geometry for the determination of the integral of Eq. (34), the element of surface in Eq. (35) is represented by that of the cone tangent to the plume at (z,R), where R is the radius of the plume at the axial station z, φ is the cone angle of the tangent cone, r is the radial location of the point P of the base and ψ is the aximuth angle of the point Q of the plume (see Fig. 1). Since the configuration is axisymmetric, the azimuth angle of point P is arbitrary and can be set equal to zero. Using an ijk triad of unit vectors, the directions of the vector D, the normal n at the plume and the normal n_B at the base can be specified.

$$\bar{D} = \hat{i}\, z - \hat{j}\, R \sin \psi + \hat{k}\, (r - R \cos \psi).$$

$$\bar{n} = \hat{i} R \tan \varphi + \hat{j}\, R \sin \psi + \hat{k}\, R \cos \psi.$$

$$\bar{n}_B = -\hat{i}.$$

Then the cosine values μ and cos θ can be determined:

$$\mu = \frac{\bar{D}}{|D|} \cdot \frac{\bar{n}}{|n|} = \frac{\cos \varphi\, [z \tan \varphi - R + r \cos \psi]}{\sqrt{z^2 + r^2 + R^2 - 2rR \cos \psi}} ,$$

$$\cos\,\theta = \frac{\bar{D}\cdot\bar{n}_B}{|D|\;|n_B|} = \frac{z}{\sqrt{z^2 + r^2 + R^2 - 2rR\cos\,\psi}}\;;$$

and the element of area dA can be expressed

$$dA = R d\psi \frac{dz}{\cos\,\varphi} \quad .$$

The flux at point P therefore can be written

$$q(r) = \int_z R(z) z \int_\psi \frac{I(\mu, z)[z\,\tan\,\varphi - R + r\,\cos\,\psi]d\psi}{[z^2 + r^2 + R^2 - 2rR\,\cos\,\psi]^2}\,dz\,. \tag{36}$$

The domain of integration includes all values of ψ for which

$$z\,\tan\,\varphi - R + r\,\cos\,\psi > 0\;.$$

Since the integrand is regular the evaluation of Eq. (36) is straightforward and the increment dq(r) due to the plume segment centered at $z_n \pm \frac{\Delta z}{2}$ will be added to the sum generated from $z = 0$ to z_{n-1}. The radius term R(z) in the integrand is known as a result of the calculation of the plume boundary layer. The intensity $I(\mu, z) = I(\mu, y_2(z))$ at the outer edge of the plume boundary layer can be obtained according to the plane stratified model by setting $\xi = \xi_L$ in Eq. (10) for $\mu > 0$. This yields

$$I^+ (\xi_L, \mu) = 2e^{-\xi_L/\mu} \int_0^{\xi_L} B(\zeta) \cosh\left(\frac{\zeta}{\mu}\right) \frac{d\zeta}{\mu} . \quad (37)$$

Equation (37) is not adequate to describe the general case of radiation to the base, because for $\mu \to 0$, it goes to the limit

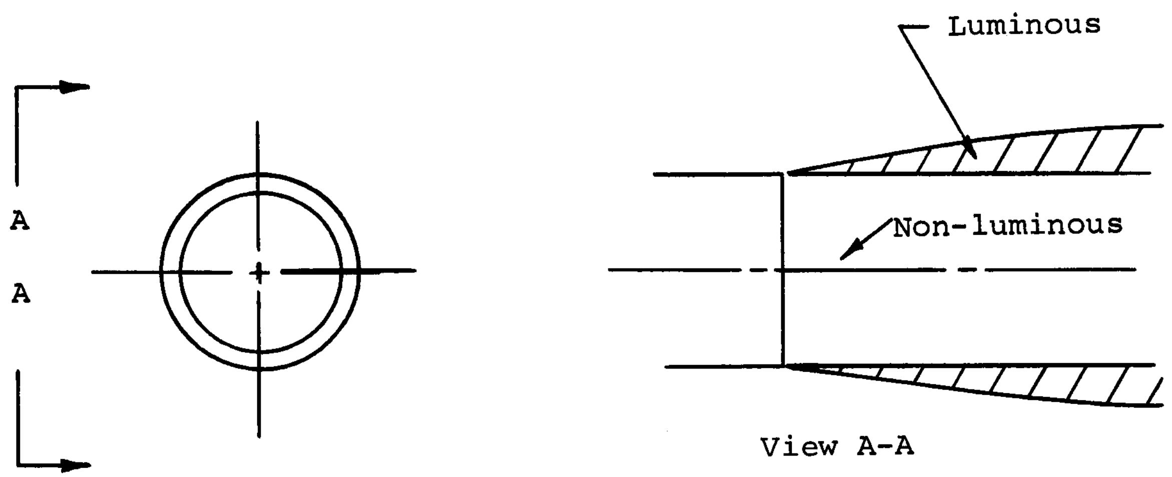
$$I^+ (\xi_L, \mu) \xrightarrow[\mu \to 0]{} B(\xi_L) \quad .$$

Indeed the in-plane (μ=0) component of intensity for any ξ also goes to the black body value $B = \sigma T^4$. This defect in the intensity distribution is a result of the infinite lateral extent of the emitting and absorbing layer of the plane stratified model.

It will be shown in the next section that the flux and therefore the energy transfer is essentially correct, but that the intensity field radiated at fringe angles to the plume surface normal (near 90°) needs to be computed with due consideration of the plume surface curvatures.

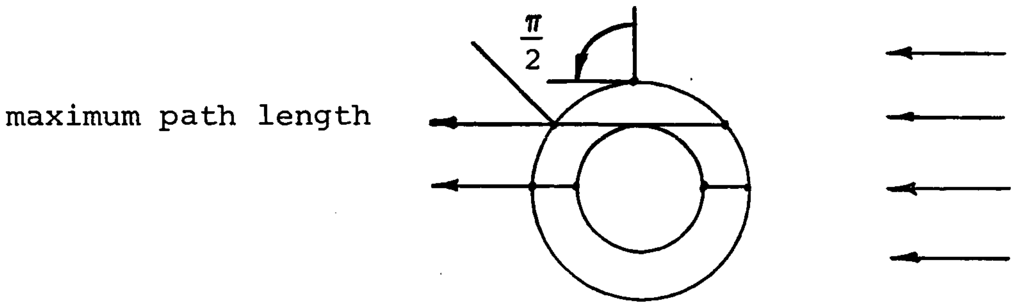
V. RADIATION FROM A CURVED STRATIFIED REGION

If one examines a photograph of an ordinary Bunsen burner flame, which is a good example of an axisymmetric flame with an annular zone of luminosity near the base, one will notice that the luminosity appears intense near the edge of the flame region of the photograph, and is almost negligible near the centerline.



View A-A

Since the flow is axisymmetric the greater luminosity at the edges indicates that (for optically non-thick flames at least) the emergent intensities at large angles from the normal cone is relatively much more important than those components leaving at small angles to the normal. For optically thin flames this is simply due to the greater geometric path length traversed by a ray which cuts near the inner edge of the annular region of optical activity. Nevertheless this path length does not become



infinite as $\mu \rightarrow o$ as in the planar model. Instead, it goes to zero after reaching a maximum. This behavior will be presented somewhat more quantitatively below.

Let us consider the intensity of the ray from point Q of the plume surface to point P of the base. The ray QP and the normal to the plume at Q define a plane which cuts the plume in a section as shown in Fig. 2. At the point of intersection of the ray with the outer plume we can replace the curve locally by the osculating circle which is tangent to the local plume curve and has the same radius of curvature. The intensity along the ray Q'P' is still given in Eq. (5).

$$\frac{1}{\rho_c \bar{\varkappa}} \frac{dI(s)}{ds} + I(s) = B(s) \tag{5}$$

and the emergent radiation at Q in the direction QP can be written formally as

$$I^{+}(\xi_Q) = 2e^{-\xi_Q} \int_0^{\xi_Q} B(\zeta) \cosh \zeta \, d\zeta , \qquad (38)$$

where

$$\xi_Q = \int_0^s \rho_c \bar{\varkappa} \, ds' . \qquad (39)$$

The origin s=0 can be either the inner boundary of the plume, or the point of symmetry on the ray path where I^{+} is equal to I^{-} (point Q" of Fig. 3). Unfortunately s is not a convenient variable to use. It is seen from Fig. 4 that all plume geometric and thermodynamic properties can be conveniently stated in terms of the local radius of curvature R_c, the distance t inward from the outer plume boundary, and μ, the direction cosine of the emergent ray QP.

We obtain for the transformation from s to t

$$ds = - \frac{(R_c - t) \, dt}{\sqrt{R_c^2 \mu^2 - 2R_c t + t^2}} \qquad t \leqslant t_o \qquad (40)$$

$$t_o = R_c \left(1 - \sqrt{1 - \mu^2} \right) \qquad (41)$$

Then the emergent radiation intensity along the ray QP is

$$I^{+}(QP) = I_{Q}^{+}(\mu) = 2e^{-\xi_Q} \int_{o}^{t_o} \frac{\rho_c \bar{\kappa}(R_c - t)B(t)}{\sqrt{R_c^2 \mu^2 - 2R_c t + t^2}} \cosh \xi(t,\mu)dt, \tag{42}$$

where

$$\xi(t,\mu) = \int_{t}^{t_o} \frac{\rho_c \bar{\kappa} (R_c - t_1)dt_1}{\sqrt{R_c^2 \mu^2 - 2R_c t_1 + t_1^2}} , \tag{43}$$

$$\xi_Q = \xi(o,\mu) = \int_{o}^{t_o} \frac{\rho_c \bar{\kappa}(R_c - t)dt}{\sqrt{R_c^2 \mu^2 - 2R_c t + t^2}} . \tag{44}$$

It is of interest to note that the "in-plane" component of radiation at an interior point of the plume (i.e., the value of Q" along Q"Q) is

$$I_{Q''}^{+}(\mu) = I_{Q''}^{-}(\mu) = \int_{o}^{t_o} \frac{\rho_c \bar{\kappa} (R_c - t) B(t)}{\sqrt{R_c^2 \mu^2 - 2R_c t + t^2}} e^{-\xi(t,\mu)}dt . \tag{45}$$

In the limit when the optical layer thickness T is very small compared to the local radius of curvature ($T \ll R_c$), some interesting comparisons with the plane stratified approximation can be made. Two cases can be distinguished. First if emergent angle is not too close to 90°, i.e., when

$$\mu > \sqrt{\frac{2T}{R_c}} \quad , \tag{46}$$

and when the thickness T is less than $t_o = R_c(1-\sin\beta)$ defined by Eq. (41). This set of conditions is exemplified by the ray Q_2P_2 of Fig. 4.

In this case $ds \to -\frac{dt}{\mu}$ which is exactly the transformation appropriate to the plane stratified model.

However, when t_o is of the same order or less than T, then we should write

$$ds \to -\sqrt{\frac{R_c}{2t_o - t}}\, dt \tag{47}$$

so that

$$\xi(t,\mu) = \int_t^{t_o} \rho_c \bar{\varkappa} \sqrt{\frac{R_c}{2t_o - t}} \, dt \,, \tag{48}$$

$$I_Q^+(\mu) = 2e^{-\xi_Q} \int_0^{t_o} \rho_c \bar{\varkappa} \sqrt{\frac{R_c}{2t_o - t}} \, B(t) \cosh \xi \, dt \tag{49}$$

The emergent intensity thus goes smoothly to zero as $\sqrt{t_o}$, or even more.

The question of the amount of error incurred in the use of the plane stratified model for computing radiative flux divergence in the coupled flow energy equation can now be answered. Thus, it is noted that the normal flux depends on those components of the intensity with direction cosine μ which are not small, since the flux depends on the product $\mu I(\mu)$. Thus, if $\mu > \sqrt{\frac{2T}{R_c}}$, then the plane approximation is applicable. When $\mu \leqslant \sqrt{\frac{2T}{R_c}}$, the contribution to the flux becomes small. It is therefore concluded that the formulation used in Section III is adequate.

For the purposes of radiation to the base, the expression of Eq. (49) or of Eq. (42) is needed for the plume edge radiative components. This is not difficult to carry out in a digital computation scheme and will be included in the computer program.

REFERENCES

1. R.V. Stull and G.N. Plass, J. Optical Society of America, 50, 121 (1960).

2. R.G. Siddall, and I.A. McGrath, "The Emissivity of Luminous Flames," Ninth Symposium on Combustion (Academic Press, New York, 1963).

3. P.G.W. Hawksley, British Coal Utilization Research Association Monthly Bulletin, XVI, Nos. 4 & 5 (1952).

4. R.P. Main and E. Bauer, J. Quantitative Spectroscopy and Radiative Transfer 6, 1 (1966).

5. C.C. Ferriso and C.B. Ludwig, J. Quantitative Spectroscopy and Radiative Transfer 1, 215 (1964).

6. S.S. Penner and P. Varanasi, J. Quantitative Spectroscopy and Radiative Transfer 4, 799 (1964).

7. W.S. Benedict and E.K. Plyler, Energy Transfer in Hot Gases, NBS Circular 523, March 1954.

8. R.M. Huffaker, "Inhomogeneous Radiant Heat Transfer from Saturn Rocket Exhaust Plumes," AIAA Paper No. 66-652 (1966).

APPENDIX

Determination of $F(\lambda)$:

$$F(\lambda) = \frac{n_1^2 n_2}{[(n_1^2 + n_1^2 n_2^2)^2 + 4(n_1^2 - n_1^2 n_2^2 + 1)]}$$

where

$$n_1^2 - n_1^2 n_2^2 = 1 + \frac{6.448 \times 10^{32}}{4.062 \times 10^{35} - \omega^2}$$

$$+ \frac{3.224 \times 10^{32}}{9.549 \times 10^{33} - \omega^2} + \frac{3.224 \times 10^{32}}{5.217 \times 10^{33} - \omega^2}$$

$$+ \frac{6.348 \times 10^{32} \, (1.966 \times 10^{32} - \omega^2)}{(1.956 \times 10^{32} - \omega^2)^2 + 1.369 \times 10^{33} \omega^2}$$

$$- \frac{3.05 \times 10^{31}}{2.323 \times 10^{31} + \omega^2} \equiv \theta(\omega)$$

$$2n_1^2 n_2 = \frac{6.347 \times 10^{32} \, (3.70 \times 10^{16} \omega)}{(1.956 \times 10^{32} - \omega^2)^2 + 1.369 \times 10^{33} \omega^2}$$

$$+ \frac{3.05 \times 10^{31} \, (4.82 \times 10^{15})}{(2.323 \times 10^{31} + \omega^2)\,\omega} \equiv \varphi(\omega)$$

$$\omega = \frac{2\pi c}{\lambda}$$

c = speed of light. ω has units sec^{-1} if c is in $cm\text{-}sec^{-1}$ and λ is in cm.

These equations can be written in terms of the $\theta(\omega)$ and $\varphi(\omega)$ functions as

$$n_1^2 - n_1^2 n_2^2 = \theta(\omega) \quad ,$$

$$2n_1^2 n_2 = \varphi(\omega) \quad .$$

The solution is

$$n_1 = \sqrt{\frac{\varphi(\omega)}{2 n_2}} \quad ,$$

$$n_2 = \sqrt{\left(\frac{\theta(\omega)}{\varphi(\omega)}\right)^2 + 1} - \frac{\theta(\omega)}{\varphi(\omega)} \quad .$$

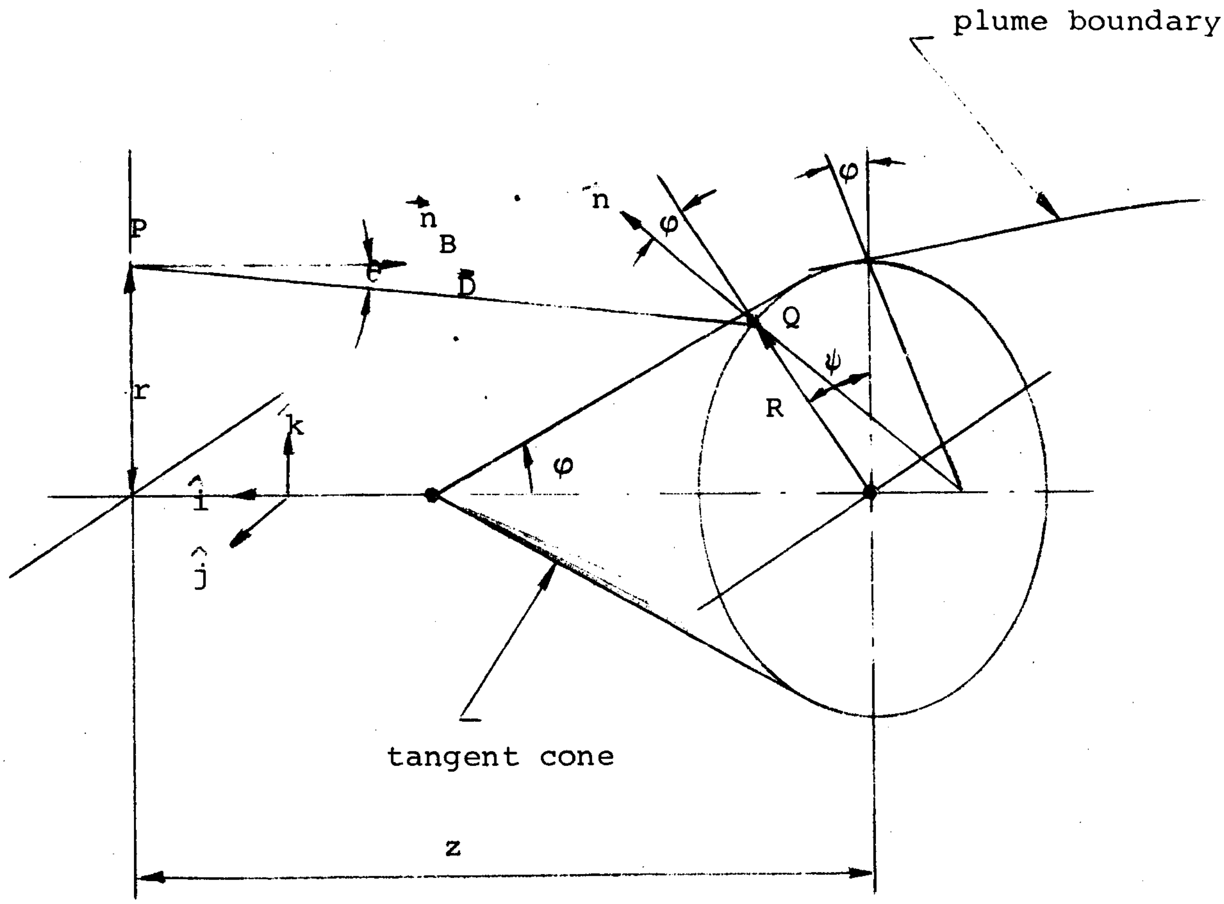


FIG. 1. PLUME GEOMETRY

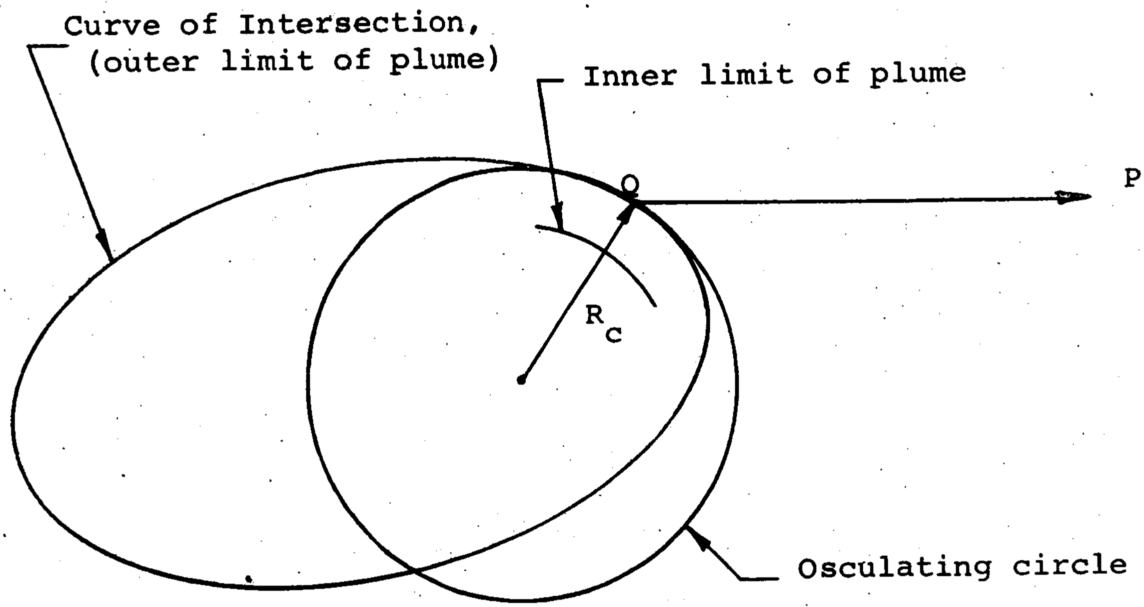


FIG. 2. CURVE OF INTERSECTION & OSCULATING CIRCLE

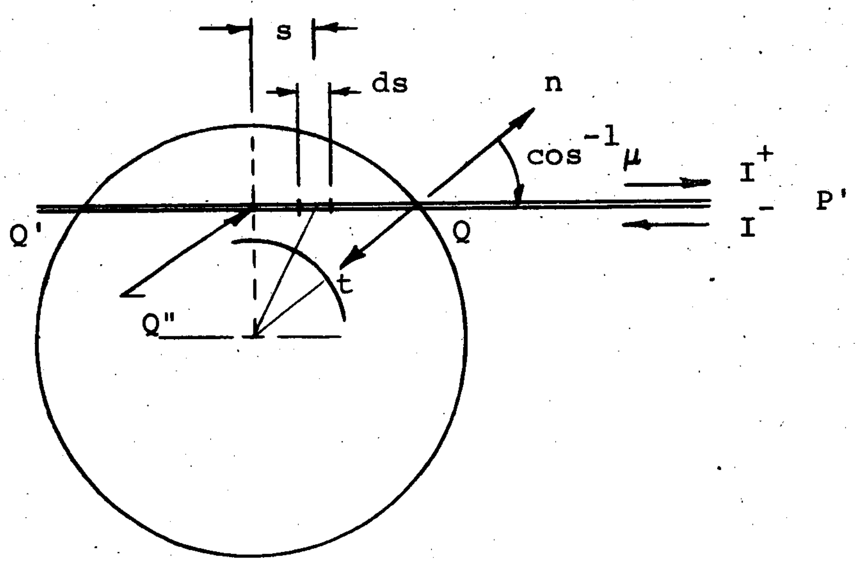


FIG. 3. RAY PASSAGE THROUGH PLUME SECTION

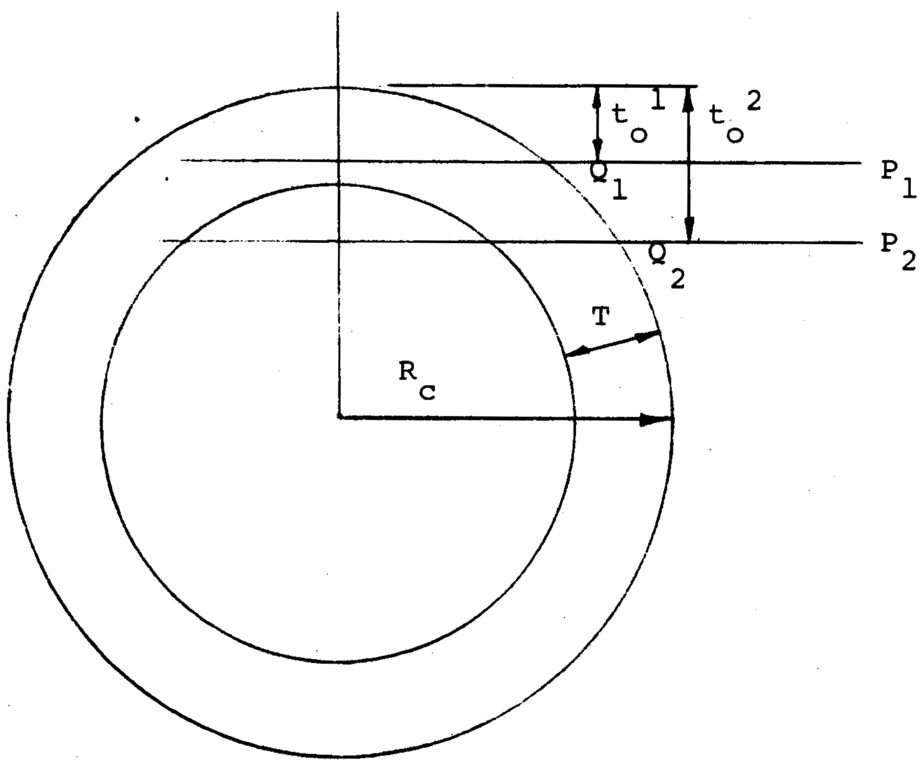


FIG. 4. GEOMETRY OF A CURVED STRATIFIED REGION

November 8, 1966

GASL Job No. 8043
Contract No. NAS8-20298

ERRATUM

TECHNICAL REPORT NO. 628

RADIATION FROM CARBON IN A ROCKET PLUME MIXING REGION WITH COUPLED CONVECTIVE AND RADIATIVE ENERGY FLUXES AND GENERAL OPTICAL THICKNESS

by S. Slutsky
J. D. Melnick

Equation (10) on Page 11 should read:

$$I^{+}(\xi,\mu) = e^{-\xi/\mu}\left[\int_{0}^{\xi_L} B(\zeta)e^{-\zeta/\mu}\,\frac{d\zeta}{\mu} + \int_{0}^{\xi} B(\zeta)\,e^{\zeta/\mu}\,\frac{d\zeta}{\mu}\right], \quad \mu > 0 .$$

$$I^{-}(\xi,\mu) = -\,e^{-\xi/\mu}\int_{\xi}^{\xi_L} B(\zeta)e^{\zeta/\mu}\,\frac{d\zeta}{\mu}, \quad \mu < 0 .$$