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A THEORETICAL STUDY OF ANTENNAS IN MOVING IONIZED MEDIA

PART II:

THE RECEIVING AREA OF A DIPOLE ANTENNA IN A MOVING MEDIUM

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ABSTRACT

The receiving area of a dipole antenna immersed in a moving medium is calculated by employing Minkowski's electrodynamics of moving media together with the power conservation law. Approximate results accurate up to the first order in v/c are obtained, v being the speed of the medium and c, the vacuum speed of light. Two cases are studied in detail: in case (i) the moving medium is simple, and in case (ii) the moving medium is an ionized gas (plasma). It is found that the receiving and transmitting patterns are identical in case (i), but not in case (ii). In both cases the effect of the motion of a medium on the receiving characteristics of a dipole antenna is brought out explicitly in the receiving-area formula which is then compared with the corresponding one when the medium is at rest.

CONTENTS

I. INTRODUCTION

In antenna theory the receiving area of an antenna plays a very important role, since it directly represents the power extracted from the incident wave by the antenna. The receiving area of an antenna in vacuum or in stationary simple media has been known for a long time[1, 2], and more recent investigations into this subject have dealt with more complicated situations such as receiving antennas in magneto-ionic media[3, 4]. However, no attention has been given in the literature to the problem of finding the receiving area of an antenna immersed in moving media. The reason may be that the complementary problem of a dipole antenna radiating in the presence of moving media had not been solved until very recently[5, 6, 7]. The solution of the complementary problem makes possible a calculation of the receiving area of a dipole antenna in moving media. In the present paper such a calculation will be given for the case where the dipole antenna is in a moving ionized gas (plasma). The method of calculations will invoke Minkowski's electrodynamics of moving media together with the power conservation law.

II. A GENERALIZATION OF THE RECIPROCITY THEOREM FOR MOVING MEDIA

In this section we shall examine the conventional method of calculating the receiving area of an antenna in a stationary medium and see if the conventional method can be extended to the present case where the receiving antenna is immersed in a moving medium.

The conventional method is based on the well-known reciprocity theoram. Let $\underline{E}_1$, $\underline{H}_1$ be the electromagnetic field radiated by a current $\underline{J}_1$ occupying a finite volume V_1, and let $\underline{E}_2$, $\underline{H}_2$ be the electromagnetic field radiated by a current $\underline{J}_2$ occupying a finite volume V_2. The two current sources oscillate harmonically at the same frequency and the medium occupying the space outside of V_1 and V_2 is isotropic and linear. Then it follows directly from Maxwell's equations,

$$\int_{V_2} \underline{J}_1 \cdot \underline{E}_2 \, dV = \int_{V_2} \underline{J}_2 \cdot \underline{E}_1 \, dV, \tag{1}$$

or

$$\int_S (\underline{E}_1 \times \underline{H}_2 - \underline{E}_2 \times \underline{H}_1) \cdot \underline{n} \, dS = 0, \tag{2}$$

where $\underline{n}$ is the outward unit normal to the closed surface S excluding the sources. Either of these two equations has served as the starting point for many investigations into the problem of finding the effective receiving area A of an antenna when the receiving antenna is perfectly matched to its load for maximum power absorption. These investigations have led to the following well-known formula:

$$A = \frac{\lambda^2}{4\pi} g(\theta_o, \phi_o) \; K \; (\theta_o, \phi_o), \tag{3}$$

where λ is the wave length of the incident radiation in the medium, (θ_o, ϕ_o) is the direction of the incident propagation vector, g is the gain of the antenna when it is in transmission, and K is the polarization loss factor and is equal to the square of the cosine of the angle between the incident electric vector and the far-zone scattered electric vector.

When the surrounding medium is anisotropic, however, the reciprocity theorem in the form of eq. 1 or 2 no longer holds. In the present case of interest, the anisotropy of the medium is caused by a uniform motion and the reciprocity theorem can be generalized into the following form (see the appendix):

$$\int_{V_1} \underline{J}_1 \cdot \underline{E}_2(-\underline{v}) \; dV = \int_{V_2} \underline{J}_2 \cdot \underline{E}_1 \; (\underline{v}) \; dV, \tag{4}$$

or

$$\int_S \left[\underline{E}_1 \; (\underline{v}) \times \underline{H}_2 \; (-\underline{v}) - \underline{E}_2 \; (-\underline{v}) \times \underline{H}_1 \; (\underline{v}) \right] \cdot \underline{n} \; dS = 0. \tag{5}$$

Here $\underline{E}_1 (\underline{v})$, $\underline{H}_1 (\underline{v})$ is the electromagnetic field radiated by a source $\underline{J}_1$ when the medium passes by the source at a velocity $\underline{v}$; $\underline{E}_2 (-\underline{v})$, $\underline{H}_2 (-\underline{v})$ is the electromagnetic field radiated by a source $\underline{J}_2$ when the medium is moving at a velocity $-\underline{v}$ with respect to the source. Since the generalized reciprocity theorem (eq. 4 or 5), unlike the usual reciprocity theorem (eq. 1 or 2), relates $\underline{E}_1$, $\underline{H}_1$ and $\underline{E}_2$, $\underline{H}_2$ under different conditions, namely, for opposite directions of the velocity $\underline{v}$, it no longer lends itself to the solution of the problem of finding the receiving area of a dipole antenna in a moving medium. Hence, the present problem has to be approached from a different starting point. In the next section the power conservation law will be formulated and this law will serve as the point of departure.

III. THE POWER CONSERVATION LAW

Consider a plane monochromatic wave incident on a dipole antenna which is immersed in a moving simple medium or in a moving ionized gas (plasma). Then, the time-average power removed from the incident wave must equal the time-average power scattered by the antenna plus the time-average power absorbed by it. This is the essence of the power conservation law, and it holds so long as the surrounding medium is non-dissipative. Let $\underline{E}^{inc}$ be the electric vector of the incident wave and $\underline{J}$ the current density induced on the dipole antenna. Then the power conservation law takes the following form:

$$\frac{1}{2} \mathrm{Re} \int \underline{J}^*(\underline{r}) \cdot \underline{E}^{inc}(\underline{r}) \, dV = P_{sc} + P_{ab}, \tag{6}$$

where P_{sc} and P_{ab} denote respectively the time-average scattered power and the time-average absorbed power, and Re denotes the real part of the expression following it. The term on the left-hand side of eq. 6 represents the interaction of the dipole antenna with the incident wave. We now go on to investigate if this interaction can be interpreted as interferences between the incident field and the far-zone scattered field.

For harmonic-time dependence of the form $e^{-i\omega t}$, Maxwell's equations are

$$\nabla \times \underline{E} = i\omega \, \underline{B} \tag{7}$$

$$\nabla \times \underline{H} = -i\omega \, \underline{D} + \underline{J} \tag{8}$$

from which Poynting's theorem follows:

$$\nabla \cdot (\underline{E} \times \underline{H}^*) - i\omega(\underline{B} \cdot \underline{H}^* - \underline{E} \cdot \underline{D}^*) = -\underline{J}^* \cdot \underline{E}. \tag{8}$$

Taking the real part of eq. 8 and integrating the resulting equation over a volume V bounded by a closed surface S we obtain

$$\text{Re} \int_S (\underline{E} \times \underline{H}^*) \cdot \underline{n} \, dA + \text{Re} \int_V \underline{J}^* \cdot \underline{E} \, dV$$

$$= -\omega \, \text{Im} \int_V (\underline{B} \cdot \underline{H}^* - \underline{E} \cdot \underline{D}) \, dV, \qquad (9)$$

where Im denotes the imaginary part of the expression following it. Let the source-free solution of eq. 7 be the incident wave and the particular solution be the scattered wave. Then the field vectors $\underline{E}$, $\underline{B}$, $\underline{D}$, $\underline{H}$ in eq. 9 can be taken as the sum of the incident plus the scattered field vectors. Substituting $\underline{E} = \underline{E}^{inc} + \underline{E}^{sc}$, $\underline{H} = \underline{H}^{inc} + \underline{H}^{sc}$, etc, into eq. 9 we obtain

$$\frac{1}{2} \text{Re} \int_S \left[(\underline{E}^{inc} + \underline{E}^{sc}) \times (\underline{H}^{inc} + \underline{H}^{sc})^* \right] \cdot \underline{n} \, dA$$

$$= -\frac{1}{2} \text{Re} \int_V \underline{J}^* \cdot (\underline{E}^{inc} + \underline{E}^{sc}) \, dV, \qquad (10)$$

provided that

$$\text{Im} \int_V \left[(\underline{B}^{inc} + \underline{B}^{sc}) \cdot (\underline{H}^{inc} + \underline{H}^{sc})^* - \right.$$

$$\left. (\underline{E}^{inc} + \underline{E}^{sc}) \cdot (\underline{D}^{inc} + \underline{D}^{sc})^* \right] dV = 0. \qquad (11)$$

We shall show that eq. 11 holds for the case of moving simple media but not for the case of moving ionized gases. When a simple medium having permittivity ϵ' and permeability μ' in its rest frame is moving at a velocity $\underline{v}$ with respect to the laboratory frame, the constitutive equations are[5]

$$\underline{D} + \frac{1}{c^2}\,\underline{v} \times \underline{H} = \epsilon'\,(\underline{E} + \underline{v} \times \underline{B})$$

$$\underline{B} - \frac{1}{c^2}\,\underline{v} \times \underline{E} = \mu'\,(\underline{H} - \underline{v} \times \underline{D}), \qquad (12)$$

where c is the vacuum speed of light. Solving eqs. 12 for $\underline{D}$ and $\underline{B}$ in terms of $\underline{E}$ and $\underline{H}$, we obtain

$$\underline{D} = \underline{\epsilon}\cdot\underline{E} + \underline{\Omega} \times \underline{H}$$

$$\underline{B} = \underline{\mu}\cdot\underline{H} - \underline{\Omega} \times \underline{E}, \qquad (13)$$

where

$$\underline{\Omega} = \frac{n'^2-1}{c^2}\,\underline{v},$$

$$\underline{\epsilon} = \frac{\epsilon'}{\gamma^2(1-n'^2\beta^2)}\left[\underline{U} + \gamma^2(1-n'^2)\underline{\beta\beta}\right],$$

$$\underline{\mu} = \frac{\mu'}{\gamma^2(1-n'^2\beta^2)}\left[\underline{U} + \gamma^2(1-n'^2)\underline{\beta\beta}\right], \qquad (14)$$

$\beta = v/c$, $\gamma = (1-\beta^2)^{-\frac{1}{2}}$, $n' = c\sqrt{\mu'\epsilon'}$, $\underline{U}$ = unit dyadic. Using eq. 13 for $\underline{B}^{inc}$, $\underline{D}^{inc}$ and $\underline{B}^{sc}$, $\underline{D}^{sc}$ separately, one can easily see that the integrand in eq. 11 has no imaginary part. Consequently, eq. 10 is valid for moving simple media.

In the case of moving ionized gases, ϵ' and n' are functions not only of frequency ω but also of wave vector $\underline{k}$, and therefore, eqs. 13 hold only in $(\omega, \underline{k})$ space. Transforming eqs. 13 into $(\omega, \underline{r})$ space, one obtains for a moving ionized gas,

$$\underline{D} = \hat{\underline{\epsilon}}\cdot\underline{E} + \hat{\underline{\Omega}} \times \underline{H} \qquad (15)$$

$$\underline{B} = \hat{\underline{\mu}}\cdot\underline{H} - \hat{\underline{\Omega}} \times \underline{E}$$

Where $\underline{\epsilon}$, $\underline{\mu}$ and Ω are integral operators whose spatial Fourier transforms are respectively given by eqs. 14. Because the constitutive eqs. 15 for a moving ionized gas are in integral form, the imaginary part of the integrand in eqs. 11 is no longer identically zero, and consequently, eqs. 10 does not hold for the case of moving ionized gases.

For a moving simple medium we obtain by substituting eq. 10 into eq. 6

$$\frac{1}{2} \text{Re} \int_S \left[(\underline{E}^{inc} + \underline{E}^{sc}) \times (\underline{H}^{inc} + \underline{H}^{sc})^* \right] \cdot \underline{n} dA + P_{ab} = 0 \qquad (16)$$

where we have used

$$-\frac{1}{2} \text{Re} \int \underline{J}^* \cdot \underline{E}^{sc} \, dV = P_{sc}$$

In eq. 16, the term corresponding to the incident wave alone integrates to zero, and the term corresponding to the scattered wave alone integrates to give the total time-average scattered power P_{sc}. Hence, eq. 16 reduces to

$$\frac{1}{2} \text{Re} \int_S (\underline{E}^{inc} \times \underline{H}^{sc*} + \underline{E}^{sc} + \underline{H}^{inc*}) \cdot \underline{n} dA + P_{sc} + P_{ab} = 0 \qquad (17)$$

This is the usual form of the power conservation law described in the literature for stationary simple media and is taken as the starting point for deducing the well-known optical theorem in electromagnetic scattering problems[6]. Here we have established the validity of eq. 17 for the case of moving media. For moving dispersive media (e.g., moving ionized gases), however, eq. 17 no longer holds and the power conservation law takes the more general form given by eq. 16.

We shall begin with eq. 6 or 17 and proceed to calculate the receiving area of a dipole antenna immersed in a moving medium. The receiving area will be sought in terms of the incident flux and the gain of the antenna when the antenna is in transmission. However, before such a calculation can be carried out, it is necessary to specify the form of the incident wave and to examine the nature of far-zone scattered field in moving media. In the following two sections we shall study plane and spherical wave solutions of Maxwell's equations in moving simple media and in moving ionized gases.

IV. PLANE WAVES IN MOVING MEDIA

We shall study plane wave solutions of Maxwell's equations in moving simple media and in moving ionized gases. Assuming a plane electromagnetic wave of the form exp $(i\underline{k}\cdot\underline{r}-i\omega t)$ we have from Maxwell's equations,

$$\underline{k} \times \underline{E} = \omega\underline{B} \tag{18}$$

$$\underline{k} \times \underline{H} = -\omega\underline{D},$$

together with the constitutive eqs. 13,

$$\underline{D} = \underline{\epsilon}\cdot\underline{E} + \underline{\Omega} \times \underline{H} \tag{13}$$

$$\underline{B} = \underline{\mu}\cdot\underline{H} - \underline{\Omega} \times \underline{E}.$$

$\underline{E}$, $\underline{B}$, $\underline{D}$, $\underline{H}$ are constant field vectors and $\underline{\epsilon}$, $\underline{\mu}$, $\underline{\Omega}$ are defined by eqs. 14. Elimination of $\underline{B}$ and $\underline{D}$ from eqs. 18 by means of eqs. 13 gives

$$\underline{s} \times \underline{E} = \omega\underline{\mu}\cdot\underline{H} \tag{19}$$

$$\underline{s} \times \underline{H} = -\omega\underline{\epsilon}\cdot\underline{E},$$

where

$$\underline{s} = \underline{k} + \omega\underline{\Omega}.$$

Here and henceforth we shall assume that the speed of the medium v is much smaller than the vacuum speed of light c, i.e., $\beta = v/c \ll 1$, and we shall retain terms up to the order of β.

In the case of moving simple media we have from eqs. 19, to a first order in β,

$$\underline{s} \times \underline{E} = \omega\mu'\underline{H} \qquad (20)$$

$$\underline{s} \times \underline{H} = -\omega\epsilon'\underline{E},$$

where the permittivity ϵ' and the permeability μ' are constant in the rest frame of the medium. Equations 20 show that $E/H = \sqrt{\epsilon'/\mu'}$ and that $\underline{E}$, $\underline{H}$, $\underline{s}$ are mutually perpendicular. This means that the wave impedance is not affected by the motion of the medium in this approximation. However, the Poynting vector, which is in the direction of $\underline{s}$, no longer coincides with the direction of the wave vector $\underline{k}$. From eqs. 20 we find

$$k = \frac{n'\omega}{c}(1-a\cos\theta) \qquad (21)$$

$$\sin\chi = \sin\theta\,(1-a\cos\theta),$$

where $a = \beta\,(n'^2-1)/n'$, θ = angle between $\underline{k}$ and $\underline{v}$, and χ = angle between $\underline{s}$ and $\underline{v}$. Hence, to a first order in β, plane waves can be represented as

$$\underline{E} = \underline{E}_o\, e^{i(1-a\cos\theta)\underline{k}\cdot\underline{r}} \qquad (22)$$

$$\underline{H} = \sqrt{\epsilon'/\mu'}\;\underline{e}_s \times \underline{E}.$$

Here we have defined $k = n'\omega/c$, $\underline{e}_s$ = unit vector of $\underline{s}$, and $\underline{E}_o$ = constant vector.

In the case of moving ionized gases we have

$$\mu' = \mu_o, \quad \epsilon' = \epsilon_o\left[1-\frac{(1-\beta^2)\omega_p^{\,2}}{(\omega-\underline{v}\cdot\underline{k})^2}\right], \qquad (23)$$

where ω_p is the plasma frequency and is a Lorentz invariant. To a first order in β, eqs. 19 become

$$\underline{s} \times \underline{E} = \omega\mu_o \underline{H}$$

$$\underline{s} \times \underline{H} = -\omega\epsilon' \underline{E}.$$

Here

$$\underline{s} = \underline{k} - \frac{k_p^2}{k_o} \underline{\beta}$$

$$\frac{\epsilon'}{\epsilon_o} = 1 - \frac{k_p^2}{k_o^2} - 2\,\frac{\underline{\beta}\cdot\underline{k}\; k_p^2}{k_o^3} ,$$

and $k_o = \omega/c$, $k_p = \omega_p/c$. From eqs. 24 we find

$$k = \sqrt{k_o^2 - k_p^2} \qquad (25)$$

$$\sin \chi = \sin\theta\,(1 + b\cos\theta),$$

where $b = \beta k_p^2/(k\, k_o)$, χ and θ have the same meaning as in the case of moving simple media. Hence, to a first order in β, plane waves in moving ionized gases are of the form

$$\underline{E} = \underline{E}_o\, e^{i\underline{k}\cdot\underline{r}} \qquad (26)$$

$$\underline{H} = \frac{k}{k_o}\sqrt{\frac{\epsilon_o}{\mu_o}}\,(1 - b\cos\theta)\,\underline{e}_s \times \underline{E}.$$

Here k is given by eq. 25, $\underline{e}_s$ is a unit vector, and $\underline{E}_o$ is a constant vector. Equations 26 show that, to a first order in β, the motion of a ionized gas does not affect the propagation vector but does affect the wave impedance and the direction of energy flow.

V. DIPOLE FIELD IN MOVING MEDIA

The problem of finding the radiation field of an oscillating dipole immersed in a moving medium has been solved recently[7,8]. The formulation of the problem is based on the covariance of Maxwell's equations. A differential equation for the potential 4-vector is first deduced in the rest frame of the dipole with respect to which the medium is moving at a velocity $\underline{v}$. The differential equation is then solved by the Green's function technique. With a knowledge of the potential 4-vector, the dipole field is obtained by differentiation.

It was found that, in the case of moving simple media, the far-zone dipole field[7] is, to a first order in β,

$$\underline{E} = \omega^2 \mu' \, \underline{e}_r \times (\underline{p} \times \underline{e}_r) \, e^{-iakr \cos \theta} \, \frac{e^{ikr}}{4\pi r} \tag{27}$$

$$\underline{H} = \sqrt{\frac{\epsilon'}{\mu'}} \, \underline{e}_r \times \underline{E} \, ,$$

where $\underline{p}$ = dipole moment, $\underline{e}_r$ = unit radial vector, $a = \beta(n'^2 - 1)/n'$, $k = n'\omega/c$, and θ = angle between $\underline{e}_r$ and $\underline{v}$. The gain is defined as

$$g(\theta,\varphi) = \frac{4\pi r^2 \, \mathrm{Re}\,(\underline{E} \times \underline{H}^*) \cdot \underline{e}_r}{\int_S \mathrm{Re}\,(\underline{E} \times \underline{H}^*) \cdot \underline{e}_r \, dA} \, . \tag{28}$$

Substitution of eq. 27 into eq. 28 gives

$$g(\theta,\varphi) = \frac{3}{2} \left[\underline{e}_r \times (\underline{e}_p \times \underline{e}_r) \right]^2 , \tag{29}$$

where $\underline{e}_p$ is the unit vector in the direction of $\underline{p}$.

In the case of a moving ionized gas, we make use of the results in Reference 8 and find that, to a first order in β, the far-zone dipole field is given by

$$\underline{E} = \omega^2\mu_o \left[\underline{e}_r \times (\underline{p} \times \underline{e}_r) - b\,\underline{p} \cdot (\underline{e}_r\underline{e}_\theta + \underline{e}_\theta\underline{e}_r)\sin\theta\right] \frac{e^{ikr}}{4\pi r}$$

$$\underline{H} = \omega k\left[\underline{e}_r \times \underline{p} - b\,(\cos\theta\,\underline{e}_r \times \underline{p} - \sin\theta\,\underline{e}_\theta \times \underline{p})\right] \frac{e^{ikr}}{4\pi r}, \tag{30}$$

where $b = \beta k_p^2/(k_o k)$, and $k^2 = k_o^2 - k_p^2$. Introducing the unit vector $\underline{e}_s$

$$\underline{e}_s = (1 + b\cos\theta)\underline{e}_r - b\,\underline{e}_z = \underline{e}_r + b\sin\theta\,\underline{e}_\theta, \tag{31}$$

we can rewrite eqs. 30 as

$$\underline{E} = \omega^2\mu_o \left[\underline{e}_s \times (\underline{p} \times \underline{e}_s)\right] \frac{e^{ikr}}{4\pi r}.$$

$$\underline{H} = \omega k(1 - b\cos\theta)\,(\underline{e}_s \times \underline{p})\,\frac{e^{ikr}}{4\pi r}. \tag{32}$$

Equations 32 show that $\underline{E}$, $\underline{H}$ and $\underline{e}_s$ are mutually perpendicular, and hence, $\underline{e}_s$ is in the direction of energy flow. The gain is now given by

$$g(\theta,\varphi) = \frac{4\pi r^2\,\mathrm{Re}\,(\underline{E} \times \underline{H}^*)\cdot\underline{e}_s}{\int \mathrm{Re}\,(\underline{E} \times \underline{H}^*)\cdot\underline{e}_s\,dA}. \tag{33}$$

Insertion of eqs. 32 in eq. 33 gives

$$g(\theta,\varphi) = \frac{4\pi(1 - b\cos\theta)\left[\underline{e}_s \times (\underline{e}_p \times \underline{e}_s)\right]^2}{\int(\underline{e}_p \times \underline{e}_s)^2\,(1 - b\cos\theta)\,d\Omega}.$$

The integral can be evaluated as follows, keeping only terms of the order of β.

$$\int(\underline{e}_p \times \underline{e}_s)^2(1 - b\cos\theta)d\Omega = \int\left[1-(\underline{e}_s\cdot\underline{e}_p)^2\right](1-b\cos\theta)\,d\Omega$$

$$= \frac{8\pi}{3} + b\int \cos\theta\,(\underline{e}_s\cdot\underline{e}_p)^2\,d\Omega$$

$$= \frac{8\pi}{3} + b\int \cos\theta\,(\underline{e}_r\cdot\underline{e}_p)^2\,d\Omega$$

$$= \frac{8\pi}{3}\,.$$

Thus, we obtain for the gain

$$g(\theta,\varphi) = \frac{3}{2}(1 - b\cos\theta)\left[\underline{e}_s \times (\underline{e}_p \times \underline{e}_s)\right]^2\,. \tag{34}$$

In terms of $\underline{e}_r$, $\underline{e}_\theta$ and $\underline{e}_\varphi$ eq. 34 takes the form

$$g(\theta,\varphi) = \frac{3}{2}\left[\underline{e}_r \times (\underline{e}_p \times \underline{e}_r)\right]^2 - \frac{3}{2}\,b\left\{\cos\theta\left[\underline{e}_r \times (\underline{e}_p \times \underline{e}_r)\right]^2 + 2\sin\theta(\underline{e}_r\cdot\underline{e}_p)(\underline{e}_\theta\cdot\underline{e}_p)\right\}. \tag{35}$$

Examination of eq. 29 shows that, to a first order in β, the motion of a simple medium has no effect on the gain of the dipole antenna. However, the motion of an ionized gas, as shown in eq. 35, has a first order effect on the gain.

VI. POWER ABSORPTION AND RECEIVING AREA OF A DIPOLE ANTENNA IN MOVING SIMPLE MEDIA

We shall now calculate the power absorption of a dipole antenna in terms of the incident flux, the antenna gain, and the polarization loss factor when the dipole antenna is immersed in a moving simple medium.

Our point of departure is the conservation law (eq. 17) together with eqs. 22 for the incident wave and eqs. 27 for the scattered field. Without loss of generality, we choose the polar axis of the spherical coordinate system (r,θ,ϕ) to be in the direction of $\underline{v}$. Then from eqs. 27 we have

$$(\underline{E}^{inc} \times \underline{H}^{sc^*})\cdot\underline{e}_r = \omega k \frac{e^{-ikr}}{4\pi r} \underline{E}_o\cdot\left[\underline{e}_r \times (\underline{p}^* \times \underline{e}_r)\right]e^{ikrf(\theta,\phi)}$$

$$(\underline{E}^{sc} \times \underline{H}^{inc^*})\cdot\underline{e}_r = \omega k \frac{e^{ikr}}{4\pi r}\left[\underline{e}_r \times (\underline{E}_o^* \times \underline{e}_s)\right]\cdot\left[\underline{e}_r \times (\underline{p} \times \underline{e}_r)\right]e^{-ikrf(\theta,\phi)}, \tag{36}$$

where

$$f(\theta,\phi) = a\cos\theta + (1 - a\cos\theta_o)\times\left[\sin\theta_o\sin\theta\cos(\phi_o - \phi) + \cos\theta_o\cos\theta\right], \tag{37}$$

(θ_o, ϕ_o) being the direction of propagation of the incident wave in spherical coordinates.

Let us now consider an integral of the form

$$\int_0^{2\pi}\int_0^{\pi} h(\theta,\phi)\, e^{\pm ikrf(\theta,\phi)}\sin\theta\, d\theta\, d\phi \tag{38}$$

for $kr >> 1$. To evaluate this integral we shall invoke the method of stationary phase. The stationary points (θ_s,ϕ_s)

are obtained by solving the following equations:

$$\frac{\partial f}{\partial \theta} = - a \sin \theta + (1 - a \cos \theta_o) \times \left[\sin \theta_o \cos \theta \cos (\phi_o - \phi) - \cos \theta_o \sin \theta \right] = 0$$

$$\frac{\partial f}{\partial \phi} = (1 - a \cos \theta_o) \sin \theta_o \sin \theta \sin (\phi_o - \phi) = 0 \qquad (39)$$

From eqs. 39 we find, to a first order in β,

$$\theta_s = \chi \ , \ \phi_s = \phi_o \ , \qquad (40)$$

and

$$\theta_s = \pi - \chi \ , \ \phi_s = \pi + \phi_o, \qquad (41)$$

where χ is given by eq. 21; i.e.,

$$\sin \chi = \sin \theta_o \ (1 - a \cos \theta_o).$$

Recalling that a is of the order of β, we then have

$$\chi = \theta_o - a \sin \theta_o \ . \qquad (42)$$

Evaluating integral 38 around the two stationary points, we obtain, after some lengthy calculations[9],

$$\int_0^{2\pi} \int_0^{\pi} h(\theta,\phi) e^{\pm ikrf(\theta,\phi)} \sin \theta \ d\theta \ d\phi$$

$$\approx \frac{-2\pi i}{kr} \left[\pm \ h(\chi,\phi_o) e^{\pm ikr} \mp h(\pi - \chi, \ \pi + \phi_o) e^{\mp ikr} \right]. \qquad (43)$$

Making use of formula 43 and noting that $\underline{e}_r = \underline{e}_s$ when $\theta_s = \chi$, $\phi_s = \phi_o$, and that $\underline{e}_r = -\underline{e}_s$ when $\theta_s = \pi - \chi$, $\phi_s = \pi + \phi_o$, we get

$$\frac{1}{2} \text{Re} \int (\underline{E}^{inc} \times \underline{H}^{sc^*}) \cdot \underline{e}_r \, dA$$

$$= \frac{\omega}{4} \text{Im} \left\{ \underline{E}_o \cdot \left[\underline{e}_s \times (\underline{p}^* \times \underline{e}_s) \right] \right.$$

$$\left. - \underline{E}_o \cdot \left[\underline{e}_s \times (\underline{p}^* \times \underline{e}_s) \right] e^{-2ikr} \right\} , \tag{44}$$

and

$$\frac{1}{2} \text{Re} \int (\underline{E}^{sc} \times \underline{H}^{inc^*}) \cdot \underline{e}_r \, dA$$

$$= - \frac{\omega}{4} \text{Im} \left\{ \underline{E}_o^* \cdot \left[\underline{e}_s \times (\underline{p} \times \underline{e}_s) \right] \right.$$

$$\left. + \underline{E}_o^* \cdot \left[\underline{e}_s \times (\underline{p} \times \underline{e}_s) \right] e^{2ikr} \right\} . \tag{45}$$

In deriving eq. 45 we have used the fact that $\underline{e}_s \cdot \underline{E}_o^* = 0$. Adding eqs. 44 and 45 we find that the power conservation law 17 gives

$$\frac{\omega}{2} \text{Im} \, \underline{E}_o^* \cdot \left[\underline{e}_s \times (\underline{p} \times \underline{e}_s) \right] = P_{sc} + P_{ab} \, . \tag{46}$$

Let the dipole antenna be perfectly matched for maximum power absorption. Then the power absorbed by the antenna equals the power scattered by it. The scattered power can be found by integrating the Poynting vector obtained from the far field expressions 27. Hence

$$P_{ab} = P_{sc} = \frac{\omega^4 \mu' \sqrt{\mu' \epsilon'}}{12 \pi} |p|^2 . \tag{47}$$

The condition of maximum power absorption further implies that p be 90 degrees out of phase with E_o, as can be seen from the interaction term in eq. 6. Here $\underline{p} = \underline{e}_p\, p$ and $\underline{E}_o = \underline{e}_o\, E_o$, $\underline{e}_p$ and $\underline{e}_o$ being unit vectors and the incident wave being taken to be linearly polarized. Now we can solve eq. 46 for $|p|$, and then substituting the resulting $|p|$ into eq. 47, we find

$$P_{ab} = \frac{\lambda^2}{4\pi} S^{inc} \frac{3}{2} \left[\underline{e}_s \times (\underline{e}_p \times \underline{e}_s) \right]^{-2} K(\chi, \phi_o) \,, \tag{48}$$

where the incident energy flux S^{inc} is given by

$$S^{inc} = \frac{1}{2} \sqrt{\frac{\mu'}{\epsilon'}}\, \underline{E}^{inc} \cdot \underline{E}^{inc^*} \,,$$

and the polarization loss factor K is defined as

$$K(\chi, \phi_o) = \frac{|\,\underline{E}^{inc}(\chi,\phi_o) \cdot \underline{E}^{sc}(\chi,\phi_o)\,|^2}{|\,E^{inc}(\chi,\phi_o)\,|^2 |\,E^{sc}(\chi,\phi_o)\,|^2} \,.$$

By virtue of eq. 29 we finally obtain

$$P_{ab} = \frac{\lambda^2}{4\pi}\, g(\chi,\phi_o)\, K\,(\chi,\phi_o)\, S^{inc} \,, \tag{49}$$

whence the effective receiving area A is

$$A = \frac{\lambda^2}{4\pi}\, g(\chi,\phi_o)\, K\,(\chi,\phi_o) \,. \tag{50}$$

Comparing our formulas 49 and 50 for the power absorbed by and the receiving area of a dipole antenna in moving simple media with the corresponding well-known formulas in stationary simple media, we come to the following conclusion. To a first order in β, the formulas in the two cases have the same formal structure except for one difference, i.e., in the case where the medium is in motion it is the direction of the incident Poynting vector, not the direction of the propagation vector of the incident wave, which enters the formulas explicitly.

Let us now consider very briefly how we can re-derive eqs. 49 and 50 starting from the power conservation law 6. This consideration will provide us a guideline when we calculate the receiving area of a dipole antenna in a moving ionized gas.

For a receiving dipole antenna situated at the origin, eq. 6 gives

$$\frac{\omega}{2} \text{Im } \underline{p} \cdot \underline{E}_o^* = P_{sc} + P_{ab} \; .$$

Under the condition of maximum power absorption this equation yields a unique solution for $|p|$. With a knowledge of $|p|$ we obtain

$$P_{ab} = \frac{3\lambda^2}{8\pi} (\underline{e}_o \cdot \underline{e}_p)^2 \, S^{inc} \qquad .$$

This is as far as we can get from conservation law 6. To proceed further we note that, for any unit vector $\underline{e}$ perpendicular to $\underline{e}_o$, we have $\underline{e}_o \cdot \underline{e}_p = \underline{e}_o \cdot \left[\underline{e} \times (\underline{e}_p \times \underline{e}) \right]$. To obtain eqs. 49 and 50 we must set $\underline{e} = \underline{e}_s$. This choice of $\underline{e}$ will be used in the following section.

VII. POWER ABSORPTION AND RECEIVING AREA OF A DIPOLE ANTENNA IN MOVING IONIZED GASES

Let us now calculate the receiving area of a dipole antenna immersed in a moving ionized gas. We shall first determine the induced dipole moment from the power conservation law under the condition of maximum power absorption. Then we shall calculate the power absorbed by the dipole antenna in terns of the incident flux, the gain, and the polarization loss factor.

As was shown in Section III, the power conservation law in a moving ionized gas takes the form of eq. 6. Writing $\underline{J}(r) = -i\omega \underline{p}\, \delta(\underline{r})$ and using eq. 26 for $\underline{E}^{inc}$, we obtain from eq. 6,

$$\frac{\omega}{2} \operatorname{Im} \underline{p}\cdot\underline{E}_o^* = P_{sc} + P_{ab} \; . \tag{51}$$

P_{sc} can be obtained by integrating the Poynting vector formed from the far field expressions eqs. 30 and is found to be

$$P_{sc} = \frac{\omega^4 \mu_o \sqrt{\mu_o \epsilon_o}}{12\pi} (1 - X)^{\frac{1}{2}} |p|^2 \quad , \tag{52}$$

where $X = \omega_p^2/\omega^2$. Assume that the dipole antenna is perfectly matched for maximum power absorption. Then $P_{ab} = P_{sc}$ and hence eq. 51 gives

$$\operatorname{Im} \underline{p}\cdot\underline{E}_o^* = \frac{\omega^3 \mu_o \sqrt{\mu_o \epsilon_o}}{3\pi} (1 - X)^{\frac{1}{2}} |p|^2 \quad . \tag{53}$$

Since the condition of maximum power absorption demands that p and E_o be 90 degrees out of phase, eq. 53 suffices for the determination of p. Writing $p = i\, \alpha\, E_o$ (α being a constant), we find from eq. 53 that α, and hence p, is given by

$$p = i\, \alpha\, E_o = \frac{3\pi\, \underline{e}_p \cdot \underline{e}_o}{(1 - X)^{\frac{1}{2}}\, \omega^3 \mu_o \sqrt{\mu_o \epsilon_o}} E_o \quad , \tag{54}$$

where $\underline{e}_p$ and $\underline{e}_o$ are unit vectors in the directions of $\underline{p}$ and $\underline{E}_o$, respectively. Insertion of eq. 54 in eq. 52 gives

$$P_{ab} = P_{sc} = \frac{3\lambda^2}{16\pi} \sqrt{\frac{\epsilon_o}{\mu_o}} (1 - X)^{\frac{1}{2}} | E_o |^2 (\underline{e}_p \cdot \underline{e}_o)^2 , \tag{55}$$

where $\lambda^{-1} = \omega \sqrt{\mu_o \epsilon_o} (1 - X)^{\frac{1}{2}} / 2\pi$. The incident energy flux S^{inc} is given by

$$S^{inc} = \frac{1}{2} \mathrm{Re} \, (\underline{E}^{inc} \times \underline{H}^{inc^*}) \cdot \underline{e}_s .$$

substitution of eqs. 26 into this equation gives, to a first order in β,

$$\frac{1}{2} \sqrt{\frac{\epsilon_o}{\mu_o}} (1 - X)^{\frac{1}{2}} | E_o |^2 = S^{inc} (1 + b \cos \theta_o) \tag{56}$$

from which eq. 55 becomes

$$P_{ab} = \frac{3\lambda^2}{8\pi} S^{inc} (1 + b \cos \theta_o) (\underline{e}_p \cdot \underline{e}_o)^2 , \tag{57}$$

where, as before, $b = \beta k_p^2/(k k_o)$ and θ_o = angle between the propagation vector of the incident wave and the velocity $\underline{v}$ or, which amounts to the same thing, the polar axis of the spherical coordinate system.

We shall now proceed to express eq. 57 in terms of the gain and the polarization loss factor. To do this we note that, since $\underline{e}_s^{inc}$, the unit vector in the direction of the incident flux, is perpendicular to $\underline{e}_o$, we can write

$$\underline{e}_o \cdot \underline{e}_p = \underline{e}_o \cdot \left[\underline{e}_s^{inc} \times (\underline{e}_p \times \underline{e}_s^{inc}) \right] . \tag{58}$$

From eqs. 30 we find that the unit vector $\underline{e}_{sc}$ in the direction of the scattered electric field vector is given by

$$\underline{e}_{sc} = \frac{\underline{e}_s \times (\underline{e}_p \times \underline{e}_s)}{\sqrt{[\underline{e}_s \times (\underline{e}_p \times \underline{e}_s)]\cdot[\underline{e}_s \times (\underline{e}_p \times \underline{e}_s)]}} \quad ;$$

whence

$$(\underline{e}_o \cdot \underline{e}_p)^2 = \left[\underline{e}_s^{inc} \times (\underline{e}_p \times \underline{e}_s^{inc})\right]^2 \left[\underline{e}_o \cdot \underline{e}_{sc}(\theta_s,\phi_s)\right]^2 , \qquad (59)$$

where (θ_s,ϕ_s) is the direction of $\underline{e}_s^{inc}$. Inserting eq. 59 in eq. 57 and making use of eq. 34 we can rewrite eq. 57 as

$$P_{ab} = \frac{\lambda^2}{4\pi} \frac{1 + b \cos \theta_o}{1 - b \cos \theta_o} g(\theta_s,\phi_s) K (\theta_s,\phi_s) S^{inc} . \qquad (60)$$

Here g is the gain of the dipole antenna when it is in transmission, and the polarization loss factor K is defined as

$$K = (\underline{e}_o \cdot \underline{e}_{sc})^2 = \frac{|\underline{E}^{inc} \cdot \underline{E}^{sc}|^2}{|E^{inc}|^2 |E^{sc}|^2} .$$

To the accuracy of a first order in β we can write

$$\frac{1 + b \cos \theta_o}{1 - b \cos \theta_o} = (1 + 2 b \cos \theta_o) = (1 + 2 b \cos \theta_s) ,$$

where we have used eq. 25 for the relation between θ_s and θ_o . Equation 60 now becomes

$$P_{ab} = \frac{\lambda^2}{4\pi} (1 + 2 b \cos \theta_s) g (\theta_s,\phi_s) K(\theta_s,\phi_s) S^{inc} \qquad (61)$$

from which we obtain the effective receiving area A,

$$A(\theta_s,\phi_s) = \frac{\lambda^2}{4\pi} (1 + 2 b \cos \theta_s) g (\theta_s,\phi_s) K (\theta_s,\phi_s) . \qquad (62)$$

Thus we see that the receiving pattern of a dipole antenna in a moving ionized gas is given by $(1 + 2b\cos\theta_s)g(\theta_s,\phi_s)$, while the transmitting pattern is $g(\theta_s,\phi_s)$. The two patterns become identical only when the direction of the incident energy flux is perpendicular to the motion of the ionized gas.

VIII. CONCLUDING REMARKS

Let us now review the material which has been presented. The main result in this report is the evaluation of the effective receiving area of a dipole antenna immersed in a moving medium. Two cases have been studied in detail: in case (i) the moving medium is simple, and in case (ii) the moving medium is an ionized gas (plasma). The receiving area is obtained in terms of the polarization loss factor, the antenna's gain, and the direction of the incident energy flux. Since in practice the speed of the medium v is always much smaller than the vacuum speed of light c, approximate results accurate up to the first order in v/c are given.

The method of solution departs from the conventional one where the reciprocity theorem is employed. It is shown that the reciprocity theorem can be generalized to moving media. However, the generalized reciprocity theorem relates the incident and scattered fields under different conditions, namely, the incident field in a medium moving at a velocity $\underline{v}$ and the scattered field in a medium moving at a velocity $-\underline{v}$. Thus, the generalized reciprocity theorem does not lend itself to the problem of finding the receiving area of a dipole antenna in a moving medium. The approach is found in the power conservation law which equates the time-average power removed from the incident wave to the time-average power absorbed by the antenna plus the time-average power absorbed by the antenna plus the time-average power scattered by it. For a receiving dipole antenna this conservation law is particularly useful, for it yields readily the induced dipole moment from which the receiving area can be easily calculated. For a receiving antenna of more general type the problem must be treated as a boundary-value problem which, in principle, can be solved once the Green's function in a moving medium is known.

In case (i) where the moving medium is simple, it is found that the receiving-area formula resembles that for the case of a stationary simple medium. The only difference is that the motion of the medium makes distinguishable the direction of Poynting's vector and the direction of the propagation vector. It is the direction of the incident Poynting vector, not the direction of the incident propagation vector, which enters explicitly the receiving-area formula. In case (ii) where the moving medium is an ionized gas, it is found that the receiving pattern of a dipole antenna is not identical to its transmitting pattern, in contradistinction to the case where the ionized gas is at rest. Again, the motion of the ionized gas brings into play the direction of the incident energy flux in the receiving-area formula.

The results of this investigation provide quantitative information about the effect of the motion of a medium on the receiving characteristics of a dipole antenna. The practical importance of this information stems from the need of antenna designers to know how the motions of various types of media affect the antenna operation in reception. In this study, it is found that the motion of an ionized gas has a more pronounced effect on the receiving characteristics of a dipole antenna than does the motion of a non-dispersive medium. When an ionized gas is in motion, it appears to be not only temporally dispersive but also spatially dispersive. In such a medium, the physical interpretations of the Poynting vector, the field momentum, and the field energy become very obscure. As a result, in a moving ionized gas the power conservation law takes a form more general than the usual one. It is concluded that a detailed study should be made on the electromagnetic properties of spatially dispersive media—a branch of electrodynamics which has so far been neglected in the literature.

APPENDIX

In this appendix we shall show that the generalized reciprocity theorem for moving media is given by

$$\int_{V_1} \underline{J}_1 \cdot \underline{E}_2(-\underline{v})dV = \int_{V_2} \underline{J}_2 \cdot \underline{E}_1(\underline{v})dV, \tag{A.1}$$

or by

$$\int_S \left[\underline{E}_1(\underline{v}) \times \underline{H}_2(-\underline{v}) - \underline{E}_2(-\underline{v}) \times \underline{H}_1(\underline{v})\right] \cdot \underline{n}dS = 0 . \tag{A.2}$$

The Maxwell equations for the field produced by $\underline{J}_1$ are

$$\nabla \times \underline{E}_1 = i\omega \underline{B}_1 \tag{A.3}$$

$$\nabla \times \underline{H}_1 = -i\omega \underline{D}_1 + \underline{J}_1 , \tag{A.4}$$

and the Maxwell equations for the field produced by $\underline{J}_2$ are

$$\nabla \times \underline{E}_2 = i\omega \underline{B}_2 \tag{A.5}$$

$$\nabla \times \underline{H}_2 = -i\omega \underline{D}_2 + \underline{J}_2 . \tag{A.6}$$

Here and henceforth, it is understood that the field with subscript 1 depends on $\underline{v}$ and that with subscript 2 depends on $-\underline{v}$. Multiplying A.4 by $\underline{E}_2$ and A.6 by $\underline{E}_1$ and making use of A.3 and A.5 we obtain

$$\underline{J}_1 \cdot \underline{E}_2 = i\omega \underline{D}_1 \cdot \underline{E}_2 + i\omega \underline{B}_2 \cdot \underline{H}_1 + \nabla \cdot (\underline{H}_1 \times \underline{E}_2) \tag{A.7}$$

$$\underline{J}_2 \cdot \underline{E}_1 = i\omega \underline{D}_2 \cdot \underline{E}_1 + i\omega \underline{B}_1 \cdot \underline{H}_2 + \nabla \cdot (\underline{H}_2 \times \underline{E}_1) . \tag{A.8}$$

Subtracting A.8 from A.7 and integrating the resulting equation over all space, we get

$$\int (\underline{J}_1\cdot\underline{E}_2 - \underline{J}_2\cdot\underline{E}_1)dV = \int (\underline{E}_1 \times \underline{H}_2 - \underline{E}_2 \times \underline{H}_1)\cdot\underline{n}dA +$$

$$+ i\omega\int (\underline{E}_2\cdot\underline{D}_1 - \underline{E}_1\cdot\underline{D}_2 + \underline{B}_2\cdot\underline{H}_1 - \underline{B}_1\cdot\underline{H}_2)dV = 0. \qquad (A.9)$$

By an application of Parseval's theorem the second integral on the righthand side of A.9 can be converted into one over the wave vector $\underline{k}$ space, i.e.,

$$\int (\underline{E}_2\cdot\underline{D}_1 - \underline{E}_1\cdot\underline{D}_2 + \underline{B}_2\cdot\underline{H}_1 - \underline{B}_1\cdot\underline{H}_2)dV =$$

$$\frac{1}{8\pi^3}\int \Big[\underline{D}_1(\underline{k})\cdot\underline{E}_2(-\underline{k})-\underline{E}_1(\underline{k})\cdot\underline{D}_2(-\underline{k}) + \underline{H}_1(\underline{k})\cdot\underline{B}_2(-\underline{k}) - \underline{B}_1(\underline{k})\cdot\underline{H}_2(-\underline{k})\Big]d^3k. \qquad (A.10)$$

We shall now show that the integrand on the righthand side of A.10 is identically zero. To do this we first transform the constitutive equations (eqs. 15) into wave vector-frequency space and obtain

$$\underline{D}(\omega,\underline{k}) = \underline{\epsilon}(\omega,\underline{k})\cdot\underline{E}(\omega,\underline{k}) + \underline{\Omega}(\omega,\underline{k}) \times \underline{H}(\omega,\underline{k})$$

$$\underline{B}(\omega,\underline{k}) = \underline{\mu}(\omega,\underline{k})\cdot\underline{H}(\omega,\underline{k}) - \underline{\Omega}(\omega,\underline{k}) \times \underline{E}(\omega,\underline{k}) \qquad (A.11)$$

For moving simple media, $\underline{\epsilon}$, $\underline{\mu}$, $\underline{\Omega}$ are constant, while for moving ionized gases, they are functions of $\omega-\underline{v}\cdot\underline{k}$. By virtue of A.11 and noting that $\underline{\epsilon}$, $\underline{\mu}$, $\underline{\Omega}$ are invariant under the simultaneous transformations $\underline{v} \to -\underline{v}$ and $\underline{k} \to -\underline{k}$, we can easily see that the integrand on the righthand side of A.10 is identically zero. Hence, A.9 reduces to

$$\int (\underline{J}_1\cdot\underline{E}_2 - \underline{J}_2\cdot\underline{E}_1)dV = \int_S (\underline{E}_1 \times \underline{H}_2 - \underline{E}_2 \times \underline{H}_1)\cdot\underline{n}dA\ . \qquad (A.12)$$

The surface integral on the righthand side of A.12 gives no contribution if S is chosen to be a closed surface at infinity, and we arrive at A.1. On the other hand, if S is chosen not to enclose the sources $\underline{J}_1$ and $\underline{J}_2$ the lefthand side of A.12 is zero, and we arrive at A.2.

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