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# ON THE POSSIBILITY OF REFINING BY MEANS OF OPTICAL LOCATION SOME ASTRONOMICAL PARAMETERS OF THE SYSTEM EARTH-MOON

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## SUMMARY

A mathematical treatment is performed of the possibilities of determining by the results of optical location of the Moon the following parameters, using an artificial light reflector on its surface:

- the mean radius and the eccentricity of the lunar orbit;
- the selenographic coordinates of the point being located, i. e., the latitude, longitude and the radius-vector of Moon's relative mass center;
- the radius-vector of the point of observation relative to the mass center of the Earth;
- the ephemeris time.

Analysis of the possible errors is conducted and it is shown that the precision of determination of these parameters may be significantly higher than the precision with which they are known, or may be determined by other methods. However, the attaining of high precision is possible only on the condition of preliminary investigation of physical libration, of which the insufficient knowledge of laws may lead at the present time to great errors in the parameters investigated. The possibility is indicated of libration measurement by the method of optical location.

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A method has been described in the work [1] (ST-CM-OA-LPS-10505) for the computation of a series of astrometric parameters of the Moon, Earth and the lunar orbit with the utilization of an artificial light reflector on the surface of the Moon. Analysis of errors was performed and it was shown that the possible precision of determination of the parameters sought for by measurements in the course of one lunar month is several times higher than the precision with which these parameters are presently known.

While developing below the method described in [1], we shall consider the problem of determining a series of parameters of the system Earth-Moon under the special condition of choice of optical location measurements (selenographic coordinates of the point located, position of the lunar orbit relative to the ecliptic plane, position of the moon on the orbit).

As in [1], we shall start from the fact that the current distance to the point being located (that is, to the artificial light reflector) may be measured with an error of $\sim \pm 3$ m in a single experiment.

## THE DETERMINATION OF THE CORRECTION $\Delta T$ (EPHEMERIS TIME MINUS UNIVERSAL TIME) DETERMINATION OF THE RADIUS VECTOR OF THE POINT OF OBSERVATION

The distance between the observed and the point located, measured at a certain moment of time, is linked with the Earth's and Moon's parameters by the following relations (see Fig.1):

$$\cos\mu_i D_i = (\Delta_i^2 + \rho^2 + 2\rho\Delta_i\cos\gamma_i)^{1/2} - r\cos\omega_i \quad (i = 1, 2, \ldots, n), \tag{1}$$
$$\Delta_i = \frac{a_0}{\sin\pi_{☾_i}}, \quad \cos\gamma_i = \sin\varphi_i'\sin\delta_i + \cos\varphi_i'\cos\delta_i\cos t_i,$$
$$\cos\omega_i = \sin b\sin b_{0i} + \cos b\cos b_{0i}\cos(l - l_{0i}),$$

where $a_0$ is the equatorial radius of the Earth, $\delta$, $t$, $\pi_☾$ are respectively the declination, the local hour angle and the parallax of the Moon, $l_0$, $b_0$ are the librations of the Moon in longitude and latitude, taking account of the topocentric corrections, $l$, $b$ are respectively the selenographic longitude and latitude of the point located, $\Delta_i$ is the current distance between the mass centers of the Moon and the Earth, $\Delta_i'$ is the topocentric distance from the point of observation to the center of the Moon, $\Delta_0$ is the mean distance between the mass centers, e is the eccentricity, $\phi'$ is the geographic latitude of the point of observation .

We may utilize Eq.(1) in order to make parameters $\Delta_0, r, e, l, b, \rho, t$ more precise by the measured values of $D_i$. For the linearization of the problem we shall take advantage of the fact that these parameters are known with some precision or may be determined from astronomical observations. We shall call the point in space with parameters $\Delta_0, r, e, l, b, \rho, t, \varphi', \delta$ the parametric point. According to Eq.(1) to every i-th parametric point corresponds a certain value of $D_i$, whereupon to parameters known from astronomical observations corresponds a computed value $D_{com}$ , while to that found by way of location measurements $D_{meas}$ correspond certain more precise sought for values of parameters, so that $D_{meas} - D_{com} = dD_i$. We shall call the quantities $d\Delta_0 = \Delta_{0meas} - \Delta_{0comp}$ , $dr = r_{meas} - r_{com}$ and so forth corrections to well known values of parameters sought for.

The quantity $dD_i$ may be represented in the form of expansion in Taylor series by corrections.* Differentiating Eq.(1), and eliminating the intermediate values, we shall have

$$\left(\frac{\Delta_i}{\Delta_i'}-\frac{\rho}{\Delta_i'}\cos\gamma_i\right)\frac{3422'',7}{\pi_{\text{☾}i}}d\Delta_0-\cos\omega_i dr-\frac{3422'',7}{\pi_{\text{☾}i}}\Delta_i Q_i de-$$
$$-\cos b\ \cos b_{0i}\sin(l-l_{0i})\,dl+[\sin b_{0i}\cos b-\cos b_{0i}\sin b\cos(l-l_{0i})]\,db-$$
$$-\left(\frac{\rho}{\Delta_i'}-\frac{\Delta_i}{\Delta_i'}\cos\gamma_i\right)d\rho+\frac{\Delta_i}{\Delta_i'}\rho(\cos\varphi_i'\sin\delta_i-\sin\varphi_i'\cos\delta_i\cos t_i)\,d\varphi'+$$
$$+\frac{\Delta_i}{\Delta_i'}\rho(\sin\varphi_i'\cos\delta_i-\cos\varphi_i'\sin\delta_i\cos t_i)\,d\delta$$
$$-\frac{\Delta_i}{\Delta_i'}\rho\cos\varphi_i'\cos\delta_i\sin t_i\,dt=dD_i\ \ (i=1,2,\ldots,n), \tag{2}$$

where Q denotes the series obtained as a result of differentiation of $\frac{d\sin\pi_{\text{☾}}}{de}$.

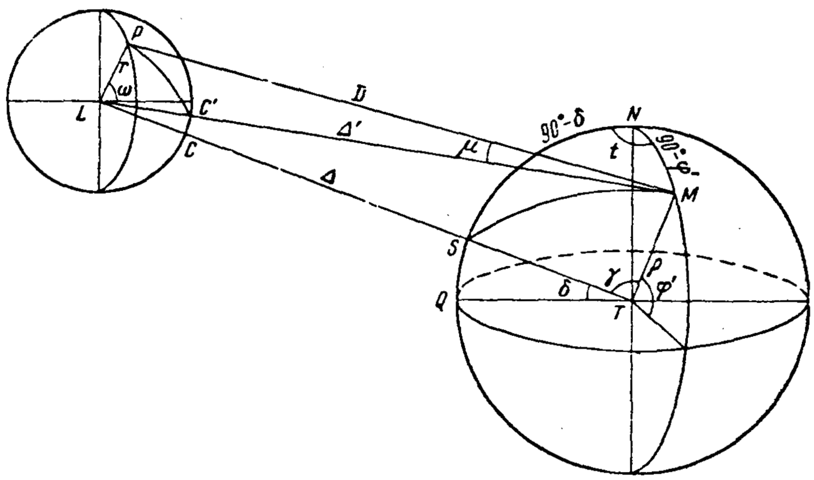


Fig.1

As is shown in [1], the consideration of the system of equations (2), composed for various points of the orbit, allows us to make the parameters sought for somewhat more precise. These parameters may be substantially refined on the basis of the results of measurements of the distance $D_i$, in the specifically matched moments of time $t_i$ of system's Earth—Moon motion. The method of assortment of these moments stems from the following considerations. The coefficients of Eq.(2) may be broken down into two groups. Related to the first group are the coefficients at $d\Delta_0$, dr, de, $dl$, db, which vary substantially only over the extent of a lunar month, changing little in the course of a few hours. To the second group belong the coefficients at $d\phi'$, $d\delta$, $d\rho$, dt, of which the values vary substantially in the course of a few hours. Taking into account the above property of the coefficients at parameters of the first group, and also the fact that parameters $\phi'$ and $\delta$ are known with great

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* Inasmuch as their relative values are small $\frac{d\Delta_0}{\Delta_0}, \frac{dr}{r}, \ldots, \frac{d\rho}{\rho} < 10^{-5}$, we may limit ourselves to their first powers.

precision, we shall determine the parameters dρ and dt; to that effect we shall consider three simplified equations of type (2), composed for three moments of time of the very same day: the moment close to Moon rise, the moment of the upper culmination and that close to Moon setting:

$$\left(\frac{\Delta_j}{\Delta_i'}-\frac{\rho}{\Delta_i'}\cos\gamma_i\right)\frac{3422'',7}{\pi_{☾ i}}\,d\Delta_0-\left(\frac{\rho}{\Delta_i'}-\frac{\Delta_i}{\Delta_i'}\cos\gamma_i\right)d\rho-$$
$$-\frac{\Delta_i}{\Delta_i'}\rho\cos\varphi_i'\cos\delta_i\sin t_i\,dt=dD_i\ (i=1,2,3). \qquad (3)$$

This system allows us to find the radius-vector of the point of observation ρ, and also to determine the correction ΔT = E.T. − U.T. The precision of these quantities will be determined by random measurement errors of $D_i$, and also by systematic errors connected with the simplification of system (2) to the form (3). In order to determine the random error in dρ and dt, it is necessary to resolve the system of equations (3), of which the right-hand parts constitute the random measurement errors of $\varepsilon_i$.

$$\varepsilon_{d\rho}=\sum_{i=1}^{3}\frac{(S_{d\rho})_i}{S}(\pm\varepsilon_i),\qquad \varepsilon_{dt}=\sum_{i=1}^{3}\frac{(S_{dt})_i}{S}(\pm\varepsilon_i),$$

where S is the determinant of system (3), $S_{d\rho}, S_{dt}$ are minors in system's (3) determinant, corresponding to determinant's elements equal to the coefficients at dρ or dt in the i-th equation. The root-mean-square errors in dρ and dt are

$$\delta(d\rho)_\varepsilon=\sigma_{\text{изм}}\sqrt{\sum_{i=1}^{3}\left[\frac{S_{d\rho}}{S}\right]^2},\qquad \delta(dt)_\varepsilon=\sigma_{\text{изм}}\sqrt{\sum_{i=1}^{3}\left[\frac{S_{dt}}{S}\right]^2},$$

where $\sigma_{\text{изм}}$ is the root-mean square error in the measurement of the distance. In order to evaluate the systematic errors on account of the simplification of system's (2), it is necessary to resolve the system of equations (3), of which the right-hand parts constitute the rejected terms of (2). For example, the error in dρ on account of rejection of terms with dr is

$$\delta(d\rho)_r\leqslant\frac{\begin{matrix}A_1R_1\cos\omega_1\\A_2R_2\cos\omega_2\\A_3R_3\cos\omega_3\end{matrix}}{S}\,\delta r,$$

where δr is the error in the well known value of <u>r</u> (dr < δr),

$$A_i=\left(\frac{\Delta_i}{\Delta'_i}-\frac{\rho}{\Delta'_i}\cos\gamma_i\right)\frac{3422'',7}{\pi_{☾ i}},\quad R_i=\left(\frac{\rho}{\Delta'_i}-\frac{\Delta_i}{\Delta'_i}\cos\gamma_i\right).$$

It is possible to show that the systematic errors will be so much the smaller as the point located is closer to the center of the visible disk of the Moon. As an example, we present below an estimate of errors, with which corrections δρ and dt can be determined according to measurements at 1800 h and 2300 h on 15 December 1967, and at 0400 hours on 16 December 1967 (*) on the condition that the located point have the selenographic coordinates

(*) [ It is presumed that the date of 16 November 1967 is a misprint in the original text].

$l = b = 10°$. The type-(3) equations, with terms in the right-hand part determining the systematic errors on account of simplification of system (2) to the form (3), are as follows:

$$
\begin{aligned}
&1{,}0557d\Delta_0 - 0{,}3747d\rho - 0{,}6513dt = \\
&= dD_1 + 0{,}9997dr + 0{,}0189r\,dl - 0{,}0108r\,db - 448720de + \\
&\quad + 0{,}8903\rho d\delta + 0{,}0313\rho d\varphi', \\
&1{,}0552d\Delta_0 - 0{,}8664d\rho + 0{,}0329dt = \\
&= dD_2 + 0{,}9997dr - 0{,}0011r\,dl - 0{,}0169r\,db - 450440de + \\
&\quad + 0{,}0942\rho d\delta + 0{,}4898\rho d\varphi', \\
&1{,}0551d\Delta_0 - 0{,}3592d\rho + 0{,}6557dt = \\
&= dD_3 + 0{,}9997dr - 0{,}0165r\,dl - 0{,}0189r\,db - 451000de + \\
&\quad + 0{,}8863\rho d\delta + 0{,}0312\rho d\varphi'.
\end{aligned}
$$

We compiled in Table 1 the values of errors in $d\rho$ and dt on condition that the parameters introduced into the equations are known with errors $\delta r \sim 3$ km, $\delta$ , $\delta b \sim 40''$, $\delta e \sim 2\ \ 10^{\ 7}$, $\delta\phi' \sim 0.1''$, $\delta(\delta) \sim 0.01''$. We assume $\sigma_{изм} \sim \pm 3$ m.

TABLE 1

| | r | $l$ | b | e | $\delta$ | $\phi'$ | $\sigma_{изм}$ |
|---|---|---|---|---|---|---|---|
| $\delta(d\rho)$ m<br>$\delta(dt)$ | 0.6<br>1" | 1.3<br>0.3" | 1.2<br>0.1" | 0.1<br>0.01" | 1.2<br>negl. | 3<br>negl. | 7<br>negl. |

A further decrease of errors in $d\rho$ and dt may be conducted by way of refining parameters e, r, $l$, b. The method of making these parameters more precise is considered below.

## DETERMINATION OF THE ECCENTRICITY OF LUNAR ORBIT AND OF THE MEAN DISTANCE BETWEEN THE MASSES OF THE EARTH AND THE MOON. DETERMINATION OF THE RADIUS VECTOR AND OF SELENOGRAPHIC COORDINATES OF THE POINT LOCATED

The method of refining parameters e, $\Delta_0$, r, $l$ (for making b more precise see below) is analogous to the above-described. For the determination of corrections to these parameters we shall compose four equations of the type (2) according to the following principle.

The first equation corresponds to an orbit for which the line of apsides is close to the line of nodes, and the position of the Moon on the orbit is near perigee; the second equation corresponds to the case when the angle between the line of apsides and the line of nodes is nearly straight and the distance between the centers of masses of the Earth and the Moon is close to average; the third equation, when the line of apsides is close to that of the nodes, the Moon is near apogee and the fourth equation may be composed for the same lunar month as the second for the point of orbit at which the distance between the mass centers of the Earth and of the Moon is close to average, but with opposite libration by latitude, i. e. just about after one half of the lunar month.

For the moment of time when the Moon is near the upper culmination (that is, for hour angles from -1 h to +1 h) and the coefficients at db are close, i. e., for the moments of time of identical libration by latitude (with a precision to $\sim 0.1°$).

TABLE 2

| | b | ρ | φ' | δ | t | $\sigma_{пзм}$ |
|---|---|---|---|---|---|---|
| $\delta(d\Delta_0)$ | ∿ 100 m | ∿ 300 m | ∿ 1 m | neglect. | neglect. | ∿ 50 m |
| $\delta(dr)$ | ∿ 300 m | ∿ 10 m | ∿ 4 m | neglect. | neglect. | ∿ 50 m |
| $\delta(de)$ | $0.02 \cdot 10^{-7}$ | $0.15 \cdot 10^{-7}$ | $0.07 \cdot 10^{-7}$ | neglect. | neglect. | $0.07 \cdot 10^{-7}$ |
| $\delta(d\ )$ | ∿ 4" | ∿ 4" | ∿ 1" | ∿ 0.1" | neglect. | ∿ 0.3" |

Without bringing here the system of equations in a general form, let us compose the four indispensable equations for the following concrete moments of time: 2281 h on 15 November 1967, 0583 h on 12 February 1966, 1739 h on 27 Jan. 1966 and 2352 h on 28 April 1967. The selenographic coordinates of the point located will be taken equal to $l = b = 10°$ (the most advantageous point is that with coordinates $l = b = 0$):

$$
\begin{gathered}
1{,}0562 d\Delta_0 + 0{,}9998 dr - 450950 de - 0{,}0004 r\, dl = \\
= dD_1 - 0{,}0166 r\, db - 0{,}8671 d\rho - 0{,}4912 \rho (d\varphi' + d\delta), \\
0{,}9999 d\Delta_0 + 0{,}9905 dr + 36030 de + 0{,}1367 r\, dl = \\
= dD_2 - 0{,}0034 r\, db - 0{,}4506 d\rho - 0{,}8659 \rho (d\varphi' + d\delta), \\
1{,}0047 d\Delta_0 + 0{,}9919 dr + 2533 de - 0{,}1265 r\, dl = \\
= dD_3 - 0{,}0094 r\, db - 0{,}9054 d\rho - 0{,}4177 \rho (d\varphi' + d\delta), \\
0{,}9310 d\Delta_0 + 0{,}9999 dr + 417690 de + 0{,}0126 r\, dl = \\
= dD_4 - 0{,}0088 r\, db - 0{,}5763 d\rho - 0{,}8088 \rho (d\varphi' + d\delta).
\end{gathered}
$$

Assuming $\delta b \approx 40''$, $\delta\phi \sim 0.1''$, $\delta(\delta) \sim 0.01''$ with $\delta\rho \sim 15$ m and $\delta t \sim 0.1''$ as indicated in Table 1, we shall obtain the following errors in the parameters $d\Delta_0$, dr, de, $dl$ (the method of evaluation is analogous to that described earlier).

The correction db may be found with about the same precision as that with which the correction $dl$ was determined above (see Table 2). To that effect it is necessary to examine two equations composed for the points in the orbit, for which the distance Earth-Moon is close to average and the line of apsides is close to the line of nodes. The required distance measurements may be conducted in the same lunar months as for the first and 3rd equations. Utilizing the data of Table 2 when resolving the system of equations (3), it is possible to perform a further refining of parameters $dl$ and dt, and so forth.

Thus, there are two interconnected groups of parameters. The refining of parameters of one group may be utilized for finding new, refined values of parameters of the second group. The introduction of these new values into the equations for the determination of parameters of the first group allows in its turn to refine the latter. After the two-fold mutual refinement of parameters the aggregate errors in them are obtained as follows:

TABLE 3

| $\delta(\delta\Delta_0)$ | $\delta(dr)$ | $\delta(de)$ | $\delta(dl)$ | $\delta(d\rho)$ | $\delta(dt)$ | $\delta(db)$ |
|---|---|---|---|---|---|---|
| $\sim$300 m | $\sim$300 m | $\sim 2 \cdot 10^{-8}$ | $\sim$15" | 7 ÷ 10 m | 0.02" | $\sim$15" |

It is necessary to remark that the analysis conducted is valid only in the case when all the quantities entering into Eq.(1) (aside from the parameters sought for) are known with a sufficient precision. The number of possible mutual refinements may be limited, first of all, by the insufficient precision of determination of physical libration and to a lesser degree by the error in the inclination of the lunar orbit, the oscillations of the terrestrial axis and so forth. In particular, the errors in the determination of the physical libration ($\sim$ 10") do not allow us to conduct the calculation of parameters sought for even in the first approximation. For the indicated computation the required error in the physical libration must be no more than 1"(one) selenocentric second.

Let us note beforehand that such a precision of measurement of aggregate libration may be attained by optical location. Analysis of this question and also of the question of the possibility of third etc. approximations will be given by us in subsequent works.

## CONCLUSION

The above considerations show that the application of optical location with the use of a light reflector on the Moon will allow us:

1) to increase by approximately one order the precision of the determination of Moon's orbit parameters $\Delta_0$, $\underline{e}$;
2) to determine with high precision the "frontal" radius of the Moon and the selenographic coordinates $\underline{r}$, $\underline{l}$, $\underline{b}$ of the point located;
3) to determine the correction $\overline{\Delta}T$ (ephemeris time U.T.) by the results of measurements in the course of 24 hours.

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*** T H E E N D ***

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