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USE OF VARIABLE LIFT CONTROL TO OPTIMIZE AERODYNAMIC BRAKING FOR A MARS ENTRY VEHICLE

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FOREWORD

The research described in this report, "Use of Variable Lift Control to Optimize Aerodynamic Braking for a Mars Entry Vehicle," Number 67-34, by R. A. Niemann, was carried out under the direction of C. T. Leondes, A. R. Stubberud, E. B. Stear, and D. M. Wilberg, Co-principal Investigators in the Department of Engineering, University of California, Los Angeles.

This research was supported jointly by the National Aeronautics and Space Administration under Grant NsG-237-62 to the Institute of Geophysics and Planetary Physics of the University, and the Jet Propulsion Laboratory under Contract Number 951889 (Subcontract under NASA Contract NAS 7-100, Task Order RD-30).

ABSTRACT

The use of variable lift control to obtain maximum velocity reduction through aerodynamic braking for Mars entry is considered. The method of lift control used is that of rolling the lift vector about the vehicle's stability axis so that the lift vector may either point upward or downward in the vertical plane. Through this method of lift control, it is shown that the maximum allowable ballistic coefficient which may be used to obtain a specific terminal velocity can be doubled over that which may be used with a constant L/D lifting vehicle. Also it is shown that the minimum velocity attainable for a given ballistic coefficient and atmosphere is the same for all entry conditions.

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NOMENCLATURE

B_c	=	ballistic coefficient (m/C_DA), slugs/ft^2
a_T	=	total deceleration, earth g's
A	=	frontal area, ft^2
C_L, C_D	=	lift, drag coefficient, respectively
g_m	=	acceleration of gravity at surface of Mars, ft/sec^2
	=	12.1 ft/sec^2
h	=	altitude, ft
L/D	=	lift-drag ratio
m	=	mass of vehicle, slugs
$\dot{q}_c$	=	convective heating rate, BTU/ft^2-sec
Q_c	=	total convective heating, BTU/ft^2
R_m	=	radius of Mars, ft
	=	11,200,000 ft
v	=	velocity, ft/sec
β	=	inverse scale height of Martian atmosphere, ft^{-1}
γ, γ_s	=	flight path angle, skip out angle, respectively, deg.
ρ, ρ_0	=	density, surface density of Martian atmosphere, respectively, slugs/ft^3

INTRODUCTION

One of the main problems in accomplishing a soft landing on Mars is that of decreasing the extremely high velocity of entry to a velocity at which parachutes and crushable impact absorbers can be employed. Aerodynamic braking can accomplish a large part of this velocity reduction, but due to the extremely thin atmosphere of Mars (surface pressure as low as 5 mb according to Mariner IV) the selection of the entry trajectory is extremely critical if one is to avoid having to use an extremely low ballistic coefficient, resulting in a low payload, in order to accomplish the necessary velocity reduction.

Several papers[1,2] have discussed the use of a lifting vehicle with constant L/D in order to increase the amount of aerodynamic braking obtained over that using a simple ballistic entry capsule. The next step is thus to investigate the possibility of using variable lift control to further increase the efficiency of the aerodynamic braking. This problem is amenable to solution using the optimization techniques of modern control theory, and this approach will be considered in this paper.

In considering the direct entry problem, i.e., entry without first going into orbit, the terminal guidance accuracy (the accuracy in entry angle, primarily) becomes a problem. For the constant L/D lifting vehicle, this problem is solved by designing for the steepest possible entry angle, and this design will thus work for all other possible entry angles. For example, if the terminal guidance accuracy is $\pm 5^{\circ}$, and the skip out angle (the minimum entry angle for which the vehicle is captured by the planet's atmosphere, or, to be more specific, the minimum angle for which the vehicle, after initial pull up, skips up to no higher than a specified altitude) is γ_s, the ballistic coefficient is determined so that, for the given L/D, at an entry angle of $\gamma_s + 10^{\circ}$, the desired terminal velocity is reached at the desired terminal altitude. Then, at any other entry angle between γ_s and $\gamma_s + 10^{\circ}$, the terminal velocity will be at least as small as the desired terminal velocity, since for the constant L/D trajectories, the shallower the entry, the smaller is the terminal velocity. However, if we use some controlled lift program, it does not follow that a lift program designed for the steepest entry angle will work for a shallower entry angle. Therefore, in this paper, rather than finding an optimal lift program for some specific entry angle, we will determine a sub-optimal lift program which will work for all entry angles within the entry corridor.

FORMULATION OF THE PROBLEM

The equations of motion for a vehicle entrying the atmosphere of Mars are given by

$$\begin{aligned}
\dot{\gamma} &= \left(- \frac{v}{R_m} + \frac{g_m}{v} \right) \cos\gamma - L/D \, \frac{\rho v}{2B_c} \\
\dot{v} &= g \sin\gamma - \rho v^2 / 2B_c \qquad (1) \\
\dot{h} &= -v \sin\gamma
\end{aligned}$$

where a spherical, non-rotating planet, two-dimensional planar motion and $R_m >> h$ have been assumed. We will further assume an exponential atmosphere, $\rho = \rho_0 e^{-\beta h}$.

In determining the method of lift control to use, we want a fairly simple control program, and also one which can be designed to work for a range of entry angles. The lift control to be used here will be to allow the vehicle to be rolled about its stability axis so that the lift vector is either positive upward or negative downward. That is, for any specific lifting vehicle configuration, there is a stability angle between velocity vector and axis of symmetry at which the vehicle will tend to fly. The axis through the velocity vector is thus the stability axis. The vehicle can be rolled about this stability axis, and each orientation around the stability axis is stable. Thus by rolling about the stability axis, we can roll the lift vector and change its direction without changing its magnitude or the direction or magnitude of the drag vector. For this paper, we assume that the lift vector may either be kept in the positive upward direction or rolled to the negative downward position, but we will not make use of any of the other possible orientations. Mathematically, this amounts to allowing the sign of the L/D term in the first of Equations (1) to be changed to plus or minus in the form of a bang-bang control (we will assume this change can be made instantaneously, although future studies will have to take into account the finite time required to roll the vehicle). This method of control is fairly simple, since we will only have to determine a few switching points, as opposed to using a continuously varying lift control. Also, we can determine the switching points as a function of the state variables, and thus more readily find a control program that will work for a whole range of entry angles.

The initial conditions for Mars entry will be taken to be v_0 = 26,000 ft/sec, h_0 = 360,000 ft, γ_0 unspecified. The terminal conditions will be h_f = 20,000 ft, γ_f unspecified, v_f to be minimized. That is, we want to initiate the terminal maneuver (releasing parachutes) at h_0 = 20,000 ft., and we want to make the velocity low enough to allow the performance of the desired terminal maneuver. In order to be able to use subsonic parachutes, we need v_f at least as low as 1,000 ft/sec. For any given ballistic coefficient B_c we can obtain a certain minimum v_f. We want to determine the largest B_c for which the minimum v_f is below 1,000 ft/sec. Eventually we will want to use a B_c which will give us a v_f enough below 1,000 ft/sec that we can choose a suboptimal control program which, for a range of γ_0, will still give $v_f \leq$ 1,000 ft/sec. We will also eventually have to consider the uncertainties in the density of the Martian atmosphere. However, initially we will consider the basic optimization problem of minimizing v_f given B_c and $\rho(h)$.

THE OPTIMIZATION PROBLEM

We are given the state variable equations (1), with initial conditions h_0 = 360,000 ft., v_0 = 26,000 ft/sec, and terminal condition h_0 = 20,000 ft. The control variable L/D will be taken to be of constant magnitude $|L/D| = 0.5$, but may assume either a positive or negative sign, with a finite but unspecified number of switching points. The admissible set of the control variable is thus a closed set. Since the possible switching times vary continuously over a finite interval, there are an infinite number of admissible control programs. Among all those trajectories with admissible control programs which start out at v_0 = 26,000 ft/sec and h_0 = 360,000 ft., and reach h_f = 20,000 ft. at some unspecified final time T, we want to choose that one which has a minimum v_f. We will use Pontryagin's maximum principle[3] to solve this optimization problem.

We first must derive the Hamiltonian defined by

$$H = \underline{f}^T \cdot \underline{p} = p_1 \dot{\gamma} + p_2 \dot{v} + p_3 \dot{h} \tag{2}$$

where $\underline{f}$ is the three-dimensional state variable derivative vector $\dot{\underline{x}} = \underline{f}(x,u), \underline{x} = \{\gamma, v, h\}$, $\underline{f} = \{\dot{\gamma}, \dot{v}, \dot{h}\}$, $u = L/D$, the control variable, and $\underline{p}$ is the three-dimensional adjoint variable defined by

$$\dot{p}_i = -\frac{\partial H}{\partial x_i} \qquad i = 1, 2, 3 \tag{3}$$

We thus have six differential equations, three state variable equations (1) and three adjoint equations (3). We have three state variable boundary conditions v_o, h_o and h_f. We thus need three additional adjoint boundary conditions. Since γ_0 and γ_f are unspecified, the corresponding adjoint conditions p_{1_0} and p_{1_f} are zero. Since v_f is the quantity to be minimized, the corresponding adjoint condition is $p_{2_f} = -1$.[4] This completes the necessary boundary conditions.

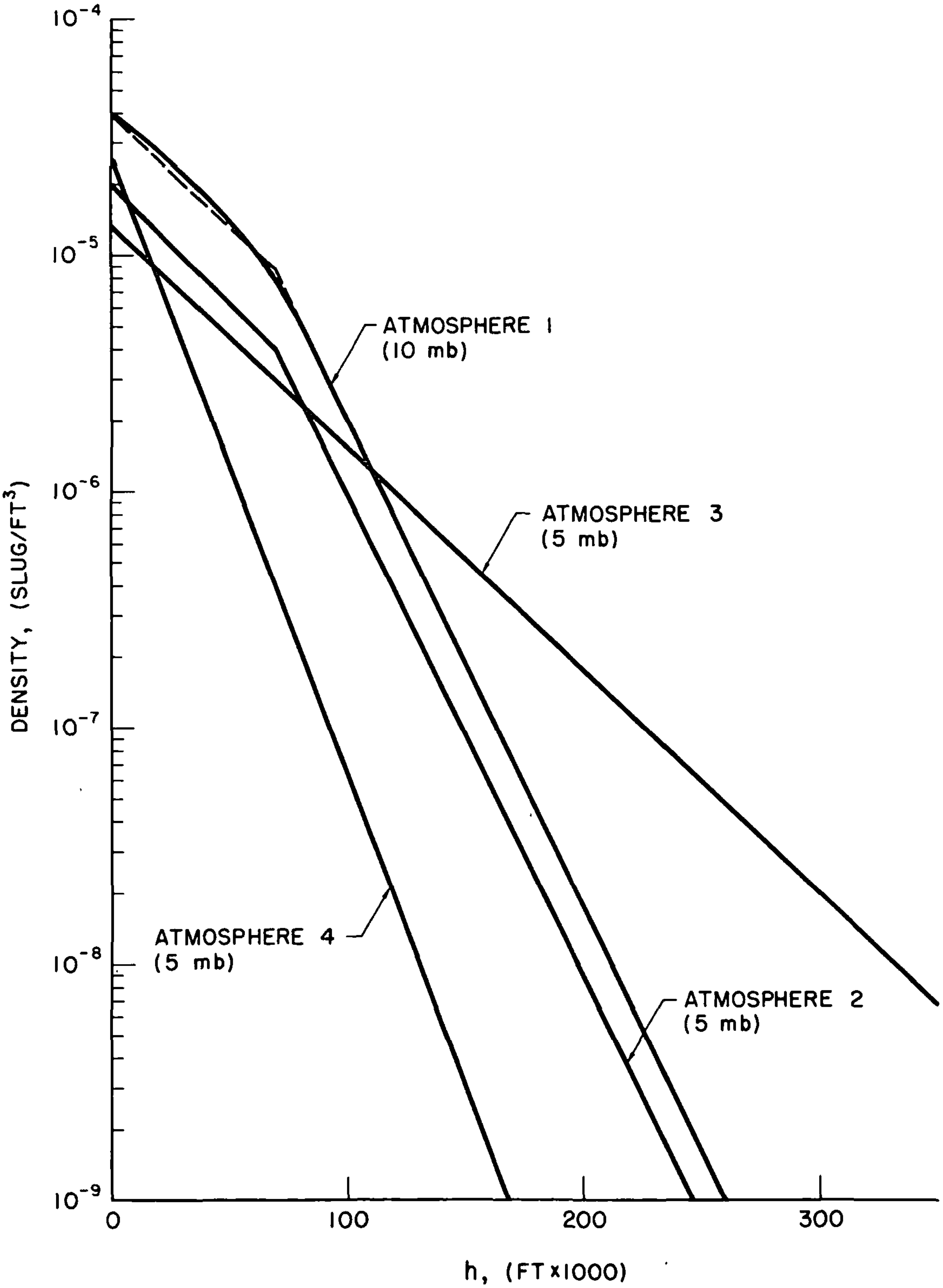
According to Pontryagin, the optimal control L/D is that control as a function of time which minimizes the Hamiltonian as a function of time. If we substitute (1) into (2), we see that H is maximized over all admissible L/D (L/D = ± 0.5) if

$$L/D = -\ 0.5\ \mathrm{sgn}\ (p_1) \qquad (4)$$

For our first results, we will assume the 10 mb atmosphere shown in Figure 1 (obtained from Reference 1). This atmosphere will be approximated by the two straight lines segments shown; thus, for our exponential approximation $\rho = \rho_0 e^{-\beta h}$, we have $\rho_0 = 4 \times 10^{-5}$ slug/ft^2, $\beta = 2.4 \times 10^{-5} ft^{-1}$ (for $h < 70,000$ ft.), and $\rho_0 = .78 \times 10^{-5}$ slug/ft^2, $\beta = 4.8 \times 10^{-5} ft^{-1}$ (for $h > 70,000$ ft.). We will initially assume $B_c = 1.0$ slugs/ft^2.

The problem we are now confronted with is a two-point boundary value problem. Among the many existing techniques for solving this problem, quasilinearization will be employed in this paper. Since adequate discussion of the application of this technique exists in the literature,[5] no details on its application will be given here.

Solution of the two-point boundary value problem given above for a few values of final time T points to a further problem in solving the practical velocity minimization problem. With no bounds placed on the state variable, the optimal trajectory for a sufficiently large final time T is one which descends to as low an altitude as possible, not only below h = 20,000 ft., but below h = 0 for the mathematical formulation given, and then skips back up to h = 20,000 ft. at the end of the trajectory. This stands to reason since the trajectory would seek to make use of as high a density as possible. However, for our practical problem, we must put some lower bound on the altitude. That is, the trajectories will make one or more pull ups or skips, and we must constrain the depth of the pull ups. The bound must at least be greater than zero, and it could also conceivably be greater than 20,000 ft., except for the final descent to the terminal condition.



ATMOSPHERE DENSITY PROFILES

FIGURE 1

However, in this paper, a state variable constraint of $h \geqq 20,000$ ft. will be used.

According to Pontryagin, for the bounded state variable problem, the optimal trajectory consists of segments for which the trajectory is inside the constraint, in which case the optimal control satisfies the maximum principle, and segments for which the trajectory is on the constraint, in which case the optimal control is determined to hold the trajectory on the constraint. In our problem since the control can only have two values (L/D ± 0.5) the control cannot hold the trajectory on the constraint, i.e., fly at constant altitude, for a finite time. Thus for our method of control, the optimal bounded state variable trajectory will merely touch the constraint at at least one point in between the initial and final point, and immediately come off the constraint. Also according to Pontryagin, there is a jump in the value of the adjoint vector at the point where the trajectory touches the constraint, and for our problem, the jump condition is indeterminate.

Thus it becomes impossible to apply the optimization procedure to the entire trajectory at once. However, according to Bellman's principle of optimality,[6] any segment of an optimal trajectory from any point on the trajectory to the end of the trajectory is the optimal trajectory from that point to the end point. Therefore, we can optimize the last segment of the trajectory, from the last point at which the trajectory touches the constraint to the end point, and then match up the first part of the trajectory to this last segment. That is, since we know that at the point where the trajectory touches $h = 20,000$ ft., γ must be zero (in order for the trajectory to be tangent to $h = 20,000$ ft. and immediately pull up), we can apply the maximum principle to the problem with initial conditions $h_0 = 20,000$ ft., $\gamma_0 = 0$, v_0 unspecified, final condition $h_f = 20,000$ ft., final time T unspecified, and minimize v_f, and this will be the last segment of our trajectory for the given entry conditions (assuming the v_0 determined for the problem starting at 20,000 ft. is attainable from the initial conditions for entry).

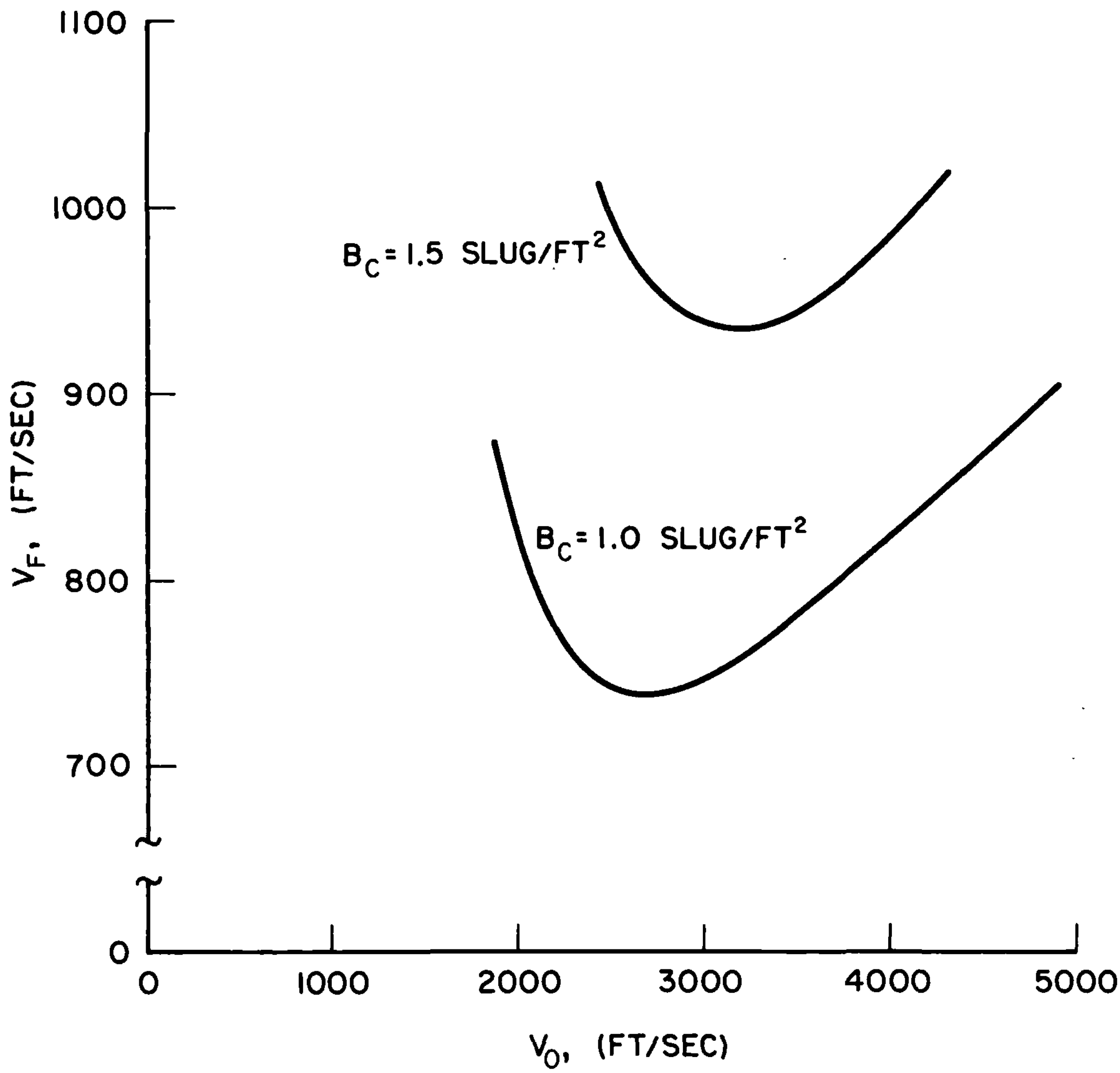
We can also make an intuitive guess at what the optimal control for this final segment will be, i.e., that L/D will be positive (equal to +0.5) for the entire last segment, because it stands to reason that the vehicle should hold above 20,000 feet for as long as possible after the last pull up to allow the velocity to decrease as much as possible. Or, to follow another line of reasoning, the last segment of the trajectory will be the optimal trajectory for the unbounded state variable problem, even though the trajectory all lies above 20,000 ft., for if there were an optimal trajectory starting at $h = 20,000$ ft. and $\gamma = 0$ which dropped below 20,000 ft. at some point, then the optimal bounded state variable trajectory would touch $h = 20,000$ ft. at some point in between, which conflicts with the segment's being the last segment after touching 20,000 ft. Thus, for the last segment, the optimal trajectory for bounded state variable is the same as that for unbounded state variable. However, if L/D is not +0.5 for this full last segment, i.e., if L/D = -0.5 for any part of the last segment, it stands to reason that the way to obtain the lowest v_f would be to have L/D = -0.5 during the first part of the last segment of the trajectory and allow the vehicle to take advantage of the lower densities, and then hold L/D = +0.5 for the remainder of the trajectory. Then, of course, the trajectory falls below 20,000 ft. which is not allowable. This again would lead one to believe that the L/D holds the constant value of +0.5 over the entire last segment of the optimal trajectory. Of course this reasoning does not rigorously prove this, it only serves as a good guess, a guess which will later prove to be correct.

Thus, proceeding under the assumption that L/D = +0.5 over the last segment, our only unknown is the initial velocity for the last segment, since we know h_0, γ_0, and L/D. Thus, we can simply guess several values of v_0, integrate Equations (1) from $h_0 = 20,000$ ft.,

$\gamma_0 = 0$, and stop when h returns to 20,000 ft., and by trial and error, choose the trajectory which results in the lowest v_f. This eliminates the problem of having to determine the final time T, since T is automatically determined by when the trajectory falls back to $h_f = 20,000$ ft.

Figure 2 shows the final velocity v_f plotted as a function of the initial velocity v_0 at $h_0 = 20,000$ ft., $\gamma_0 = 0$ (for both $B_c = 1.0$ and 1.5 slug/ft^2). The minimum v_f of 738 ft/sec for $B_c = 1.0$ slugs/ft^2 occurs at approximately $v_0 = 2650$ ft/sec. The final time T for this trajectory is approximately 102 seconds. We can now verify our assumption of L/D = +0.5 for the entire last segment by applying our optimization technique with $h_0 = 20,000$ ft., $\gamma_0 = 0$, $h_f = 20,000$ ft., T = 102 sec, and v_0 unspecified. The optimal trajectory should have L/D = +0.5 over the entire segment with $v_0 \cong 2650$ ft/sec. The terminal conditions on the adjoint equations are the same as for the full trajectory ($p_{1_f} = 0$, $p_{2_f} = -1$) but the initial condition $p_{1_0} = 0$ is replaced by $p_{2_0} = 0$, since here v_0 instead of γ_0 is unspecified.

Solving the resulting two point boundary value problem using quasilinearization, the optimal trajectory does indeed turn out to have a control program of L/D = +0.5 over the whole segment, with $v_0 = 2636$ ft/sec and $v_f = 738$ ft/sec. If we increase T from 102 seconds to 102.5 seconds, the optimal trajectory turns out to have a very short initial segment with L/D =-0.5 (thus dropping below 20,000 ft.), and then the remainder of the trajectory has L/D = +0.5. For T < 102 seconds, L/D =+0.5 over the whole trajectory. Thus our original assumption has been verified. Although this does not rigorously prove that the optimal last segment of the trajectory has L/D =+0.5 for all atmosphere density profiles and all B_c, we will assume this is the case and calculate optimal last segments by trial and error variation of v_0 at $h_0 = 20,000$ ft. and $\gamma_0 = 0$. It is interesting to note that the nature of the final last segment is completely

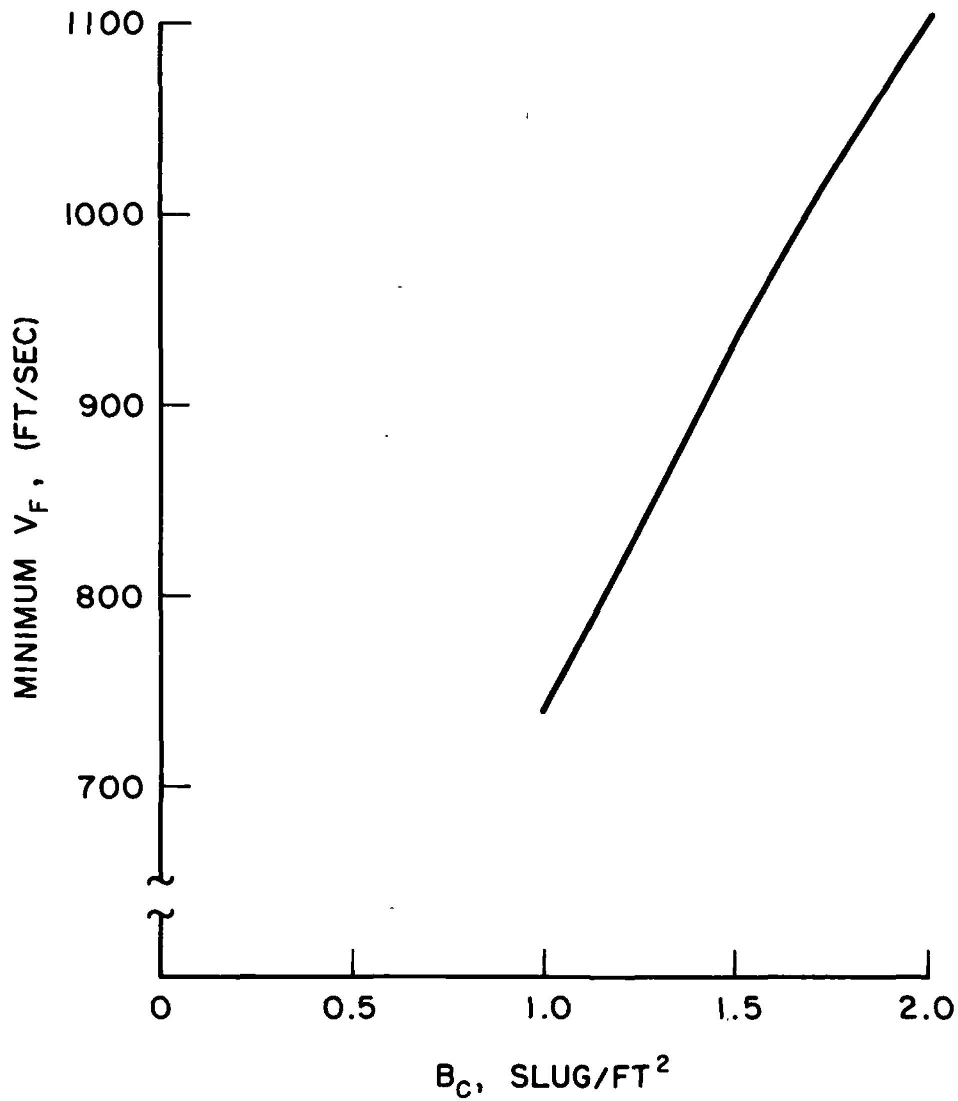


FINAL VELOCITY VS. INITIAL VELOCITY FOR
h_o = 20,000 ft., γ_o = 0, L/D = + 0.5
B_c = 1.0, 1.5, slugs/ft^2, Atmosphere 1
FIGURE 2

independent of the initial conditions of entry for the full trajectories, i.e., regardless of the initial entry conditions all optimal trajectories have the same final segment, and thus the same minimum v_f, as long as the point h_0 = 20,000 ft., γ_0 = 0, and v_0 = 2636 ft/sec (for B_c = 1.0 slugs/ft^2) is attainable from the initial entry conditions, which is probably the case for all possible direct entry and orbital entry cases. Also, the determination of the first portion of the trajectory is no longer an optimization problem, but merely a two-point boundary value problem, since merely assuring the final boundary conditions h_f = 20,000 ft., γ_f = 0, and v_f = 2636 ft/sec on the initial portion yields an optimal full trajectory. Thus there are probably several ways of solving the two point boundary value problem for the initial segment all constituting optimal trajectories for the full trajectory.

This situation only occurs when the function to be minimized is a function of the terminal state. If it were an integral of a function of the state variables over the whole trajectory the situation would be quite different. This multiplicity of solutions is the reason for the fact that the adjoint conditions at the constraint boundary is indeterminate, that is, there are several values of the jump in adjoint conditions which will yield optimum trajectories for the given boundary conditions.

Figure 3 shows a graph of the minimum final velocity versus ballistic coefficient obtained by determining the final segment of the trajectory assuming L/D =+0.5 over the entire segment. It can be seen that in order to insure $v_f \leq$ 1,000 ft/sec, the maximum allowable B_c is 1.7 slugs/ft^2. However, since we are going to try to find a suboptimal control program which will work for all trajectories of an entry corridor of 10^o, we will use a B_c of 1.5 slug/ft^2 in our search for suboptimal controls. A B_c of 0.75 slug/ft^2 is the maximum attainable B_c using a constant L/D =.5 over the whole trajectory for atmosphere 1 of Figure 1. Thus if we can find a suboptimal control



MINIMUM FINAL VELOCITY VS. BALLISTIC COEFFICIENT
Atmosphere 1

FIGURE 3

program which works for a 10° entry corridor for a B_c = 1.5 slugs/ft^2, we will have achieved a 100% increase over the B_c attainable using a constant L/D entry vehicle.

CONTROL PROGRAM DETERMINATION

In determining the first segment of the trajectory, we want to find a series of switching points for the control as a function of the state variables which will result in a final segment with a final velocity less than 1000 ft/sec for a range of entry angles over an entry corridor of 10°. From Figure 2, for B_c = 1.5 slugs/ft^2, we see that at the beginning of the final segment, and thus at the end of the initial segment, i.e., at h = 20,000 ft and γ=0, we need a velocity of between 2500 and 4200 ft/sec to insure a v_f of no more than 1000 ft/sec. Also, if we assume the final segment starts at h = 25,000 ft and γ = 0, and determine final segments for various initial velocities and L/D =+0.5 over the entire segment, we find that $v_f \leq$ 1000 ft/sec for initial velocities between 2500 and 3700 ft/sec. Thus, v_f is not too sensitive to the values of h and v at the last pull up (γ=0), i.e., while v_f and h at the last pull up vary between 2500 and 3700 ft/sec and between 20,000 and 25,000 ft, respectively, v_f varies only between 930 and 1000 ft/sec. Thus, in determining an L/D program for the first segment, we need only insure that the end of the first segment goes through a pull up at γ=0 in the region around and above h = 20,000 ft and v around 2500-4000 ft/sec.

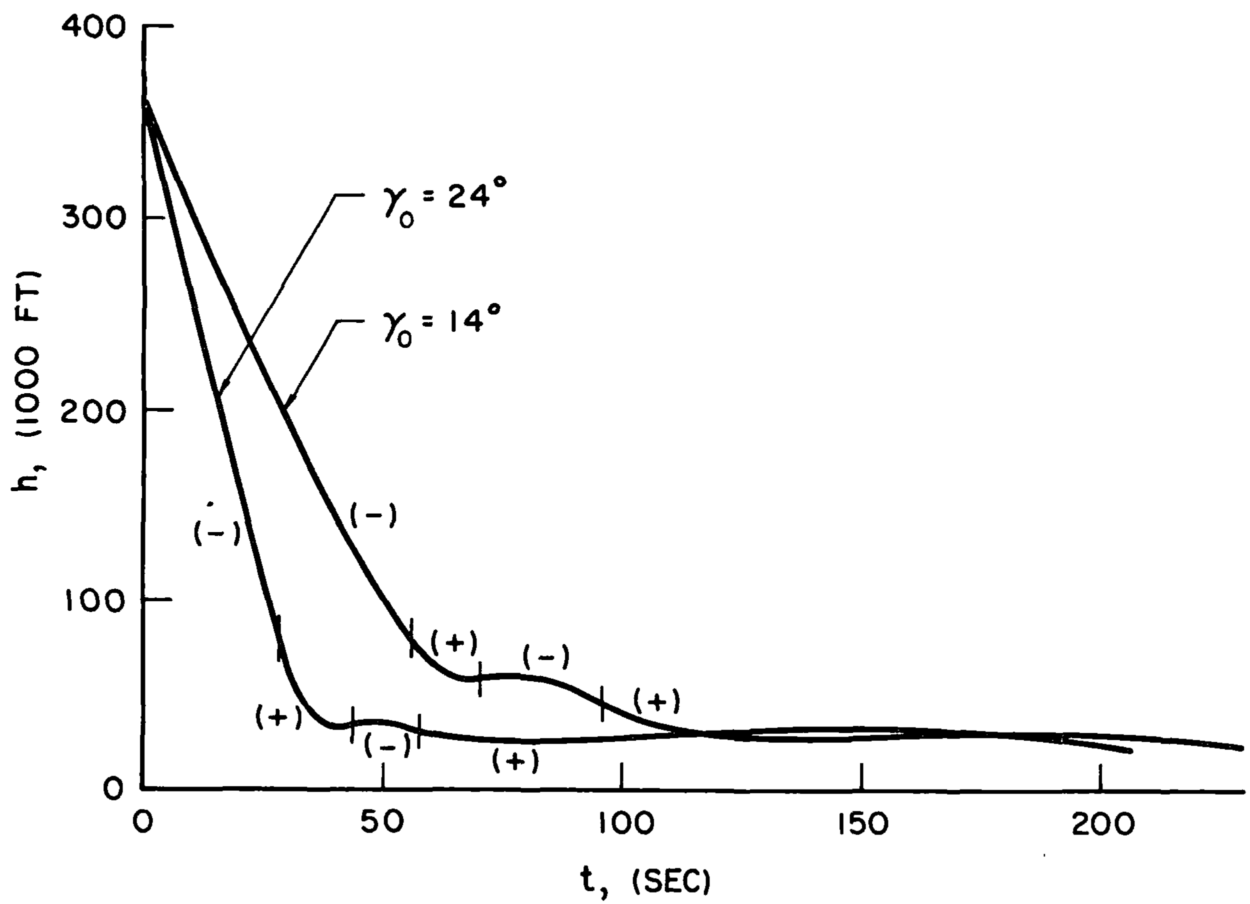
In the first form of control program we will try, we will use three switching points in L/D, i.e., L/D will start out at -0.5, at some point will change to +0.5, then back to -0.5, and finally back to +0.5. Using L/D =-0.5 at the start, we can use smaller initial entry angles, since the skip out angle for a constant L/D =-0.5 is less than 10°, compared to around 24° for constant L/D =+0.5.

After some trial and error manipulation of switching points, the following control program was determined:

Initial Entry Conditions:	L/D =-0.5
h = 80,000 ft.:	L/D =+0.5
γ = - 0.1 rad.:	L/D =-0.5
v = 5200/ft/sec or γ = .275 rad.: (whichever comes first)	L/D =+0.5

Thus, at the beginning of entry, L/D =-0.5. When the vehicle descends to h = 80,000 ft., L/D is switched to +0.5. This causes the vehicle to make its first pull up at an altitude below 80,000 ft., but considerably above 20,000 ft. When the vehicle pulls up and reaches an angle γ = -0.1 rad. (γ measured positive below horizontal, negative above), L/D again switches to -0.5, causing the trajectory to reach a peak and start down again. When either the velocity falls below 5200 ft/sec or the angle exceeds 0.275 rad., L/D switches to +0.5, initiating the final pull up and leading to the final segment of the trajectory. For this control program, for entry angles between 14^{o} and 24^{o}, the final pull ups occur between h = 25,000 and 27,000 ft and v = 2500 and 3000 ft/sec, and v_f is less than 1000 ft/sec for all angles in the range. Thus we can employ this control program for a nominal entry angle of 19^{o} and a terminal guidance accuracy of $\pm 5^{o}$, and all possible entry trajectories will have a $v_f \leq$ 1000 ft/sec (for atmosphere 1 of Figure 1). Figure 4 gives the altitude versus time profiles for this control program for $\gamma_0 = 14^{o}$ and 24^{o}. The slashes on the trajectories indicate the switching points, and the signs in parenthesis indicate whether L/D is +0.5 or -0.5.

It is also possible to determine a control program for the first segment of the trajectory using only two switching points, i.e., with the vehicle starting with L/D = +0.5 at the initial conditions, at some point switching to L/D =-0.5, and then back to L/D =+0.5. The following program was determined by trial and error:



ALTITUDE VS. TIME
CONTROL PROGRAM #1

FIGURE 4

Initial Entry Conditions: $L/D = +0.5$

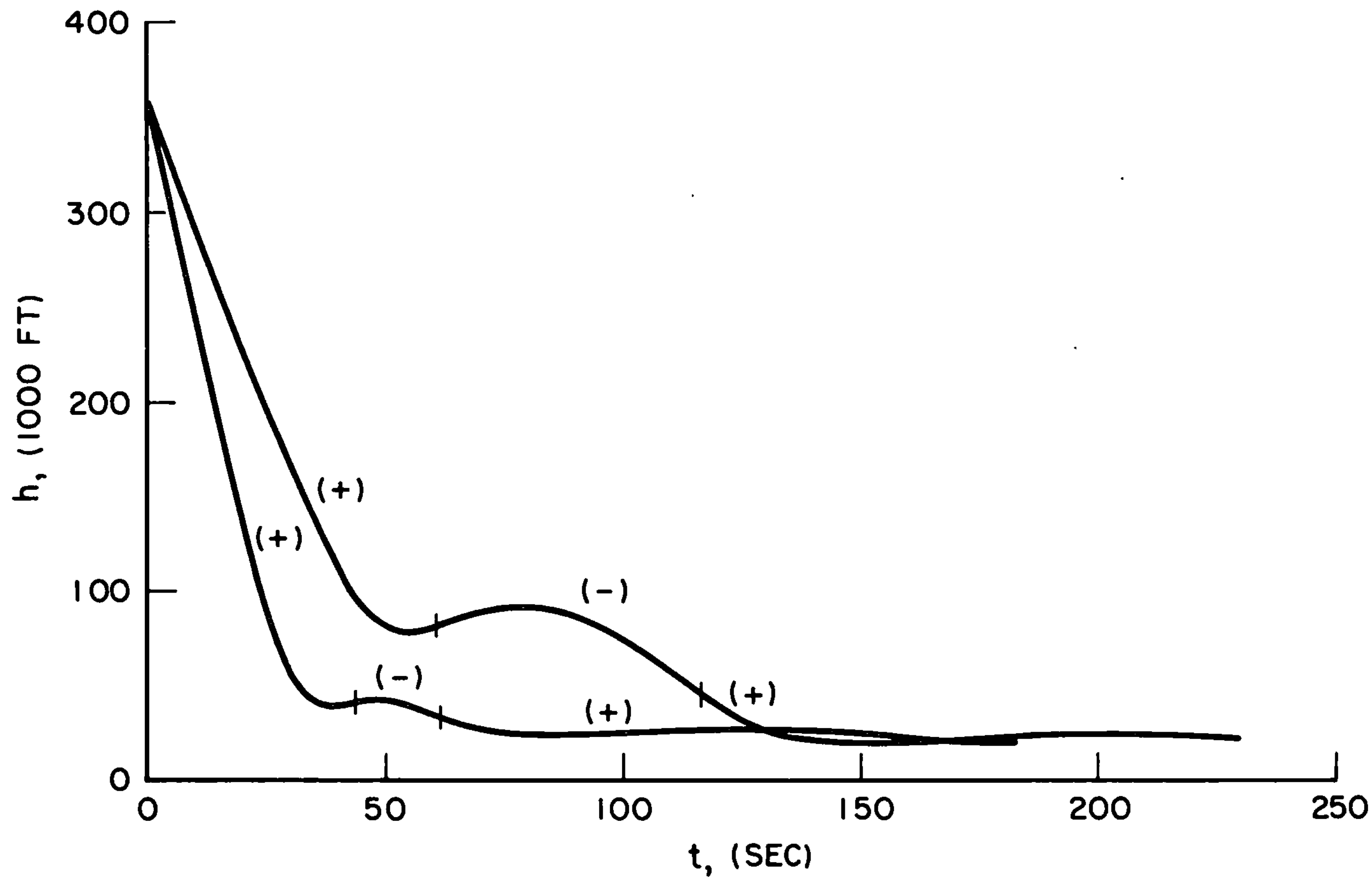
$\gamma = -0.1$ rad.: $L/D = -0.5$

$[(h \leq 45,000 \text{ ft}) \text{ AND } (\gamma \geq 0.3 \text{ rad.})]$ OR $[(h \leq 40,000 \text{ ft}) \text{ AND } (\gamma \geq 0.25 \text{ rad.})]$: $L/D = +0.5$

In this program the final switching point is more complex than for the first program, but this is to be expected due to the smaller number of switching points in this program. However, the main difficulty in mechanizing these control programs is in measuring the current values of the state variables; once one is able to generate the values of the state variables, the design of the switching circuits is fairly straightforward.

This control program will produce a $v_f \leq 1000$ ft/sec for γ_0 between 16^o and 26^o. The final pull up points for this range of γ_0 occurs between $h = 20,000$ and $27,000$ ft and $v = 2500$ and 2800 ft/sec. The altitude versus time curves for $\gamma = 16^o$ and 26^o are given in Figure 5.

These trajectories have all been determined for the 10 mb atmosphere 1 in Figure 1. However, our best measurements to date on the density of the Mars atmosphere indicate that the atmosphere is between 5 and 10 mb. Thus, we should design our control program to take into account this uncertainty in density. If we assume that the 5 mb atmosphere has the same shape, i.e., the same β's, as the 10 mb atmosphere, with simply half the density values at each point, and all atmospheres in between 5 and 10 mb also have the same β's, we can simply design for the lowest density and the control program will still work for the higher densities. For example, ρ_0 is always divided by B_c wherever it occurs in Equations (1), i.e., $\rho/B_c = \rho_0 e^{-\beta h}/B_c$. Thus, if ρ_0 is only half as large as for the previously used atmosphere, if we also take B_c only half as large, the trajectories for the control programs determined earlier will be unchanged. (Also, the maximum B_c attainable using constant $L/D = +0.5$ will be only



ALTITUDE VS. TIME
CONTROL PROGRAM #2

FIGURE 5

half as large, so that our relative increase in B_c obtained using $L/D = \pm 0.5$ will be the same). Thus, the B_c for the 5 mb atmosphere will be $B_c = 0.75$ slugs/ft^2. If we use this B_c for any atmosphere greater than 5 mb, but with the same β's, the terminal velocity will still be less than 1000 ft/sec. The 5 mb atmosphere is shown as atmosphere 2 in Figure 1.

However, there are also uncertainties in the values of β. For example, Reference 7 gives these two possibilities for a 5 mb atmosphere

$$\rho_0 = 1.32 \times 10^{-5} \text{ slugs/ft}^3, \quad \beta = 2.15 \times 10^{-5} \text{ ft}^{-1}$$

$$\rho_0 = 2.56 \times 10^{-5} \text{ slugs/ft}^3, \quad \beta = 6.07 \times 10^{-5} \text{ ft}^{-1}$$

These atmospheres are given as atmospheres 3 and 4, respectively, in Figure 1. Of course, the plot of atmosphere 1 is probably a more accurate representation of the shape of an atmosphere than the single straight lines of atmospheres 3 and 4, but these latter atmospheres do give some indication of the possible variations in the β's.

Since the β for atmosphere 3 is approximately the same as for the low altitudes of atmosphere 1, we can assume that the maximum B_c for atmosphere 3 is proportional to the ρ_0, as we did for atmospheres 1 and 2. Thus, since ρ_0 for atmosphere 3 is approximately 1/3 of that of atmosphere 1, we can assume that the maximum allowable B_c for atmosphere 3 is 1/3 x 1.5 or 0.5 slugs/ft^2.

However, if we apply either of the two control programs derived earlier to atmosphere 3 with $B_c = 0.5$ slugs/ft^2, we do not get $v_f \leq 1000$ ft/sec, due to the fact that for higher altitudes atmosphere 3 does not have the same shape as atmosphere 1. Thus we must vary our original control program, using more complicated switching points, so that the control program will work for atmospheres of the shape of both atmosphere 1 and 3. By trial and error, the following varied version of the first control program was obtained

Initial Entry Conditions:	L/D =-0.5
(h $\leq$ 80,000 ft.) OR ($\gamma \geq$ 0.45 rad.):	L/D =+0.5
γ =-0.1 rad.:	L/D =-0.5
[(v $\leq$ 5200 ft/sec) AND ($\gamma \geq$ 0.17 rad.)]}: OR ($\gamma \geq$ 0.275 rad.)	L/D =+0.5

This program will result in $v_f \leq$ 1000 ft/sec for γ_0 between 14°-24°, the same as for the first control program for atmosphere 1. The variations from the first control program derived earlier are in the first and third switching points (not counting the initial conditions as a switching point). Owing to the higher densities at higher altitudes for atmospheres shaped as atmosphere 3 over atmospheres shaped as atmosphere 1, the L/D =-0.5 causes the trajectory to dive more rapidly. Thus we add the upper bound on γ in the first switching point. Also, a lower bound on γ has been added to the last switching point. It should be noted that a term such as ($\gamma \geq$ 0.45 rad.) in a switching point means only that when γ becomes greater than 0.45, the control switches; it does not mean that if γ then falls below 0.45, the control switches back. The control doesn't switch back until the next indicated switching point.

Atmosphere 4 of Figure 1 has approximately the same density values in the neighborhood of h = 20,000 ft as atmosphere 3, and thus the same maximum B_c of 0.5 slugs/ft^2 should be applicable to this atmosphere. However, due to the high value of β and thus the extremely rapid fall off in density with altitude, it is much more difficult to shape the first segment of the trajectory than for the other atmospheres. Therefore, a further decrease in B_c would be necessary to obtain $v_f \leq$ 1000 ft/sec. (Also, the maximum B_c for constant L/D =+0.5 would be less than it would for atmospheres with similar surface density but smaller β's.) However, determining a final control program for this atmosphere was not attempted in this paper, since the intention here is merely to demonstrate the advantages and feasibility of using variable lift control; more detailed studies will be left to future investigators, presumably equipped with better data on the range of uncertainties in the Martian atmosphere.

It is also desirable to look at the heating and deceleration problems for the Mars entry vehicle. The deceleration is given by

$$a_T = \sqrt{\left(\frac{L}{m}\right)^2 + \left(\frac{D}{m}\right)^2} \tag{5}$$

The convective heating rate is given by

$$\dot{q}_c \sqrt{R_N} = (20.4 \times 10^{-9})\ \rho^{1/2}\ v^3 \tag{6}$$

where $\dot{q}_c$ is in units of BTU/ft^2-sec and R_N, the nose radius, is in feet. The total heat input Q_c is the integral of $\dot{q}_c$ with respect to time.

For the first control program for atmosphere 1, with B_c = 1.5 slug/ft^2, the maximum values of a_T, $\dot{q}_c\sqrt{R_N}$, and $Q_c\sqrt{R_N}$ over all values of γ_0 within the 10^o entry corridor are 50 earth g's, 630 BTU/ft$^{3/2}$-sec, and 11,000 BTU/ft$^{3/2}$, respectively. The maximum a_T and $\dot{q}_c$ occur for the steepest entry, the maximum Q_c for the shallowest entry (since the total time is longer for the shallower entries). For the second control program, the respective maximum values are 46 g's, 620 BTU/ft$^{3/2}$-sec and 12,500 BTU/ft$^{3/2}$. For a constant L/D =+0.5 and B_c = 1.5 slug/ft^2, over an entry corridor of 24-34^o (γ_s = 24^o for constant L/D =+0.5) respective maximum values are 60 g's, 730 BTU/ft$^{3/2}$-sec, and 9170 BTU/ft$^{3/2}$. (For the latter case, v_f was as high as 1500 ft/sec). It can be seen that there is little difference between the maximum values of a_T, $\dot{q}_c\sqrt{R_N}$ and $Q_c\sqrt{R_N}$ for the two control programs, and there is improvement in obtaining low a_T and $\dot{q}_c$ over that for constant L/D =+0.5 for the same B_c. The reason why $Q_c\sqrt{R_N}$ is less for constant L/D is that the variable L/D programs hold the vehicle at low densities for a larger time to get the greatest velocity reduction, and thus the constant

L/D program, which gets less velocity reduction, also gets less total heating. However, the maximum heating rate, which always occurs before the first pull-up, is greater for constant L/D, since the entry angles are steeper. However, the results for constant L/D data are only presented for comparison purposes for heating and deceleration; since the v_f's are too high, the constant L/D trajectories for $B_c = 1.5$ slug/ft^2 are not admissible trajectories.

Since the peak heating and deceleration occur before the first pull up, it is not possible to obtain too much improvement in heating and deceleration by varying the control program. That is, before the peak heating and deceleration occur, there is only a period of about ten seconds in which the density is high enough to obtain substantial lift control, and thus the trajectories can't be shaped too much in this period. Thus, for the two control programs, even though one has negative lift and one positive lift over much of the period before the first pull up, there is still not too much difference in the maximum heating and deceleration. These maxima are more determined by entry angle and ballistic coefficient than by control program. However, since the optimum v_f is entirely determined by the final segment of the trajectories, and the first segment is only a two-point boundary value problem, it is possible to optimize the first segment in terms of heating and deceleration and still get the minimum v_f for the last segment, if a small improvement in deceleration and heating is desirable.

SUMMARY AND CONCLUSIONS

We have shown how a simple scheme of variable lift control can be used to double the magnitude of the ballistic coefficient which can be used to obtain a given terminal velocity for a given atmosphere density profile. We have also been shown that by determining the control program as a function of altitude, velocity and flight path angle, control programs can be designed to take into account some of the uncertainties in the Martian atmosphere; however, further studies on the possible variation in the atmosphere are needed. Also, obtaining a more precise knowledge of the Martian atmosphere will of course facilitate the design of the control program. Although the control programs in this paper were determined as a function of γ, v, and h, if it is decided that the on board measurement of these variables is too difficult, it is likely that control programs can be determined just as readily as a function of other variables, such as deceleration, dynamic pressure, and derivatives of the state variables. Finally, although an entry velocity of 26,000 ft/sec was used for the studies in this paper, the procedures developed may very easily be applied to other entry velocities, including orbital velocity, and some of the results, such as the minimum terminal velocity for a given ballistic coefficient and atmosphere, were shown to be the same for all entry velocities.

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