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# SEMI-ANNUAL STATUS REPORT ON RESEARCH PROGRESS

NASA Research Grant NsG-2-59
October 1, 1966, to March 31, 1967

During the subject reporting period, faculty members have derived support from NASA Research Grant NsG-2-59 as follows:

1. Professor Frederick J. Beutler was supported on a 25% basis.
2. Professor Lawrence L. Rauch was supported on a 25% basis.
3. Professor William L. Root was supported on a 50% basis.

In addition, approximately half-time support was furnished Mr. Robert Bayma who is working on his doctoral dissertation research under the direction of Professor Rauch.



In the subject reporting period, Professor Beutler has obtained still further new results treating applications related to the theory of stationary point processes. In collaboration with Dr. O. Leneman, he has submitted a paper, "The Spectral Analysis of Impulse Processes " for the International Symposium on Information Theory, to be held in Athens, Greece in September 1967. This paper improves the results found in Dr. Leneman's "Random Sampling of Random Processes: Impulse Processes," Information and Control, 9 (1966, pp. 347-363. A summary of the new paper by Professor Beutler and Dr. Leneman follows:

> An expression for the spectral density of the impulse process $s(t) = \sum_{-\infty}^{\infty} \alpha_n \delta(t - t_n)$ is derived under the assumption that $\{\alpha_n\}$ is a stationary point process [B4] independent of $\{\alpha_n\}$. The spectral density appears as an infinite series in terms of the correlation of $\{\alpha_n\}$ and the interval statistics of $\{t_n\}$. The same result was obtained by Leneman [L2] by a different argument under considerably more restrictive conditions of validity.
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> Various models of impulse processes are discussed relative to random sampling of random processes. Random and systematic loss of samples, separate read-in and read-out jitters, and correlated random scaling errors can all be represented by appropriate assumptions on $\{\alpha_n\}$ and $\{t_n\}$.
>
> Finally, closed form expressions are calculated for the

spectral density of s(t) and the sampled process under combinations of the sampling errors mentioned in the preceding paragraph.

Professor Beutler has also been investigating the possibility of accomplishing alias-free sampling of random processes by sampling at random instants. A sampling sequence $\{t_n\}$ is said to be alias-free if the spectrum of a random process x(t) can be recovered from the discrete parameter random process $\{x(t_n)\}$. It is already known that recovery is in some instances possible if sampling is at a rate lower than the Nyquist rate. Professor Beutler is using a new approach to the problem to obtain additional results.

The report* "The Use of Statistical Dependence Between Data Samples in Binary PCM Demodulation," describing the work of Professor Rauch with James K Strozier on optimum and suboptimum demodulation of uncoded PCM, was issued in November 1966. An abstract of the work appears in the previous Semi-Annual Status Report.

The work on improved analog demodulation with Professor Rauch's doctoral student, Robert W. Bayma, has continued. Professor Rauch and Mr. Bayma attended the special conference at the Ames Research Center on March 30 and 31 and April 1, 1967.

Professor Root completed a paper, "Estimation of $\epsilon$-Capacity for Certain Linear Communication Channels," and submitted it for publication. Part of the work on this paper was supported by the grant. This investigation was concerned with obtaining bounds on possible communication rates in environments where the signal is distorted by influences that cannot be treated probabilistically; a fuller discussion was given in the previous Progress Report. A briefer, more mathematically oriented paper on the same subject entitled, "The $\epsilon$-Entropy and $\epsilon$-Capacity of Certain Time-Invariant Channels," was co-authored by Root and Professor R. T. Prosser (not supported on the grant) of Dartmouth College. This second paper has been accepted for publication in the Journal of Mathematical Analysis and Application.

A paper was given by Root, which also was co-authored by R. T. Prosser, at the Princeton Conference on Information Sciences and Systems, March 30, 31, 1967. This paper entitled, "On the Identification of Unknown Systems," will appear in the Conference Proceedings. Root's work on this paper was supported by the grant. The summary of this paper is as follows:

> "A procedure is given for the identification from input-output data of systems which can be represented by integral operators, linear or non-linear, time-invariant or time-varying. The procedure is not a practical one, but it serves to classify those situations where there is sufficient a priori information to make

*Doctoral Thesis of James K. Strozier.

an identification possible. The procedure is deterministic, but is mathematically somewhat analogous to a known stochastic technique, and a comparison between the two is made."

Professor Root and Professor P. P. Varaiya (not supported by the grant) of the University of California have been working on the problem of finding the capacity (in the sense of Shannon) of channels with additive Gaussian noise when there is an unknown linear operation (belonging to a certain specified class) in the channel. A report entitled "Capacity of Classes of Gaussian Channels: Part 1," has been completed and will be submitted for publication. Root's work on this report was supported by the grant. The abstract of the paper reads as follows:

> "The usual definition of the capacity of a discrete-time, memoryless Gaussian channel is generalized to the case of a collection of such channels. Each member of the collection is specified by a pair $(A, \Omega)$ where A represents the deterministic transmission matrix, possibly infinite-dimensional, and $\Omega$ is the covariance matrix of the additive Gaussian noise. The definition is justified by showing that the capacity is the supremum of the attainable rate."

Root and Varaiya are continuing work in the same problem area.

During the period covered by this report a paper, "Determinable Classes of Channels," by Root and R. T. Prosser appeared in the Journal of Mathematics and Mechanics, v 16, October 1966, pp 365-398. This paper has been reported earlier; Root's work was partly supported by the grant.