# A THEORETICAL STUDY OF METEORIC TRAJECTORIES AND PROCESSES, INCLUDING EXAMINATION OF THE INCIDENCE AND CHARACTERISTICS OF PHOTOGRAPHIC METEORS BY REDUCTION OF ABOUT 600 DATA POINTS <br> By Annette Posen and Richard E. McCrosky 

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## FORFWORD

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#### Abstract

This report describes a newly developed, efficient, automatic-reduction procedure for photographic meteor data. The time required to process a pair of films is reduced from 24 to 3 man-hours, and this is due mainly to an electronic computer program that automatically identifies the stars on a photographic film. It is necessary to know the approximate ( $\pm 5^{\circ}$ ) hour angle and declination of the center of the star field of the film (the direction of the camera's optical axis) and the focal length of the camera.


This automatic star-identification technique can be widely applied to astronomical photographic data.

New, precisely computed orbits for 357 meteors are presented.

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Annette Posen ${ }^{1}$ and Richard E. McCrosky ${ }^{2}$

## 1. INTRODUCTION

During a constant nighttime patrol, from February 1952 to January 1959, a total of 6000 meteors were simultaneously photographed by two Super -Schmidt cameras of the Harvard Meteor Project in New Mexico.

Because of the length of time required to process the films, only a limited number of precise reductions have been made in the past (Jacchia and Whipple, 1961; Hawkins and Southworth, 1961), mainly from the data collected between February 1952 and July 1954. Much of the early data was also treated in an approximate fashion ( $\pm 5 \%$ in velocity) to yield a statistically significant distribution of orbits of meteoric particles (McCrosky and Posen, 1961). However, as computation of meteoroid dimensions requires very careful reduction, only the small sample of precise reductions can be used for any reliable information on this parameter.

The number, size, momentum, and energy distributions of meteoric particles in the solar system are of vital interest to designers of space vehicles.

[^0]We reviewed the entire reduction procedure, including the film handing and measuring process, and investigated new mathematical and logical techniques for data manipulation. We now have available precision-measuring engines with card-punch capability. Large catalogs of star positions have been compiled on magnetic tape and are thus readily accessible by electronic computer.

We developed a precise-reduction procedure of practical length, and can now base our knowledge of the distribution of meteoric particles upon a much larger sample of data.

## 2. THE NEW TECHNIQUES

## 2. 1 General Approach to Meteor Reduction

The nature of the Super-Schmidt photographic data and of the results derived from them are described in detail by Whipple and Jacchia (1957).

The films are curved, conforming to the focal surface of the camera. They are photographically copied, by gnomonic projection, onto a flat glassbacked emulsion, for measuring. The cameras are driven at the sidereal rate; the meteor trails across the "stationary" star background. A great circle in the sky (the meteor trajectory) projects into a straight line on our film. The great circle is reconstructed against a background of stars seen from two known stations. Triangulation from the two stations yields ranges and heights above the mean earth for specific points along the trail. A focalplane shutter in the camera interrupts the meteor trail to produce a time scale, and the midpoints of the segments into which the shutter chops the trailed image of the moving meteor are, in practice, the points along the trail for which coordinates are computed. A straight line is fit to these points by the method of least squares, and represents the trail in the remainder of the reduction.

The heights and distances along the trajectory are computed separately for the trail as seen from each station, because point-by-point correspondence cannot be made in the absence of an obvious "common point" in the luminosity curve. However, distances along the trail are referred to a common beginning point, the higher of the two beginning points.

Extrapolated preatmospheric velocities should match for the two plates. This velocity, with the direction and position within the atmosphere, and the date and time of encounter with the earth, serve as input data to compute the heliocentric orbit of the body.

The data that are fundamental to photographic astronomy are the precise positions of the stars photographed as a background to the event of interest. Determination of the coordinate transformation (plate constants) from plate coordinates to sky coordinates of these known stars permits the transformation of the plate coordinates of any other points on the film to known directions in space, as seen from a particular station.

### 2.2 Star Identification

The most time-consuming operation in the reduction of Super-Schmidt films has been the identification of the star field of the film. This step was followed by the selection of appropriate reference stars for measuring, catalog lookups, the manual transcription of positions, and the preparation of appropriate punched cards. This process was long, tedious, and error-prone.

The prime contribution of our project was the automation of these steps in the procedure. This accounts for the major part of the difference in time between 24 and 3 man-hours spent per meteor reduction.

The Smithsonian Astrophysical Observatory (Staff SAO, l966) has produced a catalog of 250,000 stars that is available on magnetic tape and suitable for machine processing. Having the data in this form may enable one to eliminate printed catalog lookup from all astronomical photographic work. However, there are several levels of sophistication in possible approaches to this problem.

For example, one might simply wish to continue to identify stars in the field visually, extract their GC numbers, and search a tape ordered by GC number to find the precise position data at a standard epoch and equinox. If the number of stars of interest is small, the catalog on tape is best for a simple selection by well-established criteria. On the other hand, one could quickly organize a straightforward but inefficient tape search, where an approximate position is matched by trial and error within a long list ordered on only one coordinate at a time (e.g., Wagner, 1963).

The aim of our investigation was to eliminate completely hand reference to star charts and star catalogs and at the same time to develop within a computer an efficient reference system (more efficient than those cited above) that could be incorporated within the body of the photographic meteor-reduction procedure.

## 2. 3 The New Approach to Star Identification

Given the nature of the projection of the image of the sky or the photographic plate, the focal length of the camera, the direction of the axis of the camera, and the plate coordinates of a known direction in the sky, one can compute, from the plate coordinates of anknown star relative to the center, an approximate sky position for that star (see Appendix A).

The uncertainty in the computed position is a function of the quality of the image measured, the accuracy to which the position of the plate center is known, both on the plate and in the sky, and the uniformity of scale across the photograph. The predicted sky position must be sufficiently close to the true position of the star so that it can be matched with the appropriate catalog position. The permissible "circle of confusion" is limited by the population density of the stars in the catalog consulted and by the sensitivity of the method used to discard invalid identifications.

As Super-Schmidt films are of excellent quality, multiple- and invalidstar identifications can be easily identified by inspection of deviations from a coordinate transformation computed for two stars that gives generally good residuals when the transformation is applied to all the tentative positions assigned. Typical deviations from a plate-constants fit in the least-squares sense are of the order of $10 \mu$ ( 10 arcsec ).

To implement the new approach to star identification, we organized the data in the SAO Star Catalog on magnetic tape in the following form.

The stars are visualized as falling within a two-dimensional grid in right ascension and declination. The dimensions of the grid in each coordinate are established by the expected uncertainty in the prediction computation, the population density of the stars within the magnitude range being considered, and by the expected range of success in discriminating between close possibilities. A band one grid-step deep in declination comprises a $360^{\circ}$ spread in right ascension, divided by an appropriate scale into rectangular grid elements. Each of these elements can be assigned a unique address, computed from its right ascension and declination, and each star can be assigned such an address, even though some multiple occupancy will occur, depending on grid size relative to population density. This grid-address codeword attached to the accurate position can be used to simulate a two-dimensional sort. The resulting tape catalog of star positions is a one-dimensional list, and can be ordered on this one variable.

However, the search-match must proceed in a circular fashion about the predicted position; therefore, an auxiliary representation is necessary. The star catalog is actually "plotted" in the computer as a series of on-or-off binary bits corresponding to occupied or unoccupied grid positions. Each of these bits has an address within the storage block that corresponds to the codewords attached to the stars. The logical operations of the computer are admirably suited to testing very quickly the condition of specific addressable bits, such as the eight grid areas surrounding the predicted position. The addresses of "on" bits are used to search later the ordered codewords on tape and extract the accurate positions of possible matches.

The general approach has wide applicability as an efficient automatic star-identification procedure, readily adaptable to a specific camera and problem.

## 2. 4 Star Identification for Super-Schmidt Films

Faint stars (8th or 9th mag) are most desirable for use as fiducial points in the immediate vicinity of the meteor trail because of the generally good quality of their images, which are of optimum size for accurate measurement. The population of such faint stars in the sky is very dense. Star positions cannot be predicted on the basis of the initial data (see Appendix A) sufficiently closely to discriminate particular stars of 8 th and 9 th mag from their neighbors. Because we cannot identify these faint stars directly, we must use a two-stage approach to the identification of stars in the immediate vicinity of the trail.

We first prepared a plotted "map" and corresponding list (see section 2. 3) for stars of 4.5 mag or brighter. Approximate positions are computed for six of these stars distributed over the plate, and these stars are identified by the process described above. The grid size for this catalog is $1^{\circ}$ on a side, and the total number of stars is 820.

With the known sky coordinates of these stars and their plate coordinates relative to the center, a closer approximation to the direction of the camera axis can be computed (see Appendix B).

With this new center and the newly identified bright stars, plate constants are computed (coordinate transformation matrix from plate to sky), which can be used to predict sky coordinates of trail stars to an approximation of better than 6 arc min. This small grid size is then suitable for a search-and-match identification of the trail stars.

The grid size used for this computer plot of the stars in the sky is 6 arc min in declination and approximately ( $6 \mathrm{sec} \delta$ ) arc min in right ascension. The scale in $a$ is not changed with every band in $\delta$, but sufficiently frequently to maintain reasonably equal search areas over the sky.

## 2. 5 The Mechanics of Processing

For efficient use of this new form of star catalog data, the reduction procedure should no longer be thought of as a case-by-case process. If the search data are appropriately ordered, any number of predicted star positions from various unrelated plates can be matched on one sequential tape reading and a once-through match test of any segment of the catalog. This minimizes computer time per case. A compromise must be made, however, in view of several other considerations.

We do not wish to process too large a batch at any one time, for reasons other than ease of handling. If an error by the automatic card punch attached to the measuring engine, or by the man who assembles the decks, or by the computer should stop a run where work to that point is unrecoverable, it should not be allowed to waste a large amount of preparation or machine time. There is a limitation also on the length of a list of approximate positions and matched positions that can be contained in the machine with a convenient segment of the star catalog.

The entire bright-star map and list reside in the computer core at once, and a list of predicted faint-star positions is generated from this information. The faint-star map is read in so that three contiguous records reside in core ( $3^{\circ}$ in $\delta$ ). The circular search can thus extend across bands in $\delta$. An ordered list of matched occupied map positions is compiled with case number and star number attached for later selection. The faint-star list is read sequentially, $1^{\circ}$ in $\delta$ at a time, and a list of precise positions is compiled.

This new star-identification procedure effects the major timesaving over previous methods. We also investigated every other aspect of the reduction procedure to find possible shortcuts.

Electronic computers have been used in the past to perform the arithmetic involved in the reduction, but there were always many breaks for intervention by the human computer. Human intervention is time-consuming and also error-prone, particularly in instances where tasks such as table lookup and data transcription are involved. The following "bookkeeping" operations are most readily assumed by the automatic computer: averaging of repeated measures; selection of the appropriate station coordinates for the date of the meteor (the stations were moved several times); selection of precession constants to bring the catalog star position to the equinox of the meteor year; "drawing" of the best straight line through the trail measures; applying of field corrections to trail measures (corrections derived from residuals from the plate-constants fit as a function of distance along the trail); and plotting of a retardation curve (deviation from constant velocity trajectory) to illustrate anomalies in the meteor behavior or in the measures (see Appendix C).

Measuring engines now available can be attached to automatic card-punching readout. Such a device eliminates the need for the measurer to look away from the machine repeatedly to transcribe vernier readings and then later to prepare these data into appropriately formatted punched cards. The engine screws have been motorized because the new star-identification technique requires measures over a wide area of the plate, rather than just in the vicinity of the meteor. The measuring engine used here has been fitted with a projection screen. There is less fatigue for the measurer than with the conventional eyepiece, but to suit the new conditions, new approaches had to be tried to discover the optimum density of image and focus on the copy plate in the region of the meteor.

The timesaving in incorporating such simple and uncompromising automation is very significant. See Appendix $D$ for a step-by-step account of the procedure.

## 3. EVALUATION OF PROCEDURE

We achieved an eightfold reduction in handling time per meteor pair without loss of accuracy.

The star identification is not always successful. The reasons for this are not obvious; there is no apparent inherent reason for failure. We do have the difficulty in this data sample of greater uncertainty in the center of the field than we would like. At the time this information (HA ${ }_{c}, \delta_{c}$ ) was recorded, it was not anticipated that it would be used in any other way than as a guide to the appropriate region of the star chart. A further displacement of the center occurs during the copying process where the projection center is displaced from the shutter center in the attempt to optimize the focus in a particular region of the field. This can be translated only approximately into a displacement in the sky, and a new direction center-North Pole. Sometimes there is no visual observation to fit the time of the meteor, and so there is uncertainty in the hour angle of the center at the appropriate time.

There are unanticipated omissions in the catalogs. A new version has just been received, and it will be processed into the appropriate form.

The measurer cannot always judge which of the bright-appearing stars on a film are indeed brighter than visual magnitude 4.5 , or 9 , the limits for the two catalogs used.

The failure rate at the extreme edges of the field is unexpectedly high in view of the generally high quality of the Super-Schmidt films.

When the stars have been successfully identified, the reduction proceeds in the standard fashion, except for a few new tasks assigned to the machine. All measured trail points are included in the determination of the straightline trajectory. When this is done visually, obvious spurious points are
omitted (e.g., in faint regions where the scatter of the measures must be high). The curve is fitted by machine to the plate-constants deviations and is used as a field-correction curve. However, the field correction is rarely large enough to be troublesome except for very long meteors or those crossing the center of the field. The retardation curve (see Appendix C) is plotted by machine. It should be inspected for obviously misnumbered dashes (where there are faint gaps, difficult to count), or obviously bad measures, and the effects of shutter flutter and camera vibration. (See Figure C-1.) This winnowing of the dash measures was not done for our sample because these irregularities occur infrequently. However, the velocities used as orbit input were determined by careful inspection rather than directly from the derived relation between $D$ and $t$ along the trail (see Appendix $C$ ).

However, meteor data cannot be handled entirely in a routine fashion. Expert judgment is required at several points. Where we have made an attempt to circumvent this, we have programmed a computer plot for ready reference to judge the success of the programmed decision. Unfortunately, decisions that help to compute meaningful initial velocities and midtrajectory decelerations have not yet been successfully programmed. We are encouraged by the progress made, but are not yet prepared to apply the method to large quantities of data.

The probable errors of the velocities have been deliberately omitted since we have not yet devised a consistent and satisfactory scheme for extrapolating the observed velocity to the preatmospheric velocity required to compute the orbit. At present, all measured dashes are included in a least-squares fit to a relation between the distance $D$ along the trail and time $t$ :

$$
D=a+b t+c e^{k t}
$$

from which we derive

$$
\mathrm{V}_{\infty}=\left(\frac{\mathrm{dD}}{\mathrm{dt}}\right)_{\mathrm{t}=-\infty}=\mathrm{b}
$$

If the meteor duration is great enough and the basic data are accurate enough, this formulation is most satisfactory. But spurious results are often obtained if these conditions are not fulfilled. It appears that the derived quantity $D$ must be inspected to determine the best method of computing the velocity and probable errors for each individual case. Velocities were individually considered in our sample because of their importance in orbit computations. Decelerations also are not available on an automated impersonal basis for consistent computation of masses.

We have made no photometric measures on our sample of meteors, but we have tried to devise a reasonably simple impersonal method of machine densitometry, as opposed to the visual method used in the past.

## 4. THE DATA

A total of 415 pairs of Super-Schmidt meteor camera films have been measured and reduced by our new semiautomated procedures described above. For 357 of these we were able to compute orbital elements. The remaining $14 \%$ failed at some preliminary stage of the computations, mostly for one of the following reasons:

1. The star catalog was incomplete.
2. Stars of negative declination were not identified because of an uncorrected error in the machine program.
3. Meteor trails selected as a pair were actually photographs of two different objects.
4. Meteors near the edge of the field presumably suffered from a general degradation of the star-prediction process for stars very far from the projection center and in poor focus as the extreme edge of the film is reached.

Tables 1 and 2 present the orbital and trajectory data for these 357 meteors.

Figures 1 to 5 illustrate the frequency distributions of the orbital elements l/a, e, $q$, $i$, and the meteor velocity $V_{\infty}$ used to compute the orbits. The characteristics of these distributions are similar to those obtained by McCrosky and Posen (1961) and by Hawkins and Southworth (1961), the latter indicated by solid-line plots on Figures 1 to 5 .

The large number of hyperbolic cases relative to the sample of Hawkins and Southworth reflects our difficulty in selecting the appropriate $V_{\infty}$ without the very careful examination customarily used. (See Section 3.)

The following is an explanation of column heads used in Tables 1 and 2. Orbital data are referred to Equinox 1950.0.

| film | Film number in SL camera series - always used as station A of the pair |
| :---: | :---: |
| mo/day /yr | Date and time of day in U.T. for meteor occurrence |
| a | Semimajor axis (a.u.) |
| e | Eccentricity |
| q | Perihelion distance (a. u.) |
| $q^{\prime}$ | Aphelion distance (a. u.) |
| $\omega$ | Argument of perihelion (degrees) |
| $\Omega$ | Longitude of ascending node (degrees) |
| i | Inclination of the orbit plane to the ecliptic (degrees) |
| c. w. | Cosmic weight (see Whipple, 1954) |
| $\lambda$ | Elongation of the true radiant from the apex of earth's motion (degrees) |
| $a^{\circ}, \delta^{\circ}$ | True radiant of meteor, right ascension and declination (degrees) |
| $\mathrm{CZ}_{\mathrm{R}}$ | Cosine of the zenith angle of the apparent radiant |
| $\mathrm{n}_{\mathrm{A}}$ | Number of shutter breaks visible on the SL trail ( $0.0167 \mathrm{sec} / \mathrm{break}$ ) |
| ${ }^{\text {n }}$ ML | Number of shutter break at maximum light on SL film |
| H BEG, END | Beginning and end heights ( km ) computed for the trail at each station denoted by A, B |
| $\mathrm{H}_{\mathrm{ML}}$ | Height at maximum light, computed for SL trail |
| $\sin Q$ | $Q$ is the angle between the apparent great circles of motion as seen from the two stations |
| $V_{\infty}$ | Meteor velocity ( $\mathrm{km} / \mathrm{sec}$ ) outside earth's atmosphere |
| $V_{G}, V_{H}$ | Meteor velocity ( $\mathrm{km} / \mathrm{sec}$ ), relative to earth and to sun, respectively |











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[^2]Film










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Figure 3. Distribution of meteor orbits in perihelion distance. Line plot: Hawkins and Southworth (1961).


Figure 4. Distribution of meteor orbits in inclination. Line plot: Hawkins and Southworth (1961).


Figure 5. Distribution of meteor velocities. Line plot: Hawkins and Southworth (1961).

HAWKINS, G. S., AND SOUTHWORTH, R. B.
1961. Orbital elements of meteors. Smithsonian Contr. Astrophys., vol. 4, pp. 85-95.
JACCHIA, L. J., AND WHIPPLE, F. L.
1961. Precision orbits of 413 photographic meteors. Smithsonian Contr. Astrophys., vol. 4, pp. 97-129.
McCROSKY, R E., AND POSEN, A.
1961. Orbital elements of photographic meteors. Smithsonian Contr. Astrophys., vol. 4, pp. 15-84.

STAFF, SMITHSONIAN ASTROPHYSICAL OBSERVATORY
1966. Smithsonian Astrophysical Observatory Star Catalog.

Smithsonian Institution, Washington, D C.
W AGNER, A.
1963. A computer program for automatic star identification. BRL Memorandum No. 1535, 33 pp.
WHIPPLE, F. L
1954. Photographic meteor orbics and their distribution in space. Astron. Journ., vol. 59, pp. 201-217.
WHIPPLE, F. L., AND JACCHIA, L. J.
1957. Reduction methods for photographic meteor trails. Smithsonian Contr. Astrophys., vol. l, pp. 183-206.

## APPENDIX A

There is in the Super-Schmidt camera a focus post, which is photographed as a shadow on the periphery of the film. The line joining this marker (N) to the plate center defines the center-North Pole direction. The position of the shutter center on the film is well-defined. The observer records the approximate ( $\pm 5^{\circ}$ ) direction of the camera axis when he sets the position circles of the camera at the beginning of each exposure.

A gnomonic projection is used in copying the spherical Super-Schmidt films onto flat glass plates. Angles are preserved at $C$, the center of projection. Distances along a radial direction $r$ transform into angular separation in the sky, according to $\theta=\tan ^{-1} r / f$, where $f$ is the focal length of the camera. Thus, we can plot the center of the field, the north direction, and the position of the star relative to these on a representation of the celestial sphere. See Figures A-1 through A-3.

Applying spherical trigonometry, we can compute for the unknown star (*), $90-\delta_{*}$ and $c a_{*}$. The sign of $c_{c}^{\Delta a_{*}}$ is determined by the right- or left-handedness of the set NC*.


Figure A-1. In the camera.


Figure A-2. On the film.


Figure A-3. In the sky.

## APPENDIX B

Given the measured plate coordinates and the sky coordinates of three to six bright stars, as well as the plate coordinates of the center, one can solve the set of equations,

$$
\lambda_{*} \lambda_{c}+\mu_{*} \mu_{c}+v_{*} \nu_{c}=\cos \theta_{*}
$$

where $\lambda_{*}, \mu_{*}, \nu_{*}$ are direction cosines representing the spherical coordinates of the stars; the $\theta_{*}$ are angular separations of the stars from the center as computed from their radial distances and from the focal length of the camera; and $\lambda_{c}, \mu_{c}, \nu_{c}$ are the required direction cosines representing the spherical coordinates of the center.

A measure of the success of the fit can be obtained by comparing the $\cos \theta$ computed from the new center coordinates with the input $\cos \theta_{*}$.

A further check should be made by comparing the plate-constants fit referred to the new center with that referred to the original input center.

## APPENDIX C

The basic data computed for the trajectory are the distances from some zero point for each time segment along the trail.

The form of the relation between distance and time is assumed to be

$$
D=a+b t+c e^{k t}
$$

where $b$ is identified with the no-atmosphere velocity, $V_{\infty}=(d D / d t)_{t=-\infty}$, and $\mathrm{ck}^{2} \mathrm{e}^{\mathrm{kt}}$ with deceleration of the body.

The so-called "retardation curve" (see Fig. C-1) is a plot of

$$
\Delta \mathrm{D}=\left(\mathrm{D}-\mathrm{D}_{0}\right)-\mathrm{V}_{\mathrm{b}}\left(\mathrm{t}-\mathrm{t}_{0}\right),
$$

where the subscript 0 refers to the first measured dash on this plate, and $V_{b}$ is a mean beginning velocity computed from $\Delta D / \Delta t$ for the first several dashes.

There is a departure from the method outlined in Whipple and Jacchia (1957). The search for k is made by machine, not by drawing a smooth curve by eye. From a given starting value, a search is initiated for a $k$ for which a least-squares fit to $a, b$, and $c$ produces a minimum rms deviation.

The minimum is, in practice, very shallow and broad, and often oscillatory. This is not entirely satisfactory. However, the critical quantities $b$ and $d V / d t$ are relatively stable.


## APPENDIX D

THE METHOD IN DETALL

Films are selected in a convenient batch size from the library of 6000 pairs. They are examined, appropriately paired, and marked for copying onto glass plates. While they are being copied, the time and region data are transcribed and punched into cards. The glass plates are further marked to guide the measurer when the plate is on the measuring engine. A convenient morning's or afternoon's measuring is gathered for a sitting. A man can easily prepare in the morning the three or four plates that he can comfortably measure in the afternoon; the number depends on the quality of the images, the lengths of the trails, and related factors.

The original observers' records contain data on the approximate center of the field, the beginning time of the exposure, and the time of any visual event. The measurer, while preparing his films, must extract these data. He must establish that a mate exists, and then collect all the pertinent information He marks the spherical film for the photographer so that the projection onto a flat plate by the copying camera has optimum density and focus in the region of the meteor. He then marks the plates for identification of points to be measured other than along the trail: the north point, shutter center, projection center, and six bright stars.

The initial division of the computer program is as follows:
A. Read in "library" data:
up to a total of 25 stations' coordinate cards;
up to a total of 25 precession matrix cards;
bright star map, list, and key.
B. Process each pair:

Identify station by film number.
Extract precession constants appropriate to year of meteor.
Compute sidereal time corresponding to input LST
Take means of repeated measures.
Identify the nature of each measure by a code on its card.
Compute effective (HA) $c_{c}$ by applying the time lapse between the exposure beginning and the meteor time, and the displacement of the projection center from the shutter center on the plate.

Compute predicted sky coordinates for each bright star, one plate at a time, and its corresponding grid number.

Search the nine grid positions of the map centered at this point and accumulate possibilities.

If fewer than four stars are matched with possible positions, displace the center by $2^{\circ}$ and try again (four times).

If at least four stars are picked up, proceed to extract from the list precise corresponding positions.

Compute a plate-to-sky coordinate transformation for pairs of these stars two-by-two, selecting for each solution the set of the other star positions that gives the "best fit," discarding individual stars with deviations $>50 \mu$, or solutions with mean deviations greater than this.

Select the best fit, and use these stars as the basis for a leastsquares plate-constants solution.

If no satisfactory fit exists, move the center and try new predictions where the center is allowed to move by $2^{\circ}$ in each coordinate ( $a, \delta$ ).

If the effective focal length from this solution is satisfactory (within 3 . $5 \%$ of the accepted value), use these stars to compute an improved position of the projection center (Appendix B).

With this new center, perform a new least-squares plate-constants computation for use to predict sky coordinates of the measured faint stars.

Compute the grid codewords corresponding to these positions, and attach the plate number and star number to them.

Write on a scratch tape the plate identification, direction cosines of center and standard plate coordinate axes, focal length, plate coordinates of faint stars and trail dashes, and precession constants.

Compute relative station coordinates of the pair.
Write on tape for use in trajectory computation: all date and time information, station coordinates, camera and shutter information, and trail measures for the pair.
C. Order the list of predicted positions for faint stars for all 20 pairs of the batch by declination and codeword to correspond to the catalog order.

Read in three records of faint-star map at a time flanking the next $\delta$ of interest, and extract codewords of occupied grid positions surrounding the predicted positions.

Sort again on $\delta$ and codeword.
D. Read the faint-star list, degree by degree, and extract positions for the stars picked up from the map.

Order the entire batch on plate number and star number.
E. Do the following for each plate:

Read in the data previously written on the scratch tape.
Set up the faint-star measures with the corresponding possible star positions.

Perform trial plate-constants solutions and sort out valid identifications as for bright stars.

Apply precession.
Compute a fit to the field-correction curve.
Apply field corrections, according to this curve, to the trail measures.

Compute constants for a least-squares straight line through both corrected and uncorrected trails.

Produce reference plots to help judge the success of the se curvefitting operations.
F. Write on tape for use in the trajectory computation: plate constants, trail measures altered by the field corrections, and constants for equation of line with and without field corrections.

At this point, the computer program terminates. For sample output, see pages 42 through 44. The two tapes for further processing are saved. The output to date is examined. One must consider: Were there any failures of plates to proceed to this point? Do the plate-constants solutions indeed look reasonable? Does application of the computed field correction improve the appearance (straightness) of the trail? Are there scattered trail measures that unrealistically affect the determination of the trail equation? (Computer plots of field correction and of trails are provided for perusal.)

Having decided which pairs warrant continued processing, prepare the following input to the next programmed section:
a. In the order of the previous section, one card for each plate, with the film number in the first eight columns, and a tag on those cards for plates for which the computed field correction should be omitted.
b. The two data tapes prepared by the first programmed section.

The second program proceeds with a standard meteor-trajectory computation. For sample output, see page 45 .

These data are now checked for inner consistency, for reliability of the solution for velocity and deceleration, and for any atypical behavior.

A "best" no-atmosphere velocity is computed to be used as input to the orbit computation.

The input to the orbit-computation program consists of the following:
a. card " 2 " punched by the trajectory program, with $\mathrm{V}_{\infty}$ inserted, and
b. card "l" punched by the star-identification program.
Sample output from initial computer run.
$\begin{array}{ccc}\text { DINECIITN COSINFS OF PLATE CENTRE ANO AXES } \\ -0.6 E 5635 \mathrm{C} & -0.5713959 & 0.4494876 \\ 0.639655 & -0.7686615 & 0 . \\ 0.3455038 & 0.287517 . & 0.8932865\end{array}$

## PLATE CONSTANIS ANO INVERSES

$\begin{array}{rrrrrr}2 C 5.341234 & -7.556333 & 898.205215 & -7.124273 & -206.137558 & 118.695469 \\ 0.06486375 & -C . C 0 C 17878 & -4.34759647 & -0.00016809 & -0.00484496 & C .72315271\end{array}$

| 1 | $N$ |  | X | Y | DX | DY | EXI | ETA |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -0.85807C-C.4328C7 | 0.276394 |  | 953.322 | 157.039 | 0.045 | -C.004 | -C.225C27 | -0.181121 |
| -C.P36C07-C.445932 | 0.319745 |  | 858.687 | 147.360 | 0.004 | -C.00C | -C. 197403 | -C. 135141 |
| -C.746844-C.615818 | 0.340734 |  | 899.739 | 164.812 | -0.054 | C. 009 | -C.CO1326 | -0.226628 |
| -0.585932-C.785757 | 0.185750 |  | 749.929 | 174.384 | 0.045 | -C.OC4 | C. 241754 | -0.281396 |
| OIRECIIOV COSINES OF | PLATE CEATRE | ANO | AXES |  |  |  |  |  |
| -C.6. 143515 | -0.5831527 |  | 0.4529713 |  |  |  |  |  |
| 0.6541069 | -0.7564022 |  | 0. |  |  |  |  |  |
| 0.3426285 | 0.2962917 |  | 0.8915251 | ETA |  |  |  |  |

[^4]$\begin{array}{rrrrrr}205.172145 & -9.218458 & 901.786484 & -7.922633 & -205.040838 & 117.384529 \\ 0.00486669 & -C . C 0021875 & -4.36304110 & -0.00018805 & -0.00486862 & C .741 C 7824\end{array}$

$\begin{array}{rr} & \\ \text { PREDICTIONS OF FAINTETARS } \\ 2 C 9.7316 & 9.5041 \\ 21 C .2121 & 5.7579 \\ 210.26 \epsilon 0 & 9.3210 \\ 211.6588 & 7.5622 \\ 212.6 C 5 C & 1 C .0478 \\ 214.46 \epsilon 4 & 8.7642 \\ 215.4584 & 8.7588 \\ 715.9140 & 9.3953 \\ 211.8111 & 8.4941 \\ 218.3932 & 9.9383 \\ 215.09 C 3 & 8.6752 \\ 219.833 C & 11.8702 \\ 219.9780 & 11.2761 \\ 220.5441 & 1 C .9603 \\ 22 C .74 C 5 & 1 C .7947\end{array}$
n
10C31.
$\begin{array}{ccl}\text { DIREC IION COSINES OF PLATE CENTRE AND AXES } \\ -0.6749547 & -0.5820972 & 0.4534302 \\ 0.6530938 & -0.7572770 & 0 . \\ 0.3433722 & 0.2961325 & 0.8912918\end{array}$


$44$




[^0]:    ${ }^{1}$ Smithsonian Astrophysical Observatory.
    ${ }^{2}$ Harvard College Observatory and Smithsonian Astrophysical Observatory.

[^1]:    
    $\stackrel{\circ}{8}$
    
    

[^2]:    

[^3]:    

[^4]:    PLATE CONSTANTS AND INVERSES

