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STRUCTURAL DESIGN SYNTHESIS APPROACH TO FILAMENTARY COMPOSITES

by George Gerard and C. Lakshmikantham

Prepared by
ALLIED RESEARCH ASSOCIATES, INC.
Concord, Mass.

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Summary

The first part of this paper is in the nature of a progress report on recent developments of analysis methods for filamentary composites. Theoretical predictions of the stiffness and strength properties of a unidirectional composite based on a knowledge of the constituent properties are correlated with experiments for both tensile and compressive loadings. The analysis of multilayer or laminated composites based upon the unidirectional composite properties then requires the rather straight forward use of classical anisotropic shell theory.

Some structural aspects of filamentary composites designed for biaxial loads are considered in the second part. In particular, certain design restrictions inherent in the use of such composites become evident when compared to the more familiar isotropic sheet. Some of these restrictions can be overcome by a close matching of filament orientations and stress field. These factors serve to emphasize the overwhelming importance of creative structural concepts in the design of successful filamentary composites.

Symbols

A_f	cross sectional area of filament
C	contiguity factor
E	elastic modulus
G	shear modulus
n	number of filaments per unit width
N_1, N_2	loading per unit width referred to total composite thickness
N_f	loading per unit width in filamentary direction referred to thickness of unidirectional composite
t_f	equivalent filamentary sheet thickness
T	shear strength of composite
V	volume fraction
W	weight fraction
X, Y, Z	uniaxial composite strengths in filamentary, transverse and thickness directions, respectively
β	angle between loading and filamentary axes
ν	Poisson's ratio
σ	stress component
σ_f	tensile strength of filaments
$\bar{\sigma}_{cr}$	microbuckling strength
τ	shear stress

Subscripts:

f	filamentary
m	matrix
x, y	coordinates
1, 2	principal directions
—	denotes composite properties

STRUCTURAL DESIGN SYNTHESIS APPROACH TO FILAMENTARY COMPOSITES

Introduction

In Ref. 1, the essential roles of, and research opportunities for the constituent elements of the composite (matrix, reinforcement and interface) were presented in considerable depth. These areas essentially encompass the materials aspects of composites. When we consider the composite under various loading conditions, then we necessarily shift from a materials to a structural viewpoint. In this regard, it was noted in Ref. 1 that since a composite is a combination of materials selected to obtain specified design objectives, a structural mechanics approach is essential to the successful design of composites tailored for specific applications.

The treatment of the structural aspects of the composite presented in Ref. 1 consisted of an identification of fundamental problem areas and an assessment of the state of knowledge in those areas as of 1963. Considerable progress in the development of methods of analysis of filamentary composites and their experimental confirmation has been achieved since that time as a result of an expanding research effort. One of the objectives here is to highlight some significant developments in this area and this information is contained in Section 2.

A second, and perhaps more important objective, is to present some recent information on structural design aspects of filamentary composites particularly from a minimum weight viewpoint. Here, starting with the mechanics of a unidirectional filamentary sheet, the strength/weight characteristics of laminates composed of variously oriented filamentary sheets are considered for various loading conditions. The unidirectional filamentary sheet is considered to be the composite material which becomes the fundamental building block of the structural composite or laminate.

As compared to the familiar isotropic metallic sheet in a biaxial stress field, certain restrictions are inherent in the design of filamentary sheet laminates with regard to the magnitude and orientation of the stress components. This aspect constitutes a major difference in the structural applications of composites when compared to familiar metallic sheets. These and other design considerations for filamentary composites are presented in Section 3.

2. Structural Analysis Methods for Filamentary Composites Under Uniaxial Loads

Elementary Considerations

The only significant load that a single continuous filament is capable of sustaining is one that is collinear with the filament axis and tensile in character. The following quantities are associated with the filament: filament strength (σ_f), filament modulus (E_f), cross sectional area (A_f).

The fundamental filamentary composite sheet consists of a series of parallel filaments (unidirectional) of the same material properties (homogeneous) uniformly stressed (isotensoid) in a suitable matrix. For a large volume fraction of filaments, their role is primarily that of load transmission. Functionally, the matrix positions the filaments, provides the desired geometric contours and acts as a sealant. For tensile loading, it also provides a shear path around fractured filaments. For compressive loading, the matrix provides an effective foundation modulus against buckling of the filamentary columns.

As indicated in Fig. 1, the filamentary composite sheet has an equivalent filamentary thickness, $t_f = nA_f$ where n is the number of filaments per inch width. Based on the filament stress, we can define the loading, $N_f = \sigma_f t_f$. Similarly, we can define an effective composite thickness $\bar{t}_c$ based on the composite stress $\bar{\sigma}_c$ such that

$$N_f = \bar{\sigma}_c \bar{t}_c = \sigma_f t_f \qquad (1)$$

The properties based on the filamentary characteristics neglect the matrix whereas the composite properties include the volume fraction of the matrix.

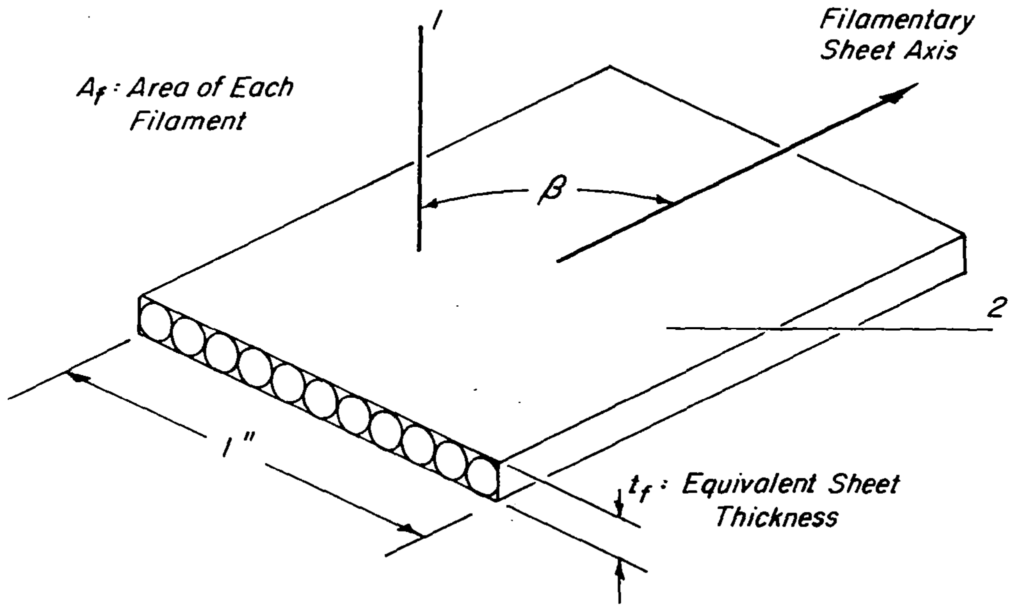


Figure 1 Elements of a unidirectional filamentary sheet

Elastic Properties of Filamentary Sheets

The elastic stiffness properties of composites are of fundamental importance in the design of structures subject to buckling as well as those subject to deflection limitations. Tsai[2] has surveyed various methods of analysis for predicting the elastic constants of unidirectional filamentary composites from the constituent properties (filament: E_f, ν_f; matrix: E_m, ν_m). Particularly noteworthy are the test results that he has obtained and the correlation obtained with theory.

To summarize his results and conclusions, a unidirectional filamentary composite can be represented as an anisotropic sheet and characterized in terms of four composite elastic constants: $\overline{E}_1$, $\overline{E}_2$, $\overline{\nu}_{12}$, and $\overline{G}$. Test results and theory for the four constants as a function of matrix weight fraction are shown in Fig. 2. It was observed that $\overline{E}_1$ depends primarily upon E_f and that filament unalignment caused the scatter in the data of Fig. 2. On the other hand, $\overline{E}_2$, and $\overline{G}$ are strongly influenced by E_m as well as the degree of filament contiguity, a factor not readily predictable although C = 0.2 represents the test data of Fig. 2. The major Poisson's ratio, ν_{12} is somewhat influenced by the contiguity factor and is well predicted by the theory.

Once the properties of a unidirectional filamentary sheet have been established, classical theories of anisotropic layered plates can be used to predict the elastic constants of laminated filamentary composites. By a series of experiments, Azzi and Tsai[3] have demonstrated the successful application of such theory to cross-ply and angle-ply filamentary composite laminates. Further experimental work by NASA[4] on filament wound cylinders confirm the use of existing theories for the elastic properties.

Tensile Strength of Filamentary Sheets

Tensile strength data constitute basic material property data in the aerospace field. Thus, we return again to the unidirectional filamentary composite for the purpose of establishing the strength properties that are required to characterize

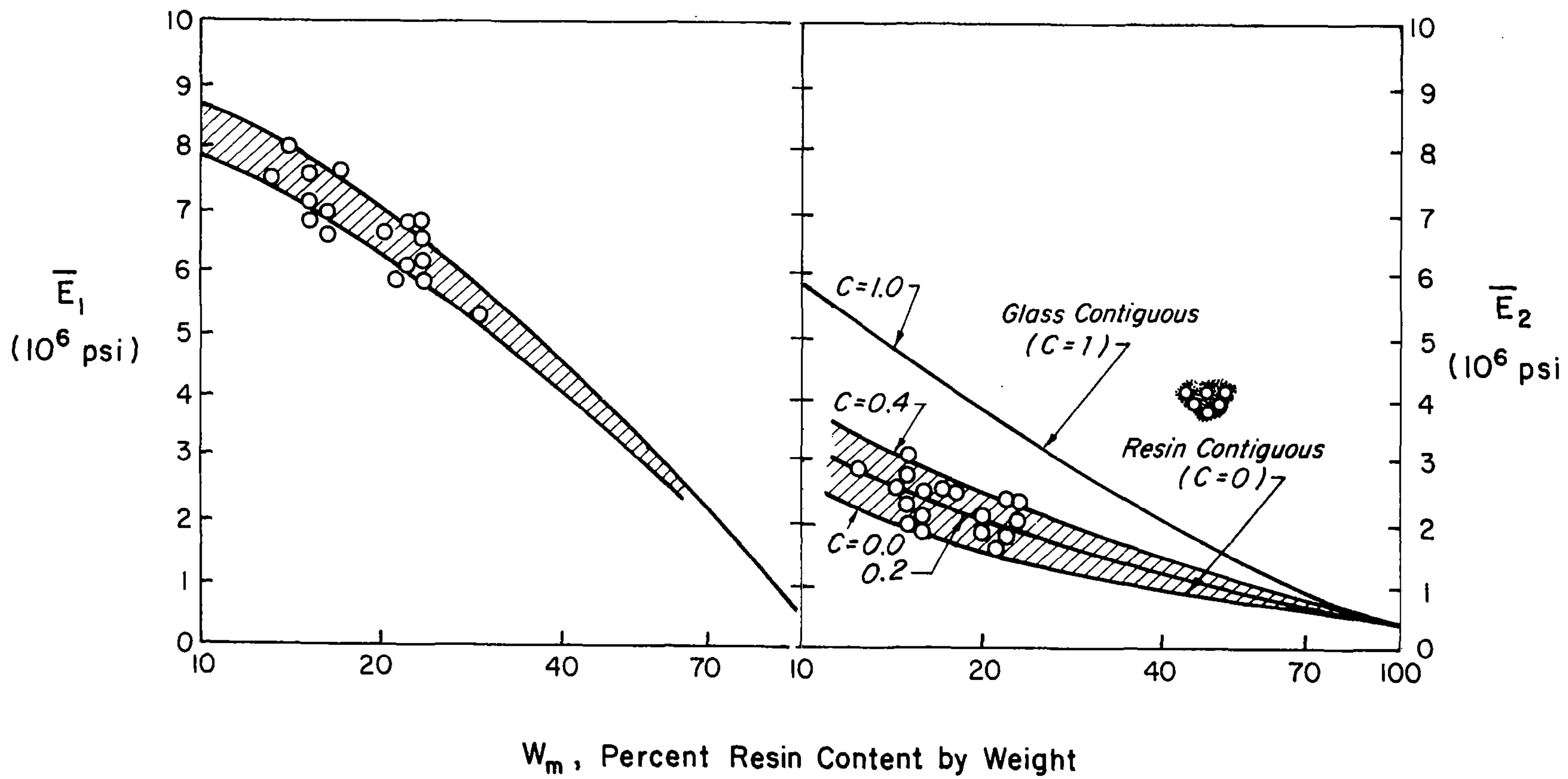


Figure 2a Glass-epoxy composites

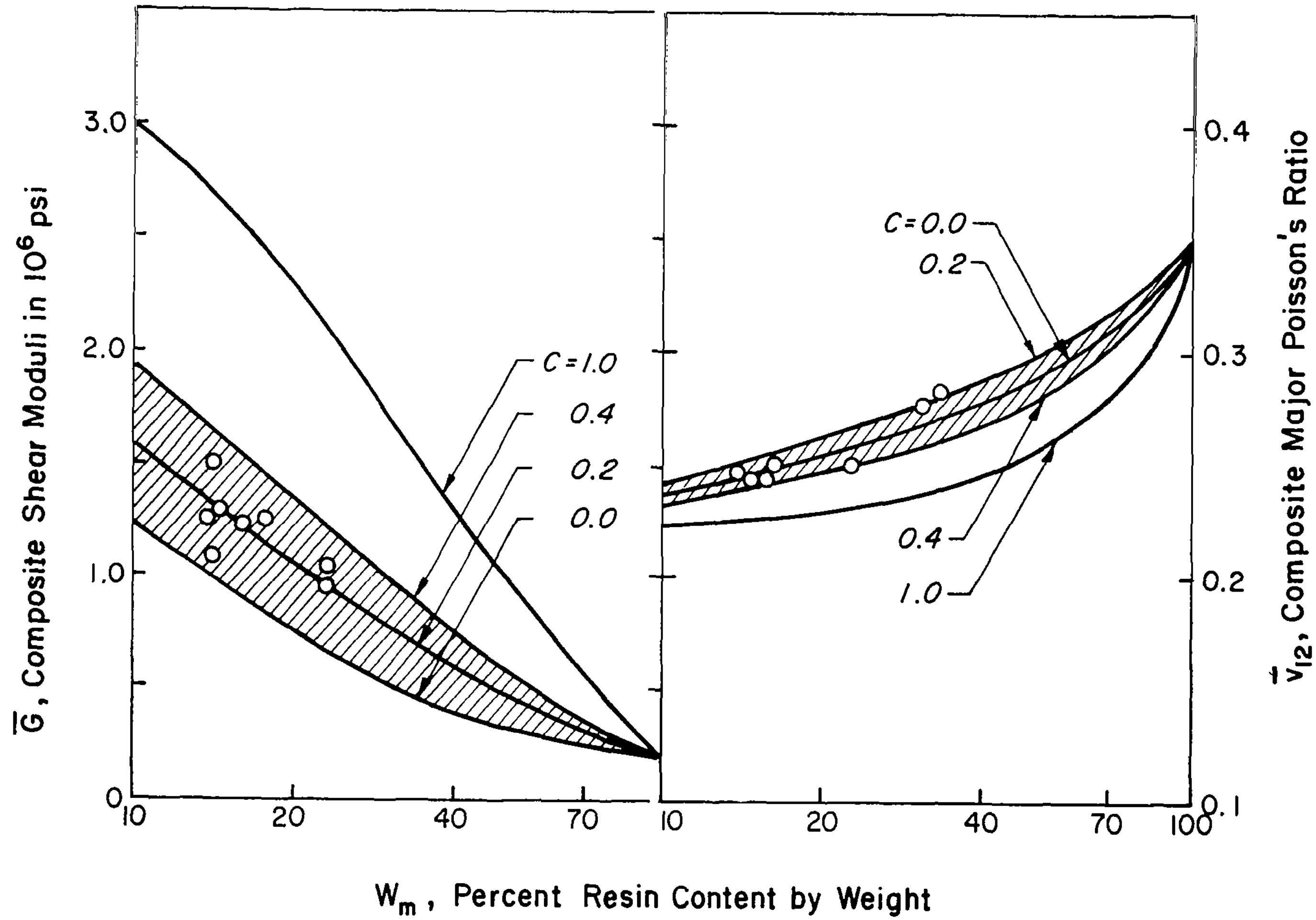


Figure 2b Glass-epoxy composites

the behavior of such a composite under tensile loads at various angles to its filamentary axis.

Azzi and Tsai[5] have examined this problem for the unidirectional filamentary sheet and by assuming plane stress conditions have taken the thickness stress components to be zero. Under this condition, Hill's*generalized anisotropic strength law becomes

$$\sigma_x^2 - \left[1 + \frac{X^2}{Y^2} - \frac{X^2}{Z^2}\right]\sigma_x\sigma_y + \frac{X^2}{Y^2}\,\sigma_y^2 + \frac{X^2}{T^2}\,\tau_{xy}^2 = X^2 \tag{2}$$

Here, the filamentary axis is in the x-direction, y is transverse and z is the thickness direction. The axial strength properties in the filamentary, transverse and thickness directions are X, Y, and Z, respectively, and T is the shear strength.

By assuming that the unidirectional filamentary composite is transversely isotropic, Y = Z, Eq. (2) reduces to

$$\sigma_x^2 - \sigma_x\sigma_y + \left(\frac{X}{Y}\right)^2 \sigma_y^2 + \left(\frac{X}{T}\right)^2 \tau_{xy}^2 = X^2 \tag{3}$$

Thus, Azzi and Tsai[5] have used three experimentally determined composite strength values to characterize the unidirectional filamentary sheet: the tensile strength in the filamentary direction (X), the tensile strength transverse to the filamentary direction (Y), and the shear strength on a plane of anisotropic symmetry (T). The latter can be obtained on a torsion tube using a specimen with only circumferential windings, or in a pure shear loading frame.

In cases where the tensile loading axis (1) may be at an angle β with the filamentary axis (x), the well known coordinate transformation equations must be employed. For the uniaxial tension case the following relation is obtained[5]

---

*Hill, R., <u>The Mathematical Theory of Plasticity</u>, Oxford University Press, London, 1950, pp. 318-320.

$$\sigma_1/X = \left[\left(\frac{X}{Y}\right)^2 \sin^4\beta + \left\{\left(\frac{X}{Y}\right)^2 - 1\right\} \sin^2\beta\cos^2\beta + \cos^4\beta\right]^{-1/2} \tag{4}$$

Experimental confirmation of the validity of Eq. (4) for a glass filament-reinforced resin unidirectional composite is shown in Fig. 3.

From a practical standpoint, Fig. 3 demonstrates conclusively the intolerance of a unidirectional composite to load misalignment. It also demonstrates that a convenient "filamentary approximation" for low strength matrices is the following:

$$\sigma_1/X = 1 \text{ for } \beta = 0; \qquad \sigma_1/X = 0 \text{ for } \beta > 0 \tag{5}$$

While the foregoing serves to identify three experimentally determined composite strength properties for characterizing unidirectional composites, it is apparent that there has been a great interest in predicting the composite tensile strength (X) directly from a knowledge of the constituent properties. The effort in this area has proceeded for quite some time since the most simple and direct measure of composite efficiency is the tensile test. Thus, knowledge in this area is perhaps most widespread of all composite properties. The work of Kelly and Tyson, Cratchley, and Weeton on metal filament/metal matrix composites is well known. Some representative recent work on glass filament/organic matrix composites, in which various statistical distributions of glass filaments strength have been used to predict composite tensile strength, includes that of Rosen[6] and Ekvall.[7] The latter has also treated matrix materials exhibiting a variable strength law by utilizing the Mohr strength envelope.

Tsai and Azzi[8] have generalized the unidirectional results for laminates subjected to combined external loads as well as the thermomechanical stresses

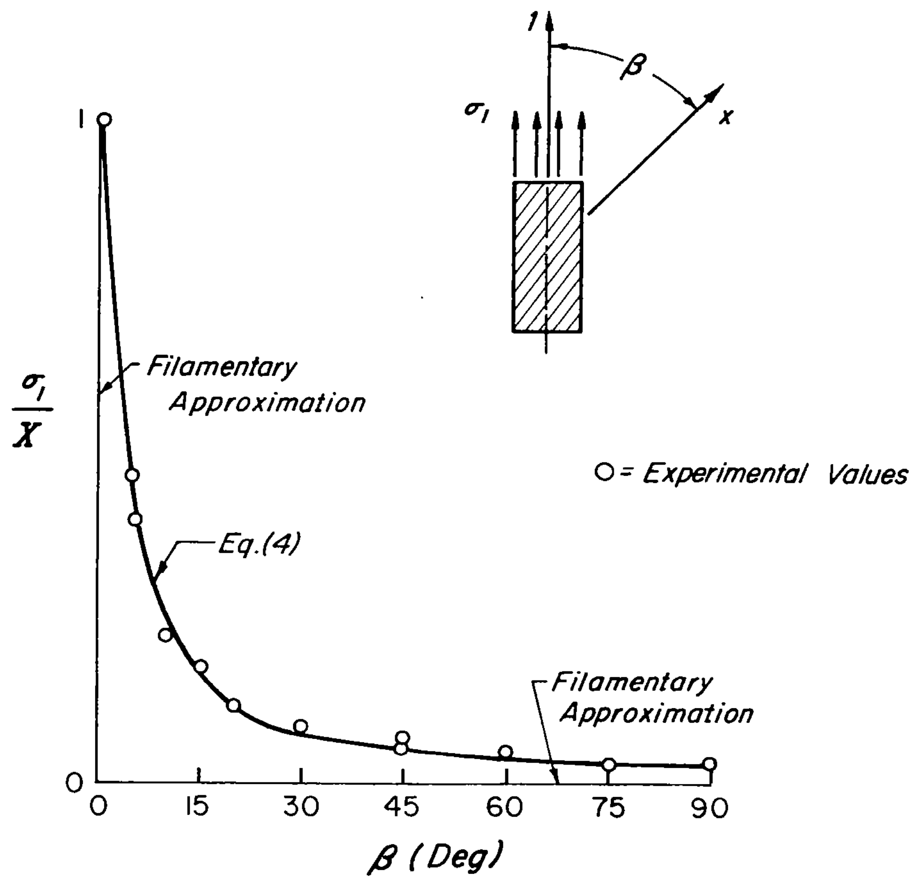


Figure 3 Glass filaments/reinforced resin unidirectional filamentary composite. Composite X=150 ksi, Y=5.30, T=6.05 (from Ref. 5)

induced by the lamination process. Test data on cross-ply and angle-ply composites under uniaxial tensile loads correlate quite well with theory.

Compressive Strength of Filamentary Sheets

The design of structures subject to compressive loads is generally governed by either of the following limitations: buckling in a general instability mode in which the elastic stiffness properties are of basic importance or compressive strength which for a filamentary composite depends upon a filamentary microbuckling phenomenon.

Presently, there are not much data available on the elastic buckling characteristics of filamentary composites. However, Fig. 4 is an example of the correlation achieved between orthotropic shell stability theory and test data on filament wound cylinders under compressive loads.[9] As additional test data become available and are correlated with orthotropic stability theory, it will permit available orthotropic shell solutions to be applied directly to filamentary structures with confidence.

The compressive strength of filamentary composites involves buckling on a far smaller scale: microbuckling of the filaments in the presence of the foundation effect provided by the matrix. Well known analogs include buckling of a column on an elastic foundation and wrinkling of the sandwich face upon the core. Ekvall[7] has shown some interesting photomicrographs of filament microbuckling in compressively failed composites which provide dramatic evidence of the phenomenon.

Rosen[6] has provided a rather simple and direct analysis of filament microbuckling of a unidirectional composite by considering the two analytical models illustrated in Fig. 5. For a completely elastic filament and matrix, the composite compressive stresses for microbuckling in terms of the constituent properties are as follows:

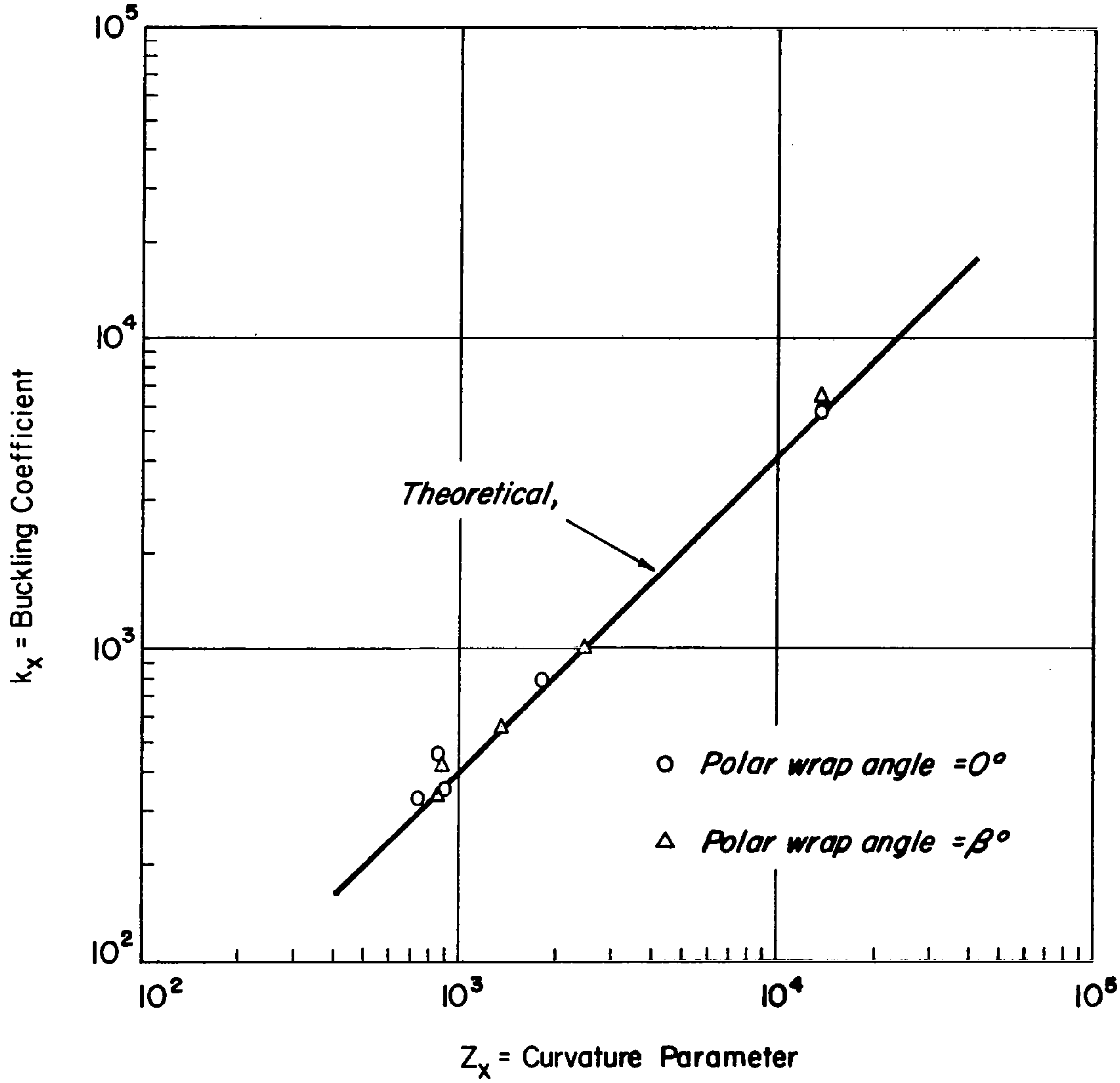


Figure 4 Comparison of orthotropic stability theory with experimental data of Ref. 9 on filament wound cylinders under compression

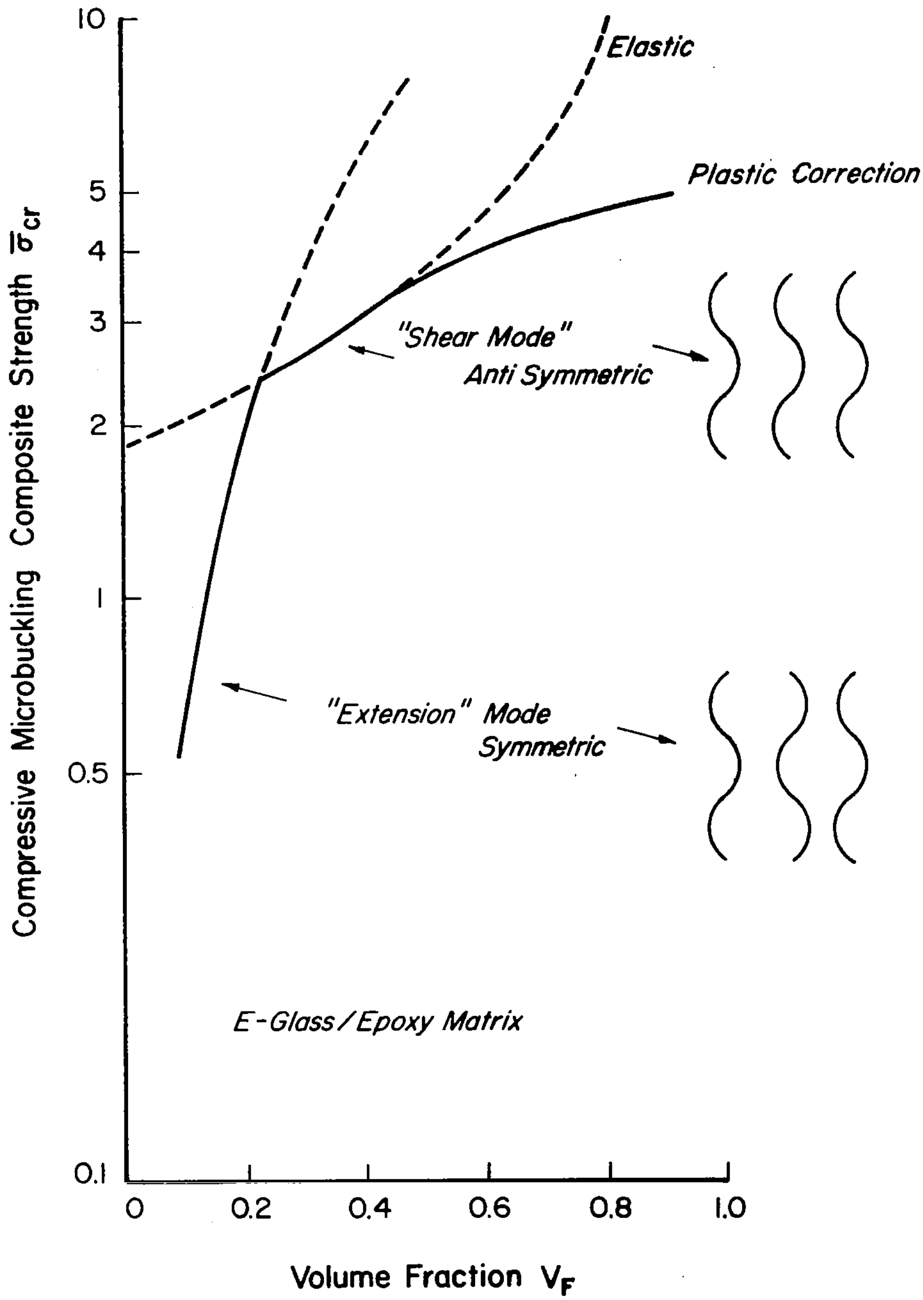


Figure 5 Micro-buckling strength of E-glass/epoxy composite

extension mode:

$$\bar{\sigma}_{cr} = 2\, V_f \left[V_f E_m E_f / 3(1 - V_f) \right]^{1/2}$$

shear mode: (6)

$$\bar{\sigma}_{cr} = G_m / (1 - V_f)$$

The relative importance of each mode as a function of filament volume fraction (V_f) is illustrated in Fig. 5 for glass-epoxy composites. At $V_f \simeq 0.7$, the shear mode dominates and the strength is such that the matrix undergoes an average shortening of over 5 percent. Obviously, the matrix is no longer elastic and G_m in Eq. (6) must be replaced by an inelastic value to produce the results shown in Fig. 5. Rosen has pointed out that the composite compressive strengths predicted by Fig. 5 have not been achieved experimentally and has suggested several refinements for the analytical model.

Schuerch[10] has recently presented an independent analysis of filament microbuckling in unidirectional composites and has obtained essentially the same results given by Eq. (6). For the extension mode, however, the additional term $[1 + (1 - V_f)E_m / V_f E_f]$ appears on the right side which can be of importance at low V_f where this mode governs. Of particular importance are the test results obtained for boron filament/magnesium matrix unidirectional composites and shown in Fig. 6. Not only are the test data in rather remarkable agreement with the theory but also illustrate that high composite compressive strengths can indeed be achieved.

Review of Analysis Methods

Before proceeding to design considerations for filamentary composites, it appears desirable to summarize the current state of knowledge of structural analysis methods for filamentary composites under uniaxial loads. For this purpose, Table 1 has been prepared.

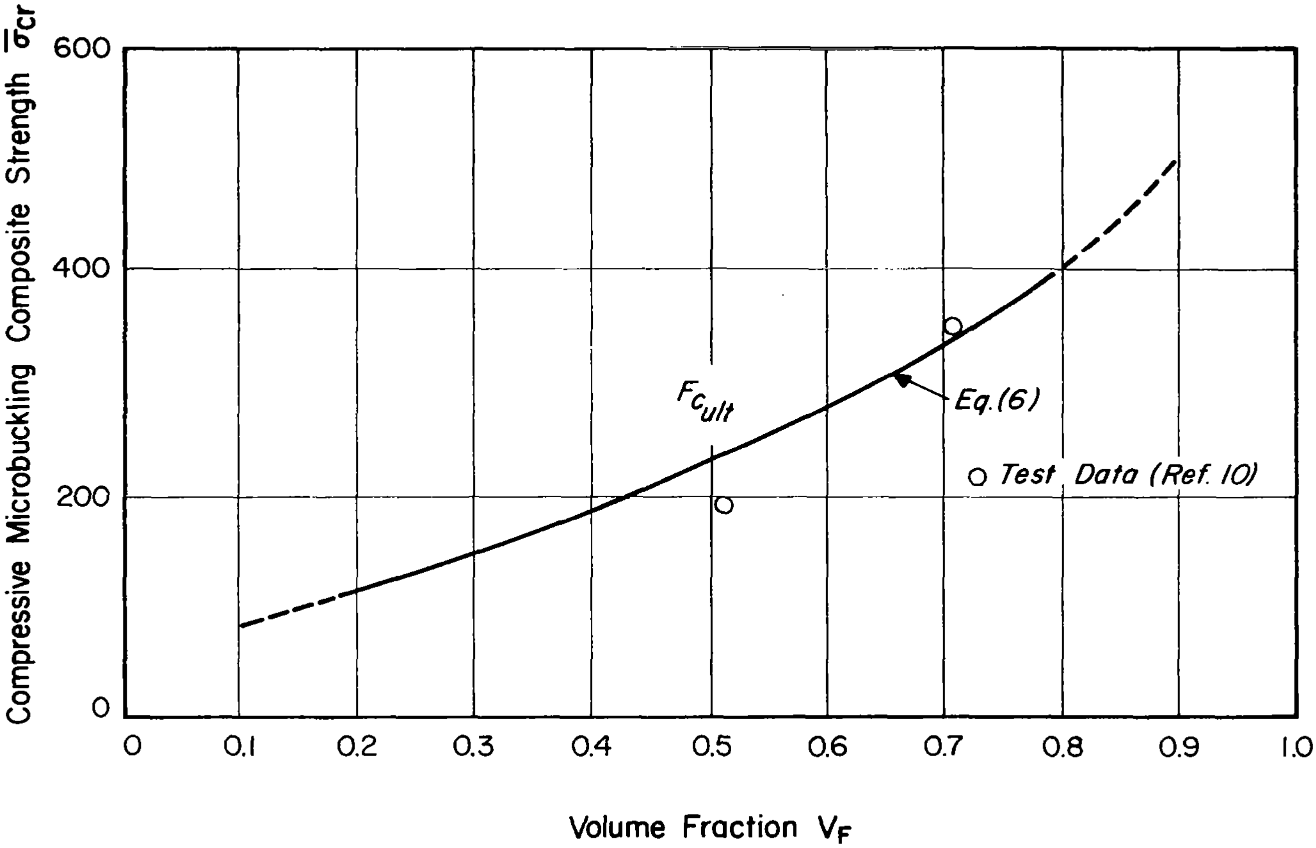


Figure 6 Micro-buckling strength of boron/magnesium composite

Table 1

Summary of Analysis Methods for Composites

Property	Prediction of unidirectional composite properties from constituent properties	Prediction of laminate properties from unidirectional composite properties
Elastic	theory satisfactory - experimental data needed for composites other than glass/organic matrix	classical anisotropic theory of plates is available and can be used directly
Tensile strength	for brittle filaments with large strength variations, theory is unsatisfactory	theory satisfactory when experimentally determined strength properties are used for unidirectional composite
Compressive strength	exploratory work is promising - considerable theoretical and experimental work required	buckling: orthotropic stability theory can be used - test data required to confirm this microbuckling: no published work on laminates

As is usual today, analysis outruns experiment. While it appears that available analysis methods provide at least a satisfactory first order prediction, experimental confirmation has been obtained primarily on glass/organic matrix composites. It is obvious that experimental work on more advanced filamentary composites is a most desirable research objective.

3. Design of Filamentary Composites Under Biaxial Loads

The methods of analysis reviewed in Section 2 for unidirectional filamentary sheets and laminates composed of unidirectional composites were basically concerned with uniaxial load applications. It is understandable that the early emphasis would be on uniaxial loads in order to build up our level of knowledge of the behavior of composites. However, the major successful applications of filamentary composites have been in pressure vessels where biaxial loads govern. Other applications will likewise require consideration of biaxial loads and consequently it is important for us to turn our attention now to the design of filamentary composites under biaxial loads.

Since it is rather important to understand some of the design possibilities and restrictions of filamentary composites, we will employ for this purpose a particularly simple method of analysis. We will assume a matrix of zero strength thus permitting us to use the filamentary approximation given by Eq. (5). In this manner, rather simple and direct design results can be obtained with the recognition that the influence of the matrix has been neglected. It is believed, however, that inclusion of matrix effects will not substantially alter any of the principal conclusions derived herein.

Mechanics of a Bilayer Composite Sheet

In order to design composite laminates for biaxial loads it is convenient to base the design upon the properties of the unidirectional filamentary sheets comprising the laminate and characterized by a filamentary strength σ_f, thickness t_f and filament density as indicated in Fig. 1. The fundamental laminate is assumed to be the bilayer shown in Fig. 7 consisting of two identical unidirectional filamentary sheets oriented symmetrically at an angle $\pm\beta$ with respect to the reference axis. For convenience, we refer to a unidirectional composite stress resultant

N_f along a filament direction and always associated with the unidirectional sheet properties, $N_f = \sigma_f t_f$. For the bilayer laminate, we define the following stress resultants in terms of the total laminate thickness, $2t_f$.

$$N_1 = 2\sigma_1 t_f; \qquad N_2 = 2\sigma_2 t_f; \qquad N_{12} = 2\sigma_{12} t_f \tag{7}$$

By use of Fig. 8, the following equilibrium equations can be written for the bilayer:

$$\begin{aligned} N_1 &= 2\sigma_f t_f \cos^2\beta = 2N_f \cos^2\beta \\ N_2 &= 2\sigma_f t_f \sin^2\beta = 2N_f \sin^2\beta \\ N_{12} &= 0 \end{aligned} \tag{8}$$

By manipulation of Eqs. (8), we can obtain the following restriction for an isotensoid bilayer

$$\tan^2\beta = N_2/N_1 \tag{9}$$

Further, since $N_{12} = 0$, the directions (1, 2) become the principal directions in the sense of plane elasticity.

In addition, we obtain from Eq. (8)

$$N_1 + N_2 = 2N_f$$

or (10)

$$\sigma_1 + \sigma_2 = \sigma_f$$

If in Eq. (10) we let σ_f be equal to X the uniaxial filament strength we obtain the following failure law for a filamentary bilayer under plane stress conditions:

$$\sigma_1 + \sigma_2 = X \tag{11}$$

The neglect of matrix strength necessarily implies the uniformity of strain. Hence, we find for the strain state

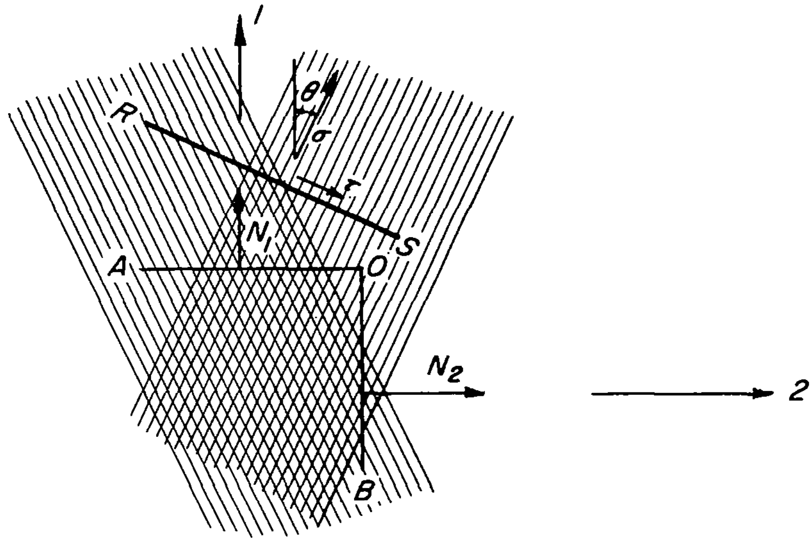


Figure 7 Stress resultants in a bilayer

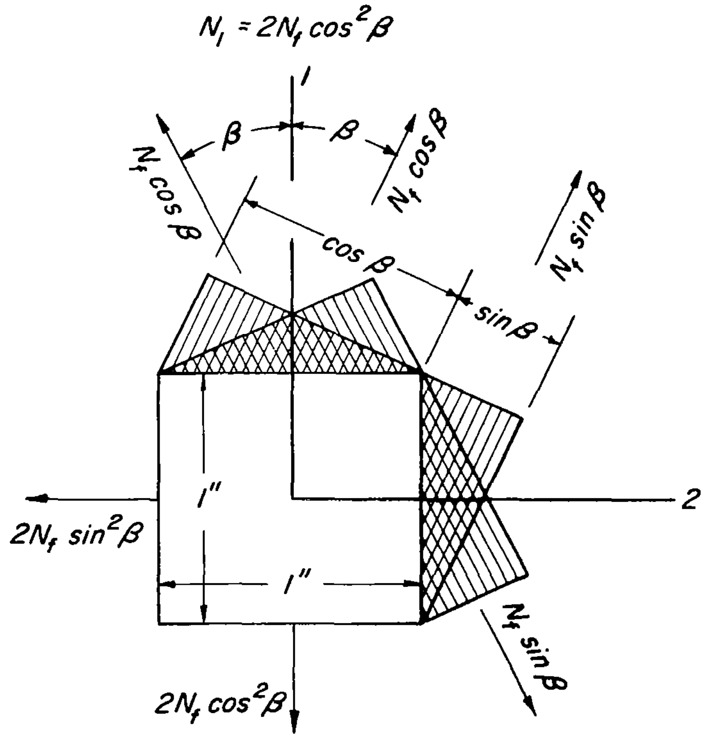


Figure 8 Equilibrium of a bilayer

$$\epsilon_1 = \epsilon_2 = \epsilon_f = \epsilon \tag{12}$$

Utilizing Eq. (12) in Eq. (11) and writing $E_1 = \sigma_1 / \epsilon_1$, $E_2 = \sigma_2 / \epsilon_2$ we find

$$E_1 + E_2 = E_f \tag{13}$$

Eqs (11) and (13) constitute the failure and rigidity laws for filamentary bilayers.

The failure law given by Eq. (11) is shown in Fig. 9 together with the specific bilayer orientations corresponding to Eq. (9). It can be observed that the strength law for a filamentary bilayer is quite different than that for an isotropic sheet not only in shape but also in the fact that the axis of the composite must be specified as well as the specific stress ratio for which it was designed. A bilayer designed for one stress ratio cannot be used arbitrarily for another without a serious degradation in strength properties.

A complete strength envelope for filamentary composites can be as indicated in Fig. 10. In the compression quadrant, microbuckling governs the compressive strength which for illustrative purposes is taken to be equal to the tensile strength. Also shown are the shear strength quadrants which are governed primarily by the interlaminar shear strength of the matrix. This boundary can be strongly influenced by the addition of whiskers or other fillers to the matrix. It can be observed that the overall strength envelope for the filamentary composite is considerably different from the ellipse usually obtained for an isotropic sheet.

Idealized Model of a Bilayer with Matrix

In comparing a filamentary bilayer with an istotropic sheet we have seen that in the case of the bilayer the axis of the composite as well as the design-stress-ratio (N_2/N_1) must be specified. In an isotropic sheet if N_1, N_2 are the principal stress resultants associated with the principal directions 1 and 2, then with any other pair of orthogonal directions θ, ξ ($= \theta + \pi/2$) there are three stress resultants N_θ, N_ξ

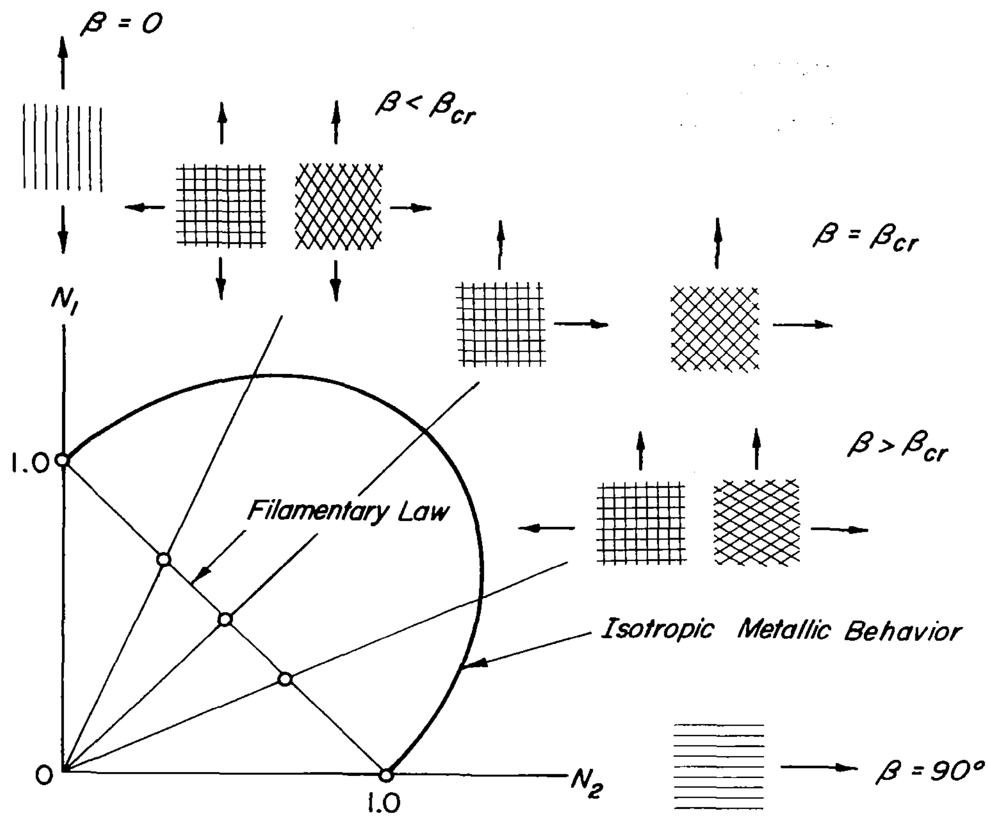


Figure 9 Strength surfaces for filamentary and isotropic sheets

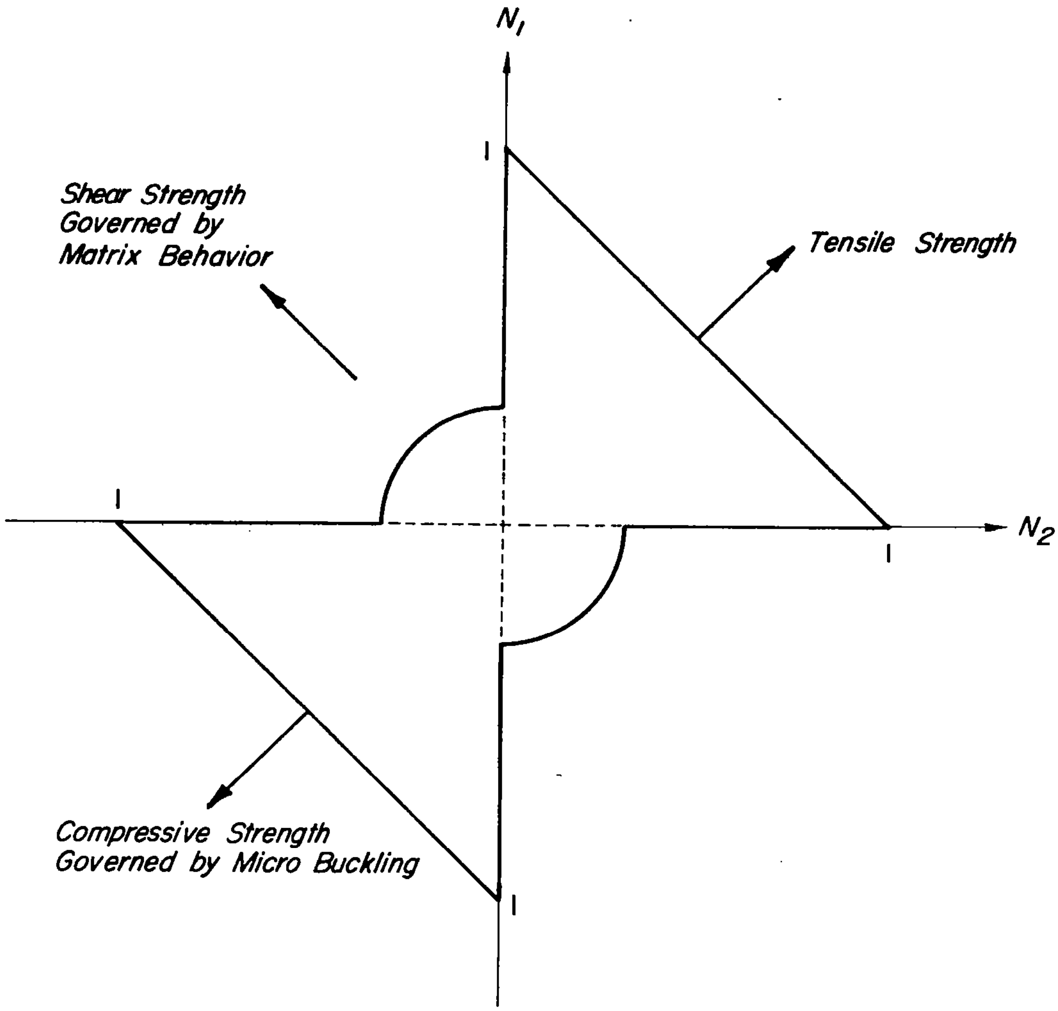


Figure 10 Possible strength envelope for filamentary composite

and $N_{\theta\xi}$ associated with the orthogonal axes, θ, ξ. Hence, in an isotropic system one could prescribe either N_1, N_2 as the design loads or N_θ, N_ξ, $N_{\theta\xi}$ along any two orthogonal directions. However, in a bilayer such equivalent loading is inconsistent with the omission of the shear resistance of the matrix.

In order to take into account the presence of shearing components, and yet idealize the picture so that matrix strength can be neglected, we construct the following model. We consider the bilayer to be imbedded in a frame and the loading applied externally to the frame. The frame itself may carry shear resultants such as $N_{\theta\xi}$, while the filaments develop only axial stresses. The matrix, as a load carrying agent, is then visualized as the frame. It carries the shearing components while the filaments develop the axial stresses.

Taking into account the shear carrying capacity, we can write for a bilayer the following equations, for a general orthogonal system of θ, ξ, Fig. 8:

$$N_\theta = \sigma_f t_f \, (1 + \cos 2\beta \cos 2\theta)$$

$$N_\xi = \sigma_f t_f \, (1 - \cos 2\beta \cos 2\theta) \qquad (14)$$

$$\left| N_{\theta\xi} \right| = \sigma_f t_f \cos 2\beta \sin 2\theta. \qquad (15)$$

Fig. 11 shows a plot of the stress resultants, normalized with respect to $\sigma_f t_f$, as a function of θ for a bilayer angle $\beta = 30^\circ$.

Mechanics of Bilayer Laminates

It is of interest to note that the conclusions concerning a bilayer composite apply equally well to bilayer laminates of such sheets. If each bilayer is denoted by its orientation angle β_i and thickness $2t_i$, then the equilibrium equations corresponding to Eq. (8) become:

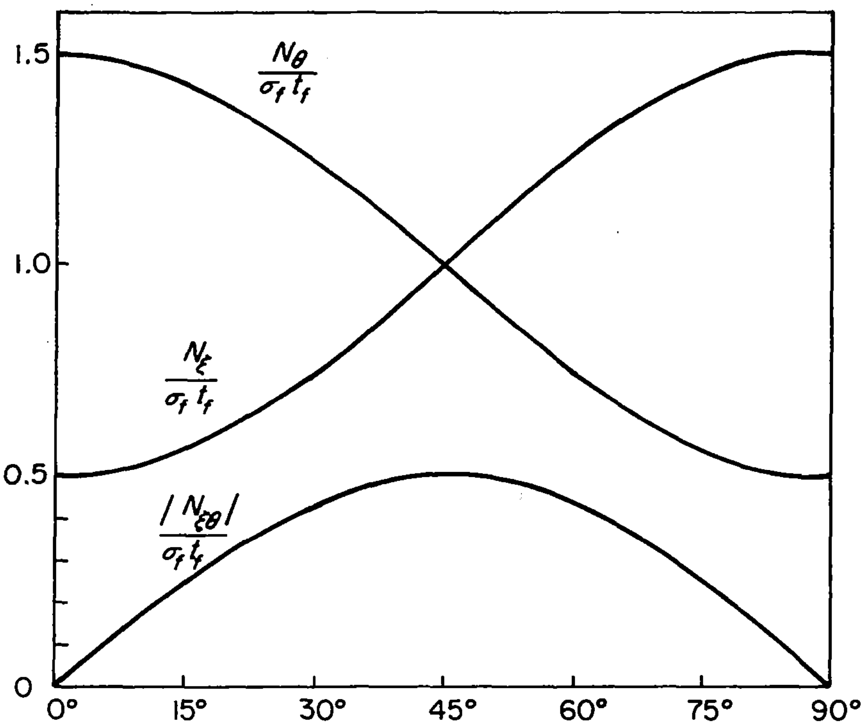


Figure 11 Stress resultant distribution in a bilayer with orientation angle of 30°

$$N_1 = \sum_{i=1}^{n} 2\sigma_f t_{f_i} \cos^2 \beta_i$$

$$N_2 = \sum_{i=1}^{n} 2\sigma_f y_{f_i} \sin^2 \beta_i \tag{16}$$

By suitably combining Eqs. (16), the same failure law Eq. (11) is obtained. However, Eq. (9) is replaced by the condition

$$N_2/N_1 = \sum_{i=1}^{n} \sin^2 \beta_i / \sum_{i=1}^{n} \cos^2 \beta_i \tag{17}$$

In a given design problem where it is assumed that N_1 and N_2 are given, Eq. (17) implies that the bilayer angles $\beta_1, \beta_2 \ldots \beta_{n-1}$ may be arbitrarily chosen while β_n can be adjusted to satisfy Eq. (17). In a general filamentary system of n bilayers designed for a given N_1 and N_2, there are (n-1) degrees of freedom in selecting the β_i angles of individual bilayers. This is an important consideration which permits some design freedom to overcome the restrictions inherent in a single bilayer composite sheet.

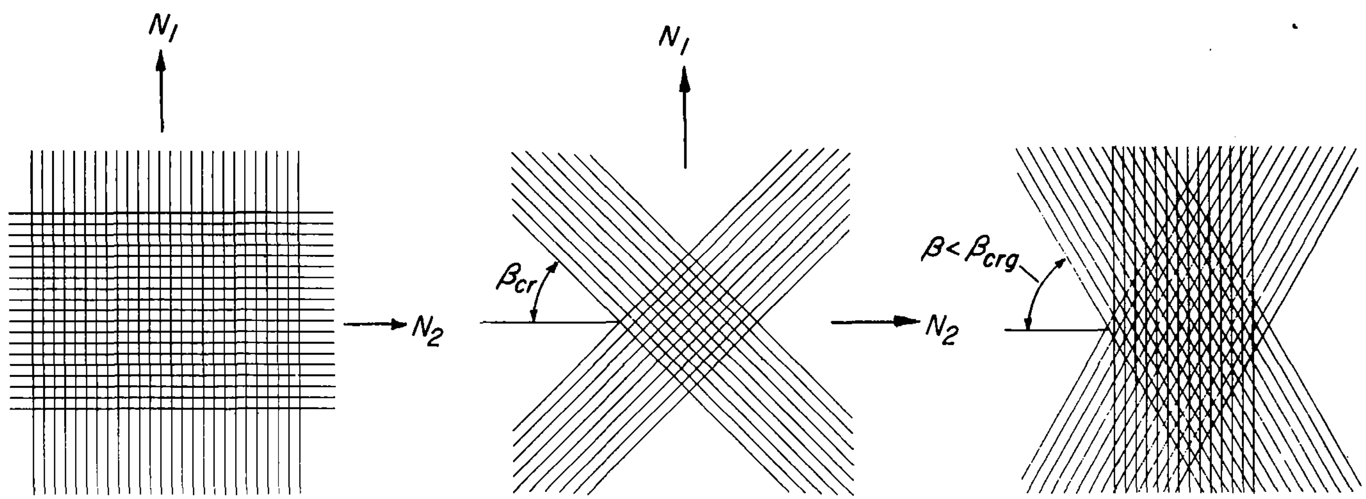
Weight Efficiency of Filamentary Bilayer Laminates

In the cylindrical pressure vessel design, manufacturing techniques have produced three filamentary network patterns for the associated 2:1 biaxial stress field; as shown in Fig. 12. We shall compare the weight/strength efficiencies of these arrangements, given the design loads N_1 and N_2.

Let the laminate be part of a shell whose thickness h is made up of several bilayers each of thickness $2t_i$. Hence,

$$h = \sum_{i=1}^{n} 2t_i$$

The weight of a shell element of length, width and thickness l, b, and h respectively, and density $\bar{\rho}$, (which may be either the matrix density or the filament density



2:1 Filamentary Design for 0°-90° Bi-layer Cross-ply Laminate

Filamentary Design for $\pm \beta_{cr}$ Bilayer Angle-ply Laminate

Filamentary Design for $\pm \beta$ Bilayer Along With Hoop Winding

Figure 12 Filamentary network arrangements for pressure vessels or a linear combination of these) is given by

$$W = \bar{\rho}\, l\, b\, h = 2\, \bar{\rho}\, l\, b \sum_{i=1}^{n} t_i \tag{18}$$

If β_i is the orientation of each bilayer, utilizing Eq. (18) we have

$$W = \bar{\rho}\, l\, b\, (N_1 + N_2)/\sigma_f \tag{19}$$

Thus, Eq. (19) shows that the weight of a uniform element of a bilayer laminate with arbitrary individual bilayer orientation is independent of the orientation and is dependent only upon the external loading invariant $(N_1 + N_2)$ and the filament strength. Hence, all the three designs of Fig.11 have the same theoretical weight/strength efficiency.

A further interesting feature of the weight efficiency study concerns the strength weight efficiency of an "isotropic" filamentary laminate. By an "isotropic" filamentary laminate we mean a multiple bilayer system with the orientations $\beta_1, \beta_2 \ldots$ arranged in such a manner that there is planar symmetry of stress. That is, the stress resultant along any direction θ from axis 1 is the same,, i.e., $N_1 = N_2 = N_\theta = N$. This is achieved either by taking $\beta_1 = \beta_2 \ldots \pi/4$ or by considering the generalizations of Eqs. (14) for the multiple bilayer systems and letting $\beta_1, \beta_2 \ldots$ be the appropriate combinations. For instance, if the laminate consists of even (2n) bilayers, then $\beta_1 + \beta_2 = \beta_2 + \beta_3 = \beta_3 + \beta_4 \ldots = \beta_{2n-1} + \beta_{2n} = \pi/2$ is a suitable arrangement.

Having thus defined an isotropic laminate, we may compare the strength/ weight efficiency of an isotropic filamentary laminate with that of a monolayer system, where all the filaments are lined along the direction of the loading, under the same loading N.

Let an elementary parallelepiped of the filamentary sheet of length l, width b and thickness h be considered. If it is composed of 2n bilayers, each of thickness

$2\bar{t}$, where $\bar{t}$ is the bilayer thickness, then the thickness of element $h = 4n\bar{t}$. If the orientations are $\beta_1 \ldots \beta_{2n}$, then from the equilibrium of multilayer systems we have,

$$N = N_1 = 2\sigma_f \bar{t} (\cos^2 \beta_1 + \cos^2\beta_2 +, \ldots \cos^2\beta_{2n})$$
$$= \sigma_f \frac{h}{2n} (\cos^2\beta_1 + \cos^2\beta_2 + \ldots \cos^2\beta_{2n}) \tag{20}$$

But from the requirement of planar isotropy $\beta_1 + \beta_2 = \beta_2 + \beta_3 = \ldots \beta_{2n-1} + \beta_{2n} = \pi/2$

$$N = \sigma_t (h/2n) (1/2) 2n = \sigma_f h/2 \tag{21}$$

Since the weight of the parallelepiped W, is given by $W = \rho\, l\, b\, h$ where ρ is the density of material of the filament, we find from Eq. (21)

$$[N/(W/b)]_{iso} = (1/2) (\sigma_f/\rho l) \tag{22}$$

Now if the laminate were a monolayer of the same material and the same dimensions, then evidently

$$[N/(W/b)]_{mono} = (\sigma_f/\rho l) \tag{23}$$

Hence, the strength/weight efficiency of an "isotropic" filamentary laminate is half that of a monolayer laminate.

Some Design Possibilities With Filamentary Bilayer Laminates

Now that it is evident that certain degrees of freedom can be attained by use of multiple bilayer composites, it is of interest to examine how this can be exploited. One obvious case is the manufacturing freedom offered by the third design of Fig. 12. Another example is that a laminate can be designed to carry alternative loads in addition to the basic biaxial loads (N_1 and N_2) for which it was primarily designed. Thus we can pose the design question: given a filamentary bilayer laminate designed for the principal stress resultants N_1 and N_2, what are other possible load combinations that the laminate can carry?

Starting first with a single bilayer, we see that it is designed primarily for the biaxial loads which according to Eq. (10) are $N_1 + N_2 = 2N_f$ where N_f is the filamentary strength of a unidirectional composite referred to its thickness. In this discussion it is convenient to use N_f as a reference value. As indicated in Fig. 13, the following additional load combinations are possible:

a. combined loads N_f applied along $\pm\beta$

b. single loads N_f applied along $\pm\beta$.

A similar situation is obtained for the bilayer laminate shown in Fig. 14. Note that there are two additional degrees of freedom here although the strength/weight ratio of a unidirectional composite is obviously degraded relative to the complete laminate. Finally, we turn to the n-bilayer laminate and obtain the results shown in Fig. 15. Also shown in an alternative form in Fig. 16 is a representation of the biaxial loads as well as single load combinations possible with an n-bilayer laminate and for an isotropic sheet of the same uniaxial strength. Again the difference in behavior of the filamentary sheet and isotropic sheet is considerable.

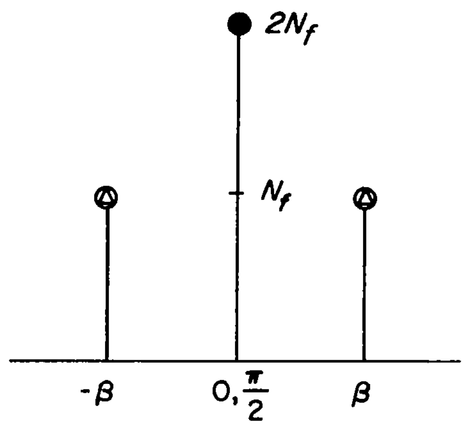


Figure 13 Loading combinations for a single bilayer system

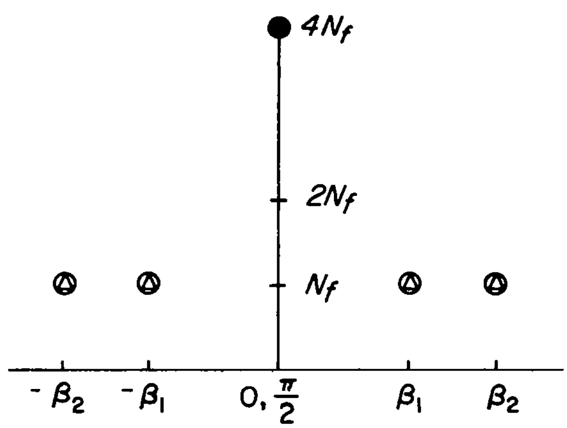


Figure 14 Loading combinations for a two bilayer system

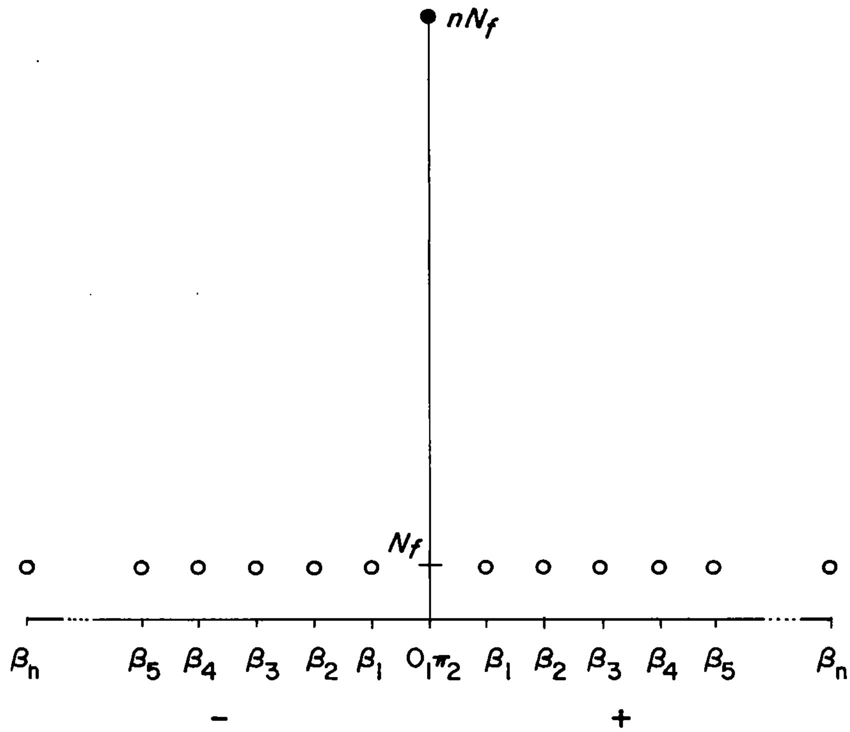


Figure 15 Loading combinations for an n-bilayer system

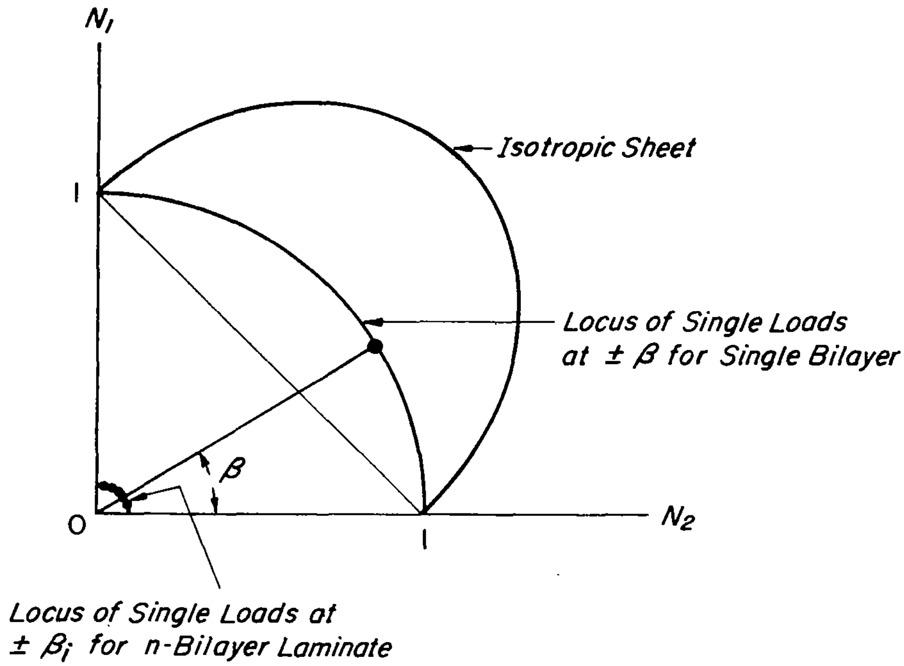


Figure 16 Loading combinations: Alternative representation

4. Concluding Remarks

An underlying objective of Section 3 has been to demonstrate the folly of "substitution", the favorite device of the designer in which he substitutes a panel of new material for one which has been tested by time in the design environment. Filamentary materials require an accurate matching of filament orientation and stress field in order to achieve superior strength/weight properties. The important and creative job required of the structural designer is to arrange the unidirectional filamentary composite material into an efficient filamentary structural laminate.

While the preceding discussion has been concerned primarily with strength limited structures, it is important to recognize that many types of aerospace structures are stability limited. Such structures are governed by elastic buckling considerations because of their geometry in combination with the relatively low magnitude of the applied loads. In such cases, filament orientation and stress field matching is required to achieve maximum stiffness/weight. As is well known, cross-sectional shaping by the use of stiffening and sandwich concepts provides a very effective means of achieving high structural efficiency for isotropic sheet materials. Creative ideas on cross-sectional shaping that can utilize the unique features of filamentary composites will be required to exploit this concept for filamentary structures.

Finally, some words on the selection of materials for composites in terms of filament, matrix and volume fraction. It is obvious that the selection of constituent materials for unidirectional composites and their volume fraction will depend upon whether the structural application is strength or stability as well as the role of the matrix under the applied loads. For example, it is well known that glass filaments are fine for strength limited applications, although they are relatively poor in stability limited applications. On the other hand, beryllium and boron filaments exhibit very superior stiffness properties although their strength properties represent little, if any, improvement over glass filaments.

Thus, we can expect that the same situation will prevail with filamentary composites as exists with sheet metallics. Specific criteria should be developed to define the materials efficiency aspects of the composite, its constituents and volume fraction for designs governed by strength or stability limitations. Approximate analyses based on criteria derived from isotropic sheet materials can already provide an effective screening tool for composites.

References

1. Dixmier, G., and Gerard, G., "Composite Materials," AGARD Report No. 483, July 1964.
2. Tsai, S. W., "Structural Behavior of Composite Materials," NASA CR-71, July 1964.
3. Azzi, V. D., and Tsai, S. W., "Elastic Moduli of Laminated Anisotropic Composites," Experimental Mechanics, Vol. 5, No. 6, pp. 177-185, June 1965.
4. Card, M. F., "Experiments to Determine Elastic Moduli for Filament-Wound Cylinder," NASA TN D-3110, Nov. 1965.
5. Azzi, V. D. and Tsai, S. W., "Anisotropic Strength of Composites," Experimental Mechanics, Vol. 5, No. 9, pp. 283-288, Sept. 1965.
6. Rosen, B. W., "Mechanics of Composite Strengthening," Fiber Composite Materials, American Society for Metals, Metals Park, Ohio, 1965, pp. 37-75.
7. Ekvall, J. C., "Structural Behavior of Monofilament Composites," AIAA Sixth Annual Structures and Materials Conference, pp. 250-263, April 1965.
8. Tsai, S. W. and Azzi, V. D., "Strength of Laminated Composite Materials," AIAA Journal, Vol. 4, No. 2, pp. 296-301, Feb. 1966.
9. Milligan, R., Gerard, G., and Lakshmikantham, C., "General Instability of Orthotropically Stiffened Cylinders Under Axial Compression," AIAA Preprint 66-139, Jan. 1966.
10. Schuerch, H., "Prediction of Compressive Strength in Uniaxial Boron Fiber-Metal Matrix Composite Materials," AIAA Journal, Vol. 4, No. 1, pp. 102-106, Jan. 1966.

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