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## SUMMARY

A method utilizing the direction of pen motion during sequential increments of time is presented as a means of recognizing hand drawn line patterns. During the time a pattern is drawn, $d+1$ discrete points are measured. From these, d quantized directions are obtained and used as parameters in a discriminant function. The coefficients, or weights, of the function are determined through a learning algorithm. Experiments were performed using ten different sets of children's writing. The complete English Alphabet, in capital letters, and the numerals comprised each set. These patterns had various degrees of distortion, rotation, and continuity. After a training phase in which all ten sets were used, the error during recognition was $6.0 \%$.

## INTRODUCTION

One of the major problems in any pattern recognition system is that of feature extraction. In most line pattern recognition systems, feature extraction involves data processing of input coordinate points to produce features such as length of line pattern, center of gravity, curvature, etc. In this paper feature extraction is simplified since only one type of feature, based on approximated spatial derivative information, is extracted. The spatial derivatives are resolved into directions, coded with "Direction Numbers", and assembled into a feature vector. The element positions of particular features in the vector coincide with the time sequence in which the features were extracted.

## DIRECTION TIME FEATURES

In order to understand how the features are extracted, it is helpful to briefly consider the construction of the input device. ${ }^{1}$ A pad simulates a portion of the first quadrant of a rectangular coordinate system. As a pen moves along the pad, the $x, y$ coordinates of the pen are stored into successive locations of a memory system each time the coordinates change. The points in Figure $1(a)$ can be considered as being on the pad. If a pen is placed at point 1 and moved through points 3 and 5 to point 7 , as in Figure 1(b), a direction sequence is obtained with elements "down, right, up". If the points 7, 5, 1, 3, 7 are traversed in that order, as in Figure $1(c)$, the sequence obtained is
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Figure 1. Direction sequence illustration
"down, up left, down, up right". The directions down, right, etc. are quantized into sectors and each sector is represented by a number. Figure 2 illustrates this idea with eight quantized sectors. Each sector is 45 degrees wide and sectors 0 and 4 are centered about the


Figure 2. Quantization of directions
horizontal. Applying this numerical representation of directions to the direction sequence "down, right, up" results in the sequence " 6 , $0,2^{\prime \prime}$, where each number will be called a direction number. Similarly, the sequence "down, up left, down, up right" becomes the sequence of direction numbers "6, 3, 6, 1". This last sequence can be used to represent the pattern of Figure $1(c)$ when drawn as indicated. It cannot be used to represent a pattern whose final appearance is the same as that in Figure $l(c)$ but whose stroke sequence is different. This is
a seeming disadvantage of the method. It has been verified, however, that there is a considerable amount of invariance in the order of strokes used in constructing a hand-drawn line pattern ${ }^{2}$. Furthermore, in some applications the sequence of strukes in grawing the patterns is intended to be invariant ${ }^{3}$.

Since, in fact, a large number of coordinate points are stored for any typical pattern (the coordinate grid is structured with 100 lines per inch in the device under consideration), it is evident that any pattern drawn on the pad can be represented as precisely as Cesired by using a sequence of direction numbers (within the resolving ability of the coordinate grid and quantization shown in Figure 2)--the precision increasing as the number of direction numbers increases. For example, using the additional points 2, 4 , and 6 in Figure $1(a)$. It is also evident that such a sequence represents the direction in which the pen was moving during successive increments of time. It therefore can be considered a feature vector with direction numbers as elements--a direction-time feature vector. Hereafter, such vectors will be denoted as
$\underline{x}=\left[x_{1}, x_{2}, \ldots, x_{j}, \ldots, x_{d}\right]^{t}$
where
$x_{j}$ is a direction number indicating the direction of motion of the pen in the jth increment of time.

## THE RECOGNITION METHOD

The development which follows assumes the existence of a prototype vector for each of the recognizable categories. Each prototype is the standard for its category. The simplest way of obtaining such vectors is to construct well drawn patterns and then store their feature
vectors as the prototype vectors. The ith recognizable pattern category is denoted by $X_{i}$ while its prototype vector is denoted by $\underline{P}_{i}$.

## Normalization

As implied previously, a prime consideration in developing this method was simplicity. Accordingly, only size normalization is done. A simple scheme, based on the fact that the number of data points is indicative of pattern size, is used. For example, drawing the letter " $L$ " one inch high might result in approximately 180 coordinate points stored in the memory system whereas an " $L$ " two inches high would have approximately 360 coordinate points stored.

Normalization could be achieved if some number of points, less than the total number stored, were used to represent the pattern. The number chosen must be one greater than $d$, the number of elements desired in the feature vector. Furthermore, the points must be chosen
such that they are in fact representative of the pattern. A promising, convenient, and simple choice is to include the first and last coordinate points and $d-1$ approximately equally spaced points in between.

## Extracting Features

If the $d+l$ points representing a size normalized pattern are considered in the same time sequence in which they were traversed during the drawing of the pattern, ad dimensional direction-time feature vector can be developed. Let the sequence $s_{1}, s_{2}, \cdots, s_{d}, s_{d+1}$ represent the points. This implies that the pen first contacted the pad at $s_{1}$. Then possibly after moving some distance, itt passed over $s_{2}$ and similarly for $s_{3}, \ldots, s_{d}$. Finally, the pattern was completed with the pen stopping at $s_{d+1}$. No restriction is made on the number of times the pen may have left the pad and returned in between $s_{1}$ and $s_{d+1}$. Lack of this restriction suggests a need for a signal from the user to the system to indicate that a pattern has been completed and recognition should commence. In order to extract feature $x_{j}$, the slope,

$$
m_{j}=\left(a_{j}^{-a_{j+1}}\right) /\left(b_{j}-b_{j+1}\right)=N_{j} / D_{j}
$$

of an imaginary line drawn from $s_{j}$ to $s_{j+1}$
is obtained and quantized utilizing the slopes of the four lines in Figure 2. The quantization can be expressed as

$$
\begin{array}{r}
x_{j}=0 \text { or } 4 \text { if } M_{4}<m_{j} \leqslant M_{1} \\
x_{j}=1 \text { or } 5 \text { if } M_{1}<m_{j} \leqslant M_{2} \\
x_{j}=2 \text { or } 6 \text { if }\left\{\begin{array}{l}
M_{2}<m_{j}<\infty \\
\infty<m_{j} \leqslant M_{3}
\end{array}\right\} \\
x_{j}=3 \text { or } 7 \text { if } M_{3}<m_{j} \leqslant M_{4}
\end{array}
$$

Discrimination between the two possible choices for $x_{j}$ is accomplished using the following relationships:

$$
\begin{aligned}
& x_{j}=0,1,2,6, \text { or } 7 \text { if } D_{j}<0 \\
& \mathbf{x}_{j}=0,2,2,3, \text { or } 4 \text { if } N_{j}<0
\end{aligned}
$$

After d passes through this decision structure, the direction-time feature vector $X$ is obtained.

## Recognition

Frequently recognition is based on a set of linear discriminant functions of the form $\left\{g_{i}(\underline{X}): \quad g_{i}(\underline{X})=W_{i} \cdot \underline{X}, \quad i=1,2, \ldots, R\right\}$. To be effective such functions require the elements of $X$ to possess an ordering relationship similar
to the real numbers. Since in the method presented here the elements of $\bar{X}$ are in fact elements of $z_{8}$, the set of integers mod 8 , such
functions do not suffice (i.e. the ordering has zero in between seven and one). Instead, a set of functions of the form
$\left.\left\{g_{i}\left(\underline{P}_{i}, \underline{X}\right): \quad g_{i}\left(\underline{P}_{i}, \underline{X}\right)=\underline{W}_{i} \not \underline{\left(P_{i}\right.}, \underline{X}\right), i=1,2, \ldots, R\right\}$ is used where $g_{i}$ is a metric in the d dimensional space of the feature vectors. The output is determined by $\min _{i}\left\{\mathrm{~g}_{\mathrm{i}}\left(\underline{P}_{\mathrm{i}}, \underline{\mathrm{X}}\right)\right\}$. Clearly, $\mathrm{g}_{\mathrm{i}}$ is a metric if the individual components in the linear combination which comprises $g_{i}$ are metrics. Accordingly, if the jth elements of ${\underset{W}{i}}, \underline{P}_{i}$, and $X$ are denoted by $w_{i j}, p_{i j}$, and $x_{j}$. Then one can write

$$
g_{i}\left(\underline{p}_{i}, \underline{x}\right)=\sum_{j=1}^{1} w_{i j} \emptyset_{j}\left(p_{i j}, x_{j}\right) .
$$

If the restriction that at least one of the $w_{i j}$
$>0$ is imposed, then $g_{i}$ is a metric if $\phi_{j}$ is a metric. When $\emptyset_{j}$ is defined as
$\phi_{j}\left(p_{i j}, x_{j}\right)=\left\{\begin{array}{ll}\left|p_{i j}-x_{j}\right| & \text { if }\left|p_{i j}-x\right|<4 \\ \left|6-p_{i j}-x_{j}\right| & \text { if }\left|p_{i j}-x_{j}\right|\end{array}\right\}$
its metric properties are easily obtained.

## Training

While the function $\varnothing\left(\underline{P}_{i}, \underline{X}\right)$ is itself worthy
of some note as a discriminant function, the errors it produces can be decreased by a suitable selection of weight vectors. Some strides have been taken toward determining these weight vectors when assumptions can be made about the parameters which characterize the probability distributions of the patterns. 4 When no such assumptions can be made non-parametric methods must be employed. Non-parametric methods involve attempts to recognize known patterns presented sequentially to the system. After each attempt the weight vectors are either modified if the response was incorrect or not modified if the response was correct. In the method employed here for example, if $X$ is the $k$ th pattern in the sequence and it is known to be in category i and $\underline{W}_{i} \bullet \underline{\phi}\left(\underline{P}_{i}, \underline{X}\right)<\underline{W}_{j} \cdot \underline{\phi}\left(\underline{P}_{j}, \underline{X}\right), j=1,2, \ldots, R: \quad j \neq 1$, then $\underline{X}$ is correctly recognized as being in $X_{i}$ and no modification takes place. If, however, $\underline{X}$ is known to be in category $i$ and $\underline{W}_{j} \cdot \underline{\varnothing}\left(\underline{P}_{j}, \underline{X}\right) \leqslant \underline{W}_{i} \cdot \underline{\phi}\left(\underline{P}_{d} \cdot \underline{X}\right)$ for $j$ in some non-empty set of integers $J$, then the set of weight vectors $\left\{W_{h}: h=1,2, \ldots, R\right\}$ is modified according to the following training rule:

[^0]\[

$$
\begin{array}{cc}
\underline{W}_{i}(k)-c(k) \varnothing\left(\underline{P}_{i}, \underline{X}(k)\right) & h=i \\
\left.W_{h}(k+1)=\underline{W}_{j}(k)+\alpha_{j}(k) c(k) \underline{\left(P_{j}\right.}, \underline{X}(k)\right) & \begin{array}{c}
h=j, \\
j \in J \\
\underline{W}_{h}(k)
\end{array} \\
\text { otherwise }
\end{array}
$$
\]

where $k$ denotes the number of modifications, $c(k)>0, \alpha_{j}(k)>0, \sum \alpha_{j}(k)=1$, and at least one $w_{h m}(k)>0 . \quad w_{h m}(k)$ is the mth element of $W_{h}(k)$.

The proof that this training rule results in a set of weight vectors which can correctly recognize all the patterns used in training, if such a set exists, is given in reference 8.

## EXPERIMENTS

The method was simulated on an IBM 7090 computer using the MAD language. Lack of the aforementioned input device necessitated drawing the patterns on graph paper and storing the $X, Y$ coordinate data on punched cards.

An arbitrary, interesting, and convenient choice for the patterns to be recognized was the set of alphanumeric characters with no distinction between " $O$ " and zero. This placed $R$, the number of patterns recognizable by the system, at 35 . More intuitive than arbitrary was the selection of $d$, the dimension of the feature vector, as 10 and the direction quantization as shown in Figure 2. Also, for simplicity $c(k)=1 / 8, \alpha_{j}(k)=1$ for the $j$ which denotes the minimum of $\left\{\underline{W}_{j} \cdot \underline{Q}_{j}: j \in J\right\}$, and all other $\alpha_{j}(k)=0$, for all $k$.

## Data.

Data for the program consisted of 10 sets of alphanumeric characters. Representative samples are shown in Figure 3. Set 1 was reproduced from a leading text on handwritting ${ }^{6}$, and was used as the prototype set. The sequence of strokes used in drawing each of the other sets was the same as that shown on the prototype set.

## Results

In one experiment the program trained on two sets of characters (Sets 2 and 3), and then it attempted to recognize all 10 sets. Initially all weight factors were set equal to unity. The results are viewed with mixed emotions. For although the weights were modified such that Sets 2 and 3 were recognizable with no errors

[^1]Set 1




Set 3


Set 4


Set 5


Set 6






Figure 3. Samples of Datterns used in experiments
after the system had observed them nine times, the error rate after attempting to recognize the 350 patterns in the 10 sets (using the modified weights) was considerably higher at $16 \%^{* * *}$. The convergence of the error rate during the training phase is shown in Figure 4.

More significant and gratifying results were obtained when the program was allowed to train on all 10 pattern sets. Again all weight factors were initially set at unity. Then after having observed each of the 350 patterns 15 times, an attempt was made to recognize them. The error rate for this attempt was $6.0 \%$. Figure 4 depicts the convergence of the error rate during the learning phase.

The execution time of the IBM 7090 in recognizing the 350 patterns was 87 seconds. This implied a recognition rate of one pattern per 0.25 seconds.

[^2]

Figure 4. \% error during training phase

## CONCLUSIONS

Aside from the basic conclusions that pattern recognition is feasible using directiontime feature vectors, it is evident that use of a more efficient machine language program, as opposed to the compiled version used here, and the faster computers available today should pr duce a significant increase in speed. Accordingly, it can also be concluded that use of directiontime feature vectors permits rapid recognition of patterns.

The fact that the method is fast, simple, and able to recognize reasonably well draw alphanumeric characters with a low error rate, suggests its use as a man-machine communicating device. The particular application in mind is that in which a user takes a few moments to teach the system his style of drawing the alphanumerics, mathematical symbols, etc., then proceeds to write, edit, or insert data into a computer program. Such an application suggests several areas for extending this research.

First, it could not have gone unnoticed that a great many parameters were set arbitrarily or intuitively, i.e. the number of sectors, the slopes of the lines for extracting features and the number of elements in the feature vector. A method of optimally determining some or all of these parameters based on the user's style of

- drawing seems both beneficial and interesting to pursue. Second, the development of functional units capable of performing the functions implied in the method also appears to be a challenging and worthwile task. Finally, the learning scheme appears to converge slowly. This may or may not be a function of the patterns used. Nevertheless, in view of the fact that learning methods with varying rates of convergence exist for other discriminant functions 7 , it seems reasonable to conduct further investigation in this area.

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[^0]:    * With all elements of all weight vectors set equal to one the error rate is $22 \%$.

[^1]:    ** Duda and Fossum ${ }^{5}$ describe a similar nonparametric method. Their discriminant functions are of the form $g_{j}(X)=W_{i} \cdot X$ and their training The is appliud when, for some $j \in J, W_{j} \cdot X \geqslant W_{i} \cdot X$. The trainjog ( $k$ 位 $+c$ ( $k$ ) $x$ (k) $W_{h}(k+1)=\left\{\begin{array}{l}\bar{W}_{1}^{1}(k)-\alpha_{j}(k) c(k) X(k) \\ \bar{W}_{h}^{1}(k)\end{array}\right.$ $h=j, \quad j \in J\}$ otherwise $\}$

[^2]:    A correct response is considered as any response which correctiy identifies the unknown pattern and only that pattern. Any other response is an error.

