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A METHOD OF CHARACTERISTICS SOLUTION FOR THE EQUATIONS GOVERNING THE UNSTEADY FLOW OF LIQUIDS IN CLOSED SYSTEMS

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ABSTRACT

This report is a study of the unsteady flow of liquids in closed, unbranched systems. It includes the derivations of the governing equations and their method of characteristics solution for systems containing both distributed and concentrated losses. The appendix contains the resulting computer program and sample outputs from this program.

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NOMENCLATURE

a - Acceleration, speed of sound in fluid in pipe

f - Moody friction factor

g_c - Constant relating force, mass, and acceleration (32.174) $\left(\frac{\text{ft-lbm}}{\text{lbf-sec}^2}\right)$

m - Mass

t - Time

t_W - Pipe wall thickness

x - Spatial coordinate

A - Pipe cross-sectional area

A_W - Wetted pipe wall area

C_F - Fanning friction factor

C_1 - Constant which relates the influence of constraints on longitudinal movements of the pipe to stresses on a transverse cross-section of the pipe

D - Pipe diameter

E	-	Pipe wall material modulus of elasticity
F	-	Force
K	-	Flow loss coefficient
L	-	Length
P	-	Pressure
R	-	Pipe radius
V	-	Velocity, volume
β	-	Fluid bulk modulus
ϵ	-	Strain
ζ	-	Slope of characterictic curves
λ	-	Constant multiplier
μ	-	Poisson ratio for pipe wall material
ρ	-	Fluid density
σ	-	Stress
τ_W	-	Shearing stress at wall

SUBSCRIPTS

f	-	Friction term
o	-	Over-all
t	-	Time t
t + dt	-	Time t + dt
x	-	Longitudinal axis of pipe
Ave	-	Average
B	-	Body force
H	-	Hydrostatic
L	-	Left-hand face of fluid element
R	-	Right-hand face of fluid element
Tot	-	Total
W	-	Wetted area, pipe wall
1	-	Longitudinal, conditions upstream of a concentrated loss
2	-	Lateral, conditions downstream of a concentrated loss

SUPERSCRIPTS

'	-	Denotes static pressure including the hydrostatic pressure

I. PHENOMENOLOGICAL DESCRIPTION OF THE UNSTEADY FLOW OF LIQUIDS IN CLOSED SYSTEMS

1.1 The unsteady flow of liquids in closed systems is most commonly known as waterhammer. Before beginning a detailed mathematical exploration of waterhammer, it will prove informative to discuss some of the more salient features of this phenomenon on a physical basis. This task is best accomplished by considering the academic case of instantaneous valve closure for the simple frictionless system shown in Figure 1(a).

1.2 Initially the fluid in the pipe is flowing with a uniform velocity V and at the reservoir pressure. At time $t = 0$, the valve is closed instantaneously thereby initiating the following sequence of events:

1. The fluid immediately adjacent to the valve is brought to rest and its kinetic energy is changed to elastic strain energy which compresses the fluid and stretches the pipe walls. This process, called a positive wave, propagates with acoustic velocity back along the pipe until it reaches the reservoir, Figure 1(b). The fluid behind the wave is thus at zero velocity and at a pressure higher than that of the reservoir, Figure 1(c).

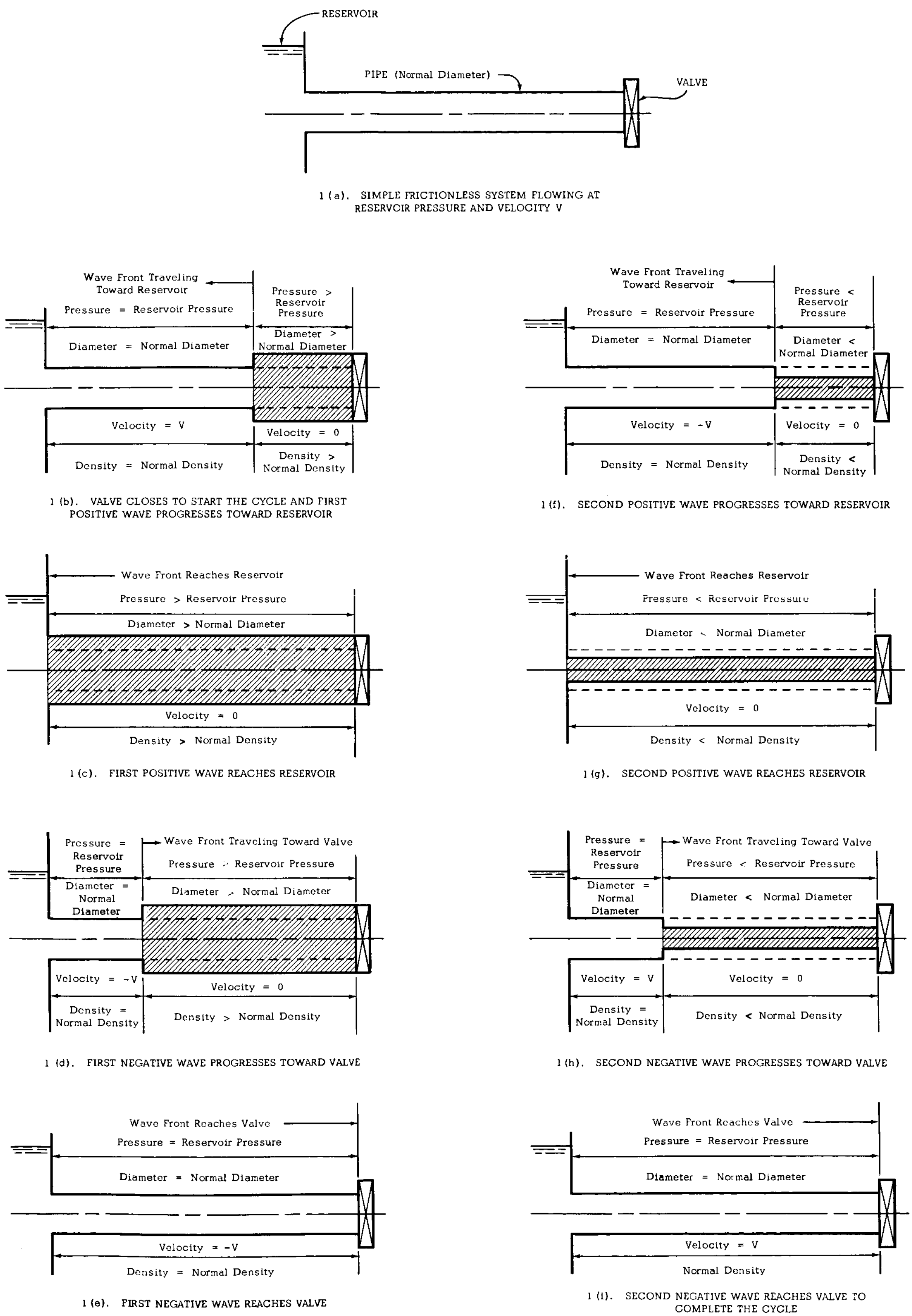
2. When this wave reaches the reservoir, the higher pressure in the pipe causes the fluid at the pipe inlet to flow back into the reservoir, thus lowering the pressure to that of the reservoir and causing

the pipe walls to return to their initial size. Since the system is assumed to be frictionless, the elastic strain energy is recovered entirely resulting in a velocity -V. This so-called negative wave propagates back to the valve with acoustic velocity, Figure I(d). Behind the wave, the fluid is at the reservoir pressure, flows with velocity -V, and the pipe walls are at their initial size, Figure 1(e).

3. After the negative wave reaches the valve, the inertia of the fluid and the pipe walls causes the fluid at the valve to drop below the reservoir pressure and the pipe walls to contract below their originaly size, Figure 1(f). This wave propagates back to the reservoir at the acoustic velocity leaving the fluid in the pipe at zero velocity, with a pressure less than that of the reservoir, and with the pipe walls contracted to less than their original size, Figure 1(g).

4. When this wave reaches the reservoir, the higher pressure in the reservoir causes a flow from the reservoir into the pipe, thus returning the fluid to its initial velocity and pressure and the pipe walls to their initial size, Figures 1(h) and 1(i).

1.3 Since the system being considered is frictionless, this cycle would be repeated indefinitely without attenuation. In a real system viscous dissipation would damp out the pressure surging in several cycles and the fluid would eventually come to rest at the reservoir pressure.



1 (a). SIMPLE FRICTIONLESS SYSTEM FLOWING AT RESERVOIR PRESSURE AND VELOCITY V

1 (b). VALVE CLOSES TO START THE CYCLE AND FIRST POSITIVE WAVE PROGRESSES TOWARD RESERVOIR

1 (f). SECOND POSITIVE WAVE PROGRESSES TOWARD RESERVOIR

1 (c). FIRST POSITIVE WAVE REACHES RESERVOIR

1 (g). SECOND POSITIVE WAVE REACHES RESERVOIR

1 (d). FIRST NEGATIVE WAVE PROGRESSES TOWARD VALVE

1 (h). SECOND NEGATIVE WAVE PROGRESSES TOWARD VALVE

1 (e). FIRST NEGATIVE WAVE REACHES VALVE

1 (i). SECOND NEGATIVE WAVE REACHES VALVE TO COMPLETE THE CYCLE

FIGURE 1. WATERHAMMER WAVE CYCLE FOR A SIMPLE, FRICTIONLESS SYSTEM WITH AN INSTANTANEOUS VALVE CLOSURE

II. ANALYTICAL DESCRIPTION OF WATERHAMMER

INTRODUCTORY REMARKS

2.1 In general, a fluid flow field is completely described by the simultaneous solutions of the momentum, continuity, energy, and state equations. It is well known, however, that for an isothermal liquid flow field, or a liquid flow field having relatively small temperature gradients, the momentum and continuity equations become uncoupled from the energy and state equations. This report presents a study of these specific cases. Thus, the spatial distributions of pressure and velocity in the flow field will be determined by the simultaneous solutions of the momentum and continuity equations alone.

BASIC ASSUMPTIONS

2.2 The derivations of the momentum and continuity equations are based on the following assumptions:

a. The flow is one-dimensional, that is, the flow variables and fluid properties vary in the flow direction only and are therefore constant across any transverse cross section of the pipe.

b. The static pressure at every point in the flow field always exceeds the vapor pressure of the fluid.

c. The pipes are full at all times.

Assumptions (a) and (b) are necessary to assure a single-phase (liquid) flow field.

d. Pipes have circular cross sections.

e. Stresses in pipes are always below the elastic limit.

f. "End effects" on stresses in pipes are negligible.

g. Pipe geometry is such that the "thin wall" case is valid.

h. Pipe and liquid are perfectly elastic (all energy dissipation is due to shearing stresses at the walls).

DERIVATION OF THE MOMENTUM EQUATION

2.3 Since the fluid under consideration is a continuous media, it is reasonable to assume that the fluid properties vary in a smooth, continuous manner in a given region. If these variations and their derivatives are continuous, it is possible to express a fluid property or flow variable at a given point in the flow field in terms of the same property or flow variable at a neighboring point. This relationship is given by a Taylor series expansion (References [1] and [2])[1/] as:

$$f(b) = f(a) + \left(\frac{\partial f}{\partial x}\right)_{x=a} (b-a) + \frac{1}{2!}\left(\frac{\partial^2 f}{\partial x^2}\right)_{x=a} (b-a)^2 + \frac{1}{3!}\left(\frac{\partial^3 f}{\partial x^3}\right)_{x=a} (b-a)^3 + \ldots\ldots \qquad (2.1)$$

1/ Numbers in brackets designate references at end of report.

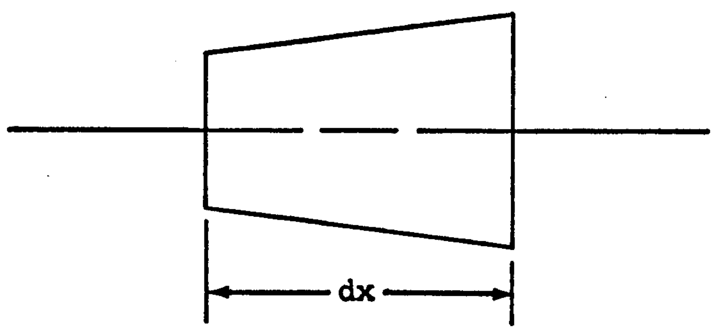
As point b approaches point a, the quantity (b - a) approaches the infinitesimally small variation dx and the higher order terms in the series become negligible when compared with the first two terms. Therefore, the series may be approximated by:

$$f(b) = f(a) + \left(\frac{\partial f}{\partial x}\right)_{x=a} (b-a) \tag{2.2}$$

or, in general terms of the spatial coordinate x,

$$f(x + \Delta x) = f(x) + \left(\frac{\partial f}{\partial x}\right) (dx) \ . \tag{2.3}$$

These variations for an incremented length of pipe and fluid are shown in Figure 2.



Spatial coordinate	x	$x + dx$
Diameter	D	$D + \frac{\partial D}{\partial x} dx$
Cross-sectional area	A	$A + \frac{\partial A}{\partial x} dx$
Pressure	P	$P' + \frac{\partial P'}{\partial x} dx$
Density	ρ	$\rho + \frac{\partial \rho}{\partial x} dx$

FIGURE 2. VARIATIONS IN FLUID PROPERTIES AND FLOW VARIABLES FOR AN INCREMENTAL LENGTH OF LINE AND FLUID

Newton's Second Law For the Element

$$\sum F = \frac{m a}{g_c} \tag{2.4}$$

where: the positive direction for forces, accelerations, and velocities is to the right.

Forces Acting on the Element

The force on the left-hand face is given by:

$$F_L = P'A \quad . \tag{2.5}$$

Similarly, the force on the right-hand face is given by:

$$F_R = \left(P' + \frac{\partial P'}{\partial x} dx\right)\left(A + \frac{\partial A}{\partial x} dx\right)$$

or,

$$F_R = (P')(A) + \left(\frac{\partial P'}{\partial x}\right)\left(\frac{\partial A}{\partial x}\right)(dx)^2$$

$$+ (A)\left(\frac{\partial P'}{\partial x}\right)(dx) + (P')\left(\frac{\partial A}{\partial x}\right)(dx) \quad . \tag{2.6}$$

Neglecting the higher order term:

$$F_R = (P')(A) + (A)\left(\frac{\partial P'}{\partial x}\right)(dx) + (P')\left(\frac{\partial A}{\partial x}\right)(dx) \quad . \tag{2.7}$$

The frictional force due to shearing stresses at the wall is:

$$F_f = (\tau_W)(\Delta A_W) . \tag{2.8}$$

The Fanning friction factor is defined as:

$$C_F = \frac{\tau_W}{\left(\frac{\rho V^2}{2 g_c}\right)} . \tag{2.9}$$

Solving for the shearing stress at the wall:

$$\tau_W = (C_F)\left(\frac{\rho V^2}{2 g_c}\right) . \tag{2.10}$$

The relationship between the Fanning and Moody friction factors is known to be:

$$C_F = \frac{f}{4} . \tag{2.11}$$

Wetted area:

$$\Delta A_W = \text{(average circumference)(length of element)},$$

or

$$\Delta A_W = \left[\frac{1}{2}\right]\left[\pi D + \pi D + \left(\frac{\partial D}{\partial x}\right)(dx)\right]\left[dx\right] . \tag{2.12}$$

$$\Delta A_W = (\pi)(D)(dx) + \left(\frac{1}{2}\right)\left(\frac{\partial D}{\partial x}\right)(dx)^2 \qquad (2.13)$$

Neglecting the higher order term:

$$\Delta A_W = (\pi)(D)(dx) \quad . \qquad (2.14)$$

Substituting Eqs. (2.10), (2.11), and (2.14) in Eq. (2.8):

$$F_f = \left(\frac{f}{4}\right)\left(\frac{\rho V^2}{2 g_c}\right)(\pi)(D)(dx) \qquad (2.15)$$

or, finally

$$F_f = \left(\frac{\pi f D \rho V^2}{8 g_c}\right)(dx) \quad . \qquad (2.16)$$

<u>Mass of the Element</u>

$$m = \text{(average density)(volume of element)} \qquad (2.17)$$

$$m = \left[\frac{1}{2}\right]\left[\rho + \rho + \left(\frac{\partial \rho}{\partial x}\right)(dx)\right]\left[\frac{1}{2}\right]$$

$$\left[A + A + \left(\frac{\partial A}{\partial x}\right)(dx)\right]\left[dx\right] \qquad (2.18)$$

Expanding this expression:

$$m = \left[\frac{1}{4}\right]\left[(4)(\rho)(A)(dx) + \left(\frac{\partial \rho}{\partial x}\right)\left(\frac{\partial A}{\partial x}\right)(dx)^3 + (2)(A)\left(\frac{\partial \rho}{\partial x}\right)(dx)^2 + (2)(\rho)\left(\frac{\partial A}{\partial x}\right)(dx)^2\right] \quad (2.19)$$

Neglecting the higher order terms:

$$m = (\rho)(A)(dx) \quad (2.20)$$

Body force on element — assume: component of the body force along the axis of the element against the direction of the flow.

$$F_B = \frac{m g_x}{g_c} \quad (2.21)$$

Substituting Eq. (2.20) in Eq. (2.21):

$$F_B = \left(\frac{\rho A g_x}{g_c}\right)(dx) \quad (2.22)$$

Acceleration of the Element

$$a = \frac{dV}{dt} \quad (2.23)$$

Since the unsteady flow case is being considered:

$$V = f(x, t) \quad , \tag{2.24}$$

so that

$$\frac{dV}{dt} = \left(\frac{\partial V}{\partial x}\right)\left(\frac{dx}{dt}\right) + \left(\frac{\partial V}{\partial t}\right) . \tag{2.25}$$

But

$$V = \frac{dx}{dt} \quad , \tag{2.26}$$

which gives:

$$a = \frac{dV}{dt} = \left[\left(\frac{\partial V}{\partial t}\right) + (V)\left(\frac{\partial V}{\partial x}\right)\right] . \tag{2.27}$$

Substituting Eqs. (2.5), (2.7), (2.16), (2.20), (2.22), and 2.27) in Eq. (2.4):

$$F_L - F_R - F_f - F_B = \left[\frac{\rho A}{g_c}\right]\left[\left(\frac{\partial V}{\partial t}\right) + (V)\left(\frac{\partial V}{\partial x}\right)\right]\left[dx\right] \tag{2.28}$$

or

$$\left[(P')(A) - (P')(A) - (A)\ \frac{\partial P'}{\partial x}\ (dx) - (P')\left(\frac{\partial A}{\partial x}\right)(dx)\right.$$

$$\left. - \left(\frac{\pi f D \rho V^2}{8 g_c}\right)(dx) - \left(\frac{\rho A g_x}{g_c}\right)(dx)\right]$$

$$= \left[\frac{\rho A}{g_c}\right]\left[\left(\frac{\partial V}{\partial t}\right) + (V)\left(\frac{\partial V}{\partial x}\right)\right]\left[dx\right] \tag{2.29}$$

which reduces to:

$$(A)\left(\frac{\partial P'}{\partial x}\right) + (P')\left(\frac{\partial A}{\partial x}\right) + \left(\frac{\pi f D \rho V^2}{8 g_c}\right)$$

$$= -\left[\frac{\rho A}{g_c}\right]\left[\left(\frac{\partial V}{\partial t}\right) + (V)\left(\frac{\partial V}{\partial x}\right)\right] - \left[\frac{\rho A g_x}{g_c}\right] . \qquad (2.30)$$

It has been empirically determined that the second term in the left-hand member of Eq. (2.30) is negligibly small when compared with the remaining terms in the equation. This gives:

$$(A)\left(\frac{\partial P'}{\partial x}\right) + \left(\frac{\pi f D \rho V^2}{8 g_c}\right)$$

$$= -\left[\frac{\rho A}{g_c}\right]\left[\left(\frac{\partial V}{\partial t}\right) + (V)\left(\frac{\partial V}{\partial x}\right)\right] - \left[\frac{\rho A g_x}{g_c}\right] . \qquad (2.31)$$

Since the line has a circular cross-section:

$$A = \frac{\pi D^2}{4} . \qquad (2.32)$$

Dividing both sides of Eq. (2.31) by the area and substituting Eq. (2.32):

$$\frac{\partial P'}{\partial x} + \frac{f \rho V^2}{2 D g_c}$$

$$= -\left[\frac{\rho}{g_c}\right]\left[\left(\frac{\partial V}{\partial t}\right) + (V)\left(\frac{\partial V}{\partial x}\right)\right] - \left[\frac{\rho g_x}{g_c}\right] . \qquad (2.33)$$

But;

$$P' = P + P_H \tag{2.34}$$

so that:

$$\frac{\partial P'}{\partial x} = \frac{\partial P}{\partial x} + \frac{\partial P_H}{\partial x} \quad . \tag{2.35}$$

Substituting Eq. (2.35) in Eq. (2.33):

$$\frac{\partial P}{\partial x} + \frac{\partial P_H}{\partial x} + \frac{f\rho V^2}{2Dg_c}$$

$$= - \left[\frac{\rho}{g_c}\right] \left[\left(\frac{\partial V}{\partial t}\right) + (V) \left(\frac{\partial V}{\partial x}\right)\right] - \left[\frac{\rho g_x}{g_c}\right] \quad . \qquad (2.36$$

The definition of hydrostatic pressure gives:

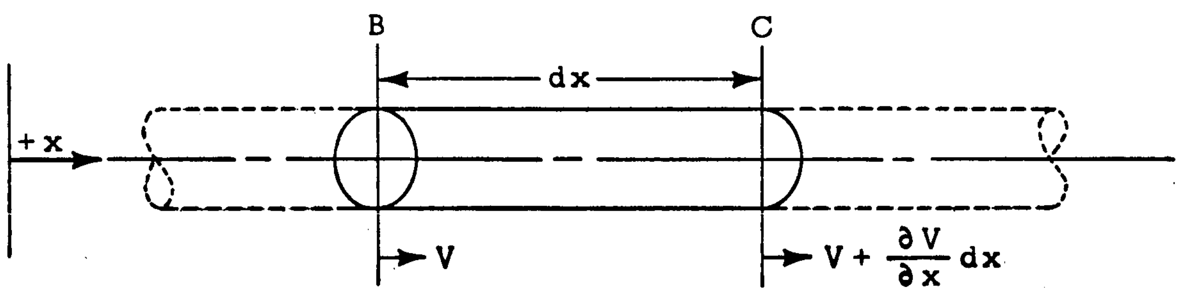
$$\frac{\partial P_H}{\partial x} = - \frac{\rho g_x}{g_c} \quad . \tag{2.37}$$

Substituting this result in Eq. (2.36) (Ref. [3]), gives the final form of the momentum equation:

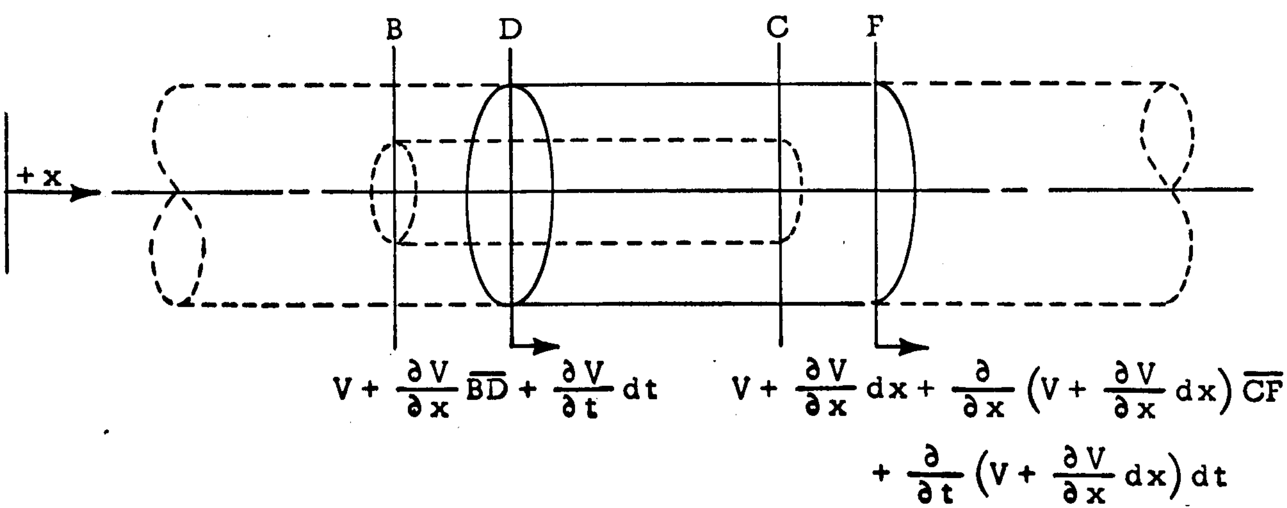
$$\boxed{\frac{\partial P}{\partial x} + \frac{f\rho V^2}{2Dg_c} = - \left[\frac{\rho}{g_c}\right]\left[\left(\frac{\partial V}{\partial t}\right) + (V) \left(\frac{\partial V}{\partial x}\right)\right]} \quad . \tag{2.38}$$

DERIVATION OF THE CONTINUITY EQUATION

2.4 This derivation follows that given by Parmakian (Ref. [4]). It consists of equating two expressions for the change in length of a differential fluid element during an infinitesimal time step, dt. The first of these expressions is derived from purely kinematic considerations, while the second expression is obtained by considering the changes in shape of the element due to the stretching of the pipe walls and compression of the fluid. The fluid element in shown in Figure 3:



3(a). Fluid element at time t



3(b). Fluid element at time t + dt

FIGURE 3. CHANGE IN CONFIGURATION OF A FLUID ELEMENT

Change in Length of the Element from Kinematic Considerations

The total change in length of the element dx in moving from $\overline{BC}$ to $\overline{DF}$ is:

$$(\Delta L)_{Tot} = \overline{BD} - \overline{CF} \quad . \tag{2.39}$$

The average velocity of face B in moving from B to D during the time interval dt may be computed as:

$$V_{Ave} = \frac{V_t + V_{t+dt}}{2} \quad , \tag{2.40}$$

or, substituting the velocities shown in Figure 3:

$$V_{Ave} = \frac{(V) + \left[V + \frac{\partial V}{\partial x}\overline{BD} + \frac{\partial V}{\partial t}dt\right]}{2} \quad , \tag{2.41}$$

$$V_{Ave} = V + \frac{1}{2}\frac{\partial V}{\partial x}\overline{BD} + \frac{1}{2}\frac{\partial V}{\partial t}dt \quad . \tag{2.42}$$

The distance $\overline{BD}$ may now be computed as:

$$\overline{BD} = \left(V_{Ave}\right)dt = \left[V + \frac{1}{2}\frac{\partial V}{\partial x}\overline{BD} + \frac{1}{2}\frac{\partial V}{\partial t}dt\right]dt \quad , \tag{2.43}$$

$$\overline{BD} = V\,dt + \frac{1}{2}\frac{\partial V}{\partial x}\overline{BD}\,dt + \frac{1}{2}\frac{\partial V}{\partial t}(dt)^2 \quad . \tag{2.44}$$

The average velocity of face C in moving from C to F during the time interval dt may be computed in a similar manner as follows:

$$V_{Ave} = \frac{V_t + V_{t+dt}}{2} \quad . \qquad (2.45)$$

Substituting the velocities shown in Figure 3:

$$V_{Ave} = \left[\tfrac{1}{2}\right]\left[V + \frac{\partial V}{\partial x}dx + V + \frac{\partial V}{\partial x}dx + \frac{\partial}{\partial x}\left(V + \frac{\partial V}{\partial x}dx\right)\overline{CF} + \frac{\partial}{\partial t}\left(V + \frac{\partial V}{\partial x}dx\right)dt\right], \qquad (2.46)$$

which reduces to:

$$V_{Ave} = V + \frac{\partial V}{\partial x}dx + \tfrac{1}{2}\frac{\partial}{\partial x}\left(V + \frac{\partial V}{\partial x}dx\right)\overline{CF} + \tfrac{1}{2}\frac{\partial}{\partial t}\left(V + \frac{\partial V}{\partial x}dx\right)dt \quad , \qquad (2.47)$$

and $\overline{CF}$ is given by:

$$\overline{CF} = \left(V_{Ave}\right)dt \qquad (2.48)$$

$$\overline{CF} = \left[V + \frac{\partial V}{\partial x}dx + \tfrac{1}{2}\frac{\partial}{\partial x}\left(V + \frac{\partial V}{\partial x}dx\right)\overline{CF} + \tfrac{1}{2}\frac{\partial}{\partial t}\left(V + \frac{\partial V}{\partial x}dx\right)dt\right]\left[dt\right] \qquad (2.49)$$

or;

$$\overline{CF} = V\,dt + \frac{\partial V}{\partial x} dx\,dt + \tfrac{1}{2}\frac{\partial V}{\partial x}\overline{CF}\,dt + \tfrac{1}{2}\frac{\partial^2 V}{\partial x^2}\overline{CF}\,dx\,dt$$

$$+ \tfrac{1}{2}\frac{\partial V}{\partial t}(dt)^2 + \tfrac{1}{2}\frac{\partial^2 V}{\partial x \partial t}dx(dt)^2 \quad . \qquad (2.50)$$

Finally, noting that the distances $\overline{BD}$ and $\overline{CF}$ are of a smaller order than dx and dt, the total change in length of the element may be computed as:

$$(\Delta L)_{Tot} = (\overline{BD} - \overline{CF}) = -\frac{\partial V}{\partial x} dx\,dt \qquad (2.51)$$

Change in Length of the Element Due to Stretching of the Pipe Walls and Compression of the Fluid

The change in the length of the element dx is caused by two factors:

(1) A change in the internal pressure causes the pipe to expand or contract, and the resulting change in cross-sectional area produces a change in length in order to contain the same volume of fluid.

(2) The change in internal pressure causes a change in the volume of the fluid and therefore a further change in the length of the element.

This total change $(\overline{BD} - \overline{CF})$ in the length of the element dx is now computed considering these two effects:

(1) This analysis assumes the pipe geometry is such that the thin-wall case is valid. "A pressure vessel is described as thin walled when the ratio of the wall thickness to the radius of the vessel is so small that the distribution of normal stress on a plane perpendicular to the surface of the shell is essentially uniform throughout the thickness of the shell. Actually this stress varies from a

maximum value at the inside surface to a minimum value at the outside surface of the shell, but it can be shown that if the ratio of the wall thickness to the inner radius of the vessel is less than .1, the maximum normal stress is not more than 5 per cent greater than the average." (Ref. [5])

For elastic deformations of a solid, the change in a linear dimension is given by:

$$\Delta L = (L)(\epsilon) \quad . \tag{2.52}$$

But;

$$\epsilon = \frac{\sigma}{E} \tag{2.53}$$

so that;

$$\Delta L = (L)\left(\frac{\sigma}{\epsilon}\right) \quad . \tag{2.54}$$

Applying this to the radius of a thin-wall cylinder:

$$\Delta R = R_{Ave}\left[\frac{(\Delta\sigma_2)_o}{E}\right] \quad . \tag{2.55}$$

where; the average radius is defined as:

$$R_{Ave} = R + \frac{t_w}{2} \quad . \tag{2.56}$$

In general, a change in the internal pressure in a cylinder will produce a change in the stresses on both the longitudinal and transverse planes. These stresses (Figure 4) are not independent, however, but are coupled through the Poisson ratio.

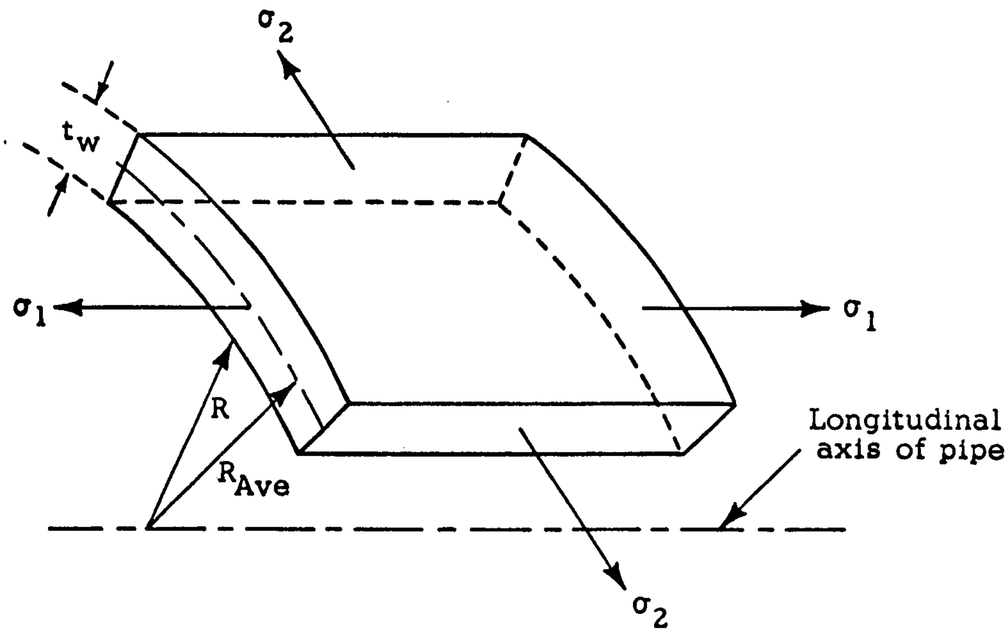


FIGURE 4. STRESSES ON LONGITUDINAL AND TRANSVERSE PLANES OF PIPE WALL ELEMENT

From the assumption of deformations in the elastic range only, it follows that:

$$\frac{\sigma_1}{\epsilon_1} = E \tag{2.57}$$

and,

$$\frac{\sigma_2}{\epsilon_2} = E \quad . \tag{2.58}$$

Equating (2.57) and (2.58):

$$\frac{\sigma_1}{\epsilon_1} = \frac{\sigma_2}{\epsilon_2} \tag{2.59}$$

or,

$$\sigma_2 = \left(\frac{\epsilon_2}{\epsilon_1}\right) \sigma_1 \quad . \tag{2.60}$$

But the ratio of the lateral strain ϵ_1 to the longitudinal strain ϵ_2 is defined as the Poisson ratio (μ), which is an empirically determined constant for each material. Thus,

$$\sigma_2 = \mu \sigma_1 \quad , \tag{2.61}$$

and therefore the over-all change in the stress on a longitudinal plane is:

$$\left(\Delta \sigma_2\right)_o = \left(\Delta \sigma_2 - \mu \Delta \sigma_1\right) \quad . \tag{2.62}$$

Substituting Eqs. (2.56) and (2.62) in Eq. (2.55):

$$\Delta R = \left(\frac{R + \frac{t_w}{2}}{E}\right) \left(\Delta \sigma_2 - \mu \Delta \sigma_1\right) \quad . \tag{2.63}$$

However, since the pipe is assumed to be thin walled, by definition:

$$t_w \leq (.1)(R) \tag{2.64}$$

or,

$$\frac{t_w}{2} \leq (.05)(R) \quad , \tag{2.65}$$

so that to a good approximation:

$$\Delta R = \left(\frac{R}{E}\right) \left(\Delta \sigma_2 - \mu \Delta \sigma_1\right) \quad . \tag{2.66}$$

Similarly, the change in the axial length of the element is found to be:

$$\Delta x = \left(\frac{dx}{E}\right)\left(\Delta\sigma_1 - \mu\Delta\sigma_2\right) \quad . \tag{2.67}$$

The volume enclosed by the stressed element is:

$$dV = (\pi)(R + \Delta R)^2 (dx + \Delta x) \quad . \tag{2.68}$$

The change in length of the original element compatible with this change in volume is:

$$\Delta L = \left(\frac{\text{change in volume}}{\text{original area}}\right) = \frac{(\pi)(R + \Delta R)^2 (dx + \Delta x) - \pi R^2 dx}{\pi R^2} \quad . \tag{2.69}$$

Expanding Eq. (2.69) and neglecting the higher order terms:

$$\Delta L = \Delta x + \frac{(2)(\Delta R)}{R} dx \quad . \tag{2.70}$$

Substituting Eqs. (2.66) and (2.67) in Eq. (2.70):

$$\Delta L = \left[\left(\Delta\sigma_1\right)\left(1 - 2\mu\right) + \left(\Delta\sigma_2\right)\left(2 - \mu\right)\right]\left[\frac{dx}{E}\right] \quad . \tag{2.71}$$

The stress developed on the longitudinal plane of a pressurized pipe depends on the manner in which the movement of the pipe is constrained in the longitudinal direction. If the pipe is anchored at one end, free to move in the longitudinal direction throughout its length and has no expansion joints, then the stresses produced by a change of internal pressure dP are given by:

$$\Delta \sigma_1 = \left(\frac{R}{2t_w}\right) dP \tag{2.72}$$

and,

$$\Delta \sigma_2 = \left(\frac{R}{t_w}\right) dP \tag{2.73}$$

Substituting Eqs. (2.72) and (2.73) in Eq. (2.71) yields:

$$\Delta L = \left[\left(\frac{R}{2t_w}\right)(dP)(1-2\mu) + \left(\frac{R}{t_w}\right)(dP)(2-\mu)\right]\left[\frac{dx}{E}\right] \tag{2.74}$$

or,

$$\Delta L = \left(\frac{5}{2} - 2\mu\right)\left(\frac{R}{Et_w}\right)(dP)(dx) \tag{2.75}$$

If the pipe is anchored against longitudinal movement throughout its length:

$$\Delta \sigma_2 = \left(\frac{R}{t_w}\right)(dP) \tag{2.76}$$

and,

$$\Delta \sigma_1 = \mu\left(\Delta \sigma_2\right) \tag{2.77}$$

or,

$$\Delta \sigma_1 = \left(\frac{\mu R}{t_w}\right)(dP) \tag{2.78}$$

Substituting Eqs. (2.76) and (2.78) in Eq. (2.71):

$$\Delta L = \left[\left(\frac{\mu R}{t_w}\right)(dP)(1-2\mu) + \left(\frac{R}{t_w}\right)(dP)(2-\mu)\right]\left[\frac{dx}{E}\right] \tag{2.79}$$

or,

$$\Delta L = (2)(1 - \mu^2)\left(\frac{R}{E t_w}\right)(dP)(dx) \quad . \qquad (2.80)$$

If the pipe has an expansion joint between anchors, then:

$$\Delta \sigma_1 = 0 \qquad (2.81)$$

and,

$$\Delta \sigma_2 = \left(\frac{R}{t_w}\right)(dp) \quad . \qquad (2.82)$$

Substituting Eqs. (2.81) and (2.82) in Eq. (2.71):

$$\Delta L = (2 - \mu)\left(\frac{R}{E t_w}\right)(dP)(dx) \quad . \qquad (2.83)$$

Thus, by defining the constant C_1, Eqs. (2.75), (2.80), and (2.83) may be summarized as:

$$\Delta L = \left(\frac{C_1 R}{E t_w}\right)(dP)(dx) \quad . \qquad (2.84)$$

where: $C_1 = \left(\frac{5}{2} - 2\mu\right)$ (anchored at one end only)

$C_1 = (2)\left(1 - \mu^2\right)$ (anchored throughout entire length)

$C_1 = (2 - \mu)$ (expansion joint between anchors)

(2) The change in the volume of fluid in the element due to a change in pressure dP is:

$$dV = \left(\frac{V}{\beta}\right)(dP) \tag{2.85}$$

or,

$$dV = \left(\frac{\pi R^2}{\beta}\right)(dP)(dx) \quad . \tag{2.86}$$

The corresponding change in the length of the element is:

$$\Delta L = \frac{dV}{A} \quad . \tag{2.87}$$

Substituting Eq. (2.86) in Eq. (2.87):

$$\Delta L = \left(\frac{1}{\beta}\right)(dP)(dx) \quad . \tag{2.88}$$

The total change in the length of the fluid element due to stretching of the pipe walls and a change in pressure may now be found by adding Eqs. (2.84) and (2.88).

$$(\Delta L)_{Tot} = \left(\frac{C_1 R}{E t_w} + \frac{1}{\beta}\right)(dP)(dx) \quad . \tag{2.89}$$

Equations (2.51) and (2.89) give two independently derived expressions for the change in length of the fluid element during the time interval dt. Equating these expressions:

$$-\left(\frac{\partial V}{\partial x}\right)(dt) = \left(\frac{C_1 R}{E t_w} + \frac{1}{\beta}\right)(dP) \quad . \tag{2.90}$$

But, since the unsteady flow case is being considered:

$$P = P(x, t) \tag{2.91}$$

Taking the differential of P:

$$dP = \left(\frac{\partial P}{\partial x}\right)(dx) + \left(\frac{\partial P}{\partial t}\right)(dt) \tag{2.92}$$

and,

$$dP = \left(\frac{\partial P}{\partial x}\right)\left(\frac{dx}{dt}\right)(dt) + \left(\frac{\partial P}{\partial t}\right)(dt) \quad , \tag{2.93}$$

but,

$$V = \frac{dx}{dt} \quad . \tag{2.94}$$

Substituting Eqs. (2.93) and (2.94) in Eq. (2.90) and rearranging gives:

$$\frac{\partial P}{\partial t} + (V)\left(\frac{\partial P}{\partial x}\right) = -\left(\frac{1}{\frac{C_1 R}{E t_w} + \frac{1}{\beta}}\right)\left(\frac{\partial V}{\partial x}\right) \quad . \tag{2.95}$$

Multiply and divide the right-hand member of Eq. (2.95) by $\frac{\rho}{g_c}$ to get:

$$\frac{\partial P}{\partial t} + (V)\left(\frac{\partial P}{\partial x}\right) = -\left[\frac{\rho}{g_c}\right]\left[\frac{1}{\left(\frac{\rho}{g_c}\right)\left(\frac{C_1 R}{E t_w} + \frac{1}{\beta}\right)}\right]\left[\frac{\partial V}{\partial x}\right] . \tag{2.96}$$

Define the speed of sound in the fluid as:

$$a \equiv \sqrt{\frac{1}{\left(\frac{\rho}{g_c}\right)\left(\frac{C_1 R}{E t_w} + \frac{1}{\beta}\right)}} \quad . \tag{2.97}$$

Substitute Eq. (2.97) in Eq. (2.96) to obtain the continuity equation:

$$\boxed{\frac{\partial P}{\partial t} + (V)\left(\frac{\partial P}{\partial x}\right) = -\left(\frac{a^2 \rho}{g_c}\right)\left(\frac{\partial V}{\partial x}\right)} \quad . \qquad (2.98)$$

III. SOLUTION OF THE GOVERNING EQUATIONS BY THE METHOD OF CHARACTERISTICS

CLASSIFICATION OF THE GOVERNING EQUATIONS

3.1 The momentum and continuity equations as derived in the previous chapter are:

$$\frac{\partial P}{\partial x} + \frac{f \rho V^2}{2 D g_c} = - \left[\frac{\rho}{g_c}\right] \left[\frac{\partial V}{\partial t} + (V) \left(\frac{\partial V}{\partial x}\right)\right] \quad \text{(momentum)} \qquad (3.1)$$

and,

$$\frac{\partial P}{\partial t} + (V) \left(\frac{\partial P}{\partial x}\right) = - \left(\frac{a^2 \rho}{g_c}\right) \left(\frac{\partial V}{\partial x}\right) \qquad \text{(continuity)} \qquad (3.2)$$

These equations may be described as a set of quasi-linear, hyperbolic, partial differential equations of the first order in two dependent (P, V) and two independent (x, t) variables. As previously stated, their simultaneous solution, taken with the appropriate boundary conditions, gives a complete description of the flow field.

3.2 Because the equations are hyperbolic, both characteristics are real, and thus the equations are amenable to solution by the method of characteristics.

GENERAL REMARKS ON THE METHOD OF CHARACTERISTICS

3.3 The methods of solving quasi-linear partial differential equations generally fall in one of two main categories:

(1) Methods which reduce the complexity of the equations.

(2) Methods which reduce the partial differential equations to ordinary differential equations.

The first category contains the small disturbance (perturbation) methods which usually serve to linearize the equations.

The second category contains the method of characteristics, along with other methods such as self-similar solutions and integral relations.

3.4 The fundamental idea behind the method of characteristics is to find certain "characteristic curves" in the space-time domain along which the partial differential equation simplifies to an ordinary differential equation. This ordinary differential equation plus the ordinary differential equation which defines the characteristic curves themselves form a pair of equations which are entirely equivalent to the original partial differential equation.

3.5 In general, for a system of n equations with n dependent variables there will be n characteristic curves through every point in the space-time domain. Thus, for the system of Eqs. (3.1) and (3.2) there will be two characteristic curves through each point in the x-t plane.

APPLICATION OF THE METHOD OF CHARACTERISTICS TO THE WATERHAMMER EQUATIONS (Ref. [6])

3.6 Equations (3.1) and (3.2) may be rewritten respectively as:

$$J_1 = \left(\frac{g_c}{\rho}\right)\left(\frac{\partial P}{\partial x}\right) + \frac{fV^2}{2D} + \frac{\partial V}{\partial t} + (V)\left(\frac{\partial V}{\partial x}\right) = 0 \qquad (3.3)$$

and,

$$J_2 = \left(\frac{a^2 \rho}{g_c}\right)\left(\frac{\partial V}{\partial x}\right) + (V)\left(\frac{\partial P}{\partial x}\right) + \frac{\partial P}{\partial t} = 0 \qquad (3.4)$$

Forming a linear combination of Eqs. (3.3) and (3.4):

$$J = J_1 + \lambda J_2 \tag{3.5}$$

$$J = \left(\frac{g_c}{\rho}\right)\left(\frac{\partial P}{\partial x}\right) + \frac{fV^2}{2D} + \frac{\partial V}{\partial t} + (V)\left(\frac{\partial V}{\partial x}\right)$$

$$+ \lambda\left[\left(\frac{a^2\rho}{g_c}\right)\left(\frac{\partial V}{\partial x}\right) + (V)\left(\frac{\partial P}{\partial x}\right) + \frac{\partial P}{\partial t}\right] = 0 \ . \tag{3.6}$$

Grouping terms:

$$J = \lambda\left[\left(\frac{g_c}{\lambda\rho} + V\right)\left(\frac{\partial P}{\partial x}\right) + \frac{\partial P}{\partial t}\right]$$

$$+ \left[\left(V + \frac{\lambda a^2\rho}{g_c}\right)\left(\frac{\partial V}{\partial x}\right) + \frac{\partial V}{\partial t}\right] + \frac{fV^2}{2D} = 0 \ . \tag{3.7}$$

The simultaneous solutions of Eqs. (3.3) and (3.4) may be written in the form:

$$V = V(x, t) \tag{3.8}$$

and,

$$P = P(x, t) \ , \tag{3.9}$$

from which,

$$\frac{dV}{dt} = \left(\frac{\partial V}{\partial x}\right)\left(\frac{dx}{dt}\right) + \frac{\partial V}{\partial t} \tag{3.10}$$

and,

$$\frac{dP}{dt} = \left(\frac{\partial P}{\partial x}\right)\left(\frac{dx}{dt}\right) + \frac{\partial P}{\partial t} \quad . \tag{3.11}$$

Thus, inspection of Eq. (3.7) shows that the term in the first bracket may be replaced by $\frac{dP}{dt}$ if

$$\frac{g_c}{\lambda \rho} + V = \frac{dx}{dt} \quad , \tag{3.12}$$

and that the term in the second bracket may be replaced by $\frac{dV}{dt}$ if

$$\frac{\lambda a^2 \rho}{g_c} + V = \frac{dx}{dt} \quad . \tag{3.13}$$

Equating (3.12) and (3.13):

$$\frac{dx}{dt} = \frac{g_c}{\lambda \rho} + V = \frac{\lambda a^2 \rho}{g_c} + V \tag{3.14}$$

or,

$$\lambda^2 = \left(\frac{g_c}{a \rho}\right)^2 \quad , \tag{3.15}$$

which gives:

$$\lambda = \pm \frac{g_c}{a \rho} \quad . \tag{3.16}$$

Therefore, when λ assumes one of the values given by Eq. (3.16), Eq. (3.7) reduces to the general characteristic equation for P and V as follows:

$$J = \frac{dV}{dt} + \lambda \frac{dP}{dt} + \frac{fV^2}{2D} = 0 \tag{3.17}$$

or,

$$(J)(dt) = dV + \lambda dP + \left(\frac{fV^2}{2D}\right)(dt) = 0 \quad . \tag{3.18}$$

Substituting the values of λ given by Eq. (3.16) in Eq. (3.18) gives the two equations for P and V along the characteristic curves in the x-t plane ,

$$(J)(dt) = dV + \left(\frac{g_c}{a\rho}\right)(dP) + \left(\frac{fV^2}{2D}\right)(dt) = 0 \tag{3.19}$$

and

$$(J)(dt) = dV - \left(\frac{g_c}{a\rho}\right)(dP) + \left(\frac{fV^2}{2D}\right)(dt) = 0 \quad . \tag{3.20}$$

The ordinary differential equations which describe the slopes of the characteristic curves in the x-t plane are found by substituting the values for λ in Eqs. (3.12) and (3.13).

$$\frac{dx}{dt} = V + \left(\frac{g_c}{a\rho}\right)\left(\frac{a^2\rho}{g_c}\right) \tag{3.21}$$

$$\frac{dx}{dt} = V + a \tag{3.22}$$

or,

$$\frac{dt}{dx} = \frac{1}{V + a} \equiv \zeta_+ \tag{3.23}$$

and,

$$\frac{dx}{dt} = V + \left(\frac{-a\rho}{g_c}\right)\left(\frac{g_c}{\rho}\right) \tag{3.24}$$

or,

$$\frac{dt}{dx} = \frac{1}{V - a} = \zeta_- \quad . \tag{3.25}$$

Thus, the original partial differential Eqs. (3.1) and (3.2) have been replaced by two ordinary differential equations [(3.19) and (3.20)] which describe the behavior of P and V along the curves in the x-t plane described by Eqs. (3.23) and (3.25). Summarizing these equations:

<u>Along the Positive Characteristic (C_+ direction)</u>

$$dt - \frac{dx}{V + a} = 0 \tag{3.26}$$

and,

$$dV + \left(\frac{g_c}{a\rho}\right)(dP) + \left(\frac{fV^2}{2D}\right)(dt) = 0 \quad . \tag{3.27}$$

<u>Along the Negative Characteristic (C_- direction)</u>

$$dt - \frac{dx}{V - a} = 0 \tag{3.28}$$

and,

$$dV - \left(\frac{g_c}{a\rho}\right)(dP) + \left(\frac{fV^2}{2D}\right)(dt) = 0 \quad . \tag{3.29}$$

NUMERICAL SOLUTION OF THE CHARACTERISTIC EQUATIONS

Method of Integration

3.7 The set of characteristic equations, (3.26) through (3.29), will be integrated by using the first-order finite difference approximation:

$$\int_{x_o}^{x_1} f(x)\,dx \approx f(x_o)(x_1 - x_o) \quad , \tag{3.30}$$

in conjunction with a simple extrapolation procedure. It has been shown by Roberts (Ref. [7]) that this procedure, which is discussed in Section 3.10 of this report, increases the accuracy of the first-order approximation to that of a second-order approximation of the type:

$$\int_{x_o}^{x_1} f(x)\,dx \approx \left[\tfrac{1}{2}\right]\left[f(x_o) + f(x_1)\right]\left[x_1 - x_o\right] \quad . \tag{3.31}$$

Furthermore, the second-order approximation would require the use of a relatively time-consuming iterative technique.

Integration of the Characteristic Equations

Consider the intersection of two characteristics, C_+ and C_-, as shown in Figure 5.

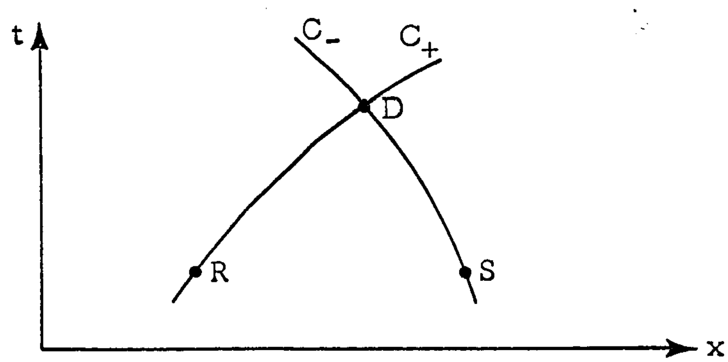


FIGURE 5. INTERSECTION OF TWO CHARACTERISTICS AT A GENERAL POINT D IN THE x-t PLANE

Assuming the value of x at R is denoted by x_R, and so on, if the values of x_R, x_S, t_R, t_S, V_R, V_S, P_R, and P_S are known, then the values of x_D, t_D, V_D, and P_D can be determined by using the first-order approximation to integrate the characteristic equations. Integrating along the right-running (C_+) characteristic:

$$\int_{t_R}^{t_D} dt - \int_{x_R}^{x_D} \left(\frac{1}{V+a}\right) dx = 0 \tag{3.32}$$

or,

$$\left(t_D - t_R\right) - \left(\frac{1}{V+a}\right)_R \left(x_D - x_R\right) = 0 \tag{3.33}$$

and,

$$\int_{V_R}^{V_D} dV + \left(\frac{g_c}{a\rho}\right) \int_{P_R}^{P_D} dP + \int_{t_R}^{t_D} \left(\frac{fV^2}{2D}\right) dt = 0 \tag{3.34}$$

or,

$$\left(V_D - V_R\right) + \left(\frac{g_c}{a\rho}\right) \left(P_D - P_R\right) + \left(\frac{fV^2}{2D}\right)_R \left(t_D - t_R\right) = 0 \tag{3.35}$$

Integrating along the left-running (C_-) characteristic:

$$\int_{t_S}^{t_D} dt - \int_{x_S}^{x_D} \left(\frac{1}{V-a}\right) dx = 0 \tag{3.36}$$

or

$$\left(t_D - t_S\right) - \left(\frac{1}{V-a}\right)_S \left(x_D - x_S\right) = 0 \tag{3.37}$$

and,

$$\int_{V_S}^{V_D} dV - \left(\frac{g_c}{a\rho}\right)\int_{P_S}^{P_D} dP + \int_{t_S}^{t_D}\left(\frac{fV^2}{2D}\right) dt = 0 \qquad (3.38)$$

or,

$$\left(V_D - V_S\right) - \left(\frac{g_c}{a\rho}\right)\left(P_D - P_S\right) + \left(\frac{fV^2}{2D}\right)_S\left(t_D - t_S\right) = 0 \qquad (3.39)$$

Equations (3.33), (3.35), (3.37), and (3.39) have the four unknowns X_D, t_D, V_D, and P_D and are therefore solvable. This solution is accomplished by using the method of specified time intervals as developed by Lister (Ref. [8]).

SOLUTION OF THE INTEGRATED CHARACTERISTIC EQUATIONS BY THE METHOD OF SPECIFIED TIME INTERVALS

3.8 This method employs specified intervals in the t-direction and uses Eqs. (3.33), (3.35), (3.37), and (3.39) to relate the values of P and V at the beginning of each time interval to those at the end. Notation for this method is shown in Figure 6.

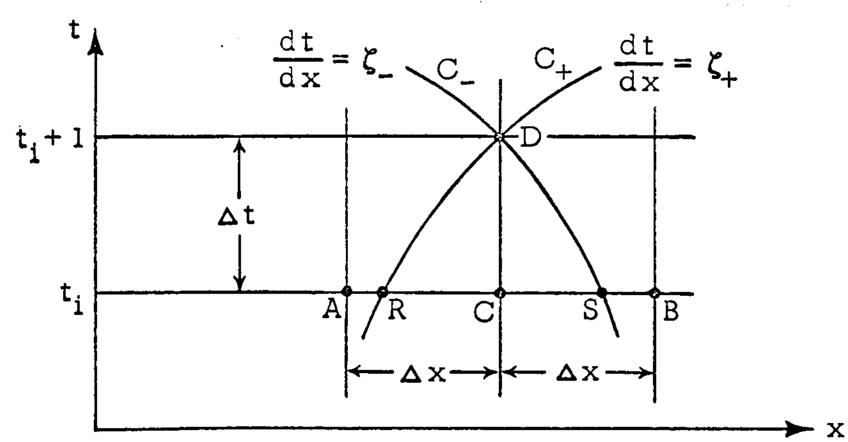


FIGURE 6. NOTATION FOR SOLVING THE INTEGRATED CHARACTERISTIC EQUATIONS BY THE METHOD OF SPECIFIED TIME INTERVALS

The points A, B, and C are three adjacent points on the line $t = t_i$ (beginning of the time interval) and are spaced Δx apart. Point D from Figure 5 is located at the intersection of the lines $t = t_{i+1}$ (end of time interval) and $x = x_C$. At points A, B, and C the pressures and velocities are known quantities, and the problem is to predict the pressure and velocity at point D in terms of these quantities. This is accomplished by using the integrated characteristic equations; however, it should be noted that the points R and S do not, in general, coincide with nodal points. It is therefore necessary to compute the pressures, velocities, and spatial coordinates of points R and S in terms of these quantities at points A, B, and C before the integrated characteristic equations can be used.

Spatial Coordinates of Points R and S

Solving Eq. (3.33) for x_R:

$$x_R = x_D - (V + a)_R \left(t_D - t_R\right) \quad . \tag{3.40}$$

Assume that:

$$(V + a)_R = (V + a)_C \quad . \tag{3.41}$$

This assumption is valid because "a" is constant throughout the flow field, $V << a$, and Δx is sufficiently small. Noting that

$$\Delta t = t_D - t_R \ , \quad \text{and} \quad x_C = x_D \quad , \tag{3.42}$$

and substituting Eqs. (3.41) and (3.42) in Eq. (3.40) gives:

$$x_R = x_C - (V + a)_C (\Delta t) \quad . \tag{3.43}$$

Similarly, solving Eq. (3.37) for x_S and assuming that

$$(V - a)_S = (V - a)_C \tag{3.44}$$

gives:

$$x_S = x_C - (V - a)_C (\Delta t) \quad . \tag{3.45}$$

Velocities at Points R and S

The slope of the positive characteristic has been defined as:

$$\zeta_+ = \frac{1}{V + a} = \frac{dt}{dx} \quad . \tag{3.46}$$

Assume: Δx is chosen small enough such that

$$(\zeta_+)_C = (\zeta_+)_R \quad , \tag{3.47}$$

and that velocity is a linear function of x over the range $x = x_R$ to $x = x_C$.

Then, the following linear interpolation can be made:

$$(\zeta_+)_C = (\zeta_+)_R = \frac{\Delta t}{(x_C - x_R)} \quad . \tag{3.48}$$

From the assumption of a linear velocity distribution:

$$\frac{x_C - x_R}{x_C - x_A} = \frac{x_C - x_R}{\Delta x} = \frac{V_C - V_R}{V_C - V_A} \quad . \tag{3.49}$$

Therefore,

$$x_C - x_R = \left(\frac{V_C - V_R}{V_C - V_A}\right)(\Delta x) \quad . \tag{3.50}$$

Substituting Eq. (3.50) in Eq. (3.48) gives:

$$(\zeta_+)_C = \frac{(\Delta t)}{\left(\frac{V_C - V_R}{V_C - V_A}\right)(\Delta x)} \quad . \tag{3.51}$$

After defining the variable,

$$\theta \equiv \frac{\Delta t}{\Delta x} \quad , \tag{3.52}$$

Eq. (3.51) becomes:

$$(\zeta_+)_C = \left(\frac{V_C - V_A}{V_C - V_R}\right)(\theta) \quad . \tag{3.53}$$

But, from the defining equation:

$$(\zeta_+)_C = \left(\frac{1}{V + a}\right)_C \quad . \tag{3.54}$$

Substituting Eq. (3.54) in Eq. (3.53),

$$\left(\frac{1}{V + a}\right)_C = \left(\frac{V_C - V_A}{V_C - V_R}\right)(\theta) \tag{3.55}$$

or, solving for V_R:

$$V_R = \left[V_C\right]\left[1 - (\theta)(V + a)_C\right] + (\theta)\left(V_A\right)(V + a)_C \quad . \qquad (3.56)$$

Similarly, it can be shown that:

$$V_S = \left[V_C\right]\left[1 + (\theta)(V - a)_C\right] - (\theta)\left(V_B\right)(V - a)_C \quad . \qquad (3.57)$$

Pressures at Points R and S

The derivations of the pressures at points R and S are similar to those for the velocities at these points. That is, if pressure is assumed to be a linear function of x over the ranges $x = x_R$ to $x = x_C$ and $x = x_S$ to $x = x_C$, then it can be shown that:

$$P_R = \left[P_C\right]\left[1 - (\theta)(V + a)_C\right] + (\theta)\left(P_A\right)(V + a)_C \quad , \qquad (3.58)$$

$$P_S = \left[P_C\right]\left[1 - (\theta)(V - a)_C\right] - (\theta)\left(P_B\right)(V - a)_C \quad . \qquad (3.59)$$

Pressure and Velocity at Point D

Assuming that

$$\left(\frac{fV^2}{2D}\right)_R = \left(\frac{fV^2}{2D}\right)_S = \left(\frac{fV^2}{2D}\right)_C \qquad (3.60)$$

and noting that

$$t_R = t_S = t_C \tag{3.61}$$

and

$$\left(t_D - t_C\right) = \Delta t \quad , \tag{3.62}$$

Eqs. (3.35) and (3.39) may be written as:

$$\left(V_D - V_R\right) + \left(\frac{g_c}{a\rho}\right)\left(P_D - P_R\right) + \left(\frac{fV^2}{2D}\right)_C (\Delta t) = 0 \tag{3.63}$$

and,

$$\left(V_D - V_S\right) - \left(\frac{g_c}{a\rho}\right)\left(P_D - P_S\right) + \left(\frac{fV^2}{2D}\right)_C (\Delta t) = 0 \quad . \tag{3.64}$$

Equations (3.63) and (3.64) are now solved simultaneously for P_D and V_D. Subtracting (3.64) from (3.63) gives:

$$V_D - V_R - V_D + V_S + \left[\frac{g_c}{a\rho}\right]\left[P_D - P_R + P_D - P_S\right] = 0 \quad , \tag{3.65}$$

$$\left(V_S - V_R\right) + \left[\frac{g_c}{a\rho}\right]\left[2P_D - P_R - P_S\right] = 0 \quad , \tag{3.66}$$

or,

$$\boxed{P_D = \left(\frac{a\rho}{2g_c}\right)\left(V_R - V_S\right) + \left(\tfrac{1}{2}\right)\left(P_R + P_S\right)} \quad . \tag{3.67}$$

Substituting Eq. (3.67) in Eq. (3.64):

$$\left(V_D - V_S\right) - \left[\frac{g_c}{a\rho}\right]\left[\left(\frac{a\rho}{2g_c}\right)\left(V_R - V_S\right) + \left(\tfrac{1}{2}\right)\left(P_R + P_S\right) - P_S\right] + \left(\frac{fV^2}{2D}\right)_C (\Delta t) = 0 \quad . \qquad (3.68)$$

Expanding and grouping terms:

$$\left(V_D - V_S\right) - \left(\tfrac{1}{2}\right)\left(V_R - V_S\right) - \left(\frac{g_c}{2a\rho}\right)\left(P_R + P_S\right) + \left(\frac{g_c}{a\rho}\right)\left(P_S\right) + \left(\frac{fV^2}{2D}\right)_C (\Delta t) = 0 \;, \qquad (3.69)$$

$$V_D - V_S - \tfrac{1}{2}V_R + \tfrac{1}{2}V_S - \left(\frac{g_c}{2a\rho}\right)\left(P_R + P_S - 2P_S\right) + \left(\frac{fV^2}{2D}\right)_C (\Delta t) = 0 \;, \qquad (3.70)$$

$$V_D - \left(\tfrac{1}{2}\right)\left(V_R + V_S\right) - \left(\frac{g_c}{2a\rho}\right)\left(P_R - P_S\right) + \left(\frac{fV^2}{2D}\right)_C (\Delta t) = 0 \;, \qquad (3.71)$$

or,

$$\boxed{V_D = \left(\tfrac{1}{2}\right)\left(V_R + V_S\right) + \left(\frac{g_c}{2a\rho}\right)\left(P_R - P_S\right) - \left(\frac{fV^2}{2D}\right)_C (\Delta t)} \quad . \qquad (3.72)$$

BASIC CALCULATION PROCEDURE

3.9 The set of equations, (3.56), (3.57), (3.58), (3.59), (3.67) and (3.72), forms the nucleus of all computations made by any computer program using this method. (See Figure 7.)

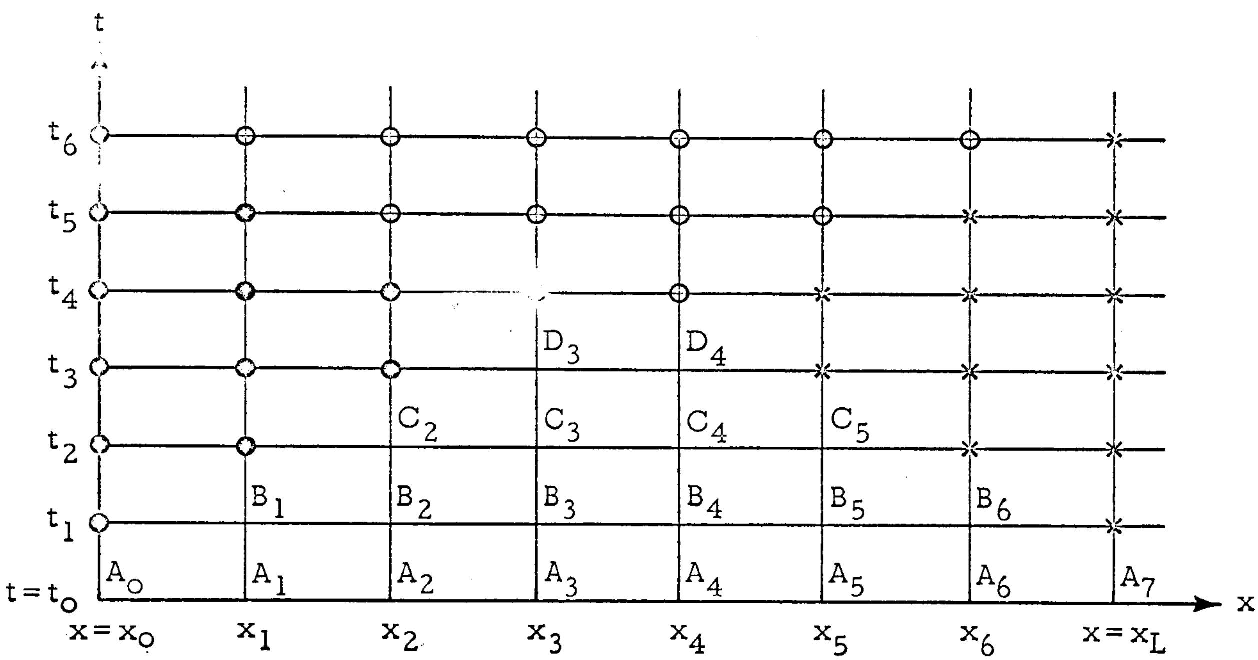


FIGURE 7. NOMENCLATURE FOR DISCUSSION OF CALCULATION PROCEDURE IN x-t PLANE

The basic calculation procedure is as follows:

(1) **Pressures and velocities are given at points A_o** through A_7.

(2) Pressures and velocities are calculated at points B_1 through B_6 using the above-mentioned set of equations.

(3) The set of equations is used to calculate pressures and velocities at points C_2 through C_5 and then to make these same calculations at points D_3 and D_4.

Boundary Conditions

The area in the x-t plane encompassed by the points calculated in these three steps is known as the "area of determinacy." The extent of the area of determinacy is determined by the number of known points at $t=t_o$. It is obvious that calculations cannot proceed beyond this area without making use of boundary conditions. When suitable boundary conditions are given at $x=x_o$; pressures and velocities may be calculated at all points in Figure 7 marked by black dots. Similarly, if boundary conditions are given at $x=x_L$, pressures and velocities may be found at all points in Figure 7 marked by crosses. Thus, the computations may proceed for as long as desired if the boundary conditions are known at both ends of the system.

The boundary conditions are incorporated in the calculation procedure as follows:

(a) At the left-hand, or reservoir, end of the system (Figure 8(a)), Eqs. (3.57) and (3.59) are used to calculate V_S and P_S, respectively. Then, if P_D is the known boundary condition, Eq. (3.64) is rearranged as follows to calculate V_D.

$$V_D = V_S + \left(\frac{g_c}{a\rho}\right)\left(P_D - P_S\right) - \left(\frac{fV^2}{2D}\right)_C (\Delta t) \quad . \tag{3.73}$$

If, however, V_D is known, then Eq. (3.64) is rearranged to give P_D as:

$$P_D = P_S + \left(\frac{a\rho}{g_c}\right)\left(V_D - V_S\right) + \left(\frac{a\rho}{g_c}\right)\left(\frac{fV^2}{2D}\right)_C (\Delta t) \quad . \tag{3.74}$$

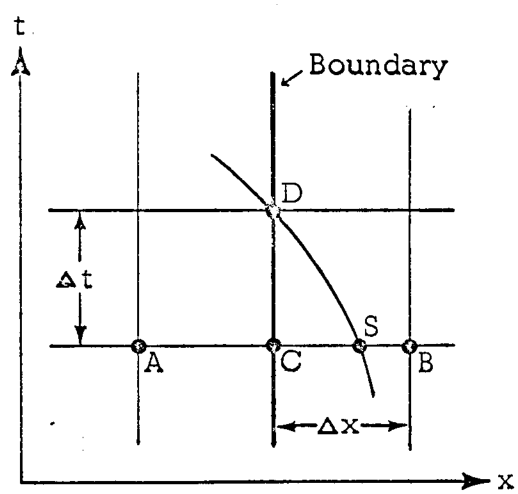
(b) Similarly, at the right-hand, or valve, end of the system (Figure 8(b)), Eqs. (3.56) and (3.58) are used to calculate V_R and P_R. Then, if P_D is the known boundary condition, Eq. (3.63) is used to calculate V_D as:

$$V_D = V_R - \left(\frac{g_c}{a\rho}\right)\left(P_D - P_R\right) - \left(\frac{fV^2}{2D}\right)_C (\Delta t) \quad . \qquad (3.75)$$

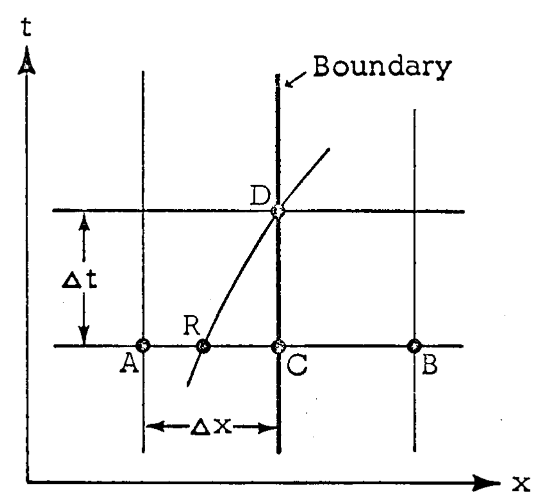
If V_D is known, then Eq. (3.63) gives P_D as:

$$P_D = P_R - \left(\frac{a\rho}{g_c}\right)\left(V_D - V_R\right) - \left(\frac{a\rho}{g_c}\right)\left(\frac{fV^2}{2D}\right)_C (\Delta t) \quad . \qquad (3.76)$$

It should be noted at this point that Subroutine SURGE uses P_D and mass flowrate, respectively, as the left- and right-hand boundary conditions. P_D is used in Eq. (3.73) (as described above), while the mass flowrate is first used to calculate V_D, which in turn is used in Eq. (3.76).



8(a). Left-hand Boundary



8(b). Right-hand Boundary

FIGURE 8. BOUNDARY CONDITIONS

EXTRAPOLATION PROCEDURE

3.10 The use of a simple first-order finite difference approximation to integrate the characteristic equations necessitates the employment of an extrapolation technique due to Roberts (Ref. [7]) to increase the accuracy of the computations.

Roberts' Technique

Consider a function $f(x, t)$ which is to be determined at $t = 2n\Delta t$ in terms of its known value at $t = 0$. The function $f(x, 2n\Delta t)$ may be found by repeating a linear process at a constant value of x for n steps of $2\Delta t$ or for $2n$ steps of Δt. The results of these two series of computations are denoted by $f_2(x, 2n\Delta t)$ and $f_1(x, 2n\Delta t)$, respectively. Roberts has shown that the average of the results given by

$$\bar{f}(x, 2n\Delta t) = 2f_1(x, 2n\Delta t) - f_2(x, 2n\Delta t) \qquad (3.77)$$

agrees with the true value $f(x, 2n\Delta t)$ if terms of the order $(\Delta t)^2$ are neglected.

Computer Calculation Procedure Using Roberts' Technique

Since the set of equations in the basic calculation procedure comprises a linear process of the type mentioned above, Roberts' technique was chosen for use in Subroutine SURGE. A rough outline of the procedure is as follows:

(1) Pressures and velocities are calculated for a time interval of $2\Delta t$ using Eqs. (3.56), (3.57), (3.58), (3.59), (3.67), (3.73), and (3.76).

(2) Pressures and velocities are calculated for a time interval of Δt using the same equations.

(3) Using the results of step (2) as initial values, the pressures and velocities are calculated for a second time interval of Δt.

(4) The pressures and velocities obtained in steps (2) and (3) are combined using Eq. (3.77) to obtain the desired results.

Steps (1) through (4) are repeated until the calculation time limit is reached.

IV. APPLICATION OF THE WATERHAMMER EQUATIONS AND THEIR METHOD OF CHARACTERISTICS SOLUTION TO SYSTEMS CONTAINING CONCENTRATED LOSSES

INTRODUCTION

4.1 The waterhammer equations and their method of characteristics solution as presented in Chapters II and III are applicable only to systems in which the energy losses are uniformly distributed. A method (Ref. [8]) is now presented which extends the applicability of these equations and their solution to systems which contain concentrated losses (i.e., valves, orifices, and area changes) in addition to distributed losses.

CONTRACTOR'S METHOD FOR CONCENTRATED LOSSES

4.2 Basically, this method divides a system into sections having only distributed losses. These sections are separated by concentrated losses that are assumed to take place across the transverse planes connecting the sections. Thus, the upstream and downstream conditions at a concentrated loss are represented as boundary conditions for the equations comprising the method of characteristics solution. This is shown in Figure 9.

The four unknown boundary conditions at a loss are P_{D_1}, P_{D_2}, V_{D_1}, and V_{D_2}. These variables are given in each case by the simultaneous solution of the following equations:

(1) Right-running characteristic equation

(2) Left-running characteristic equation

(3) Continuity equation

(4) Energy equation

Solutions are presented for the following cases:

(1) Area change with a loss (i.e., valves, conical sections)

(2) Loss without an area change (i.e.,valves, orifices)

(3) Area change without a loss (frictionless case)

(4) Change in line wall thickness and/or wall material without a change in cross-sectional area

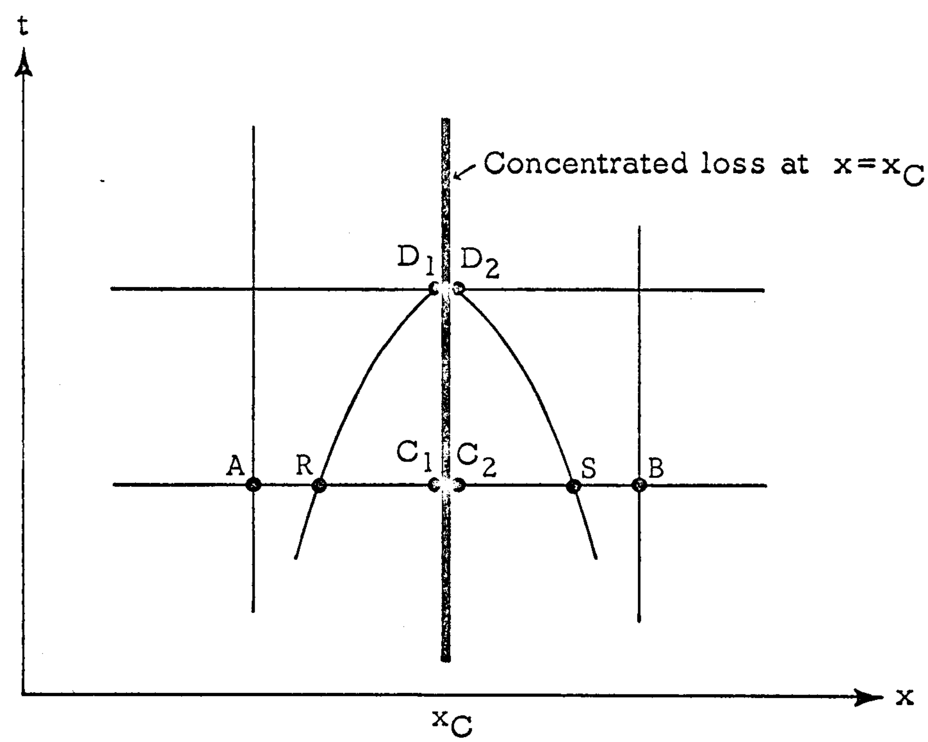


FIGURE 9. NOMENCLATURE FOR FLOW ACROSS A PLANE CONTAINING A CONCENTRATED LOSS

Case 1 - Area Change with a Loss

The governing equations are:

$$\left(V_{D_1} - V_R\right) + \left(\frac{g_c}{a_{C_1}\rho}\right)\left(P_{D_1} - P_R\right)$$

$$+ \left(\frac{fV^2}{2D}\right)_{C_1}(\Delta t) = 0 \quad \text{(right-running characteristic)} \qquad (4.1)$$

$$\left(V_{D_2} - V_S\right) - \left(\frac{g_c}{a_{C_2}\rho}\right)\left(P_{D_2} - P_S\right)$$

$$+ \left(\frac{fV^2}{2D}\right)_{C_2}(\Delta t) = 0 \quad \text{(left-running characteristic)} \qquad (4.2)$$

$$A_{D_1} V_{D_1} = A_{D_2} V_{D_2} \quad \text{(continuity)} \qquad (4.3)$$

$$P_{D_2} = P_{D_1} - (K)\left(\frac{\rho V_{D_1}^2}{2 g_c}\right) \quad \text{(energy)} \qquad (4.4)$$

These equations are solved simultaneously as follows:

From Eq. (4.3):

$$V_{D_2} = \left(\frac{A_{D_1}}{A_{D_2}}\right)\left(V_{D_1}\right) \qquad (4.5)$$

Substituting Eqs. (4.4) and (4.5) in Eq. (4.2) gives:

$$\left[\left(\frac{A_{D_1}}{A_{D_2}}\right)\left(V_{D_1}\right) - V_S\right] - \left[\frac{g_c}{a_{C_2}\rho}\right]\left[P_{D_1} - (K)\left(\frac{\rho V_{D_1}^2}{2g_c}\right) - P_S\right] + \left(\frac{fV^2}{2D}\right)_{C_2}(\Delta t) = 0 \,. \quad (4.6)$$

From Eq. (4.6):

$$P_{D_1} = \left[\frac{a_{C_2}\rho}{g_c}\right]\left[\left(\frac{A_{D_1}}{A_{D_2}}\right)\left(V_{D_1}\right) - V_S\right] + \left(\frac{a_{C_2}\rho}{g_c}\right)\left(\frac{fV^2}{2D}\right)_{C_2}(\Delta t) + (K)\left(\frac{\rho V_{D_1}^2}{2g_c}\right) + P_S \quad (4.7)$$

Substituting Eq. (4.7) in Eq. (4.1):

$$\left(V_{D_1} - V_R\right) + \left[\frac{g_c}{a_{C_1}\rho}\right]\left[\left(\frac{a_{C_2}\rho}{g_c}\right)\left(\frac{A_{D_1}}{A_{D_2}}\right)\left(V_{D_1}\right) - \left(\frac{a_{C_2}\rho}{g_c}\right)\left(V_S\right) + \left(\frac{a_{C_2}\rho}{g_c}\right)\left(\frac{fV^2}{2D}\right)_{C_2}(\Delta t) + (K)\left(\frac{\rho V_{D_1}^2}{2g_c}\right) + P_S - P_R\right] + \left(\frac{fV^2}{2D}\right)_{C_1}(\Delta t) = 0 \,. \quad (4.8)$$

Expanding and grouping terms:

$$\left(\frac{K}{2a_{C_1}}\right)\left(V_{D_1}^2\right) + \left[1 + \left(\frac{a_{C_2}}{a_{C_1}}\right)\left(\frac{A_{D_1}}{A_{D_2}}\right)\right]\left[V_{D_1}\right] - \left[V_R + \left(\frac{a_{C_2}}{a_{C_1}}\right)(V_S)\right] + \left(\frac{g_c}{a_{C_1}\rho}\right)\left(P_S - P_R\right) + \left[\left(\frac{fV^2}{2D}\right)_{C_1} + \left(\frac{a_{C_2}}{a_{C_1}}\right)\left(\frac{fV^2}{2D}\right)_{C_2}\right]\left[\Delta t\right] = 0 \tag{4.9}$$

An iterative technique is used to solve Eq. (4.9) for V_{D_1}.

Thus, the set of equations, (4.3), (4.4), (4.7), and (4.9), are used to calculate the four unknowns for Case 1.

Case 2 - Loss without an Area Change

The governing equations are:

$$\left(V_{D_1} - V_R\right) + \left[\frac{g_c}{a_{C_1}\rho}\right]\left[P_{D_1} - P_R\right] + \left(\frac{fV^2}{2D}\right)_{C_1}(\Delta t) = 0 \quad \text{(right-running characteristic)} \tag{4.10}$$

$$\left(V_{D_2} - V_S\right) - \left(\frac{g_c}{a_{C_2}\rho}\right)\left(P_{D_2} - P_S\right)$$

$$+ \left(\frac{fV^2}{2D}\right)_{C_2}(\Delta t) = 0 \quad \text{(left-running characteristic)} \qquad (4.11)$$

$$V_{D_2} = V_{D_1} \quad \text{(continuity)} \qquad (4.12)$$

$$P_{D_2} = P_{D_1} - (K)\left(\frac{\rho V_{D_1}^2}{2g_c}\right) \quad \text{(energy)} \qquad (4.13)$$

These equations are solved simultaneously as follows:

Substitute Eqs. (4.12) and (4.13) in Eq. (4.11):

$$\left(V_{D_1} - V_S\right) - \left[\frac{g_c}{a_{C_2}\rho}\right]\left[P_{D_1} - (K)\left(\frac{\rho V_{D_1}^2}{2g_c}\right) - P_S\right] + \left(\frac{fV^2}{2D}\right)_{C_2}(\Delta t) = 0 \qquad (4.14)$$

Solving Eq. (4.14) for P_{D_1} gives:

$$P_{D_1} = \left(\frac{a_{C_2}\rho}{g_c}\right)\left(V_{D_1} - V_S\right) + \left(\frac{a_{C_2}\rho}{g_c}\right)\left(\frac{fV^2}{2D}\right)_{C_2}(\Delta t) + (K)\left(\frac{\rho V_{D_1}^2}{2g_c}\right) + P_S \qquad (4.15)$$

Substituting Eq. (4.15) in Eq. (4.10):

$$\left(V_{D_1} - V_R\right) + \left[\frac{g_c}{a_{C_1}\rho}\right]\left[\left(\frac{a_{C_2}\rho}{g_c}\right)\left(V_{D_1} - V_S\right) + \left(\frac{a_{C_2}\rho}{g_c}\right)\left(\frac{fV^2}{2D}\right)_{C_2}(\Delta t) + (K)\left(\frac{\rho V_{D_1}^2}{2g_c}\right) + P_S - P_R\right] + \left(\frac{fV^2}{2D}\right)_{C_1}(\Delta t) = 0 \; . \quad (4.16)$$

Expanding and grouping terms:

$$\left(\frac{K}{2a_{C_1}}\right)\left(V_{D_1}^2\right) + \left[1 + \left(\frac{a_{C_2}}{a_{C_1}}\right)\right]\left[V_{D_1}\right] - \left[\left(\frac{a_{C_2}}{a_{C_1}}\right)\left(V_S\right) + V_R\right] + \left(\frac{g_c}{a_{C_1}\rho}\right)\left(P_S - P_R\right) + \left[\left(\frac{fV^2}{2D}\right)_{C_1} + \left(\frac{a_{C_2}}{a_{C_1}}\right)\left(\frac{fV^2}{2D}\right)_{C_2}\right]\left[\Delta t\right] = 0 \quad (4.17)$$

An iterative technique is used to solve Eq. (4.17) for V_{D_1}. Thus, the set of equations, (4.12), (4.13), (4.15), and (4.17) are used to calculate the four unknowns for Case 2.

Case 3 - Area Change without a Loss

The governing equations are:

$$\left(V_{D_1} - V_R\right) + \left(\frac{g_c}{a_{C_1}\rho}\right)\left(P_{D_1} - P_R\right)$$

$$+ \left(\frac{fV^2}{2D}\right)_{C_1}(\Delta t) = 0 \quad \text{(right-running characteristic)} \tag{4.18}$$

$$\left(V_{D_2} - V_S\right) - \left(\frac{g_c}{a_{C_2}\rho}\right)\left(P_{D_2} - P_S\right)$$

$$+ \left(\frac{fV^2}{2D}\right)_{C_2}(\Delta t) = 0 \quad \text{(left-running characteristic)} \tag{4.19}$$

$$A_{D_1} V_{D_1} = A_{D_2} V_{D_2} \quad \text{(continuity)} \tag{4.20}$$

$$P_{D_1} + \frac{\rho V_{D_1}^2}{2g_c} = P_{D_2} + \frac{\rho V_{D_2}^2}{2g_c} \quad \text{(energy)} \tag{4.21}$$

These equations are solved simultaneously as follows:

From Eq. (4.20):

$$V_{D_2} = \left(\frac{A_{D_1}}{A_{D_2}}\right)\left(V_{D_1}\right) \tag{4.22}$$

From Eq. (4.21):

$$P_{D_2} = P_{D_1} + \left(\frac{\rho}{2g_c}\right)\left(V_{D_1}^2 - V_{D_2}^2\right) \tag{4.23}$$

Substituting Eq. (4.22) in Eq. (4.23) gives:

$$P_{D_2} = P_{D_1} + \left[1 - \left(\frac{A_{D_1}}{A_{D_2}}\right)^2\right]\left[\frac{\rho V_{D_1}^2}{2g_c}\right] \tag{4.24}$$

Substituting Eqs. (4.22) and (4.24) in Eq. (4.19) gives:

$$\left[\left(\frac{A_{D_1}}{A_{D_2}}\right)\left(V_{D_1}\right) - V_S\right] - \left[\frac{g_c}{a_{C_2}\rho}\right]\left\{P_{D_1} + \left[1 - \left(\frac{A_{D_1}}{A_{D_2}}\right)^2\right]\left[\frac{\rho V_{D_1}^2}{2g_c}\right] - P_S\right\} + \left(\frac{fV^2}{2D}\right)_{C_2}(\Delta t) = 0 \tag{4.25}$$

Solving Eq. (4.25) for P_{D_1} gives:

$$P_{D_1} = \left[\frac{a_{C_2}\rho}{g_c}\right]\left[\left(\frac{A_{D_1}}{A_{D_2}}\right)\left(V_{D_1}\right) - V_S\right] + \left(\frac{a_{C_2}\rho}{g_c}\right)\left(\frac{fV^2}{2D}\right)_{C_2}(\Delta t) + \left[\left(\frac{A_{D_1}}{A_{D_2}}\right)^2 - 1\right]\left[\frac{\rho V_{D_1}^2}{2g_c}\right] + P_S \tag{4.26}$$

Substituting Eq. (4.26) in Eq. (4.18) gives:

$$\left(V_{D_1} - V_R\right) + \left[\frac{g_c}{a_{C_1}\rho}\right]\left\{\left(\frac{a_{C_2}\rho}{g_c}\right)\left(\frac{A_{D_1}}{A_{D_2}}\right)\left(V_{D_1}\right)\right.$$

$$- \left(\frac{a_{C_2}\rho}{g_c}\right)\left(V_S\right) + \left(\frac{a_{C_2}\rho}{g_c}\right)\left(\frac{fV^2}{2D}\right)_{C_2}(\Delta t)$$

$$\left. + \left[\left(\frac{A_{D_1}}{A_{D_2}}\right)^2 - 1\right]\left[\frac{\rho V_{D_1}^2}{2g_c}\right] + P_S - P_R\right\} + \left(\frac{fV^2}{2D}\right)_{C_1}(\Delta t) = 0 \qquad (4.27)$$

Expanding and grouping terms:

$$\left[\left(\frac{A_{D_1}}{A_{D_2}}\right)^2 - 1\right]\left[\frac{1}{2a_{C_1}}\right]\left[V_{D_1}^2\right] + \left[1 + \left(\frac{a_{C_2}}{a_{C_1}}\right)\left(\frac{A_{D_1}}{A_{D_2}}\right)\right]\left[V_{D_1}\right]$$

$$- \left[\left(\frac{a_{C_2}}{a_{C_1}}\right)\left(V_S\right) + V_R\right] + \left(\frac{g_c}{a_{C_1}\rho}\right)\left(P_S - P_R\right) \qquad (4.28)$$

$$+ \left[\left(\frac{fV^2}{2D}\right)_{C_1} + \left(\frac{a_{C_2}}{a_{C_1}}\right)\left(\frac{fV^2}{2D}\right)_{C_2}\right]\left[\Delta t\right] = 0$$

An iterative technique is used to solve Eq. (4.28) for V_{D_1}. Thus, the set of equations, (4.20), (4.21), (4.26), and (4.28), are used to calculate the four unknowns for Case 3.

Case 4 - Change in Line Wall Thickness and/or Wall Material without a Change in Cross-sectional Area

The governing equations are:

$$\left(V_{D_1} - V_R\right) + \left(\frac{g_c}{a_{C_1}\rho}\right)\left(P_{D_1} - P_R\right)$$

$$+ \left(\frac{fV^2}{2D}\right)_{C_1}(\Delta t) = 0 \quad \text{(right-running characteristic)} \qquad (4.29)$$

$$\left(V_{D_2} - V_S\right) - \left(\frac{g_c}{a_{C_2}\rho}\right)\left(P_{D_2} - P_S\right)$$

$$+ \left(\frac{fV^2}{2D}\right)_{C_2}(\Delta t) = 0 \quad \text{(left-running characteristic)} \qquad (4.30)$$

$$V_{D_1} = V_{D_2} \quad \text{(continuity)} \qquad (4.31)$$

$$P_{D_1} = P_{D_2} \quad \text{(energy)} \qquad (4.32)$$

These equations are solved simultaneously as follows:

Substitute Eqs. (4.31) and (4.32) in Eq. (4.30):

$$\left(V_{D_1} - V_S\right) - \left(\frac{g_c}{a_{C_2}\rho}\right)\left(P_{D_1} - P_S\right) + \left(\frac{fV^2}{2D}\right)_{C_2}(\Delta t) = 0 \qquad (4.33)$$

Solving Eq. (4.33) for P_{D_1} gives:

$$P_{D_1} = \left(\frac{a_{C_2}\rho}{g_c}\right)\left(V_{D_1} - V_S\right) + \left(\frac{a_{C_2}\rho}{g_c}\right)\left(\frac{fV^2}{2D}\right)_{C_2}(\Delta t) + P_S \qquad (4.34)$$

Substituting Eq. (4.34) in Eq. (4.29) gives:

$$\left(V_{D_1} - V_R\right) + \left[\frac{g_c}{a_{C_1}\rho}\right]\left[\left(\frac{a_{C_2}\rho}{g_c}\right)\left(V_{D_1} - V_S\right) + \left(\frac{a_{C_2}\rho}{g_c}\right)\left(\frac{fV^2}{2D}\right)_{C_2}(\Delta t) + P_S - P_R\right] + \left(\frac{fV^2}{2D}\right)_{C_1}(\Delta t) = 0 \qquad (4.35)$$

Expanding and grouping terms:

$$\left[1 + \left(\frac{a_{C_2}}{a_{C_1}}\right)\right]\left[V_{D_1}\right] - \left[V_R + \left(\frac{a_{C_2}}{a_{C_1}}\right)\left(V_S\right)\right] + \left(\frac{g_c}{a_{C_1}\rho}\right)\left(P_S - P_R\right) + \left[\left(\frac{fV^2}{2D}\right)_{C_1} + \left(\frac{a_{C_2}}{a_{C_1}}\right)\left(\frac{fV^2}{2D}\right)_{C_2}\right]\left[\Delta t\right] = 0 \qquad (4.36)$$

The set of equations, (4.31), (4.32), (4.34), and (4.36), are used to calculate the four unknowns for Case 4.

Comparison of the Four Cases

A comparison of the four sets of governing equations shows that the equations for Case 1 may be regarded as a "general" set of equations that applies to each of the four cases; that is, when the Case 1 equations are applied to Cases 2, 3, and 4, the inapplicable terms automatically drop out. The only anomaly which occurs when using the equations in this fashion is that the flow loss coefficient K in Case 3 must be set equal to the factor $\left[\left(\frac{A_{D_1}}{A_{D_2}}\right)^2 - 1\right]$.

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APPENDIX

SUBROUTINE SURGE

SUBROUTINE WMAIN

SUBROUTINE PRNT

SAMPLE RESULTS FROM SUBROUTINE SURGE

SUBROUTINE SURGE

This subroutine uses the method of characteristics to solve the full nonlinear momentum and continuity equations which describe the unsteady flow of liquids in closed, unbranched systems.

CALLING SEQUENCE

CALL SURGE

SOLUTION METHOD

<u>Initialize the following variables</u>

Hydrostatic pressure.

1. PDELH = 0

Time.

2. TIME = 0

Integration time interval.

3. DELT2 = 576

4. Do 14, I = 1, NS

Number of divisions in each section.

5. XN (I) = N (I)

Number of divisions in each section plus one.

6. NN = N (I) + 1

7. Do 8, J = 1, NN

Velocity.

8. $V(I, J) = ((576)/(3.1416))(WDOT)/((RHO)(D(I))^2)$

Length of each division in a section.

9. DELX (I) = (XL (I))/(XN (I))

Pressure drop for each division of a section.

10. $PDELF(I) = ((F(I))/(12))((DELX(I))/(D(I)))(RHO)(V(I, NN))^2/((2)(GC))$

Speed of sound in the liquid.

11. A (I) = SQRT ((144)/((RHO)/(GC) (C1 (I)) (R (I))/((E (I)) (PWT (I))) + (1)/(BETA)))

Integration time interval.

12. DELT1 = (DELX (I))/(A (I))

Pick smallest DELT1 of all sections and set into DELT2.

13. If DELT1 < DELT2, DELT2 = DELT1, go to 14

If DELT1 ≥ DELT2, go to 14

14. Continue

Integration time intervals.

15. DELT1 = DELT2

16. DELT2 = (DELT2)(.5)

Distances from the reservoir along the longitudinal axis of the pipe to the beginning of each section.

17. AA(1) = 0

18. Do 19, I = 2, NS

19. AA(I) = AA(I - 1) + XL(I - 1)

Pick off pressure for first section using Subroutine NTERP.

20. IL = 0

21. IK = 0

22. T = 0

23. CALL NTERP(TIMT, PDT, 100, 0, P(1, 1), S, 1, IK, L)

Check if error return from NTERP.

24. If L $\neq$ 0, go to 119

If L = 0, go to 25

Reset NTERP indicator.

25. IK = 0

26. Do 32, I = 1, NS

Set NI = number of divisions in this section plus one.

27. NI = N(I) + 1

Calculate initial pressure distribution throughout the system

28. Do 29, J = 2, NI

Initial pressure distribution along each section.

29. P (I, J) = P (I, J - 1) - PDELF (I)

Check if this is last section.

30. If I = NS, go to 32

If I ≠ NS, go to 31

Pressure drop between sections.

31. P (I + 1, 1) = P (I, NI) - ((FLC (I)) (RHO)/ ((288) (GC)))

(ABS (V (I, NI))) (V (I, NI))

32. Continue

Set integration time interval.

33. DELT = DELT1

Set indicator.

34. JJ = 0

Set initial pressure distribution.

35. PD (1, 1) = P (1, 1)

36. Go to 96

Print out pressures and velocities at time = 0.

37. CALL PRNT

38. JJ = 1

39. Go to 12

Pick off pressure at reservoir for this time step.

40. CALL NTERP (TIMT, PDT, 100, T, PD (1, 1), S, 1, IK, L)

Check if error return from NTERP.

41. If L $\neq$ 0, go to 119

If L = 0, go to 42

Velocity at reservoir at end of integration time interval

Ratio of integration time interval to increment of spatial coordinate.

42. DELTDX = (DELT)/(DELX (1))

Dummy variable.

43. PSVS = (DELTDX)(V (1, 1) - A(1))

Velocity at start of integration time interval at point on a left-running characteristic.

44. VS = (V (1, 1)) (1 + PSVS) - (PSVS) (V (1, 2))

Pressure at start of integration time interval at point on a left-running characteristic.

45. PS = (P (1, 1)) (1 + PSVS) - (PSVS) (P (1, 2))

Velocity at reservoir at end of integration time interval.

46. VD (1, 1) = VS + ((144) (GC)/((A (1)) (RHO)))

(PD (1, 1) - PS) - (F (1)) (V (1, 1))

(ABS (V (1, 1))) (12)/((2) (D (1))) (DELT)

Pressure and velocity at end of integration time interval for each nodal point in a section

47. Do 71, I = 1, NS

Set NI = number of divisions in this section plus one.

48. NI = N(I) + 1

Ratio of integration time interval to increment of spatial coordinate.

49. DELTDX = (DELT)/(DELX(I))

50. Do 58, J = 2, NI

Dummy variable.

51. PSVS = (DELTDX)(V(I, J) + A(I))

Dummy variable

52. PSVT = (DELTDX)(V(I, J) - A(I))

Velocity at start of integration time interval at point on a right-running characteristic.

53. VR = (V(I, J))(1 - PSVS) + (PSVS)(V(I, J - 1))

Velocity at start of integration time interval at point on a left-running characteristic.

54. VS = (V(I, J))(1 + PSVT) - (PSVT)(V(I, J + 1))

Pressure at start of integration time interval at point on a left-running characteristic.

55. PS = (P(I, J))(1 + PSVT) - (PSVT)(P(I, J + 1))

Pressure at start of integration time interval at point on a right-running characteristic.

56. PR = (P (I, J)) (1 - PSVS) + (PSVS) (P (I, J - 1))

Velocity at nodal point at end of integration time interval.

57. VD (I, J) = (.5) (VR + VS) + ((144) (GC)/ ((2) (RHO) (A (I))))
(PR - PS) - (F (I)) (V (I, J)) (ABS (V (I, J))) ((12)/
((2) (D (I)))) (DELT)

Pressure at nodal point at end of integration time interval.

58. PD (I, J) = (A (I)) ((RHO)/ ((288) (GC))) (VR - VS)
+ (.5) (PR + PS)

Check if this is last section of the system.

59. If I = NS, go to 72
If I $\neq$ NS, go to 60

Pressure and velocity at end of integration time interval for last nodal point in a section and first nodal point in the succeeding section

Dummy variable.

60. PSVT = ((DELT)/ (DELX (I + 1))) (V (I + 1, 1) - A (I + 1))

Velocity at start of integration time interval at point on a left-running characteristic.

61. VS = (V (I + 1, 1)) (1 + PSVT) - (PSVT) (V (I + 1, 2))

Pressure at start of integration time interval at point on a left-running characteristic.

62. PS = (P (I + 1, 1)) (1 + PSVT) - (PSVT) (P (I + 1, 2))

Clear ESTM loop control counter.

63. IDL = 0

Estimate of velocity at last nodal point in a section at end of integration time interval.

64. VD (I, NI) = V (I, NI)

Dummy variable.

65. DUM = (FLC (I)) ((.5)/ (A (I))) (ABS (VD (I, NI))) (VD (I, NI))

+ (1 + ((D (I))/(D (I + 1))^2) (A (I + 1))/(A(I)))(VD(I, NI))

- (((A (I + 1))/(A (I))) (VS) + VR) + ((F (I))((6)/ (D (I)))

((ABS (V (I, NI))) (V (I, NI)) + ((A (I + 1))/(A(I)))(F(I + 1))

- ((6)/(D (I + 1))) (ABS (V (I + 1, 1)))(V (I + 1, 1))) (DELT)

+ (GC) ((144)/ ((A (I)) (RHO))) (PS - PR)

Velocity at last nodal point in a section at end of integration time interval.

66. CALL ESTM (IDL, 1, TAB1, TAB2, DUM, VD (I, NI), LUPNAM)

67. If ABS (DUM) > CMULT, go to 65

If ABS (DUM) ≤ CMULT, go to 68

Velocity at first nodal point in succeeding section at end of integration time interval.

68. VD (I + 1, 1) = (VD (I, NI)) ((D (I))/ (D (I + 1)))^2

Pressure at last nodal point in a section at end of integration time interval.

69. PD(I, NI) = (A(I + 1))((RHO)/((144)(GC)))((D(I))/(D(I+1)))^2

(VD(I, NI) - VS) + (((A(I + 1))(RHO)/((12)(GC)))

((F(I+1))/((2)(D(I+1))))(ABS(V(I+1, 1)))(V(I+1, 1)))

(DELT) + ((FLC(I))(RHO)/((288)(GC)))

(ABS(VD(I, NI)))(VD(I, NI)) + PS

Pressure at first nodal point in succeeding section at end of integration time interval.

70. PD(I + 1, 1) = PD(I, NI) - ((FLC(I))(RHO)/((288)(GC)))

(VD(I, NI))(ABS(VD(I, NI)))

71. Continue

Pressure and velocity at last nodal point in system at end of integration time interval

Mass flowrate at valve end of system.

72. CALL NTERP(TITA, WDOTT, 100, T, WDOT, S, 1 IL, L)

Check if error return from NTERP.

73. If L $\neq$ 0, go to 121

If L = 0, go to 74

Velocity at last nodal point in system at end of integration time interval.

74. VD(I, NI) = ((576)/(3.1416))(WDOT)/((RHO)(D(I))^2)

Pressure at last nodal point in system at end of integration time interval.

75. PD (I, NI) = (A (I)) ((RHO)/((144) (GC))) (VR - VD (I, NI))

- ((A (I)) (RHO) (F (I)) (V (I, NI)) (ABS (V (I, NI)))

/ ((24) (D (I)) (GC))) (DELT) + PR

76. Go to (77, 87), JJ

<u>Store velocities and pressures</u>

77. Do 81, I = 1, NS

Set NI = number of divisions in this section plus one.

78. NI = N (I) + 1

79. Do 81, J = 1, NI

Store velocities.

80. V1 (I, J) = VD (I, J)

Store pressures.

81. P1 (I, J) = PD (I, J)

Increment dummy time.

82. T = TIME + DELT2

Set integration time interval.

83. DELT = DELT2

Set indicator.

84. JJ = 2

Set control variable.

85. K = 1

86. Go to 40

87. Go to (88, 96), K

88. Do 92, I = 1, NS

Set NI = number of divisions in this section plus one.

89. NI = N (I) + 1

90. Do 92, J = 1, NI

Store velocities.

91. V (I, J) = VD (I, J)

Store pressures.

92. P (I, J) = PD (I, J)

Increment dummy time.

93. T = TIME + DELT1

Set control variable.

94. K = 2

95. Go to 40

96. Do 110, I = 1, NS

Set NI equal to number of division in section plus one.

97. NI = N (I) + 1

98. Do 110, J = 1, NI

Check if entry is for initialization only.

99. If JJ = 0, go to 37

If JJ ≠ 0, go to 100

Velocity at each nodal point.

100. V(I, J) = (2)(VD(I, J)) - V1(I, J)

Pressure at each nodal point (not including hydrostatic pressure).

101. P(I, J) = (2)(PD(I, J)) - P1(I, J)

Hydrostatic pressure calculations

Check to determine if hydrostatic pressure calculations will be performed.

102. If HYDFL = 0, go to 110

If HYDFL ≠ 0, go to 103

Set acceleration.

103. G = G1

Check to determine if constant or time varying acceleration will be used in hydrostatic pressure calculations.

104. If HYDFL < 2, go to 106

If HYDFL ≥ 2, go to 105

Pick value of acceleration.

105. CALL NTERP(GIND, GDEP, 100, TIME, G, S, 1, KK, LL)

Number of spatial increments along pipe section up to nodal point J.

106. XJ = J - 1

Distance along longitudinal axis of a section of pipe up to nodal point J.

107. XJD = (XJ) (DELX (I))

Distance below bottom of reservoir.

108. $X = AA(I) + (AC(1, I))(XJD) + (AC(2, I))(XJD)^2$

$+ (AC(3, I))(XJD)^3 + (AC(4, I))(XJD)^4$

Hydrostatic pressure.

109. PDELH = (RHO) (X) (G)/ ((144) (GC))

Pressure at each nodal point (including hydrostatic pressure).

110. PP (I, J) = P (I, J) + PDELH

Check if entry to here was for initialization.

111. If JJ = 0, go to 37

If JJ $\neq$ 0, go to 112

Check if pressure less than allowable minimum.

112. If PP (I, J) $<$ PMIN, go to 126

If PP (I, J) $\geq$ PMIN, go to 113

Increment time.

113. TIME = TIME + DELT1

Print for this time step.

114. CALL PRNT

Set integration time interval.

115. DELT = DELT1

Set indicator.

116. JJ = 1

Determine if time limit achieved.

117. If TIME ≤ TLIMT, go to 40

If TIME > TLIMT, go to 118

118. Return

Set up error printouts

119. JKL = 1

120. Go to 122

121. JKL = 2

122. If L > 0, go to 124

If L ≤ 0, go to 123

123. L = 2

Print error message.

124. CALL ERPRNT (L, T, JKL)

Error return.

125. CALL CTROL

Print error message.

126. CALL PRNTER (I, J)

Error return.

127. CALL CTROL

End

SUBROUTINE SURGE NOMENCLATURE

Symbol	Description	Units	Reference
A	Speed of sound in fluid in pipe	$\frac{ft}{sec}$	/WHAM/, 11, 12, 43, 46, 51, 52, 57, 58, 60, 65, 69, 75
AA	Array containing distances from reservoir along longitudinal axis of the system to the beginning of each section	ft	DIM, 17, 19, 108
AC	Array containing coefficients of a fourth order polynomial which gives the distance below the beginning of a pipe section as a function of axial distance along the section	—	/WHAM/, 108
BETA	Fluid bulk modulus	$\frac{lbf}{in^2}$	/WHAM/, 11
C1	Constant which relates the influence of constraints on longitudinal movements of the pipe to stresses on a transverse cross-section of pipe	—	/WHAM/, 11
CMULT	Tolerance used in computing fluid velocity at last nodal point in a section	—	67
D	Pipe diameter	in	/WHAM/, 8, 10, 46, 57, 65, 68, 69, 75
DELT	Integration time interval	sec	33, 42, 46, 49, 60, 65, 69, 75, 83

SUBROUTINE SURGE NOMENCLATURE (cont.)

Symbol	Description	Units	Reference
DELT1	Integration time interval	sec	/WHAMR/, 13, 15, 33, 93, 113, 115
DELT2	Integration time interval	sec	3, 13, 15, 16, 82, 83
DELTDX	Ratio of integration time interval to increment of spatial coordinate	$\frac{sec}{ft}$	42, 43, 49, 51, 52
DELX	Length of each division of a section of pipe	ft	/WHAMR/, 9, 10, 12, 42, 49, 60, 107
DUM	Dummy variable	ft/sec	65, 66, 67
E	Modulus of elasticity of pipe wall material	$\frac{lbf}{in^2}$	/WHAM/, 11
F	Pipe (Moody) friction factor	—	/WHAM/, 10, 46, 57, 65, 69, 75
FLC	Flow loss coefficient between sections	—	/WHAM/, 31, 65, 69, 70
G	Acceleration	$\frac{ft}{sec^2}$	103, 105, 109
G1	Acceleration	$\frac{ft}{sec^2}$	/WHAM/, 103
GC	Constant relating force, mass, and acceleration - (32.174)	$\frac{ft-lbm}{lbf-sec^2}$	/WHAM/, 10, 11, 31, 46, 57, 58, 69, 70, 75, 109

SUBROUTINE SURGE NOMENCLATURE (cont.)

Symbol	Description	Units	Reference
GDEP	Array containing the dependent variables of acceleration for interpolation of acceleration	$\frac{ft}{sec^2}$	/WHAM/, 105
GIND	Array containing the independent variables for interpolation of acceleration	sec	/WHAM/, 105
HYDFL	Variable used to denote method of hydrostatic pressure calculations HYDFL = 0, no hydrostatic pressure calculations are performed HYDFL = 1, hydrostatic pressure calculations are performed with a constant acceleration HYDFL = 2, hydrostatic pressure calculations are performed with a time-varying acceleration	—	/WHAM/, 102, 104
I	Loop control counter denoting pipe section	—	4, 5, 6, 8, 9, 10, 11, 12, 18, 19, 26, 27, 29, 30, 31, 47, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 68, 69, 70, 74, 75, 77, 78, 80, 81, 88, 89, 91, 92, 96, 97, 100, 101, 107, 108, 110, 112, 126

SUBROUTINE SURGE NOMENCLATURE (cont.)

Symbol	Description	Units	Reference
IDL	ESTM loop control counter	—	63, 66
IK	Used by Subroutine NTERP	—	21, 23, 25, 40
IL	Used by Subroutine NTERP	—	20, 72
J	Loop control counter denoting nodal point in a section	—	7, 8, 28, 29, 50,51, 52, 53, 54, 55, 56, 57, 58, 79, 80, 81, 90, 91, 92, 98, 100, 101, 106, 110, 112
JJ	Indicator	—	34, 38, 76, 84, 99, 111, 116
JKL	Error indicator	—	119, 121, 124
K	Control variable	—	85, 87, 94
KK	Used in Subroutine NTERP	—	105
L	Error indicator for Subroutine NTERP	—	23, 24, 40, 41, 72, 73, 122, 123, 124
LL	Used in Subroutine NTERP	—	105
LUPNAM	Used by Subroutine ESTM	—	66
N	Number of divisions in a section of pipe	—	/WHAM/, 5, 6, 27, 48, 89, 97
NI	Number of division in a section of pipe plus one	—	27, 28, 31, 48, 50, 64, 65, 66, 68, 69, 70, 74, 75, 77, 78, 79, 88, 89, 90, 97, 98

SUBROUTINE SURGE NOMENCLATURE (cont.)

Symbol	Description	Units	Reference
NN	Number of divisions in a section of pipe plus one	—	6, 7
NS	Number of sections in system	—	/WHAM/, 4, 18, 26, 30, 47, 59, 77, 88, 96
P	Pressure at nodal point (not including hydrostatic pressure)	$\frac{lbf}{in^2}$	/WHAMR/, 23, 29, 31, 35, 45, 55, 56, 62, 92, 101, 110
PP	Pressure at nodal point (including hydrostatic pressure)	$\frac{lbf}{in^2}$	110
P1	Array used for temporary storage of pressures	$\frac{lbf}{in^2}$	/WHAMR/, 81, 101
PD	Pressure at nodal point at end of integration time interval	$\frac{lbf}{in^2}$	/WHAMR/, 35, 40, 46, 58, 69, 70, 75, 81, 92, 101
PDELF	Steady-state pressure drop for each division of a section of pipe	$\frac{lbf}{in^2}$	/WHAMR/, 10, 29
PDELH	Hydrostatic pressure	$\frac{lbf}{in^2}$	1, 109, 110

SUBROUTINE SURGE NOMENCLATURE (cont.)

Symbol	Description	Units	Reference
PDT	Array containing the dependent variable of pressure for interpolating pressure at reservoir	$\frac{lbf}{in^2}$	/WHAM/, 23, 40
PMIN	Fluid vapor pressure	$\frac{lbf}{in^2}$	/WHAM/, 112
PR	Pressure at start of integration time interval at point on a right-running characteristic	lbf/in^2	56, 57, 58, 65, 75
PS	Pressure at start of integration time interval at point on a left-running characteristic	$\frac{lbf}{in^2}$	45, 46, 55, 57, 58, 62, 65, 69
PSVS	Dummy variable	—	43, 44, 45, 53, 56
PSVT	Dummy variable	—	52, 54, 55, 60, 61, 62
PWT	Pipe wall thickness	in	/WHAM/, 11
R	Pipe radius	in	/WHAM/, 11
RHO	Fluid density	lbm/ft^3	/WHAM/, 8, 10, 11, 31, 46, 57, 58, 65, 69, 70, 74, 75, 109

SUBROUTINE SURGE NOMENCLATURE (cont.)

Symbol	Description	Units	Reference
S	Dummy variable for Subroutine NTERP	—	23, 40, 72, 105
T	Dummy time	sec	22, 40, 72, 82, 93
TAB1	Array used in Subroutine ESTM	—	66
TAB2	Array used in Subroutine ESTM	—	66
TIME	Time	sec	/WHAMR/, 2, 82, 93, 105, 113, 117
TIMT	Array containing the independent variables of time for interpolating pressure at reservoir	sec	/WHAM/, 23, 40
TITA	Array containing the independent variables of time for interpolation of fluid mass flowrate	sec	72
TLIMT	Calculation time limit	sec	/WHAM/, 117
V	Velocity at nodal point	$\frac{ft}{sec}$	/WHAMR/, 8, 10, 31, 43, 44, 46, 51, 52, 53, 54, 57, 60, 61, 64, 65, 69, 75, 91, 100
V1	Array used for temporary storage of velocities	$\frac{ft}{sec}$	/WHAMR/, 80, 100

SUBROUTINE SURGE NOMENCLATURE (cont.)

Symbol	Description	Units	Reference
VD	Velocity at nodal point at end of integration time interval	$\frac{ft}{sec}$	/WHAMR/, 46, 57, 64, 65, 66, 68, 69, 70, 74, 75, 80, 91, 100
VR	Velocity at start of integration time interval at point on a right-running characteristic	$\frac{ft}{sec}$	53, 57, 58, 65, 75
VS	Velocity of start of integration time interval at point on a left-running characteristic	$\frac{ft}{sec}$	44, 46, 54, 57, 58, 61, 65, 69
WDOT	Fluid mass flowrate	$\frac{lbm}{sec}$	/WHAM/, 8, 72, 74
WDOTT	Array containing the dependent variables of mass flowrate for interpolation of mass flowrate at valve end of system	$\frac{lbm}{sec}$	72
X	Distance below bottom of reservoir	ft	108, 109
XJ	Loop control counter J, minus one. Denotes number of spatial increments along a section of pipe up to nodal point J	—	106, 107
XJD	Distance along longitudinal axis of pipe to nodal point J	ft	107, 108

SUBROUTINE SURGE NOMENCLATURE (cont.)

Symbol	Description	Units	Reference
XL	Length of a section of pipe	ft	/WHAM/, 9, 19
XN	Array containing the number of divisions in a section of pipe	—	DIM, 5, 9

```
$IBFTC SURGED  FULIST,DECK
      SUBRØUTINE SURGE
      CØMMØN /WHAMR/ V(20,101), P(20,101),PD(20,101), VD(20,101), TIME,
     1    P1(20,101), V1(20,101),PP(20,101),  DELX(20),PDELF(20),A(20)
     2,  DELT1
      CØMMØN /WHAM/NS,N(20),WDØT,XL(20),F(20),D(20),RHØ,GC,C1(20),
     1    R(20),E(20),PWT(20),BETA,TIMT(100),PDT(100), G1,    FLC(20),
     2            CMULT,TITA(100),WDØTT(100),TLIMT,PDTIM,PVØPT,HYDFL,
     3     GDEP(100), GIND(100),AC(4,20),PMIN
      INTEGER PVØPT,HYDFL
      DIMENSIØN TAB1(20),TAB2(20),XN(20) ,AA(20)
      DATA LUPNAM /2HVD/
      PDELH= 0.
      TIME= 0.
C          SET DUMMY VALUE INTØ DELT2
      DELT2= 576.
      DØ 2 I=1,NS
      XN(I)=N(I)
      NN= N(I) +1
      DØ 3 J=1,NN
    3 V(I,J)= 576./3.1416*WDØT/(RHØ*D(I)**2)
      DELX(I)= XL(I)/XN(I)
      PDELF(I)= F(I)/12. * DELX(I)/D(I) * RHØ*V(I,NN) **2/(2.*GC)
      A(I)= SQRT(144./(RHØ/GC *(C1(I)*R(I)/(E(I)*PWT(I)) + 1./BETA) ) )
      DELT1= DELX(I) / A(I)
      IF(DELT1 .LT. DELT2)  DELT2 = DELT1
    2 CØNTINUE
      DELT1= DELT2
      DELT2= DELT2*.5
      AA(1)= 0.
      DØ 4 I= 2,NS
    4 AA(I)= AA(I-1) + XL(I-1)
C ***      PICK ØFF PRES FØR FIRST SECTIØN USING NTERP
       IL= 0
       IK= 0
      T= 0.
      CALL NTERP(TIMT,PDT,100,0.,P (1,1),S,1,IK,L)
      IF(L .NE. 0) GØ TØ 700
      IK= 0
      DØ 15 I= 1,NS
      NI= N(I)+1
      DØ 9  J= 2,NI
    9 P(I,J) =   P(I,J-1) - PDELF(I)
C ***      DETERMINE WHICH EQS. TØ USE FØR PRES. AT START ØF NEXT SECT.
      IF (I .EQ. NS) GØ TØ 15
   10 P(I+1,1)= P(I,NI) - FLC(I)*RHØ/(288.*GC) *ABS(V(I,NI))*V(I,NI)
   15 CØNTINUE
      DELT= DELT1
      JJ= 0
      PD(1,1)= P(1,1)
```

FIGURE A.1. SYMBOLIC LISTING OF SUBROUTINE SURGE

```
      GØ TØ 109
   14 CALL PRNT
      JJ= 1
      GØ TØ 12
C ***       PICK ØFF PRES FØR FIRST SECT.
   13 CALL NTERP(TIMT,PDT,100, T,PD(1,1),S,1,IK,L)
      IF(L .NE. 0) GØ TØ 700
   12 DELTDX= DELT/DELX(1)
      PSVS= DELTDX*(V(1,1)-A(1) )
      VS= V(1,1) * (1.+PSVS) - PSVS* V(1,2)
      PS= P(1,1) * (1.+PSVS) - PSVS* P(1,2)
      VD(1,1)= VS + 144.*GC/(A(1)*RHØ)*(PD(1,1)-PS)-F(1)*V(1,1) *
     1        ABS(V(1,1))*12./(2.*D(1)) * DELT
      DØ 200 I= 1,NS
      NI= N(I)+1
      DELTDX=DELT/DELX(I)
      DØ  90 J= 2,NI
      PSVS= DELTDX *(V(I,J) + A(I))
      PSVT= DELTDX *(V(I,J) - A(I))
      VR=   V(I,J) * (1.-PSVS) + PSVS*V(I,J-1)
      VS=   V(I,J) * (1.+PSVT) - PSVT*V(I,J+1)
      PS=   P(I,J) *(1.+PSVT) - PSVT*P(I,J+1)
      PR=   P(I,J) *(1.-PSVS) + PSVS*P(I,J-1)
      VD(I,J)= .5*(VR+VS)+144.*GC/(2.*RHØ*A(I))*(PR-PS)-F(I)*V(I,J) *
     1        ABS(V(I,J)) * 12./(2.*D(I))*DELT
   90 PD(I,J)= A(I)*RHØ/(288.*GC)*(VR-VS) + .5*(PR+PS)
C           CHECK IF THIS IS LAST SECTIØN ØF THE SYSTEM
      IF(I.EQ. NS) GØ TØ 92
      PSVT= DELT/DELX(I+1)* (V(I+1,1) - A(I+1)   )
      VS= V(I+1,1) *(1.+PSVT) - PSVT*V(I+1,2)
      PS= P(I+1,1) *(1.+PSVT) - PSVT*P(I+1,2)
   96 IDL=0
      VD(I,NI)  = V(I,NI)
   97 DUM=FLC(I)*.5 /A(I)*ABS(VD(I,NI ))*VD(I,NI  )+(1.+(D(I)/D(I+1))**2
     A      *A(I+1)/A(I) ) *VD(I,NI   ) -(A(I+1)/A(I)*VS +VR) + (
     2    F(I)*6./D(I)*ABS(V(I,NI))*V(I,NI)+A(I+1)/A(I)*F(I+1)*6./D(I+1)
     3    *ABS(V(I+1,1))*V(I+1,1) )*DELT + GC*144./(A(I)*RHØ)*(PS -PR)
      CALL ESTM(IDL,1,TAB1,TAB2,DUM,VD(I,NI),LUPNAM)
      IF( ABS(DUM) .GT. CMULT) GØ TØ 97
      VD(I+1,1)= VD(I,NI) *(D(I)/D(I+1))**2
      PD(I,NI)= A(I+1)*RHØ/(144.*GC)*((D(I)/D(I+1))**2
     A                                   *VD(I,NI)-VS)       +(A(I+1)*RHØ /
     1          (12.*GC)*F(I+1)/(2.*D(I+1))*ABS(V(I+1,1))*V(I+1,1) )  *
     2         DELT+ FLC(I)*RHØ/(288.*GC)*ABS(VD(I,NI))*VD(I,NI)+PS
      PD(I+1,1)= PD(I,NI)-FLC(I)*RHØ/(288.*GC)*VD(I,NI)*ABS(VD(I,NI))
  200 CØNTINUE
   92 CALL NTERP(TITA,WDØTT,100,T,WDØT,S,1,IL,L)
      IF(L .NE. 0) GØ TØ 701
      VD(I,NI)= 576./3.1416 * WDØT/(RHØ*D(I)**2)
      PD(I,NI)= A(I)*RHØ/(144.*GC) * (VR-VD(I,NI)) -  A(I)*RHØ*F(I) *
```

FIGURE A.1. SYMBOLIC LISTING OF SUBROUTINE SURGE (cont.)

```
     1          V(I,NI) * ABS(V(I,NI))/(24.*D(I)*GC) * DELT + PR
 100 GØ TØ(104,105),JJ
 104 DØ 106 I= 1,NS
     NI= N(I)+1
     DØ 106 J=1,NI
     V1(I,J)= VD(I,J)
 106 P1(I,J)= PD(I,J)
     T= TIME + DELT2
     DELT = DELT2
     JJ= 2
     K= 1
     GØ TØ 13
 105 GØ TØ(108,109),K
 108 DØ 110 I= 1,NS
     NI= N(I)+1
     DØ 110 J=1,NI
     V(I,J)= VD(I,J)
 110 P(I,J)= PD(I,J)
     T= TIME + DELT1
     K= 2
     GØ TØ 13
 109 DØ 111 I= 1,NS
     NI= N(I)+1
     DØ 111 J=1,NI
     IF(JJ .EQ. 0) GØ TØ 112
     V(I,J)= 2.*VD(I,J) - V1(I,J)
     P(I,J)= 2.*PD(I,J) - P1(I,J)
 112 IF(HYDFL .EQ. 0)  GØ TØ 111
     G= G1
     IF(HYDFL .LT. 2)  GØ TØ 300
     CALL NTERP(GIND,GDEP,100,TIME,G,S,1,KK,LL)
 300 XJ= J-1
     XJD= XJ*DELX(I)
     X= AA(I) + AC(1,I)*XJD + AC(2,I)*XJD**2 + AC(3,I)*XJD**3 + AC(4,I)
    1   *XJD**4
     PDELH= RHØ * X * G / (144.*GC)
 111 PP(I,J)= P(I,J) + PDELH
     IF(JJ .EQ. 0) GØ TØ 14
     IF( PP(I,J) .LT. PMIN  )  GØ TØ 900
     TIME= TIME + DELT1
     CALL PRNT
     DELT = DELT1
     JJ= 1
     IF(TIME .LE. TLIMT) GØ TØ 13
     RETURN
 700 JKL= 1
     GØ TØ 702
 701 JKL= 2
 702 IF(L .GT. 0) GØ TØ 703
     L= 2
```

FIGURE A.1. SYMBOLIC LISTING OF SUBROUTINE SURGE (cont.)

```
703 CALL ERPRNT(L,T,JKL)
    CALL CTRØL
900 CALL PRNTER(I,J)
    CALL CTRØL
    RETURN
    END
```

FIGURE A.1. SYMBOLIC LISTING OF SUBROUTINE SURGE (cont.)

SUBROUTINE WMAIN

This subroutine controls the inputs of Subroutine SURGE.

CALLING SEQUENCE

CALL WMAIN

SOLUTION METHOD

Read inputs using NAMELIST feature.

1. READ

Initialize page number for printing inputs.

2. IP = 1

Write page heading.

3. WRITE, IP

Print inputs.

4. WRITE WDOT, RHO, GC, BETA, TLIMT, CMULT, PDTIM, PMIN, G1, PVOPT, HYDFL, NS

5. WRITE OUTP(1), (XL(I), I = 1, NS)

6. WRITE OUTP (2), (F (I), I = 1, NS)

7. WRITE OUTP (3), (D (I), I = 1, NS)

8. WRITE OUTP (4), (C1 (I), I = 1, NS)

9. WRITE OUTP (5), (R (I), I = 1, NS)

10. WRITE OUTP (6), (E (I), I = 1, NS)

11. WRITE OUTP (7), (PWT (I), I = 1, NS)

12. WRITE OUTP (8), (FLC (I), I = 1, NS)

Increase page number.

13. IP = 2

Eject page, print heading and continue printing inputs.

14. WRITE IP

15. WRITE OUTP (9), (TIMT (I), I = 1, 100)

16. WRITE OUTP (10), (PDT (I), I = 1, 100)

Increase page number.

17. IP = 3

Eject page, print heading and continue printing inputs.

18. WRITE IP

19. WRITE OUTP (11), (TITA (I), I = 1, 100)

20. WRITE OUTP (12), (WDOTT (I), I = 1, 100)

Check whether hydrostatic inputs are present.

21. If HYDFL = 0, go to 29

If HYDFL ≠ 0, go to 22

Increase page number.

22. IP = 4

Eject page, print heading and print hydrostatic inputs.

23. WRITE IP

24. WRITE OUTP(13), (GDEP(I), I = 1, 100)

25. WRITE OUTP(14), (GIND(I), I = 1, 100)

Increase page number.

26. IP = 5

Eject page, print heading and finish printing inputs.

27. WRITE IP

28. WRITE OUTP(15), (AC(I, J), I = 1, 4), J = 1, 20)

29. CALL SURGE

Return.

End

SUBROUTINE WMAIN NOMENCLATURE

Symbol	Description	Units	Reference
AC	Array containing coefficients of a fourth order polynomial which gives the distance below the beginning of a pipe section as a function of axial distance along the section	—	/WHAMM/, /WHAM/, 28
BETA	Fluid bulk modulus	$\frac{lbf}{in^2}$	/WHAMM/, /WHAM/, 4
C1	Constant which relates the influence of constraints on longitudinal movements of a pipe to stresses on a transverse cross section of the pipe	—	/WHAMM/, /WHAM/, 8
CMULT	Tolerance used in computing fluid velocity at last nodal point in a section	$\frac{ft}{sec}$	/WHAMM/, /WHAM/, 4
D	Pipe diameter	in	/WHAMM/, /WHAM/, 7
E	Modulus of elasticity of of pipe wall material	$\frac{lbf}{in^2}$	/WHAMM/, /WHAM/, 10
F	Pipe (Moody) friction factor	—	/WHAMM/, /WHAM/, 6
FLC	Flow loss coefficient between sections	—	/WHAMM/, /WHAM/, 12
G1	Acceleration	$\frac{ft}{sec^2}$	/WHAMM/, /WHAM/, 4

SUBROUTINE WMAIN NOMENCLATURE (cont.)

Symbol	Description	Units	Reference
GC	Constant relating force, mass, and acceleration — (32.174)	$\frac{\text{ft-lbm}}{\text{lbf-sec}^2}$	/WHAMM/, /WHAM/, 4
GDEP	Array containing the dependent variables of acceleration for interpolation of acceleration	$\frac{\text{ft}}{\text{sec}^2}$	/WHAMM/, /WHAM/, 24
GIND	Array containing the independent variables of time for interpolation of acceleration	sec	/WHAMM/, /WHAM/, 25
HYDFL	Variable used to denote method of hydrostatic pressure calculations HDFL = 0, no hydrostatic pressure calculations are performed HYDFL= 1, hydrostatic pressure calculations are performed with a constant acceleration HYDFL= 2, hydrostatic pressure calculations are performed with a time-varying acceleration	—	/WHAMM/, /WHAM/, INT, 4, 21

SUBROUTINE WMAIN NOMENCLATURE (cont.)

Symbol	Description	Units	Reference
I	Loop control counter denoting pipe section	—	5, 6, 7, 8, 9, 10, 11, 12, 15, 16, 19, 20, 24, 25, 28
IP	Page number	—	2, 3, 13, 14, 17, 18, 22, 23, 26, 27
J	Loop control counter denoting nodal point in a section	—	28
NS	Number of sections in system	—	/WHAMM/, /WHAM/, 4, 5, 6, 7, 8, 9, 10, 11, 12
OUTP	Array containing input names for printout	—	DIM, DATA, 5, 6, 7, 8, 9, 10, 11, 12, 15, 16, 19, 20, 24, 25, 28
PDT	Array containing the dependent variable of pressure for interpolating pressure at reservoir	$\frac{lbf}{in^2}$	/WHAMM/, /WHAM/, 16
PDTIM	Print time multiplier	—	/WHAMM/, /WHAM/, 4
PMIN	Fluid vapor pressure	$\frac{lbf}{in^2}$	/WHAMM/, /WHAM/, 4
PVOPT	Print option indicator	—	/WHAMM/, /WMAIN/, INT, 4
PWT	Pipe wall thickness	in	/WHAMM/, /WHAM/, 11
R	Pipe radius	in	/WHAMM/, /WHAM/, 9

SUBROUTINE WMAIN NOMENCLATURE (cont.)

Symbol	Description	Units	Reference
RHO	Fluid density	lbm/ft^3	/WHAMM/, /WHAM/, 4
TIMT	Array containing the independent variables of time for interpolating pressure at reservoir	sec	/WHAMM/, /WHAM/, 15
TITA	Array containing the independent variables of time for interpolation of mass flowrate	sec	/WHAMM/, /WHAM/, 19
TLIMT	Calculation time limit	sec	/WHAMM/, /WHAM/, 4
WDOT	Fluid mass flowrate	$\frac{lbm}{sec}$	/WHAMM/, /WHAM/, 4
WDOTT	Array containing the dependent variables of mass flowrate for interpolation of mass flowrate at valve end of system	$\frac{lbm}{sec}$	/WHAMM/, /WHAM/, 20
XL	Length of a section of pipe	ft	/WHAMM/, /WHAM/, 5

```
$IBFTC WHMAIN  FULIST,DECK
      SUBRØUTINE WMAIN
      CØMMØN /WHAM/NS,N(20),WDØT,XL(20),F(20),D(20),RHØ,GC,C1(20),
     1    R(20),E(20),PWT(20),BETA,TIMT(100),PDT(100), G1,    FLC(20),
     2             CMULT,TITA(100),WDØTT(100),TLIMT,PDTIM,PVØPT,HYDFL,
     3     GDEP(100), GIND(100),AC(4,20),PMIN
      INTEGER PVØPT,HYDFL
      NAMELIST /WHAMM/ NS,N,WDØT,XL,F,D,RHØ,GC,C1,R,E,PWT,BETA,TIMT,PDT,
     1        FLC,    CMULT,TITA,WDØTT,TLIMT ,PDTIM,PVØPT,HYDFL,GDEP,
     2       GIND,AC ,PMIN ,G1
      DIMENSIØN ØUTP(15)
      DATA (ØUTP(I), I=1,15)  /
     1                    90H    XL=     F=     D=    C1=     R=     E=  PWT=  FL
     2C= TIMT=  PDT= TITA=WDØTT= GDEP= GIND=   AC=         /
   11 READ(5,WHAMM)
      IP=1
      WRITE(6,1)      IP
    1 FØRMAT(1H1 40X,22H  WATERHAMMER  INPUTS    35X,5HPAGE  I1////)
      WRITE(6,2)    WDØT,RHØ,GC,BETA,TLIMT,CMULT,PDTIM,PMIN, G1,
     1          PVØPT,HYDFL,    NS
    2 FØRMAT(1H0 /5X,                      7H WDØT= E16.8,2X,7H  RHØ= E16.8,
     1  2X,   7H   GC= E16.8,2X,7H BETA= E16.8/5X,7HTLIMT= E16.8,2X,
     2        7HCMULT= E16.8,2X,7HPDTIM= E16.8,2X,7H PMIN=  E16.8/
     3  5X,7H   G1=  E16.8, 2X,7HPVØPT=  I1,17X,
     4       7HHYDFL= I1,17X,7H   NS=    I2)
      WRITE(6,6)  (N(I), I=1,NS)
    6 FØRMAT(1H 6X, 6H  N=   20I6 )
      WRITE  (6,3)  ØUTP(1),(XL(I),I=1,NS)
      WRITE  (6,3)  ØUTP(2),(F(I),I=1,NS)
      WRITE  (6,3)  ØUTP(3),(D(I),I=1,NS)
      WRITE  (6,3)  ØUTP(4),(C1(I),I=1,NS)
      WRITE  (6,3)  ØUTP(5),(  R(I),I=1,NS)
      WRITE  (6,3)  ØUTP(6),(  E(I),I=1,NS)
      WRITE  (6,3)  ØUTP(7),(PWT(I),I=1,NS)
      WRITE  (6,3)  ØUTP(8),(FLC(I),I=1,NS)
      IP= 2
      WRITE(6,1)      IP
      WRITE  (6,3)  ØUTP( 9),(TIMT(I),I=1,100)
      WRITE  (6,3)  ØUTP(10), (PDT(I),I=1,100)
      IP= 3
      WRITE(6,1)    IP
      WRITE  (6,3)  ØUTP(11),(TITA(I),I=1,100)
      WRITE  (6,3)  ØUTP(12),(WDØTT(I),I=1,100)
    3 FØRMAT(1H04X,A6, E17.8,4E25.8/(11X,E17.8,4E25.8  )  )
      IF(HYDFL .EQ. 0)   GØ TØ 5
      IP = 4
      WRITE(6,1)    IP
      WRITE(6,3)ØUTP(13), (GDEP(I),I=1,100)
      WRITE(6,3) ØUTP(14), (  GIND(I),I=1,100)
      IP = 5
```

FIGURE A.2. SYMBOLIC LISTING OF SUBROUTINE WMAIN

```
      WRITE(6,1)      IP
      WRITE(6,4) ØUTP(15), ((AC(I,J), I=1,4),J= 1,20)
    4 FØRMAT(1H 4X,A6,E17.8,3E25.8/(11X,E17.8,3E25.8) )
    5 CØNTINUE
      CALL SURGE
      RETURN
      END
```

FIGURE A.2. SYMBOLIC LISTING OF SUBROUTINE WMAIN (cont.)

SUBROUTINE PRNT

This subroutine controls the output for Subroutine SURGE.

CALLING SEQUENCE

CALL PRNT

The following are entries for error print-outs:

CALL ERPRNT(L6, T, JKT)

CALL PRNTER(IJ, JI)

SOLUTION METHOD

Check if first time into the routine.

1. If TIME $\neq$ 0, go to 59

   If TIME = 0, go to 2

Initialize line counter, page number and time to print.

2. LC = 0

3. IP = 1

4. TT = 0

Set up array NI to contain the number of divisions plus one for each section.

5. Do 6, J = 1, NS

6. NI(J) = N(J) + 1

Write page heading.

7. WRITE IP, DELT1

Determine print option.

8. Go to (9, 26, 41), PVOPT

Set K = section, J = number of divisions in section.

9. K = 1

10. J = NI(1)

Write out print line.

11. WRITE TIME, PP(1, 1), PP(1, J), V(1, J)

Increase line count.

12. LC = LC + 1

Check if more than one section.

13. If NS < 2, return

If NS ≥ 2, go to 14

Set K2 = section number, KNS = number of sections.

14. K2 = 2

15. KNS = NS

Check if more room on this page.

16. If (LC + NS-1) < 54, go to 18

If (LC + NS-1) ≥ 54, go to 17

Set KNS to maximum lines to print on page.

17. KNS = 54 - LC

18. Do 21, K = K2, KNS

Increase line count.

19. LC = LC + 1

20. J = NI(K)

21. WRITE PP(K, J), V(K, J)

Check if all sections have been output.

22. If K = NS, return

If K ≠ NS, go to 23

Initialize for rest of section.

23. K2 = KNS + 1

24. KNS = NS

25. Go to 66

Set up print constants

26. K = 2

27. J = 1

Print a line for print option 2.

28. WRITE TIME, PP(1, 1), PP(2, 1), V(2, 1)

See if more sections to be output.

29. If NS < 3, return

If NS ≥ 3, go to 30

Set up print constants for remaining sections.

30. K2 = 3

31. KNS = NS

Determine if more room on this page.

32. If (LC + NS - 2) < 54, go to 34

Set KNS to maximum lines for this page.

33. KNS = 54 - LC

34. Do 36, K = K2, KNS

Increase line count.

35. LC = LC + 1

36. WRITE PP(K, 1), V(K, 1)

Determine if more sections to be output.

37. If K = NS, return

If K ≠ NS, go to 38

Set print constants.

38. K2 = KNS + 1

39. KNS = NS

40. Go to 66

Set up print constants for print option 3.

41. J = NI(1)

42. L = 1

43. K = 2

Increase line count.

44. LC = LC + 1

45. WRITE TIME, PP(1, 1), PP(1, J), V(1, J)

Check if only one section in system.

46. If NS $<$ 2, return

If NS $\geq$ 2, go to 47

Initialize for looping, K2 = current section number, KNS = maximum section number.

47. K2 = 2

48. KNS = NS

Determine if all sections can be printed on this page.

49. If (LC + (2)(NS) - 2) $<$ 54, go to 52

If (LC + (2)(NS) - 2) $\geq$ 54, go to 50

50. KNS = (54 - LC)/2

51. If KNS $<$ 3, KNS = 2

52. Do 55, K = K2, KNS

Set J = number of divisions in this section.

53. J = NI(K)

Increase line count.

54. LC = LC + 2

55. WRITE PP(K, 1), V(K, 1), PP(K, J), V(K, J)

56. If K = NS, return

If K $\neq$ NS, go to 56

Increase for remaining sections.

57. K2 = KNS + 1

58. KNS = NS

59. Go to 66

Determine if it is time for print.

60. If TIME < (TT + (PDTIM)(DELT1)) and
TIME < TLIMT, return

If not, go to 61

Reset time for next print.

61. TT = TIME

Check if line count exceeded.

62. If LC < 52, go to 8

If LC $\geq$ 52, go to 62

Increase page number and clear line count.

63. IP = IP + 1

64. LC = 0

Write page heading.

65. WRITE IP, DELT1

66. Go to 8

Increase page number.

67. IP = IP + 1

Write page heading.

68. Write IP, DELT1

Clear line count.

69. LC = 0

Determine correct entry back.

70. If PVOPT = 1, go to 18

If PVOPT = 2, go to 34

If PVOPT = 3, go to 52

ENTRY ERPRNT

Write NTERP error message.

71. WRITE TIME, T, TBL(L6), TBL(L6 + 4), TBL(JKT + 2)

72. Return

ENTRY PRNTER

Write error message.

73. WRITE PP(IJ, JI), PMIN

74. Return

End

SUBROUTINE PRNT NOMENCLATURE

Symbol	Description	Units	Reference
DELT1	Integration time interval	sec	/WHAMR/, 7, 65, 68
IJ	Index counter	—	CALL, 73
IP	Page number	—	3, 7, 63, 65, 67, 68
J	Loop control counter denoting nodal point in a section	—	5, 6, 10, 11, 20, 21, 27, 41, 45, 53, 55
JI	Index counter	—	CALL, 73
JKT	Index counter	—	CALL, 71
K	Loop control counter	—	9, 18, 20, 21, 22, 26, 34, 36, 37, 43, 52, 53, 55, 56
K2	Loop control counter	—	14, 18, 23, 30, 34, 38, 47, 52, 57
KNS	Section number for loop control	—	15, 17, 18, 24, 31, 33, 34, 38, 39, 48, 50, 51, 52, 57, 58
L	Print line constant	—	42
L6	Index counter	—	CALL, 71
LC	Line counter	—	2, 12, 16, 19, 32, 35, 44, 49, 50, 54, 62, 64, 69
N	Array containing number of divisions in a section	—	/WHAM/, 6

SUBROUTINE PRNT NOMENCLATURE (cont.)

Symbol	Description	Units	Reference
NI	Array containing number of divisions in a section plus one	—	DIM, 6, 10, 20, 41, 53
NS	Number of sections in system	—	/WHAM/, 5, 13, 15, 16, 22, 24, 29, 31, 32, 37, 39, 46, 48, 49, 56, 58
PDTIM	Print time multiplier	—	/WHAM/, 60
PMIN	Fluid vapor pressure	$\frac{lbf}{in^2}$	/WHAM/, 73
PP	Pressure at nodal point (including hydrostatic pressure)	$\frac{lbf}{in^2}$	/WHAMR/, 11, 21, 28, 36, 45, 55, 73
PVOPT	Print option flag	—	/WHAM/, INT, 8, 70
T	Dummy time	sec	CALL, 71
TBL	Array containing Hollerith print words for error message	—	DIM, DATA, 71
TIME	Time	sec	/WHAMR/, 1, 11, 28, 45, 60, 61, 71
TLIMT	Calculation time limit	sec	/WHAM/, 60
TT	Contains time of last print	sec	4, 60, 61
V	Velocity at nodal point	$\frac{ft}{sec}$	/WHAMR/, 11, 21, 28, 36, 45, 55

```
$IBFTC PRNTWH  FULIST,DECK
      SUBRØUTINE PRNT
      CØMMØN /WHAMR/ V(20,101), P(20,101),PD(20,101), VD(20,101), TIME,
     1    P1(20,101), V1(20,101),PP(20,101),  DELX(20),PDELF(20),A(20)
     2,  DELT1
      DIMENSIØN NI(20)
      DIMENSIØN TBL(6)
      DATA TBL /6H .GT. ,6H .LT. ,6H TIMT ,6H TITA ,6H MAX  ,6H MIN  /
      CØMMØN /WHAM/NS,N(20),WDØT,XL(20),F(20),D(20),RHØ,GC,C1(20),
     1    R(20),E(20),PWT(20),BETA,TIMT(100),PDT(100), G1,    FLC(20),
     2            CMULT,TITA(100),WDØTT(100),TLIMT,PDTIM,PVØPT,HYDFL,
     3     GDEP(100), GIND(100),AC(4,20),PMIN
      INTEGER PVØPT,HYDFL
      IF(TIME   .NE. 0.)  GØ TØ 50
      LC= 0
      IP= 1
      TT= 0.
      DØ 4 J= 1,NS
    4 NI(J)= N(J)+1
    2 WRITE(6,1) IP,DELT1
    1 FØRMAT(1H135X,20H WATERHAMMER ØUTPUT  35X,5HPAGE I3/ 8X,7HDELT1=
     1  E16.8//)
    5 GØ TØ (10,20,30),PVØPT
   10 K= 1
      J= NI(1)
      WRITE(6,100) TIME,PP(1,1),K,J,PP(1,J),K,J,V(1,J)
      LC= LC+1
      IF(NS .LT. 2) RETURN
      K2= 2
      KNS= NS
      IF(LC+NS-1 .LT. 54) GØ TØ 12
      KNS= (54 - LC)
   12 DØ 11 K= K2,KNS
      LC= LC+1
      J= NI(K)
   11 WRITE(6,101) K,J,PP(K,J),K,J,V(K,J)
      IF(K .EQ. NS) RETURN
      K2= KNS+1
      KNS= N2
      GØ 3Ø 5
   2 K= 2
      J= 1
      LL= LL+1
      6RR3N(6,1) 3IME,PP(1,1),K,J,PP(2,1),K,J,V(2,1)
      IF(NS .LT. 3) RETURN
      K2 = 3
      KNS= NS
      IF(LC+NS-2 .LT. 54) GØ TØ 22
      KNS= (54-LC)
   22 DØ 21 K= K2,KNS
```

FIGURE A.3. SYMBOLIC LISTING OF SUBROUTINE PRNT

```
      LC= LC+1
   21 WRITE(6,101) K,J,PP(K,1),K,J,V(K,1)
      IF( K.EQ. NS) RETURN
      K2= KNS+1
      KNS= NS
      GØ TØ 500
  100 FØRMAT(7H TIME= E16.8,11H  PP(1,1)= E16.8, 2X,
     1          5H  PP(I2,1H,I3,3H)= E16.8,2X,4H  V(I2,1H,I3,3H)= E16.8)
  101 FØRMAT(1H 51X,5H  PP(I2,1H,I3,3H)= E16.8, 2X,
     1                  4H  V(I2,1H,I3,3H)= E16.8)
   30 J= NI(1)
      L= 1
      K= 2
      LC= LC+1
      WRITE(6,100) TIME,PP(1,1),L,J,PP(1,J),L,J,V(1,J)
      IF(NS.LT. 2) RETURN
      K2= 2
      KNS= NS
      IF(LC + 2*NS-2 .LT. 54)  GØ TØ 32
      KNS= (54-LC)/2
      IF(KNS .LT. 3)  KNS= 2
   32 DØ 31 K=K2,KNS
      J= NI(K)
      LC= LC+2
   31 WRITE(6,301) K,L,PP(K,1),K,L,V(K,1),K,J,PP(K,J),K,J,V(K,J)
      IF(K .EQ. NS) RETURN
      K2= KNS+1
      KNS= NS
      GØ TØ 500
  301 ØØRMA3(1H 52X,4H PP(I2,1H,I3,3H)= E16.8,2X,4H  V(I2,1H,I3,3H)=
     1 E16.8/53X,4H PP(I2,1H,I3,3H)= E16.8,2X,4H  V(I2,1H,I3,3H)= E16.8)
   50 IF(TIME .LT. (TT+PDTIM*DELT1) .AND. TIME .LT. TLIMT) RETURN
      TT= TIME
      IF(LC .LT. 52) GØ 3Ø 5
      IP= IP+1
      LC = 0
      WRITE(6,1 )  IP,DELT1
      GØ TØ 5
  500 IP= IP+1
      WRITE(6,1) IP,DELT1
      LC= 0
      IF(PVØPT .EQ. 1)  GØ TØ 12
      IF(PVØPT.EQ. 2) GØ TØ 22
      GØ TØ 32
      ENTRY ERPRNT(L6,T,JKT)
      WRITE(6,700)TIME,T,TBL(L6),TBL(L6+4),TBL(JKT+2)
  700 FØRMAT(45H0 *****  ERRØR ENCØUNTERED IN NTERP AT TIME=  E16.8,1H./
     1    10X,2HT(E16.8,5H) IS A6,4HTHE A6,14HVALUE ØF TABLEA6,1H.)
      RETURN
      ENTRY PRNTER(IJ,JI)
```

FIGURE A.3. SYMBOLIC LISTING OF SUBROUTINE PRNT (cont.)

```
      WRITE(6,900) IJ,JI,PP(IJ,JI) , PMIN
      RETURN
  900 FØRMAT(12H1 *****  PP(I2,1H,I3,3H)= E16.8,19H IS LESS THAN PMIN(
     1  E16.8,26H)  CASE TERMINATED.   *****    )
      END
```

FIGURE A.3. SYMBOLIC LISTING OF SUBROUTINE PRNT (cont.)

SAMPLE RESULTS FROM SUBROUTINE SURGE

The output from Subroutine SURGE is shown graphically for the following cases:

<u>Case 1 (Figure A.4)</u>

Linear valve closure in a 50-foot long horizontal pipe with friction included.

<u>Case 2 (Figure A.5)</u>

Linear valve opening in a 50-foot long horizontal pipe with friction included.

<u>Case 3 (Figure A.6)</u>

Instantaneous valve closure in a 50-foot long horizontal pipe with no friction.

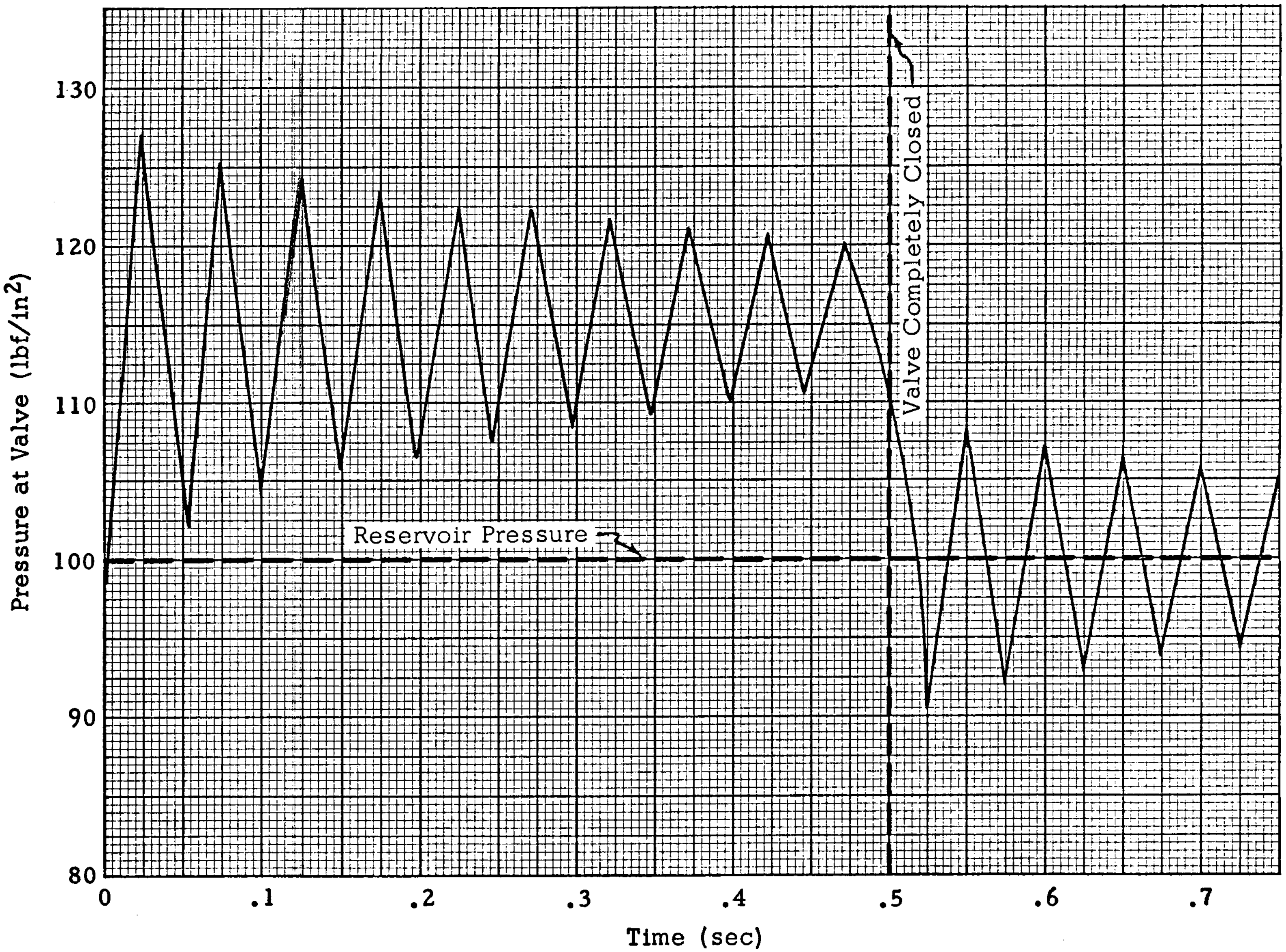

FIGURE A.4. LINEAR VALVE CLOSURE

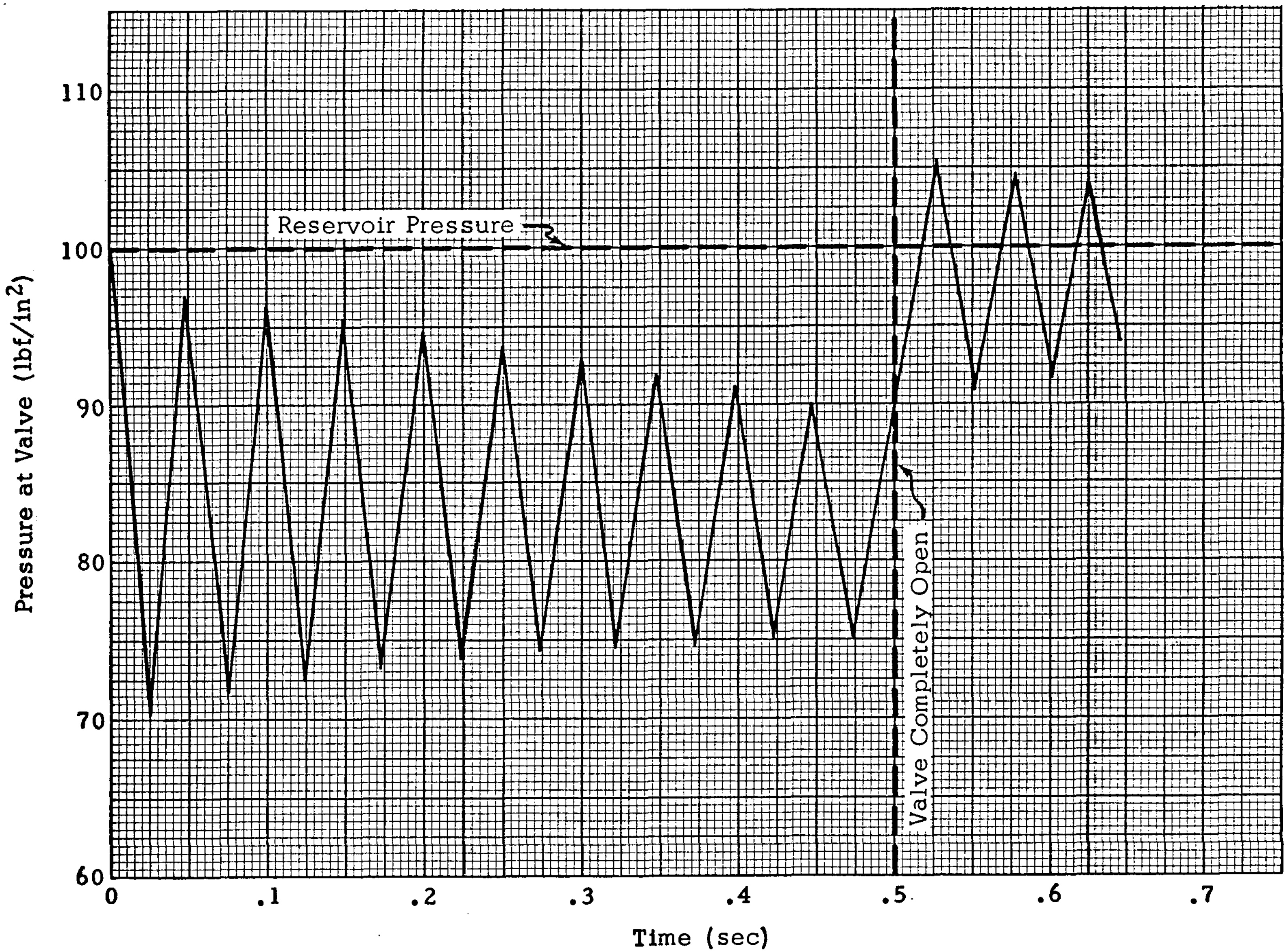

FIGURE A.5. LINEAR VALVE OPENING

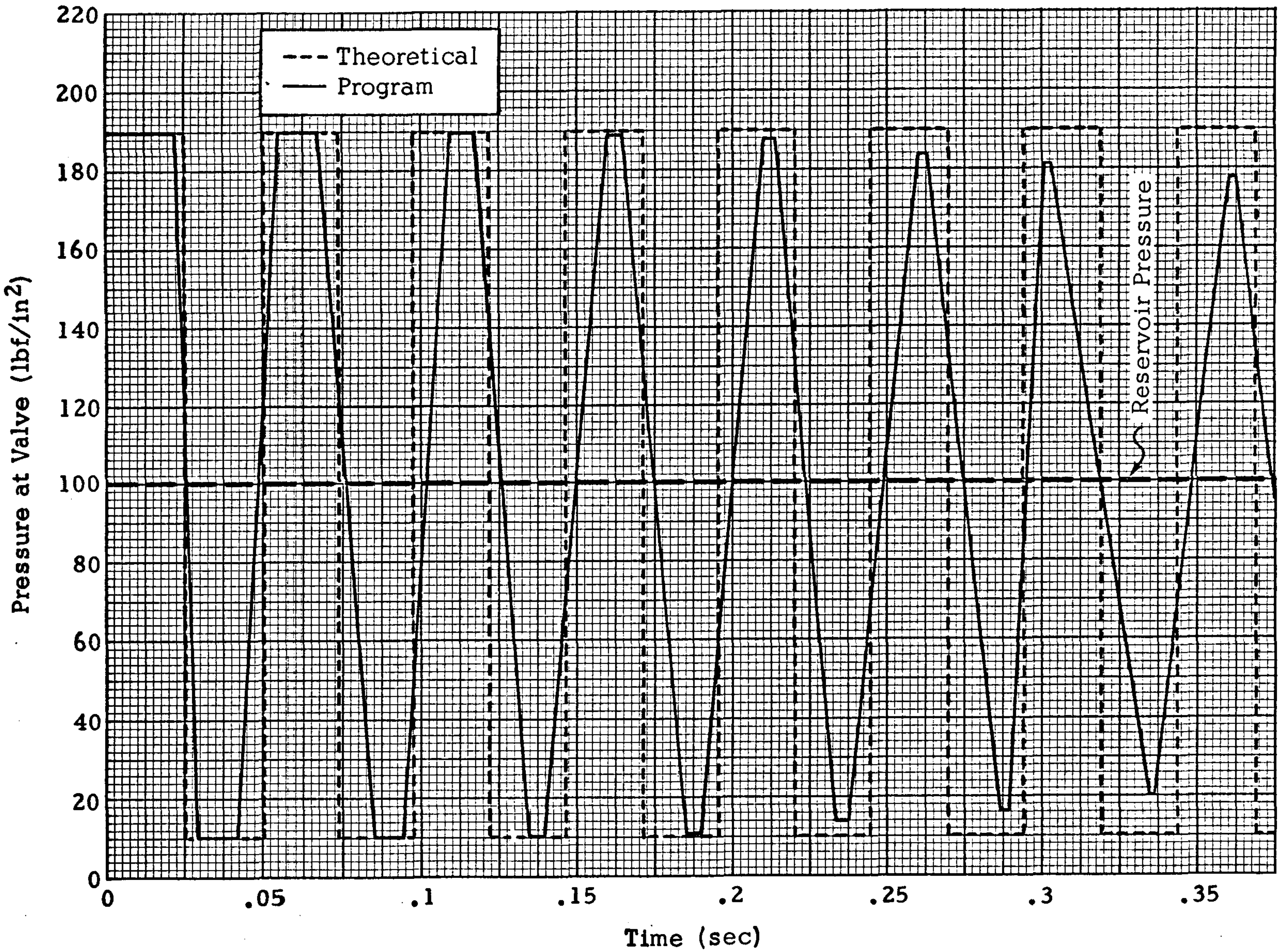

FIGURE A.6. INSTANTANEOUS VALVE CLOSURE