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SURFACE INSTABILITY AND DISINTEGRATION OF LIQUID IN A LONGITUDINALLY EXCITED CONTAINER

by

C. R. Gerlach

Technical Report No. 9
Contract No. NAS8-11045
Control No. TP3-85175 & S-1 (1F)
SwRI Project No. 02-1391

Prepared for

National Aeronautics and Space Administration
George C. Marshall Space Flight Center
Huntsville, Alabama

15 March 1967

SOUTHWEST RESEARCH INSTITUTE
SAN ANTONIO HOUSTON

SOUTHWEST RESEARCH INSTITUTE
8500 Culebra Road, San Antonio, Texas 78206

Department of Mechanical Sciences

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ABSTRACT

Results are presented for an investigation of liquid surface disintegration and associated phenomena resulting from the longitudinal excitation of a tank containing liquid.

First, details of a combined experimental and theoretical study of liquid surface disintegration are given. The conditions necessary to give surface disintegration, found experimentally for several liquids, show that the required input acceleration increases roughly in proportion to frequency. A theory is developed which predicts a large amplitude liquid surface instability and is shown to give conservative estimates of the experimentally obtained disintegration conditions. The theory appears to be valid for low gravity environments.

Results of several experiments performed to verify Yarymovych's hypothesis of spray excited modes are concluded to be adequate proof of the validity of this hypothesis. Further, an excess surface acceleration is defined and shown to be an important parameter in correlating experimental data for the input conditions necessary to give spray excited, low frequency modes.

Finally, the results of some limited testing performed to determine the influence of a random input acceleration on surface disintegration and spray excited modes are presented. It is found that the spray excited mode phenomena exhibits trends very similar to the sinusoidal excitation case. However, it is concluded that a comprehensive understanding of disintegration for a random input will require further investigation involving extensive use of a very sensitive spray transducer.

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LIST OF SYMBOLS

a_n	amplitude of the nth axisymmetric component of the liquid velocity potential
a_s	effective acceleration at liquid surface, surface acceleration
a_{se}	excess surface acceleration
a_{st}	threshold surface acceleration
A	amplitude of dimensionless surface wave
A'	amplitude of spray excited mode with spray suppression baffle
A'_{∞}	amplitude of spray excited mode with no baffle
b_n	amplitude of the nth axisymmetric liquid surface mode
B_d	Bond number based on container diameter, $\rho g d^2/\sigma$
B_L	Bond number based on wavelength, $\rho g L^2/\sigma$
$C_{1,2}$	arbitrary constants
C_c	a constant factor to account for system compressibility in Eq. (20)
d	container diameter
$D_n, F_n, H_n(a_k, b_k)$	nonlinear functions of a_k and b_k, $k = 1 \rightarrow \infty$
g	local acceleration of gravity
g_o	standard acceleration of gravity
$G(t)$	effective gravity of excited liquid-tank system
h	liquid depth
$h_{\ell, u}$	limits of baffle height which influence amplitude of spray excited mode

LIST OF SYMBOLS (Cont'd)

J_m	mth order Bessel function of the first kind
ℓ	dimensionless wavelength, L/d
L	wavelength
m, n	mode numbers corresponding to the eigenfunctions $J_m(\lambda_{mn}r) \cdot \cos m\theta$ which describe the liquid surface
r	radial coordinate
R	hypothetical tank radius, L/2
$S(\Omega)$	normalized spectral density
t	time
$T_{\ell, u}$	trajectory times corresponding to trajectory heights h_ℓ and h_u
$\overline{v}$	vector velocity
x_o	excitation amplitude
z	axial coordinate position
α_1	dimensionless amplitude of liquid velocity potential for λ_1 mode, $\lambda_1^2 a_1/\omega_1$
β_1	dimensionless amplitude of λ_1 mode surface wave, $\lambda_1 b_1$
δ	dimensionless frequency ω/ω_1
ϵ	dimensionless excitation amplitude, $\lambda_1 x_o$
$\eta(r, t)$	height of liquid surface above reference plane $z = 0$
λ_n	eigenvalues for axisymmetric modes, roots of $J_1'(\lambda_n) = 0$
ρ	liquid density

σ	surface tension
τ	dimensionless time, $\omega_1 t$
$\phi(r, z, t)$	liquid velocity potential
ω	angular frequency
ω_1	natural frequency of λ_1 mode, $(\lambda_1 g)^{1/2}$
Ω	dimensionless angular frequency of excitation $\omega(d/g)^{1/2}$

I. INTRODUCTION

Plans for future activities in space have brought into sharp focus the fact that little is presently known about the behavior of liquids which exist under reduced gravity conditions for extended periods of time. One very important aspect of this behavior concerns the response of the liquid-vapor interface to various types of disturbances, such as the stopping or starting of rocket motors and vibration resulting from on-board movement. If these disturbances are of sufficient magnitude, instability leading to disintegration of the interface may result. In terms of potential effects upon the performance of a space vehicle, the occurrence of propellant surface disintegration can be very detrimental under low gravity conditions. Some possible reasons for this are:

(1) Globules of propellant, floating free from the main body of the liquid, are susceptible to loss during venting.

(2) Gross disintegration of the liquid can result in restart problems by allowing the entrainment of vapor in the suction line.

(3) It is possible for the liquid particles, by impacting back onto the liquid surface, to excite large amplitude surface waves.

While there is a rather large amount of literature dealing with various types of surface waves and "sloshing" of liquids contained in vibrating containers, much of which has been recently summarized in a comprehensive monograph [1][1], no information is currently available to allow prediction of the breakup or disintegration of the liquid surface. It is the purpose of this report to discuss surface disintegration of a liquid in a longitudinally excited container for what will be termed high frequency excitation; that is, frequencies such that the corresponding surface wavelengths are small compared to the container diameter. Specifically, the details of an experimental and a theoretical study are given, the results of which are felt to be useful for the prediction of disintegration under arbitrary gravity conditions. In addition, the results of a study of spray excited low frequency waves are also presented.

[1] Numbers in brackets refer to references listed at the end of this report.

II. PRELIMINARY OBSERVATIONS AND SCOPE OF STUDY

Experimental Observations

As a prelude to more detailed discussions of the problem at hand, it is desirable to first describe, in a qualitative fashion, what happens when a cylinder containing liquid is oscillated longitudinally at high frequency. Starting with an initially quiescent surface, as the excitation amplitude is increased from zero for a given frequency, a point is reached at which the interface suddenly begins to exhibit 1/2 subharmonic response in the form of individual wavelets, equally spaced over the liquid surface[2]. Figure 1 shows a photograph of such a response. Increasing the input amplitude further causes these wavelets to respond with greater amplitude until a point is reached at which the wavelets begin to disintegrate and form globules or spray particles as shown in Figure 2. The amount of spray and its trajectory height above the free surface are observed to depend, logically, upon the excitation amplitude. Once significant spray is formed as a result of further increasing the input amplitude, the response of the surface is noted to change in character, with the impacting of the particles back onto the interface seemingly exciting lower order modes (surface waves with longer wavelength and lower oscillation frequency). The wavelength of the spray-excited modes is observed to increase as the spray trajectory height is increased, and excitation of the first few modes to large amplitudes is easily accomplished under the right conditions. See Figure 3.

Summarizing, if a cylinder containing liquid is arbitrarily excited longitudinally at high frequency, the free surface may respond in one of the following ways, depending upon excitation amplitude:

(1) Harmonic symmetric response for sufficiently small excitation amplitudes for some input conditions;

(2) 1/2 subharmonic response in the form of equally spaced wavelets;

(3) 1/2 subharmonic response with some spray;

(4) 1/2 subharmonic response with considerable spray and attendant spray-excited lower order modes of possibly large amplitudes.

[2]Under some conditions, the surface was observed to respond harmonically in a symmetric fashion with standing waves in the form of concentric circles before the 1/2 subharmonic response set in.

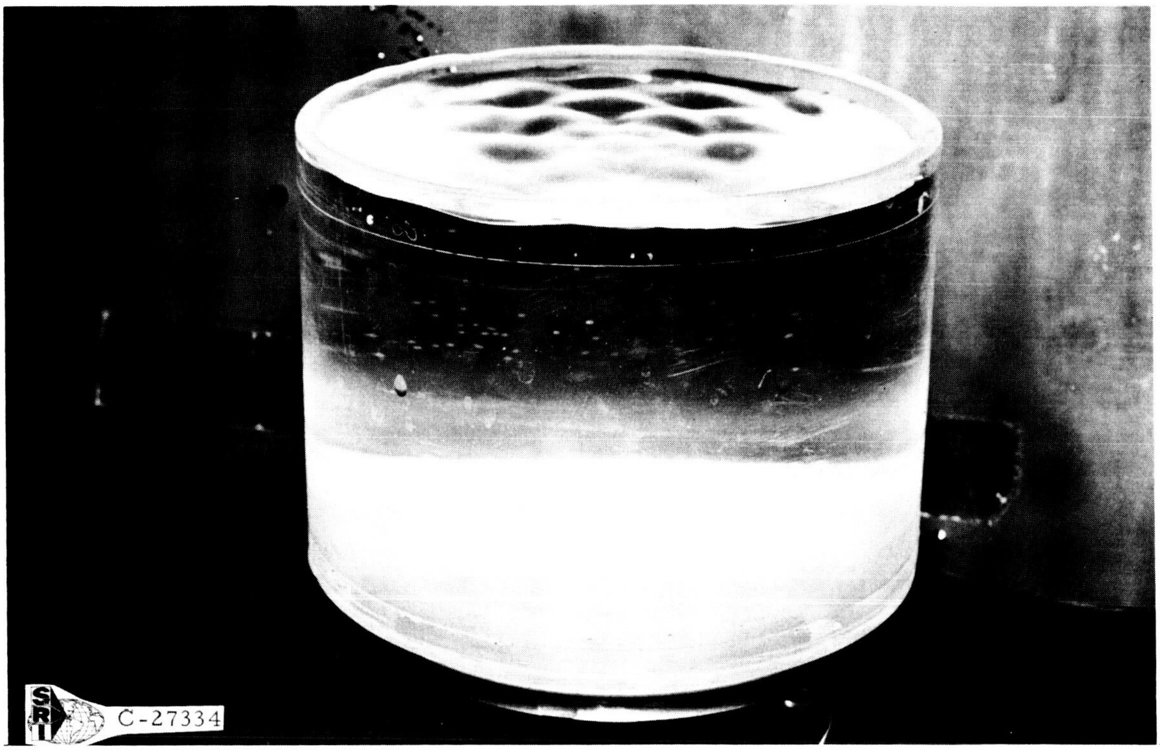


Figure 1. High Frequency 1/2 Subharmonic Response - Stable Waveform

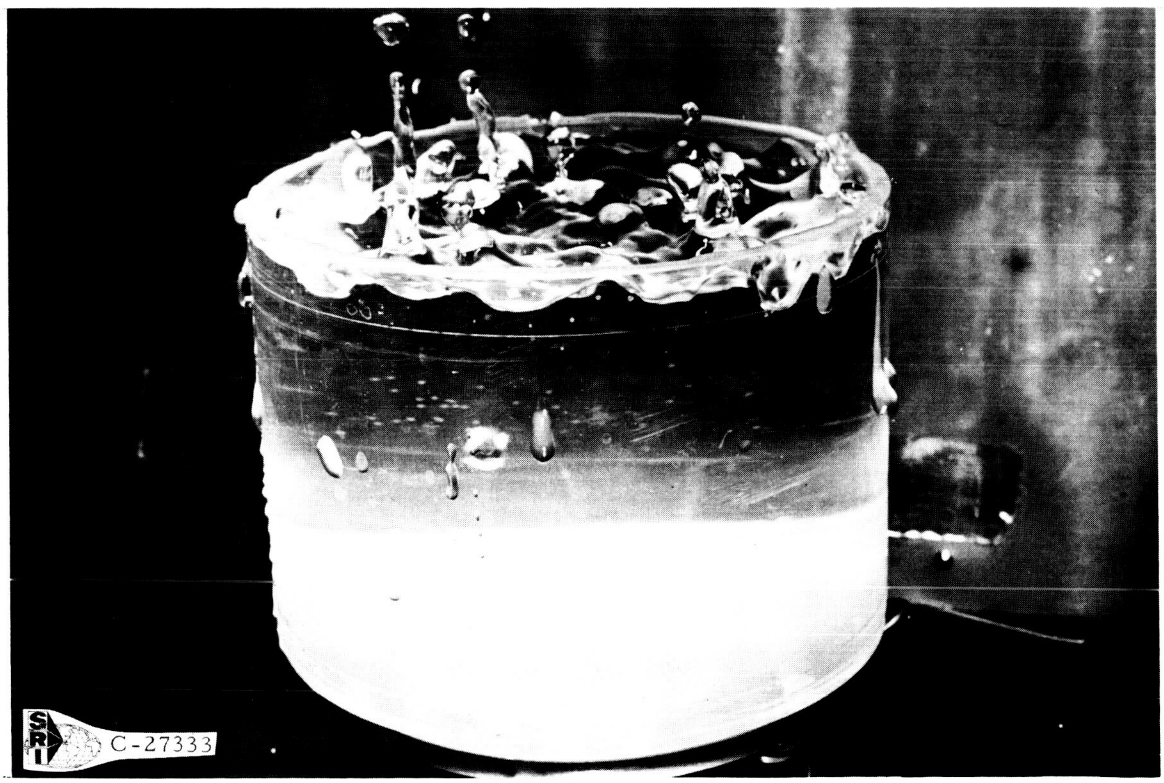


Figure 2. High Frequency 1/2 Subharmonic Response With Spray

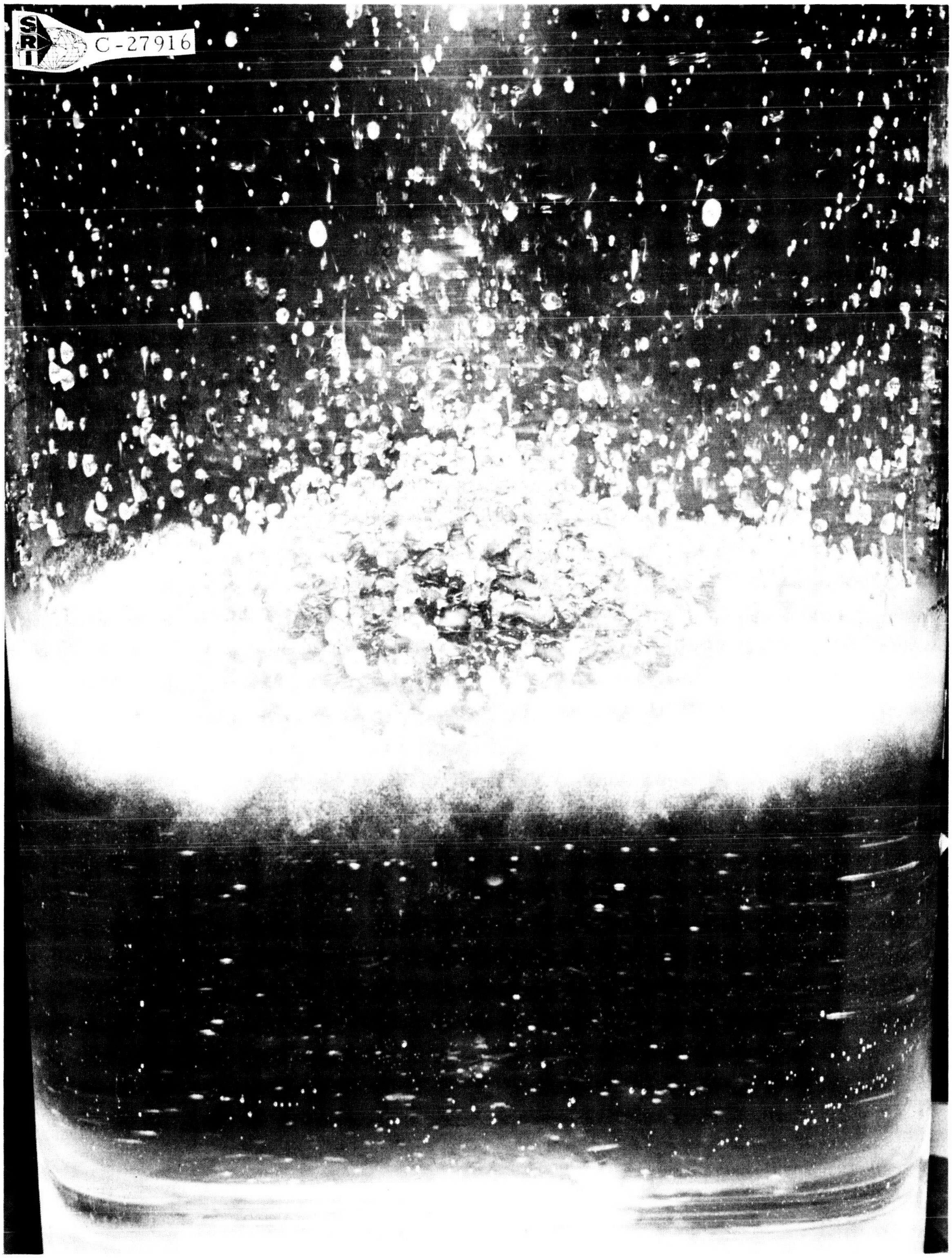


Figure 3. Spray Excited First Symmetric Mode In A
24.8-cm Cylindrical Container

Discussion of Previous Work

The various types of liquid surface responses recorded above will now be discussed in light of the work of previous investigators.

The first recorded observations of standing waves on the surface of a vertically vibrating liquid are due to Faraday [2] who noted a 1/2 subharmonic response of the surface. This was contrary to the experiments of Matthiessen [3] who later observed synchronous or harmonic response. This disagreement prompted Rayleigh [4] to further look into the matter, and he, like Faraday, observed 1/2 subharmonic response of the liquid surface.

A satisfactory theory to describe the stability of the surface of a liquid being vertically oscillated seems to have been first discussed by Benjamin and Ursell [5] and was based on a system of Mathieu equations. It should be noted that Rayleigh [6], as mentioned by Benjamin and Ursell, had earlier suggested that a special application of his theory of maintained vibrations, which resulted in a single Mathieu equation, might be extended to explain the 1/2 subharmonic surface response which had been observed. The work of Benjamin and Ursell does, however, explain both the synchronous response observed by Matthiessen and the 1/2 subharmonic response observed by Faraday and Rayleigh, hence, clearing up the disagreement.

Although it is possible, based on small amplitude theories such as those of Benjamin and Ursell, to define stability boundaries predicting input conditions required to initiate surface oscillations, successful determination of response amplitudes can only be made from a large-amplitude or nonlinear theory. Apparently, the first nonlinear analysis for liquid surface forced response was made by Yarymovych [7] for a rectangular tank. He used an analytical method similar to that earlier employed by Penney and Price [8] in their analysis of finite-amplitude standing waves. A recent study of the response of the liquid surface in a longitudinally excited cylindrical container has been reported by Dodge, Kana, and Abramson [9]. In this work, the stability boundaries for several modes were obtained analytically and verified experimentally. Further, the response amplitude was predicted from a nonlinear theory and found to compare rather well with test results.

In view of the work reported by previous investigators, as summarized above, methods for prediction of the instability and response of the first few modes in longitudinally excited containers can be considered rather well established at this time. Knowledge of the instability and response at high frequency, however, has received considerably less attention.

As noted in [9], the unstable regions for various surface wave modes overlap more and more as excitation frequency increases, hence, at high frequency, the 1/2 subharmonic response of some mode is virtually a certainty. This fact is readily displayed in Figure 4, which was taken from a report by Kana [10], and shows the general stability boundary of 1/2 subharmonic surface wave response for a longitudinally excited cylinder. One important bit of information which is lacking at this time is: What mode(s) will most probably be excited at high frequency in this region of considerable overlap of the stability boundary? In other words, for a given high frequency excitation, what is the mean wavelength of the surface waves? Another important question, for which no answer can be presently found in the literature, is: How can the conditions necessary for disintegration of the interface resulting from high frequency excitation be predicted?

As seen in Experimental Observations, the occurrence of small wavelength surface waves and spray or disintegration may not be the only consequences of high frequency excitation of a liquid containing cylinder. The discovery of spray-induced long wavelength surface waves of a very low order compared to the exciting frequency was apparently made by Yarymovych [7] in connection with his study of large-amplitude response. In addition, Yarymovych set down a hypothesis in an attempt to explain how the spray particles impacting back onto the liquid surface might cause a forcing of the low-order modes. He did not, however, present any experimental verification of this hypothesis.

In summary, it has been found through a brief examination of previously reported work that some of the surface phenomena observed during high frequency excitation of a liquid containing cylinder can be explained while some cannot. In particular, the following questions remain unanswered regarding high frequency excitation:

(1) What are the conditions necessary for surface disintegration?

(2) What modes are most probably excited for given input conditions?

(3) Can Yarymovych's spray hypothesis be experimentally verified?

(4) Can the occurrence of low-frequency spray-excited modes be predicted?

(5) What is the influence of random excitation upon surface disintegration and low-frequency spray-excited mode phenomena?

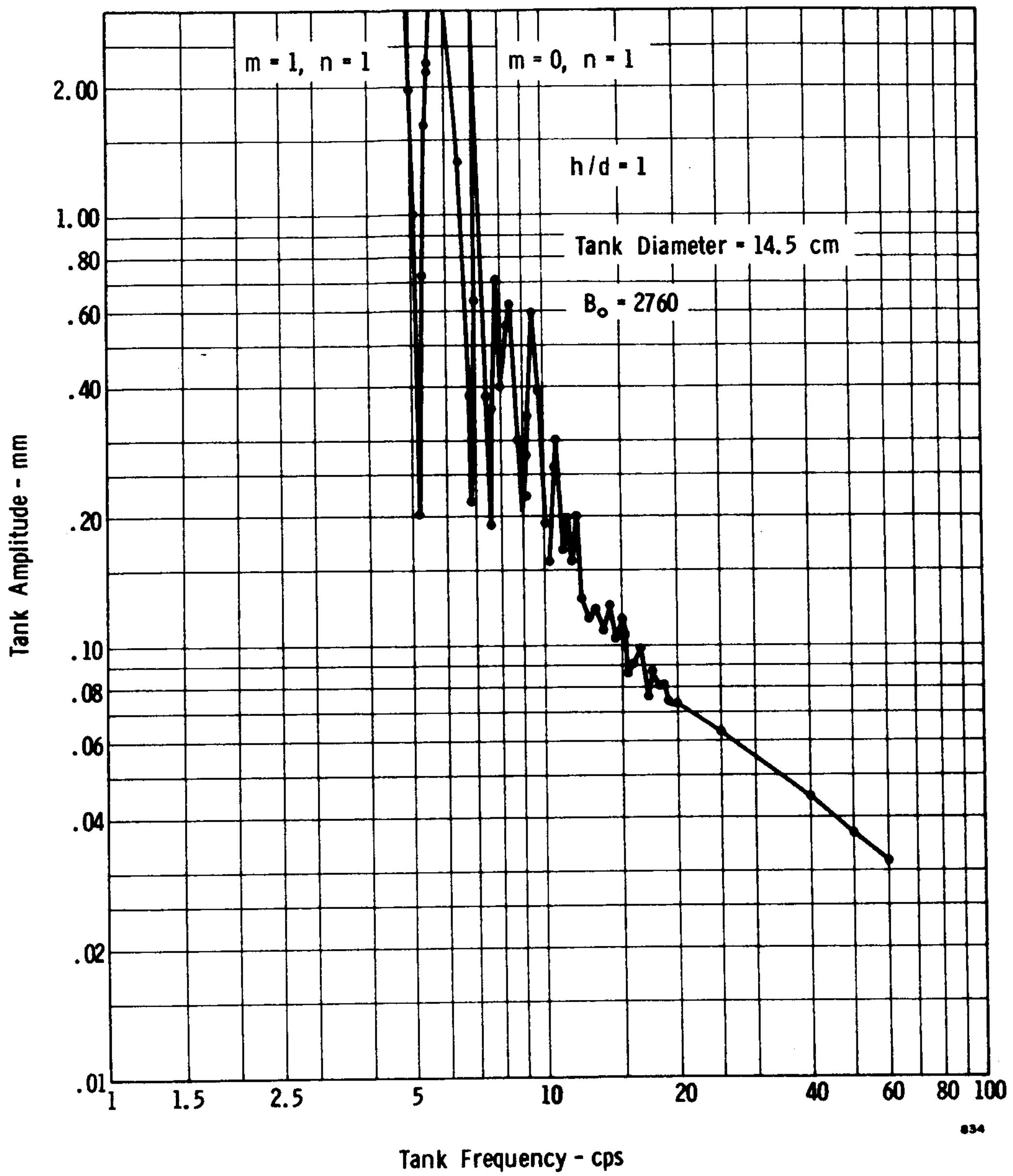


Figure 4. General Stability Boundary For Principal Parametric Resonance Of Liquid Surface In A Longitudinally Excited Rigid Cylinder

Scope of Present Study

The objective of the present study is to obtain answers to the previous questions and to interpret the results in such a manner as to be useful for low-gravity work.

In an effort to achieve this objective, the following scope of study has resulted:

(1) Experimentally determine the conditions necessary for surface disintegration.

(2) Develop a theory for the prediction of surface disintegration.

(3) Try to answer the question of what modes are most probably excited (most probable state) as a result of arbitrary high frequency excitation.

(4) Obtain experimental verification or negation of Yarymovych's hypothesis of the spray excitation of lower order modes.

(5) Define quantitatively the conditions necessary for the existence of spray-excited low-frequency modes.

(6) Gain some insight into the effect of random excitation.

III. EXPERIMENTAL STUDY OF DISINTEGRATION

The purpose of this section is to describe the apparatus, test procedure, and results of an experimental study conducted to determine the conditions necessary for disintegration of the liquid surface resulting from a high frequency excitation. Also, some additional observations are presented to help answer the question of what is the most probable state of a liquid surface for a high frequency excitation.

Apparatus and Test Method

The apparatus used for this study is shown in Figure 5. It consisted of a 9.5-cm diameter acrylic plastic cylinder, containing the test liquid and excited longitudinally by an electrodynamic shaker. A few tests were also conducted with a 24.8-cm cylindrical tank excited by a larger shaker (not shown). All tests were conducted in the following manner:

(1) The liquid to be tested was put into the test tank with sufficient depth being maintained so that the wavelength was smaller than the depth. Hence, damping resulting from nearness of the bottom could be considered negligible. For most tests, about a 2.5-cm depth was used. It should be noted that the use of greater depths indicated that coupled liquid-tank compressibility affected the results at high frequency--an undesirable complication.

(2) The liquid-tank system was excited longitudinally at a given frequency with the amplitude being carefully varied until the minimum amplitude necessary to give surface disintegration (the threshold of spray) was found. This procedure was repeated several times to ensure accurate data.

Experimental Results

The threshold of spray was obtained in the above manner for four liquids; water, ethyl alcohol, 50 percent by volume ethyl alcohol in water, and 50 percent by volume glycerine in water. The range of input frequency was 20 to 200 cps for most tests, although some tests were conducted somewhat above or below this range. The results of all tests are shown in Figure 6. The ordinate of this figure is dimensionless input acceleration, i.e., the input acceleration divided by the standard acceleration of gravity.

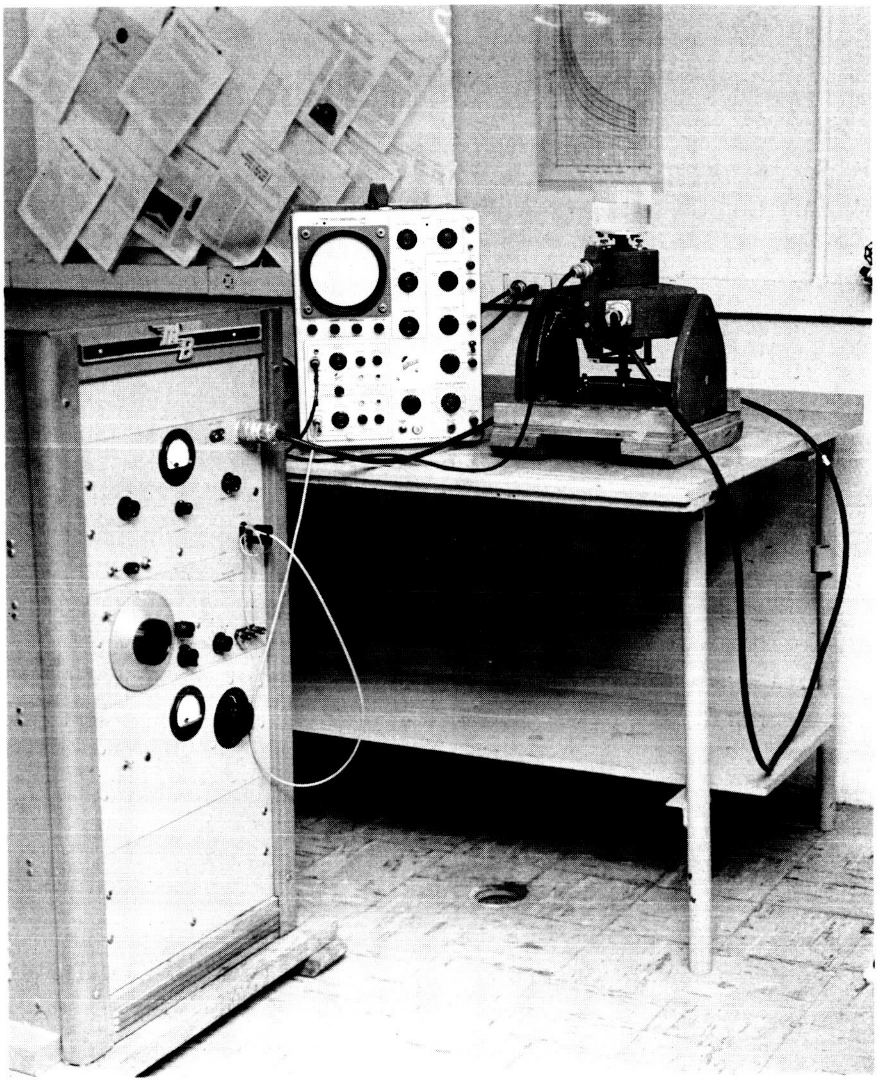


Figure 5. Experimental Apparatus Used To Obtain Threshold Of Spray

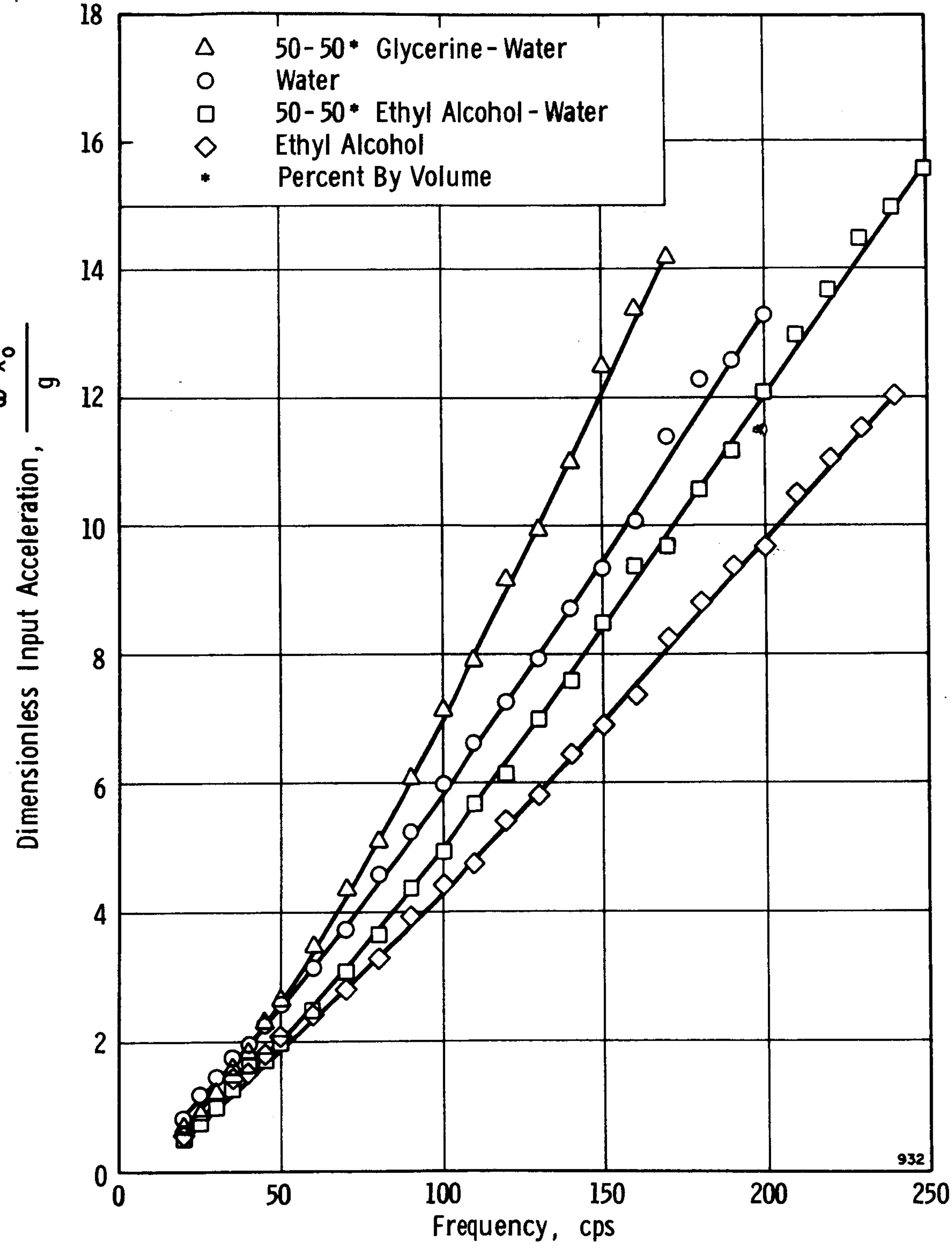


Figure 6. Experimental Threshold Of Spray

Examination of Figure 6 prompts the following observations:

(1) For a given liquid, the excitation acceleration necessary to produce disintegration is seen to increase roughly in direct proportion to frequency, indicating that the phenomena is highly dependent on excitation velocity.

(2) For a given input frequency, reducing the surface tension reduces the input acceleration needed to give spray.

(3) Increasing viscosity, for a given frequency, increases the threshold input acceleration.

(4) Below an input frequency of about 50 cps for the liquid tested, the effects of surface tension and viscosity seem to become very small, as all data tend to approach one curve, indicating gravity effects are predominant. This is logical since both the Reynolds number based on wavelength and the Bond number based on wavelength increase as frequency decreases.

Observations of the Most Probable State

While conducting the experiments to determine the threshold of spray, it was observed that, when the light conditions were properly adjusted, individual wavelets on the excited surface could be readily distinguished from overhead. Hence, overhead photographs might be used to determine the mean wavelength as a function of frequency and thus learn something about the most probable state for an arbitrary high frequency excitation. A series of photographs was therefore made of the liquid surface in a 24.8-cm cylindrical container for a rather limited range of excitation frequencies from 20 to 80 cps, water being the test liquid. Figure 7 shows a typical photograph which resulted, in this case, from a 20-cps excitation. Each picture was made with an excitation amplitude just below the threshold of spray value. The following observations were made from the photographs:

(1) Remembering that this was high frequency excitation so that the mean wavelength was at least one order of magnitude smaller than the tank diameter, it was found that at each frequency the surface could be described by "a rather equal distribution of wavelets, each of approximately the same wavelength."

(2) While not so obvious from the photographs as from actual observations of the liquid surface, the action of each wavelet looks qualitatively like the action of the first symmetric mode in a cylindrical, longitudinally excited tank.

Figure 7. Overhead Photograph Showing 1/2 Subharmonic Response With Equal Distribution Of Wavelets

(3) Measured values of wavelength taken from the photographs corresponded well with values calculated for the first symmetric, small-amplitude capillary wave in a cylindrical container with a diameter corresponding to the wavelength.

From these observations, it was concluded that the most probable state of the liquid surface responding to an arbitrary high frequency excitation is one for which:

(1) There is an equal distribution of more or less symmetric individual wavelets over the surface responding at 1/2 the excitation frequency.

(2) The mean wavelength corresponds approximately to the calculatedlinear or small-amplitude value for a single symmetric capillary wave.

IV. THEORY FOR SURFACE DISINTEGRATION

The purpose of this section is to present the details of a theory developed to predict the minimum conditions necessary for surface disintegration, i.e., the threshold of spray and to show a comparison of this theory with the experimental results of Section III.

Briefly, a solution is sought to the governing equations which describes the large-amplitude, nonlinear response of the liquid surface with surface tension effects being important. As will be seen, the solution obtained predicts a threshold input acceleration, for a given frequency and wavelength, beyond which the large-amplitude response is unstable and (it is hypothesized) results in surface disintegration. It is very important to distinguish between the large-amplitude instability to be discussed here and the small-amplitude instability which results in the onset of surface oscillation for an initially plane surface.

Formulation

For the purposes of this analysis, the fluid will be assumed nonviscous and incompressible. A gas of negligible density will be taken as the upper fluid, the interfacial surface tension will be included, and wall contact effects will be neglected, i.e., the mean surface is assumed flat. The liquid surface will be assumed responding to a longitudinal excitation of the liquid-tank system with an equal distribution of wavelets, oscillating at 1/2 the excitation frequency.

As a result of the occurrence of an equal distribution of wavelets on the surface, it is necessary only to analyze the action of a single wavelet as though it were contained in a hypothetical cylindrical tank with a diameter corresponding to the wavelength. Hence, the wavelet is the first symmetric mode in the hypothetical tank. Figure 8 shows the liquid surface to be analyzed and the coordinate system to be used, which is a symmetric cylindrical system fixed at the mean liquid level and corresponding to the hypothetical tank for one wavelet. Note that this is a coordinate system moving with the tank. Unless otherwise indicated, all further discussion will be for the "liquid-tank system" of a single wavelet.

The motion of the liquid under consideration, being incompressible and irrotational, is obtainable from a velocity potential $\phi(r, z, t)$ which is related to the vector velocity by

$$\overline{v} = \nabla\phi \quad (1)$$

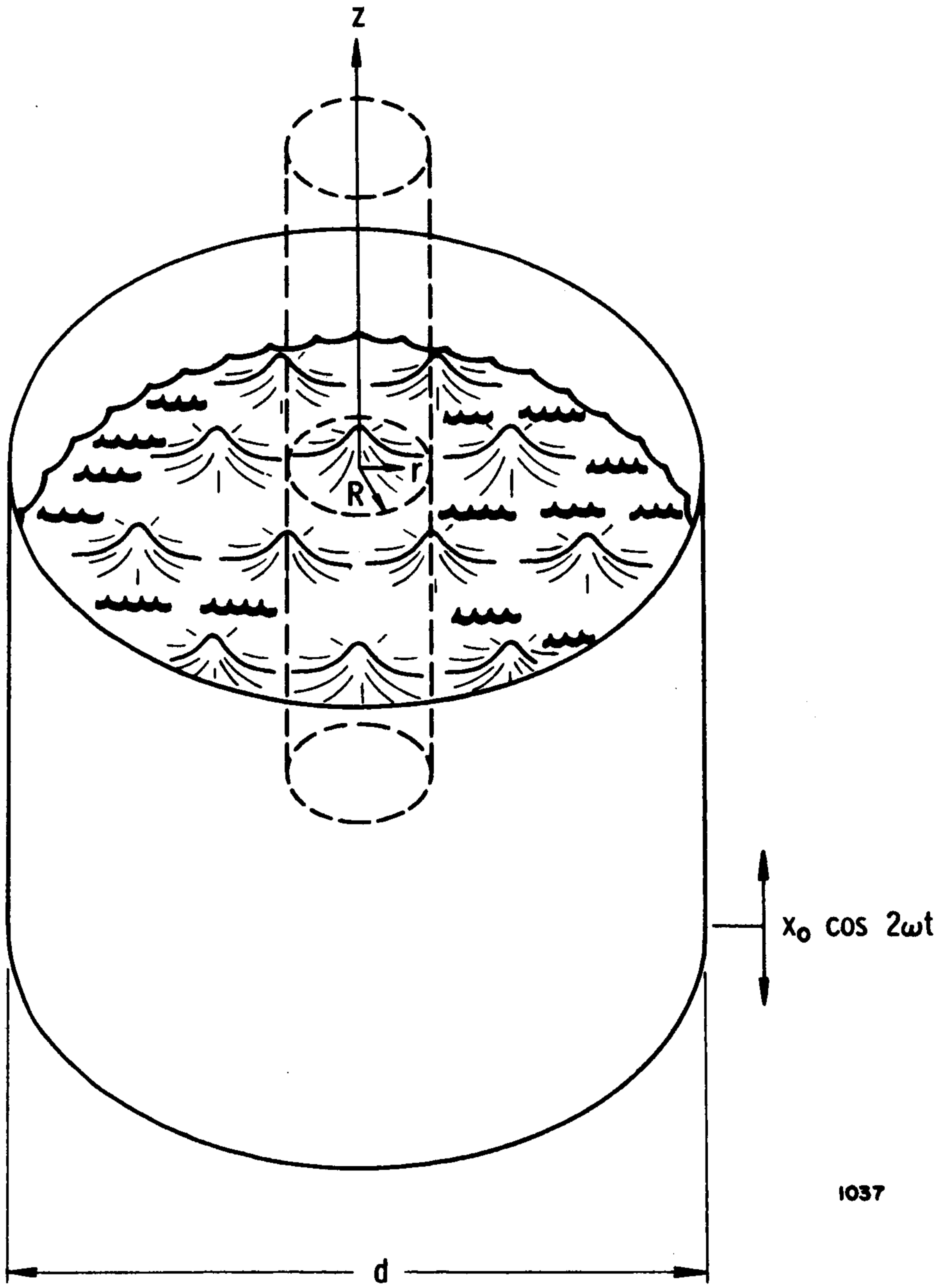


Figure 8. Coordinate System For Hypothetical Wavelet

where ϕ is governed by Laplace's equation

$$\nabla^2\phi = 0 \tag{2}$$

within the confines of the tank and the free surface; i.e., $r \leq R$ and $-h \leq z \leq \eta(r,t)$, η defining the height of the surface above the reference plane $z = 0$.

The boundary conditions which must be satisfied are:

(1) The liquid velocity normal to the tank walls and bottom must be zero, hence,

$$\frac{\partial\phi}{\partial r} = 0 \quad \text{at} \quad r = R \tag{3}$$

$$\frac{\partial\phi}{\partial z} = 0 \quad \text{at} \quad z = -h \tag{4}$$

(2) The liquid surface moves with the liquid (kinematic surface boundary condition) which implies

$$\frac{D\eta}{Dt} = 0 \quad \text{on} \quad z = \eta$$

or

$$\frac{\partial\eta}{\partial t} - \frac{\partial\phi}{\partial z} + \frac{\partial\phi}{\partial r}\frac{\partial\eta}{\partial r} = 0 \quad \text{on} \quad z = \eta \tag{5}$$

(3) The pressure across the interface results from surface tension and the surface curvature, hence,

$$\frac{\partial\phi}{\partial t} + \frac{1}{2}(\nabla\phi \cdot \nabla\phi) + G(t)\eta = \frac{\sigma}{\rho}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{r\frac{\partial\eta}{\partial r}}{\sqrt{1+\left(\frac{\partial\eta}{\partial r}\right)^2}}\right]\right\} \tag{6}$$

where $G(t) = g - 4\omega^2 x_0 \cos 2\omega t$ represents the effective gravity for the moving-tank system. The excitation frequency is 2ω, and ω is the wavelet oscillation frequency.

Solution

The method of solution essentially follows that used by Dodge, Kana, and Abramson [9] for analysis of the nonlinear response of a liquid surface in a longitudinally excited cylinder. The difference will be that surface tension effects have been included in the present analysis.

Assuming the tank is effectively infinite in depth (h >> R), a solution of Eq. (2) satisfying (3) and (4) is

$$\phi = \sum_{n=1}^{\infty} a_n J_o(\lambda_n r)\, e^{\lambda_n z} \tag{7}$$

where the λ_n are the roots of $J_1(\lambda_n R) = 0$. Similarly, it is assumed that η is of the form

$$\eta = \sum_{n=1}^{\infty} b_n J_o(\lambda_n r) \tag{8}$$

The coefficients in Eqs. (7) and (8), a_n and b_n, will be taken as functions of time.

The procedure at this point is to substitute the forms for ϕ and η, as given by Eqs. (7) and (8), into the nonlinear boundary conditions, Eqs. (5) and (6), and then expand the resulting two equations in terms of a series of Bessel functions. This results in an equation corresponding to Eq. (5) of the form

$$\sum_{n=1}^{\infty} \left\{ \frac{db_n}{dt} + a_n D_n(a_k, b_k) \right\} J_o(\lambda_n r) = 0, \qquad k = 1 \rightarrow \infty \tag{9}$$

and for Eq. (6), the form

$$\sum_{n=1}^{\infty} \left\{ \frac{da_n}{dt} + G(t) b_n + F_n(a_k, b_k) + \frac{\sigma}{\rho} H_n(a_k, b_k) \right\} J_o(\lambda_n r) = 0 \tag{10}$$

Here D_n, F_n and H_n are nonlinear functions of a_k and $b_k (k = 1 \rightarrow \infty)$, thus, nonlinear functions of time. Eqs. (9) and (10) are satisfied by setting the coefficients equal to zero, hence resulting in two infinite sets of ordinary nonlinear differential equations to be solved simultaneously. Obviously, only some truncated set of these equations can be solved. In [9], only the simultaneous effects of three modes (λ_1, λ_2, λ_3) were considered in studying the 1/2 subharmonic response of the first symmetric mode (λ_1). The results of [9] seem to indicate that the principal benefit gained from the inclusion of the λ_2 and λ_3 mode effects in the study of the λ_1 mode was a better description of the wave shape. Otherwise, the response of the λ_1 mode seems influenced little by λ_2 and λ_3 mode effects. In view of this, the symmetric response of a single wavelet in its hypothetical container will be assumed adequately described by the action of the λ_1 mode only. Hence, ϕ and η will be taken as

$$\phi = a_1 J_o(\lambda_1 r) e^{\lambda_1 \eta} \qquad \text{on } z = \eta \tag{11}$$

and

$$\eta = b_1 J_o(\lambda_1 r) \tag{12}$$

Also, use will be made of the dimensionless quantities

$$\delta = \frac{\omega}{\omega_1} \quad \text{with } \omega_1 = (\lambda_1 g)^{1/2}$$

$$\tau = \omega_1 t$$

$$\alpha_1 = \frac{\lambda_1^2}{\omega_1} a_1$$

$$\beta_1 = \lambda_1 b_1$$

and

$$\epsilon = \lambda_1 x_o$$

In view of the assumed forms of ϕ and η as given by Eqs. (11) and (12), the resulting dimensionless form of Eq. (9), correct to seventh order, is[3]

$$\dot{\beta}_1 - \alpha_1(1.00 + 0.1761\beta_1 + 0.06898\beta_1^2 + 0.01198\beta_1^3 + 0.002195\beta_1^4 + + 0.0003069\beta_1^5 + 0.00003929\beta_1^6) = 0 \tag{13}$$

[3]The numerical constants in Eqs. (13) and (14) resulted from the integrations necessary to obtain the Bessel function expansions indicated by Eqs. (9) and (10). Essentially, then, these constants arise from using a Galerkin procedure with respect to the λ_1 liquid surface mode. They were obtained numerically with the aid of a digital computer.

The corresponding dimensionless form of Eq. (10) correct to seventh order (with polynomial approximation for the surface tension contribution)[4] is

$$\ddot{\alpha}_1(1.00 + 0.3523\beta_1 + 0.2070\beta_1^2 + 0.04792\beta_1^3 + 0.01097\beta_1^4 +$$
$$+ 0.001842\beta_1^5 + 0.0002750\beta_1^6) + (1 - 4\delta^2\epsilon \cos 2\delta\tau)\beta_1 +$$
$$+ \alpha_1^2(0.5519\beta_1 + 0.2107\beta_1^3 + 0.03017\beta_1^5) +$$
$$+ \left(\frac{58.73}{B_L}\right)(\beta_1 - 0.0892\beta_1^3 + 0.00442\beta_1^5) = 0 \qquad (14)$$

where B_L represents the Bond number based on wavelength, L (L corresponds to the diameter of the hypothetical container or 2R). The next step will be to substitute α_1 from Eq. (13) into (14); however, before doing so it is well to point out the following: In obtaining Eqs. (13) and (14), a balance has been maintained by retaining up to seventh order contributions on all nonlinear terms (except the surface tension). It is important to maintain this balance by retaining only product terms up to seventh order which result upon substitution of Eq. (13) into (14). Elimination of α_1 between Eqs. (13) and (14) gives

$$\ddot{\beta}_1(1 + 0.3388\beta_1^2 + 0.04027\beta_1^4 + 0.002541\beta_1^6) +$$
$$+ \dot{\beta}_1^2(0.3519\beta_1 + 0.1523\beta_1^3 + 0.02402\beta_1^5) +$$
$$+ (1 - 4\delta^2\epsilon \cos 2\delta\tau)\beta_1(1.0 + 0.1689\beta_1^2 + 0.01337\beta_1^4 +$$
$$+ 0.0006330\beta_1^6) + \frac{58.73}{B_L}(\beta_1 + 0.07972\beta_1^3 + 0.002633\beta_1^5) = 0 \qquad (15)$$

Eq. (15) is then the nonlinear governing relation for the temporal response of the first symmetric mode (λ_1) in a hypothetical container of diameter L and correct to seventh order. In the previous study [9] of the forced nonlinear

[4]This representation of the surface tension contribution was obtained through a "polynomial fit" of the exact representation, which was calculated numerically. This procedure was necessary because of a divergence problem with a conventional power series representation.

response of the first symmetric mode (excluding surface tension) in a cylindrical container, it was shown that the most elementary stable steady-state solution is the approximation $\beta_1 = A \sin \delta\tau$.[5] This will be the form of solution sought here, the reason being to, hopefully, result in an algebraically simple yet accurate form. Now, substituting the assumed steady-state solution $\beta_1 = A \sin \delta\tau$ into Eq. (15) gives

$$\delta^2\epsilon = \frac{\delta^2(1.0 + 0.1661 A^2 + 0.006131 A^4 - 0.0004869 A^6)}{(2.0 + 0.3378 A^2 + 0.02507 A^4 + 0.001108 A^6)}$$

$$- \frac{(1.0 + 0.1267 A^2 + 0.008356 A^4 + 0.0003462 A^6)}{(2.0 + 0.3378 A^2 + 0.02507 A^4 + 0.001108 A^6)}$$

$$- \left(\frac{58.73}{B_L}\right) \frac{(1.0 + 0.05979 A^2 + 0.001646 A^4)}{(2.0 + 0.3378 A^2 + 0.02507 A^4 + 0.001108 A^6)} \qquad (16)$$

Eq. (16) now gives the theoretical dimensionless input acceleration, $\omega^2 x_o / g = 4\delta^2\epsilon$, in terms of δ (dimensionless wavelet frequency), A (dimensionless wavelet amplitude), and B_L. However, before use can be made of Eq. (16), it is necessary to specify wavelength in terms of frequency for the liquid surface. Based on the information gained in Section III concerning the most probable state, it is postulated that the wavelength of each wavelet corresponds to a linear natural frequency which is 1/2 the frequency of excitation; hence,

$$\delta^2 = \frac{58.73}{B_L} + 1.0 \qquad (17)$$

with, it will be recalled

$$\delta^2 = \frac{\omega^2}{\lambda_1 g} = \frac{\omega^2 d\ell}{7.66\, g} \qquad (18)$$

and

$$B_L = \frac{\rho g d^2 \ell^2}{\sigma} = B_d \cdot \ell^2 \qquad (19)$$

Here, ℓ is the dimensionless wavelength based on actual container diameter, d.

---

[5] The form $\beta_1 = A \sin \delta\tau$ is the first term of

$$\beta_1 = \sum_{j=1}^{\infty} \left\{ A_j \sin j\delta\tau + B_j \cos j\delta\tau \right\} \quad , \quad j = 1, 3, 5$$

with $B_1 = 0$, since $\cos \delta\tau$ is an unstable contribution.

Using Eqs. (16) and (17), the dimensionless input acceleration has been calculated as a function of response amplitude and excitation frequency for three different container Bond numbers, B_d (corresponding to the three liquids tested experimentally). Figure 9 shows a plot of dimensionless input acceleration versus response amplitude for several input frequencies with a container Bond number corresponding to water in the test tank. Note that there is a maximum acceleration point for each curve, indicating that once a certain input acceleration is achieved, corresponding to this maximum point, no increase in input acceleration is needed to give further increases in wavelet amplitude. Hence, for an input acceleration smaller than the critical (maximum) value, the wavelet response will be stable. For an input acceleration greater than the critical value, the response is unstable and should lead to surface disintegration. It is therefore hypothesized that the critical acceleration value predicted by this theory will be an indication of the input necessary to give surface disintegration.

Comparison of Theory with Experiments

In order to test the validity of the previous stated hypothesis, a comparison has been made between the theory and experiment by plotting the predicted critical acceleration as a continuous function of frequency along with the test results. This comparison is shown in Figures 10, 11, and 12, corresponding to three of the liquids tested. It may be immediately noted that, in each case, the experimental threshold of spray results are greater than the predicted instability values by approximately a constant factor (the order of 1 g). Recall, however, that the theory predicts a condition of instability for which the wave amplitude tends to grow; this might or might not correspond to a condition for which liquid globules will actually leave the surface. The validity and significance of the theory therefore remains, at this point, undetermined.

In view of this question concerning the meaning and usefulness of the theory, it was decided to investigate experimentally the possibility of observing the predicted instability and see if it corresponded to a distinct and separate condition from the threshold of spray. Utilizing the test apparatus shown in Figure 5, the surface response was carefully observed while the excitation amplitude was being slowly increased at a given frequency. It was found that there is a large amplitude instability of the type predicted by the theory, the features of which can be qualitatively described as follows:

(1) The instability is obvious in that, as the excitation amplitude is slowly increased, a point is reached (focusing attention on a single wavelet) for which the wavelet response suddenly changes from a uniform 1/2 subharmonic motion with a mode

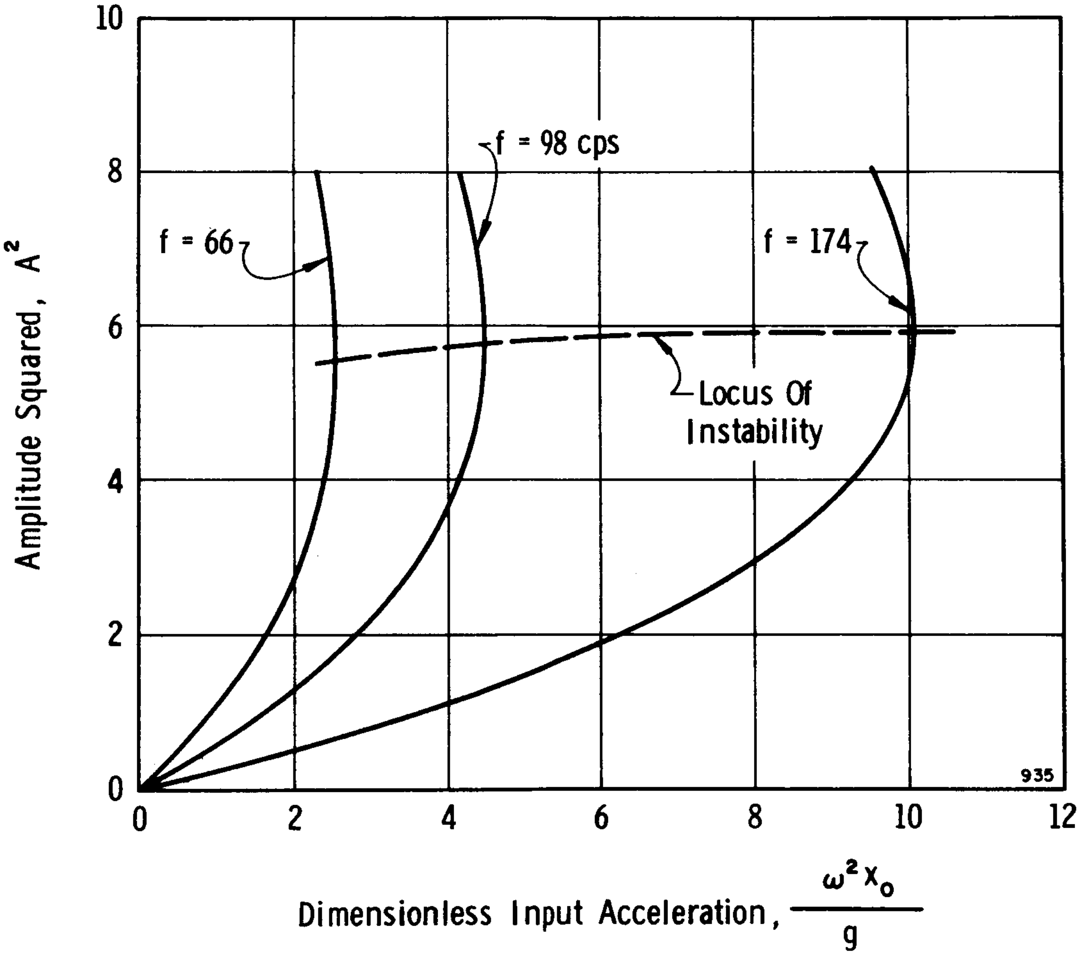


Figure 9. Theoretical Dimensionless Wavelet Amplitude Squared As A Function Of Dimensionless Input Acceleration

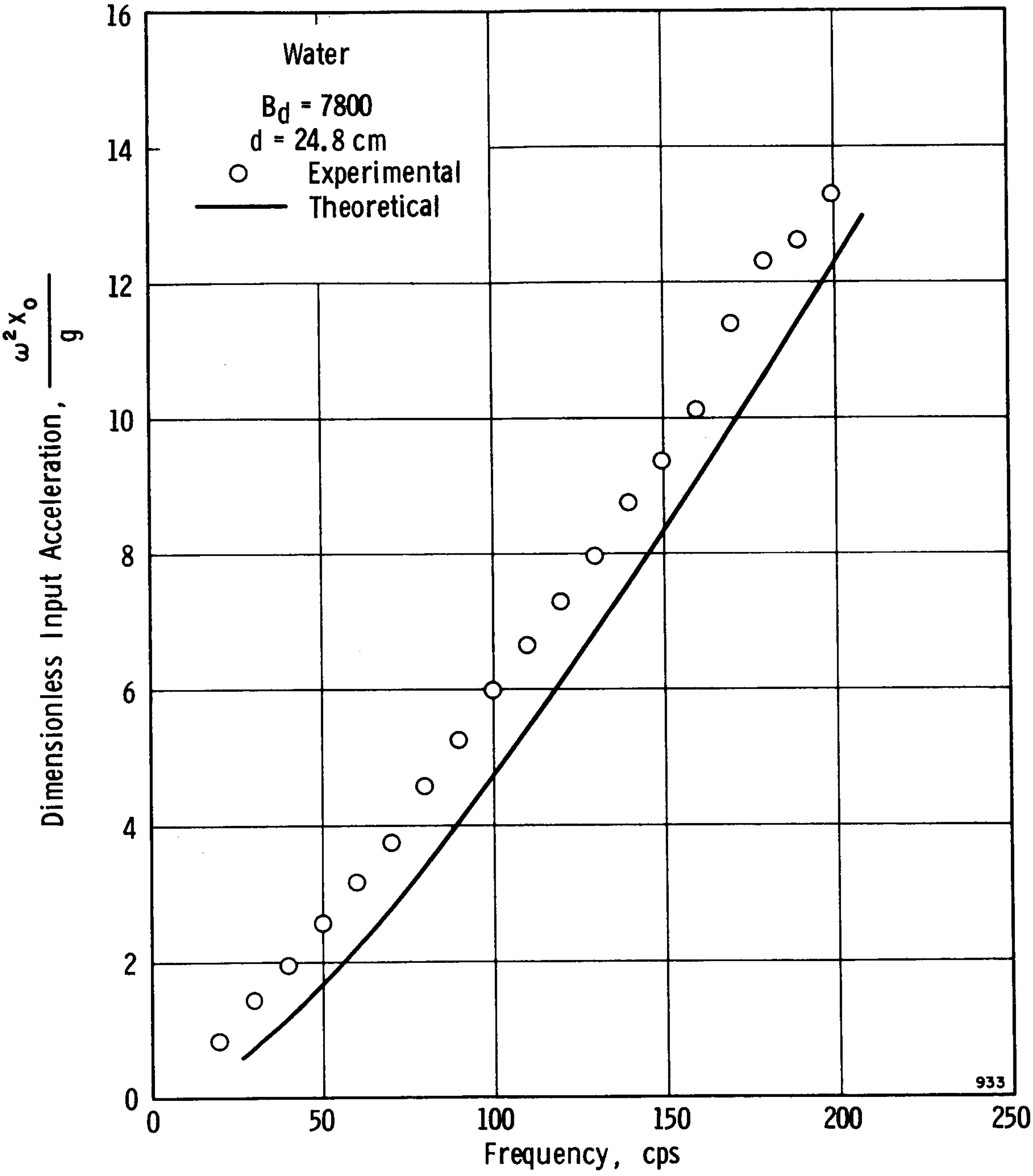


Figure 10. Comparison Of Experimental And Theoretical Threshold Of Spray For Water

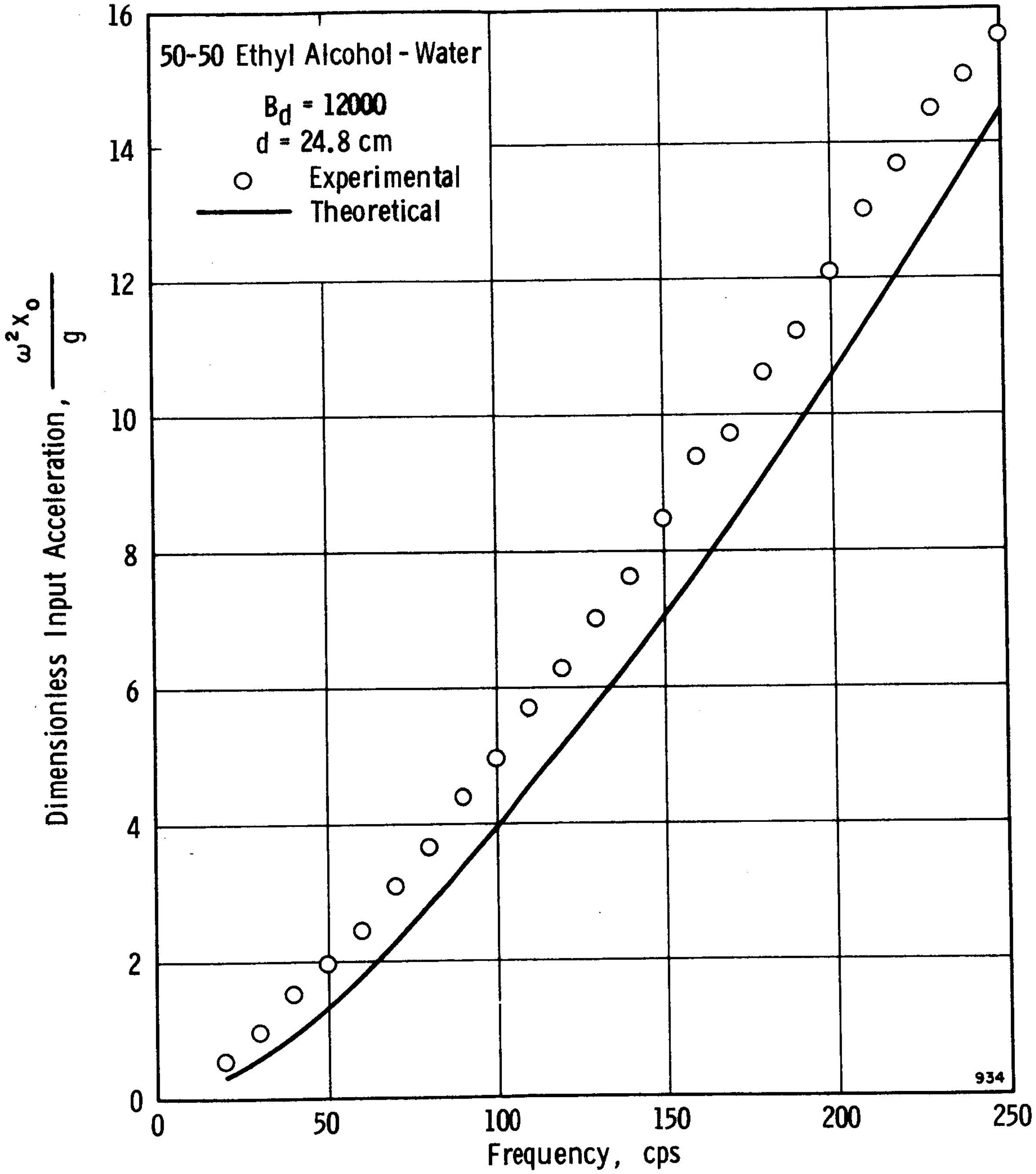


Figure 11. Comparison Of Experimental And Theoretical Threshold Of Spray For Ethyl Alcohol - Water Mixture

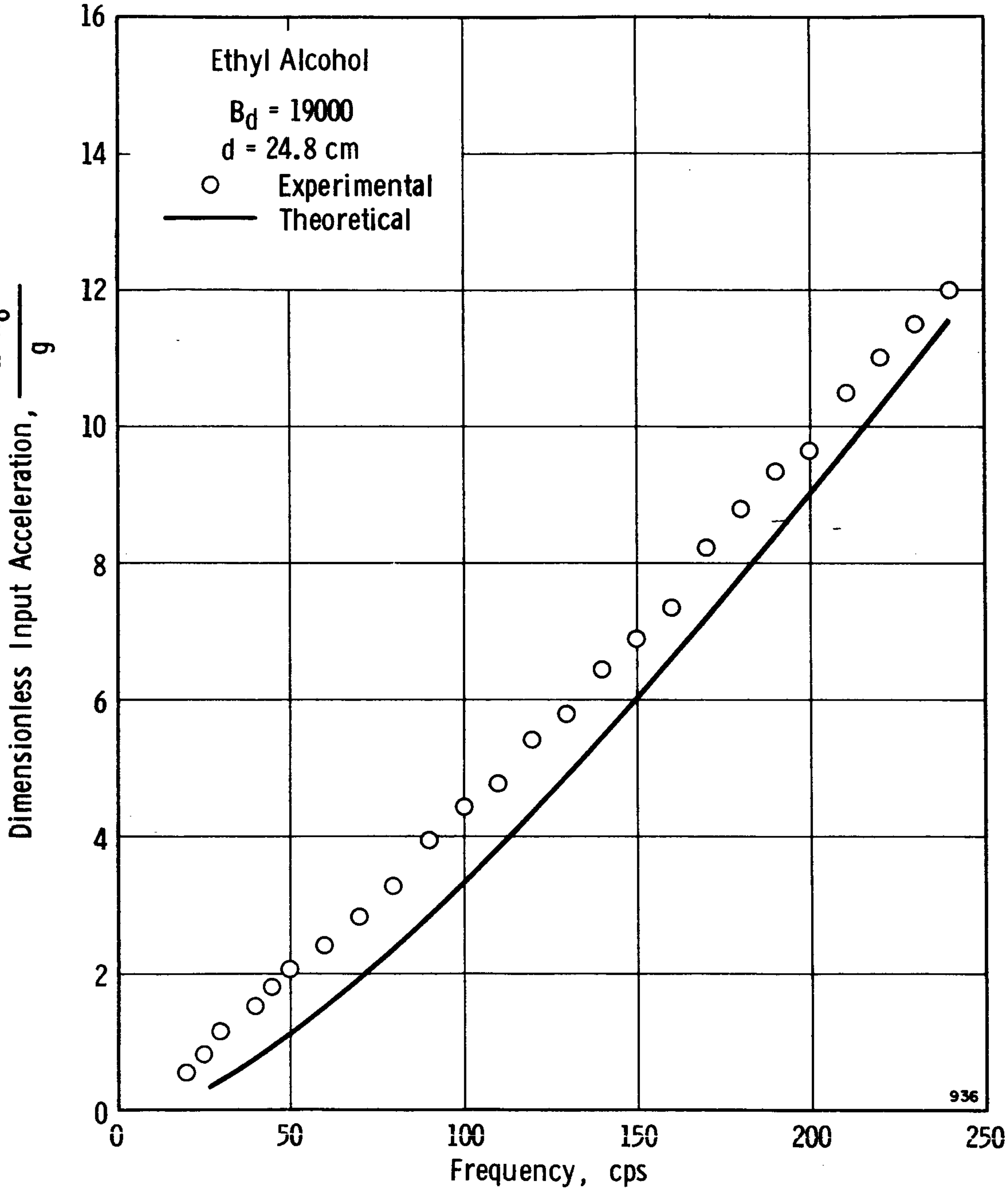


Figure 12. Comparison Of Experimental And Theoretical Threshold Of Spray For Ethyl Alcohol

shape like that of Figure 13a to a nonuniform "beating motion" with a mode shape similar to that of Figure 13b.

(2) There is no clearcut input amplitude for which the wavelets over the entire surface suddenly begin to show this instability. Rather, the condition builds up over some input range, and, when the majority of the surface seems to be exhibiting this phenomena, a gradual transition to a spray condition occurs upon further increasing input amplitude. Figure 14 is a photograph of the surface response in the 9.5-cm diameter test container with a condition for which part of the wavelets are exhibiting this instability and part are not (arrows indicate the unstable wavelets).

(3) It was found that the theoretical instability predictions corresponded to a condition for which the majority of the surface wavelets demonstrated this instability, but for which there was no actual surface disintegration (no spray).

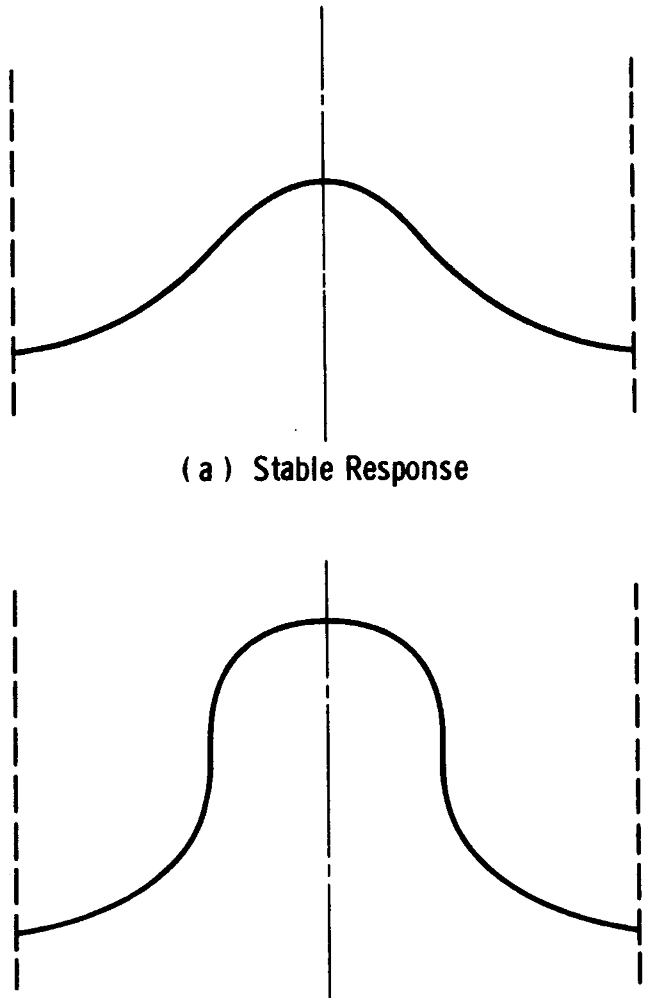
Judging from the results of the previous comparison and discussion, it is believed that the theory is of value in making <u>conservative predictions</u> of surface disintegration due to forced vibration of the liquid. In particular, the theory allows such predictions to be made for <u>any gravity</u> condition and should be beneficial for low gravity studies.

<u>Low Gravity Application of Theory</u>

To aid in the rapid prediction of surface disintegration under low gravity conditions, Figure 15 has been prepared from the theory. As can be seen from the figure, one curve is generally valid for all Bond numbers (B_d) less than about 100. For Bond numbers greater than this, separate curves are required but have not been plotted. To make use of this figure, it is necessary only to calculate the quantities $(\omega^2 x_o/g) \cdot B_d$ and $(\omega^2 d/g) \cdot B_d$ from known information and then determine if the corresponding point lies above or below the curve. Points above the curve generally indicate there will be surface disintegration, and points below indicate there will not be disintegration. It must be remembered that the theory does not predict the degree of severity of disintegration, only whether it will or will not occur. In this light, the following rules of thumb may help in assessing the severity of disintegration:

(1) Points on or slightly above the curve indicate little, if any, disintegration.

(2) Values of $(\omega^2 x_o/g) \cdot B_d$ about 50 percent greater than the curve values indicate gross disintegration with the majority of the wavelets participating. Even larger values of $(\omega^2 x_o/g) \cdot B_d$ indicate increasing particle trajectory heights.



(a) Stable Response

(b) Unstable Response

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Figure 13. Mode Shapes For Stable And Unstable Large Amplitude Wavelet Response

Figure 14. Photograph Of Unstable 1/2 Subharmonic Large Amplitude Response

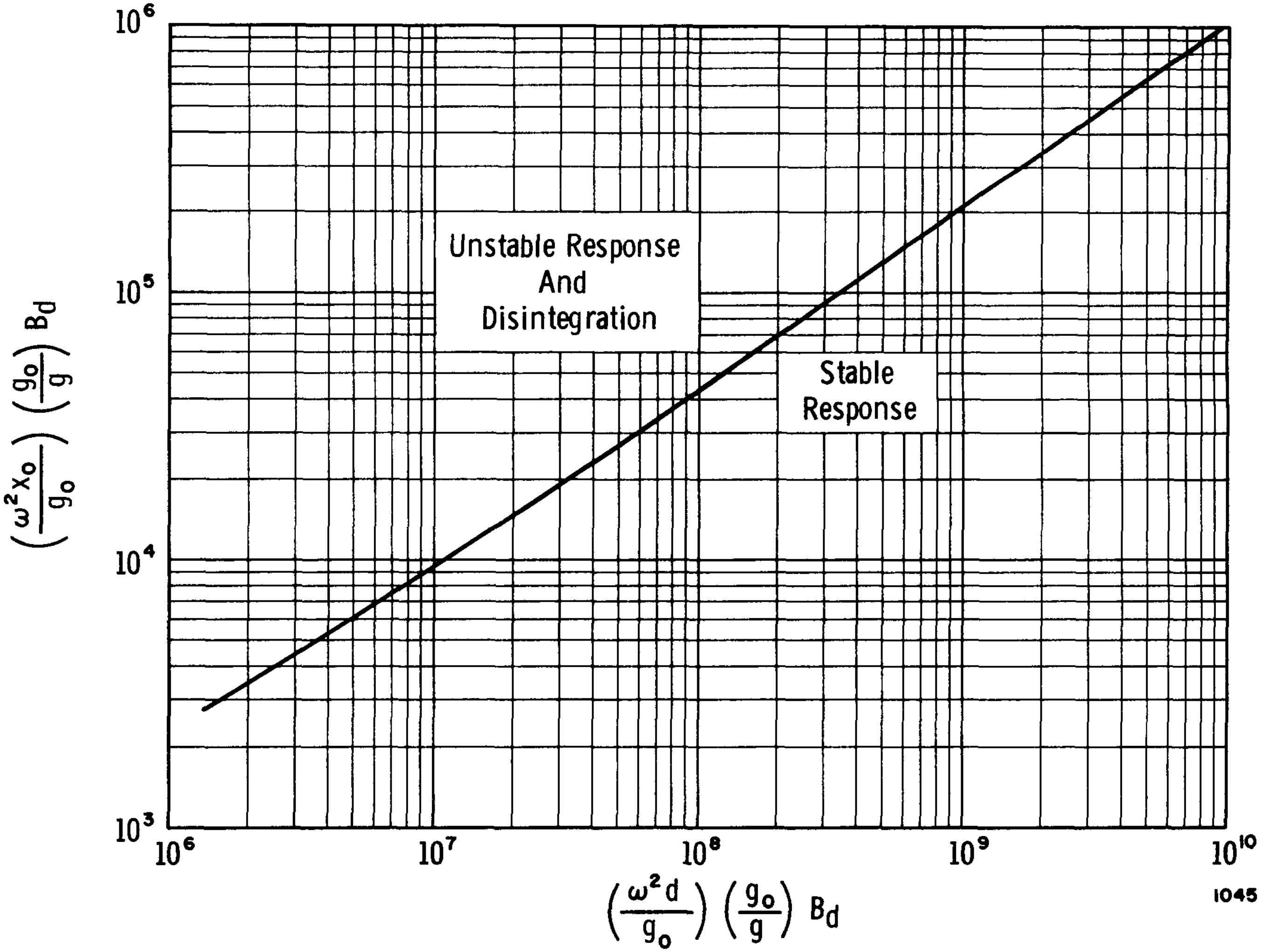
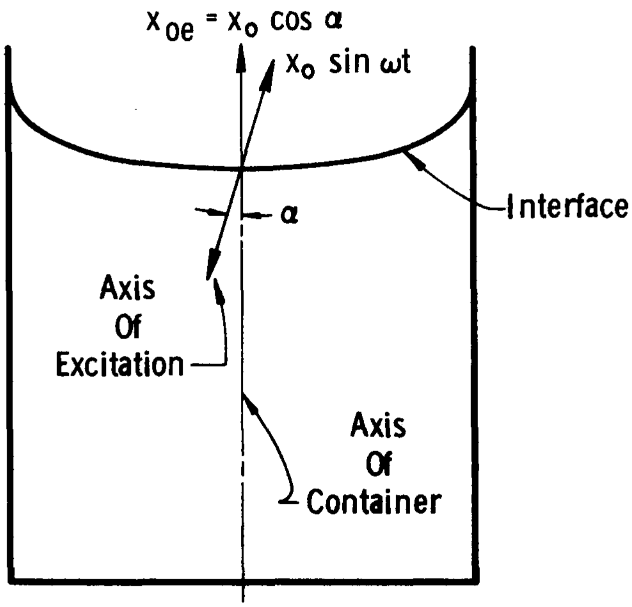


Figure 15. Dimensionless Threshold Of Surface Disintegration Valid For Bond Numbers $B_d < 100$

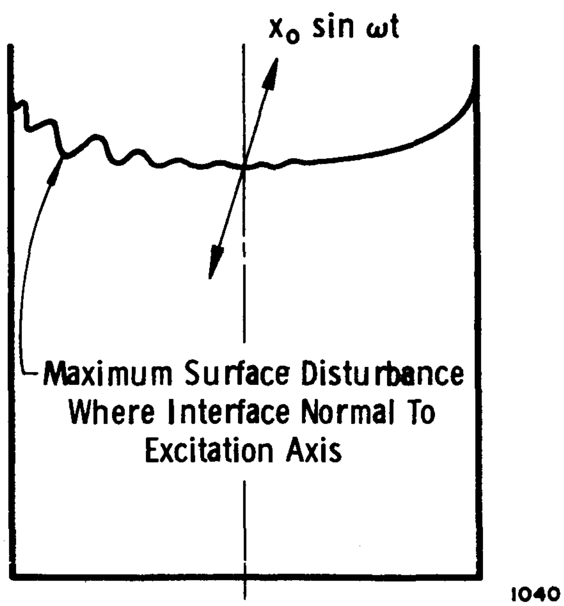
One final bit of discussion is in order concerning the use of the theory. Recall that the mean surface was assumed to be plane and perpendicular to the axis of excitation. However, for small Bond number conditions, the mean surface may assume a more nearly spherical shape with only one point being tangent to a plane perpendicular to the excitation axis. This does not appear to detract from the usefulness of the theory if the following considerations are made:

(1) The wavelengths under consideration here are small compared to the container diameter and hence to the radius of curvature of the meniscus. This means that, to a first approximation, the surface, as it appears to a single wavelet, is plane.

(2) Because of the nonplanar meniscus, the excitation axis is perpendicular only to a portion or to a single point of the surface as shown in Figure 16a. It seems logical to assume that only those portions of the surface which are perpendicular (or nearly so) to the excitation axis can be treated directly with the theory. Predictions of surface disintegration for those portions of the meniscus not approximately normal to the excitation axis can probably be made by using the local effective or normal component of the excitation amplitude, x_{oe}, in the computation of $(x_o\omega^2/g) \cdot B_d$; hence, $(x_{oe}\omega^2/g) \cdot B_d$. The local effective component is evaluated as shown in Figure 16a. Figure 16b illustrates the type of surface response which might be expected, with the largest amplitude perturbations in the regions with the largest effective excitation amplitudes.



(a) Effective Excitation For Nonplanar Interface



(b) Expected Surface Response For Nonplanar Interface

Figure 16. Effect Of Nonplanar Interface On Application Of Disintegration Criteria

V. SPRAY EXCITATION OF LOWER ORDER MODES

Introduction

The occurrence of spray-excited large-amplitude surface waves with frequencies of a very low order compared to the excitation frequency was first reported by Yarymovych [7]. His investigation of this phenomena included the formulation of a hypothesis to explain this low-order instability, the essential features of which are summarized below:

(1) Excitation of the low frequency wave results from an impacting of the spray particles onto the surface in such a manner as to cause a positive forcing.

(2) The low frequency wave near its maximum amplitude point produces a higher and denser spray than the depressed portion; hence, there is a periodic variation in spray height and density near the wave crest of the form shown in Figure 17.

(3) Positive forcing of the low frequency wave can occur only if the total momentum of the particles impacting during the downward movement of the wave is greater than the momentum of particles impacting during upward motion.

Other than the formulation of this hypothesis from visual observation, Yarymovych presented no experimental verification in support of his ideas.

In this section, the results of an experimental investigation of spray excitation of the first symmetric mode in a 24.8-cm longitudinally excited cylindrical container are given. These results are used to give support to Yarymovych's hypothesis and to provide some quantitative information about the conditions necessary for the existence of spray excited low frequency modes.

Basically, it has been observed that, by exciting a container of liquid longitudinally so that there is significant spray, the initial state of the surface (shortly after excitation begins) is a 1/2 subharmonic response (with spray). As time progresses, however, lower modes begin to appear. These modes are ordinarily absent for the same excitation frequency at an excitation amplitude for which no spray is produced.

The lowest order mode excited by the spray seems to be highly dependent upon the input conditions. Some preliminary observations showed,

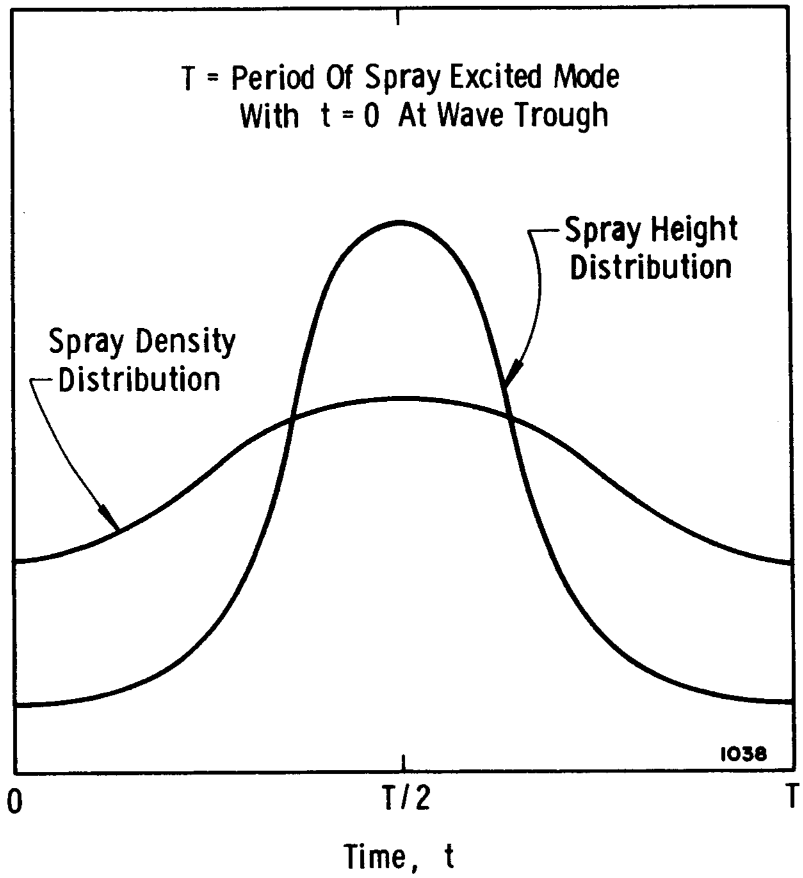


Figure 17. Spray Height And Density Distribution

for example, that, if the input amplitude was held constant and frequency slowly increased, the following low order modes could be identified superimposed on the high frequency surface[6]:

(1)	$m = 1, \; n = 2$	↓ Increasing excitation frequency (constant amplitude); hence, increasing input acceleration.
(2)	$m = 0, \; n = 1$	
(3)	$m = 2, \; n = 1$	
(4)	$m = 1, \; n = 1$	

It is very significant to note that this order of appearance of the spray excited modes also corresponds to a decreasing order of their natural frequencies[7]. One other important observation made was that, as noted by Yarymovych, the action of the low frequency mode definitely seems to modulate the spray, with particles appearing to leave the surface in the greatest numbers and with the greatest velocities when the motion of the low order mode was upward. This gave a maximum spray density above the surface when the wave was in the vicinity of its maximum upward position.

Although it was found possible to spray excite a number of low order modes, it was decided to study experimentally only one such mode and, since the first symmetric mode ($m = 0, \; n = 1$) appeared easily excitable and also its amplitude easily measured, this one was chosen for study.

Description of Experimental Apparatus and Methods

Seeking to gain enough information to answer whether Yarymovych's hypothesis is correct and to define, if possible, the conditions necessary for excitation of the ($m = 0, \; n = 1$) mode, we performed the following experiments:

(1) Production of high-speed motion pictures of the spray excited mode to allow better observation of the phenomena.

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[6] Here, m and n are mode numbers corresponding to the eigenfunctions $J_m(\lambda_{mn} r) \cdot \cos m\theta$ which describe the surface.

[7] Increasing input acceleration for a constant frequency implies greater spray trajectory heights (also greater trajectory times) which is in correspondence with the trend of decreasing natural frequencies of the spray excited modes.

(2) Study of the influence of a spray suppression baffle suspended above the liquid on the (m = 0, n = 1) mode amplitude.

(3) Monitoring of the amplitude of the spray forced (m = 0, n = 1) mode as a function of excitation amplitude and frequency.

(4) Monitoring of the spray density (particles per unit time per unit area) at various levels above the liquid surface both with and without the low frequency mode present.

Most of the experiments described above were conducted with a 24.8-cm diameter cylindrical tank containing water, the depth depending on the experiment[8]. Excitation was provided by a 1600-lb electrodynamic shaker. Special instrumentation used included:

(1) High-speed motion picture camera capable of operation at the desired rate of 500 fps.

(2) Wave height transducer for monitoring the amplitude of the spray excited wave. This transducer was identical to the one developed and described by Kana [11].

(3) A spray transducer to monitor the particle density at various heights above the liquid surface. This transducer, which was specifically built for this study, works on a variable conductance principle much the same as the wave height transducer. Figure 18 shows a block diagram of the electronic system for this transducer. Particles impacting on the probe cause a conductance change in the corresponding leg of the bridge circuit, hence, generating pulses which can be counted after proper filtering of the signal.

Results of High-Speed Motion Pictures

Viewing the slowed-down interaction between the high frequency surface with spray and the low frequency mode from the motion pictures proved very beneficial and yielded the following observations:

(1) The modulation of the spray is such that the greatest percentage of particles leave the surface during the time the low order mode

[8]Some testing was also done in a 11.4-cm tank, and, further, some unreported experimental results obtained by Kana with a 14.5-cm tank will be presented.

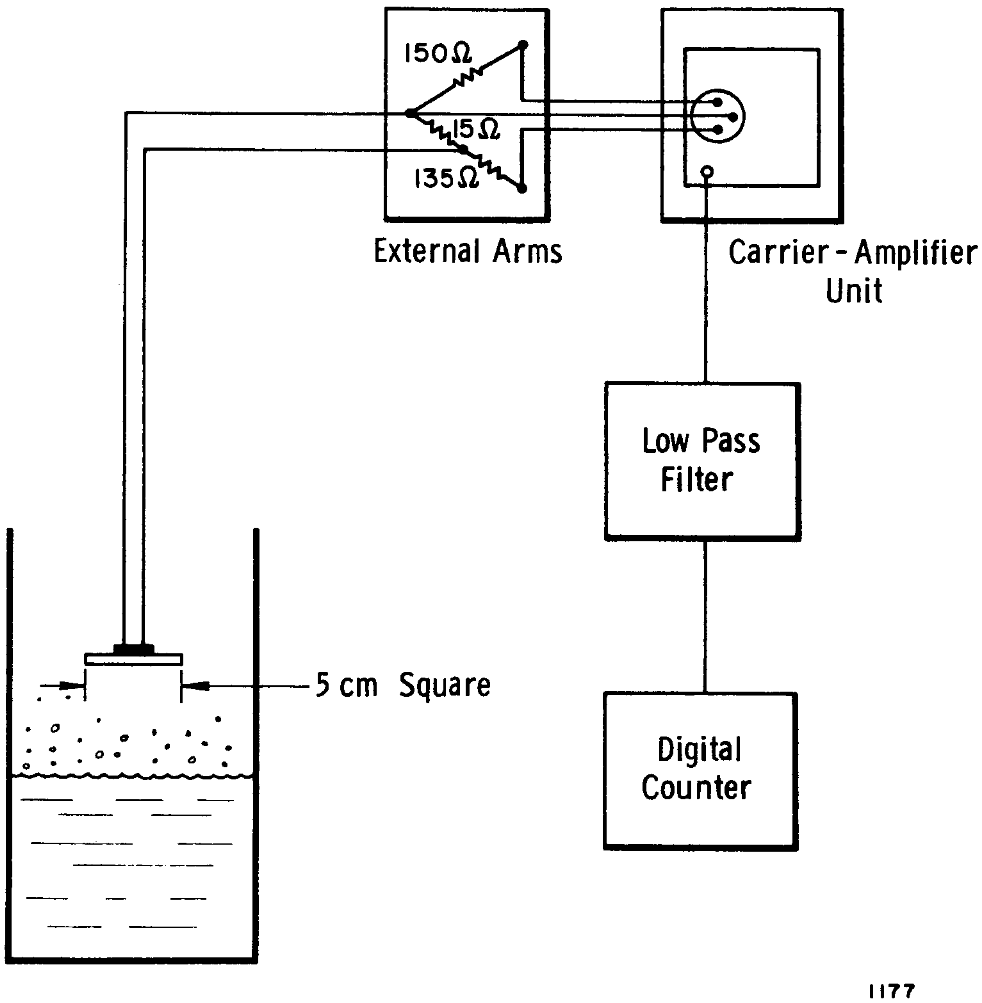

Figure 18. Block Diagram Of Spray Transducer System

is progressing from its minimum amplitude point to its maximum amplitude point. The particles leaving during this period also have the greatest trajectory heights. It should also be noted that the greatest modulation occurred in regions of greatest activity of the low frequency mode, i.e., near the center since this was the ($m = 0$, $n = 1$) wave.

(2) These particles, leaving in the period and manner described in (1) above, generally impacted back onto the surface when the low frequency mode was moving downward, hence reinforcing the motion.

These observations simply add further credence to Yarymovych's hypothesis.

Baffle Influence on Spray Excited Mode

Results of the experimental study to determine the effect of a spray suppression baffle upon the amplitude of the ($m = 0$, $n = 1$) mode are shown in Figure 19. The ordinate of this plot is amplitude of the low frequency mode. A', normalized by dividing by the amplitude with the baffle removed, A'_{∞}; thus, a normalized amplitude of unity means the baffle has no effect. Tests were performed at two excitation frequencies, 30 cps and 50 cps, with data being obtained for two input acceleration levels at each frequency. While the results of Figure 19 are rather limited and restrictive with respect to test conditions, the following comments can be made regarding these tests:

(1) A spray suppression baffle definitely does influence the spray excited mode amplitude.

(2) There is an upper value of baffle height above the liquid surface, h_u, which defines the limit of influence. For heights greater than this limit, the low frequency mode experiences no effect from the baffle. The data show h_u to increase as the excitation frequency increases.

(3) There is also a lower limit of baffle height, h_ℓ, below which the low frequency mode is completely suppressed. The data show h_ℓ to be a constant independent of input conditions.

(4) Baffle influence appears to be independent of input acceleration (at constant frequency) in the range investigated.

From these results, it is possible to infer some interesting information about the coupling between the low frequency mode and the spray. First of all,

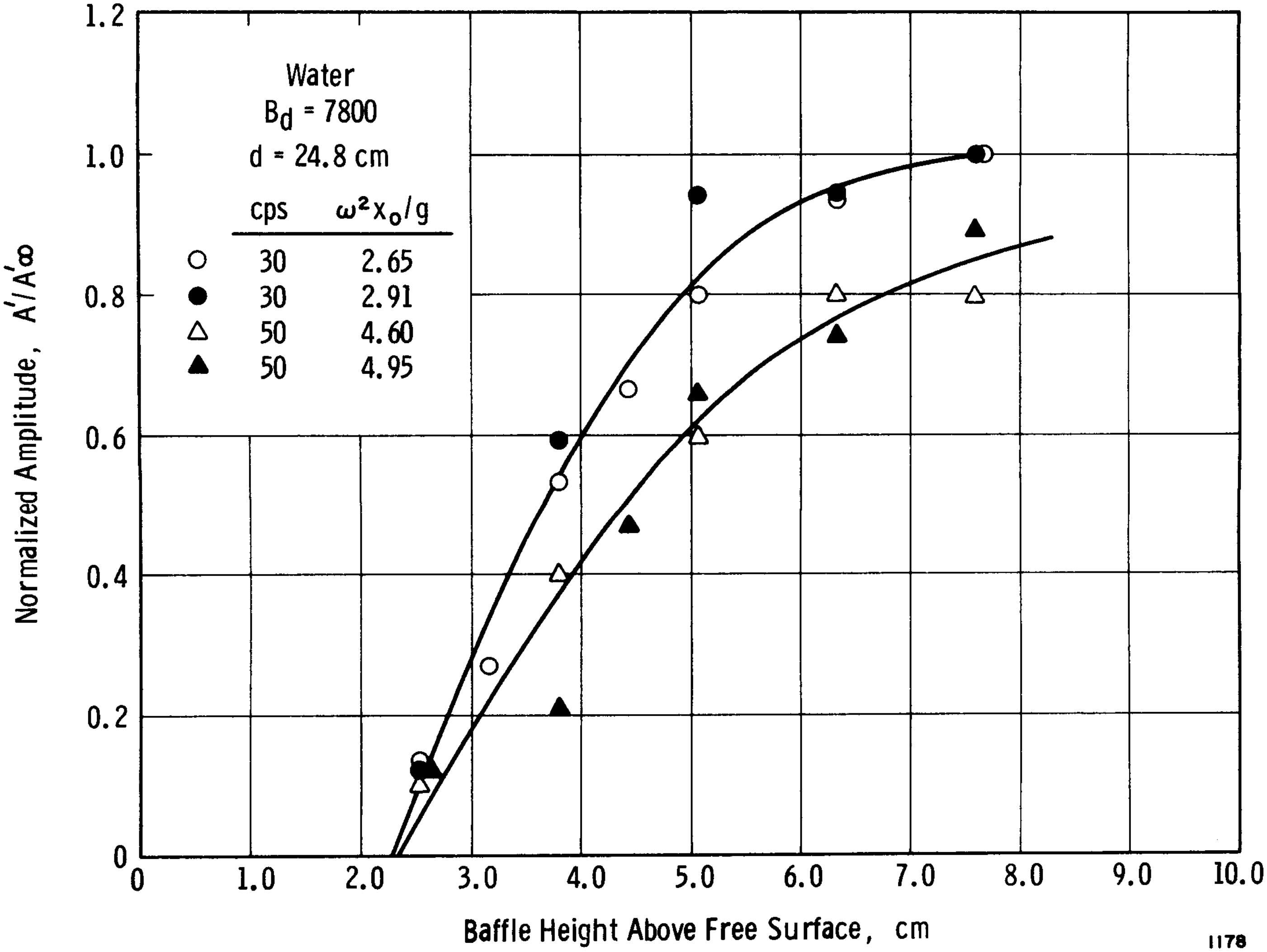

Figure 19. Normalized (m = 0, n = 1) Mode Amplitude As A Function Of Spray Suppression Baffle Height

since it is possible to completely suppress the low frequency mode by simply interrupting the spray with a baffle placed sufficiently close to the water surface, it must be concluded that the spray definitely provides the forcing mechanism. Considering the data for the 30-cps case from Figure 19, it can be seen that $h_\ell \approx 2.3$ cm and $h_u \approx 7.5$ cm. Spray trajectory times corresponding to these trajectory heights are $T_\ell = 0.14$ and $T_u = 0.25$ second, respectively. The period of the low frequency mode is $T = 0.36$ second; thus it can be seen that the trajectory time T_ℓ is somewhat less than $T/2$ while T_u is somewhat greater than $T/2$. This information can be correlated with the observations of the high-speed motion pictures as follows:

(1) Assuming that T_u is an indication of the maximum trajectory time for which there will be a positive forcing of the low frequency mode (impacting when the wave is moving downward), then the corresponding particles will leave at the time t_{ou} as shown in Figure 20.

(2) In the same way, if T_ℓ is an indication of the minimum trajectory time for which there will be a positive forcing, then these particles will leave at the time $t_{o\ell}$ shown in Figure 20.

(3) Although the above observations are somewhat hypothetical and no strong conclusions should be drawn from them, it can be further observed that the time period corresponding to the shaded region in Figure 20 correlates well with the time period over which spray particles are actually observed to leave the surface.

The results of the baffle experiments seem to further enhance the validity of Yarymovych's hypothesis.

Experimentally Determined Conditions for Spray Excitation of the (m = 0, n = 1) Mode

The excitation conditions, over the frequency range 25 cps to 80 cps, necessary to cause spray excitation of the (m = 0, n = 1) mode in a 24.8-cm container, with water as the test liquid, are shown plotted in Figure 21. The ordinate of this figure is the total amplitude of the (m = 0, n = 1) mode, and the abscissa is the dimensionless input acceleration, $\omega^2 x_o/g$. Each curve represents a single excitation frequency. Figure 22 displays a plot of dimensionless excitation acceleration versus excitation frequency for the threshold of spray and for the conditions of a 2-cm and 4-cm peak-to-peak amplitude of the spray excited mode. It may be seen that two curves for the threshold of spray are plotted in Figure 22. One curve represents the experimental

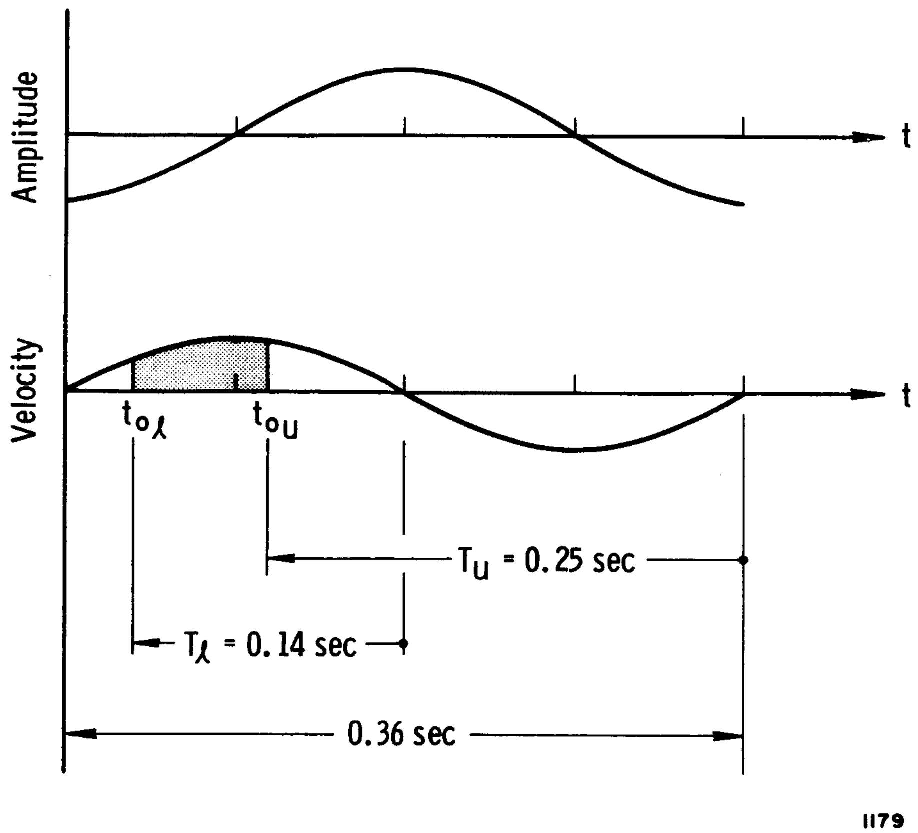

Figure 20. Possible Relationship Of T_u And T_ℓ To Wave Amplitude And Velocity

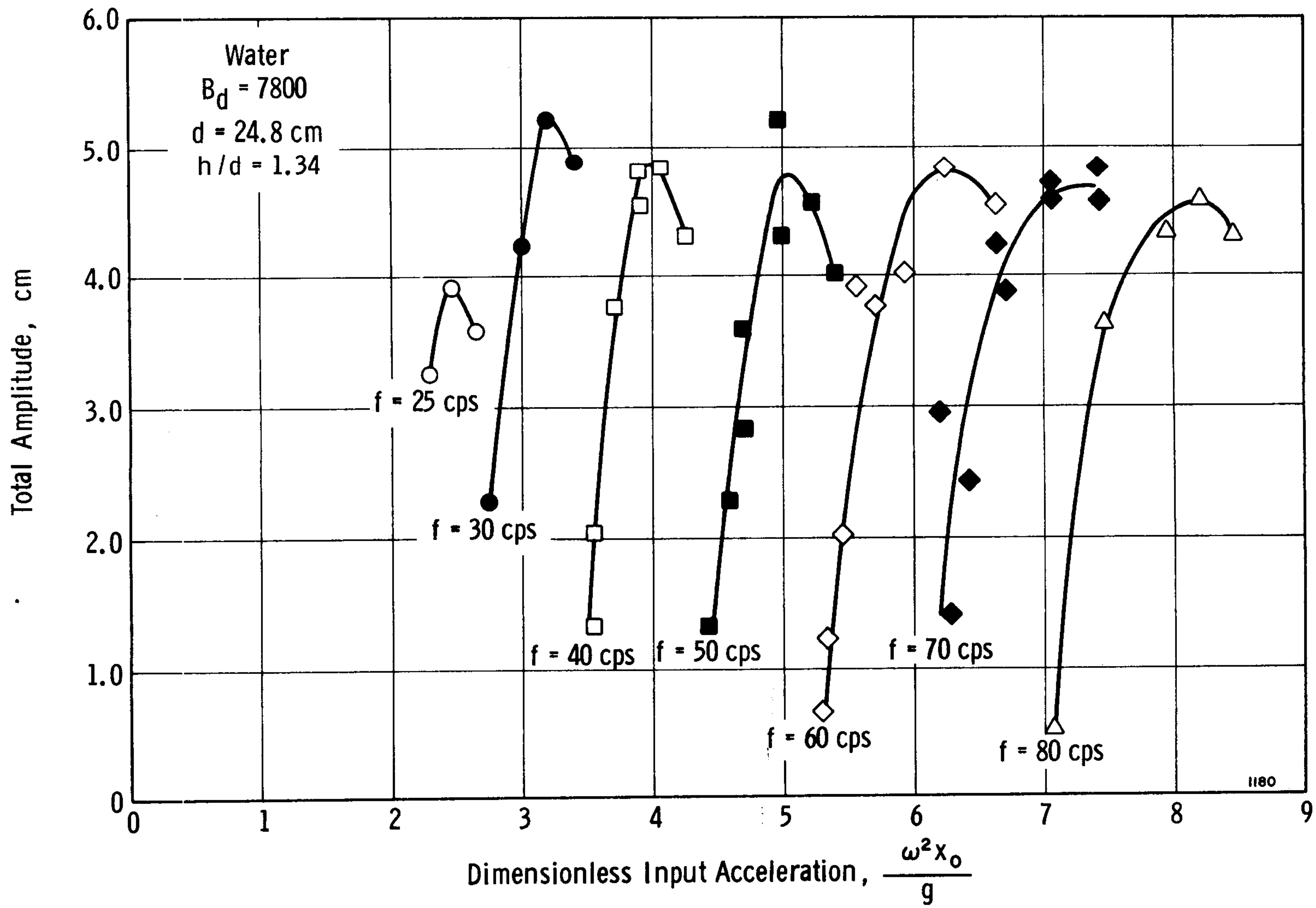

Figure 21. Total Amplitude Of Spray Excited First Symmetric Mode As A Function Of Input Acceleration And Frequency For Water In A 24.8 cm Container

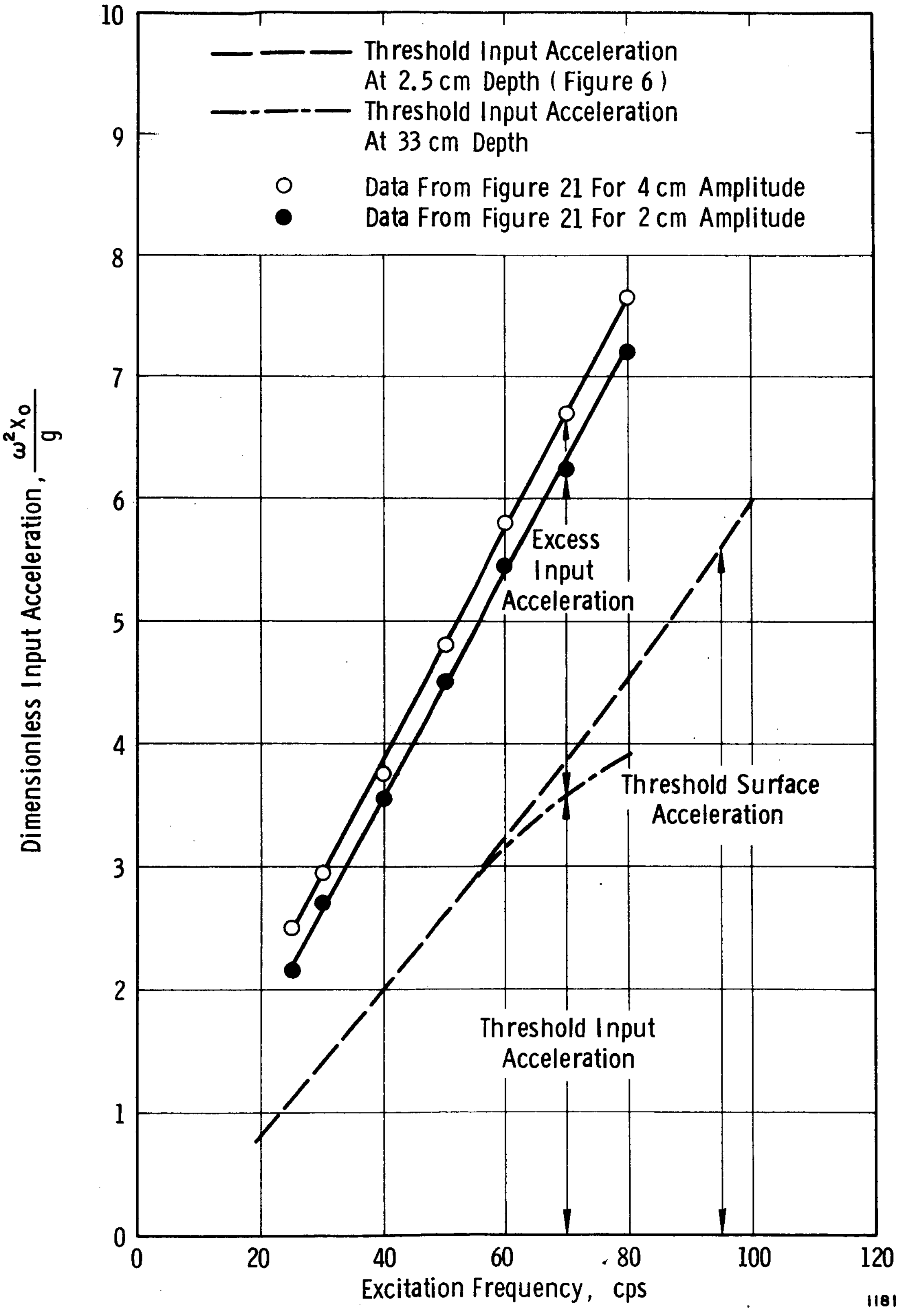

Figure 22. Dimensionless Input Acceleration (Threshold Plus Excess) Plotted As A Function Of Excitation Frequency - 24.8 cm Container

data as presented in Figure 6. It may be recalled that, in obtaining the data for Figure 6, the liquid depth was maintained small to minimize the effect of system compressibility. The other curve gives the input threshold of spray for the greater liquid depth and was also obtained experimentally. The influence of system compressibility is responsible for the divergence of the two threshold curves in the range from 50 cps to 80 cps. It will be important to account for this effect when attempting to make quantitative predictions or correlations of the spray excitation of low order modes.

Some important observations concerning Figures 21 and 22 are:

(1) In general, the amplitude of the spray excited mode is very sensitive to changes in the input acceleration for a given excitation frequency.

(2) The maximum amplitude of the spray excited mode seems to vary in a yet undetermined manner, depending upon the frequency.

(3) The input acceleration necessary to cause spray excitation of the (m = 0, n = 1) mode is of the form input acceleration equals threshold input acceleration plus excess input acceleration.

It appears that the proper approach in the correlation or prediction of spray excited mode phenomena is to work with the excess acceleration since this will be a governing factor on particle trajectory heights, etc. It should be emphasized, however, that the proper acceleration to work with is the surface acceleration, not input acceleration values. Thus, the surface acceleration could be written as

$$\frac{a_s}{g} = C_c\left(\frac{\omega^2 x_o}{g}\right) = \frac{a_{st}}{g} + \frac{a_{se}}{g} \qquad (20)$$

where

$a_{st} \equiv$ threshold surface acceleration

$a_{se} \equiv$ excess surface acceleration

$C_c \equiv$ a factor to account for system compressibility

Figure 23 displays the excess surface acceleration obtained from the information in Figure 22 and plotted as a function of the dimensionless input frequency parameter, $\omega^2 d/g$. It is immediately apparent that for these test results the excess surface acceleration can be written, to a first approximation, in the form:

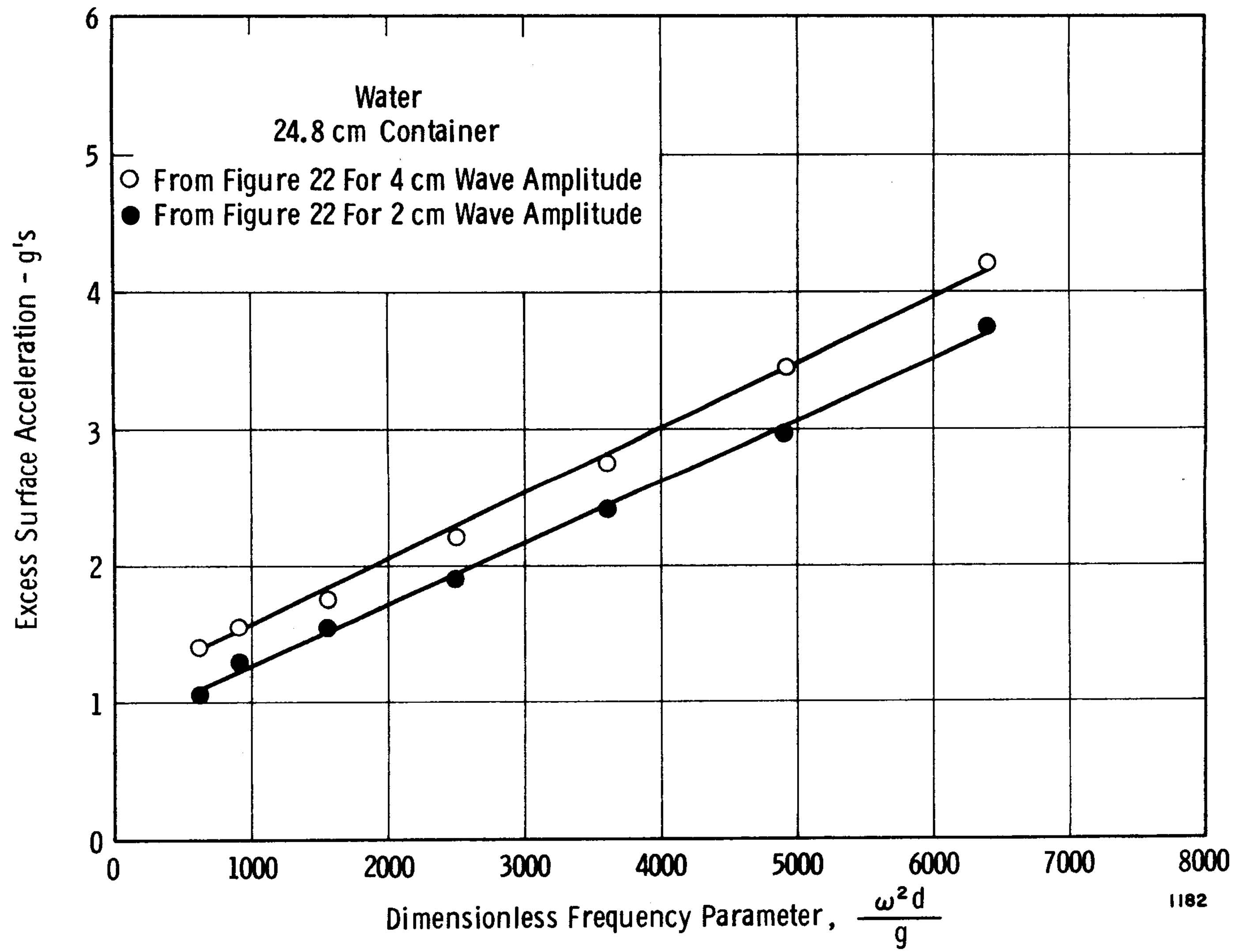

Figure 23. Excess Surface Acceleration As A Function Of The Frequency Parameter $\frac{\omega^2 d}{g}$

$$a_{se} = C_1 + C_2\left(\frac{\omega^2 d}{g}\right) \qquad (21)$$

In an effort to learn more about how tank dimensions influence the input and surface acceleration, an examination will now be made of some other limited test results. Figure 24 displays the total amplitude of the (m = 0, n = 1) spray excited mode in an 11.4-cm container as a function of input acceleration for 30, 50 and 80-cps excitation frequencies. In Figure 25, a plot is shown of the excess surface acceleration as a function of the dimensionless frequency parameter ($\omega^2 d/g$) from the data of Figure 24 for the 11.4-cm container. These results should be compared with Figure 23, and again it is noted that, to a first approximation, the excess surface acceleration is of the form of Eq. (21).

Figure 26 shows some previously unreported results obtained by Kana in his preliminary work on spray excited modes which preceded the work reported here. The ordinate of Figure 26 is the dimensionless input acceleration, $\omega^2 x_o/g$, while the abscissa is Bond number based on container diameter, B_d. All tests were performed in a 14.5-cm container, and the Bond number was varied by changing the test liquid from water at one extreme to pure ethyl alcohol at the other extreme; various mixture ratios of water to alcohol were maintained in between. Amplitude of the low frequency mode remained constant during all tests; also held constant was the input frequency parameter, $\omega^2 d/g$. As can be seen from this figure, input acceleration decreases as the Bond number increases. It is also interesting to note that the ratio of input acceleration for pure water to that of pure ethyl alcohol from Figure 26 is very close to the same ratio of input spray threshold accelerations reported for these two liquids in Figure 6. While the range of experimental data presented in Figure 26 is small, the above observation suggests that (for a constant low order mode amplitude) correlation of spray excited mode input accelerations for changes in liquid surface tension can be made from threshold acceleration data.

Despite the fact that some indications have been noted of ways to correlate experimental data for test results with varying size tanks (24.8 cm and 11.4 cm) using water and, also, for test results with varying liquid surface tension, not enough is presently known about the spray excited mode phenomena to yield a universal correlation or prediction method valid for all parameter conditions. For this reason, we have refrained from deriving empirical equations from the test results expressing input or excess acceleration as a function of the very limited test parameters. The indications are very strong that these equations would be valid only if the exact test conditions were duplicated; therefore, their utility would be very limited.

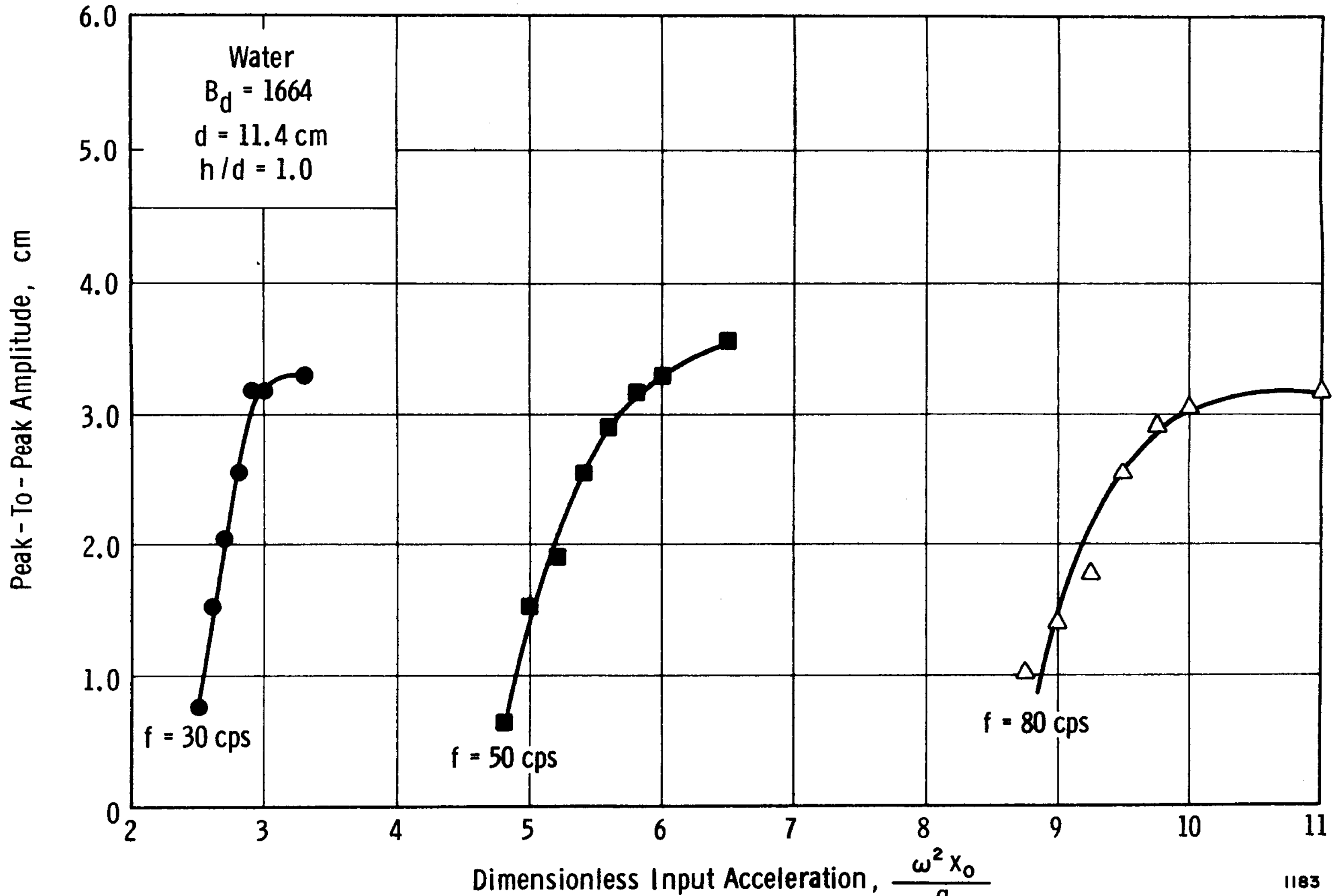

Figure 24. Total Amplitude Of Spray Excited First Symmetric Mode As A Function Of Input Acceleration And Frequency For Water In A 11.4 cm Container

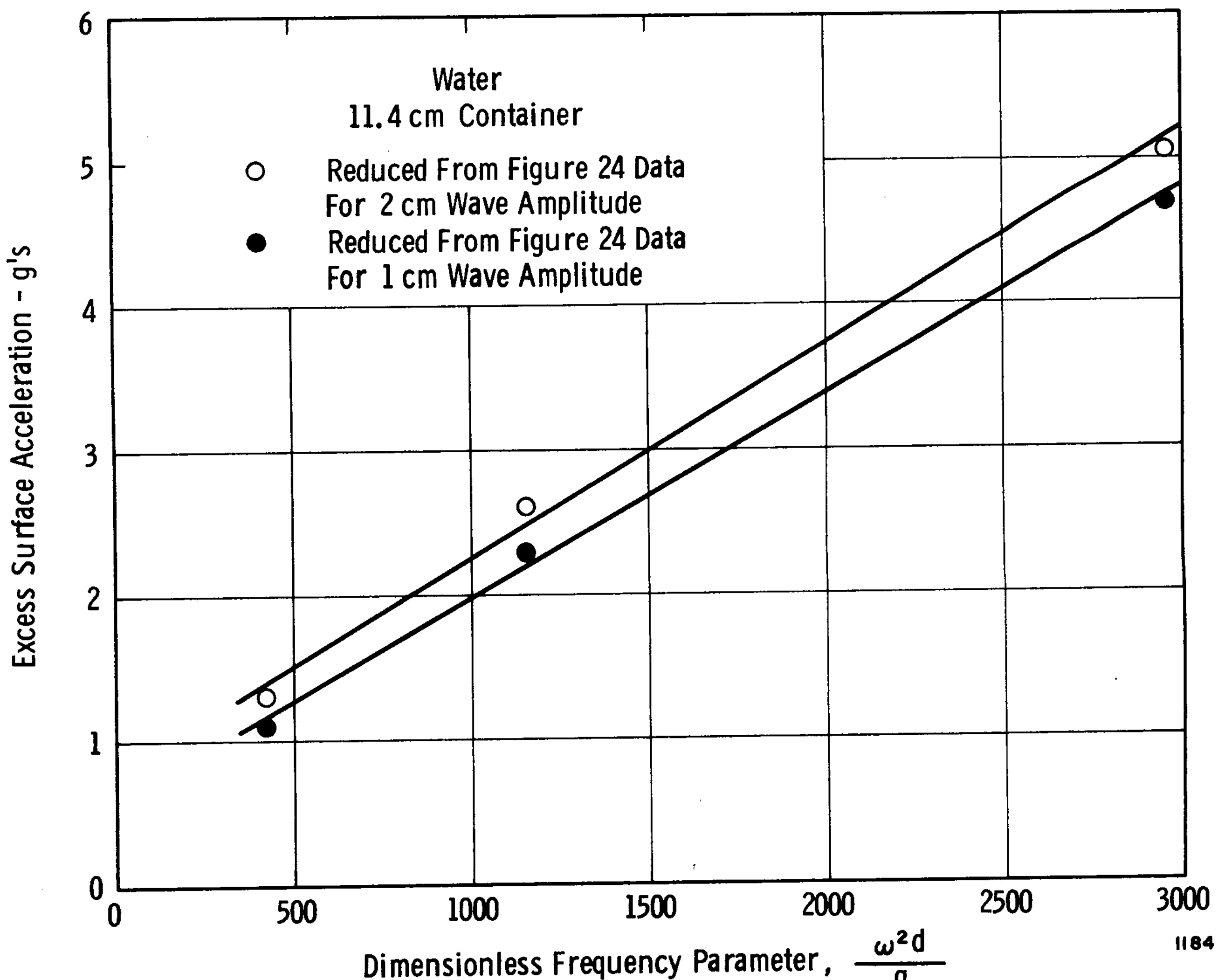

Figure 25. Excess Surface Acceleration As A Function Of The Frequency Parameter $\frac{\omega^2 d}{g}$
Container Diameter 11.4 cm

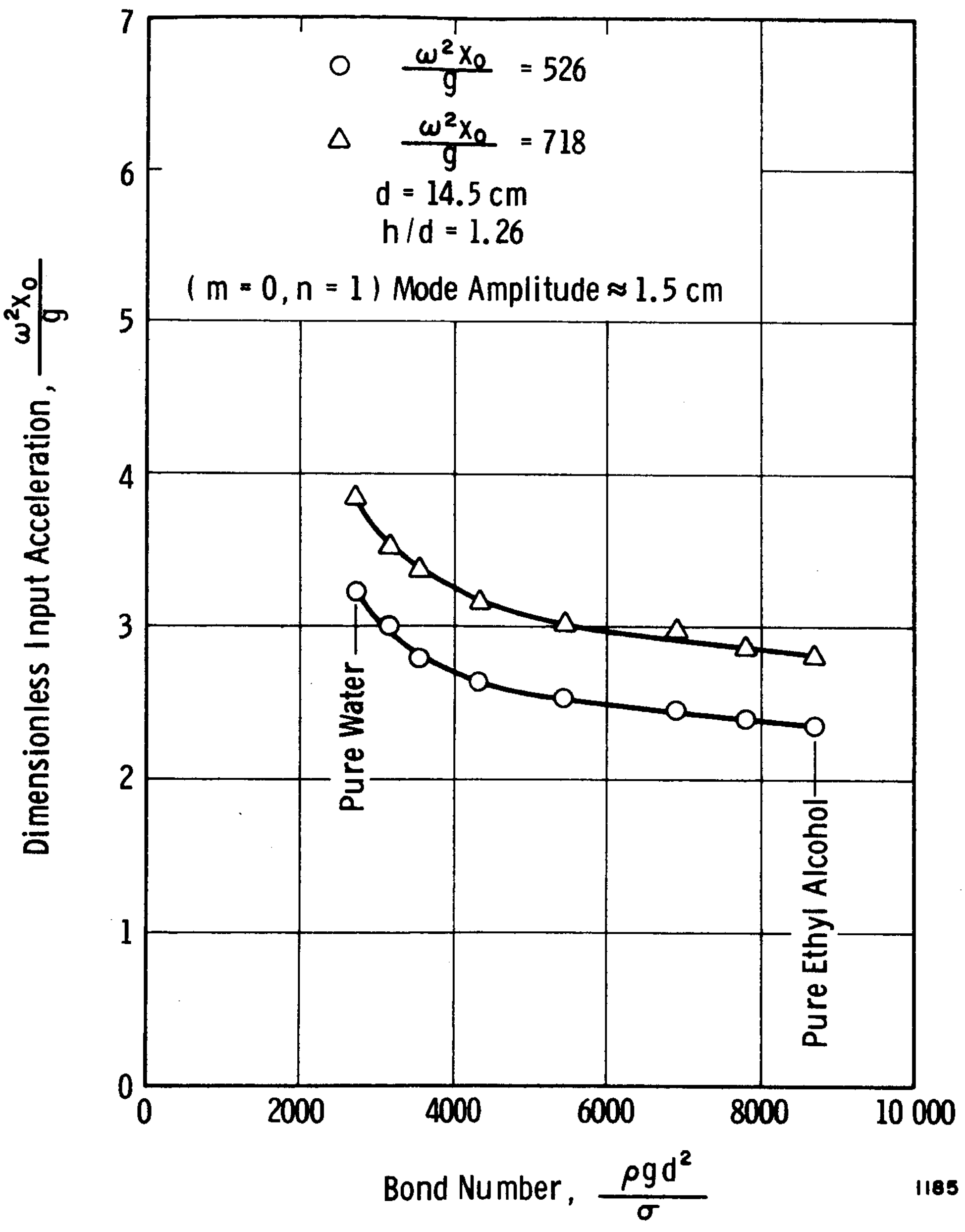

Figure 26. Input Acceleration As A Function Of Container Bond Number At Constant Low Order Mode Amplitude - From Unreported Results By Kana

Results of Spray Density Measurements

The principal objective of the limited experiments performed to obtain spray particle density information was to provide further verification of the Yarymovych hypothesis. Testing was done with only one input frequency, 50 cps, but the input acceleration was varied over a range from 4.8 g's to 8.8 g's. For these tests, the liquid depth was 2.5 cm so that damping prevented a low order mode from developing. Although the spray was uniform over the surface (except very near the wall), the monitoring was done near the tank center. For each acceleration level, the spray was monitored at several heights above the liquid surface. Test results are shown in Figure 27 with the ordinate indicating particles per unit time striking the spray transducer, hence, in reality, the number of particles per unit time which pass a given height. The abscissa is height of the transducer above the liquid surface.

Since the particles impacting on the liquid surface provide a forcing of the low frequency mode by virtue of their momentum prior to impact, the information of Figure 27 could more usefully be presented as total momentum versus height rather than just particle counts. Figure 28 shows such an interpretation of these results and was obtained by multiplying each point of the smooth curves in Figure 27 by the square root of the corresponding height[9]. A line of maximum momentum density has also been plotted in Figure 28. Note that Figure 28 represents momentum passage just as Figure 27 represented particle passage; thus, the derivative of the curves in Figure 28 would yield momentum density so points of maximum slope correspond to maximum momentum density points, i.e., maximum force density with respect to the liquid surface. Comparing Figure 28 with the results of the baffle experiments, Figure 19, and of the experimental results shown in Figure 21, the following point is noted: accepting 2.3 cm as an indication of the minimum spray trajectory height necessary for (m = 0, n = 1) mode excitation in a 24.8-cm tank, Figure 28 shows this will correspond to the maximum particle momentum density height if $\omega^2 x_o/g$ is about 5. Figure 21 shows an input acceleration of 5 to indeed provide spray excitation of the (m = 0, n = 1) mode for this excitation frequency.

It is important to point out another bit of information here, however, namely that the action of the low frequency mode provides an additional amount of surface acceleration which causes particle trajectory heights to vary or be modulated over the period of the mode. This may explain why the above comparison (based on spray measurements with no low frequency

[9]The square root of the trajectory height is proportional to impact velocity.

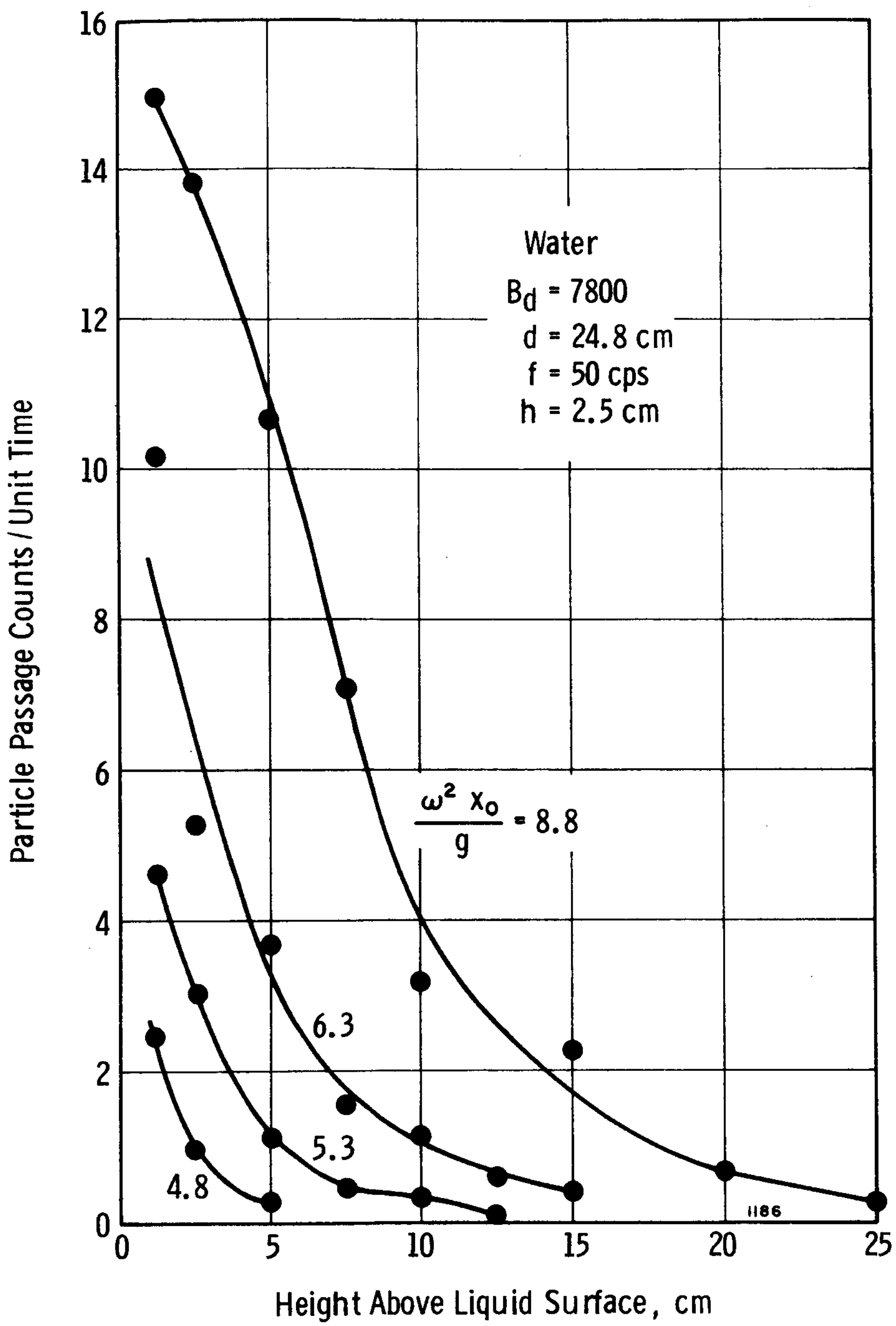

Figure 27. Spray Passage Counts Per Unit Time As A Function Of Spray Transducer Height Above The Liquid Surface And Input Acceleration

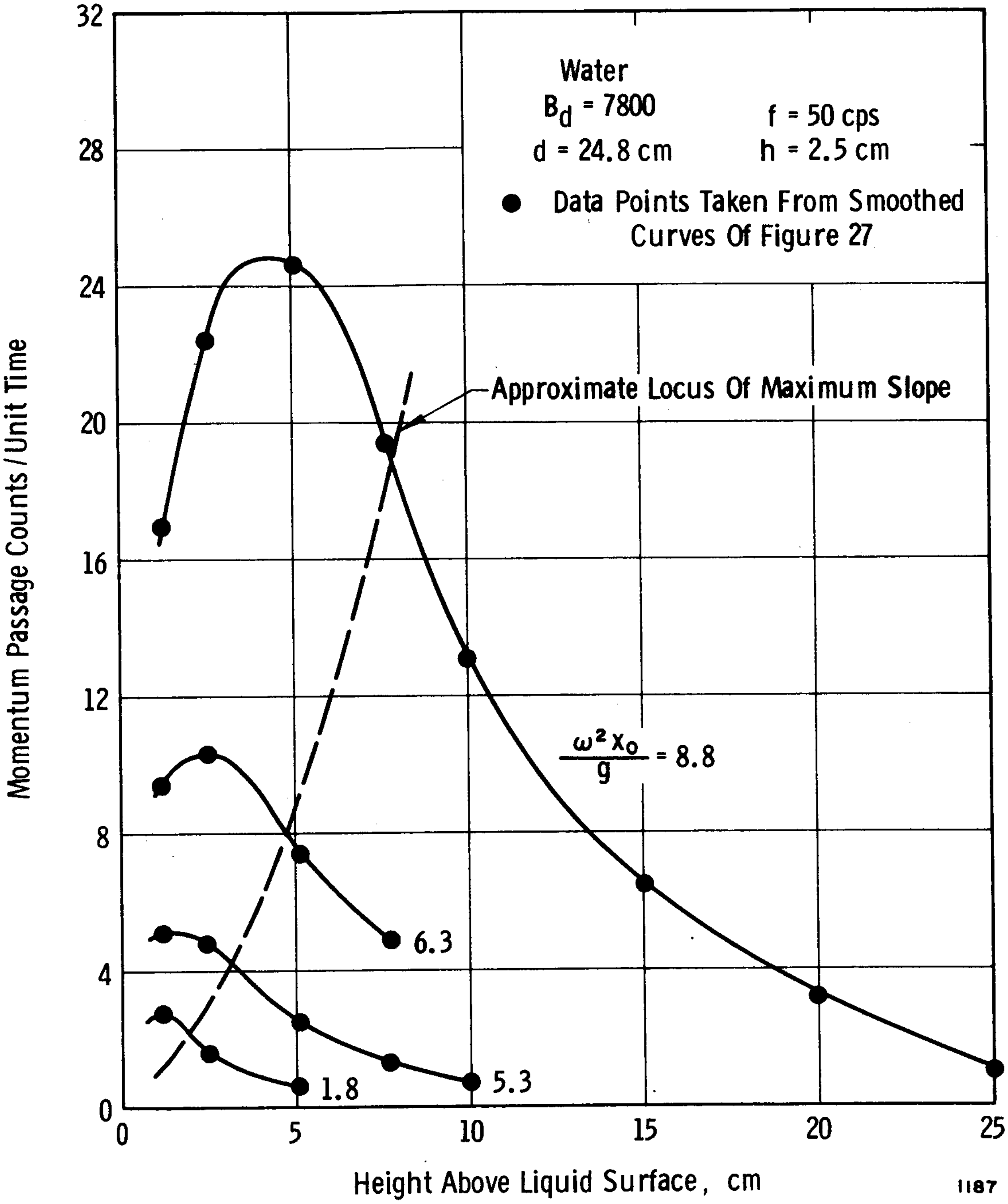

Figure 28. Momentum Passage Counts As A Function Of Height Interpreted From Figure 27

mode present) showed an input acceleration of $\omega^2 x_o/g = 5$, which normally gives the most favorable forcing of the low frequency mode (Figure 21), only gives maximum momentum spray trajectory heights of 2.3 cm (possibly minimum necessary for excitation). This also explains why it takes a greater input acceleration level to initiate mode excitation than is required to maintain it, a phenomena which has been experimentally observed.

One other rather interesting experiment was performed utilizing the spray transducer. It involved monitoring particle passage counts while the spray excited ($m = 0$, $n = 1$) wave was in existence; also the amplitude of the wave itself was monitored. The results of this experiment reiterated the fact that the spray is modulated by the low frequency mode. This experiment also demonstrated, as was inferred by the information in Figure 20, that the particles which provide the forcing energy actually leave the liquid surface somewhere during the quarter period following the time that the low frequency mode is at its lowest position.

Sorokin [12] reported the results of an interesting investigation which should be mentioned here since they are somewhat related to the spray experiments described above. He was able to show that spray leaving the surface influences the system excitation energy requirement in the same manner as an added friction force. He considered this added force to depend upon the square of the velocity of particles leaving the surface.

Summary

Most of the experimental evidence presented in the previous discussions of the spray excited mode phenomena have been, admittedly, probably more qualitative than quantitative. However, it is believed that this evidence can be considered as conclusive proof of the validity of Yarymovych's basic hypothesis. This was one of the objectives of the study. Further, while insufficient experimental data were obtained to completely describe the conditions necessary for spray excitation of low frequency modes, the following information should be of further value:

(1) The excess surface acceleration is an important correlating parameter and was found to be, generally, of the form

$$a_{se} = C_1 + C_2\left(\frac{\omega^2 d}{g}\right)$$

for a constant amplitude wave.

(2) For constant input frequency, wave amplitude and container diameter, changes in the liquid surface tension generally seem to produce changes in required input acceleration in the same ratio as produced in the input threshold of spray acceleration.

(3) All experimental evidence seemed to verify the Yarymovych spray hypothesis, and virtually no indication was found to the contrary.

VI. SOME EXPERIMENTS WITH RANDOM EXCITATION

Introduction

In the interest of rounding out the picture of surface disintegration and spray excited mode phenomena, some limited experiments were performed with a random vertical excitation. This short investigation was made possible because the excitation and readout equipment necessary for random work had already been assembled and developed on a separate and concurrent research project at SwRI. That project, under the sponsorship of Marshall Space Flight Center (Contract NAS8-20319), is in the final stages of completion and a Technical Report is forthcoming, Dalzell [13]. The objective of this other research project was to perform an exploratory study of both lateral and longitudinal random excitation of rigid cylindrical tanks. It is instructive here to summarize an important result of that study for the longitudinal case.

Qualitatively, the large amplitude free surface response to random excitation is about what is observed for the sinusoidal case; that is, it appears to be 1/2 subharmonic in nature. The magnitude of the free surface response is influenced by acceleration level and frequency distribution. Figure 29 illustrates this behavior, and, as may be seen, the predominant frequency contained in the free surface response is at about half the center frequency of the random excitation. The reader further interested in behavior of the liquid surface response to random excitation should refer to the upcoming report by Dalzell [13].

Experimental Results

Experiments performed to determine the effect of random excitation on the surface disintegration problem and the spray excited mode phenomena were conducted with the equipment developed by Dalzell. Briefly, excitation was provided by a 1600-lb electrodynamic shaker which had been "equalized" so that a given input voltage yielded a constant acceleration output amplitude over a desired frequency range. The random source driving into the shaker was a special low frequency unit (Elgenco Corp.) providing a Gaussian noise with frequency content from DC to 40 cps. A variable band-pass filter was utilized to allow changes to be made in the frequency distribution of the shaker input signal.

The first testing with the random apparatus for the present study was done to determine information on the spray excited mode for comparison with corresponding results of the sinusoidal excitation case. This testing involved

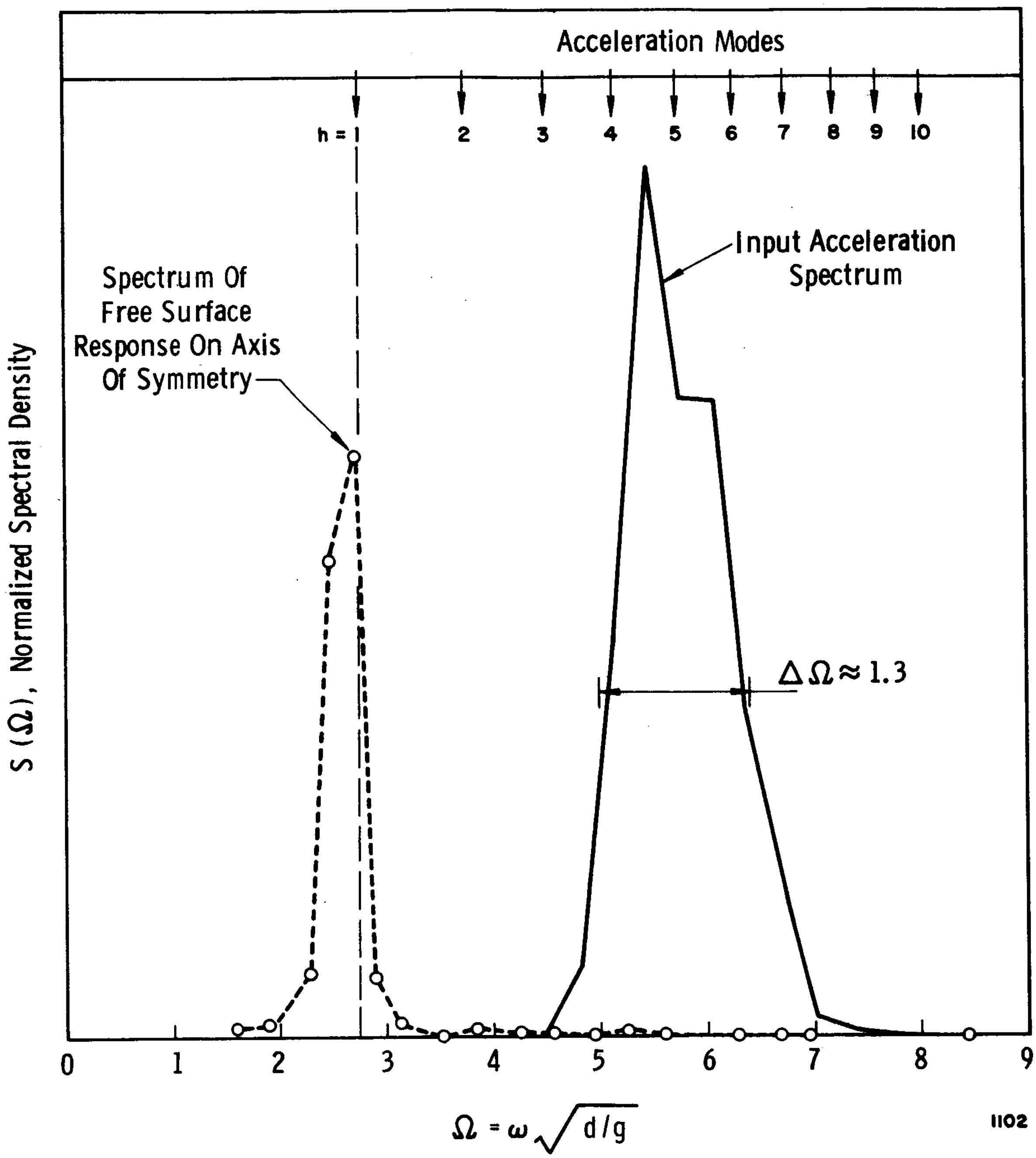

Figure 29. Spectra Of Moderate Band Random Longitudinal Excitation And Resulting Lowest Symmetric Mode Subharmonic Response

monitoring the RMS amplitude of the low frequency mode as a function of RMS input acceleration for three values of band-pass center frequency, namely 30, 50 and 80 cps. Two bandwidths were employed; 2 cps and 20 cps. Figure 30 shows the results for the 2-cps bandwidth case and should be compared with data shown in Figure 24 for sinusoidal excitation, noting that the tank diameter is the same for both and that frequency information is also comparable. Figure 31 displays results for the 20-cps bandwidth case which should be compared with both Figures 24 and 30.

While there does not appear to be any correlation between the results of Figure 24 and those of Figures 30 and 31, some trends appear similar, namely:

(1) The input acceleration required generally increases with center frequency.

(2) Spray excitation is possible over a wider range of input acceleration as excitation frequency (or center frequency) increases.

(3) Also, as was observed for the sinusoidal case (see Figure 21), there appears to be an optimum RMS input acceleration which yields a maximum spray excited wave amplitude.

Comparison of Figure 30 and Figure 31 also reveals an interesting trend. Notice that, with the exception of the region between 3 and 5-RMS g's for the 50 cps case, all of the curves compare well, both qualitatively and quantitatively. It would be interesting to determine if this trend continues as the width of the random frequency band is increased still farther.

The results of the experimenting performed to determine the random excitation effect on the threshold of spray are purely qualitative. The following information was noted:

(1) For a narrow band random excitation, the spray was generated generally as might be expected. That is, depending upon the time varying random amplitude, spray was or was not produced. If a very sensitive spray detector were available[10] to monitor spray, a correspondence could very probably be found between the percentage of time the acceleration is above the sinusoidal threshold values predicted earlier and the percentage of time spray is produced.

[10] A spray detector more sensitive than the one described earlier in this report would be needed.

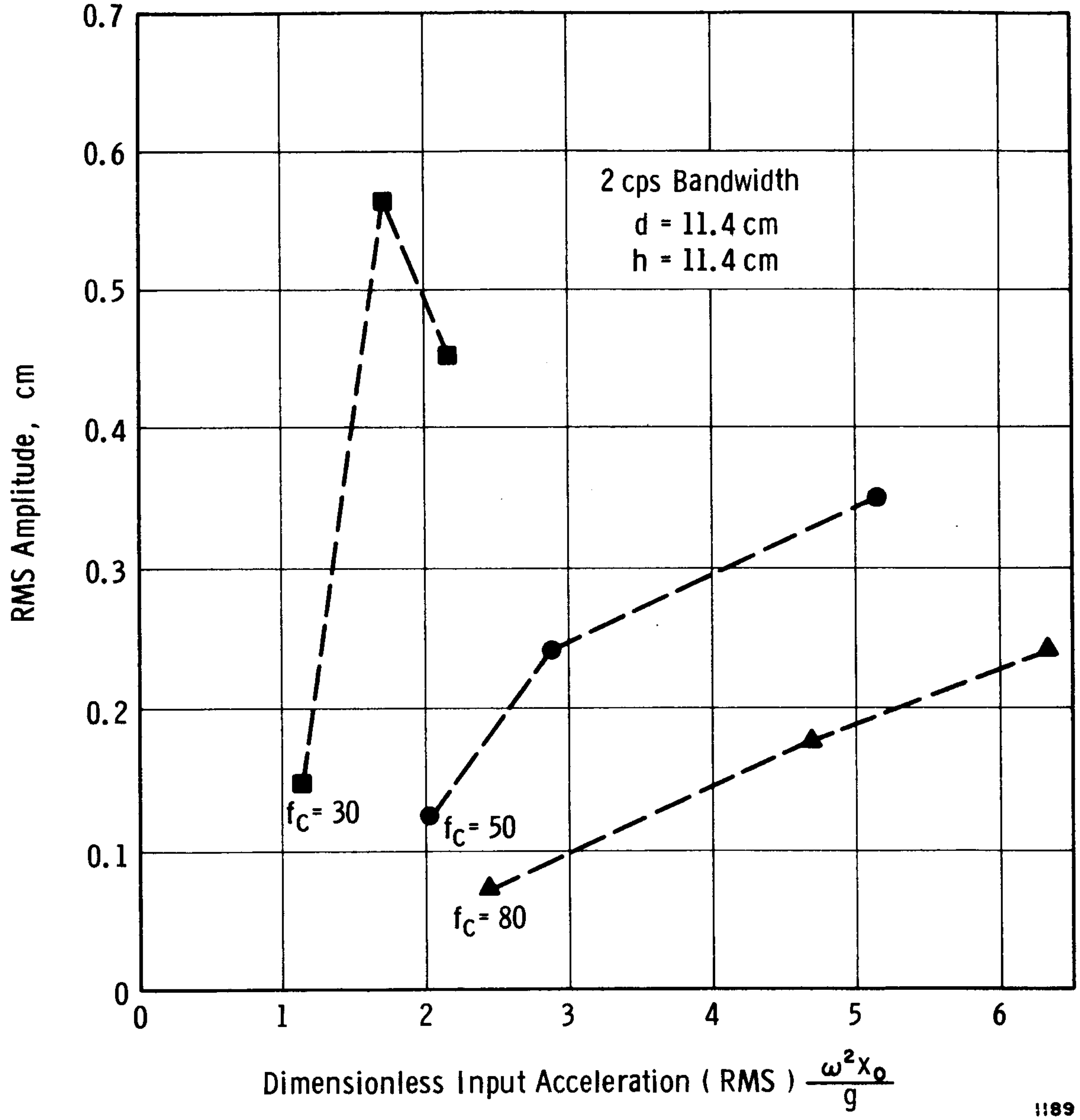

Figure 30. Total RMS Amplitude Of Spray Excited First Symmetric Mode As A Function Of RMS Input Acceleration And Frequency - 2 cps Bandwidth

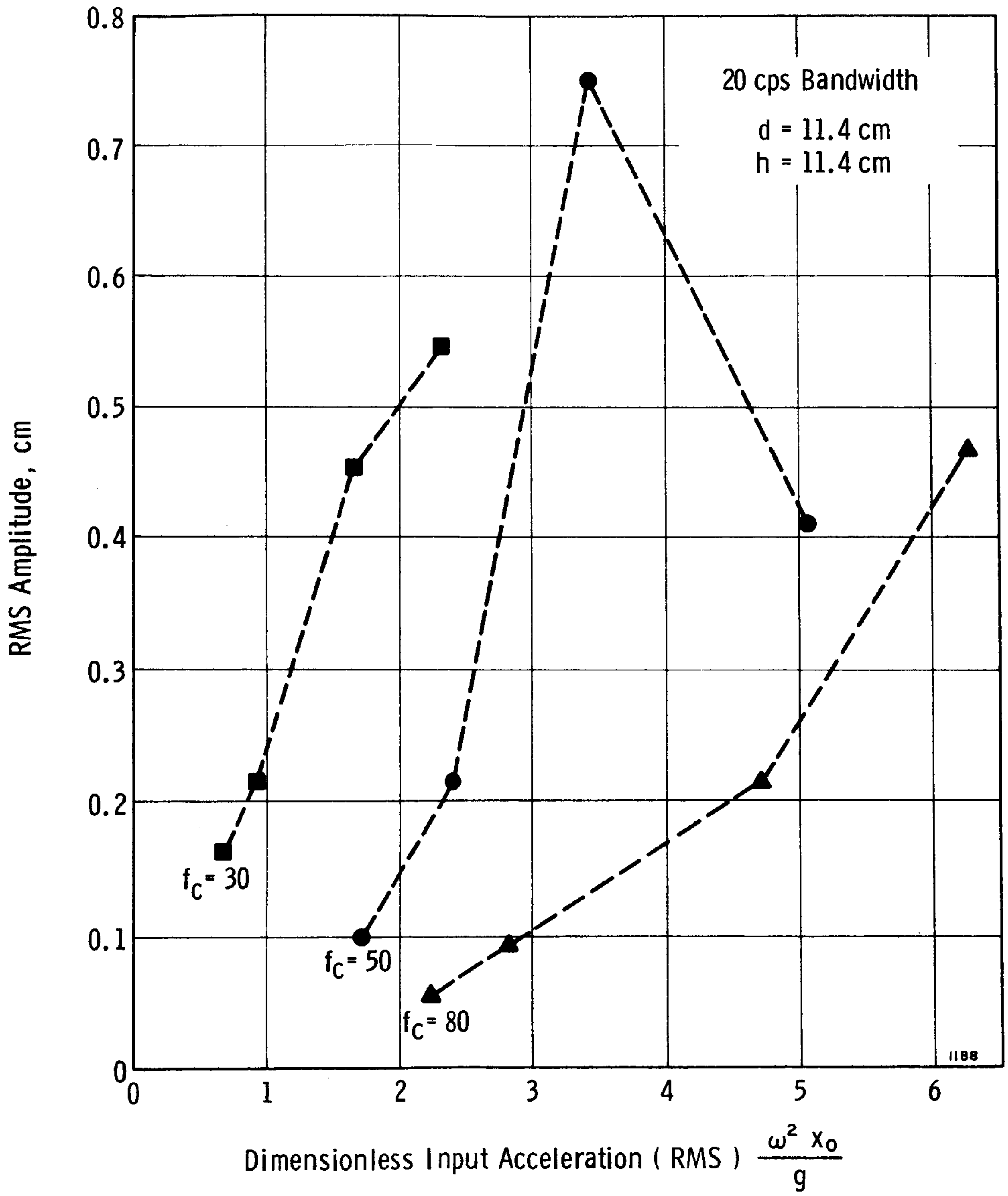

Figure 31. Total RMS Amplitude Of Spray Excited First Symmetric Mode As A Function Of RMS Input Acceleration And Frequency - 20 cps Bandwidth

(2) For broadband random excitation, the picture becomes confused; however, again it is felt that by using a sensitive spray transducer, a correspondence, as noted above, could be found except that, in this case, one would need to monitor the percentage of time all frequency components in the excitation band are above their corresponding sinusoidally predicted threshold.

The experiments performed with random excitation as described above have yielded, admittedly, rather sparce information. However, the major benefit gained was to demonstrate that, although knowledge of liquid surface behavior under the action of sinusoidal disturbances is quite great and probably adequate for many engineering purposes, the analogous random excitation problem is only at the very beginning stages of being studied and understood.

VII. DISCUSSION, CONCLUSIONS AND RECOMMENDATIONS

Discussion and Conclusions

In this report, results have been presented of a combined experimental and analytical study of liquid surface disintegration resulting from longitudinal excitation of a tank containing liquid. Also presented were the results of an experimental investigation of the spray excited low frequency mode phenomena first reported by Yarymovych. Accomplishments and conclusions can be summarized as follows:

(1) The conditions necessary for surface disintegration have been determined experimentally for several liquids. The required input acceleration increases roughly in proportion to frequency; thus, the phenomena is highly dependent on excitation velocity. Reducing the liquid surface tension reduces the required input acceleration; also, increasing viscosity increases the required acceleration. However, below a certain frequency, the effects of surface tension and viscosity become small, and gravitational effects predominate.

(2) A theory has been developed which predicts a large amplitude instability of the liquid surface. This theory gives conservative predictions of the experimentally determined disintegration and, it is concluded, should be useful in making estimates of this phenomena. The theory appears to be valid for arbitrary gravity conditions (including low gravity), although no experimental verification is available.

(3) Observations made of a high frequency excited liquid surface indicated that the most probable state is one with an equal distribution of wavelets responding at 1/2 the excitation frequency. The mean wavelength corresponds approximately to the calculated linear capillary wave value.

(4) Several experiments were performed in an attempt to verify Yarymovych's hypothesis of spray excited low frequency waves. It is believed that the evidence presented, resulting from these experiments, is conclusive proof of this hypothesis.

(5) Testing was done in an attempt to define the conditions necessary for spray excitation of low frequency modes. While the information obtained was insufficient to allow a complete prediction of

this phenomena, some of the results should be of further value. First, the excess surface acceleration defined in this report is an important correlating parameter and found to be generally of the form

$$a_{se} = C_1 + C_2\left(\frac{\omega^2 d}{g}\right)$$

for a constant amplitude wave. Secondly, changes produced in the required input acceleration, as surface tension alone is varied, is in the same ratio as corresponding changes produced in the input threshold of spray acceleration.

(6) Some very limited testing was done with a narrow band random input acceleration. It was found that a low frequency mode could indeed be spray excited. A plot of RMS amplitude of the spray excited wave as a function of RMS input acceleration for the random case resembled qualitatively a similar plot for the sinusoidal case. Spray production with very narrow band excitation occurred as might be expected; that is, depending upon the time varying random input amplitude, spray was or was not produced. For broader band excitation, the picture became confused and would be difficult to describe. The availability of a sensitive spray detector would probably allow a correlation to be made between the percentage of time the acceleration is above the sinusoidal threshold level and the percentage of time spray is produced.

Recommendations

Very briefly, some areas which appear to hold the most promise for further study are summarized below:

(1) Extend the surface disintegration theory to deal with:

(a) Transient disturbances generated by main engine cutoff, etc.

(b) The effects of system compressibility (wall elasticity, liquid compressibility, etc.). Since neglecting compressibility tends to make the results less conservative, inclusion of this effect would be important for prediction purposes.

(c) Wall contact effects and curved interfaces, which are usually important under low gravity conditions.

(2) Study the relationship between surface disintegration and vapor entrainment.

(3) Conduct experiments in accordance with the above ideas.

(4) Initiate an analytical and experimental investigation of liquid surface disintegration and associated phenomena for the random excitation case.

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