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C ANGULAR DISTRIBUTIONS AND ENERGY SPECTRA OF ELECTRONS TRANSMITTED THROUGH AND REFLECTED FROM ELEMENTAL FOILS
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ABSTRACT

Experimental electron energy sprectra at various angles of emergence or reflection from foils of carbon, aluminum, copper, silver, and gold have been measured. Thicknesses used ranged from 0.2 to 0.6 of the electron range. In all cases the incident radiation was a broad, monodirectional, monoenergetic electron beam perpendicular to the foil. Beam energies were $0.5,0.75,1.0,1.5$, and 1.95 MeV . The spectrometer used an anthracene crystal as the detector.

## SUMIMARY

Electrons in the Van Allen radiation belt are a hazard to space exploration. This report studies the scattering and degradation of electrons by various materials that may serve as shielding.

The energy spectra of electrons transmitted or reflected from thin targets of various materials are known only for a few cases. Experimental measurements with an anthracene acintillation spectrometer are presented here for many additional cases using targets of thicknesses ranging from 0.2 to 0.6 of the incident electron range. Materials studied were carbon, aluminum, copper, silver, and gold. The incident radiation was a broad circular beam perpendicular to the foil. Energies of $0.5,0.75,1.5$, and 1.95 MeV from a Van de Graaff generator were used. The spectrometer, with strict collimation, could be set at any angle up to 135 degrees from the incident direction. Provision was made for removing the bremsstrahlung and the scintillator response from the data. Comparisons with published spectra at 1.0 MeV indicate that our spectra are slightly lower in energy.

## INTRODUCTION

The degradation and scattering of electrons have been studied for many years both expeximentally and theoretically. Reference (I) reviews these studies extensively and provides a list of eighty-six references. In general, little information exists on the differentialangle spectra of electrons after multiple scattering in various materials (targets). This report provides such spectra as measured with a collimated anthracene spectrometer at various angles of emergence both in forward directions (transmitted) and at angles greater than 90 degrees (backscattered). Elemental targets of graphite, aluminum, copper, silver and gold of various unfform thicknesses were studied. Incident electron energies were $0.5,1.0,1.5$, and 1.95 MeV .

## APPARATUS

The experiments were performed within a scattering chamber which was connected directly to the drift tube of a $2-\mathrm{MeV}$ Van de Graaff generator. The chamber was semi-circular in shape and designed so that the targets to be studied could be placed at the radial center (see Fig. 1). A graphite collimator with a $3 / 4$ in. diameter aperture placed an upper limit on the size of the beam striking the material to be studied.

A second collimator system and the detector together could be rotated from the direction of the incident electron beam around to angles behind the foll. By this means spectra could be measured at any angle from the incident direction, $O$ degrees to 135 degrees.

Iead shielding was provided against bremsstrahlung generated by the relatively broad incident beam upon the target and surroundings. Figure 2 is a photograph of the target-collimator-spectrometer system and the lead shielding. The lead shield surrounding the target was slotted in the plane of the beam and spectrometer collimator.

The spectrometer collimator consisted of three gold disks each 1.18 mm thick and with a hole through its center. The disks were located 20,98 and 165 mm from the scattering foil and the holes were $0.560,0.545$, and 0.536 mm in diameter respectively. The area of the crystal exposed was $1.14 \times 10^{-2} \mathrm{~cm}^{2}$ and the acceptance solid angle was $3.6 \times 10^{-5}$ steradian. Gold was chosen for the collimator slits in order that very thin slits could be used. This reduced scattering at the surface of the collimator hole which would provide degraded energy to the spectrometer. The gold slits were covered with thick carbon on the front surface to reduce bremsstrahlung production from electrons hitting the slits. The hole in the carbon was not visible to the detector. The middle collimator was a combined anti-scattering and bremsstrahlung-reducing slit.

A graphite shutter could be introduced between the two collimating slits nearer the detector (see Fig. 1). The shutter was interposed remotely by means of a stepping relay operated by 115 volt ac power. Since this sort of relay requires a current only during switching, stray magnetic fields were avoided. Altogether three steps of the relay were used. Besides "shutter-in" and "shutter-out" position there was a "calibration" position. Calibrations were made with a weak $\mathrm{Sr}^{90}$ - $\mathrm{Y}^{90}$ beta-ray source mounted on the relay armature.

When the "shutter-in" position was used, the electrons coming through the collimator were blocked from reaching the detector but the bremsstrahlung intensity rematned essentially unchanged. By subtracting data "shutter-in" from "shutter-out" the relatively large bremsstrahlung background particularly at the high incident electron energies; could be eliminated.

The spectrometer system consisted of the gold collimators, lead shielding, an anthracene crystal, a photomultiplier and an R.C.L. Model 20611, 256 channel pulse height analyzer. The crystal was 30 mm in diameter and 0.80 mm thick. It was attached directly to a $1-1 / 2 \mathrm{in}$. diameter Dumont Type 629 photomultiplier tube and held in place by a light aluminum housing cemented to the tube with an epoxy resin. An aluminum coated 25 gauge mylar* window of areal density $1.8 \mathrm{mg} / \mathrm{cm}^{2}$ allowed the electrons to pass into the central surface of the crystal with about $3-\mathrm{keV}$ loss.

[^0]A valve in the drift tube of the Van de Graaff permitted the chamber to be opened to the atmosphere when scattering foils (targets) were changed. Roughing and diffusion pumps rapidiy reduced the inside pressure to $5 \times 10^{-6}$ torr, the degree of vacuum used for these experiments.

It was important to integrate the incident beam current during all runs in order to subtract the pulse height distribution due to bremsstrahlung from that due to both bremsstrahlung and the electrons. In these experiments we succeeded to do this only on a relative basis since the chamber geometry allowed a fraction of the incident electrons to escape collection. The fraction varied with energy and perhaps slightly with detector angle. The scattering foil and holder, the first collimator, and the horseshoemshaped lead shleld were insulated from the rest of the chamber and connected to an external 10. 4 microfarad capacitor. A vibrating reed electrometer connected to the capacitor measured the voltage produced by the collected electrons. Accurate readings were obtained by using a Leeds and Northrup Type K-3 potentiometer to read the potential across a 25 ohm resistor in the meter circuit of the electrometer.

All electrical leads passed through Stupakoff connectors near the axis of rotation of the collimator-detector assembly. Somewhat more than $50 \%$ of the incident electrons were collected and monitored by this means.

The detector assembly was connected rigidly to an outside arm. Tho O-rings seals permitted manual rotation of the arm and assembly to desired angles without affecting the vacuum.

## EXPERTMENTAL PROCEDURE

The beam-collimation, target, and detector system was first aligned with the electron beam axis. Beam axis was determined as a line between the filament and a spot burned by the electron beam at the chamber entrance. The crystal with photomultiplier was removed. By sighting along the beam axis through a window 180 degrees from the beam port (see Fig. 1) the chamber was adjusted until the Van de Graaff filament could be seen through the detector and entrance collimator slits with the detector arm set at zero degrees. In order that the scattering region be effectively infinite in directions perpendicular to the incident beam direction it was desirable that the diameter of the incident beam spot be at least the sum of the diameter
of the spot on the target received by the detectior plus twice the maximum electron range in the material for the incident energy. This determined the size of the beam collimator. The beam was defocused and unfocused to fill the aperture as well as possible. Measurements of the unfocused beam at the position of the scattering foils was made with DuPont Type 510 photographic film calibrated with Co ${ }^{60}$ gamma rays so that density could be related to the exposure. An Ansco-MacBeth Densitometer with a 0.003 in . diameter aperture was used for reading the densities. Results obtained at 1.95 MeV are shown in Fig. 3. Generally, at lower energies the beam becomes broader; consequently these results represent the least uniform condition. However, numerous specific measurements were not made and it is quite possible that in some instances less uniformity of the beam might have existed.

Before each run calibrations of gain stability were made with the internal $\mathrm{Sr}^{90}$ - $\mathrm{Y}^{90}$ source. Since the photomultiplier gain was sensitive to counting rate, the incident electron beam was adjusted each time so that the counting rate remained nearly constant. The calibration provided a means for adjusting gain shifts.

A typical run consisted of the following steps. The electron energy, foil material, and the scattering angle were selected. The beam current was slowly increased until a count rate occurred which gave about 70 percent livetime as read on the livetime meter of the pulse height analyzer. The incident beam current was next monitored by introducing a graphite beamcatcher pneumatically into the drift tube of the Van de Graaff. We assumed that for a given energy the number of electrons striking the target was always proportional to this measured current.

The $\mathrm{Sr}^{90}-\mathrm{Y}^{90}$ source was moved into calibration position next. After the calibration run was made, the relay was stepped into the "shutter-out" position. Spectral measurements were then made until at least 10,000 counts appeared in channels representing energies of primary interest. The total collected charge, as represented by the voltage across the capacitor was noted. The livetime (time during which a count could be accepted by the analyzer) and the actual time period of each run were also recorded.

A beam catcher was next introduced into the drift tube and the beam current (cc) measured. If this was consistent with the previous measurement, the current was recorded and measurements were next made of the bremsstrahlung background. This was accomplished by stepping the relay to the "shutter-in" position. Counts were recorded until the charge collected was approximately equal to that of the previous measurement. Again, voltage representing this charge, the time interval for the run, and the true livetime were recorded.

ANALYSIS OF DATA

A typical puise height spectrum and the associnted background run are shown in Fig. 4. Before we subtracted the background the number of counts in each channel was corrected by the following factor so that both the counting time and charge collected were comparable.

$$
F=\frac{(R T)_{b}}{(R T)_{s}} \cdot \frac{(I T)_{s}}{(L I)_{b}} \cdot \frac{V_{s}}{V_{b}}
$$

where

$$
\begin{aligned}
& \mathrm{RT}=\text { actual time period of the run } \\
& \mathrm{IT}=\text { analyzer livetime } \\
& \mathrm{V}=\text { charge collected }
\end{aligned}
$$

and subscripts

$$
s=\text { spectrum run }
$$

$\mathrm{b}=$ background run.
After corrected background counts were subtracted to give net, counts per chanrel, the gain correction was applied.
$\mathrm{Sr}^{90}$ The gain corrections were obtained by making plots of the $\mathrm{Sr}^{9}$ - $\mathrm{Y}^{9}$ pulse height spectrum upon linear coordinate paper. The curves were nearly linear above channel 70 (i.e., above 0.9 MeV ) except for an asymptotic tail from about channels 165 to 180. (The endpoint energy of $\mathrm{Y}^{90}$ is 2.27 MeV .) Extrapolation of the linear part of each curve to give the channel number intercept gave a sensitive and reproducible indication of gain. This intercept was not precisely proportional to gain (because of the non-linear anthracene response at low energles. However, the corrections, which were generally about 2 percent or less except for two cases which were 4 percent, were sufficiently small so that proportionality could be assumed without introducing significant error.


#### Abstract

A better method for applying gain corrections obtained from Fermi-Kurie plots was tried. A small correction at the end point energy was applifed in order to remove the effect of detector response uncertainty. $(2,3)$ Because of the tedious nature of this method and the slight improvement in accuracy to be obtained, we decided not to use it for gain correction. We did use this method to give an energy calibration independent of the Van de Ciraaff calibration.


The correction for gain requires a redistriburion of counts among the channels. This was done by assuming a swooth curve passing through the midpoint of each channel and interpolgting this curve by means of the Gauss method of divided differences. (4)

The corrected pulse height data can be unfolded to give the electron spectrum if the response of the detector system is accurateiy known. The detector response measurements had heen made much earlier usilng collimated electrons but separate from the scattering chamber. In all cases electrons from the Van de Graaft were incident normal to the crystal surface, and the calibration energies were indicated by the calibrated voltmeter which is part of the Van de Graaff control console.

The pulse height distributions due to the monoenergeilc electrons were strongly peaked but a tail extending to zero was present. This tail resulted primarily from electrons which escape (essentially backscatter) from the crystal before losing all their energy. The number of electrons in the tail was about 7 percent of the total striking the crystal. Within the accuracy of the measurements this fraction was independent of the incident energy. For generating a detector response matrix we assumed that the tail: (1) was triangular in shape, (2) was zero at zero pulse height, (3) extended up to and had its maximum value at a pulse height equal to 0.92 of that representing the incident energy, and (4) had an area equal to 0.07 of the total pulse height spectrum area. Without the tail present the peak was symmetrical and fitted a gaussian distribution satisfactorily at all incident electron energies. The experimentally measured standard deviation of the gaussian peak was found to be proportional to the square root of the incident energy. This result when combined with the findings for the tail region provided sufficient information for generating a matrix. IThe efficiency of detection was assumed to be 100 percent.

Later, in unfolding spectra we found discrepancies which suggested that the response matrix was incorrect. The collimation used for the matrix calibration was different from that of the scattering chamber, and there was also a possibility that the response characteristics of the anthracene crystal had changed during the cime between calibration
and the experiment. Fortunately during the experiments a number of IInearity checks had been performed. These provided response data that were current. At all energies the measurements showed increased width of the gaussian peak, but espectally for energies below 1 MeV . An empirical function was fitted to the data. If s repr sents the standard deviation in units of channel number and E represents energy In keV , the equation was

$$
s=\sqrt{6.2+0.0105 \mathrm{E}} .
$$

A plot of the mean channel number c for each gaussian distribum tion against calibration energies $E$, gave a curve that was linear except for a region below 200 keV . For generating the matrix the following fitted functions were used

$$
\begin{array}{ll}
\mathrm{E}=120+11.3 \mathrm{C} & \mathrm{E}>200 \mathrm{keV} \\
\mathrm{E}=77 \sqrt{\mathrm{C}} & \mathrm{E}<200 \mathrm{keV}
\end{array}
$$

again, the detection efficiency was assumed to be 100 percent, i.e., all incident electrons except at very low energies were detected.

The matrix calculated under these conditions consisted of 50 energy intervals each 40 keV wide versus 180 channels. The channels were 11.3 keV wide except at low :nercites (channel 7 and below) where they become wider due to the nom-inear response below 200 keV .

An iterative computer program ${ }^{(5)}$ was used for unfolding the spectra, the essential steps of which are given in the appendix. Each spectrum presented in this report resulted from five iterations.

RESULIS AND DISCUSSION

Table 1 is a catalog of the unfolded differential flux spectra. It assigns a number to each spectrun and identifies it in terms of beam energy, scattering angle, scattering material, and material thickness. Tables 2 through 11 present the spectra, nine in each table. The number above each column corresponds to the catalog number of that spectrum. In each column are fifty numbers each representing the fiux within a $40-\mathrm{keV}$-wide energy interval. The column at the extreme left gives the energy at the midpoint of each energy interval.

The tabulated spectral data are in arbitrary units. Ideally they would have been presented as electrons per MeV-steradian-incident electron; but, as mentioned earlier, we were unable to measure the incident current absolutely. For a given energy and foil, however, the data should be consistent if the beam current remains constant during the minute or so of time required to accumulate counts; thus changes produced by changing the scattering angle are directly comparable; e.g., Spectra 4, 5, and 6 are reasonably correct relative to each other, but may not be compared with Spectra 1, 2, and 3 except. in terms of shape.

Under some ctrcumstances where the bremsstrahlung background was high, subtraction of this background resulted in negative numbers. Such resulta are shown in the apectrum tables as blank spaces. When the incident energy was high and the scattering material was of high atomic number practically all counts in the lower energy channels were from bremsstrahlung. Small errors in adjusting the background (factor $F$ of the preceding section) could and did affect most spectra in the low-energy region. Spectrum No. 34 ( 1.95 MeV electrons on a gold foil $240 \mathrm{mg} / \mathrm{cm}^{2}$ thick) after the background was subtracted gave practically all negative values and completely unusable results even at higher energies.

The tables show data for energy intervals up to 2 MeV even when the incident, electron energy was considerably less. Such results are artifacts of the unfolding program when only a few iterations are used. With a greater number of iterations, these artificial tails are reduced systematically.

F; yure 5 shows spectra at 0,45 , and 75 degrees for 1.95 meV electrons incident upon $240 \mathrm{mg} / \mathrm{cm}^{2}$ thick aluminum foil. The spectrum remains nearly constant for angles up to 45 degrees from the normal, but becomes both attenuated and degraded in energy at 75 degrees. The
plot of the spectrum at 45 degrees has a greater area under the curve than does that at 0 degrees. Since one expects a maximum particle flux in the forward direction, it is likely that the incident beam current did not remain constant during one or the other or botin of the two exposure periods. All the spectra are subject to this sort of error.

Comparison of our data with other published spectra can be done only approximately. Figures 6, 7, and 8 show graphs taken irom reference (6). These include experimental data obtained with a lithium-drifted ailicon spectrometer and analytical data obtajned by Berger using Monte Carlo methods as described in reference (6). The ordinates given are absolute. Included in the figures are our most nearly comparable spectra. Our foils were approximately 7 percent thinner than those used by the others in each of the three graphs. Since our flux measurements were only relative, we have in each case adjusted the height of the peak so that it was roughly equal to heights of the other peaks.

Note that the poorer resolution of the anthracene spectrometer results in a somewhat broader spectrum than were obtained by the others. For thin foils and high electron energies where the transmitted spectra are still sharply peaked, one sees that good resolution becomes increasingly important.

For each of the comparisons, our spectra tend to have lower energies than the other results. When one considers also that oxr results were for thinner foils in each case, the differences appear to be significant. The detector response matrix was not as ideal as we would have liked, but its deficiencies do not provide a sufflcient basis for explaining the differences in the spectra noted above. Such differences are greater than any errors to be expected in either the calibration of Van de Graaff energy or in the determination of the analyzer channel which corresponds to the end-point energy of yttrium-90 beta rays.
TABLE I
CATALCG OF SPECTRA

| Material | $\begin{gathered} \text { Foil } \\ \text { Thickness } \\ \left(\mathrm{mg} / \mathrm{cm}{ }^{2}\right) \end{gathered}$ | Incident Energy and Angle or Scaveer* |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 1.95 MeV |  |  |  |  |  |  | $1.5 \mathrm{MeV}$ |  |  | 1.0 MeV |  |  | 0.5 MeV |  |  |
|  |  |  | $115^{\circ}$ |  | $45^{\circ}$ | $160^{\circ}$ | $0^{\circ} 75^{\circ}$ | $105^{\circ}$ | $120^{\circ}$ |  |  |  | $0^{\circ}$ | $45^{\circ}$ | $120^{\circ}$ | $0^{\circ}$ | $45^{\circ}$ | $120^{\circ}$ |
| Graphite | 93.6 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 79 | 80 |  |
|  | 153 |  |  |  |  |  |  |  |  | 36 | 37 |  |  |  |  | 81 | 82 |  |
|  | 216 | 1 |  |  | 2 |  |  |  | 3 |  |  |  | 55 | 56 |  |  |  |  |
|  | 309 |  |  |  |  |  |  |  |  | 38 | 39 |  | 57 | 58 |  |  |  |  |
|  | 511 | 4 |  |  | 5 |  |  |  | 6 |  |  |  |  |  |  |  |  |  |
| Aluminum | 34 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 83 | 84 |  |
|  | 68 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 85 | 86 |  |
|  | 103 |  |  |  |  |  |  |  |  |  |  |  | 59 | 60 | 61 | 87 | 88 |  |
|  | 131 |  |  |  |  |  |  |  |  | 40 | 41 |  |  |  |  |  |  |  |
|  | 205 |  |  |  |  |  |  |  |  |  |  |  | 62 | 63 |  |  |  |  |
|  | 240 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |  |  |  |  |  |  |  |  |  |
|  | 308 |  |  |  |  |  |  |  |  |  |  |  | 64 | 65 |  |  |  |  |
|  | 342 |  |  |  |  |  |  |  |  | 42 | 43 |  |  |  |  |  |  |  |
|  | 479 | 15 | 16 | 17 | 18 | 19 |  | 20 |  |  |  |  |  |  |  |  |  |  |
|  | 514 |  |  |  |  |  |  |  |  | 44 | 45 | 46 |  |  |  |  |  |  |
|  | 719 | 21 |  |  | 22 |  |  | 23 |  |  |  |  |  |  |  |  |  |  |
| Copper | 113 | 24 |  |  | 25 |  |  |  |  | 47 | 48 |  | 66 | 67 |  | 89 | 90 |  |
|  | 226 |  |  |  |  |  |  |  |  | 49 | 50 |  | 68 | 69 |  |  |  |  |
|  | 340 | 26 |  |  | 27 |  |  |  |  | 51 | 52 |  | 70 | 71 |  |  |  |  |
|  | 566 | 28 |  |  | 29 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Silver | 80 |  |  |  |  |  |  |  |  |  |  |  | 72 | 73 |  |  |  |  |
|  | 133 | 30 |  |  | 31 |  |  |  |  |  |  |  | 74 | 75 |  |  |  |  |
|  | 266 | 32 |  |  | 33 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Gold | 144 |  |  |  |  |  |  |  |  |  |  |  | 76 | 77 | 78 |  |  |  |
|  | 240 | 34 |  |  | 135 |  |  |  |  | 53 | 154 |  |  |  |  |  |  |  |

*Measured from forward incident direction.
























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Fig. 1 Schema of scattering chamber.


Fig. 2 Photograph slowing target, collimators and shielding within the scattering chamber.


Fig. 3 A cross-section plot of the incident electron beam at the target foil. Beam energy was 1.95 MeV .


Fig. 4 Pulse height spectrum, background and energy spectrum. This background must be multiplied by $F=0.775$ before subtraction. Note that unfolding changes the net pulse height spectrum only a small amount.


Fig. 5. Changes in spectrum as a function of scattering angle. The oscillations below 1 MeV are artifacts from the unfolding program.


Fig. 6 Spectral comparisons at 0.2 of maximum l-Mev electron range. Present results are compared with experimental results at Ling Temco Vought and analytical results at the National Bureau of Standards (references (6) and (7)). Ordinates of NRDL results are relative.


Fig. 7 Spectral comparisons at 0.4 of maximum $1-M e V$ electron range. Present results are compared with experimental results at Ling Temco Vought and analytical results at the National Bureau of Standards (references (6) and (7)). Ordinates of NRDL results are relative.


Fig. 8 Spectral comparisons at 0.6 of maximum l-MeV electron range. Present results are compared with experimental results at Ling Temco Vought and analytical results at (7)). Ordinates of NRDL results are relative.

## APPENDIX

PROCEDURE FOR UNFOLDING SPECTRA

1. Multiply the input vector (a column matrix of 180 elements) by the transpose of the response matrix to give a 50 element colurn vector designated as $S$.
2. Multiply the matrix by its transpose to obtain a 50 by 50 square matrix designated as A.

These two steps have served to reduce the response matrix and input vector to manageable size for the IBM-704 computer. In principle this reduction may introduce a slower convergence in the fitting and possibly in some cases may influence the end result.
3. Let $\mathbb{N}$ designate the energy spectrum, then $S=A N$ and we solve for N iteratively as follows:
a. Assume as a first guess that $N=S$
b. Calculate a diagonal matrix $D$ with elements $d_{i 1}=\frac{n_{i}}{s_{i}}$
d. Calculate a new $S$ by folding the new $N$ by the matrix, i.e., $S=A N$
e. Continue iteratively by calculating a new $D$ from new elements of $N$ obtained in Step ( $c$ ) and of $S$ obtained in Step (d).
4. Check the convergence at the end of step (d) by subtracting the latest value of $S$, element by element, from the original pulse height vector and summing the squares of the differences. During trial runs to see how many iterations were desirable for unfolding our spectra, this sum decreased steadily to an asymptotic value.

## APPEND.LX (conta)

In the linit the diagonal matrix $D$ converts $N$ to $S$ the same ac matrix A does. One recognizes from this the desirability of having a strong diagonal in the response matrix.

After considering the rate of convergence, required computer time, and the accuracy of the input data, a practical decision was made to use five iterations for unfolding all spectra.

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[^0]:    *Tradename of polyester synthetic film manufactured by DuPont de Nemours, Wilmington, Delaware.

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