FACILITY FORM 602

(ACCESSION NUMBER) (THRU)

(PAGES) (CODE)

(NASA CR OR TMX OR AD NUMBER) (CATEGORY) 29

INVESTIGATION OF MAGNETOSPHERIC PHYSICS

TECHNICAL STATUS REPORT

C. F. Kennel and H. E. Petschek

Contract No. NAS w-1400

Period Ending: 31 March 1967

prepared for

GRANTS AND RESEARCH CONTRACTS, CODE SC

OFFICE OF SPACE SCIENCES

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D. C. 20546

N10K

AVCO EVERETT RESEARCH LABORATORY

A DIVISION OF AVCO CORPORATION

INVESTIGATION OF MAGNETOSPHERIC PHYSICS

TECHNICAL STATUS REPORT

by

C. F. Kennel and H. E. Petschek

AVCO EVERETT RESEARCH LABORATORY
a division of
AVCO CORPORATION
Everett, Massachusetts

Contract No. NAS w-1400

Period Ending: 31 March 1967

prepared for

GRANTS AND RESEARCH CONTRACTS, CODE SC
OFFICE OF SPACE SCIENCES
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Washington, D. C. 20546

I. STRUCTURE OF THE AURORAL ZONE

Recently we have been involved in coupling the analysis by Axford, Petschek and Siscoe (1965) of the structure of the neutral sheet in the Earth's geomagnetic tail with the theory of particle precipitation, based on whistler and ion cyclotron wave turbulence, of Kennel and Petschek (1966). Axford, et al (1965) showed that the plasma in the neutral sheet flows towards the Earth's night side at a rate sufficient to account for the observed auroral dissipation. Kennel and Petschek (1966) found that whistler and ion cyclotron turbulence explains the loss of particles from the Van Allen belts, but had to postulate an unknown particle source which keeps the belts filled up. Presumably the above flow of ≈1 keV plasma from the neutral sheet could be the source needed to drive turbulent pitch angle diffusion which precipitates auroral (≈ few keV) electrons into the atmosphere.

Several observations lend credence to this view. O'Brien and Taylor (1964) showed that auroral light was due to bombarding fluxes of few keV electrons originating from above the atmosphere. Fritz and Gurnett (1965) found large fluxes of 10 keV electrons in the atmosphere on auroral lines of force. These fluxes ($\sim 10^9/cm^2 sec$) are comparable with those found on auroral lines of force near the equatorial plane (Frank, 1967). In order for the few keV electron fluxes to have the same magnitude both in the equatorial plane and in the ionosphere on the same line of force, their pitch angle distributions must be roughly isotropic. The equatorial plane results of Frank (1967) and Vasyliunas (1967, unpublished) show that the few keV electron fluxes exist all the way in from the neutral sheet to L-shells corresponding to auroral lines of force, where they suddenly disappear.

It is tempting to associate the disappearance of the fluxes in the equatorial plane with the auroral precipitation which occurs on the same lines of force.

Several features of auroral zone steady precipitation patterns can be understood without reference to specific modes of plasma turbulence. For instance, we can make a reasonable guess at plausible distortions of the particles' velocity distributions and thus identify possible sources of wave instability. Speiser and Ness (1967) have shown that the magnetic neutral sheet -- a thin region of very low magnetic field strength ($\approx 10^{-5}$ gauss) -- is always found in the geomagnetic tail beyond 10-12 Earth radii on the Earth's night side. The plasma sheet, discussed by Bame, et al (1967), is considerably thicker than the magnetic neutral sheet, but is found in the same general region of space. Within the magnetic neutral sheet, the magnetic spatial gradients are strong and we expect strong currents. However, outside the magnetic neutral sheet there are weak gradients and weak currents. Therefore, we can rule out current-driven instabilities for the auroral zone, which lies closer to the Earth than the neutral sheet. This leaves spatial density gradients and pitch angle anisotropies as possible sources for instability in the auroral zone. Spatial density gradients must have extremely short scale lengths, the order of the ion Larmor radius, to trigger instability. While such mechanisms may be important for the highly localized auroral arcs, they can hardly account for the broad diffuse auroral precipitation background we wish to discuss here.

Let us consider what pitch angle anisotropies we might expect for the auroral zone in the absence of any turbulence. Presumably the plasma deep in the geomagnetic tail is isotropic in pitch angle -- either because the geomagnetic tail is highly turbulent or because the particles are randomized each time they cross the weak field region in the magnetic neutral sheet, or both. However, according to Axford, et al (1965), this plasma must flow out of the neutral sheet into the closed but highly distorted lines of force ($R \leqslant 12\ R_E$) that comprise the auroral "cusp". In the absence of plasma turbulence in this region, the plasma particles would conserve their first and second adiabatic invariants as they flow towards the Earth. Then, very approximately, the perpendicular particle temperature $T_\perp$ will be proportional to the magnitude of the magnetic field in the equatorial plane of the moving tubes of force.

$$T_\perp \propto T_{\perp_o} (B/B_o) \tag{1}$$

where o denotes a reference point, taken to be the near end of the neutral sheet. Equation (1) follows from first invariant conservation. Similarly, second invariant conservation implies, extremely roughly, that the parallel temperature is given by

$$T_{\parallel} = T_{\parallel_o} (\ell_o/\ell)^2 \tag{2}$$

where ℓ denotes the length of the line of force. Constructing an anisotropy A , we find

$$A = \frac{T_\perp - T_\parallel}{T_\parallel} = \frac{B}{B_o}\left(\frac{\ell}{\ell_o}\right)^2 - 1 \tag{3}$$

where we have taken $T_{\perp_o} = T_{\parallel_o}$, corresponding to conditions in the neutral sheet.

Let us consider how (B/B_o) and $(\ell/\ell_o)^2$ vary following a streamline as the plasma flows towards the Earth from the neutral sheet. Observations (Cahill, 1965) suggest that B/B_o is relatively constant beyond 8 or 9 Earth radii; however, it increases rapidly within 8 R_E. (ℓ/ℓ_o) is constantly decreasing since the tubes of force into which the particles are flowing become shorter. Thus, initially $A < 0$ beyond say 8 or 9 R_E. Here we would expect whistlers to be damped and only $A < 0$ instabilities to occur. Somewhere around 8 R_E, A turns positive and whistlers and other $A > 0$ instabilities become favored. Serlemitsos (1966) found just such a progression of pitch angle anisotropies in electrons with $\gtrsim 100$ keV energies.

While a more refined analysis obviously requires computing accurate particle orbits using accurate adiabatic invariants (Eq. 2 is an approximation), the rough accuracy of the rest of our work does not warrant such a laborious step at this time. We have succeeded in guessing at a reasonable configuration for auroral zone instability sources: $A < 0$ in the outer auroral zone; $A > 0$ for the auroral maximum. At the present time we have not considered what $A < 0$ instabilities there might be. There are a variety of $A > 0$ instabilities in the literature, including those of whistlers and ion cyclotron waves (Kennel and Petschek, 1966) and anisotropic plasma oscillations (Rosenbluth and Post, 1965).

Let us assume, for at least the $A > 0$ region, there is instability, turbulence, and therefore pitch angle randomization which leads to particle precipitation into the atmosphere. We can get an idea of the coupling of the precipitation to the flow, without specifying the particular brand of plasma turbulence. For instance, Kennel and Engelmann (1966) showed that cyclotron resonance diffusion is always predominantly in pitch angle, so long as its frequency is less than the gyrofrequency of the particles involved.

An estimate of the maximum effects of pitch angle diffusion and precipitation can be made by assuming that turbulence keeps the plasma in strong diffusion. The weak and strong diffusion limits were defined by Kennel and Petschek (1966). In weak diffusion, which applies to the Van Allen zone, a particle which pitch-angle diffuses to the edge of the loss cone is immediately lost on its next trip up the line of force to the atmosphere. Thus, the loss cone is nearly empty. On the other hand, when the wave intensity is very large, particles can diffuse in and out of the loss cone before they can propagate to the atmosphere. Here the loss cone is nearly full. In the limit of very strong diffusion, the pitch angle distribution is completely isotropic, and a particle spends equal time at all pitch angles. The minimum precipitation lifetime is just given by the time to reach the atmosphere (roughly ℓ/V, where V is the particle velocity) divided by the probability/unit time that the particle be in the loss cone. This probability is just the ratio of the solid angle of the loss cone to that of the sphere. This minimum lifetime, T_{min}, calculated in Kennel and Petschek (1966) is

$$T_{min} = \frac{\ell}{V} \frac{B_E}{B} \tag{4}$$

where B_E is 1/2 gauss, the field at the foot of the line of force, and B, the field in the equatorial plane. Choosing $\ell \sim 6 \times 10^9$, and $B \sim 10^{-3}$ gauss, corresponding roughly to conditions at $L = 6$, and $V \sim 3 \times 10^9$ cm/sec corresponding to 5 keV electrons, T_{min} comes out to be 10^3 seconds. T_{min} is even larger at greater distances, and we conclude that precipitation can play only a small role in the flow dynamics in the outer auroral zone. This may be evaluated more precisely as follows: In the convective flow towards the Earth (see Axford and Hines, 1961), the total number of particles N on a tube of force is conserved, in the absence of precipitation. Thus N = constant. The number density increases, however, because the volume V of the tube of force decreases. Now the volume of a tube of force is proportional to its area (which is proportional to 1/B, since BA = constant) times its length ℓ . Thus

$$n = n_o \left(\frac{B \ell_o}{B_o \ell} \right) \tag{5}$$

where o again denotes the inner edge of the magnetic neutral sheet. On the other hand, n tends to decrease due to precipitation. We can write a simple differential equation balancing these two effects as follows

$$\frac{\partial \ln n}{\partial t} + \frac{c \underset{\sim}{E} \times \underset{\sim}{B}}{B^2} \cdot \nabla \ln \left(\frac{n \ell}{B} \right) = \frac{1}{T_{min}} \tag{6}$$

where $c\underset{\sim}{E}\times\underset{\sim}{B}/B^2$. , the so-called "electric drift velocity", is the fluid flow velocity. For simplicity, we will consider the case of a steady flow, with $\partial / \partial t = 0$. Furthermore, we will assume that the electric field across the magnetosphere is constant. Let us normalize our variables to those at the entrance to the auroral zone, i.e., at the edge of the neutral sheet,

$$b = B/B_o, \quad \ell = \ell / \ell_o \, . \tag{7}$$

We also measure distances in units of the distance to the neutral sheet

$$\underset{\sim}{S} = X/R_o \qquad (R_o \approx 12 \text{ earth radii}) \tag{8}$$

If we restrict ourselves to the steady, radially inward flow on the midnight meridian plane, Eq. (6) reduces to

$$\frac{d}{dS} \ln \left(\frac{n \ell}{b} \right) = K \frac{b^2}{\ell} \tag{9}$$

where $K = \left(\frac{B_o V}{CE} \right) \left(\frac{B_o}{B_E} \right) \left(\frac{R_o}{\ell_o} \right)$. When $V \approx 10^9$ cm/sec, $B_o \simeq 3 \times 10^{-4}$ gauss, $B_E \sim \frac{1}{2}$ gauss, $\frac{R_o}{\ell_o} \sim 4$ and guessing the electric field by assuming that the potential across the magnetosphere is ≈ 60 kV, $K \approx 10^{-1}$. Integrating (9) and assuming that $n = n_o$ at $S_o = 1$, we find

$$n = n_o \left(\frac{b}{\ell}\right) e^{K \int_1^S \frac{b^2}{\ell}(S')\, ds'} \tag{10}$$

Thus the density of the plasma in the auroral flow is critically dependent upon the magnetic field configuration in the auroral cavity, as contained in the functions $\frac{b}{\ell}$ and $\frac{b^2}{\ell}$. Some idea of the solutions may be found by assuming that the magnetic field has roughly a dipole configuration. This assumption fails for the outer auroral zone, and gets progressively better close to Earth. Under these dipole assumptions, the solution becomes

$$\frac{n}{n_o} = \frac{1}{S^4} e^{\frac{K}{7}\left(1 - \frac{1}{S^6}\right)} \tag{11}$$

The maximum density point S_{max} is given by differentiation of (11) above:

$$S_{max} = \sqrt[6]{\frac{K}{4}} = \frac{1}{\sqrt[6]{40}} \approx .55 \tag{12}$$

Because the precipitation is extremely weak at large distances, the location of the auroral maximum is not strongly dependent upon the distance to the neutral sheet. As an example, if the neutral sheet ends at 12 R_E geocentric distance, the location of the density maximum is around 7 R_E. Furthermore, the density of auroral particles falls off extremely rapidly, as e^{-1/S^6} as S decreases even further inside the auroral maximum. The slow (algebraic) build-up of density, the location of the density maximum and the exponential fall-off on the Earth side of the maximum all agree with Frank's (1967) published data on 3-10 keV electrons, and qualitatively with Vasyliunas' (unpublished) results. The precipitation, and the validity of the strong diffusion assumption ceases when the density drops below that critical value needed for instability.

Since by hypothesis and necessity the pitch angle distributions are very nearly isotropic, the density of particles in the equatorial plane is very nearly equal to that in the loss cones in the atmosphere. Therefore the latitude distribution of auroral electron precipitation is also described by (10), once a proper mapping transformation along a line of force between the equatorial plane to the auroral zone atmosphere location is defined. Assuming, once again, that the mapping is roughly that of a dipole, we can arrive at a precipitation distribution such as that of Fig. 1, where we compare with O'Brien and Taylor's (1964) auroral light measurements. The qualitative features: a sharp low-latitude border for the auroral precipitation and a more diffuse high-latitude distribution, appear to be described by the mathematical solutions based upon the dipole idealization, though of course, details, which depend upon the particular magnetospheric configuration at any given time, are not reproduced.

Similar considerations also hold for auroral zone protons. However, protons of the same energy as electrons have a longer minimum lifetime because their velocity along the lines of force is slower. The density and precipitation maximum for protons occurs at

$$S_{max}(\text{protons}) = \sqrt[6]{\frac{K}{4}\ \frac{M-}{M+}} \approx \frac{1}{\sqrt[6]{1600}} \approx 0.3 \qquad (13)$$

Thus the proton maximum occurs at roughly 4 R_E. This is in qualitative

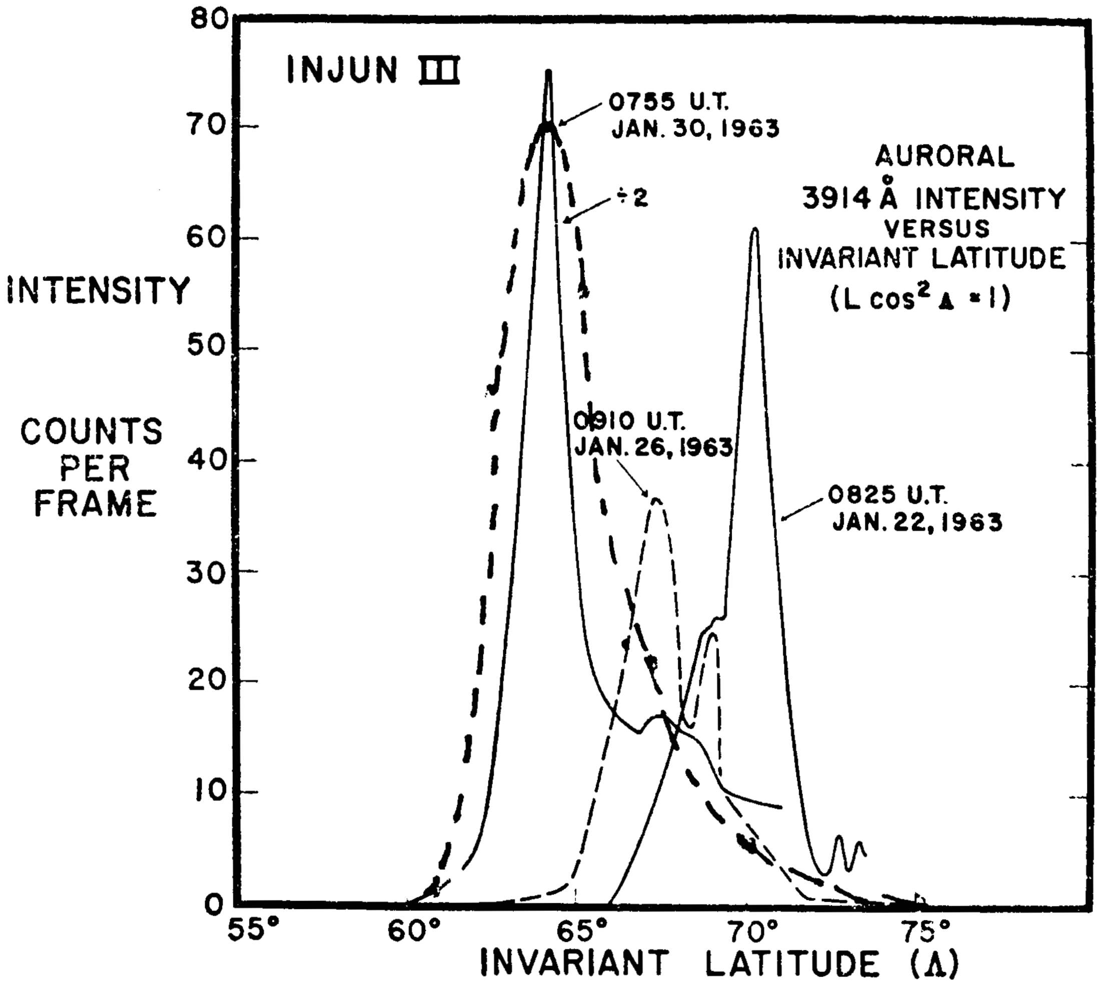

Fig. 1 The solid lines are measurements taken by Injun III (O' Brien and Taylor, 1964) of auroral light, which is due to the precipitation of few keV electrons, as a function of geomagnetic latitude. The dashed line is a theoretical estimate based upon Eq. (11), assuming the Earth's field is dipolar.

A7670

agreement with observations of Doppler shifted Balmer emissions (Omholt, 1963). These originate in the auroral proton bombardment, which charge exchanges with neutral hydrogen in the upper atmosphere. Auroral proton precipitation always occurs at lower latitudes than the auroral electron precipitation. It is curious to note that the prediction of Eq. (13) for the inmost penetration of the neutral sheet proton flow agrees with the location of the "knee" in total particle density (Angerami and Carpenter, 1966).

The discussion of the previous sections may be summarized as follows: The magnetic field configuration in the auroral zone is such that the flow from the neutral sheet generates negative pitch angle anisotropies in the outer and positive anisotropies in the inner auroral zone. Should plasma turbulence be generated by instabilities due to these anisotropies, it will presumably have a different character in the two regions. Without specifying the particular brand of turbulence, some information about the maximum coupling of turbulence and precipitation to the flow can be obtained merely by assuming that the turbulence scatters particles primarily in pitch angle at a rate sufficient to maintain strong diffusion. Maintenance of strong diffusion is not a stringent condition since we have shown that the minimum precipitation lifetime and the flow time are at least comparable throughout in the outer auroral zone. The flow scale times and precipitation lifetimes become comparable at the auroral maximum. Thereafter, the precipitation depletes the flow of auroral particles.

Now we turn from a general discussion of auroral zone structure to consideration of possible candidates for the turbulence driving the precipitation. First, we consider whistler mode turbulence, as in Kennel and Petschek (1966). For simplicity, we restrict ourselves to the special case of whistler propagation strictly parallel to the magnetic field. Then the parallel energy component E_R of the particles in cyclotron resonance with the wave is

$$E_R = \frac{B^2}{8\pi N}\left(\frac{\omega}{\Omega_-}\right)^{-1}\left(1 - \frac{\omega}{\Omega_-}\right)^3 \tag{15}$$

where ω is the whistler frequency, Ω_- is the electron gyrofrequency. Choosing conditions appropriate to the auroral precipitation maximum, $B \sim 10^{-3}$ gauss, $N \approx 1/\text{cm}^3$ (Angerami and Carpenter, 1966),

$$\frac{B^2}{8\pi N} \approx 25 \text{ keV} \tag{16}$$

Choosing as a typical frequency $\frac{\omega}{\Omega_-} \approx \frac{1}{2}$, we find that $E_R \approx 6$ keV, comparable with the energy of particles precipitated in the aurora. A necessary condition for instability is

$$A > \frac{1}{\frac{\Omega_-}{\omega} - 1} \approx 1 \tag{17}$$

This condition may be somewhat stringent, but let us assume for the sake of argument that it is obeyed. Then, the ensuing whistler turbulence scatters all electrons with energies $\gtrsim$ 6 keV in pitch angle, precipitating some into the atmosphere as a diffuse aurora. By the previous arguments, we may estimate the precipitation lifetime by the minimum lifetime.

Now, using arguments similar to those in Kennel and Petschek, we can estimate the equatorial plane whistler intensity in the auroral zone. The rate of loss of particle energy by precipitation is roughly

$$\frac{\partial E_p}{\partial t} = \frac{n\left(\frac{\omega}{\Omega_-}\right)E_R}{T_{min}} \approx \frac{\left(\frac{n}{N}\right)\frac{B^2}{8\pi N}\left(1-\frac{\omega}{\Omega_-}\right)^3}{T_{min}} \quad \frac{\text{ergs}}{\text{cm}^3/\text{sec}} \approx \frac{n}{N}\,\frac{B^2}{64\pi T_{min}} \tag{18}$$

where n is the fractional density of particles resonant with unstable whistlers, i.e., with $E_R > 6$ keV. For rough estimates possible here, we may take $\frac{n}{N} \approx 1$. Taking $B \approx 10^{-3}$ gauss and $T_{min} \approx 3 \times 10^3$ sec (corresponding to $R \approx 7 R_E$), we arrive at

$$\frac{dE_P}{dt} \approx 2 \times 10^{-12} \frac{\text{ergs}}{\text{cm}^3/\text{sec}} \quad . \tag{19}$$

This energy must be fed into whistler waves, so that

$$2\gamma \frac{B'^2}{8\pi} \approx 2 \times 10^{-12} \frac{\text{ergs}}{\text{cm}^3/\text{sec}} \tag{20}$$

as well. γ is the wave growth rate, and B'^2 is the wide band whistler amplitude. Let us suppose, after Kennel and Petschek, that the whistler distribution is in a quasi-steady state, so that

$$\gamma = \frac{V_G}{L} \ln \frac{1}{R} \tag{21}$$

where V_G is the group velocity, L the length of the line of force, and R an effective reflection coefficient for reflection of whistler back across the equatorial plane once it has propagated away. Henceforth we shall take $\ln \frac{1}{R} \approx 3$. Thus, we estimate $\gamma \approx 6$ rads/sec so that

$$B' \approx 0.2\,\gamma = 2 \times 10^{-6} \text{ gauss} \quad . \tag{22}$$

This is more than an order of magnitude greater than the whistler intensity in the equatorial plane of the Van Allen Zone.

There is no published data on the whistler intensity in the auroral zone equatorial plane. Private discussions with satellite experimentalists suggest that such a figure may be somewhat extreme for the auroral zone equatorial plane. The OGO-1 and 3 VLF experiments at Stanford

(R.A. Helliwell, private communications, 1967) indicate that the Van Allen belts are filled with whistler noise but that the auroral zone and high-regions of the outer magnetosphere are marked by an absence of whistlers. On the other hand, the UCLA group, using the same satellites, report bursts of whistler noise with up to 1γ wide band amplitude in the auroral zone. Thus, the experimental situation is unclear.

Now we turn to the precipitation of protons by ion cyclotron wave turbulence. The energy of protons in cyclotron resonance with ion cyclotron waves is

$$E_R \sim \left(\frac{\Omega_+}{\omega}\right)^2 \left(1 - \frac{\omega}{\Omega_+}\right)^3 \frac{B^2}{8\pi N} \qquad (23)$$

where Ω_+ is the ion gyrofrequency. If we take conditions corresponding to the proton precipitation maximum $(R \approx 4\ R_E)$, we may estimate $B \sim 5 \times 10^{-3}$ gauss, and assuming the precipitation maximum lies beyond the density knee, so that $N \approx 10/cm^3$, we find $\frac{B^2}{8\pi N} \approx 60$ keV. Choosing $\frac{\omega}{\Omega_+} \sim \frac{1}{2}$ again as the maximum unstable frequency, we find that protons with energies $\gtrsim 30$ keV can precipitate due to ion cyclotron wave turbulence.

The ion cyclotron wave energy may be estimated in the same way as the whistler's. The rate of particle energy loss is

$$\frac{dE_p}{dt} \approx \frac{B^2}{16\pi} \left(\frac{n}{N}\right) \frac{1}{T_{min}} \qquad (24)$$

where $T_{min} \approx 2 \times 10^3$ sec, so that

$$\frac{dE_p}{dt} \approx 3 \times 10^{-10} \frac{\text{ergs}}{\text{cm}^3/\text{sec}} \quad , \qquad (250$$

leading to an ion cyclotron wave intensity of

$$B' \approx 10^{-4} \text{ gauss} \approx 10\gamma \ . \qquad (26)$$

Once again, equatorial plane observations of the ion cyclotron wave intensity have not been published.

II. DIRECTIONS FOR FUTURE RESEARCH

(a) Change of wave polarizations; other forms of plasma turbulence

In view of the fact that the predicted magnetic intensities for whistler and ion cyclotron turbulence are large, we are checking the following two possibilities: First of all, the auroral zone contains a high β (= ratio of plasma to magnetic pressure) and low density plasma, whereas nearly all theories to date have treated the two opposite limits. We have, therefore, undertaken a study of waves in a low density, high β plasma with the aim of answering some of the following questions:

(i) Do whistlers in high β plasmas have significant electrostatic components, so that the expected wave energy is hidden in electrostatic energy not observed by magnetic detectors on satellites?

(ii) Are there new wave modes whose turbulence could be significant? We have preliminary evidence that a pitch angle anisotropy instability exists for the so-called "electro-acoustic" wave which would at times be competitive with whistlers.

(b) Non-steady Effects

So far, we have discussed a theory of the steady quiet-time auroral zone. Is there another, macroscopic, instability which leads to the violent auroral substorms?

(c) Particle Acceleration

The strong diffusion regime is an obvious candidate to accelerate particles, since they do not feel the loss cone strongly. Preliminary estimates suggest that a 1/10 - 1γ whistler intensity, can account for the observed acceleration of 50-100 keV electrons in the Van Allen Zone.

III. SUMMARY

A theory of the coupling of the neutral sheet plasma flow to particle precipitation to form the nightside auroral zone can be constructed using only the most generalized properties of plasma turbulence. Several features, the pitch angle isotropy of the auroral precipitation, the location of separate precipitation maxima for ions and electrons, and the acceleration of high energy particles, are described by this model. The properties of the specific mode of plasma turbulence are still somewhat unclear, since the auroral zone plasma has a large ratio of plasma to magnetic pressure, a case not extensively studied thus far in the fusion literature.

BIBLIOGRAPHY

Angerami, J., and D. Carpenter, Whistler studies of the plasmapause in the magentosphere, 2, Electron density and total tube electron content near the knee in magnetospheric ionization, J. Geophys. Res. 71, 3, 711-726, 1966.

Axford, W. I., H. E. Petschek, and G. L. Siscoe, Tail of the magnetosphere, J. Geophys. Res.,70(5), 1231-1236, 1965.

Bame, S. J., J. R. Asbridge, H. E. Felthauser, E. W. Hones, and I. B. Strong, Characteristics of the plasma sheet in the earth's magnetic tail, J. Geophys. Res. 72, 113-130, 1967.

Frank, L. A., Initial observations of low energy electrons in the earth's magnetosphere with OGO-3, J. Geophys. Res., 72, 185-196, 1967.

Fritz, T. A. and D. A. Gurnett, Diurnal and latitudinal effects observed for 10 kev electrons at low satellite altitudes, J. Geophys. Res, 70, 2485-2502, 1965.

Kennel, C. F. and F. Engelmann, Velocity space diffusion from weak plasma turbulence in a magnetic field, Phys. Fluids, 9, 12, 1966.

Kennel, C. F. and H. E. Petschek, A limit on stably trapped particle fluxes, J. Geophys. Res., 71, 1, 1-28, 1966.

O'Brien, B. J. and H. Taylor, High latitude geophysical studies with satellite Injun 3, 4. Auroras and their excitation. J. Geophys. Res. 69, 1, 45-64, (1964).

Speiser, T. W. and W. F. Ness, The neutral sheet in the geomagnetic tail, its motion, equivalent currents, and field line connection through it, J. Geophys. Res. 72, 131-142, 1967.