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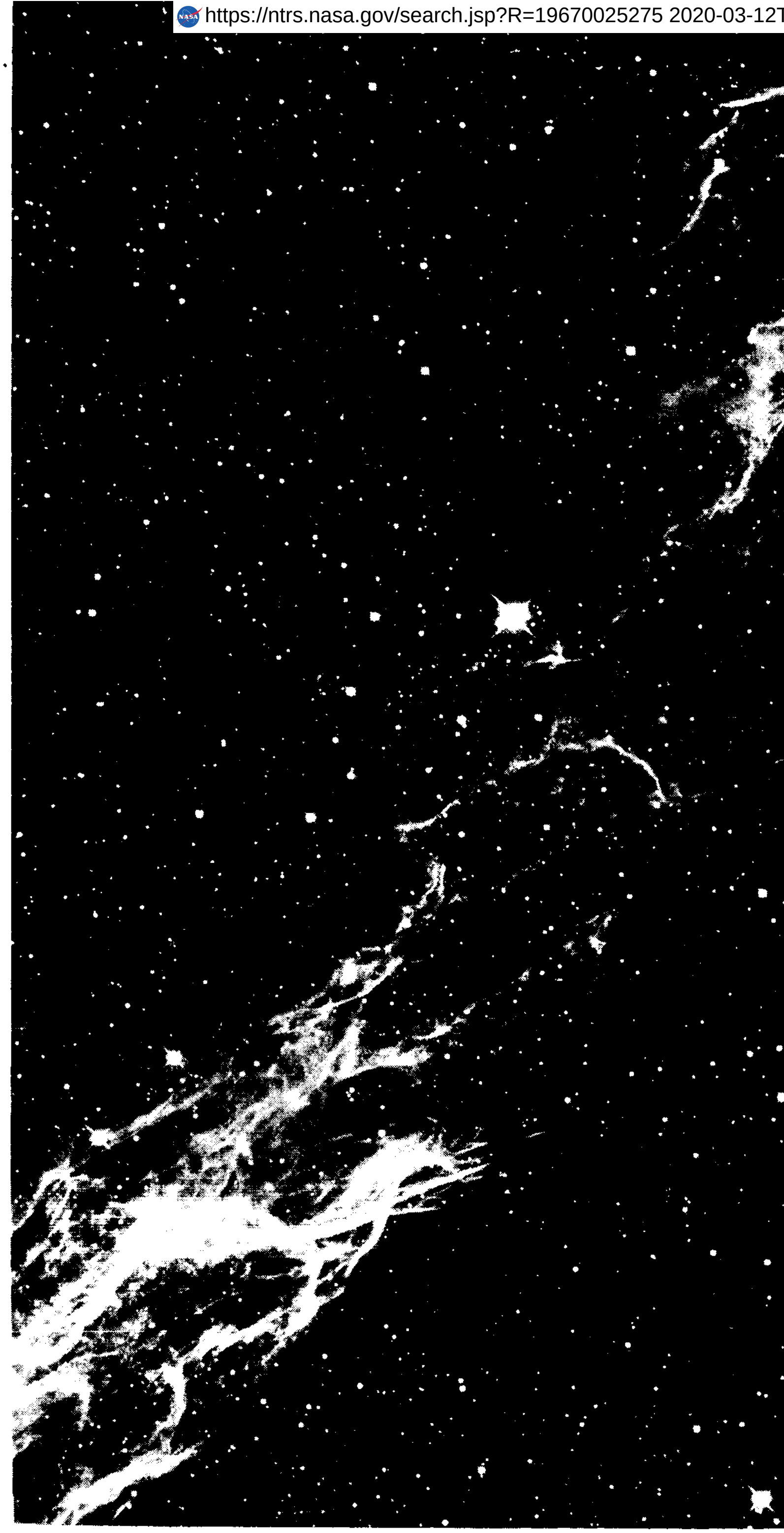

Report No. T-18

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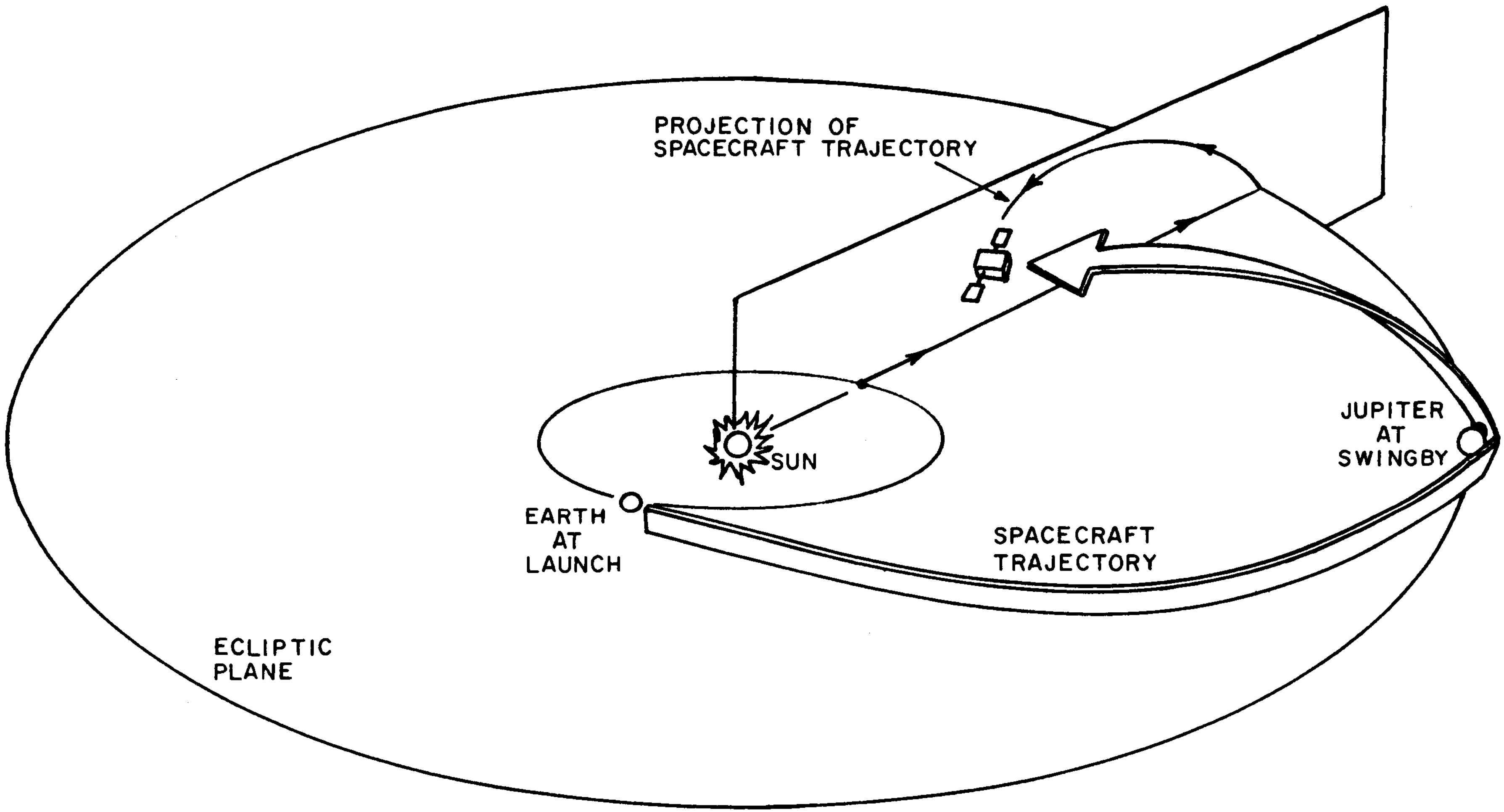
A representation of a typical Jupiter gravity-assisted trajectory. The spacecraft travels from Earth to Jupiter on a trajectory which lies very close to the ecliptic plane. The spacecraft's flight path is illustrated as being deflected up out of the ecliptic plane and in toward the Sun by the gravitational field of Jupiter. Many other maneuvers are also possible. The trace of this trajectory is shown on a plane which rotates about the Sun with the spacecraft and is normal to the ecliptic plane. The envelope of all possible traces on this plane defines that region of the solar system which is "accessible" to the spacecraft.

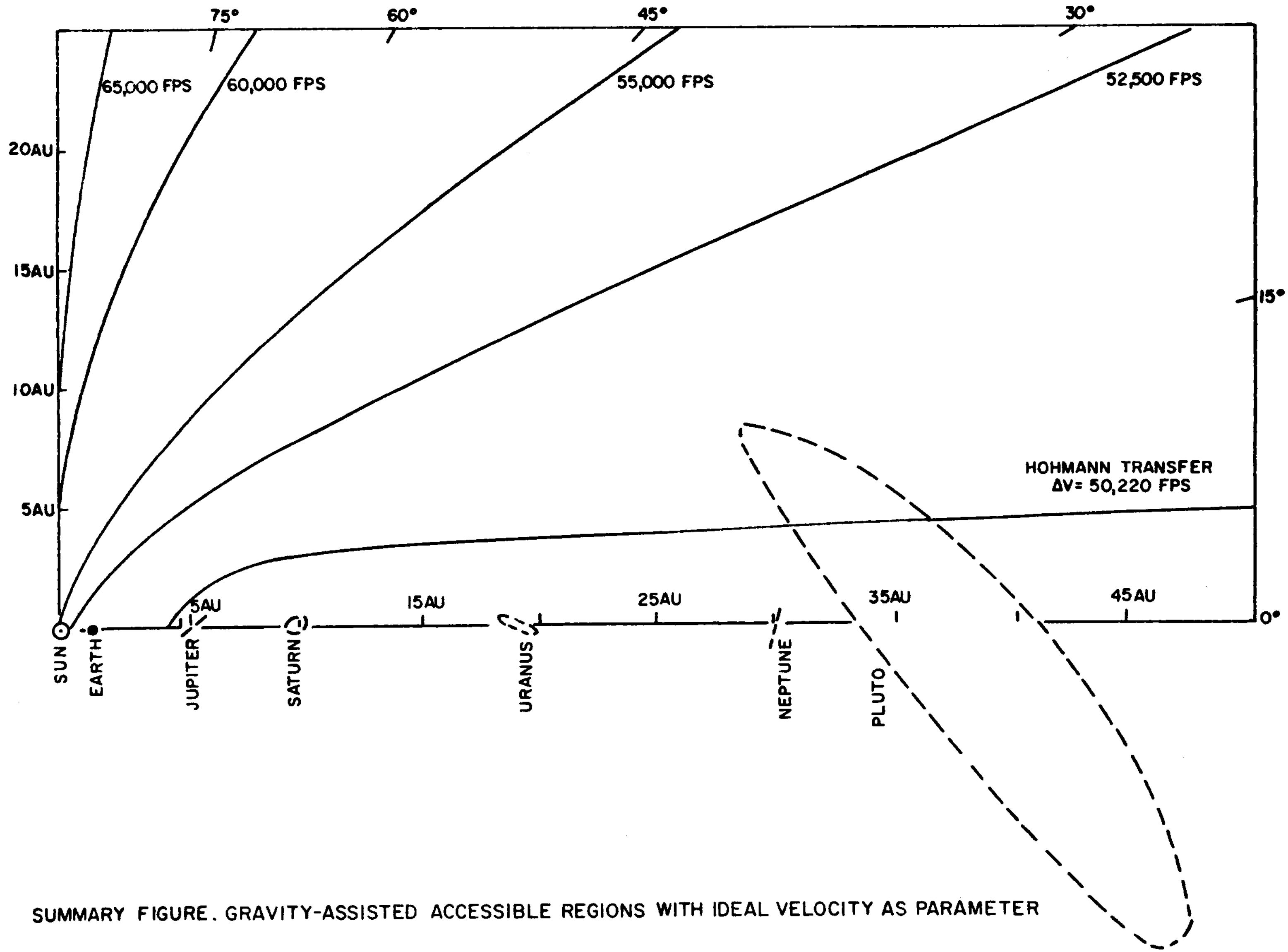


GRAVITY-ASSISTED TRAJECTORY

SUMMARY

The exploration capabilities of a given spacecraft-launch vehicle combination are greatly increased by a Jupiter gravity-assist (swingby) maneuver. Many regions of the solar system, which are inaccessible in the direct ballistic flight mode, can be reached by spacecraft using Jupiter gravity-assisted trajectories. For example, this technique permits a spacecraft, launched with an energy just sufficient to reach Jupiter, to explore regions of the solar system far beyond the orbit of Pluto and to climb as high as five astronomical units (AU) out of the ecliptic plane. Greater heights can be attained with only slightly larger launch energies. Solar impact can be achieved far more readily than in the direct ballistic flight mode.

The study results are presented as contours (regions) of accessibility on a latitude-radius (P_N) plane normal to the ecliptic plane for constant ideal velocities (launch energies). Each contour envelopes the latitude-radius space traversed by all the constant ideal velocity trajectories passing close to Jupiter. The ideal velocity is the total velocity increment given to a spacecraft launched from Earth. The summary figure shows contours on the P_N plane for ideal velocities of 50,220, 52,500, 55,000, 60,000, and 65,000 feet per second. These



SUMMARY FIGURE. GRAVITY-ASSISTED ACCESSIBLE REGIONS WITH IDEAL VELOCITY AS PARAMETER contours are symmetrical about the ecliptic plane and therefore only the upper half of the contour is shown. These contours show, for example, that the entire solar system (except for a small volume more than 10 AU above and below the Sun) can be explored by spacecraft having ideal velocities of 65,000 feet per second.

Gravity-assisted ideal velocity contours, with maximum time of flight as a parameter, are compared with analogous contours based on direct ballistic flight trajectories. A comparison of regions accessible with ballistic, Jupiter gravity-assisted and nuclear electric low thrust flight modes is also given in the text. Additional study of Venus and Mars gravity-assisted trajectories using the accessible regions method of analysis is recommended.

TABLE OF CONTENTS

LIST OF FIGURES

Report No. T-18

THE ACCESSIBLE REGIONS PRESENTATION OF GRAVITY-ASSISTED TRAJECTORIES USING JUPITER

1. INTRODUCTION

The primary objective of this study was to delineate those regions of space accessible to spacecraft launched from Earth with a known velocity onto a trajectory incorporating a Jupiter gravity-assist maneuver. A secondary objective was to determine the influence of launch velocity, Jupiter miss distance, and orbital inclination upon the post-Jupiter trajectory.

The accessible regions method (Narin 1964) of portraying energy and time of flight requirements has proven extremely useful for mission survey purposes. The results obtained by this study are presented using this method. The spacecraft trajectory is traced on a plane P_N which is normal to the ecliptic plane, and rotates about the Sun such that the spacecraft always lies in the plane. This is illustrated by the frontispiece. For orientation, Figure 1 shows the planetary orbits as projected on the plane P_N. It has been assumed in this study that the Earth and Jupiter move in the ecliptic plane in circular orbits about the Sun. An Earth-Jupiter trajectory then appears on the plane P_N as a horizontal line in the ecliptic plane, originating

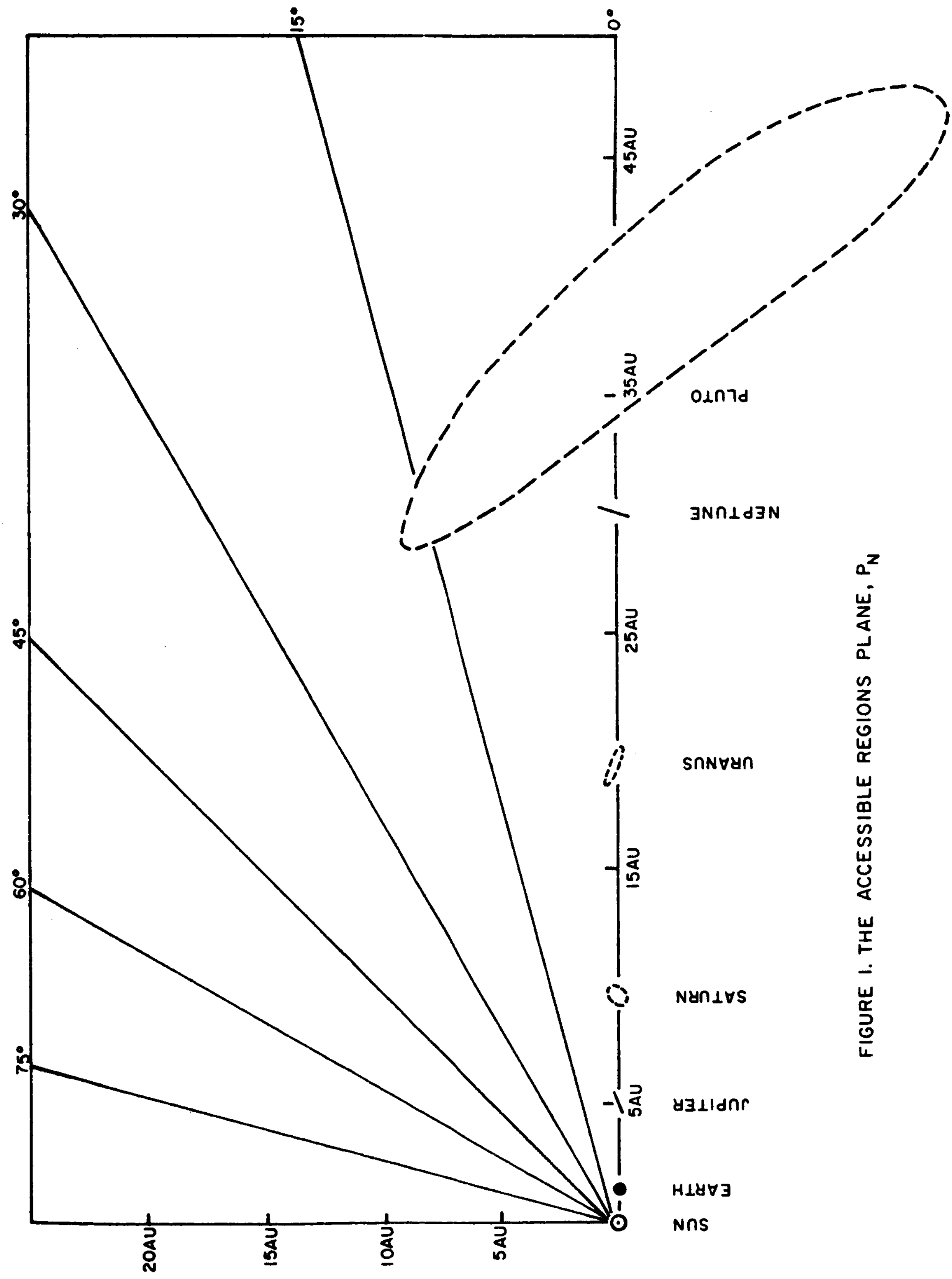


FIGURE I. THE ACCESSIBLE REGIONS PLANE, P_N

at the Earth's orbital radius and terminating at Jupiter's orbital radius. All possible spacecraft trajectories, originating with a given Earth launch energy, may be traced on the plane P_N. The envelope of these trajectory traces encloses that region of the solar system which is accessible to a spacecraft launched from Earth with the given energy. In this manner, the three-dimensional solar system is reduced to a two-dimensional side-view model. The exact date on which a specific flight may be launched is not presented by the accessible regions method, since such a date corresponds to a specific earth position.

Jupiter-assisted trajectories have been studied by Hunter (1964), Niehoff (1965), and Porter (1965). Hunter concluded that the launch energy required for a solar flyby can be reduced significantly by employing a Jupiter swingby, as compared to a direct trajectory from Earth. Porter investigated some Jupiter-assisted trajectories for deep solar system probes, Saturn flybys, solar probes, and out-of-the-ecliptic missions. His study of deep solar system, Saturn flyby, and solar probe missions was confined to ecliptic plane trajectories, while the non-ecliptic plane mission work considered only trajectories which pass directly over the Sun and trajectories which maximize the obtainable velocity component normal to the ecliptic plane, after Jupiter encounter. Niehoff generated extensive parametric data for Jupiter-assisted trajectories confined to the ecliptic plane. This study deals with a generalized presentation of post-Jupiter exploration capabilities to all regions of the solar system (ecliptic and non-ecliptic).

2. ASSUMPTIONS AND RESULTS

Various approximations have been made in obtaining the results of this study. As mentioned above, the Earth and Jupiter have been assumed to follow circular orbits in the ecliptic plane. A perihelion Earth launch has been assumed, i.e., the spacecraft heliocentric velocity resulting from the launch is tangent to the Earth's orbit. The effects of a non-circular Jupiter orbit and a non-perihelion launch have been investigated and are discussed in Section 3.3. Other assumptions which have been made include restriction to conic trajectory analysis, omission of launch or intercept time constraints, and representation of the gravity-assist phase as an instantaneous change in the spacecraft velocity, occurring when the spacecraft reaches the orbital radius of Jupiter.

The results of this study are summarized in Figure 2, where accessible region contours have been presented in terms of the ideal velocity, i.e., the total velocity increment given to the spacecraft on leaving Earth. A spacecraft launched with the specified ideal velocity cannot necessarily reach every point enclosed by the contour, since, at the larger values of ideal velocity, insufficient energy loss is accomplished in the Jupiter gravity-assist maneuver to enable the spacecraft to approach the Sun as closely as indicated by the contour. However, the solar approaches implied by the contours can be achieved by launching at a lesser ideal velocity. The contours

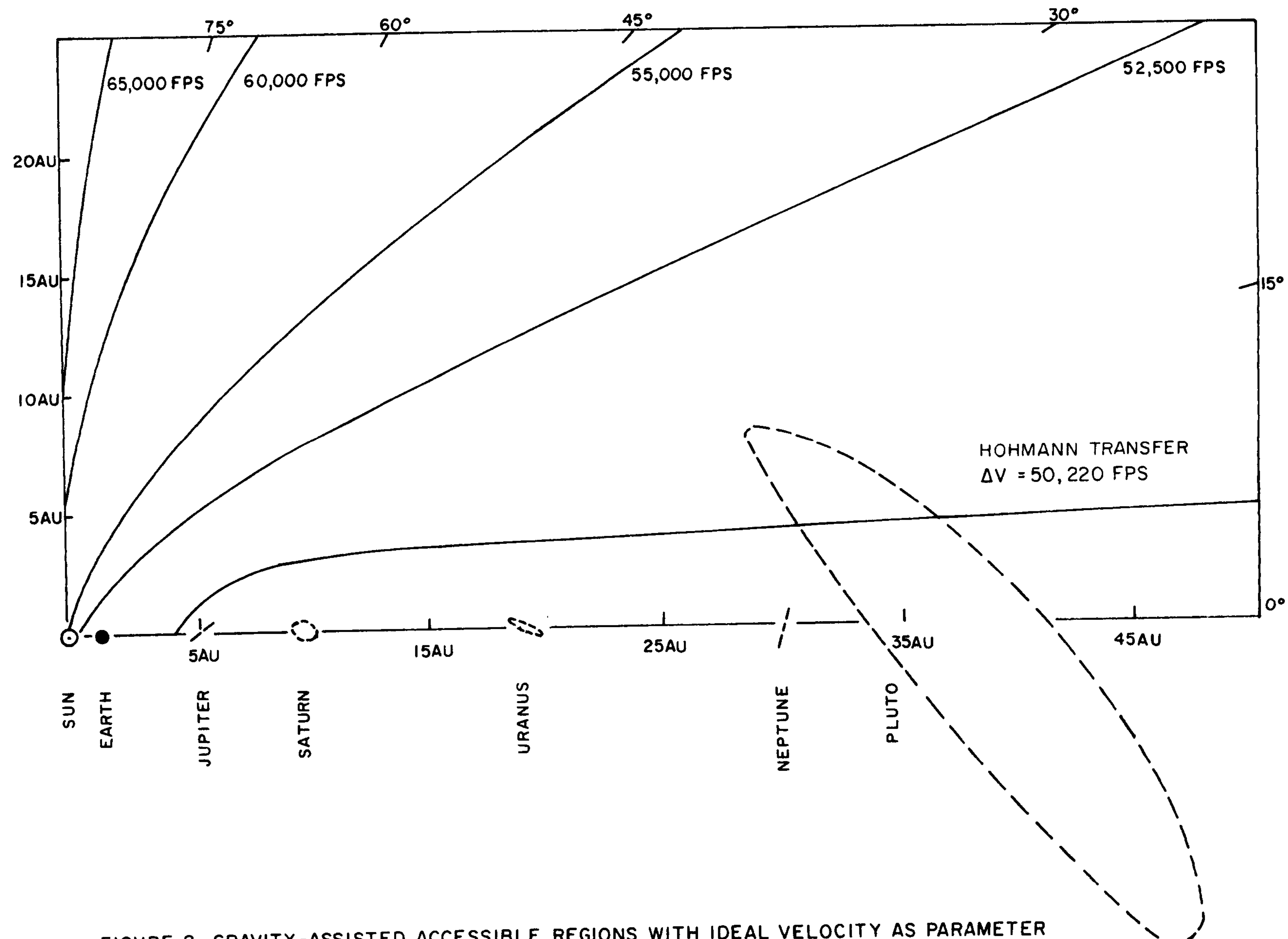


FIGURE 2. GRAVITY-ASSISTED ACCESSIBLE REGIONS WITH IDEAL VELOCITY AS PARAMETER

are symmetric with respect to the ecliptic plane, but only the upper half of the solar system has been shown for simplicity.

Comparison of Jupiter-assisted ideal velocity contours with those corresponding to direct ballistic flight emphasizes the increased exploration capabilities furnished by a Jupiter gravity-assist maneuver. Figures 3 through 7 compares Jupiter-assisted contours with the direct ballistic contours obtained by Narin (1964) and shows the effect of constraining the time of flight to 2, 4, 6, 8 or 10 years. In each case, Jupiter gravity-assist permits accomplishment of a wide variety of missions which are not feasible in the direct ballistic flight mode.

As an example, a simple Hohmann transfer to Jupiter can be transformed, by a Jupiter-assist, into an outer planet probe which proceeds far out past the orbit of Pluto. Such a probe can also attain a height of five astronomical units out of the ecliptic plane. A nominal increase in ideal velocity (from 50,220 ft/sec to 52,500 ft/sec) results in Jupiter-assisted trajectories which reach latitudes of 45 degrees near the Sun. An ideal velocity of slightly less than 55,000 ft/sec permits complete exploration of the ecliptic plane, from solar impact to solar system escape. An ideal velocity of almost 100,000 ft/sec would be required for solar impact using a direct ballistic flight. The entire solar system (except for a small volume above and below the Sun at heliocentric distances greater than 10.6 AU) can be explored by spacecraft having

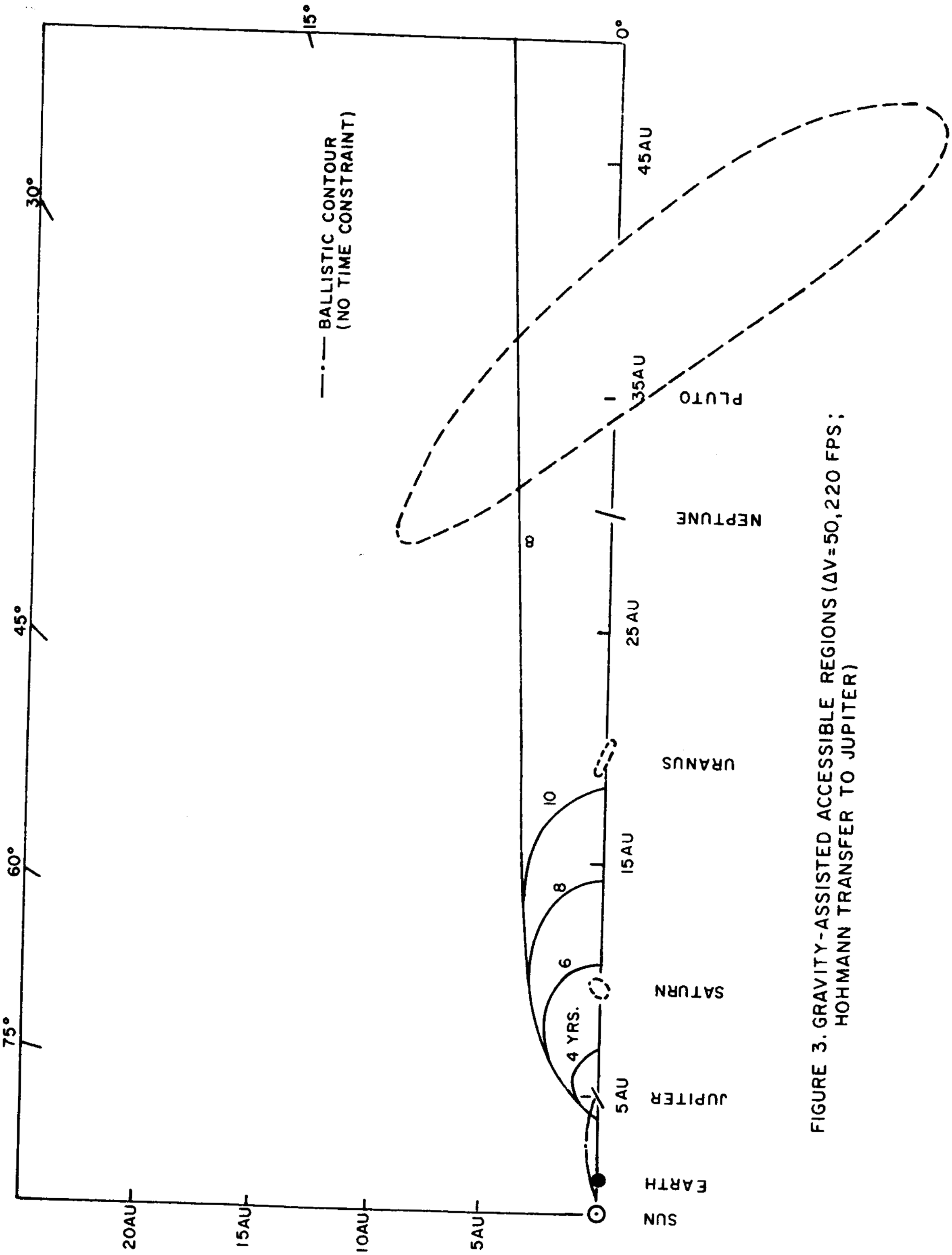


FIGURE 3. GRAVITY-ASSISTED ACCESSIBLE REGIONS ($\Delta V = 50{,}220$ FPS; HOHMANN TRANSFER TO JUPITER)

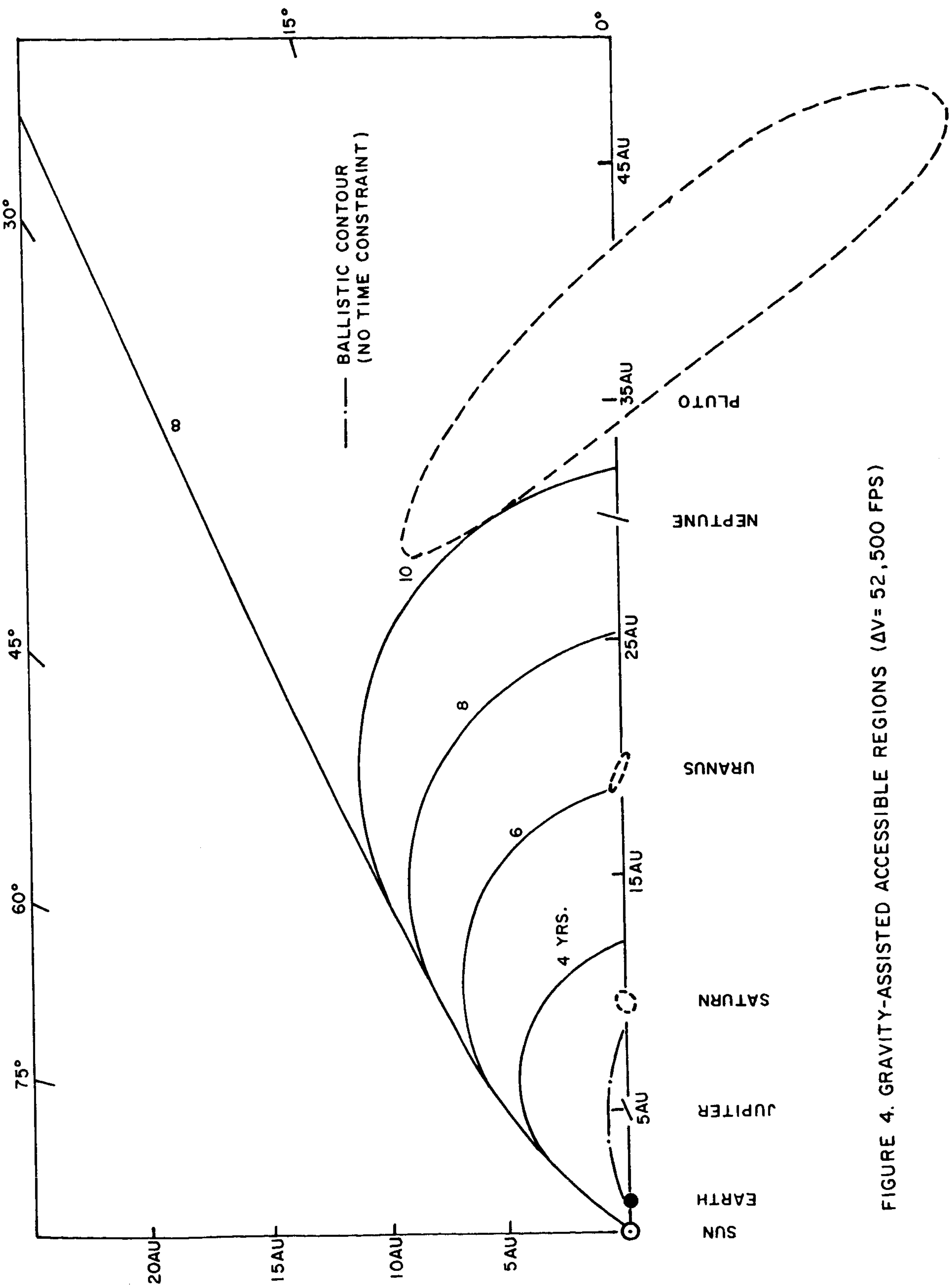


FIGURE 4. GRAVITY-ASSISTED ACCESSIBLE REGIONS (ΔV= 52,500 FPS)

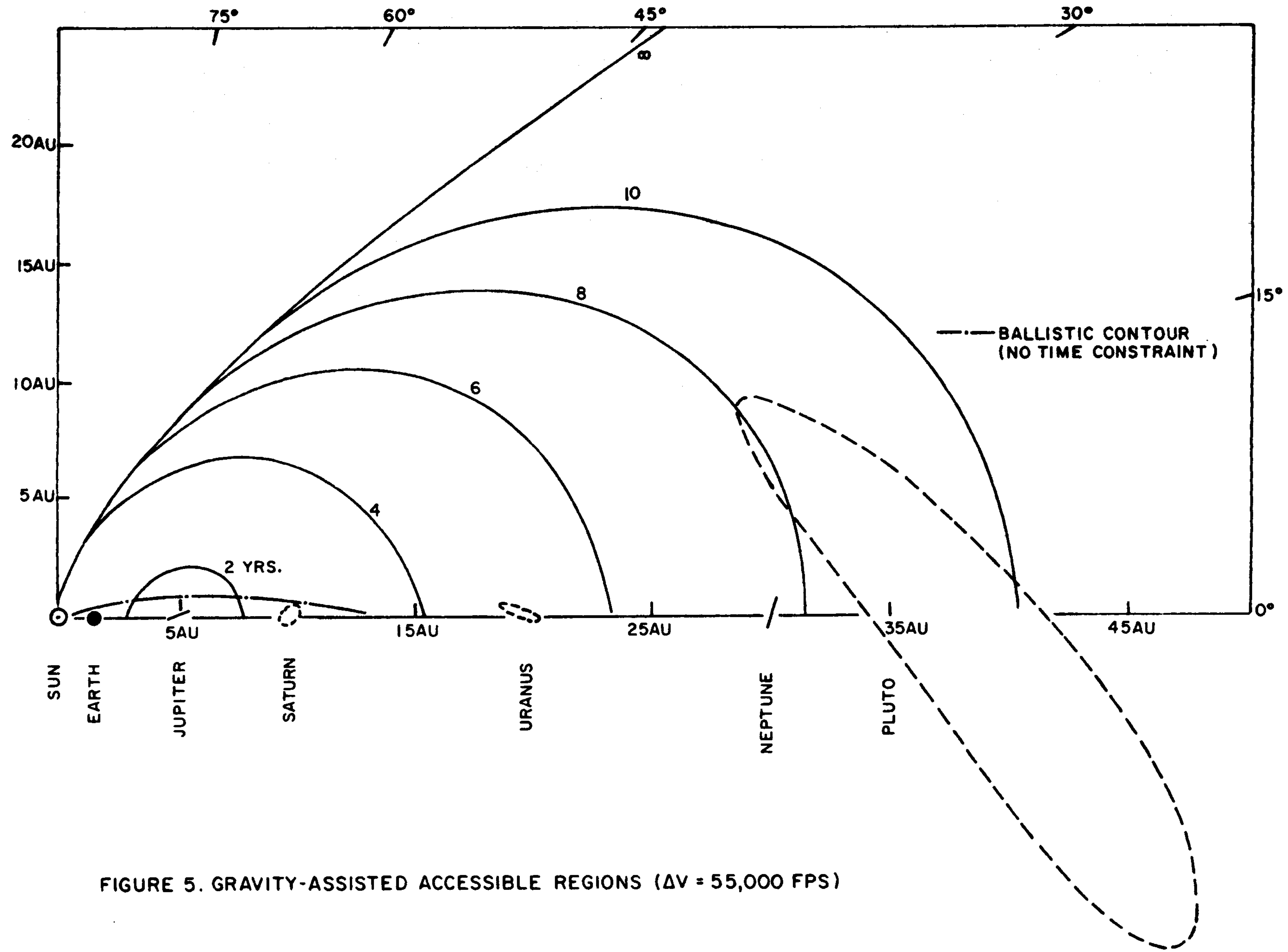


FIGURE 5. GRAVITY-ASSISTED ACCESSIBLE REGIONS (ΔV = 55,000 FPS)

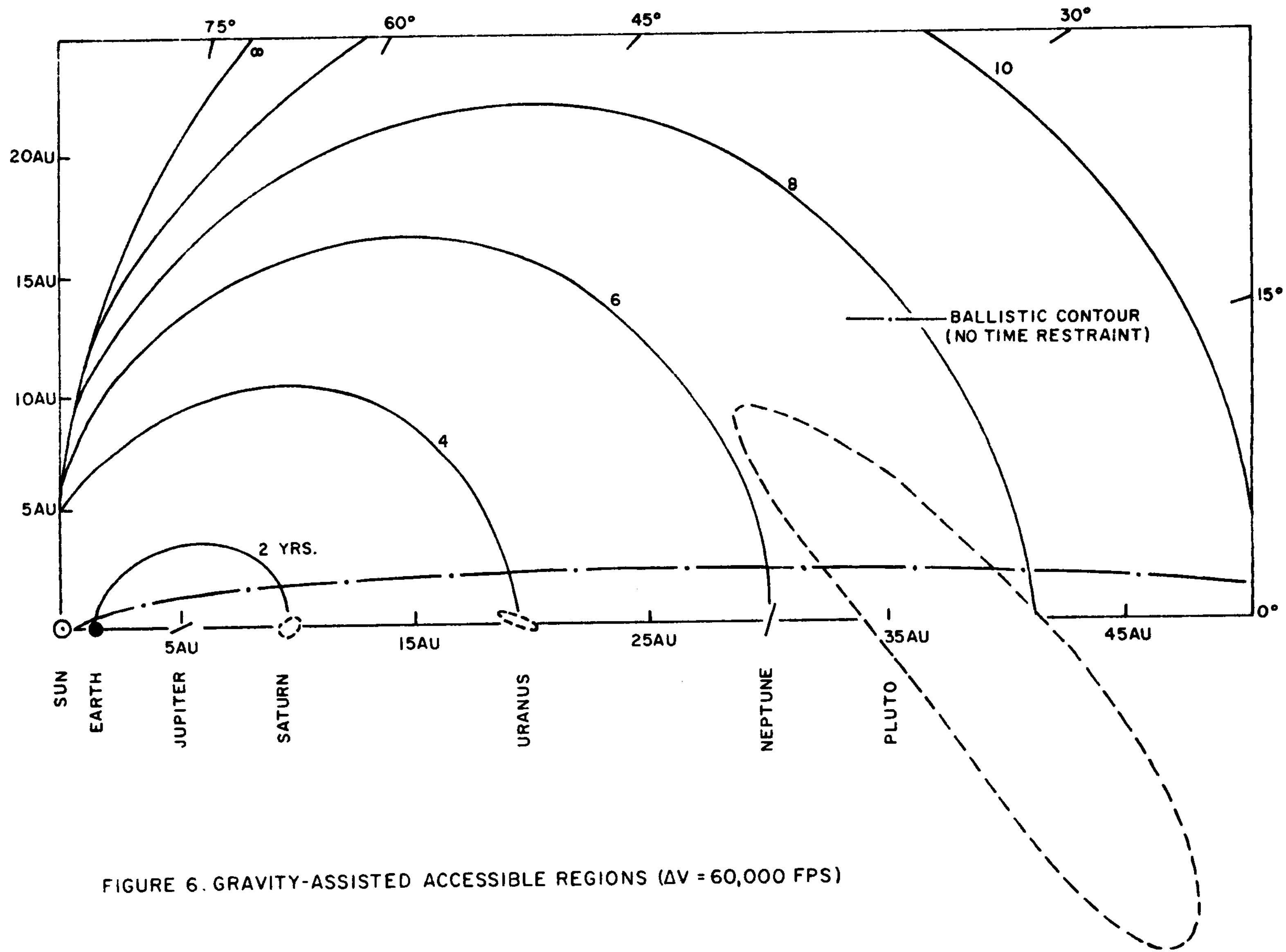


FIGURE 6. GRAVITY-ASSISTED ACCESSIBLE REGIONS (ΔV = 60,000 FPS)

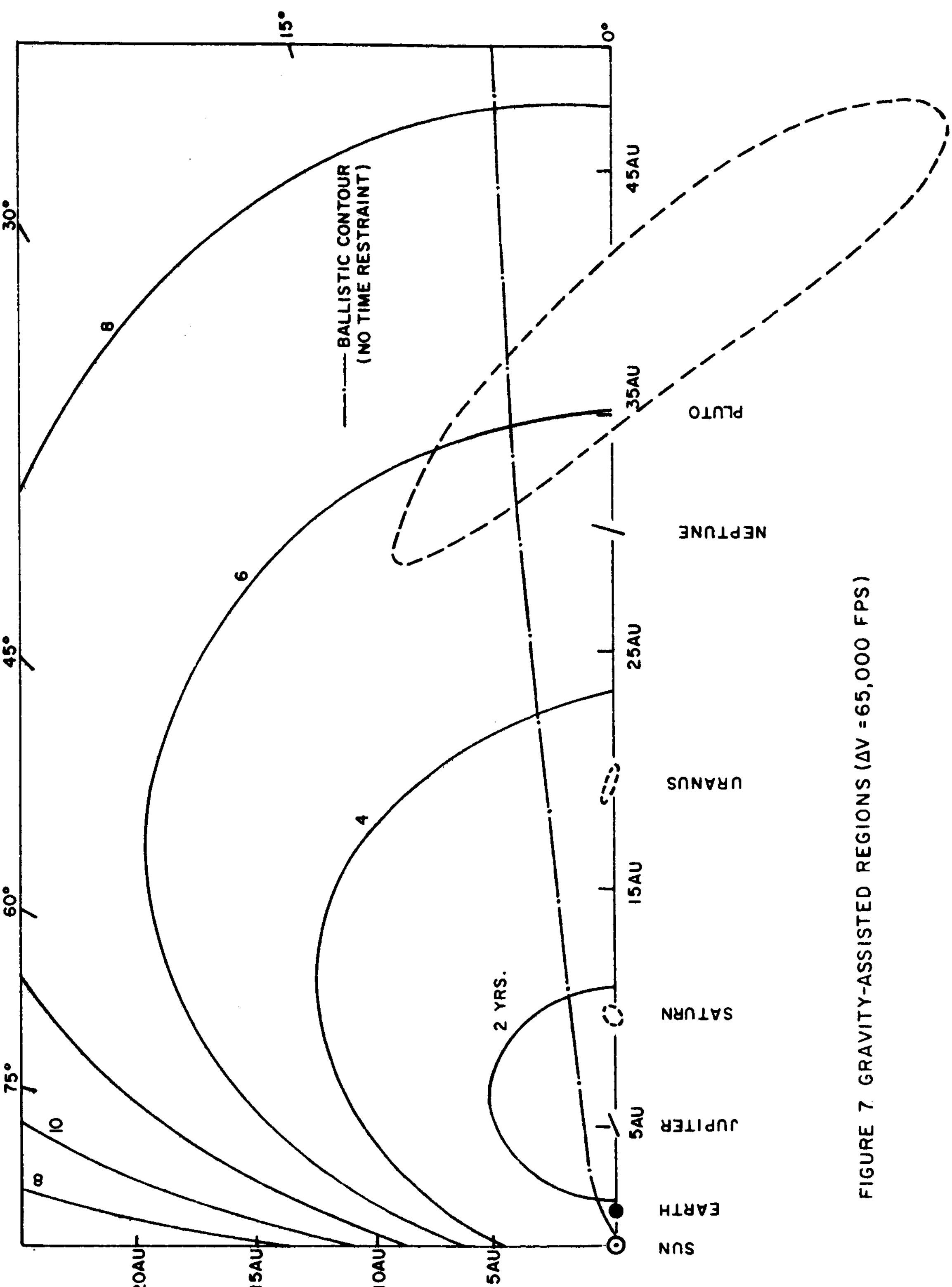


FIGURE 7. GRAVITY-ASSISTED REGIONS (ΔV = 65,000 FPS)

maximum ideal velocities of 65,000 ft/sec, provided a Jupiter gravity-assist maneuver is employed.

A comparison of regions accessible to ballistic, Jupiter gravity-assisted and nuclear electric low thrust spacecraft with approximately equal science payloads and constrained to 3 year flight times is presented in Figure 8. The trajectory data for the ballistic and low thrust contours were obtained from reports by Narin (1964) and Friedlander (1965) respectively. A science payload including experiments, experiment support, data handling and communications hardware of 500 lbs. was selected as a basis of comparison. The launch vehicle for all three flight modes is a Titan IIIC (1207). This vehicle is an improved Titan IIIC with 7-segment, 120-inch solid motors. For the ballistic and gravity-assist flight modes a Centaur upper stage was added to launch the spacecraft to an ideal velocity of 55,000 ft/sec. The total spacecraft weight including the 500 lbs. science payload, power, attitude control, guidance and structure weighs approximately 2,000 lbs. It can be seen that even with a total flight time limit of 3 years, a Jupiter gravity-assist greatly increases the ballistic region of exploration out-of-the-ecliptic as well as adding the specific target bodies, Sun and Saturn.

For the low thrust flight mode, a hypothetical nuclear electric stage, NES-1 (Friedlander 1966), was combined with the Titan IIIC (1207). The initial NES-1 stage weight is 20,000 lbs. of which 10,000 lbs. is the power plant weight, 2,400 lbs. is

3. DISCUSSION

3.1 Analytic Method

The method used to determine specific Jupiter-assisted spacecraft trajectories is an extension of that reported by Niehoff (1965). The spacecraft trajectory may be described in terms of three distinct phases:

1. Earth-Jupiter phase
2. Jupiter gravity-assist phase
3. Post-Jupiter phase.

Each of these phases will be discussed in turn.

During the Earth-Jupiter phase, the spacecraft is assumed to be under the influence of the solar gravitational field only. The spacecraft is launched from Earth, at some arbitrary time, with a given ideal velocity ΔV and injection flight path angle γ. The ideal velocity is the total velocity increment given to the spacecraft upon launch. It is related to the hyperbolic escape velocity VHL by

$$\Delta V = \quad (VHL)^2 + (36{,}178)^2 + 4000 \text{ ft/sec}, \qquad (1)$$

where 36,178 ft/sec is the characteristic velocity for Earth escape launch from Cape Kennedy and the 4,000 ft/sec term corrects for gravitational and frictional losses during launch. The injection flight path angle γ is the angle between the Earth's orbital velocity and the hyperbolic escape velocity (see Figure 9). An injection flight path angle of zero was assumed throughout most of this study. The conic section trajectory of the Earth-Jupiter phase is completely determined by specification of the ideal velocity and injection flight path angle. The Earth-Jupiter trajectory is confined to the ecliptic plane, and is terminated when the spacecraft first reaches a heliocentric radius equal to the radius of Jupiter's orbit.

The gravity-assist phase results in a change in the spacecraft velocity. This change is computed by assuming that during the gravity-assist phase Jupiter is represented by a point mass fixed at the intersection of the Earth-Jupiter trajectory with the circle representing Jupiter's orbit (see Figure 9), and that within a sphere of influence, centered at the point mass, the spacecraft is acted upon only by Jupiter's gravitational field.

The spacecraft heliocentric velocity as the spacecraft enters the sphere of invluence is $\vec{V}_2$, with the components V_{2X}, V_{2Y}, and V_{2Z} in a heliocentric-ecliptic coordinate system. In this coordinate system, the X-axis is directed outward from the Sun toward the Earth's position at launch, the Y-axis and Z-axis are oriented such that the Y-axis lies in the ecliptic plane and a right-handed coordinate set is formed. Since the spacecraft is confined to the ecliptic plane during the Earth-Jupiter phase, V_{2Z} is zero.

The details of the gravity-assist phase are most simply dealt with in a coordinate system centered at the point mass representing Jupiter, and moving with the heliocentric velocity

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$$\Delta V = \sqrt{(VHL)^2 + (36{,}178)^2} + 4000 \text{ ft/sec}, \tag{1}$$

where 36,178 ft/sec is the characteristic velocity for Earth escape from a 100 n.m. orbit and the 4,000 ft/sec term corrects for gravitational and frictional losses during launch. The injection flight path angle γ is the angle between the Earth's orbital velocity and the hyperbolic escape velocity (see Figure 9). An injection flight path angle of zero was

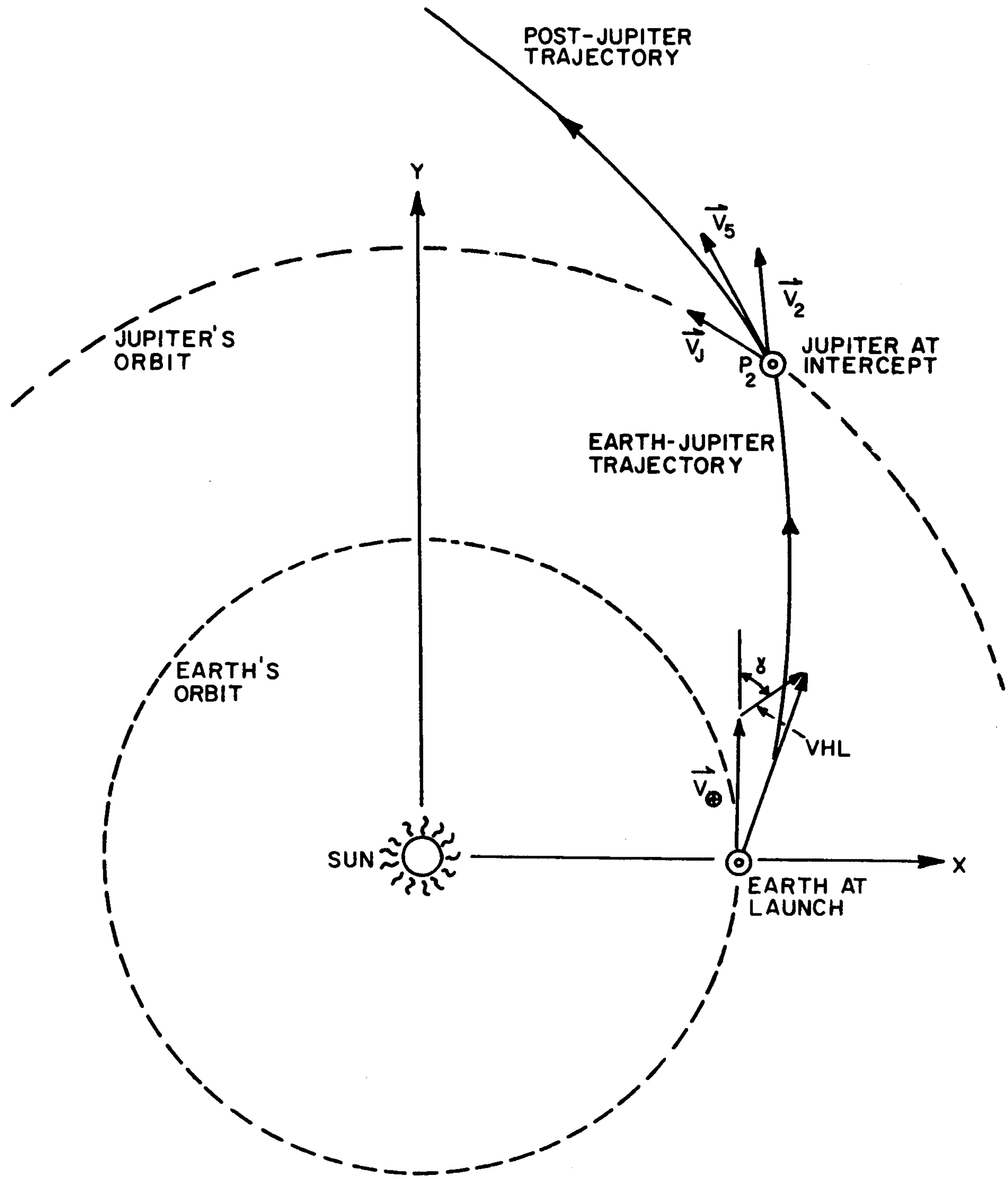

FIGURE 9. TWO-DIMENSIONAL HELIOCENTRIC GEOMETRY
(NOT TO SCALE)

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The details of the gravity-assist phase are most simply dealt with in a coordinate system centered at the point mass representing Jupiter, and moving with the heliocentric velocity

$\vec{V}_J$, which is the orbital velocity of Jupiter. In this coordinate system, the spacecraft enters the sphere of influence with the velocity $\vec{v}_3$, defined as

$$\vec{v}_3 = \vec{V}_2 - \vec{V}_J \tag{2}$$

Similarly, the spacecraft departs from the sphere of influence with the velocity $\vec{v}_4$, relative to Jupiter. The corresponding heliocentric departure velocity is

$$\vec{V}_5 = \vec{v}_4 + \vec{V}_J \tag{3}$$

During the gravity-assist phase, the spacecraft follows a hyperbolic orbit with respect to the point mass representing Jupiter. The swingby maneuver can result in a post-Jupiter trajectory which is out of the ecliptic plane only if either the approach velocity $\vec{V}_2$ or the point mass representing Jupiter lies out of the ecliptic plane. The distinction between the two cases in unimportant in this study. For convenience, imagine that the spacecraft has performed some minor mid-course maneuver such that is is slightly above or below the ecliptic plane as it approaches Jupiter. The Jupiter orbital velocity $\vec{V}_J$ is assumed to be parallel to the ecliptic plane. The plane defined by the velocities $\vec{v}_3$ and $\vec{v}_4$, and which contains the hyperbolic swingby trajectory, makes an angle θ with the ecliptic plane. The plane of the swingby trajectory is shown in Figure 10. The characteristic velocity resulting from the swingby is completely determined by the Jupiter approach velocity $\vec{V}_2$, the angle θ, and

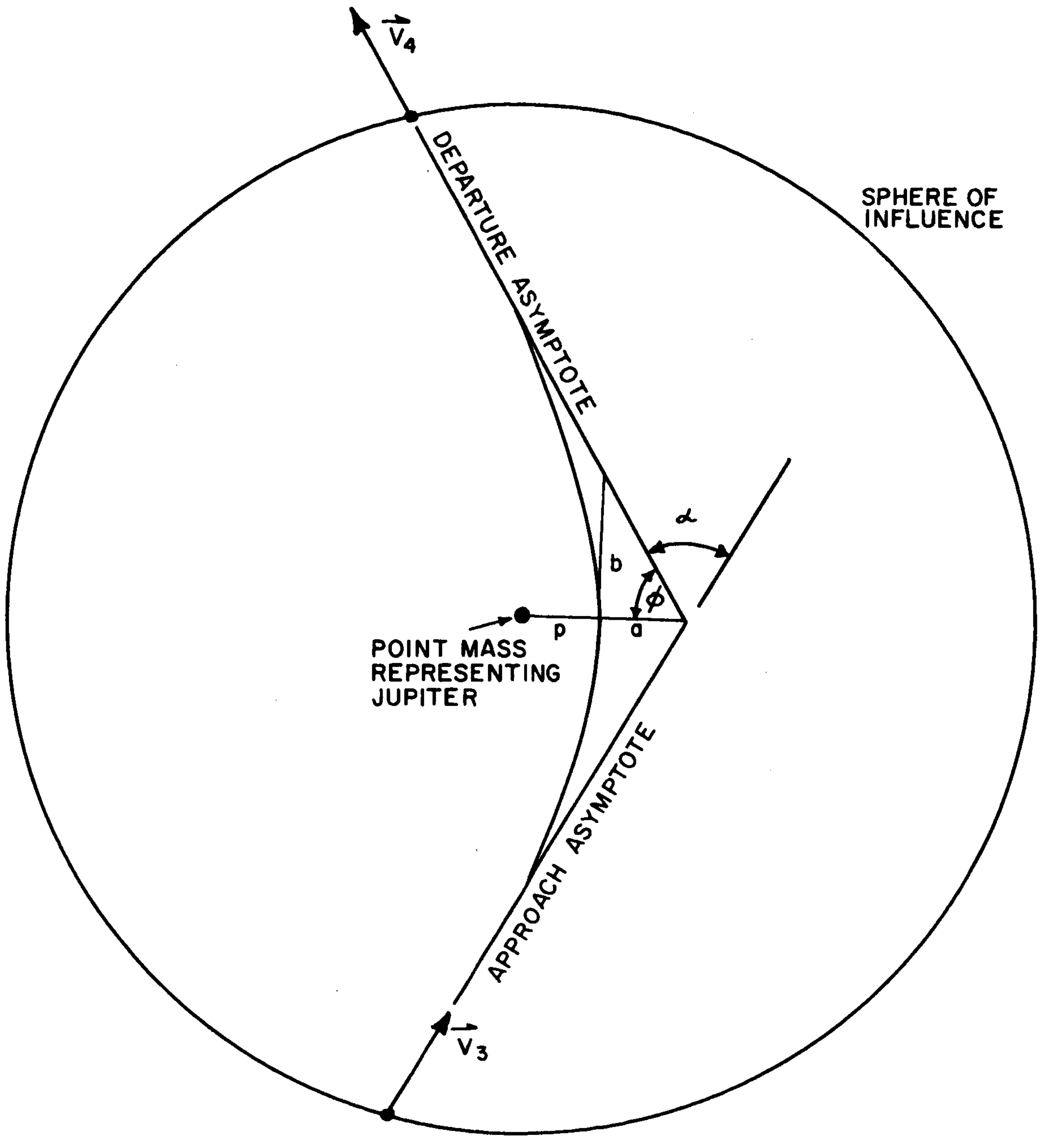

FIGURE 10. GRAVITY-ASSIST PHASE IN PLANE OF SWINGBY TRAJECTORY

the miss distance p (the periapsis distance of the hyperbolic swingby trajectory relative to the point mass representing Jupiter).

The turning angle α is the angle between the approach and departure asymptotes. From Figure 10,

$$\alpha = \pi - 2\phi \qquad (4)$$

Figure 10 also shows the semi-major axis a, the semi-minor axis b, and the miss distance p. The semi-major axis is taken as positive in this report. From the geometrical properties of a hyperbola,

$$a + p = \epsilon a \qquad (5)$$

and

$$\epsilon^2 = 1 + \left(\frac{b}{a}\right)^2 , \qquad (6)$$

where ϵ is the eccentricity. Combining equations (5) and (6), ϵ may be eliminated and b determined as

$$b = \sqrt{p^2 + 2ap} \qquad (7)$$

Figure 10 shows that

$$\phi = \tan^{-1} \left(\frac{b}{a}\right). \qquad (8)$$

Substitution of equations (7) and (8) into equation (4), and use of the trigonometric identity

$$\tan^{-1} x = \frac{\pi}{2} - \tan^{-1} \frac{1}{x} , \qquad (9)$$

yields for the turning angle,

$$\alpha = 2 \tan^{-1} \left(\frac{a}{\sqrt{p^2 + 2\,ap}} \right) \qquad (10)$$

The specific energy equation for a hyperbola is

$$E = \frac{1}{2} v^2 - \frac{k}{r} = \frac{k}{2a} , \tag{11}$$

where k is the Jupiter gravitational constant (1.267×10^8 km^3/sec^2). Since $\vec{v}_3$ is the asymptotic approach velocity, $\vec{v}_3$ may be regarded as the spacecraft velocity, relative to Jupiter, at an infinite distance from Jupiter. Using the asymptotic approach velocity, equation (11) may be solved to obtain

$$a = \frac{k}{v_3^2} \tag{12}$$

Substitution of equation (12) into equation (10) yields

$$\alpha = 2 \tan^{-1} \left(\frac{k}{v_3 \sqrt{p^2 v_3^2 + 2kp}} \right) \tag{13}$$

This result indicates that for a fixed miss distance p, increasing the magnitude of the approach velocity $\vec{v}_3$ decreases the turning angle. Thus the slowest approach velocity produces the greatest change in direction. Similarly, for a fixed value of approach speed, a small miss distance results in a large turning angle. The smallest miss distance attainable is represented by that swingby trajectory which just grazes Jupiter's atmosphere. Using 71,350 km (as measured from the center of Jupiter) for this distance yields

$$\alpha_{max} = 2 \tan^{-1} \left(\frac{1776}{v_3 \sqrt{v_3^2 + 3552}} \right) \tag{14}$$

for the maximum turning angle, where the approach speed v_3 is in km/sec. For a Hohmann transfer trajectory to Jupiter, v_3 is 5.64 km/sec and the corresponding maximum turning angle is 158 degrees.

The components of the heliocentric velocity V_5, at the end of the gravity-assist phase, depend upon the orientation of $\vec{v}_4$, the asymptotic departure velocity. Conservation of energy along the hyperbolic swingby trajectory requires that the departure speed equal the approach speed, i.e.,

$$\left|\vec{v}_4\right| = \left|\vec{v}_3\right| . \tag{15}$$

If β is the angle between the approach velocity $\vec{v}_3$ and the heliocentric X-axis, measured counterclockwise from $\vec{v}_3$, then

$$\beta = 2\pi - \tan^{-1}\left(\frac{v_{3y}}{v_{3x}}\right) \tag{14}$$

where v_{3x} is the x-component of $\vec{v}_3$ and similarly for v_{3y}. From Figure 11, which shows the relation of the swingby plane to the ecliptic plane, the components of $\vec{v}_4$ are

$$\begin{aligned} v_{4x} &= v_3 \cos\delta \cos(\lambda-\beta) \\ v_{4y} &= v_3 \cos\delta \sin(\lambda-\beta) \\ v_{4z} &= v_3 \sin\delta \end{aligned} \tag{17}$$

where

$$\sin\delta = \sin\theta \sin\alpha \tag{18}$$

$$\cos\lambda = \cos\alpha/\cos\delta , \tag{19}$$

and equation (15) has been employed.

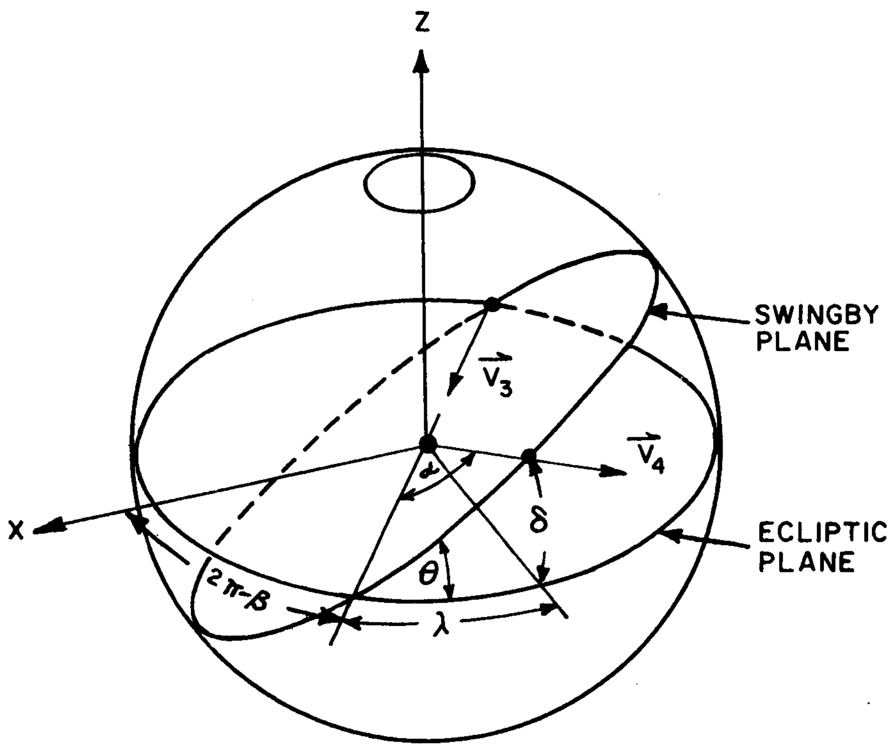

FIGURE 11. THE SWINGBY AND ECLIPTIC PLANES

Because, on a heliocentric scale, both the radius of the sphere of influence and the required departure of either Jupiter or $\vec{v}_3$ from the ecliptic plane are small, the gravity-assist phase has been assumed to occur at a single point in space. That is, the spacecraft enters the sphere of influence at that point where the ecliptic Earth-Jupiter trajectory first intersects a circle of radius equal to that of Jupiter's orbital radius, namely the point P_2 in Figure 9. The gravity-assist maneuver is then assumed to take place in zero time, and the spacecraft departs from the sphere of influence at the point P_2. This is equivalent to reducing the sphere of influence to a point.

The heliocentric velocity at the completion of the swingby phase is determined by equation (3), as discussed above. The spacecraft is assumed to begin the post-Jupiter phase of the mission with this velocity. The post-Jupiter trajectory is then completely determined using the two-body equations of motion.

3.2 Computer Program and Construction of Contours

A FORTRAN-IV computer program for the calculation and automatic plotting of gravity-assisted trajectories was developed as part of this study. The program is based on the analytic method summarized above, and provides automatic parametric variation of the ideal velocity, miss distance, angle θ, and time of flight. A basic feature of the program is automatic plotting of trajectories, as projected on the plane P_N, by use of the CalComp digital incremental plotter. The accessible regions contour, for a specific ideal velocity, is the envelope of all possible trajectories, as projected on the plane P_N, starting from Earth with the specific ideal velocity. More than 100 trajectories must be plotted, for each ideal velocity, to construct the accessible regions contour with any accuracy. Use of the CalComp plotter was essential to this study.

The computer program may be used to compute and plot gravity-assisted trajectories using any single gravity-assisting body, which has a circular orbit in the ecliptic plane. This study, however, has examined only Jupiter-assisted trajectories. Output from the program includes spacecraft heliocentric radius, latitude, longitude, and the flight time from launch, at a maximum of 100 points along the trajectory. The program requires about one second of computing time on an IBM 7094 to generate a single complete trajectory. The CalComp plotter requires about three minutes to plot ten trajectories on a single graph.

For almost all of the Jupiter-assisted trajectories generated during this study, a perihelion Earth launch was assumed (initial injection flight path angle equal to zero). The Jupiter orbital radius was taken as 5.203 astronomical units (one AU = 1.49599×10^8 kilometers), and the Jupiter planetary radius as 71,350 kilometers. The miss distance p, as used by the computer program, is in units of Jupiter planetary radii from the center of Jupiter.

A typical CalComp plot, as produced by this study, is shown in Figure 12. This particular plot shows, on the plane P_N, a set of trajectories, each starting with an Earth launch

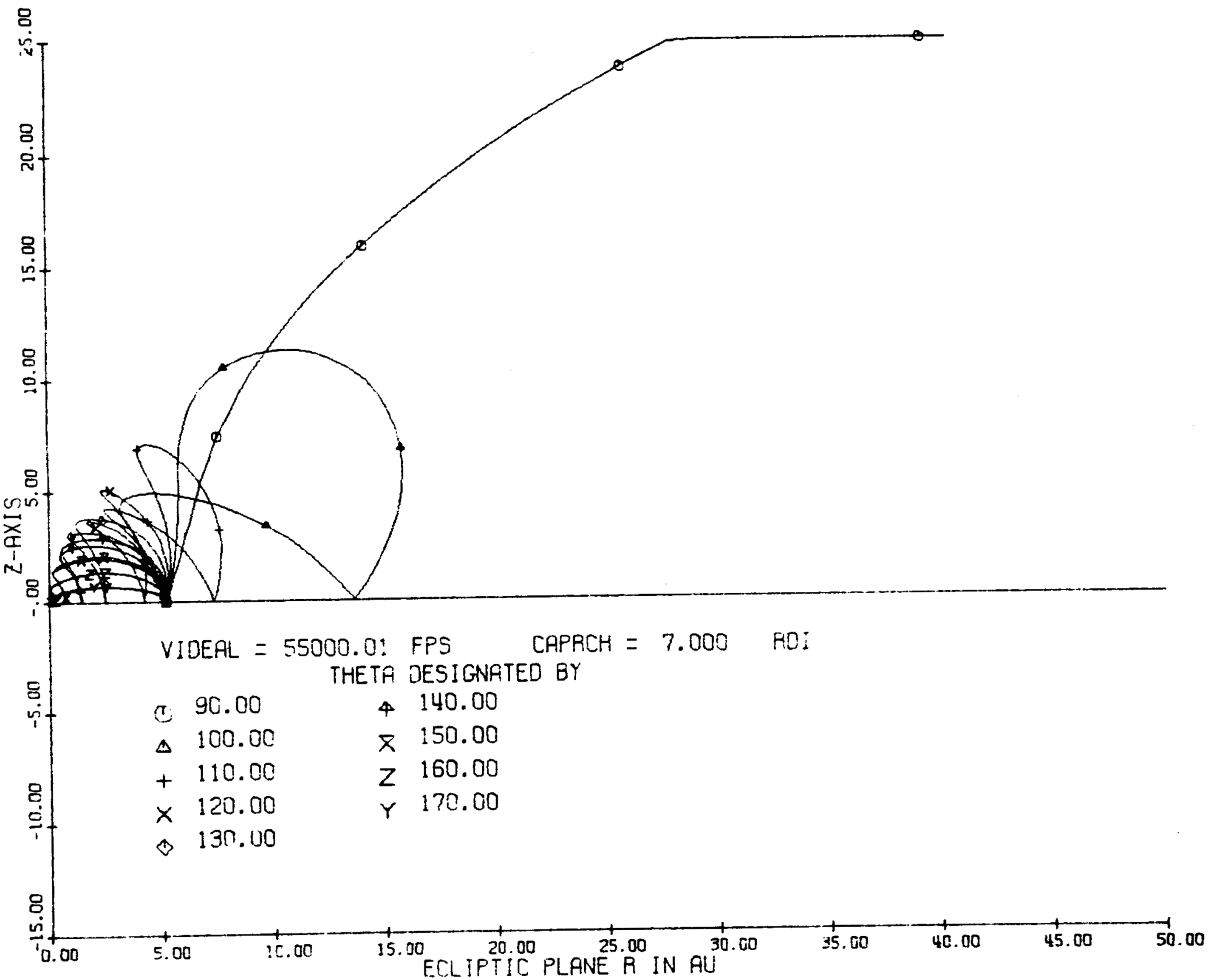

FIGURE 12. TYPICAL CALCOMP PLOT

and an ideal velocity of 55,000 ft/sec and then passing by Jupiter with a miss distance of seven Jupiter planetary radii. The Earth-Jupiter trajectory lies in the ecliptic plane. The different post-Jupiter trajectories correspond to different values of θ, the angle between the hyperbolic swingby trajectory and the ecliptic plane. Each trajectory is identified by the symbol defined on the figure. Trajectories which pass through the ecliptic plane have been reflected about the ecliptic plane, such that the portion of the trajectory which is below the ecliptic appears above the ecliptic in the figure. Any two trajectories which correspond to two θ values which are symmetric about 180 degrees are mirror image trajectories with respect to the ecliptic plane. For example, the trajectory corresponding to a θ of 190 degrees is identical to the trajectory corresponding to a θ of 170 degrees, except that the heliocentric latitude of each point on the 190 degree trajectory is the negative of the heliocentric latitude of the corresponding point on the 170 degree trajectory. Along any trajectory, those points having Z-coordinates greater than 25 AU have been plotted as if the Z-coordinate were 25 AU. This is the origin of the pronounced kink in the trajectory corresponding to a θ value of 90 degrees shown in the figure.

3.3 Sensitivity of Results

The results reported above assume that Jupiter follows a circular orbit in the ecliptic plane at a heliocentric radius of 5.203 AU. Accessible regions contours were obtained with a Jupiter orbital radius of 4.951 AU, and with a Jupiter orbital radius of 5.455 AU. The resultant contours were indistinguishable (on a 50 AU scale) from those obtained assuming a Jupiter orbital radius of 5.203 AU.

The accessible regions contours presented thus far are based upon trajectories which are initiated from an Earth perihelion launch. Non-perihelion launches reduce the accessible regions volume. As an example, Figure 13 shows accessible regions contours for an ideal velocity of 55,000 ft/sec and compares a perihelion launch with a non-perihelion launch. In the latter case, the injection flight path angle has been taken as 30 degrees. The accessible regions contour for values of initial flight path angles less than 30 degrees is nearly identical to the accessible regions contour assuming perihelion launch, provided that the original launch hyperbolic excess speed is multiplied by cos γ. For example, in the case illustrated by Figure 13, a perihelion launch and an ideal velocity of 55,000 ft/sec result in a VHL of 10.96 km/sec. For an initial flight path angle of 30 degrees, this VHL multiplied by cos γ is 9.49 km/sec. Using this value of VHL in equation (1) yields an ideal velocity of 51,700 ft/sec. Thus the accessible regions contour for an ideal velocity of 55,000 ft/sec and γ of 30 degrees is nearly identical to the accessible regions contour for a perihelion launch and an ideal velocity of 51,700 ft/sec.

Finally, the shape of the post-Jupiter trajectory is often sensitive to the gravity-assist phase parameters, such as the angle between the swingby trajectory and the ecliptic plane. This has been most noticeable for post-Jupiter trajectories which

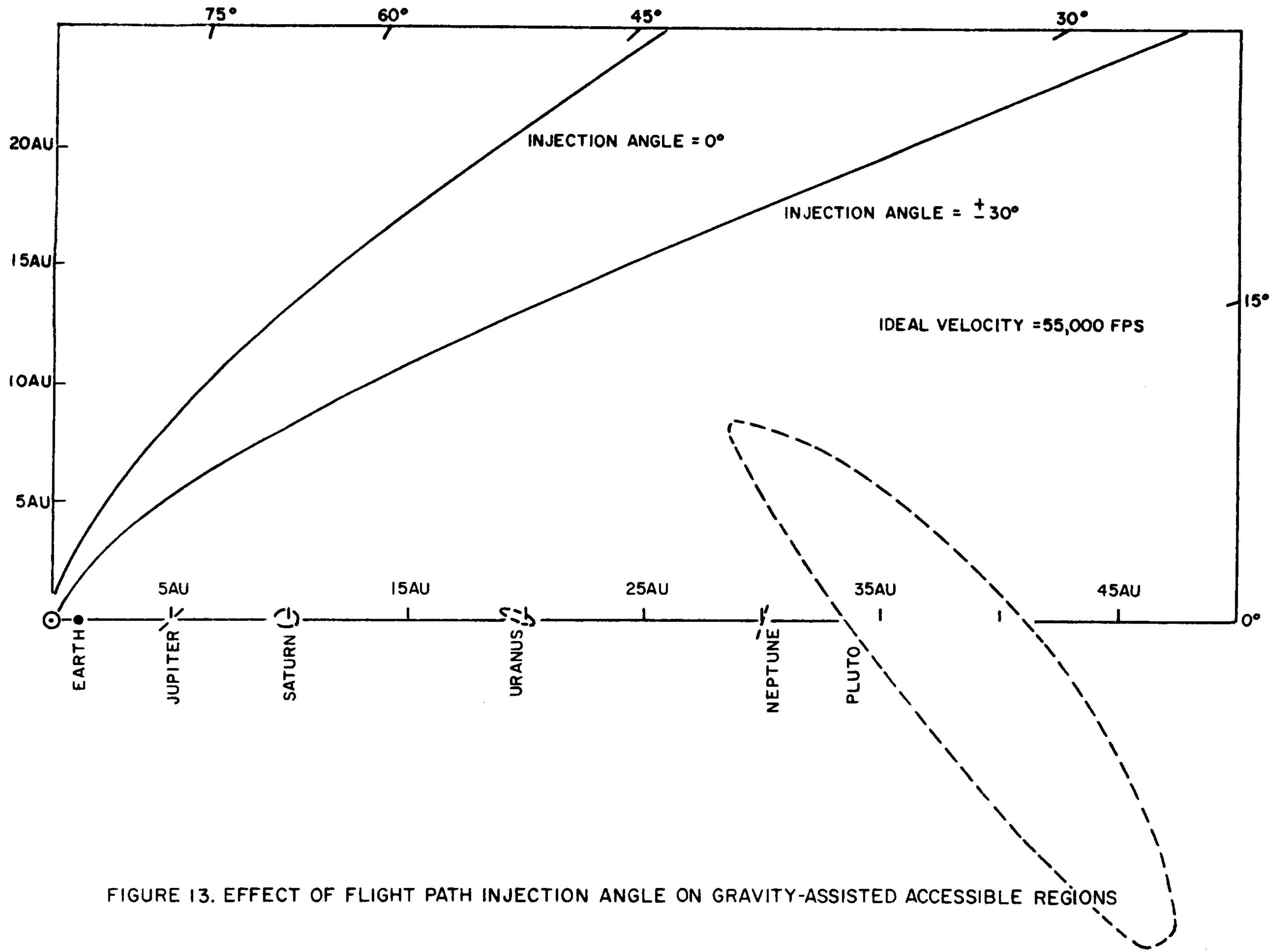

FIGURE 13. EFFECT OF FLIGHT PATH INJECTION ANGLE ON GRAVITY-ASSISTED ACCESSIBLE REGIONS

start the post-Jupiter phase with heliocentric velocities nearly normal to the ecliptic plane. An example of this sensitivity is shown in Figure 14, which shows the effect of small changes in θ on the post-Jupiter trajectory. Similarly, a small change in the Jupiter miss distance can have significant effects on the post-Jupiter trajectory. For example, increasing the miss distance from one Jupiter radii to 1.2 Jupiter radii (an increase of 14,270 kilometers) raises the Z-axis intercept of the post-Jupiter trajectory envelope shown in Figure 14 from 5.3 AU to 6.5 AU. This sensitivity of the post-Jupiter trajectories to details of the gravity-assist maneuver has important consequences in the guidance requirements for some types of missions.

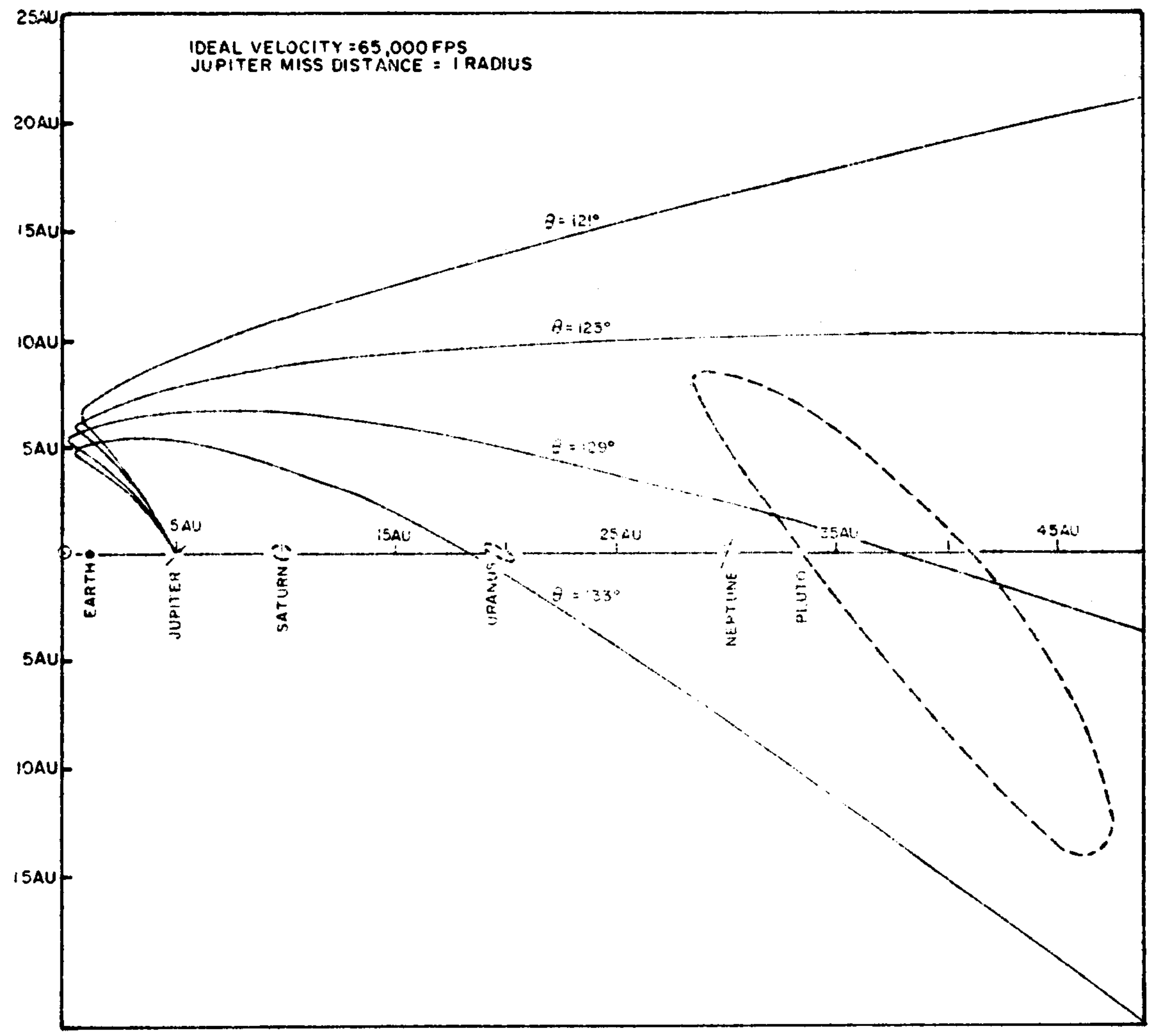

FIGURE 14. SENSITIVITY OF POST- JUPITER TRAJECTORY

4. RECOMMENDATIONS

This report has dealt only with Jupiter-assisted missions. The accessible regions method should be applied to the gravity-assisted trajectories using planets other than Jupiter. Although some specific Venus-assisted missions, such as Earth-Venus-Mercury (Niehoff 1965, Cutting and Sturms 1964) and solar probes (Casal and Ross 1965), have been examined, no comprehensive study of Venus-assisted missions has been performed. The accessible regions method is especially useful for such a study, and should be applied to Venus-assisted trajectories. Although a Mars-assisted mission from Earth to Jupiter (Niehoff 1964) appears to offer no advantages over a direct flight, there may be some class of missions which are enhanced by a Mars-assist, e.g., asteroid missions. Application of the accessible regions method to Mars-assisted trajectories would enable the mission planner to delineate clearly those missions which are aided by Mars-assist from those which are not.

Figure 14 and other data, have shown that some trajectories are particularly sensitive to conditions, such as Jupiter miss distance, of the gravity-assist phase. This sensitivity may have a profound influence on the guidance requirements for certain types of missions. Data presentation methods which portray guidance requirements, launch time constraints, or intercept time constraints should be studied.

REFERENCES

Casal, F. G. and Ross, S., "The Use of Close Venusian Passage During Solar Probe Missions," American Astronautical Society Symposium on Unmanned Exploration of the Solar System, February 8-10, 1965, Preprint No. 65-31.

Cutting, E. and Sturms, F. M., Jr., "Trajectory Analysis of a 1970 Mission to Mercury via a Close Encounter with Venus," JPL TM No. 312-505, December 7, 1964.

Friedlander, A. F., "Low-Thrust Trajectory and Payload Analysis for Solar System Exploration Utilizing the Accessible Regions Method," ASC/IITRI Report No. T-14, 1965.

Friedlander, A. F., "Low-Thrust Trajectory Capabilities for Exploration of the Solar System Using Nuclear Electric Propulsion," ASC/IITRI Report No. T-17, 1966.

Hunter, II, Maxwell, W., "Future Unmanned Exploration of the Solar System," Astronautics and Aeronautics, p. 16, May 1964.

Narin, F., "The Accessible Regions Method of Energy and Flight Time Analysis for One-Way Ballistic Interplanetary Missions," ASC/IITRI Report No. T-6, 1964.

Niehoff, J., "An Analysis of Gravity-Assisted Trajectories in the Ecliptic Plane," ASC/IITRI Report No. T-12, 1965.

Porter, R. F., Luce, R. G., and Edgecombe, D. S., "Gravity-Assisted Trajectories for Unmanned Space Exploration," Battelle Memorial Institute, Report No. BMI-NVLP-FTR-65-1, 1965.