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The Effects of Reinforcement Interval on the Acquisition of Paired-Associate Responses[1]

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Abstract

The length of the reinforcement interval (RI) in paired-associate learning was studied using a within-subjects design to eliminate confounding of presentation rate with the time between successive presentations of items. Forty Ss were run for fifteen trials on a 24-item list with RI's of $\frac{1}{2}$, 1, 2, and 4 sec. Results indicated: (a) mean errors were a decreasing function of RI; (b) mean errors for items meeting a criterion were not related to RI, but the proportion of items meeting criterion was an increasing function of RI; (c) precriterion mean latencies increased slightly for both correct and incorrect responses, whereas postcriterion latencies decreased; (d) the proportion of correct responses decreased as the number of intervening items increased, but the latency measure showed no effect.

Several alternative models dealing with RI effects are proposed and evaluated against these data. None of the models prove entirely satisfactory.

The length of the reinforcement interval in paired-associate learning has been an experimental variable in recent studies by Nodine (1963, 1965); Bugelski (1962); Bugelski & Rickwood (1963); Murdock (1965); Newman (1964); and Keppel & Rehula (1965). Most of these studies used the anticipation method which partitions an item presentation into the following intervals: (1) the stimulus-alone interval (St) during which the S is required to respond; (2) the reinforcement interval (RI) during which the stimulus and response members are presented together; (3) the interstimulus interval (ISI) during which nothing is presented.

The typical experimental design used to study the length of the reinforcement interval assigns a different value of RI to independent groups of Ss and then compares learning measures across the groups. Evaluation of data obtained using this design suffers from the fact that other variables are inseparably confounded with the effects of RI. Specifically, the total time to complete one presentation cycle of the list and the time between successive presentations of the same item are both confounded with the length of RI.

The present study eliminates the confounding of RI with other temporal variables by using a within-subjects design. RI's of $\frac{1}{2}$, 1, 2, and 4 sec. were assigned to four subsets of items with six items in each subset. On each trial the entire list of 24 items was presented to the S in a new random order. Consequently, the time required for a trial (i.e., one cycle through the list), and the average time between one presentation of an item and its next presentation are constant.

A second variable manipulated in this study was concerned with the effect of always presenting the same RI for an item versus the effect of randomly changing RI's from one presentation of the item to the next. Two conditions were used: one where the RI assigned to an item remained the same throughout the session, and one where the RI for an item was randomly assigned on each trial. This second independent variable was also handled so that a within-subjects comparison could be made.

The theoretical analysis of the data will deal primarily with an evaluation of assumptions concerning the effects of manipulating RI; these assumptions will be incorporated into existing versions of both incremental and discrete-process models for paired-associates learning.

Method

Design. Each S learned a list of 24 paired-associate items. The main independent variable was the length of RI; four values were used. For each S three items were assigned to each of the four values of RI, and will be designated $F(\frac{1}{2})$, $F(1)$, $F(2)$, and $F(4)$ to indicate that these subsets of items had fixed RI's of $\frac{1}{2}$, 1, 2, and 4 sec., respectively. The assignment of RI's for these 12 items remained fixed throughout the session. For the remaining 12 items the RI assignments were variable; i.e., for these items the RI's were reassigned randomly at the start of each trial with the restriction that each of the four RI's were assigned to exactly three of these 12 items. Thus, on every trial, each of the four values of RI always occurred with six items, three were fixed assignments and three were variable assignments.

Subjects. Forty Stanford University students from an introductory psychology class were used. They were either paid $2.00 or given credit toward a course requirement.

Materials. The stimuli used were two-digit numbers, and the responses were the letters A, B, and C. For each S 24 stimuli were randomly selected from a master pool of 38 stimuli which was constructed by the following procedure: (1) the fifty two-digit numbers with the lowest association values as described by Battig & Spera (1962) were chosen. (2) Double numbers (i.e., 11, 22, 33, ...) and numbers with consecutive digits (i.e., 12, 23, 34, ...) were eliminated reducing the sample to 44 numbers ranging from 26 to 97. (3) The largest six of these were always used as stimuli in a practice session leaving 38 numbers (26-87) available for the learning session. The stimuli and responses were drawn with black ink on a white background, photographed on microfilm, and projected on a ground glass screen during the experiment. The letters and numbers appeared as lighted figures, $\frac{3}{4}$ in. high, on a dark gray background.

Apparatus. The experiment was conducted in the Computer-Based Learning Laboratory at Stanford University. The control functions were performed by computer programs running in a modified PDP-1 computer manufactured by the Digital Equipment Corp., and under control of a time-sharing system. The S was seated at an IBM microfilm display terminal (IBM 9405). There were six terminals located in individual 7×8 ft. sound-shielded rooms. Elements of the display appeared in the following positions on a 10×13 in. ground-glass screen: (1) the stimulus was $2\frac{1}{2}$ in. from the left edge and $4\frac{1}{2}$ in. from the top edge. (2) The response areas were $\frac{1}{2}$ in. from the left and $7\frac{1}{2}$ in. from the top and consisted of a row of three boxes, 1 in. square, $\frac{1}{2}$ in. apart, which contained the letters A, B, and C, respectively. (3) The response member of the reinforcement appeared 5 in. to the right of the stimulus. (4) When used, the comment, "Please make response," was centered $1\frac{1}{2}$ in. from the top of the screen.

Responses were made by touching one of the three response boxes with a light pen. Due to the mechanical operations involved in executing slide changes there was a moderate amount of noise during the ISI and a slight noise from a fan during the entire session.

Procedure. The Ss arrived in groups of one to four and were taken as a group into one of the six booths. Instructions were read to them explaining that they were to learn a list of number-letter pairs. They were shown where the stimuli would appear on the screen and how responses were to be made. Then 12 practice items were run for the group illustrating the presentation sequence and giving each of them an opportunity to make a few practice responses with the light pen. After questions about procedures were answered each S was assigned to a booth and the session of 360 item presentations began, i.e., 15 trials of the 24-item list. For each S the computer program performed the functions of randomly selecting stimuli, assigning stimuli to fixed and variable conditions, and assigning responses to stimuli, as well as randomizing the order of the list on every trial. The format for each item was the same except for the length of the RI. The stimulus appeared on the screen and remained on until the response was made, with the exception that if the response did not occur in 3.6 sec., the stimulus was removed and the statement, "Please make response," appeared and remained until the response occurred. After the response was made the stimulus and response members of the pair appeared on the screen for the appropriate RI. Then there was an ISI of 2 sec. during which the computer selected the slide for the next item. The computer program serviced each S individually even though more than one S ran simultaneously. It should be noted that the response reminder was rarely displayed after the practice session during which the Ss became accustomed to the presentation rate.

Results

Overall performance. Figure 1 presents the mean total errors per item for each of the experimental variables. For the four subsets of fixed-assignment items the mean total errors are a decreasing function of RI (upper curve in Fig. 1); these differences are highly reliable [$F(4,39) = 3.71$, $p < .025$ for a treatments-by-subjects analysis of variance]. However, the mean number of errors over all fixed items versus variable items (5.8 and 5.9, respectively) is not significant using a paired t-test [$t(39) = 1.28$].

The learning curves presented in Fig. 2 support the results obtained for mean total errors. The curves for the fixed and variable conditions are very close to each other throughout the session; for both curves the proportion of correct responses increases from about .33 to .80 over the 15 trials. Although they are not presented, the learning curves for the four fixed-interval conditions tend to be arranged in order of increasing RI, but there is some overlapping of points over the 15 trials.

For items with variable RI assignments another analysis is needed to demonstrate the effect of RI. We considered the proportion of correct responses conditional on the fact that a specific RI was presented on the previous presentation of the item and combined these over trials and items. The conditional proportions were computed separately for certain events

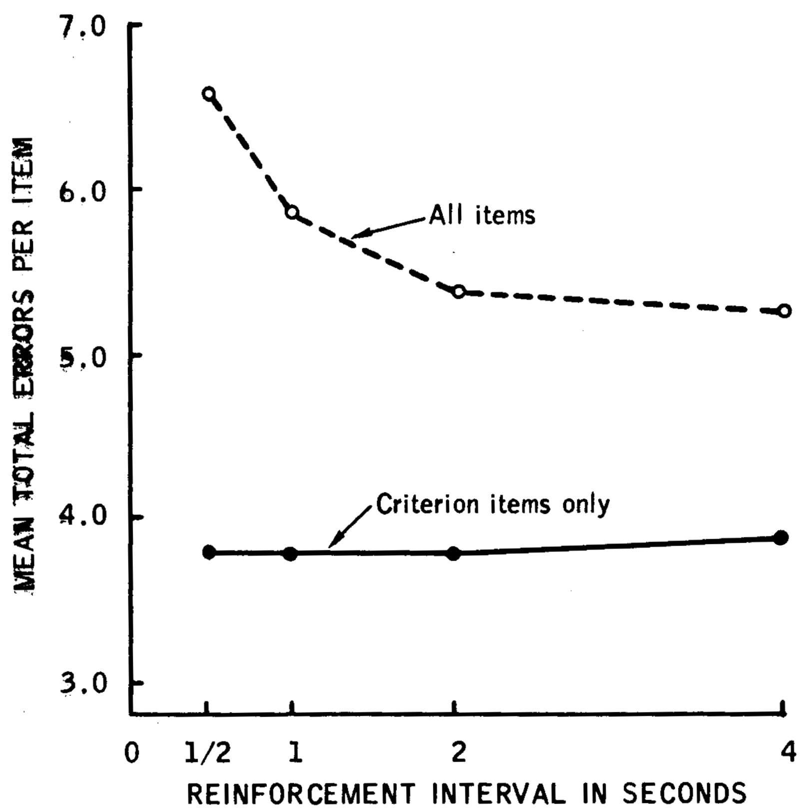

Figure 1. Mean total errors as a function of duration of the reinforcement interval plotted separately for all items and for items meeting a criterion of five consecutive correct responses.

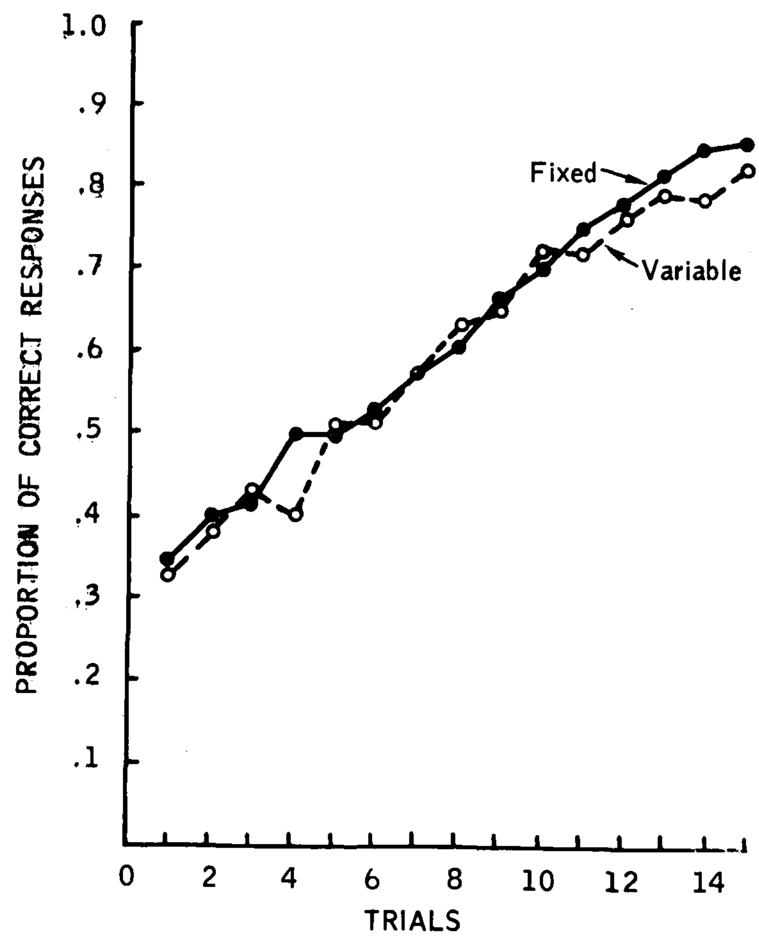

Figure 2. Mean learning curves for the overall fixed and variable conditions.

occurring on the previous trial which included the four RI's, the fixed versus variable conditions, and correct versus incorrect responses; they are displayed in Fig. 3. The proportion of correct responses is an increasing function of RI when the previous response was incorrect for both the fixed and the variable items. When the previous response was correct the RI's have less effect for the fixed items and almost no effect for the variable items. When the response on trial n is ignored and only the RI is considered we obtain the two curves in the center section of Fig. 3, which for the fixed items again indicate that proportion correct is an increasing function of the RI on the previous trial. The corresponding curve for the variable items indicates less effect of RI with possibly only the 4-sec. interval being better than the other three RI's.

Criterion analysis. Since the sessions were terminated after 15 trials a learning criterion of five consecutive correct responses was subsequently applied to each item. The proportion of items meeting the criterion was .625 and .637 respectively, for the variable and fixed conditions. However, the four RI conditions are not equally represented in the overall fixed condition since the proportions of items meeting criterion for $F(\frac{1}{2})$, $F(1)$, $F(2)$, and $F(4)$ were .52, .60, .68, and .74, respectively. When we consider only precriterion trials the proportions of errors are .65 and .63 based on 3554 and 3504 observations for the fixed and variable conditions, respectively. But for 1737 and 1734 observations which occurred after the criterion run, the corresponding error proportions are .052 and .054. While we showed earlier that mean total errors was a decreasing function of RI, further analysis shows a flat curve (see the lower curve of Fig. 1) when only criterion items

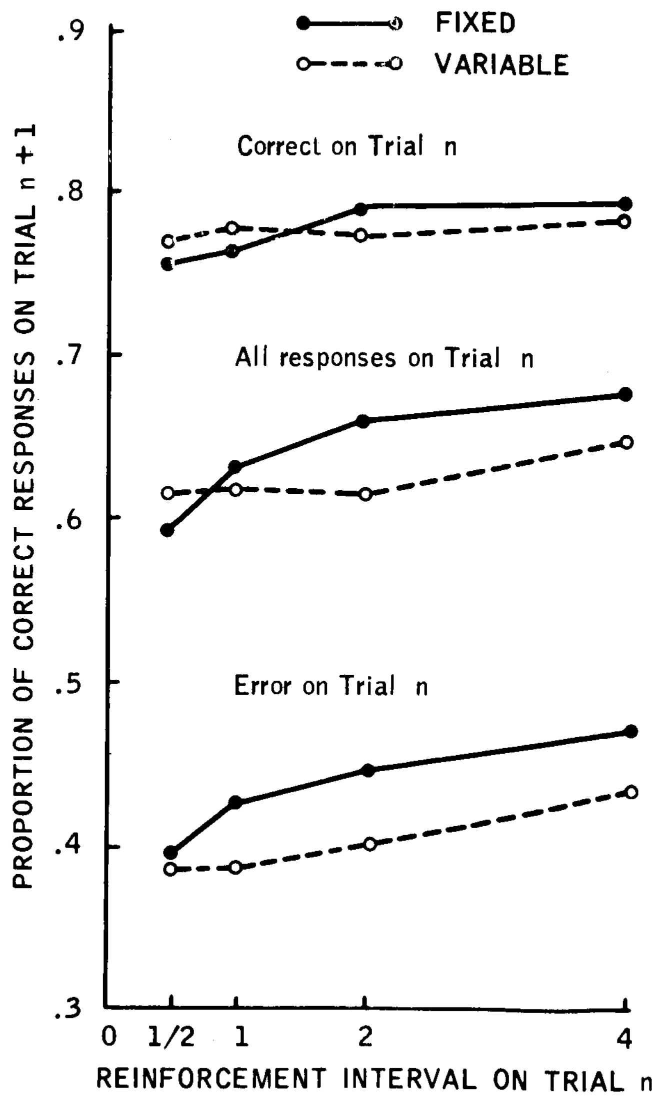

Figure 3. Proportions of correct responses conditional on the reinforcement interval on the previous trial, and conditional on whether the previous response was correct or incorrect.

are considered, indicating that for these items there is little effect of RI on performance.

The mean latency curves also tend to support the separation of item protocols into pre- and postcriterion trials. Figure 4 displays trial-by trial mean latencies separately for the fixed and variable items, where the upper curves in each panel are based on trial 1 to the trial of last error for all items. For the lower curves we renumber the trials beginning with the first trial of the criterion run for those items which met criterion. Latencies for precriterion trials for both correct responses and errors are similar to each other, and tend to increase with trials; however, latencies for correct responses in the postcriterion trials gradually decrease to about 1.5 sec.

An analysis suggested by Suppes and Ginsberg (1963) to evaluate response stationarity in the precriterion trials involves splitting the protocols into four equal Vincent quartiles. For each item, the response protocol after trial 1 and before the last error in the sequence was divided into quartiles. As shown in Fig. 5, the proportion correct is fairly stationary in the first three quartiles, but in the fourth quartile it increases for both the fixed and variable conditions.

Analysis of intervening items. One source of forgetting may be due to the amount of activity required of *S* between successive presentations of an item. When an entire list of 24 items is randomly presented in a complete cycle, the number of other items which may intervene between two successive occurrences of a given item will range from 0 to 46. If all items were independent and no time-dependent forgetting occurred we would

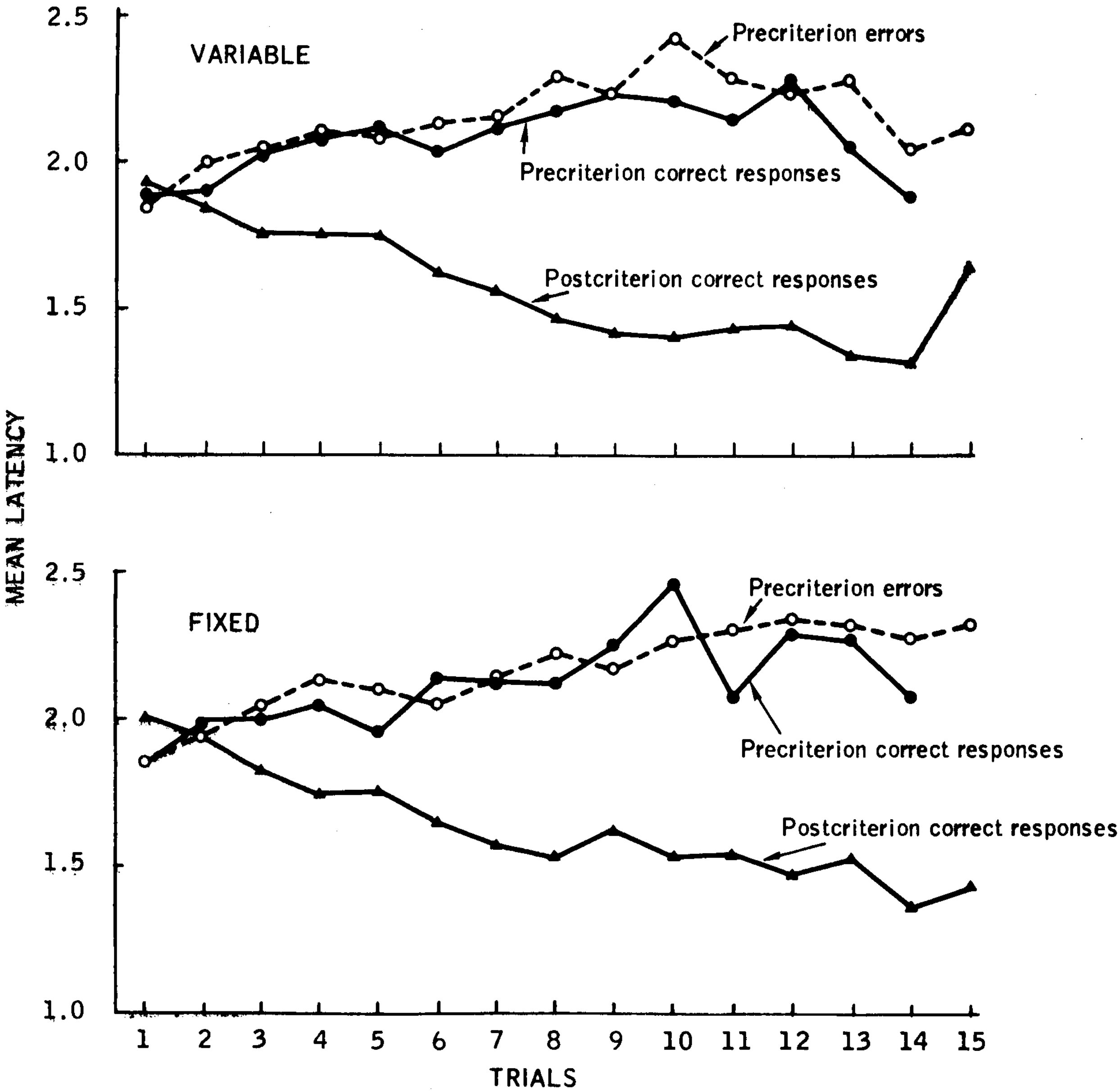

Figure 4. Mean latencies for precriterion trials (trial 1 to the last error before a run of 5 consecutive corrects) and postcriterion trials (the first trial of the criterion run is renumbered as trial 1).

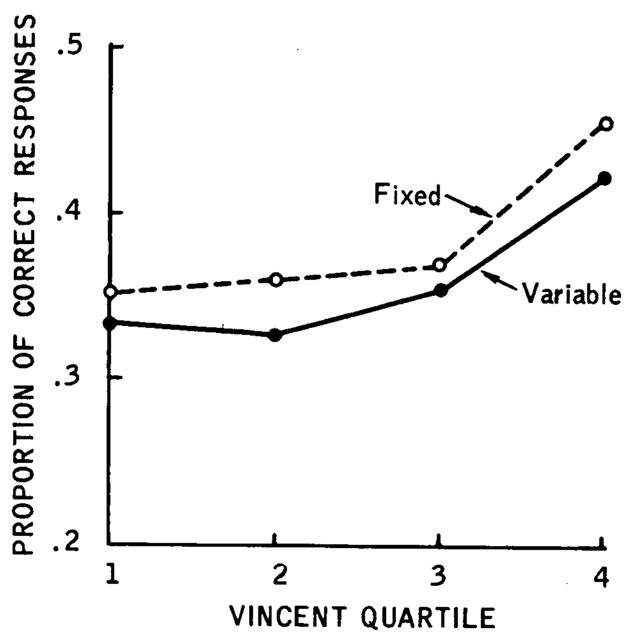

Figure 5. Proportion of correct responses for precriterion sequences divided into Vincent quartiles.

expect that the number of intervening items would not affect the probability that an item is correct. Figure 6 presents the proportion of correct responses on trial n for a given item as a function of the number of intervening items since its presentation on trial $n-1$. Each of the curves shows decreasing proportions of correct responses as the number of intervening items increases. We might also expect some change in mean latency as a function of the number of intervening items but as indicated in Fig. 7, there is almost no effect for either correct or incorrect responses.

Nonindependence of successive items. In an earlier analysis we examined the effect of a particular RI on the response to the same item on the next trial. In this analysis we consider the effect of a particular RI on the very next item presented, and find that there seems to be no effect on the likelihood of a correct response; the proportions correct were .613, .608, .607, and .610, given that the RI's on the previously presented items were $\frac{1}{2}$, 1, 2, and 4 sec. However, the mean latencies show reliable effects for both correct and incorrect responses. In Fig. 8 we see that mean latency is an increasing function of the length of RI on the previous item. This increasing function suggests that _S_ was optimally ready to respond to the next stimulus presentation when the preceding RI was $\frac{1}{2}$ sec., but the longer RI's may have initiated processes that continued into the stimulus interval of the next item.

Discussion

We shall analyze these data in terms of two fairly simple models that have been proposed to account for paired-associate learning: the linear model (Bush & Mosteller, 1955; Sternberg, 1959) and the one-element

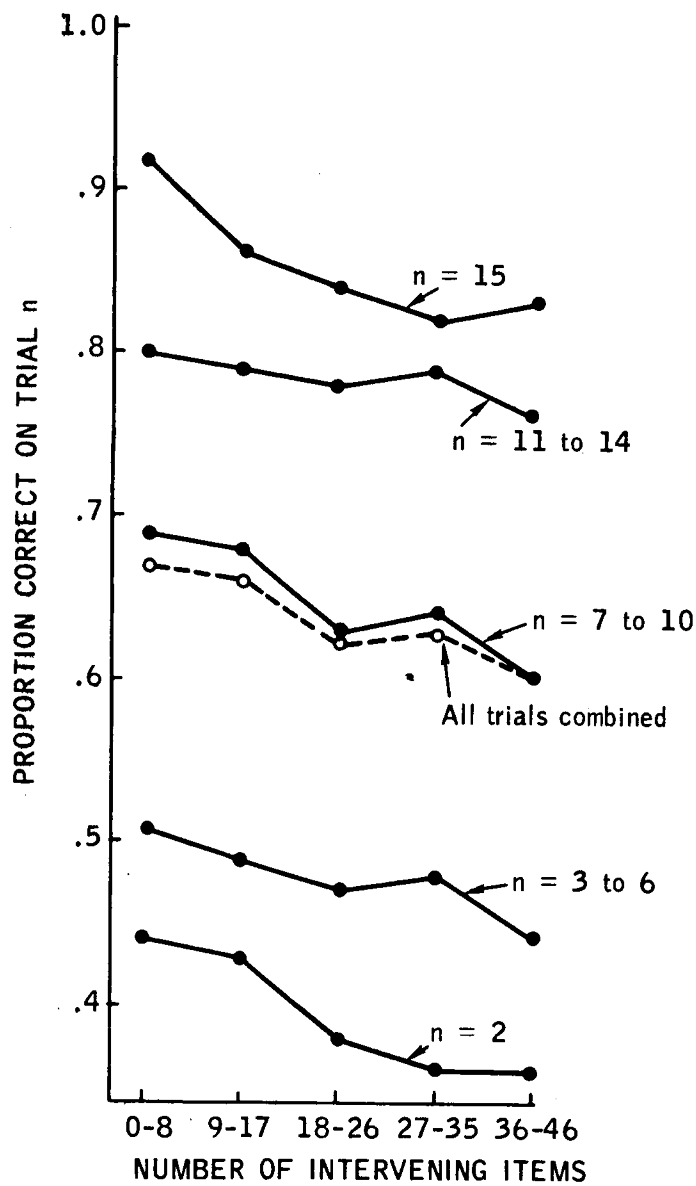

Figure 6. Proportion of correct responses on trial n as a function of the number of intervening items between trial n-1 and n.

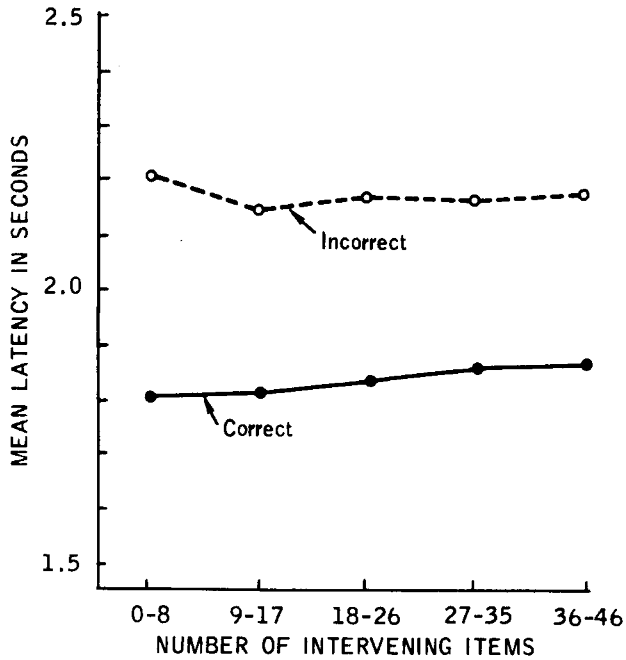

Figure 7. Mean latency on trial n as a function of the number of intervening items between trial n-1 and trial n.

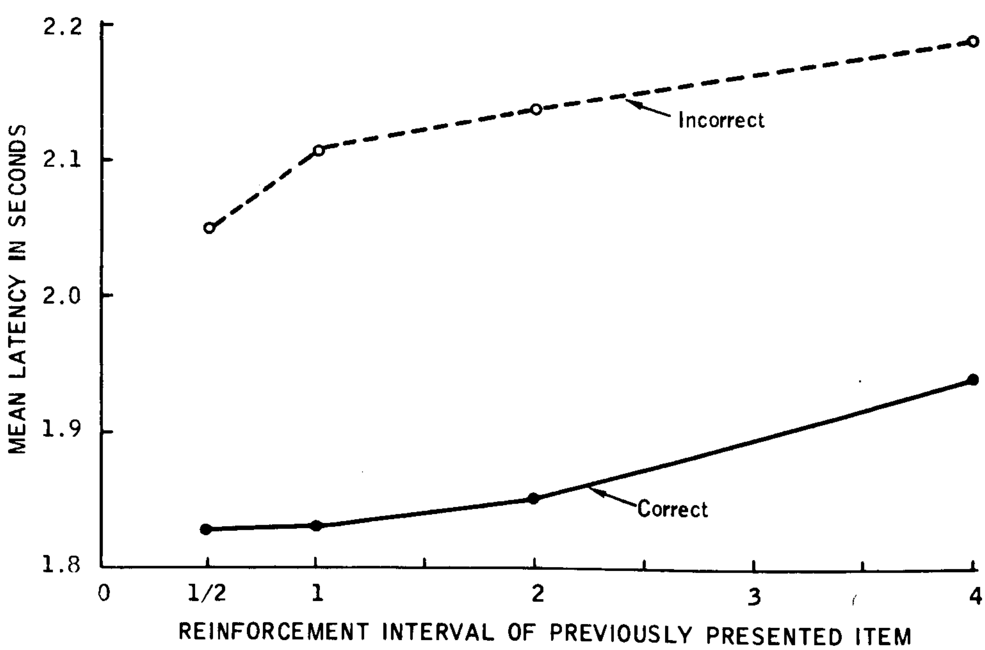

Figure 8. Mean latency as a function of the RI associated with the previous item.

model (Bower, 1961; Estes, 1959). The linear model assumes that the effect of each reinforcement is to add an increment to the strength of the association between the stimulus and the correct response. If we let p_n denote the probability of a correct response on the n^{th} presentation of a given stimulus item, then the linear model postulates that

$$p_{n+1} = (1-\theta)p_n + \theta$$

where p_1 is the initial guessing probability (which is $\frac{1}{3}$ in our experiment). The one-element model assumes that learning for any given stimulus item proceeds in an all-or-none fashion; the item is either in a learned state (where performance is perfect) or in an unlearned state (where performance is at a chance level). Stated more precisely, the one-element model assumes that

$$p_{n+1} = \begin{cases} 1 & \text{, with probability } c \\ p_n & \text{, with probability } 1-c \end{cases}$$

where again $p_1 = \frac{1}{3}$. Thus, response probability starts out at $\frac{1}{3}$, remains at that value for a series of presentations, and then jumps to one for the remaining trials. A more precise characterization of these two models can be found in Atkinson, Bower, & Crothers (1965, Ch. 3).

The models being considered make no explicit assumptions concerning the effect of RI on learning. One approach is to quantize time and express each RI as a fixed number of base-time units. If we assume that during each time unit a learning operator characterized by the parameter a is applied, then the parameter characterizing the effect of a reinforcement interval of time t, which is made up of m time units, is

$$a_t = a + a(1-a) + a(1-a)^2 + \cdots + a(1-a)^{m-1} . \quad (1)$$

We shall refer to the parameter a as the learning parameter, and it is to be interpreted as c in the one-element model, and θ in the linear model. Using Eq. 1 with a base-time unit of $\frac{1}{2}$ sec., the parameters associated with the fixed reinforcement intervals of $\frac{1}{2}$, 1, 2, 4 sec. and with the variable reinforcement condition are as follows:

$$\begin{aligned}
a_{\frac{1}{2}} &= a \\
a_1 &= a + a(1-a) \\
a_2 &= a + a(1-a) + a(1-a)^2 + a(1-a)^3 \\
a_4 &= a + a(1-a) + a(1-a)^2 + \cdots + a(1-a)^7 \\
a_v &= \tfrac{1}{4}[a_{\frac{1}{2}} + a_1 + a_2 + a_4] . \qquad (2)
\end{aligned}$$

Equations 1 and 2 assume that the learning operator applies uniformly over all time units. However, it is possible that there is some attenuation in the effectiveness of conditioning in the later parts of the longer RI's. To take this possibility into account we introduce an attenuation parameter, d, in the expressions for a_i; namely

$$\begin{aligned}
a_{\frac{1}{2}} &= a \\
a_1 &= a + (1-a)ad \\
a_2 &= a + (1-a)ad + (1-a)(1-ad)ad^2 + (1-a)(1-ad)(1-ad^2)ad^3 \\
a_4 &= a + (1-a)ad + (1-a)(1-ad)ad^2 + \cdots \\
&\quad + [(1-a)(1-ad)(1-ad^2)\cdots(1-ad^6)ad^7] . \qquad (3)
\end{aligned}$$

When d approaches one, the above equations reduce to those in Eq. 2; when d approaches zero, the expressions approach a common value, a, implying that learning is not affected differentially by the RI duration.

Another extension of this line of argument involves the introduction of a parameter, x, to allow for an estimate of learning during the ISI. Since all items, independent of RI's, have the same ISI only a single value of x is required; hence

$$a_i' = a_i + (1 - a_i)x \ , \tag{4}$$

for $i = \frac{1}{2}, 1, 2, 4, v$. In summary, the parameters a, d, and x are used to characterize the reinforcement effects; a is the learning parameter applied in each time unit of RI, d allows for attenuation in successive time units of RI, and x is applied during the ISI.

Parameter estimates for the linear and one-element models were obtained by using the chi-square minimization procedure described by Atkinson & Crothers (1964). The data used were the four-tuples of successes and errors from trials 2 through 5, 6 through 9, and 10 through 13. Following the notation of Atkinson and Crothers, let 0 denote a correct response and 1 an error. Define $O_{i,j,n}$ as the four-tuple response sequence listed in the i^{th} row of the data tables (see Tables 1, 2, and 3), for RI condition j ($j = \frac{1}{2}, 1, 2, 4, v$) where the sequence begins on trial n (in our analysis $n = 2, 6,$ and 10). Further, let $\hat{N}(O_{i,j,n})$ be the observed frequency of the four-tuple, and $Pr(O_{i,j,n};p)$ be the predicted probability given a particular choice of the parameter vector p of the model. The expected frequency may be obtained by taking the product of $Pr(O_{i,j,n};p)$ and T, the total number of item protocols for a given RI condition. The function

$$\chi^2_{i,j,n} = \frac{[T\,Pr(O_{i,j,n};p) - \hat{N}(O_{i,j,n})]^2}{T\,Pr(O_{i,j,n};p)} \ , \tag{5}$$

is a measure of the discrepancy between the predicted and observed frequencies for a particular four-tuple. A measure of the discrepancy

Table 1

Observed and Predicted Frequencies for Four-tuples of Response

Sequences from Trials 2 through 5

(L denotes the predicted column for the linear model; O-E denotes the one-element model)

Trial	$F(\frac{1}{2})$			F(1)			F(2)			F(4)			Variable		
2345	Obs.	L	O-E	Obs.	L	O-E	Obs.	L	O-E	Obs.	L	O-E	Obs.	L	O-E
0000	8.0	4.0	10.5	12.0	4.5	12.7	14.0	5.0	13.3	11.0	5.2	13.7	43.0	18.6	49.7
0001	4.0	4.3	2.4	5.0	4.5	2.5	6.0	4.6	2.2	2.0	4.7	2.2	12.0	18.1	9.0
0010	3.0	4.9	2.8	8.0	5.2	2.7	4.0	5.4	2.7	4.0	5.5	2.7	19.0	21.1	10.9
0011	8.0	5.3	4.7	5.0	5.2	4.6	2.0	5.0	4.4	4.0	5.0	4.4	27.0	20.5	18.1
0100	11.0	5.7	4.1	4.0	6.1	4.2	4.0	6.5	4.3	2.0	6.7	4.4	16.0	25.0	17.0
0101	4.0	6.1	4.7	6.0	6.1	4.6	5.0	6.0	4.4	4.0	6.0	4.4	8.0	24.3	18.1
0110	4.0	7.0	5.6	7.0	7.1	5.5	4.0	7.1	5.4	8.0	7.1	5.4	25.0	28.3	21.9
0111	7.0	7.5	9.5	5.0	7.1	9.1	5.0	6.6	8.8	10.0	6.4	8.7	33.0	27.5	36.2
1000	7.0	6.7	8.2	9.0	7.3	9.0	12.0	7.9	9.6	13.0	8.2	9.8	31.0	30.1	36.6
1001	2.0	7.2	4.7	.0	7.3	4.6	6.0	7.4	4.4	5.0	7.4	4.4	17.0	29.2	18.1
1010	6.0	8.3	5.6	4.0	8.5	5.5	4.0	8.7	5.4	7.0	8.7	5.4	27.0	34.1	21.9
1011	3.0	8.8	9.5	11.0	8.5	9.1	7.0	8.0	8.8	5.0	7.8	8.7	31.0	33.2	36.2
1100	12.0	9.6	8.1	11.0	9.9	8.4	9.0	10.3	8.7	9.0	10.5	8.8	33.0	40.4	34.1
1101	4.0	10.2	9.5	11.0	9.9	9.1	13.0	9.6	8.8	16.0	9.5	8.7	32.0	39.3	36.2
1110	8.0	11.7	11.1	7.0	11.5	11.0	9.0	11.3	10.9	5.0	11.2	10.8	51.0	45.8	43.8
1111	29.0	12.5	19.0	15.0	11.6	18.2	16.0	10.5	17.6	15.0	10.1	17.5	75.0	44.5	72.3
	X^2	48.47	34.40	X^2	29.59	24.63	X^2	31.65	14.99	X^2	30.24	16.65	X^2	82.87	23.26

Table 2

Observed and Predicted Frequencies for Four-tuples of Response

Sequences from Trials 6 through 9

(L denotes the predicted column for the linear model; O-E denotes the one-element model)

Trial	$F(\frac{1}{2})$			F(1)			F(2)			F(4)			Variable		
6789	Obs.	L	O-E	Obs.	L	O-E	Obs.	L	O-E	Obs.	L	O-E	Obs.	L	O-E
0000	26.0	11.6	32.3	31.0	13.6	36.9	37.0	16.0	40.6	43.0	17.2	41.5	145.0	58.1	151.6
0001	6.0	7.7	1.9	9.0	8.1	1.8	1.0	8.5	1.6	1.0	8.7	1.6	14.0	33.2	6.9
0010	6.0	8.6	2.2	.0	9.2	2.1	9.0	9.7	2.0	6.0	9.9	2.0	18.0	37.5	8.3
0011	5.0	5.8	3.8	4.0	5.5	3.5	3.0	5.2	3.3	5.0	5.0	3.2	12.0	21.5	13.8
0100	7.0	9.7	3.3	4.0	10.4	3.3	3.0	11.1	3.2	2.0	11.4	3.2	22.0	42.7	13.0
0101	2.0	6.5	3.8	6.0	6.2	3.5	5.0	5.9	3.3	2.0	5.8	3.2	8.0	24.4	13.8
0110	1.0	7.2	4.5	5.0	7.0	4.2	6.0	6.7	4.0	7.0	6.6	4.0	10.0	27.6	16.7
0111	4.0	4.8	7.6	3.0	4.2	7.0	1.0	3.6	6.6	3.0	3.3	6.5	19.0	15.8	27.6
1000	10.0	10.9	6.5	9.0	11.8	6.9	8.0	12.8	7.2	11.0	13.2	7.2	36.0	48.8	27.9
1001	3.0	7.3	3.8	2.0	7.1	3.5	6.0	6.8	3.3	5.0	6.7	3.2	14.0	27.9	13.8
1010	3.0	8.1	4.5	3.0	8.0	4.2	2.0	7.7	4.0	7.0	7.6	4.0	22.0	31.6	16.7
1011	5.0	5.4	7.6	4.0	4.8	7.0	5.0	4.1	6.6	1.0	3.8	6.5	15.0	18.1	27.6
1100	9.0	9.1	6.5	15.0	9.0	6.5	6.0	8.9	6.5	9.0	8.7	6.5	33.0	35.9	26.0
1101	7.0	6.1	7.6	6.0	5.4	7.0	2.0	4.7	6.6	.0	4.4	6.5	30.0	20.5	27.6
1110	10.0	6.8	8.9	7.0	6.1	8.4	8.0	5.4	8.1	8.0	5.0	8.0	29.0	23.2	33.4
1111	16.0	4.5	15.2	12.0	3.7	14.0	18.0	2.9	13.1	10.0	2.6	12.9	53.0	13.3	55.2
	X^2	65.19	30.94	X^2	67.64	52.13	X^2	133.34	40.07	X^2	88.17	32.26	X^2	327.28	45.58

Table 3

Observed and Predicted Frequencies for Four-tuples of Response

Sequences from Trials 10 through 13

(L denotes the predicted column for the linear model; O-E denotes the one-element model)

Trial	$F(\frac{1}{2})$			F(1)			F(2)			F(4)			Variable		
10,11, 12,13	Obs.	L	O-E	Obs.	L	O-E	Obs.	L	O-E	Obs.	L	O-E	Obs.	L	O-E
0000	53.0	22.4	49.8	60.0	26.4	56.0	71.0	31.3	60.9	67.0	33.6	62.1	265.0	113.4	229.4
0001	3.0	10.1	1.5	2.0	10.4	1.4	5.0	10.5	1.2	7.0	10.5	1.2	6.0	41.7	5.3
0010	4.0	11.1	1.8	2.0	11.5	1.6	7.0	11.7	1.5	3.0	11.8	1.5	17.0	46.3	6.4
0011	3.0	5.0	3.0	2.0	4.5	2.7	.0	3.9	2.4	1.0	3.7	2.4	7.0	17.1	10.5
0100	5.0	12.2	2.6	6.0	12.7	2.5	4.0	13.1	2.4	5.0	13.2	2.4	25.0	51.6	9.9
0101	5.0	5.5	3.0	2.0	5.0	2.7	.0	4.4	2.4	1.0	4.1	2.4	12.0	19.0	10.5
0110	1.0	6.0	3.6	1.0	5.5	3.3	3.0	4.9	3.0	2.0	4.6	2.9	9.0	21.1	12.7
0111	2.0	2.7	6.1	2.0	2.2	5.4	3.0	1.6	4.9	4.0	1.5	4.8	10.0	7.8	21.1
1000	6.0	13.5	5.2	11.0	14.2	5.3	10.0	14.8	5.3	12.0	15.0	5.3	26.0	57.8	21.3
1001	4.0	6.1	3.0	1.0	5.6	2.7	1.0	5.0	2.4	.0	4.7	2.4	8.0	21.3	10.5
1010	8.0	6.7	3.6	2.0	6.1	3.3	3.0	5.5	3.0	4.0	5.2	2.9	6.0	23.6	12.7
1011	4.0	3.0	6.1	4.0	2.4	5.4	.0	1.9	4.9	2.0	1.6	4.8	9.0	8.7	21.1
1100	10.0	7.3	5.2	8.0	6.8	5.0	3.0	6.2	4.8	4.0	5.9	4.8	12.0	26.3	19.8
1101	4.0	3.3	6.1	2.0	2.7	5.4	1.0	2.1	4.9	2.0	1.8	4.8	14.0	9.7	21.1
1110	3.0	3.6	7.1	6.0	3.0	6.5	4.0	2.3	6.0	4.0	2.1	5.9	23.0	10.8	25.5
1111	5.0	1.6	12.2	9.0	1.2	10.8	5.0	.8	9.8	2.0	.6	9.5	31.0	4.0	42.1
	X^2	74.36	30.80	X^2	132.45	21.91	X^2	105.76	56.72	X^2	67.30	56.15	X^2	529.15	75.35

between observed and predicted frequencies for RI condition j is found by summing Eq. 5 over the 16 possible four-tuples and the three sets of trials, i.e.,

$$\chi_j^2(a,d,x) = \sum_{i=1}^{16} \chi_{i,j,2}^2 + \sum_{i=1}^{16} \chi_{i,j,6}^2 + \sum_{i=1}^{16} \chi_{i,j,10}^2 \; . \tag{6}$$

Note that this equation generates a χ^2 value for any set of parameters a, d, and x that we choose. Hence we can minimize $\chi_j^2(a,d,x)$ with regard to these parameters to obtain an estimate of a, d, and x for condition j. However, we would prefer a single estimate of a, d, and x obtained simultaneously over the five RI conditions. To do this we define the function,

$$\chi^2(a,d,x) = \chi_{\frac{1}{2}}^2(a,d,x) + \chi_1^2(a,d,x) + \chi_2^2(a,d,x) + \chi_4^2(a,d,x) + \chi_V^2(a,d,x). \tag{7}$$

The minimization of Eq. 7 was carried out for the data presented in Tables 2, 3, and 4 by a computer program that searched a grid on the parameter space, yielding parameter estimates accurate to three decimal places. In evaluating the minimum of $\chi^2(a,d,x)$, note that each set of 16 success-error sequences yields 15 df (since the predicted frequencies are constrained to sum to the total number of observations); further, there are three sets of four-tuples and five different RI conditions. Hence, the total degrees of freedom is $15 \times 3 \times 5 = 225$, minus three for the number of parameters being estimated.[2] The parameter estimates for the linear and one-element models and the corresponding chi-squares are presented in Table 4. The predicted frequencies are presented in Tables 1, 2, and 3.

The estimates of d of .422 and .516 for the one-element and linear models, respectively, indicate that there is considerable attenuation in

Table 4

Parameter Estimates and Goodness-of-fit Measures for the One-Element and Linear Models

Model	Parameter	Estimate	Estimate (with d = 1)
One-element	c	.023	.008*
	d	.422	1.000**
	x	.031	.016
	χ^2	555.84	1255.84
Linear	θ	.016	.008*
	d	.516	1.000**
	x	.047	.008*
	χ^2	1813.47	10370.37

*Smallest value used by the minimization procedure

**Parameter held constant

the effectiveness of the longer reinforcement intervals. This result is supported by the large chi-squares shown in the second column of estimates, which were obtained by carrying out the minimization with d set equal to unity ($d = 1$ assumes no attenuation over successive time units). Estimates of x of .031 and .047 suggest a slight learning effect during the ISI. Since both the linear and the one-element models have the same number of estimated parameters, the chi-squares of 1813.47 and 555.84 indicate that the one-element model does a far better job. However, as indicated by the chi-square values, both models can be rejected on statistical grounds.

Predictions for separate RI conditions. We next estimate the parameters for the linear and one-element models separately from each of the five RI conditions in order to compare them with the modified versions of the models used in the previous section. We also applied the random-trials-incremental (RTI) model of Norman (1964) because it subsumes both the linear and one-element models as special cases. For the RTI model

$$p_{n+1} = \begin{cases} (1-\theta)p_n + \theta\ , & \text{with probability } c \\ p_n\ , & \text{with probability } 1-c\ , \end{cases} \tag{8}$$

where $p_1 = \frac{1}{3}$. If θ equals one, the RTI model reduces to the one-element model; on the other hand, if c equals one, then the model reduces to the linear model.

Table 5 presents the parameter estimates for the three models. These estimates were obtained by minimizing the chi-square function defined in Eq. 6 separately on data for each RI condition. Inspection of Table 5 reveals that all three models can be rejected on statistical grounds. Again the one-element model fits the data better than the linear model,

Table 5

Parameter Estimates and Goodness-of-fit Measures for the One-Element, Linear, and RTI Models Applied Separately to the Data of the Five Experimental Conditions

		Condition				
Model	Parameter	$F(\frac{1}{2})$	F(1)	F(2)	F(4)	Variable
One-element	c	.054	.062	.070	.070	.070
χ^2		96.30	98.61	111.54	104.75	141.94
Linear	θ	.062	.070	.078	.086	.070
χ^2		187.56	230.56	270.77	181.07	937.56
RTI	c	.086	.102	.109	.117	.094
	θ	.758	.773	.805	.781	.875
χ^2		60.97	57.95	50.45	53.40	60.14

and of course, the RTI model with its two parameters for each RI condition fits best of all. Notice that the parameter θ of the RTI model is relatively constant over conditions; whereas c appears to increase with increasing values of RI. An interesting fact that emerges from Table 5 is that the sums of the chi-squares over the five experimental conditions for both the linear model and the one-element model are only slightly lower than the chi-squares presented in Table 4. In the case of Table 5, five parameters were used, whereas in Table 4 only three parameters were used to characterize changes in RI. Thus, despite the poor fit of the models, there is some indication that the assumptions regarding the effects of variations in RI, as represented in Eq. 4, may not be too bad.

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Footnotes

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[2]The minimum of $\chi^2(a,d,x)$ is not precisely chi-square distributed, but for our purposes the approximation is adequate. For a discussion of this point, see Atkinson, Bower, and Crothers (1965, pp. 394-5).