# nasa CE6.5863 

## LIBRARY COPY

NOV 241967
MANNED SPACECRAFT CENTER HOUSTON, TEXAS


NORTH AMERICAN AVIATION, INC.
SPACE DIVISION


# PRECEDING PAGE BLANK NOT FILMED. 

## FOREWORD

This report was prepared by North American Aviation, Inc., Space Division, under NASA Contract NAS9-4552, for the National Aeronautics and Space Administration, Manned Space Flight Center, Houston, Teras.with Dr. F.C. Hung, Program Manager and Mr. P.P. Radkowski, Assistant Program Manager. This work was administered under the direction of Structural Mechanics Division, MSC, Houston, Texas with Dr. F. Stebbins as the technical monitor.

Thic report is presented in eleven volumes for convenience in handling and distribution. All volumes are unclassified.

The objective of the study was to develop methods and Fortran IV computex programs to determine by the techniques described below, the hydro-elastic response of representation of the structure of the Apollo Command Module immediately following impact on the water. The development of theory, methods and computer programs is presented $: s$ Task I Hydrodynamic Pressures, Task II Structural Response and Task III Hydroelastic Response Analysis.

Under Task I - Computing program to extend flexible sphere using the Spencer and Shiffman approach has been developed. Analytical formulation by Dr. Li using nonlinear hydrodynamic theory on structural portion is formulated. In order to cover a wide range of impact conditions, future extensions are necessary in the following items:
a. Using linear hydrodynamic theory to incluc. horizontal velocity and rotation.
b. Nonlinear hydrodynamic theory to develop computing program on spherical portion and to develop nonlineas theory on toroidal and conic sections.

Under Task II - Computing progiam and User". Manual were developed for nonsymmetrical loading on unsymmetrical elasti: shells. To fully develop the theory and methods to cover reaistic Apollo configuration the following extensions are recommended:
a. Modes of vibration and modal analysis.
b. Extension to nonsymmetric short time impulses.

## 

c. Linear buckling and elasto-plastic analysis

These technical extensions will not only be useful for Apollo and future Apollo growth configurations, but they will also be oi value to other aeronautical and spacecraft programs.

The hydroelastic response of the flexible shell is obtained by the numerical solution of the combined hydrodynamic and shell equations. The results obtained herein are compared numerically with those derived by neglecting the interaction and applying rigid body pressures to the same elastic shell. The numerical results show that for an axially symmetric impact of the particular shell studied, the interaction between the shell and the fluid produces appreciable differences in the overall acceleration of the center of gravity of the shell, and in the distribution of the pressures and responses. However the maximum responses are within $15 \%$ of those produced when the interaction between the fluid and the shell is neglected. A brief summary of results is shown in the abstracts of individual volumes.

The volume number and authors are listed on the following page.

The contractor's designation for this report is SID 67-498.

# INDEX FOR FINAL REPORT 

"Apollo Water Impact"

## Volume No.

Volume Title

## Authors

Hydrodynamic Analysis of Apollo Water Impact

Dynamic Response of Shells of Revolution During Vertical Impact Into Water - No Interaction

Dynamic Response of Shells of Revolution During Vertical Impact Into Water - Hydroelastic Interaction

Comparison With Experiments
User's Manual - No Interaction
User's Manual - Interaction

Modification of Shell of Revolution Analysis

Unsymmetric Shell of Revolution Analysis

Mode Shapes and Natural Frequencies Analysis

User's Manual for Modification of Shell of Revolution Analysis

User's Manual for Unsymmetric Shell of Revolution Analysis
T. Li and T. Sugimura
A. P. Cappelli, and J.P.D. Wilkinson
J. P. D. Wilkinson, A. P. Cappelli, and R.N. Salzman
J. P. D. Wilkinson
J.P.D. Wilkinson
J.P.D. Wilkinson and R. N. Salzman
A. P. Cappelli and S. C. Furuike
A. P. Cappelli, T. Nishimoto, P. P. Radkowski and K. E. Pauley
A. P. Cappelli
A. P. Cappelli and S. C. Furuike
E. Carriôn, S. C. Furuike and T. Nishimoto

# PRECEDING PAGE BLANK NOT FILMED. 

Volume 11<br>User's Manual<br>For the Unsymmetric Shell of Revolution Computer<br>Programs Static and Dynamic


#### Abstract

The unsymmetric shell of revolution programs described in this volume were developed as a basic tool in the static and dynamic analysis of shells of revolution having arbitrary distribution of stiffness subjected to arbitrary loads and temperature.

The analytical basis for the two computer programs, static and dynamic, is presented. The analysis is restricted to linear-elastic thin shell theory. The basic equations include the effect of shear distortion. The analysis utilizes a Fourier series technique to separate the circumferential variation. The resulting set of equations are reduced to an algebraic set by the use of finite difference forms for variations of the other variables. These algebraic equations are then solved numerically by a direct matrix elimination procedure.

The computer programs were written in FORTRAN IV. In order to keep input data at a minimum extensive use of call function subroutines have been used. Call functions are subroutines which are coded and compiled by the user. This eliminates the necessity of large tabular inputs.

Solutions obtained from the Dynamic Unsymmetric Shell program are the displacement and rotation time histories. Also included in the output are the histories of internal forces and moments.

The solutions obtained from the Static Unsymmerric Shell program include displacements, rotations, internal force and moment resultants, and stresses. This output is presented in tabular form. The users of these programs should be forewarned that the programs are only a tool and insight must be used in iolating results to an actual physical problem.

This volume is intended to supply the information necessary for the use of both the Dynamic Unsymmetric Shell program and the Static Unsymmetric Shell program. Detailed descriptions have been included in this volume to aid the user in performing extensions and modifications of these programs.




## PRECEDING PAGE BLANK NOT FILMED.

## CONTENTS

Section Page
FOREWORD ..... iii
ABSTRACT ..... vii
I. THEORY ..... 1
1.1 Introduction ..... 1
1.2 Nomenclature ..... 1
1.3 Scope and Limitations ..... 3
1.4 Shell Coordinate System ..... 4
1.5 Equations of Motion ..... 5
1.6 Formulation into Solution Variables ..... 8
1.7 Circumferential Variable Separation ..... 13
1.8 Finite Difference Formulation in the Meridional Variable ..... 26
1.9 Finite Difference Formulation in the Time Variable ..... 28
1.10 Matrix Solution of the Difference Equations ..... 30
1.11 Static Analysis ..... 32
1.12 References ..... 33
1.13 Appendix ..... 33
1.13.1 Modification of Sanders' Equations . ..... 33
1.13.2 Series Product Expansion. ..... 36
1.13.3 Coefficients ..... 36
II. GENERAL DESCRIPTION OF THE COMPUTER PROGRAMS ..... 45
2.1 Introduction ..... 45
2.2 Program Capabilities and Limitations ..... 45
2.3 Sign Conventions and Dimensions ..... 46
2.4 Reference, Inner and Outer Surfaces ..... 46
2.5 Geometry ..... 47
2.5.1 Cone-Cylinder ..... 47
2.5.2 Sphere-Toroid ..... 48
2.5.3 Discrete Point ..... 49
2.6 Function Subroutine Parameters ..... 50
2.6.1 Stiffness Properties ..... 50
2.6.2 Thermal Loads . ..... 51

## Section

2.6.3 Elastic Foundations ..... 51
2.6.4 External Damping. ..... 51
2.6.5 Mass Parameier ..... 51
2.6.6 Pressure Loads ..... 51
2.6.7 Parameters for Stresses ..... 52
2.7 Stations ..... 52
2. 8 Boundary Conditions ..... 53
2.9 Reference Quantities ..... 53
2.10 Output Indicators ..... 54
III. DETAILED USE OF THE PROGRAMS ..... 55
3.1 Introduction ..... 55
3.2 Deck Set-Up ..... 56
3.3 Data Deck Set-Up ..... 59
3.3.1 Title Cards ..... 60
3.3.2 General Shell Data, GDA ..... 60
3.3.3 Geometry Data, GMDA ..... 62
3.4 Function Subprograms ..... 64
3.5 Utility Subroutines ..... 69
3.5.1 DECRD ..... 69
3.5.2 MAD, MSU, MMY, INVMS ..... 72
3.5.3 DINTRP, ENTERP ..... 83
3.5.4 CODIMA ..... 86
3.5.5 FILE ..... 90
3.6 Sample Problem Number 1 ..... 95
3.6.1 Problem Description and Set-Up ..... 95
3.6.2 Data Sheets ..... 96
3.6.3 Function Subprograms Used ..... 98
3.6.4 Output ..... 102
3.7 Sample Problem Number 2 ..... 114
3.7.1 Problem Description and Set-Up ..... 114
3.7.2 Data Sheets ..... 116
3.7.3 Function Subprograms Used ..... 119
3. 8 Sample Problem Number 3 ..... 115
3.8.1 Problem Description and Set-Up ..... 125
3.8.2 Data Sheets ..... 126
3.8.3 Function Subprograms Used ..... 128
3.8.4 Output ..... 131
3.9 Sample Problem Number 4 ..... 167
3.9.1 Problem Description and Set-Up ..... 167
3.9.2 Data Sheets ..... 169
3.9.3 Function Subprograms Used ..... 171
3. 10 Error Indications, Pitfalls, Recommendations ..... 176
3.11 Program Listings for Static Version ..... 1.77
3.12 Program Listings for Dynamic Version; Load Map . . . . . . . . . . . 230
3.13 Nomenclature ..... 298

# PRECEDING PAGE BLANK NOT FILMED. 

## ILLUSTRATIONS

Figure . Page
1.1 Geometry and Coordinates . . . . . . . . 4
1.2 Sign Convention . . . . . . . . . . . 7
1.3 Stiffness Profile . . . . . . . . . . 15
2. 1 Cone-Cylinder Geometry . . . . . . . . . 48
2.2 Sphere-Toroid Geometry . . . . . . . . . 48
2.3 Discrete Point Geometry . . . . . . . . . 50

### 1.0 THEORY

### 1.1 INTRODUCTION

This section presents the analytical basis of the computer programs. The dynamic response problem is presented and the sperialization for the static analysis is made.

The analysis is based on a modified form of the general first order linear shell theory of Sanders. These equations have been modified to include transverse shear distortion, see Appendix 1. 13.1 The modified equilibrium equations are extended to include time dependence by D'Alembert's principle. Fourier analysis is used to separate variables in the circumferential direction and a system of finite difference approximations are used to reduce the partial differential equationss to an algebraic set. This set is solved by using a direct matrix elimination procedure.

The material presented in this section is an extension and parallels of the work of Sanders ${ }^{1}$, Budiansky and Radkowski ${ }^{2}$, and Johnson and Greif ${ }^{3}$. The notation used is identical to that of Reference 2 except where noted.

### 1.2 NOMENCLATURE

## Fourier Coefficients

$u_{n}, v_{n}, w_{n}, \phi_{\xi n}, \phi_{\theta_{n}}$
$=$ displacements and rotations
$p_{\xi n}, p_{\theta n}, p_{n}$
$t_{\xi n}^{T}, t_{\theta n}^{T}, m_{\xi n}^{T}, m_{\theta n}^{T}$
$=$ trmperature induced force terms
$t_{\xi}^{n}, t_{\theta}^{n}, t_{\xi \theta}^{n}, t_{\xi \theta}^{n}, q_{\xi}^{n}$
$q_{\theta}^{n}, m_{\xi}^{n}, m_{\theta}^{n}, m_{\xi}^{n}$
$\mathrm{b}_{\mathrm{mj}}, \mathrm{d}_{\mathrm{mj}}, \mathrm{gmj}_{\mathrm{mj}}, \mathrm{g}_{13 \mathrm{j}}$
$=$ force resultants, modified and effective
$k_{m j}, c_{m j}, m_{m j}$
$=$ elastic foundation, damping, mass


## Coordinates and Constants

$\xi, \theta, \zeta$
$=$ coordinates
$\mathbf{r}$
$\mathbf{s}$

| a, $h_{0}, \sigma_{0}, E_{0}$ | = reference constants |
| :---: | :---: |
| ${ }^{\omega} \theta \cdot \omega_{\xi}$ | = nondimensional curvatures |
| $\mathrm{u}, \mathrm{v}, \mathrm{w}, \phi_{\xi}, \phi_{\theta}$ | = displacements and rotations |
| $\sigma_{\xi}, \sigma_{\theta}, \tau_{\xi \theta}, \tau_{\xi \zeta},{ }^{\top} \theta \zeta$ | = stresses |
| ${ }^{\epsilon} \xi,{ }^{\epsilon} \theta^{\prime}{ }^{\epsilon} \xi_{\theta}, \mathrm{k}_{\xi}, \mathrm{k}_{\theta}$ | = strains |
| ${ }^{k_{\xi \theta}}{ }^{\prime} \gamma_{\xi \zeta}, \gamma_{\theta \zeta}$ |  |
| $B_{i}, C_{i}, D_{i}, G_{i} \quad K_{i}, M_{i}$ | = isotropic stiffness functions |
| E, $v, a$ | $=$ material properties |
| $q_{\xi}, q_{\theta}, q$ | $=$ loads |
| T | = temperature change |
|  | -2 - |

$$
\left.\begin{array}{lll}
N_{\xi}, & N_{\theta}, & \bar{N}_{\xi \theta}, \\
M_{\xi}, & M_{\theta}, \bar{M}_{\xi \theta}, & Q_{\xi} \\
Q_{\theta}, & \hat{N}_{\xi \theta}, & N_{\xi}^{T}, \\
N_{\theta}^{T}, & M_{\xi}^{T}, & M_{\theta}^{T}
\end{array}\right\}=\begin{aligned}
& \text { force resultants, modified, effective, } \\
& \text { and temperature induced }
\end{aligned}
$$

## Matrices

$F, G, H, K, \alpha, \beta, \Omega, \Lambda, R, S=5 K \times 5 \mathrm{~K}$ order
$\mathrm{p}, \ell, \mathrm{x}=5 \mathrm{~K} \times 1$ order

## Indices

i, j, n = Dummy
$k \quad=k^{\text {th }}$ Fourier component

### 1.3 SCOPE AND LIMITATIONS

The shell theory on which the se programs are based is restricted to linear, elastic, thin shell, theory:
.(a) The thickness of the shell at any point is small compared to the . other dimensions.
(b) Deformations of the shell are small compared to the smallest radius of curvature.
(c) All material points of the shell deform elastically, obeying Hooke's law for transverse isotropic materials.
(d) The shell is "complete," i.e., its only boundaries are at meridian ends and inner and outer surfaces.
(e) The class of shells considered has a surface of revolution reference surface which is within or in close proximity of the shell walls such that $\int E \zeta d \zeta=0$.
(f) The parameters of stiffness, e.g., in-plane stiffnesses are permitted to vary in both the meridional and circumferential directions. Implied is that parameters such as thickness, Young's modulus, etc., are permitted to vary in both the meridional and circumferential directions. The stiffness parameter's variation in the circumferential direction is restricted to those with a plane of symmetry.
(g) Arbitrary loads and temperature distributions are permissible. Excluded are problems with thermal distributions such that limitation (e) is not satisfied.
(h) The effects of transverse shear is included.
(i) Instability is not considered.
(j) Distributed mass, elastic foundation, and external damping is included. These distributions have the same symmetry conditions as the stiffness parameters (f).

### 1.4 SHELL COORDINATE SYSTEM

The geometry of a shell is defined entirely by specifying the form of the reference surface and the thickness of the shell at each point. For convenience, the shell coordinate system and gewnetrical relations used in Reference 2 will be adopted here. The reference surface is assumed to be a surface of revolution and is selected at a convenient location, within or in proximity to the shell walls. The first fundamental form of the reference sürface is defined by

$$
\begin{equation*}
d \bar{s}^{2}=d s^{2}+r^{2} d \theta^{2} \tag{1.1}
\end{equation*}
$$

where $d \bar{s}$ is a line element on the surface and $s$ and $\theta$ denote orthogonal coordinates selected along lines of principal curvature.


Figure 1. 1. Geometry and Coordinites
 the distance from the axis of revolution. The coordinate $s$ is a measure of the mexidional distance along an axisymmetric reference surface, and $\theta$ is a circumferential angle, as shown in Figure 1.1. The coordinate, $\zeta$, is selected as a measure of the ncrmai outward distance from the reference surface $(\zeta=0)$. The principal radii of curvature are

$$
\begin{equation*}
R_{s}=-\left[1-\left(\frac{d r}{d s}\right)^{2}\right]^{1 / 2} / \frac{d^{2} r}{d s^{2}} \quad R_{\theta}=:\left[1-\left(\frac{d r}{d s}\right)^{2}\right]^{-1 / 2} \tag{1.2}
\end{equation*}
$$

Introduce the nondimensional meridional coordinate $\xi=s / a$, where a is a reference iength; then, wịh $\rho=r / a$, the nondimensional curvatures $\omega \xi=a / R_{s}$ and $\omega_{\theta}=a / R_{\theta}$ car. be found from the formulas

$$
\begin{equation*}
\omega_{\xi}=\cdot\left(y^{\prime}+y^{2}\right) / \omega_{\theta} \quad \omega_{\theta}=\left[1-\left(\rho^{\prime}\right)^{2}\right]^{1 / 2} / \rho \tag{1.3}
\end{equation*}
$$

where

$$
\gamma=\rho^{\prime} / \rho
$$

In these equations, and henceforth, $\left(\gamma^{\prime}=\frac{\partial()}{\partial \xi}\right.$
Finally from the Coaazzi relation we obtain

$$
\begin{equation*}
\omega_{\theta}^{\prime}=\gamma\left(\omega_{\xi}-\omega_{\theta}\right) \tag{1.4}
\end{equation*}
$$

and the relation

$$
\begin{equation*}
\frac{\rho^{\prime \prime}}{\rho}=-\omega_{\xi} \omega_{\theta} \tag{1.5}
\end{equation*}
$$

### 1.5 EQUATIONS OF MOTION

The general equilibrium equations for an arbitrary shell based on the first-order linear shell theory of Sanders are given in Reference 1. These equations are modified to include the effect of transverse shear distortion by the procedure suggested by Sanders. * These equilibrium equations are extended to equations of motion by use of D'Alembert's principle. These equations specialized for a shell whose reference surface is a surface of revolution are given as,

[^0]\[

$$
\begin{align*}
& \mathrm{a}\left[\frac{\partial \rho}{\partial \xi} \mathrm{~N}_{\xi}+\rho \frac{\partial \mathbf{N}_{\xi}}{\partial \xi}+\frac{\partial \bar{N}_{\xi \theta}}{\partial \theta}-\frac{\partial \rho}{\partial \xi} \mathrm{N}_{\theta}\right]+ \\
& a \rho \omega_{\xi} Q_{\xi}+\frac{1}{2}\left(\omega_{\xi}-\omega_{\theta}\right) \frac{\partial \bar{M}_{\xi \theta}}{\partial \theta}+a^{2} \rho_{q_{\xi}}=a^{2} \rho\left[M \frac{\partial^{2} U_{\xi}}{\partial t^{2}}+C_{\xi} \frac{\partial U_{\xi}}{\partial t}+K_{\xi} U_{\xi}\right] \\
& a\left[\frac{\partial \mathbf{N}_{\theta}}{\partial \theta}+2 \frac{\partial \rho}{\partial \xi} \bar{N}_{\xi \theta}+\rho \frac{\partial \bar{N}_{\xi \theta}}{\partial \xi}\right]+a \rho \omega_{\theta} \mathbf{Q}_{\theta}+ \\
& \frac{\rho}{2} \frac{\partial}{\partial \xi}\left[\left(\omega_{\theta}-\omega_{\xi}\right) \bar{M}_{\xi \theta}\right]+a^{2} \rho q_{\theta}=a^{2} \rho\left[M \frac{\partial^{2} U_{\theta}}{\partial t^{2}}+C_{\theta} \frac{\partial U_{\theta}}{\partial t}+K_{\theta} U_{\theta}\right] \\
& a\left[\frac{\partial \rho}{\partial \xi} Q_{\xi}+\rho \frac{\partial Q_{\xi}}{\partial \xi}+\frac{\partial Q_{\theta}}{\partial \theta}-\right.  \tag{1.6.3}\\
& \left.\rho\left(\omega_{\xi} N_{\xi}+\omega_{\theta} N_{\theta}\right)\right]+a^{2} \rho_{q_{\zeta}}=a^{2} \rho\left[M \frac{\partial^{2} W}{\partial t^{2}}+C_{\zeta} \frac{\partial W}{\partial t}+K_{\zeta} W\right] \\
& \frac{\partial \rho}{\partial \xi} \mathrm{M}_{\xi}+\rho \frac{\partial \mathrm{M}_{\xi}}{\partial \xi}+  \tag{1.6.4}\\
& \frac{\partial \bar{M}_{\xi \theta}}{\partial \theta}-\frac{\partial \rho}{\partial \xi} M_{\theta}-a \rho Q_{\xi}=\operatorname{a\rho }\left[M^{*} \frac{\partial^{2} \Phi \underline{\underline{q}}}{\partial t^{2}}+C_{\xi}^{*} \frac{\partial \Phi{ }_{\xi}}{\partial t}+K_{\xi}^{*} \Phi_{\xi}\right] \\
& \frac{\partial \mathrm{M}_{\theta}}{\partial \theta}+2 \frac{\partial \rho}{\partial \xi} \bar{M}_{\xi \theta}+  \tag{1.6.5}\\
& \rho \frac{\partial M_{\xi \theta}}{\partial \xi}-a \rho Q_{\theta}=a \dot{\rho}\left[M_{\theta}^{*} \frac{\partial^{2} \Phi_{\theta}}{\partial t}+C_{\theta}^{*} \frac{\partial \Phi_{\theta}}{\partial t}+K_{\theta}^{*} \Phi_{\theta}\right]
\end{align*}
$$
\]

where
$\mathrm{M}(\xi, \theta)=\rho(\xi, \theta) \mathrm{h}(\xi, \theta)$ (mass/unit area of shell)
$M(\xi, \theta)^{*}=\frac{\rho(\xi, \theta) h^{3}(\xi, \theta)}{12}$ (mass moment/unit area of shell)
$C_{( },(\xi, \theta)=$ external damping coefficient
$K_{( },(\xi, \theta)=$ spring constants

Where the components of membrane force, transverse force and moment (about the reference surface) per unit length, and load per unit area (assumed to be applied at the reference surface) are shown in Figure l-2.


Figure 1-2. Sign Convention and Coordinates, Moments, Forces, Loads, Displacements, and Rotations

In the Sanders' first-order theory the inplane shearing forces $\mathrm{N}_{\xi} \theta$ and $\mathrm{N}_{\theta \xi}$ as well as twisting moments $\mathrm{M}_{\xi \theta}$ and $\mathrm{M}_{\theta \xi}$ are not handled separately, but instead are combined to form modified variables

$$
\begin{align*}
& \bar{N}_{\xi \theta}=\frac{1}{2}\left(N_{\xi \theta}+N_{\theta \xi}\right)  \tag{1.7}\\
& \bar{M}_{\xi \theta}=\frac{1}{2}\left(M_{\xi \theta}+M_{\theta \xi}\right) \tag{1.8}
\end{align*}
$$

and

$$
\begin{equation*}
\overline{\mathrm{k}}_{\xi \theta}=\frac{1}{2}\left(\mathrm{k}_{\xi \theta}+\mathrm{k}_{\theta \xi}\right) \tag{1.9}
\end{equation*}
$$

It is necessary when including the effects of transverse shear distortion to consider five equilibrium equations. Recall in Reference 2 that when shear deformation is neglected the shear forces are eliminated and resulting equilibrium equations are reduced to the consideration of three equations. The neglecting of transverse shear strains implies that normals to middle surface of the shell remain normal after deformation. The degree of error introduced by this assumption naturally depends on the magnitude of transverse shearing forces and shear rigidity of the shell. For discontinuous loads and shells having low shear rigidity, shear deformations may be comparable to bending and axial deformations and cannot be ignored.

### 1.6 FORMULATION INTO SOLUTION VARIABLES

The equations of motion are now expressed in terms of the solution variables, displacements and rotations.

The force and moment expressions in the equations of motion are determined by evaluating the following integrals through the thickness

$$
\begin{align*}
& \mathrm{N}_{\xi}=\int \sigma_{\xi} \mathrm{d} \zeta \quad \mathrm{M}_{\xi}=\int \sigma_{\xi} \zeta \mathrm{d} \zeta \\
& \mathbf{N}_{\boldsymbol{\theta}}=\int \sigma_{\boldsymbol{\theta}} \mathrm{d} \zeta \quad \mathbf{M}_{\boldsymbol{\theta}}=\int \sigma_{\boldsymbol{\theta}} \zeta \mathbf{d} \zeta  \tag{1.10}\\
& \overline{\mathbf{N}}_{\xi \theta}=\int \mathrm{T}_{\xi \theta} \mathrm{d} \zeta \quad \overline{\mathrm{M}}_{\xi \theta} \quad \int \mathrm{T}_{\xi \theta}{ }^{\boldsymbol{\mathrm { d }} \mathrm{d} \zeta} \\
& Q_{\xi}=\int{ }^{T} \xi_{\zeta} \mathrm{d} \zeta \quad Q_{\theta}=\int \tau_{\theta_{\zeta}} \mathbf{d} \zeta
\end{align*}
$$

where in the above integrals we have neglected terms of order $\zeta / R, R$ is the minimum radius of curvature. The stresses used above are defined as:

| $\sigma_{\xi},{ }^{\sigma} \theta$ | are normal stresses, acting on the <br> faces |
| ---: | :--- |
| $\cdots$ | is an in-plane shear stress acting <br> parallel to the reference surface |
| ${ }^{\top} \xi \theta$ | are transverse shear stresses acting <br> $\quad$normal to the reference surface |

By assuming that plane sections before remain plane after deformation, the strains at a distance $\zeta$ from the reference surface can be expressed in terms of the reference surface strains as follows:

$$
\begin{align*}
& \epsilon_{\xi}(\zeta)=\epsilon_{\underline{\xi}}+\zeta \mathbf{k}_{\underline{\xi}} \\
& \epsilon_{\theta}(\zeta)=\epsilon_{\theta}+\zeta \mathrm{k}_{\theta}  \tag{1.11}\\
& \epsilon_{\xi \theta}(\zeta)=\epsilon_{\xi \theta}+\zeta \bar{k}_{\xi \theta}
\end{align*}
$$

where ${ }^{\epsilon} \xi,{ }^{\epsilon} \theta$ and $\epsilon \xi \theta$ are the strains of the roferonce surface and ' $\xi \theta(i)$ is one-half the usual engineering strain.

The stress-strain-tempesature relations for an isotropic material are

$$
\begin{aligned}
& \sigma_{\xi}=\frac{E}{1-v^{2}}\left\{\epsilon_{\xi}+\nu \epsilon \theta+\zeta\left(\mathbf{k}_{\xi}+v k_{\theta}\right)-\alpha(1+v) T\right\} \\
& \sigma_{\theta}=\frac{E}{1-v^{2}}\left\{\epsilon_{\theta}+\nu \epsilon \xi\right. \\
& \\
&{ }^{\tau}{ }_{\xi \theta}\left.=G\left(\epsilon_{\xi \theta}+\zeta \bar{k}_{\theta}+\nu k_{\xi}\right)-\alpha(1+\nu) T\right\} \\
&{ }_{\xi \zeta \zeta}=G\left(\gamma_{\xi \zeta}\right) \\
&{ }^{\top} \theta \zeta=G\left(\gamma_{\theta \zeta}\right)
\end{aligned}
$$

where

$$
G=\frac{E}{2(l+v)}
$$

Substituting these equations into Eqs. 1.10 and employing appropriate integrations through the thickness yield the following stress/resultants-strain relationships.

$$
\begin{align*}
N_{\xi} & =B_{1} \epsilon_{\xi}+B_{3} \epsilon_{\theta}+C_{1} k_{\xi}+C_{3}{ }_{\theta}-N_{\xi}^{T} \\
N_{\theta} & =B_{3} \epsilon_{\xi}+B_{2} \epsilon_{\theta}+C_{3} k_{\xi}+C_{2} k_{\theta}-N_{\theta}^{T} \\
\bar{N}_{\xi \theta} & =G_{1} \epsilon_{\xi \theta}+G_{12}{ }^{k} \xi \theta \\
Q_{\xi} & =G_{2} \gamma_{\xi \zeta}  \tag{1.13}\\
M_{\xi} & =C_{1}{ }^{\epsilon} \xi+C_{3} \epsilon_{\theta}+D_{1} k_{\xi}+D_{3} k_{\theta}-M_{\xi}^{T} \\
M_{\theta} & =C_{3} \epsilon_{\xi}+C_{2} \epsilon_{\theta}+D_{3} k_{\xi}+D_{2} k_{\theta}-M_{\theta}^{T} \\
\bar{M}_{\xi \theta} & =G_{12}{ }^{\epsilon} \xi \theta+G_{13} k_{\xi \theta} \\
Q_{\theta} & =G_{3} \gamma_{\theta \zeta}
\end{align*}
$$

where in the above equations the shell stiffnesses are given by

$$
\begin{align*}
& B_{i}=B_{2}=\int \frac{E}{1-\nu^{2}} d \zeta, B_{3}=\int \frac{\nu E}{1-v^{2}} d \zeta, C_{1}=C_{2}=\int \frac{E \zeta}{1-v^{2}} d \zeta, C_{3}=\int \frac{v E \zeta}{1-v^{2}} \mathrm{~d} \zeta \\
& D_{1}=D_{2}=\int \frac{E}{1-v^{2}} \zeta^{2} d \zeta, D_{3}=\int \frac{v E \zeta^{2}}{1-\nu^{2}} d \zeta \\
& G_{1}=G_{2}=G_{3}=\int \operatorname{Gd} \zeta  \tag{1.14}\\
& G_{12}=\int \operatorname{G} \zeta d \zeta \\
& G_{13}=\int G \zeta^{2} d \zeta
\end{align*}
$$

and thermal loads are

$$
\begin{align*}
& N_{\xi}^{\mathrm{T}}=N_{\theta}^{\mathrm{T}}=\int \frac{\alpha(1+v)}{1-v^{2}} \operatorname{ETU} \zeta  \tag{1.15}\\
& M_{\xi}^{\mathrm{T}}=M_{\theta}^{\mathrm{T}}=\int \frac{\alpha(1+v)}{1-v^{2}} \operatorname{ET} \zeta \mathrm{~d} \zeta
\end{align*}
$$

For the case of discontinuous material properties through the thickness the integration is taken piece-wise through the thickness. An additional assumption of constant Poisson's ratios through the thickness is made. Therefore the isotropic nature of the layers are retained in the stiffness parameters. The shell stiffness and thermal loads take the form,

$$
\begin{align*}
& \mathrm{B}_{1}=\mathrm{B}_{2}=\frac{\mathrm{B}_{3}}{v}=\frac{1}{1-v^{2}} \int \mathrm{Ed}, \mathrm{D}_{1}=\mathrm{D}_{2}=\frac{\mathrm{D}_{3}}{v}=\frac{1}{1-v^{2}} \int \mathrm{E} \zeta^{2} \mathrm{~d} \zeta \\
& C_{1}=C_{2}=\frac{C_{3}}{v}=\frac{1}{1-v^{2}} \int E \zeta d \zeta, G_{1}=G_{2}=G_{3}=B_{1} \frac{(1-v)}{2}  \tag{1.16}\\
& G_{12}=C_{1} \frac{(1-v)}{2}, G_{13}=D_{1} \frac{(1-v)}{2}
\end{align*}
$$

It follows that because of the reference surface choice, namely that the condition $\int E \zeta d \zeta=0$ is satisfied, the integration takes the form

$$
\begin{equation*}
A=\int F(\zeta) d \zeta=\sum_{j} \int_{0}^{h_{j}} F_{j}\left(\zeta_{j}\right) d \zeta_{j} \tag{1.17}
\end{equation*}
$$



The reference surface strains and bending distortion terms may be. defined in terms of displacements and rotations by the following expressions. The membrane strains of the reference surface are given by

$$
\begin{align*}
& \epsilon_{\xi}=\frac{1}{a}\left[\frac{\partial U_{\xi}}{\partial \xi}+\omega_{\xi} W\right] \\
& \epsilon_{\theta}=\frac{1}{a}\left[\frac{1}{\rho} \frac{\partial U_{\theta}}{\partial \theta}+\gamma U_{\xi}+\omega_{\theta} W\right]  \tag{1.18}\\
& \epsilon_{\xi \theta}=\frac{1}{2 a}\left[\frac{1}{\rho} \frac{\partial U_{\xi}}{\partial \theta}+\frac{\partial U_{\theta}}{\partial \xi}-\gamma U_{\theta}\right]
\end{align*}
$$

where $U, V, W$ are displacements in the $\xi, \theta$ and $\zeta$ directions respectively. Transverse shear strains are given by

$$
\begin{align*}
& \gamma_{\xi \zeta}=\Phi_{\xi}-\frac{1}{a}\left[\omega_{\xi} U_{\xi}-\frac{\partial W}{\partial \xi}\right]  \tag{1.19}\\
& \gamma_{\theta \zeta}=\Phi_{\theta}-\frac{1}{a}\left[\omega_{\theta} U_{\theta}-\frac{1}{\rho} \frac{\partial W}{\partial \theta}\right]
\end{align*}
$$

where $\Phi_{\xi}, \Phi_{\theta}$ are rotations.

The bending distorion terms are given by

$$
\begin{align*}
& \mathrm{k}_{\xi}=\frac{1}{\mathrm{a}} \frac{\partial \Phi_{\xi}}{\partial \xi} \\
& \mathrm{k}_{\theta}=\frac{1}{\mathrm{a}}\left[\frac{1}{\rho} \frac{\partial \Phi_{\theta}}{\partial \theta}+\gamma \Phi \xi\right]  \tag{1.20}\\
& \overline{\mathrm{k}}_{\xi \theta}=\frac{1}{2 \mathrm{a}}\left[\frac{1}{\rho} \frac{\partial \Phi \xi}{\partial \theta}+\frac{\partial \Phi_{\theta}}{\partial \xi}-\gamma \Phi_{\theta}+\frac{1}{2 \mathrm{a}}\left(\omega_{\xi}-\omega_{\theta}\right)\left(\frac{1}{\rho} \frac{\partial U_{\xi}}{\partial \theta^{\prime}}-\frac{\partial U_{\theta}}{\partial \xi}-\gamma U_{\theta}\right]\right.
\end{align*}
$$

Substituting equations (1.18, 1.19, 1.20) into equation 1.13 the force terms in the equations of motion can be expressed in terms of the displacements

$$
\begin{align*}
& N_{\xi}=\frac{1}{a}\left\{B_{1} \frac{\partial U_{\xi}}{\partial \xi}+B_{3} \gamma U_{\xi}+B_{3}\left(\frac{1}{\rho}\right) \frac{\partial U_{\theta}}{\partial \theta}+\left(B_{1} \omega_{\xi}+B_{2} \omega_{\theta}\right) W\right\}-N^{T} \\
& N_{\theta}=\frac{1}{a}\left\{B_{3} \frac{\partial U_{\xi}}{\partial \xi}+B_{2} \gamma U_{\xi}+B_{2}\left(\frac{1}{\rho}\right) \frac{\partial U_{\theta}}{\partial \theta}+\left(B_{3} \omega_{\xi}+B_{2} \omega_{\theta}\right) W\right\}-N^{T} \\
& N_{\xi \theta}=\frac{1}{2 a}\left\{\frac{1}{\rho} G_{1} \frac{\partial U_{\xi}}{\partial \theta}+G_{1} \frac{\partial U_{\theta}}{\partial \xi}-\gamma G_{1} U_{\theta}\right\} \\
& Q_{\xi}=G_{2}\left\{-\frac{1}{a} \omega_{\xi} U_{\xi}+\frac{1}{a} \frac{\partial W}{\partial \xi}+\Phi_{\xi}\right\}  \tag{1.21}\\
& M_{\xi}=\frac{1}{a}\left\{D_{1} \frac{\partial \Phi \xi}{\partial \xi}+D_{3} \gamma \Phi_{\xi}+D_{3} \frac{1}{\rho} \frac{\partial \Phi_{\theta}}{\partial \theta}\right\}-M^{T} \\
& M_{\theta}=\frac{1}{a}\left\{D_{3} \frac{\partial \Phi_{\xi}}{\partial \xi}+D_{2} \gamma \Phi_{\xi}+D_{2} \frac{1}{\rho} \frac{\partial \Phi_{\theta}}{\partial \theta}\right\}-M^{T} \\
& \bar{M}_{\xi \theta}=\frac{1}{2 a}\left\{\frac{1}{2 a} G_{13}\left(\omega_{\xi}-\omega_{\theta}\right)\left(\frac{1}{\rho} \frac{\partial U_{\xi}}{\partial \theta}-\frac{\partial U_{\theta}}{\partial \xi}-\gamma U_{\theta}\right)+G_{13}\left(\frac{1}{\rho} \frac{\partial \Phi \xi}{\partial \theta}+\frac{1 \Phi \theta}{\partial \xi}-\gamma \Phi_{0}\right)\right\} \\
& Q_{\theta}=G_{3}\left\{-\frac{1}{a} \sim_{\theta} U_{\theta}+\frac{1}{a \rho} \frac{\partial W}{\partial \theta}+\Phi_{\theta}\right\}
\end{align*}
$$

By employing the relations, equations 1.21 the equations of motion can be expressed in terms of the dependent variables, displacoments and rotations.

### 1.7 CIRCUMFERENTIAL VARIABLE SEPARATION

The analysis utilizes a Fourier approach which will permit separation of variables and yie. a more tractable set of shell equations. The procedure involves expanding of the pertinent variables in Fourier series with appropriate normalization to provide nondimensional Fourier coefficients of roughi; comparable magnitudes for different variables. Letiri, $\sigma_{0}, E_{0}$, $h_{0}$ be a reference stress level, Young's modulus, and thickness, respectively, solutions for the field equations are sought in the form

$$
\begin{align*}
& U_{\xi}=\frac{a \sigma_{0}}{E_{o}} \sum_{n=0}^{\infty} u_{n}(\xi, t) \operatorname{Cos} n \theta \\
& U_{\theta}=\frac{a \sigma_{0}}{E_{0}} \sum_{n=1}^{\infty} v_{n}(\xi, t) \operatorname{Sin} n \theta \\
& W=\frac{a \sigma_{o}}{E_{0}} \sum_{n=0}^{\infty} w_{n}(\xi, t) \operatorname{Cos} n \theta  \tag{1.22}\\
& \Phi_{\xi}=\frac{\sigma_{0}}{E_{o}} \sum_{n=0}^{\infty}{ }^{\phi_{\xi_{n}}}(\xi, t) \operatorname{Cos} n \theta \\
& \Phi_{\theta}=\frac{\sigma_{0}}{E_{0}} \sum_{n=1}^{\infty}{ }^{\phi_{\theta_{n}}}(\xi, t) \operatorname{Sin} n \theta
\end{align*}
$$

These Fourier expansions are consistent with loadings : ! Sorm

$$
\begin{align*}
& q_{\xi}=\frac{\sigma_{o} h_{o}}{a} \sum_{n=0}^{\infty} p_{\xi n}(\xi, t) \operatorname{Cos} n \theta \\
& q_{\theta}=\frac{\sigma_{0} h_{o}}{a} \sum_{n=1}^{\infty} p_{\theta n}(\xi, t) \operatorname{Sin} n \theta  \tag{1.23}\\
& q_{\zeta}=\frac{\sigma_{o} h_{o}}{a} \sum_{n=0}^{\infty} p_{\zeta n}(\xi, t) \operatorname{Cos} n \theta
\end{align*}
$$

The above Fourier expansions are not the must general form. The expansions $q_{\zeta}, q_{\xi}$ are symmetrical expansions about $\theta=0$. For fu' generality, they must be augmented by additional sine series exparioions. The form $q_{\theta}$ in turn would be supplemerited by cosine series. Consistently, the series expansions for displacements and rotations must be augmented for the general case.

Expansions for the temperature distributions may be described in 2 similar manner; however, since the thermal coefficients and Young's modulus can vary in the circumferential direction, it will be more convenient to expand the thermal load in Fourier series as follows

$$
\begin{align*}
& N^{T}=\sigma_{0} h_{0} \sum_{n=0}^{\infty} t_{n}^{T} \operatorname{Cos} n \theta  \tag{1.24}\\
& M^{T}=\frac{\sigma_{0} h_{0}^{3}}{a} \sum_{n=0}^{\infty} m_{n}^{T} \operatorname{Cos} n \theta
\end{align*}
$$

Where the Fourier co. Iponents $t_{n}^{T}$ and $m_{n}^{T}$ are given by

$$
\begin{aligned}
& t_{n}^{T}=\frac{2}{\pi} \int_{0}^{\pi} \frac{N^{T}}{\sigma_{0} h_{0}} \operatorname{Cos} n \theta d \theta \\
& m_{n}^{T}=\frac{2}{\pi} \int_{0}^{\pi} \frac{2 M^{T}}{\sigma_{0} h_{0}^{3}} \operatorname{Cos} n \theta d \theta
\end{aligned}
$$

Since the stiffness mass, external damping and elastic foundation parameters are variable in the circumferential directions, these will also be expanded in a Fourier series. For example, the expansion for the extensional stiffness parameter is of the form

$$
\begin{equation*}
B=\sum_{j=0}^{\infty} b_{j} \operatorname{Cos} j \theta+\sum_{j=1}^{\infty} b_{j} \operatorname{Sin} j \theta \tag{1.25}
\end{equation*}
$$

In many problems, there exists a plane of symmetry with respect to planform geometry. See Figure 1-3.


A plane of symmetry is assumed here. The coefficients of B, viz., $b_{j}$ are found by integrations of the form

$$
\begin{equation*}
b_{j}=\frac{2}{\pi} \int_{0}^{\pi} B(\xi, \theta) \operatorname{Cos} j \theta d \theta \tag{1.26}
\end{equation*}
$$

The Fourier expansions of the shell stiffness parameters (Eqs. 1.14), consistent with previous formulation are given by

$$
\begin{array}{cc}
B_{m}=E_{o} h_{o} \sum_{j=0}^{\infty} b_{m j}(\xi) \operatorname{Cos} j \theta & \text { Inplane Stiffness } \\
D_{m}=E_{o} h_{o}^{3} \sum_{j=0}^{\infty} d_{m j}(\xi) \operatorname{Cos} j \theta & \text { Bending Stiffness } \\
C_{m}=\sum_{j=0}^{\infty} c_{m j}(\xi) \operatorname{Cos} j \theta & \text { External Damping }
\end{array}
$$

$$
\begin{array}{lll}
\mathrm{K}_{\mathrm{m}}= & \sum_{\mathrm{j} \cdot 0}^{\infty} k_{\mathrm{mj}}(\xi) \operatorname{Cos} \mathrm{j} \theta & \text { Elastic: Founclation } \\
\mathrm{M}_{\mathrm{m}}= & \sum_{j=0}^{\infty} \mathrm{m}_{\mathrm{mj}}(\xi) \operatorname{Cos} \mathrm{j} \theta & \text { Mass Property }
\end{array}
$$

Substitution of the displacement and rotation series expansions (Eqs. 1.22) and the above stiffness expansions into Eqs. (1.21) and employing the proper trigonometric identities yields the following series expressions relating forces (moments) in terms of the Fourier coefficients of the displacement variables and stiffness parameters:

$$
\begin{align*}
& N_{\xi}=\sigma_{o} h_{0} \sum_{k=0}^{\infty}\left[\sum_{j=0}^{\infty}\left\{B_{1}^{k j}\left(\frac{\partial u_{j}}{\partial \xi}+w_{\xi} w_{j}\right)+B_{3}^{k j}\left(\gamma_{j}+\frac{k}{f} v_{j}+\omega_{\epsilon} w_{j}\right)\right\}-\right. \\
& \left.t_{k}^{T}\right] \operatorname{Cos} k \theta \\
& N_{\theta}=\sigma_{o} h_{o} \sum_{k=0}^{\infty}\left[\sum_{j=0}^{\infty}\left\{B_{3}^{k j}\left(\frac{\partial u_{j}}{\partial \xi}+\omega_{\xi} w_{j}\right)+B_{2}^{k j}\left(Y_{j}+\frac{k}{\rho} v_{j}+\omega w_{j}\right)\right\}-\right. \\
& \left.t_{k}^{T}\right] \operatorname{Cos} k \theta \\
& \bar{N}_{\xi \theta}=\sigma_{o} h_{o} \sum_{k=1}^{\infty}\left[\sum_{j=0}^{\infty} \frac{G_{l}^{k j}}{2}\left(\frac{\partial v_{j}}{\partial \xi}-\frac{k}{\rho} u_{j}-Y_{v_{j}}\right)\right] \operatorname{Sin} k \theta  \tag{1.28}\\
& Q_{\xi}=\sigma_{0} h_{0} \sum_{k=0}^{\infty}\left[\sum_{j=0}^{\infty} G_{2}^{k j}\left(-\omega_{\xi} u_{j}+\frac{\partial w_{j}}{\partial \xi}+\phi_{\xi j}\right)\right] \operatorname{Cos} k 0 \\
& M_{\xi}=\frac{\sigma_{0} h_{0}^{3}}{a} \sum_{k=0}^{\infty}\left[\sum_{j=0}^{\infty}\left\{D_{1}^{k j} \frac{\partial \phi_{\xi j}}{\partial \xi}+D_{3}^{k j}\left(\gamma_{\xi j} \cdot \frac{k}{\rho} \Phi_{0 j}\right)\right\}-m T\right] \cos k 0 \\
& M_{\theta}=\frac{\sigma_{0} h_{0}^{3}}{a} \sum_{k=0}^{\infty}\left[\sum_{j=0}^{\infty}\left\{D_{3}^{k j} \frac{\partial \phi_{\xi} j}{\partial \xi}+D_{2}^{k j}\left(\gamma_{\xi j}: \frac{k}{\rho} \phi_{\theta j}\right)\right\}-m^{T}\right] \operatorname{Cos} k 0
\end{align*}
$$

$$
\begin{align*}
\bar{M}_{\xi \theta}= & \frac{\sigma_{o} h_{o}^{3}}{a} \sum_{k=1}^{\infty}\left[\sum_{j=0}^{\infty} \frac{G_{13}}{2}\left(-\frac{k}{\rho} \phi_{\xi_{j}}+\frac{\partial \phi_{\theta_{j}}}{\partial \xi}-{ }^{\gamma} \phi_{\theta_{j}}+\frac{1}{2}\left(\omega_{\xi}-\omega_{\theta}\right)\right)\right. \\
& \left.\left(-\frac{k}{\rho} u_{j}-\frac{\partial v_{j}}{\partial \xi}-\gamma_{v_{j}}\right)\right] \operatorname{Sin} k \theta  \tag{1,28}\\
Q_{\theta}= & \sigma_{o} h_{o} \sum_{n=1}^{\infty}\left[\sum_{j=0}^{\infty}\left\{c_{3}^{k j}\left(-\omega_{\theta v_{j}}-\frac{k}{\rho} w_{j}+\phi_{\theta_{j}}\right)\right\}\right] \operatorname{Sin} k \theta
\end{align*}
$$

The translational inertial force terms are expressed as

$$
\begin{align*}
& M_{\xi} \ddot{U}=\sum_{k=0}^{\infty}\left[\sum_{j=0}^{\infty} M_{l}^{k j} \ddot{u}_{j}\right] \operatorname{Cos} k \theta \\
& M_{\theta} \ddot{\mathrm{V}}=\sum_{k=1}^{\infty}\left[\sum_{j=0}^{\infty} M_{2}^{k j} \ddot{v}_{j}\right] \operatorname{Sin} k \theta  \tag{1.29}\\
& M_{\zeta} \ddot{W}=\sum_{k=0}^{\infty}\left[\sum_{j=0}^{\infty} M_{3}^{k j} \ddot{w}_{j}\right] \operatorname{Cos} k \theta
\end{align*}
$$

The translational damping force is expressible as

$$
\begin{align*}
& C_{D_{\xi}} \dot{U}=\sum_{k=0}^{\infty}\left(\sum_{j=0}^{\infty} C_{l}^{k j} \dot{u}_{j}\right) \operatorname{Cos} k \theta \\
& C_{D_{\theta}} \dot{V}=\sum_{k=1}^{\infty}\left(\sum_{j=0}^{\infty} C_{2}^{k j} \dot{v}_{j}\right) \operatorname{Sin} \kappa \theta  \tag{1.30}\\
& C_{D_{\zeta}} \dot{W}=\sum_{k=0}^{\infty}\left(\sum_{j=0}^{\infty} C_{3}^{k j} \dot{w}_{j}\right) \operatorname{Cos} k \theta
\end{align*}
$$

The translational elastic foundation force is expressible as

$$
\begin{align*}
& K_{\xi} U=\sum_{k=0}^{\infty}\left(\sum_{j=0}^{\infty} k_{1}^{k j} u_{j}\right) \operatorname{Cos} k \theta \\
& K_{\theta} V=\sum_{k=1}^{\infty}\left(\sum_{j=0}^{\infty} k_{2}^{k j} v_{j}\right) \operatorname{Sin} k \theta  \tag{1.31}\\
& K_{\zeta} W=\sum_{k=0}^{\infty}\left(\sum_{j=0}^{\infty} k_{3}^{k j} w_{j}\right) \operatorname{Cos} k \theta
\end{align*}
$$

The stiffness recursion relationships above are described in the form

$$
\begin{align*}
& A^{k j}=\frac{1}{2}\left\{a_{(k+j)}+\left\{1-\delta^{2}(j-n)+\delta(k)\right] a \quad|k-j|\right\} \\
& \bar{A}^{k j}=\frac{1}{2}\left\{-\bar{a}_{(k+j)}+\left[1-\delta^{2}(j-k)+\delta(k)\right] \bar{a}|k-j|\right\} \tag{1.32}
\end{align*}
$$

where the specific coefficients of interest (dropping $\mathrm{k}_{\mathrm{j}}$ superscript) are given by

$$
\begin{aligned}
& \left.A=\mid B_{1}, B_{2}, B_{3}, D_{1}, D_{2}, D_{3}, g_{2}, M_{1}, M_{3}, C_{1}, C_{3}, K_{1}, K_{3}\right\} \\
& a=\left\{b_{1}, b_{2}, b_{3}, d_{1}, d_{2}, d_{3}, g_{2}, m_{1}, m_{3}, c_{1}, c_{3}, k_{1}, k_{3} \mid\right.
\end{aligned}
$$

and

$$
\begin{aligned}
& \bar{A}=\left\{G_{1}, G_{3}, G_{13}, M_{2}, C_{2}, K_{2}\right\} \\
& \bar{a}=\left\{g_{1}, g_{3}, g_{13}, m_{2}, c_{2}, k_{2}\right\}
\end{aligned}
$$

In the above expressions $\delta(\mathrm{m})$ is defined as

$$
\delta(m)=\left\{\begin{array}{l}
-1, m<0 \\
0, m=0 \\
+1, m>0
\end{array}\right.
$$

(*See Appendix 1.2 (multiplication of series expressions) for a more detailed description of $A^{k j}$ and $\bar{A}^{k j}$.)

Substitution of the stress resultant expressions and dynamic force resultants (Eqs. 1:28-1.31) into the shell equilibrium equations (Eqs. 1.6) yields five finite series expressions in the circumferential coordinate relating the Fourier coefficients $u_{j}, v_{j}, w_{j}, \phi_{\xi_{j}}$ and $\phi_{\theta_{j}}$ of the displacement and rotation variables. For practical considerations we truncate the series solution of the dependent variables to K terms in the Fourier component. Employing the appropriate orthogonality relationships of Fourier series to these equilibrium expressions yields a systom of 5 K ordinary differential equations relating the 5 K unknown Fourier coefficients. These equations are presented in a form amenable for computer programming and are given as follows:

$$
\begin{align*}
& \sum_{j=0}^{K-1}\left[f_{5 k+\ell, 5 j+1} \mathbf{u}_{j}^{\prime \prime}+\right. \\
& \mathrm{f}_{5 \mathbf{k}+\ell, 5 \mathbf{j}+2} \mathrm{v}_{\mathbf{j}}^{\prime \prime}+\mathrm{f}_{5 \mathbf{k}+\ell, 5 \mathbf{j}+3} \mathbf{w}_{\mathbf{j}}^{\prime \prime}+ \\
& \mathrm{f}_{5 \mathrm{k}+\ell, 5 \mathrm{j}+4} \phi_{\mathbf{g}_{\mathrm{j}}^{\prime}}+\mathrm{f}_{5 \mathrm{k}+\ell, 5 \mathrm{j}+5}{ }^{\prime}{ }_{\theta_{\mathrm{j}}}^{\prime}+ \\
& \mathrm{g}_{5 \mathbf{k}+\ell, 5 \mathbf{j}+1} \mathrm{u}_{\mathbf{j}}^{f}+\ldots+\mathrm{g}_{5 \mathrm{k}+\ell, 5 \mathbf{j}+5}{ }^{\phi} \theta_{\mathbf{j}}+ \\
& h_{5 k+\ell, 5 j+1} u_{j}+\ldots+h_{5 k+\ell, 5 j+5} \dot{\phi}_{\theta j}+  \tag{1.33}\\
& k_{5 k+\ell, 5 j+1} u_{j}+\ldots+k_{5 k+\ell, 5 j+1}{ }^{\phi_{\theta j}}=\alpha_{5 k+\ell, 5 j+1} \ddot{u}_{j}+\ldots+ \\
& \alpha_{5 \mathrm{k}+\ell, 5 \mathrm{j}+5} \ddot{\phi}_{\boldsymbol{\theta}_{\mathrm{j}}}+ \\
& \beta_{5 k+\ell, 5 j+1} \dot{u}_{j}+\ldots+ \\
& \beta_{5 \mathrm{k}+\ell, 5 \mathrm{j}+5} \dot{\phi}_{\theta_{\mathrm{j}}}+\mathrm{p}_{5 \mathrm{k}+\ell}
\end{align*}
$$

where the $f, g, h, k, \alpha$ and $\beta$ coefficients are described in Appendix l-B. (It should be noted that the form of the above equations is more complicated than was obtained in Reference 2 for analysis of shells of revolution. This complexity arises from the fact that the equilibrium equations cannot be decoupled for each Fourier component of displacement variables for the case oi unsymmetric shell.)

The above equation can be conveniently written in matrix form as follows:

$$
\begin{equation*}
F z^{\prime \prime}+G z^{\prime}+(H+K) z=\alpha \ddot{z}+\beta \dot{z}+p \tag{1.34}
\end{equation*}
$$

where $F, G, H, K, \alpha, \beta$ are square arrays and $z, p$ are column arrays. These arrays are defined as,

$$
\begin{aligned}
& H=\left[\begin{array}{lllll}
{ }^{h_{11}} & h_{12} & \cdot & \cdot & \cdot \\
h_{21} & \cdots & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\bullet & \cdot & \cdot & \cdot & \cdot
\end{array}\right] \\
& K=\left[\begin{array}{lllll}
k_{11}, & k_{12} & \cdot & \cdot & \cdot \\
k_{21} & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
k_{5 k, 1} & \cdot & \cdot & \cdot & \cdot
\end{array}\right]
\end{aligned}
$$

$$
z=\left[\begin{array}{c}
z_{1} \\
z_{2} \\
z_{3} \\
\cdot \\
z_{k}
\end{array}\right] \quad z_{i}=\left[\begin{array}{c}
u_{i} \\
v_{i} \\
w_{i} \\
\phi_{\xi_{i}} \\
\phi_{\theta_{i}}
\end{array}\right] \quad p=\left[\begin{array}{l}
p_{1} \\
p_{2} \\
\cdot \\
\cdot \\
p_{k}
\end{array}\right] \quad p_{i}=\left[\begin{array}{c}
p_{1}^{i} \\
p_{2}^{i} \\
p_{3}^{i} \\
p_{4}^{i} \\
p_{5}^{i}
\end{array}\right]
$$

The elements of the $\mathrm{F}, \mathrm{G}$ and $\mathrm{H}, \mathrm{K}, \alpha, \beta$, and p ratrices are given in Appendix 1.13.1 and are presented there in format which is designed to ease computer programming. The coefficients of $P$ are the Fourier components of the applied external load and are known quantities for a specific loading case.

The more general case of shells having arbitrary distribution of stiffness, loads, damping, and elastic foundation can be considered in the same manner as the case of plane of symmetry of stiffness and loads. In this case the total Fourier series representation of all the variables, displacements, rotations, stiffness and loads, must be carried in the analysis. The analysis will follow the same format of the special case formulated previously.

## Boundary Conditions

Consistent with Sanders' equilibrium equations, the boundary conditions for the specification of the forces or displacements, or constraint between them are described below. On the edge where $\xi=\operatorname{constant}$ (i.e., $\xi=0$, and $\xi=\bar{s})$

$$
\begin{align*}
& \mathrm{N}_{\xi} \text { or } \\
& \mathrm{U}_{\xi} \\
& \hat{N}_{\xi \theta} \text { or }  \tag{1.35}\\
& U_{\theta} \\
& Q_{\xi} \text { or } \\
& W \\
& \mathrm{M}_{\xi} \text { or } \\
& \Phi_{\xi} \\
& \overline{\mathrm{M}}_{\xi \theta} \text { or }
\end{align*} \Phi_{\theta}
$$

where

$$
\hat{\mathbf{N}}_{\xi 0}=\overline{\mathrm{N}}_{\xi \theta}+\frac{1}{2 \mathrm{a}}\left(w_{0}-w_{\xi}\right) \overline{\mathrm{M}}_{\xi \theta}
$$

These conditions can be expressed in matrix form by,

$$
\begin{equation*}
\bar{\Omega} \bar{y}+\bar{\Lambda} \bar{z}=\bar{\ell} \tag{1.36}
\end{equation*}
$$

where $\overline{\mathrm{y}}, \bar{\ell}, \overline{\mathrm{z}}$ are column matrices and $\bar{\Omega}, \bar{\Lambda}$ are appropriate diagonal matrices
$\bar{y}=\left[\begin{array}{c}N_{\xi} \\ \hat{N}_{\xi \theta} \\ Q_{\xi} \\ M_{\xi} \\ M_{\xi \theta}\end{array}\right]$
$\bar{z}=\left[\begin{array}{c}U_{\xi} \\ U_{\theta} \\ W \\ \Phi_{\xi} \\ \Phi_{\theta}\end{array}\right]$
$\bar{\ell}=\left[\begin{array}{l}\ell_{1} \\ \ell_{2} \\ \ell_{3} \\ \ell_{4} \\ \ell_{5}\end{array}\right]$

$$
\bar{\Omega}=\left[\begin{array}{lllll}
\omega_{1} & & & & \\
& \omega_{2} & & 0 & \\
& & \omega_{3} & & \\
& & 0 & & \\
& & & & \omega_{4} \\
& & & & \omega_{5}
\end{array}\right] \quad \bar{\Lambda}=\left[\begin{array}{lllll}
\lambda_{1} & & & & \\
& & & & \\
& & & & \\
& & \lambda_{3} & & \\
& 0 & & \lambda_{4} & \\
& & & & \lambda_{5}
\end{array}\right]
$$

The logic which connects $\bar{\Omega}, \bar{\Lambda}, \bar{\ell}$ and the conditions desired are given in the following table:

Matrix Elements

| Condition at Boundary | $\omega_{i}$ | $\lambda_{1}$ | $\ell_{i}$ | $\begin{aligned} & C_{1}=\text { value of displacement } \\ & C_{2}=\text { value of force } \\ & C_{3}=\text { constant relating force } \\ & \text { and displacement } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| Displacement Prescribed | 0 | 1 | $\mathrm{C}_{1}$ |  |
| Force Prescribed | 1 | 0 | $\mathrm{C}_{2}$ |  |
| Constraint Condition | 1 | $\mathrm{C}_{3}$ | 0 |  |

For example, is $\Phi_{\xi}$ is given as a boundary condition then $\lambda_{4}=1, \omega_{4}=0$ and $\ell_{4}$ is the prescribed value of $\Phi_{\xi}$. Note $C_{i}$ is nondimensionalized with the appropriate reference constants.

It will be convenient to expand forces and moments in Fourier series in manner consistent with the previous developments. Letting

$$
\begin{align*}
N_{\xi} & =\sum t_{\xi}^{n} \operatorname{Cos} n \theta \\
\hat{N}_{\xi \theta} & =\sum \hat{t}_{\xi \theta}^{n} \operatorname{Sin} n \theta \\
Q_{\xi} & =\sum q_{\xi}^{n} \operatorname{Cos} n \theta  \tag{1.37}\\
M_{\xi} & =\sum m_{\xi}^{n} \operatorname{Cos} n \theta \\
\bar{M}_{\xi \theta} & =\sum m_{\xi \theta}^{n} \operatorname{Sin} n \theta
\end{align*}
$$

and

$$
\begin{aligned}
& \ell_{1}=\sum \ell \ell_{1}^{n} \cos n \theta \\
& \ell_{2}=\sum \ell{ }_{2}^{n} \sin n \theta \\
& \ell_{3}=\sum \ell{ }_{3}^{n} \cos n \theta \\
& \ell_{4}=\sum \ell{ }_{4}^{n} \cos n \theta \\
& \ell_{5}=\sum \ell{ }_{5}^{n} \sin n \theta
\end{aligned}
$$

The above series expressions together with Eqs. (1.21) are substituted into Eq. (1.36) and the circumferential variation separated, yielding the fcllowing matrix form for the relationship of the Fourier coefficients.

$$
\begin{equation*}
\Omega y+\Lambda z=\ell \tag{1.38}
\end{equation*}
$$

where

$$
y=\left[\begin{array}{l}
y_{1} \\
y_{2} \\
\cdot \\
\cdot \\
y_{k}
\end{array}\right] \quad y_{i}=\left[\begin{array}{c}
\hat{t}_{\xi}^{i} \\
\hat{t}_{\xi \theta}^{i} \\
q_{\xi}^{i} \\
m_{\xi}^{i} \\
m_{\xi \theta}^{i}
\end{array}\right] \quad z=\left[\begin{array}{c}
z_{1} \\
z_{2} \\
\cdot \\
\cdot \\
z_{k}
\end{array}\right] \quad z_{i}=\left[\begin{array}{c}
u_{i} \\
v_{i} \\
w_{i} \\
\phi_{\xi_{i}} \\
\phi_{\theta_{i}}
\end{array}\right]
$$

$$
\begin{aligned}
& \Omega=\left[\begin{array}{llll}
\bar{\Omega} & & \\
& \bar{\Omega} & & \\
& & & 0 \\
& C & & \\
& & & \bar{\Omega}_{\mathrm{k}}
\end{array}\right] \\
& \Lambda=\left[\begin{array}{llll}
\bar{\lambda} & & \\
\bar{\Lambda} & 0 & \\
& & \\
& 0 & & \\
& & \bar{\Lambda}_{k}
\end{array}\right] \\
& \ell=\left[\begin{array}{l}
\bar{\ell}_{1} \\
\bar{\ell}_{2} \\
. \\
. \\
\bar{\ell}_{k}
\end{array}\right] \\
& \ell_{\mathrm{i}}=\left[\begin{array}{c}
?_{1 \mathrm{i}} \\
\ell_{2 \mathrm{i}} \\
\ell_{3 \mathrm{i}} \\
\ell_{4 \mathrm{i}} \\
\ell_{5 \mathrm{i}}
\end{array}\right]
\end{aligned}
$$

It will be desirable to express boundary conditions in terms of $z$. The substitution of Eqs. (1.37) in Eq. (1.27) with appropriate orthogonality orerations yields a set of recursion expressions relating Fourier cofficients of forces and moments to the displacement and stiffness coefficionts. These relationships are given by

$$
\begin{align*}
& \mathrm{t}_{\xi}^{\mathrm{k}}=\sum_{\mathrm{j}=0}^{\mathrm{K}-1}\left[\mathrm{r}_{5 \mathrm{k}+1,5 \mathrm{j}+1} \mathrm{u}_{\mathrm{j}}^{\prime}+\mathrm{r}_{5 \mathrm{k}+1,5 \mathrm{j}+4} \phi_{\xi, \mathrm{j}}^{\prime}+\mathbf{s}_{5 k+1,5 \mathbf{j}+1} \mathbf{u}_{\mathrm{j}}\right. \\
& +s_{5 k+1,5 j+2} v_{j}+s_{5 k+1,5 j+3} w_{j}+s_{5 k+1,5 j}{ }^{\prime} \Psi^{\$}{ }^{\prime}{ }_{j} \\
& \left.+s_{5 k+1,} 5 j+5{ }^{\phi} \theta_{j}\right]+a_{5 k+1} \tag{1.30}
\end{align*}
$$

$$
\begin{aligned}
q_{\xi}^{k}= & \sum_{j=0}^{K-1}\left[r_{5 k+3,5 j+2} w_{j}^{\prime}+s_{5 k+3,5 j+1} u_{j}+s_{5 k+3,5 j+4} \phi_{\xi j}\right] \\
m_{\xi}^{k}= & \sum_{j=0}^{K-1}\left[r_{5 k+4,5 j+1} u_{j}^{\prime}+r_{5 k+4,5 j+4} \phi_{\xi_{j}}^{\prime}+s_{5 k+4,5_{j+1}} u_{j}\right. \\
& +s_{5 k+4,5 j+2} v_{j}+s_{5 k+4,5 j+3} w_{j}+s_{5 k+4,5 j+4} \phi_{\xi_{j}} \\
& \left.+s_{5 k+4,5 j+5} \phi_{\theta j}\right]+a_{-5}{ }_{5 k+4} \\
m_{\xi \theta}^{k}= & \sum_{j=0}^{K-1}\left[r_{5 k+5,5 j+2} v_{j}^{\prime}+r_{5 k+5,5 j+5} \phi_{\theta_{j}}^{\prime}+s_{5 k+5,{ }^{\prime} 5 j+1} u_{j}\right. \\
& \left.+s_{5 k+5,5 j+2} v_{j}+s_{5 k+5,5 j+4} \phi_{\xi j}+s_{5 k+5,5 j+5} \phi_{\theta_{j}}\right]
\end{aligned}
$$

where

$$
k=0,1,2, \ldots k-1
$$

Coefficients are given in Appendix 1-3.
Equation 1.39 can be written in matrix notation as

$$
\begin{equation*}
y=R z^{\prime}+S z+a \tag{1.40}
\end{equation*}
$$

Hence, the boundary conditions (1.38) become

$$
\begin{equation*}
\Omega R_{z}^{\prime}+(\Lambda+\Omega 2 s) z=\ell-\Omega a \tag{1.41}
\end{equation*}
$$

The form of Eq. (1.41) is modified if the shell nas a pole (i.e., $r=0$ ) because th coefficients of the differential equations become singular for this case. Following a similar limiting prceess as described by Creenbaum (Reference 5) the conditions supplied at the pole are:

For Fourier index $=0$

$$
\begin{equation*}
u_{0}=v_{0}=\phi \xi_{0}=\phi \varepsilon_{0}=q_{\xi_{0}}=0 \tag{1.42}
\end{equation*}
$$

For Fourser index $=1$

$$
\begin{equation*}
u_{1}+v_{1}=w_{1}=\phi_{\xi_{1}}+\phi_{\theta_{1}}=t_{\xi_{1}}-\hat{t}_{\xi \theta_{1}}=m_{\xi_{1}}-m_{\xi \theta_{1}}=0 \tag{1.42}
\end{equation*}
$$

For Fourier index $\geq 2$

$$
u_{i}=v_{i}=w_{i}=\phi_{\xi_{i}}=\phi_{\theta}=0
$$

### 1.8 FINITE DIFFERENCE FORMULATION IN THE MERIDIONAL VARIABLE

In a manner similar to that described in Reference 2 the partial differential equation in the matrix furm (Equation 1.34 ) is reduced by a system of finite difference approximations. The variation in the meridional coordinate of the Fourier coefficients are described point-wise in Eq. (1. 34). The following are finite difference forms for the partial differentials in the meridional coordinate at interior points.

$$
\begin{align*}
\frac{\partial^{2} f}{\partial \xi^{2}} & =\frac{1}{\Delta^{2}}\left(f_{i+1}-2 f_{i}+f_{i-1}\right) \\
\frac{\partial f}{\partial \xi} & =\frac{1}{2 \Delta}\left(f_{i+1}-f_{i-1}\right) \tag{1.43}
\end{align*}
$$

where $\Delta$ is the increment along $\zeta$ and subscripts denote the discrete value of the function taken.

The forms at boundary points (initial)

$$
\begin{equation*}
\left(\frac{\partial f}{\partial \xi}\right)_{1}=\frac{1}{2 \Delta}\left(3 f_{1}-4 f_{2}+f_{3}\right) \tag{1.44}
\end{equation*}
$$

terminal

$$
\begin{equation*}
\left(\frac{\partial f}{\partial \xi}\right)_{N}=\frac{1}{2 \Delta}\left(-f_{N-2}+4 f_{N-1}-3 f_{N}\right) \tag{1.45}
\end{equation*}
$$

The result of the application of the various finite differcence forms can be stated compactly as the following set of equations:

$$
\begin{align*}
A_{0} z_{2}+B_{0} z_{1}+C_{o} z_{0} & =g_{0} \\
A_{i} z_{i+1}+B_{i} z_{i}+C_{i} z_{i-1} & =g_{i}+2 \Delta\left(\alpha_{i} \ddot{z}_{i}+\beta_{i} \dot{z}_{i}\right)  \tag{1.46}\\
A_{N} z_{N}+B_{N} z_{N-1}+C_{N} z_{N-2} & =g_{N}
\end{align*}
$$

Where

$$
\begin{aligned}
& A_{0}=-\Omega_{0} R_{0} \\
& B_{0}=2 \frac{\Omega 0}{\Delta} R_{0} \\
& C_{0}=\Lambda_{0}+\Omega_{0} S_{0}-3 \frac{\Omega 0}{2 \Delta} R_{0} \\
& g_{0}=\ell_{0}-\Omega_{0} a_{0}
\end{aligned}
$$

the subscript ( 0 ) refers to the conditions at $\mathrm{s}=0$.
For $\mathrm{i} \neq \mathrm{o}, \mathrm{N}$

$$
\begin{align*}
& A_{i}=\frac{2 F_{i}}{\Delta}+G_{i} \\
& B_{i}=\frac{4 F_{i}}{\Delta}+2 \Delta\left(H_{i}+K_{i}\right) \\
& C_{i}=\frac{2 F_{i}}{\Delta}-G_{i}  \tag{1.47}\\
& g_{i}=2 \Delta p_{i}
\end{align*}
$$

Finally for $\mathrm{i}=\mathrm{N}$ or conditions at $\mathbf{s}=\overline{\mathbf{s}}$

$$
\begin{align*}
& A_{N}=\Lambda_{N}+s 2_{N} S_{N}+3 \frac{\Omega_{N}}{2 \Delta} R_{N} \\
& B_{N}=-2 \frac{\Omega_{N}}{\Delta} R_{N}  \tag{1.48}\\
& C_{N}=\frac{\Omega_{N}}{2 \Delta} R_{N} \\
& S_{N}-R_{N}-\Omega_{N} N_{N}
\end{align*}
$$

### 1.9 FINITE DIFFERENCE FORMULATION IN THE TIME VARIABLE

By the use of ilifference equations the above differential equation in matrix form may be transformed into a sel of algebraic equations involving the variable $z_{i}$ at successive values of time.

The most commonly used are the central difference forms; however, from a numerical stability aspect, the difference forms of Houbolt 3, 4 are used. These forms are

$$
\begin{align*}
& \ddot{z}=\frac{2 z_{j}-5 z_{j-1}+4 z_{j-2}-z_{j-3}}{\delta^{2}}  \tag{1.49}\\
& \dot{z}=\frac{11 z_{j}-18 z_{j-1}+9 z_{j-2}-2 z_{j-3}}{6 \delta} \tag{1.50}
\end{align*}
$$

Where the subscript j refers to the time interval $\mathrm{j}=0,1,2, \ldots$ and $\dot{6}$ is the time inrrement.

Introducing these expressions in Eq. 1.46 results in the following set of algebraic equations for the shell responsi problem.

$$
\begin{align*}
& A_{0} z_{2, j}+B_{0} z_{1, j}+C_{0} z_{0, j}=g_{0} \\
& A_{i}^{*} z_{i+1, j}+B_{i, j}^{*} z_{i, j}+C_{i}^{*} z_{i-1, j}=g_{i, j}^{*}  \tag{1.51}\\
& A_{N} z_{N, j}+B_{N}^{N z} N-1, j
\end{align*}+C_{N} z_{N-2, j}=g_{N}, ~ l
$$

where

$$
\begin{aligned}
A_{i}^{*} & =\delta A_{i} \\
B_{i, j}^{*} & =\delta B_{i}+4 \frac{\Delta}{\delta} \alpha_{i}+\frac{11}{3} \Delta \beta_{i} \\
C_{i}^{*} & =\delta C_{i} \\
g_{i, j}^{*} & =\delta g_{i}+L_{i} z_{i, j-1}+M_{i} z_{i, j-2} \mid N_{i} \not_{i, j-j}
\end{aligned}
$$

and

$$
\begin{aligned}
& L_{i}=10 \frac{\Delta}{i} \alpha_{i}+6 \Delta \beta_{i} \\
& M_{i}=-8 \frac{\Delta}{i} \alpha_{i}-3 \Delta \beta_{i} \\
& N_{i}=2 \frac{\Delta}{i} \alpha_{i}+\frac{2}{3} \Delta \beta_{i}
\end{aligned}
$$

In the real problem no values of $z_{i}$ exists for less.than zero. The assumption that $z_{i}$ does exist before $t=0$ is a means of allowing the recurrence from Eq. (1.51) to apply at the origin as well as later values of time. Furthermore, no violation is made as long as the initial conditions of $t=o$ are satisfied.

To obtain values for fictitious terms $j=-1,-2$ a procedure similar to that described by Houbolt is used. The procedure will require a modification of $B_{i, j}^{*}, g_{i, j}^{*}$ for $j=1,2$.

The difference equations for the first and second derivatives at the third increment of four successive increments are given by

$$
\begin{align*}
& \ddot{z}_{i, j} \cdot \frac{1}{2}\left(z_{i, j+1}-2 z_{i, j}+z_{i, j-1}\right) \\
& \dot{z}_{i, j}=\frac{1}{6 \delta}\left(2 z_{i, j+1} 3 z_{i, j}-6 z_{i, j-\downarrow} z_{i, j-2}\right) \tag{1.52}
\end{align*}
$$

Applying the equations at $t=0$, i.e., $j=0$

$$
\begin{align*}
& \ddot{z}_{i, 0}=\frac{1}{\delta^{2}}=\left(z_{i, 1}-2 z_{i, 0}+z_{i,-1}\right) \\
& \dot{z}_{i, 0}=\frac{1}{6 \delta}\left(2 z_{i, 1}+3 z_{i, 0}-6 z_{i,-1}+z_{i,-2}\right) \tag{1.53}
\end{align*}
$$

The initial conditions are that the displacements and velocities are prescribed at $t=0$. By application of Newton's second law, a secondary initial condition can be established, i.e., acceleration immediately following application of the initial forces. These conditions are

$$
\begin{align*}
& z_{i, o}=d_{i, o} \\
& \dot{z}_{i, o}=v_{i, o}  \tag{1.54}\\
& \ddot{z}_{i, o}=a_{i, o}
\end{align*}
$$

Where $d_{i, o}, v_{i, o}, a_{i, o}$ are column $n$ matrices formed of the respective coefficients of the Fourier expansions on $\theta$ of the initial displacements, velocities, and accelerations at the meridional location $i$.

Substitution of these values into Eq. (1.53) yields the following relations

$$
\begin{align*}
z_{i, 0} & =d_{i, 0} \\
z_{i,-1} & =\delta^{2} a_{i, 0}+2 d_{i, 0}-z_{i, 1}  \tag{1.55}\\
z_{i,-2} & =6 \delta^{2} a_{i, 0}+6 \delta v_{i, 0}+a d_{i, 0}-8 z_{i, 1}
\end{align*}
$$

Substitution of these relations in Eq. (1.50) for $\mathbf{j}=1$ yields the following change in the definitions of Eq. (1.52)

$$
\begin{align*}
\mathrm{B}_{\mathrm{i}, 1}^{*}= & \delta \mathrm{B}_{\mathrm{i}}+12 \frac{\Delta}{\delta} \alpha_{\mathrm{i}}+6 \Delta \beta_{\mathrm{i}} \\
\mathrm{~g}_{\mathrm{i}, 1}^{*}= & \delta \mathrm{g}_{\mathrm{i}}+\left(12 \frac{\Delta}{\delta} \alpha_{i}+6 \Delta \beta_{i}\right) \mathrm{d}_{\mathrm{i}, 0}+\left(4 \Delta \delta \alpha_{i}-\frac{7}{3} \delta^{2} \Delta \beta_{i}\right) a_{i, 0}+  \tag{1.56}\\
& \left(12 \Delta \alpha_{i}+4 \Delta \delta \beta_{i}\right) \mathrm{v}_{\mathrm{i}, \mathrm{o}}
\end{align*}
$$

Substitution of definitions Eq. (1.50) for $j=2$ yields the following change in definitions of Eq. (1.52)

$$
\begin{align*}
g_{i, 2}^{*}= & \delta g_{i}+\left(8 \frac{\Delta}{\delta} \alpha_{i}+\frac{16}{3} \beta_{i}\right) z_{i, 1}+\left(-4 \frac{\Delta}{\delta} \alpha_{i}-\frac{5}{3} \Delta \beta_{i}\right) d_{i, 0}+  \tag{1.57}\\
& \left(2 \frac{\Delta}{\delta} \alpha_{i}+\frac{2}{3} \Delta \beta_{i}\right) \delta^{2} a_{i, 0}
\end{align*}
$$

The set of Eq. (1.52) and the additional definitions at the first two time intervals Eqs. (1.56, 1.57) is now the algebraic statement of the dynamic response problem.

### 1.10 MATRIX SOLUTION OF THE DIFFERENCE EQUATIONS

The set of matrix equations defined in Eqs. 1.51, 1.56, 1.57 will be solved by the same procedure described in Reference 2. This procedure is essentially a Gaussian elimination performed on the partitioned arrays. A slight modification of the elimination procedure described in Reference 6 is used here. Considering the first and second equations of Eq. (1.52) at the $j^{\text {th }}$ time interval

$$
\begin{align*}
& A_{o} z_{2, j}+B_{o} z_{1, j}+C_{o} z_{o, j}=g_{o}  \tag{1.58}\\
& A_{1} z_{2, j}+B_{1}^{*} z_{1, j}+C_{1} z_{o, j}=g_{1, j}^{*}
\end{align*}
$$

PAGE 3 AND $\frac{32}{31}$ ARE MISSING FROM ORIGINAL DOCUMENT.

## F FFO NOF FAENED.

The matrix solution of these difference equations are solved by the prc edure described in Section l. 10. By defining the starred quantities, ( ) *, without the star the numerical procedure for the static analysis is completely defined.

### 1.12 REFERENCES

1. Sanders, J. Lyell, Jr., 'An Improved First-Approximation Theory for Thin Shells," NASA TR R-24, 1959.
2. Budiansky and Radkowski, 'Numerical Analysis of Unsymmetrical Bending of Shells of Revolution, ' AIAA Journal, Vol. 1, No. 8, August 1963.
3. Johnson, D. E., and Greif, R., "Dynamic Response of a Cylindrical Shell; Two Numerical Methods," AIAA J. 4, 486-494 (1966).
4. Houbolt, J. C., "A Recurrence Matrix Solution for the Dynamic Response of Aircraft in Gusts," N.ł1.A TN 206G, January 1950.
5. Greenbaum, G.A., "Comments on 'Numerieal snalysis of Unsymmetrical Bending of Shells of Revolution, "' AIAA Journal, Vol. 2, No. 3 (Niarch 1964), pp 590, 591.
6. Potters, M. L. "A Matrix Method for the Solution of a Second Order Difference Equation in Two Variables, " Mathematch Centrum, Amsterdam, Holland, Report MR 19 (1955).

### 1.13 APPENDIX

1.13.1 Modification of Sander's Equations

Virtual change in the strain energy within $C$

$$
\begin{gather*}
\overline{\mathrm{U}}=\iint\left(\mathrm{N}_{11} \delta \epsilon_{11}+\mathrm{N}_{12}{ }^{\delta \epsilon} 12+\mathrm{N}_{21} \delta \epsilon_{21}+\mathrm{N}_{22} \delta \epsilon_{22}+\mathrm{M}_{11} \delta \mathrm{k}_{11}+\right. \\
\left.\mathrm{M}_{12^{\delta i k_{12}}+\mathrm{M}_{21} \delta \mathrm{k}_{21}+}+\mathrm{M}_{22} \delta \mathrm{k}_{22}+\mathrm{Q}_{1} \delta \gamma_{1}+Q_{2} \delta \gamma_{2}\right) \mathrm{ds}  \tag{1}\\
\epsilon_{12}=\epsilon_{21}  \tag{2}\\
\mathrm{k}_{12}-\mathrm{k}_{21}=\left(\frac{1}{\mathrm{R}_{2}}-\frac{1}{R_{1}}\right) \epsilon_{12}+\frac{1}{\alpha_{1} \alpha_{2}} \partial \frac{\alpha_{2} \gamma_{2}}{\partial \underline{\xi}_{1}}-\frac{1}{\alpha_{1} \alpha_{2}} \partial \frac{\alpha_{1} \gamma_{1}}{\partial \xi_{2}} \tag{3}
\end{gather*}
$$

Substituting (2) $\rightarrow(1)$

$$
\begin{align*}
- & \iint\left\{N_{11} \delta \epsilon_{11}+\left(N_{12}+N_{21} i \delta \epsilon_{12}+N_{22} \delta \epsilon_{22}+M_{11} \delta k_{11}+\right.\right. \\
& \frac{1}{2} M_{12} \delta k_{12}+\frac{1}{2} M_{12} \delta k_{12}+\frac{1}{2} M_{21} \delta k_{21}+\frac{1}{2} M_{21} \delta k_{21}+ \\
& \frac{1}{2} M_{21} \delta k_{12}-\frac{1}{2} M_{21} \delta k_{12}+\frac{1}{2} M_{12} \delta k_{21}-\frac{1}{2} M_{12} \delta k_{21}+  \tag{4}\\
& M_{22} \delta k_{22}+Q_{1} \delta \gamma_{1}+Q_{2} \delta \gamma_{2} \mid \alpha_{1} \alpha_{2} d_{\xi 1} d_{\xi 2}
\end{align*}
$$

Substitute $3 \rightarrow 5$.

$$
\begin{aligned}
& \iint\left(N_{11} \delta \epsilon_{11}+\left(N_{12}+N_{21}\right) \delta \epsilon_{12}+N_{22}^{\delta \epsilon_{22}}+M_{11} \delta k_{11}+\right. \\
& \frac{1}{2}\left(M_{12}-M_{21}\right) \delta\left(\left(\frac{1}{R_{2}}-\frac{1}{R_{1}}\right) \epsilon_{12}+\frac{1}{\alpha_{1} \alpha_{2}} \frac{\partial \alpha_{2} \gamma_{2}}{\partial \xi_{1}}-\right. \\
& \left.\frac{1}{\alpha_{1}^{\alpha}} \frac{\partial \alpha_{1} \gamma_{1}}{\partial \xi_{2}}\right)+\frac{1}{2}\left(M_{21}+M_{12}\right) \vdots\left(k_{12}+k_{21}\right)+M_{22}{ }^{i k_{22}}+ \\
& Q_{1} \delta \gamma_{1}+Q_{2} \delta \gamma_{2} \mid \alpha_{1} \alpha_{2} d_{\xi 1} d_{\xi 2}
\end{aligned}
$$

## Group.

$$
\begin{aligned}
& \iint\left\{N_{11}{ }^{\dot{\delta} \epsilon_{11}}+2\left(\frac{1}{2}\left(N_{12}+N_{21}\right)+\frac{1}{4}\left(\frac{1}{R_{2}}-\frac{1}{R_{1}}\right)\left(M_{12}-M_{21}\right)\right) \dot{\delta_{\epsilon}}{ }_{12}+\right. \\
& N_{22} \delta \epsilon_{22}+M_{11} \delta k_{11}+\frac{i}{2 \alpha_{1} \alpha_{2}}\left(M_{12}-M_{21}\right) \delta\left(\frac{\partial \alpha_{2} \gamma_{2}}{\partial \xi_{1}}-\frac{\partial \alpha_{1} \gamma_{1}}{\partial \xi_{2}}\right)+ \\
& \left.\frac{1}{2}\left(M_{21}+M_{12}\right) \delta\left(k_{12}+k_{21}\right)+M_{22} \delta k_{22}+Q_{1} \delta \gamma_{1}+Q_{2} \dot{ } \gamma_{2}\right\} \alpha_{1} \alpha_{2} d_{\xi 1} d_{\xi 2}
\end{aligned}
$$

Expanding i $\frac{\partial}{\partial \xi}(\alpha \gamma)$

$$
\varepsilon\left(\frac{\partial \alpha_{2} \gamma_{2}}{\partial \xi_{1}}-\frac{\partial \alpha_{1} \gamma_{1}}{\partial \xi_{2}}\right)=\delta\left(\gamma_{2} \frac{\partial \alpha_{2}}{\partial \xi_{1}}+\alpha_{2} \frac{\partial \gamma_{2}}{\partial \xi_{1}}-\gamma_{1} \frac{\partial \alpha_{1}}{\partial \xi_{2}}-\alpha_{1} \frac{\partial \gamma_{1}}{\partial \xi_{2}}\right)
$$

Substitute and Group.

$$
\begin{aligned}
& \iint\left\{N_{11} \delta \epsilon_{11}+2\left[\frac{1}{2}\left(N_{12}+N_{21}\right)+\frac{1}{4}\left(\frac{1}{R_{2}}-\frac{1}{R_{1}}\right)\left(M_{12}-M_{21}\right)\right] \delta \epsilon_{12}+\right. \\
& N_{22} \delta \epsilon_{22}+M_{11} \delta k_{11}+\frac{1}{2}\left(M_{21}+M_{12}\right) \delta\left(k_{12}+k_{21}\right)+M_{22} \delta k_{22}+ \\
& {\left[Q_{1}-\frac{1}{2 \alpha_{1} \alpha_{2}} \frac{\partial \alpha_{1}}{\partial \xi_{2}}\left(M_{12}-M_{21}\right)\right] i \gamma_{1}+\left[Q_{2}+\frac{1}{2 \alpha_{1} \alpha_{2}} \frac{\partial \alpha_{2}}{\partial \xi_{1}}\left(M_{12}-M_{21}\right)\right]} \\
& \left.\delta \gamma_{2}+\frac{1}{2}\left(M_{12}-M_{21}\right) \delta\left(\frac{1}{\alpha_{1}} \frac{\partial \gamma_{2}}{\partial \xi_{1}}-\frac{1}{\alpha_{2}} \frac{\partial \gamma_{1}}{\partial \xi_{2}}\right)\right\} \alpha_{1} \alpha_{2} d_{\xi 1} d_{\xi 2}
\end{aligned}
$$

Assumption

$$
\begin{aligned}
& M_{12}-M_{21} \rightarrow 0, \frac{\delta \partial y_{i}}{\partial \xi_{j}} \ll \delta(\epsilon, k) \\
& \iint\left\{\mathrm{N}_{11}{ }^{\delta \epsilon} \epsilon_{11}+2 \bar{N}_{12}{ }^{\delta \epsilon} \epsilon_{12}+\mathrm{N}_{22}{ }^{\delta \epsilon}{ }_{22}+\mathrm{M}_{11} \delta \mathrm{k}_{11}+\mathrm{M}_{22} \delta \mathrm{k}_{22}+\right. \\
& \left.2 \bar{M}_{12}{ }^{i \bar{K}_{12}}+Q_{1}^{\delta \gamma_{1}} Q_{2}{ }^{j \gamma} \gamma_{2}\right\} o_{1}{ }^{\alpha} 2_{\xi 1} d_{\xi 2}
\end{aligned}
$$

$$
\begin{aligned}
& \bar{N}_{12}=\frac{1}{2}\left(\mathrm{~N}_{12}+\mathrm{N}_{21}\right) \\
& \overline{\mathrm{M}}_{12}=\frac{1}{2}\left(\mathrm{M}_{12}+\mathrm{M}_{21}\right) \\
& \overline{\mathrm{k}}_{12}=\frac{1}{2}\left(\mathrm{k}_{12}+\mathrm{k}_{21}\right)
\end{aligned}
$$

### 1.132 Multiplication of Series Expansions

The relationships $A^{n j}$ and $A^{n j}$ used in the text are found by multiplying term by term the series expansions of stiffness and strain and noting a recurring sequence of the resulting expressions

$$
\begin{aligned}
& \text { e.g. . }\left(\sum_{j=0}^{K-1} b_{j} \operatorname{Cos} j \theta\right)\left(\sum_{n=0}^{K-1} \epsilon_{\xi n} \operatorname{Cos} n \theta\right)=\sum_{n=0}^{K-1}\left(\sum_{j=0}^{K-1} A^{n j} \epsilon_{\xi j}\right) \operatorname{Cos} n \theta \\
& =\sum_{n=0}^{K-1}\left\{\sum_{j=0}^{K=1} \frac{1}{2}\left\{b^{(n+j)}+\left|1-\delta^{2}(j-k)+\delta(k)\right|_{b}^{|n-j|}\right]_{\xi j}\right\} \operatorname{Cos} n \theta \\
& \left(\sum_{j=0}^{K=1} b_{j} \operatorname{Cos} j \theta\right)\left(\sum_{n=1}^{K-1} \bar{\epsilon}_{\xi, n} \operatorname{Sin} n \theta\right)=\sum_{n=1}^{K=1}\left(\sum_{j=0}^{K=1} \bar{A}^{n j} \bar{\epsilon}_{\xi j}\right) \operatorname{Sin} n \theta \\
& =\sum_{n=1}^{K-1}\left\{\left.\sum_{j=0}^{K=1} \frac{1}{2} \right\rvert\,-b^{(n+j)}+\left\{1-i^{2}(j-k)+j(k)\left|b^{|n-j|}\right| \bar{\epsilon}_{\xi j}\right\} \operatorname{Sin} n \theta\right.
\end{aligned}
$$

where

$$
\delta(m)=\left\{\begin{array}{rl}
-1 & m<0 \\
0 & m=0 \\
+1 & m>0
\end{array}\right.
$$

### 1.13.3 Coefficients

Stiffness parameters related in an isotropic manncr
$B_{1}=B_{2}=\frac{B_{3}}{v}$
$D_{1}=D_{2}=\frac{D_{3}}{v}$

$$
\begin{aligned}
& G_{1}=G_{2}=G_{3}=\frac{B_{1}(1-v)}{2} \\
& G_{13}=\frac{D_{1}(1-v)}{2} \\
& f_{5 k+1,5 j+1}=B_{1}^{k j} \\
& f_{5 k+1,5 j+4}=0 \\
& f_{5 k+2,5 j+2}=\frac{G_{1}^{k j}}{2}+\frac{\lambda^{2}}{8}\left(\omega_{\theta}-\omega \xi\right)^{2} G_{13}^{k j} \\
& f_{5 k+2,5 j+5}=\frac{\lambda^{2}}{4}\left(\omega_{\theta}-\omega_{\xi}\right) G_{13}^{k j} \\
& f_{5 k+3,5 j+3}=G_{2}^{k j} \\
& f_{5 k+4,5 j+1}=0 \\
& f_{5 k+4,5 j+4}=\lambda^{2} D_{1}^{k j} \\
& f_{5 k+5,5 j+2}=\frac{\lambda^{2}}{4}\left(\omega_{\theta}-\omega_{\xi}\right) G_{13}^{k j} \\
& f_{5 k+5,5 j+5}=\frac{\lambda^{2}}{2} G_{13}^{k j} \\
& g_{5 k+1,5 j+1}=B_{1}^{k_{j}^{\prime}}+\gamma B_{1}^{k j} \\
& g_{5 k+1,5 j+2}=\frac{k}{\rho} B_{3}^{k j}+\frac{k}{2 \rho} G_{1}^{k j}-\frac{\lambda^{2} k}{8 \rho}\left(\omega_{\theta}-\omega_{\xi}\right)^{2} G_{13}^{k j} \\
& g_{5 k+1,5 j+3}=\omega_{\xi} B_{1}^{k j}+\omega_{\theta} B_{3}^{k j}+\omega_{\xi} C_{2}^{k j} \\
& g_{5 k+1,5 j+4}=0
\end{aligned}
$$

$$
\begin{aligned}
& g_{5 k+1,5 j+5}=+\frac{\lambda^{2} k}{4 \rho}\left(\omega_{\xi}-\omega_{\theta}\right) G_{13}^{k j} \\
& g_{5 k+2,5 j+1}=-\frac{k}{2 \rho} G_{1}^{k j}+\frac{\lambda^{2} k}{8 \rho}\left(\omega_{\theta}-\omega_{\xi}\right)^{2} G_{13}^{k j}-\frac{k}{\rho} B_{3}^{k j} \\
& g_{5 k+2,5 j+2}= \frac{G_{1}^{k j}}{2}+\frac{\gamma}{2} G_{1}^{k j}+\frac{\lambda^{2}}{8}\left(\omega_{\theta}-\omega_{\xi}\right)^{2} G_{13}^{k^{\prime}}{ }^{\prime}- \\
& \frac{\lambda^{2}}{8}\left(\gamma\left(\omega_{\theta}-\omega_{\xi}\right)^{2}+2 \omega_{\xi}^{\prime}\left(\omega_{\theta}-\omega_{\xi}\right)\right) G_{13}^{k j} \\
& g_{5 k+2,5 j+4}=-\frac{\lambda^{2}}{4} \frac{k}{\rho}\left(\omega_{\theta}-\omega_{\xi}\right) G_{13}^{k j} \\
& g_{5 k+2,5 j+5}=+\frac{\lambda^{2}}{4}\left(\omega_{\theta}-\omega_{\xi}\right) G_{13}^{k j \prime}+\frac{\lambda^{2}}{4}\left(2 \gamma\left(\omega_{\xi}-\omega_{\theta}\right)-\omega_{\xi}^{\prime}\right) G_{13}^{k j} \\
& g_{5 k+3,5 j+1}=-\omega_{\xi} G_{2}^{k j}-\omega_{\xi} B_{1}^{k j}-\omega_{\theta} B_{3}^{k j} \\
& g_{5 k+3,5 j+3}= G_{2}^{k j^{\prime}}+\gamma G_{2}^{k j} \\
& g_{5 k+3,5 j+4}= G_{2}^{k j} \\
& g_{5 k+4,5 j+1}= 0 \\
& g_{5 k+4,5 j+3}=-G_{2}^{k j} \\
& g_{5 k+4,5 j+4}= \lambda^{2} D_{1}^{k j \prime}+\gamma \lambda^{2} D_{1}^{k j} \\
& 4
\end{aligned}
$$

$$
\begin{aligned}
& g_{5 k+4, j j+5}=\lambda^{2} \frac{k}{\rho} D_{3}^{k j}+\frac{\lambda^{2}}{2} \frac{k}{\rho} G_{13}^{k j} \\
& g_{5 k+5,5 j+1} \quad-\frac{2}{\rho}\left(\omega_{G}-\omega_{\xi}\right) G_{1}^{k j} \\
& g_{5 k+5,5 j+2}=+\frac{\lambda^{2}}{4}\left(\omega_{\theta}-\omega_{\xi}\right) G_{13}^{k j^{\prime}}+\frac{\lambda^{2}}{4}\left(2 \gamma\left(\omega_{\theta}-\omega_{\xi}\right)-\omega_{\xi}^{\prime}\right) G_{13}^{k j} \\
& g_{5 k+5,5 j+4}=-\frac{\lambda^{2}}{2} \frac{k}{\rho} G_{13}^{k j}-\lambda^{2} \frac{k}{\rho} D_{3}^{k j} \\
& g_{5 k+5,5 j+5}=\frac{\lambda^{2}}{2} G_{13}^{k j^{\prime}}+\frac{\lambda^{2}}{2} \gamma G_{13}^{k j} \\
& h_{5 k+1,5 j+1}=\gamma B_{3}^{k j^{\prime}}-\omega_{\xi} \omega_{\theta} B_{3}^{k j}-\gamma^{2} B_{2}^{k j}-\frac{k^{2}}{2 \rho^{2}} G_{1}^{k j}-\frac{\lambda^{2}}{8} \frac{k^{2}}{\rho^{2}}\left(\omega_{\theta}-\omega_{\xi}\right)^{2} G_{13}^{k j}- \\
& \omega_{\xi}^{2} G_{2}^{k j} \\
& h_{5 k+1,5 j+2}=\frac{k}{\rho} B_{3}^{k j^{\prime}} \quad \forall \frac{k}{\rho} B_{2}^{k j}-\frac{\gamma}{2} \frac{k}{\rho} G_{1}^{k j}-(\omega \xi-\omega \theta)^{2} \frac{\gamma \lambda^{2}}{8} \frac{k}{\rho} G_{13}^{k j} \\
& h_{5 k+1,5 j+3}=\omega_{\xi} B_{1}^{k j^{\prime}}+\omega_{\theta} B_{3}^{\mathbf{k j}^{\prime}}+\left(\omega^{\prime} \xi+\gamma \omega_{\xi}\right) B_{1}^{k j}-\gamma \omega_{\theta} B_{2}^{k j} \\
& h_{5 k+1,5,+4}=+\omega_{\xi} G_{2}^{k j}+\frac{\lambda^{2}}{4} \frac{k^{2}}{\rho^{2}}\left(\omega_{\theta}-\omega_{\xi}\right) G_{13}^{k j} \\
& h_{5 k+1,5 j+5}=+\frac{\lambda^{2} \gamma}{4} \frac{k}{\rho}\left(\omega^{\prime} \theta-\omega \xi\right) G_{13}^{k j} \\
& h_{5 k+2,5 j+1}=-\frac{k}{2 \rho} G_{1}^{k j^{\prime}}-\frac{\gamma k}{2 \rho} G_{1}^{k j}-\frac{k}{\rho} \gamma B_{2}^{k j}+\frac{\lambda^{2}}{8} \frac{k}{\rho}\left(\omega_{\theta}-\omega \xi\right)^{2} G_{13}^{k j}- \\
& \frac{\lambda^{2}}{8} \frac{k}{\rho}\left(\omega_{\theta}-\omega_{\xi}\right)\left(3 \gamma\left(\omega_{\theta}-\omega_{\xi}\right)-2 \omega_{\xi}^{\prime}\right) G_{13}^{k j}
\end{aligned}
$$

$h_{5 k+2,5 j+2}=\frac{1}{2}\left(\omega_{\xi^{\omega}}{ }_{\theta}-\gamma^{2}\right) G_{1}^{k j}-\frac{\gamma}{2} G_{l}^{k j^{\prime}}-\frac{k^{2}}{\rho^{2}} B_{2}^{k j}-\omega_{0}^{2} G_{13}^{k j}+$

-     - 

$$
\begin{aligned}
& \frac{\lambda^{2} \gamma}{8}\left(\omega_{\theta}-\omega_{\xi}\right)^{2} G_{13}^{\mathrm{kj}}-\frac{\lambda^{2}}{8}\left(\left(\omega_{\theta}-\omega_{\xi}\right)^{2}\left(\omega_{\xi} \omega_{\theta}+3 \gamma^{2}\right)-\right. \\
& \left.\left(\omega_{\xi}-\omega_{\theta}\right) 2 \gamma \omega_{\xi}^{\prime}\right) G_{13}^{\mathrm{kj}}
\end{aligned}
$$

$h_{5 k+2,5 j+3}=-\frac{k}{\rho} \omega_{\xi} B_{3}^{k j}-\frac{k}{\rho} \omega_{\theta} B_{2}^{k j}-\omega_{\theta} \frac{k}{\rho} C_{3}^{k j}$
$h_{5 k+2,5 j+4}=+\frac{\lambda^{2}}{4} \frac{k}{\rho} \gamma\left(2\left(\omega_{\theta}-\omega_{\xi}\right)+\omega_{\xi}^{\prime}\right) G_{13}^{k j}-\frac{\lambda^{2}}{4} \frac{k}{\rho}\left(\omega_{\theta}-\omega_{\xi}\right) G_{13}^{k j}$
$h_{5 k+2,5 j+5}=+\omega_{\theta} G_{3}^{k j}-\frac{\lambda^{2} y}{4}\left(\omega_{\theta}-\omega_{\xi}\right) G_{13}^{k j}{ }^{\prime}+\frac{\lambda^{2}}{4}\left(\omega_{\theta}-\omega_{\xi}\right)\left(\omega_{\xi} \omega_{\theta}+2 \gamma^{2}\right) G_{13}^{k j}+$

$$
\frac{\lambda^{2} y}{4} \omega \xi G_{13}^{\prime k j}
$$

$h_{5 k+3,5 j+1}=-\left(\omega_{\xi}^{\prime}+\gamma \omega_{\xi}\right) G_{2}^{k j}-\omega_{\xi} y B_{3}^{k j}-\omega_{\theta} \gamma B_{2}^{k j}-\omega_{\xi} G_{2}^{k j^{\prime}}$
$h_{5 k+3,5 j+2}=-\frac{k}{\rho} \omega_{\xi} B_{3}^{k j}-\frac{k}{\rho} \omega_{\theta} B_{2}^{k j}-\omega_{\theta} \frac{k}{\rho} G_{3}^{k j}$
$h_{5 k+3,5 j+3}=-\frac{k^{2}}{\rho^{2}} G_{3}^{k j}-\omega_{\xi}^{2} B_{1}^{k j}-2 \omega_{\theta} \omega_{\xi} B_{3}^{k j}-\omega_{\theta}^{2} B_{2}^{k j}$
$h_{5 k+3,5 j+4}=G_{2}^{k^{\prime}}{ }^{\prime}+\gamma G_{2}^{k j}$
$h_{5 k+3,5 j+5}=\frac{k}{\rho} G_{3}^{k j}$
$h_{5 k+4,5 j+1}=+\frac{k^{2}}{4 \rho^{2}} \lambda^{2}\left(\omega_{\theta}-\omega_{\xi}\right) G_{13}^{k j}+\omega_{\xi} G_{2}^{k j}$
$h_{5 k+4,5 j+2}=+\frac{\lambda^{2}}{4} \frac{k}{\rho} v\left(\omega_{\theta}-\omega_{\xi}\right) G_{1.3}^{k j}$

$$
\begin{aligned}
& h_{5 k+4,5 j+3}=0 \\
& h_{5 k+4,5 j+4}=-\lambda^{2} \omega_{\xi^{\omega} \omega_{6}} D_{3}^{k j}+\lambda^{2} v_{D}^{k j}{ }_{3}^{\prime}-\frac{\lambda^{2} k^{2}}{2 \rho^{2}} G_{13}^{k j}-\gamma^{2} \lambda^{2} D_{2}^{k j}-G_{2}^{k j} \\
& h_{5 k+4,5 j+5}=\lambda^{2} \frac{k}{\rho} D_{3}^{k j}-\frac{\lambda^{2} \gamma}{2} \frac{k}{\rho} G_{13}^{k j}-\lambda^{2} Y \frac{k}{\rho} D_{2}^{k j} \\
& h_{5 k+5,5 j+1}=+\frac{\lambda^{2}}{4} \frac{k}{\rho}\left(\omega_{\theta}-\omega_{\xi}\right) G_{13}^{k j^{\prime}}-\frac{\lambda^{2}}{4} \frac{\mathbf{k}}{\rho} \omega_{\xi}^{\prime} G_{13}^{k j} \\
& h_{5 k+5,5 j+2}=+\frac{\lambda^{2} y^{\prime}}{4}\left(\omega_{\theta}-\omega_{\xi}\right) G_{13}^{k j}{ }^{\prime}-\frac{\lambda^{2}}{4}\left(y \omega_{\xi}^{\prime}+\omega_{\xi} \omega_{\theta}\left(\omega_{\theta}-\omega_{\xi}\right)\right) G_{13}^{k j}+ \\
& \omega_{\theta} G_{3}^{k j} \\
& h_{5 k+5,5 j+3}=+\frac{k}{\rho} G_{3}^{\mathbf{k j}} \\
& h_{5 k+5,5 j+4}=-\frac{\lambda^{2}}{2} \frac{k}{\rho} G_{13}^{k j}-\lambda^{2} \frac{k}{\rho} \gamma D_{2}^{k j}-\frac{\lambda^{2}}{2} \frac{k}{\rho} \gamma G_{13}^{k j} \\
& h_{5 k+5,5 j+5}=-\frac{\lambda^{2} \gamma}{2} G_{13}^{k j}{ }^{\prime}-\lambda^{2} \frac{k^{2}}{\rho^{2}} D_{2}^{k j}-G_{3}^{k j}+\frac{\lambda^{2}}{2}\left(\omega \xi \omega \theta-\gamma^{2}\right) G_{13}^{k j} \\
& P_{5 k+1}=-P_{\xi}^{k}+t_{\xi T}^{k^{\prime}}+\gamma\left(t_{\xi T}^{k}-t_{\theta T}^{k}\right) \\
& p_{5 k+2}=-p_{\theta}^{k}-\frac{k}{\rho} t_{\theta T}^{k} \\
& \mathbf{p}_{5 k+3}=-\mathbf{p k}-\omega_{\xi} \mathbf{t}_{\xi T}^{k}-\omega_{\theta} \mathbf{t}_{\theta T}^{k} \\
& p_{5 k+4} \quad=\lambda^{2} m_{\xi T}^{k^{\prime}}+\lambda^{2} \gamma m_{\xi T}^{k}-\lambda^{2} \gamma m_{\theta T}^{k} \\
& p_{5 k+5} \quad=-\frac{k}{\rho} \lambda^{2}{ }_{m}{ }_{\theta T}
\end{aligned}
$$

$\mathbf{r}_{5 k+1,5 j+1}=\mathbf{B}_{1}^{k j}$
$r_{5 k+1,5 j+4}=0$
$r_{5 k+2,5 j+2}=\frac{1}{2} G_{1}^{k j}+\frac{\lambda^{2}}{8}\left(\omega_{\theta}-\omega_{\xi}\right)^{2} G_{13}^{k j}$
$r_{5 k+2,5 j+5}=+\frac{\lambda^{2}}{4}\left(\omega_{\theta}-\omega_{\xi}\right) G_{13}^{k j}$
$r_{5 k+3,5 j+3}=G_{2}^{k j}$
$\mathbf{r}_{5 k+4,5 j+1}=0$
$r_{5 k+4,5 j+4}=D_{1}^{k j}$
$r_{5 k+5,5 j+2}=+\left(\frac{\omega_{\theta}-\omega_{\xi}}{4}\right) G_{13}^{k j}$
$r_{5 k+5,5 j+5}=\frac{1}{2} G_{13}^{k j}$
$\mathbf{s}_{5 \mathbf{k}+1,5 \mathbf{j + 1}}=\mathrm{YB}_{3}^{\mathrm{kj}}$
$s_{5 k+1,5 j+2}=\frac{k}{\rho} B_{3}^{k j}$
$s_{5 k+1,5 j+3}=\omega_{\xi} B_{1}^{k j}+\omega_{\theta} B_{3}^{k j}$
$\mathbf{s}_{5 k+1,5 j+4}=0$
$s_{5 k+1,5 j+5}=0$
$s_{5 k+2,5 j+1}=-\frac{k}{2 \rho} G_{1}^{k j}+\frac{\lambda^{2}}{8} \frac{k}{\rho} \cdot\left(\omega_{\theta}-\omega_{\xi}\right)^{2} G_{13}^{k j}$

$$
\begin{aligned}
& s_{5 k+2,5 j+2}=-\frac{\gamma}{2} G_{l}^{k j}+\frac{\lambda^{2}}{8} \gamma\left(\omega_{\theta}-\omega_{\xi}\right)^{2} G_{l 3}^{k j} \\
& s_{5 k+2,5 j+4}=-\frac{\lambda^{2}}{4} \frac{k}{\rho}\left(\omega_{\theta}-\omega_{\xi}\right) G_{13}^{k j} \\
& s_{5 k+2,5 j+5}=-\frac{y \lambda^{2}}{4}\left(\omega_{\theta}-\omega_{\xi}\right) G_{13}^{k j} \\
& s_{5 k+3,5 j+4}=-\omega_{\xi} G_{2}^{k j} \\
& s_{5 k+3,5 j+4}=G_{2}^{k j} \\
& s_{5 k+4,5 j+1}=0 \\
& s_{5 k+4,5 j+2}=0 \\
& s_{5 k+4,5 j+3}=0 \\
& s_{5 k+4,5 j+4}=\gamma D_{3}^{k j} \\
& s_{5 k+4,5 j+5}=\frac{k}{\rho} D_{3}^{k j} \\
& \varepsilon^{2} k+5,5 j+1=+\frac{k}{4 \rho}\left(\omega_{\theta}-\omega \xi\right) G_{13}^{k j} \\
& s_{5 k+5,5 j+2}=+\frac{Y}{q}\left(\omega_{\theta}-\omega_{\underline{\xi}}\right) G_{13}^{k j} \\
& s_{5 k+5,5 j+4}=-\frac{k}{2 \rho} G_{1 j}^{k j} \\
& s_{5 k / 5,5 j+5}=-\frac{\gamma}{2} G_{13}^{k j}
\end{aligned}
$$

$a_{5 k+1}=-t_{\xi T}^{k}$
$a_{5 k+4}=-m_{\xi T}^{k}$
All other r, $s$, a are equal to zero.

### 2.0 GENERAL DESCRIPTION OF COMPUTER P.?OGRAMS

### 2.1 INTRODUCTION

Both the Static and Dynamic analysis described in Section 1.0 are programmed for solution on the IBM 7094 digital computer using the IBM FORTRAN IV code. This section will give a general description of the two computer programs. The scope and limitations of the computer programs are outlined. The necessary quantities in selecting a mathematical model for the best utilization of the programs are described. The detailed description, the machine program, input instructions, flow diagram, sample data sheets, etc., are given in Section 3. 0.

### 2.2 PROGRAM CAPABILITIES AND LIMITATIONS

Before describing some oí the general program characteristics, it will perhaps be worthwhile to list some of the capabilities which are not generally present in other shell analysis programs. Also included in this list are limitations in the program that have resulted due to theoretical restrictions, computer storage capacity, economic considerations, etc.
a. Shells having meridional and circumferential variation in the stiffness properties can be analyzed. The circumferential variation must have a property of symmetry about some plane, $\theta=$ constant (constant $=0$ ).
b. Static sheils can be subjected to loads and temperatures which may vary meridionally and circumferentially with the limication that these distributions must exhibit the same property of symmetry as the stiffness. The time dependent loadings for the dynamic response of shells must have the same conditions on the special distributions and also can vary arbitrarily in time.
c. For static shells, elastic foundations can be considered. The distribution of these parameters are subject to the same condition cf symmetry described for stiffness. For the dynamic respc..., both elastic foundations and external dampings are considered. These are also subject to the same symmetry conditions imposed on the stiffness parameters.
d. Shells analyzed must have a surface of revolution reference surface. The middle surface of the shell has been taken as the reference surface for these computer programs, i.e., the section properties must be symmetric about the reference surface $\left(\int E \zeta d \zeta=0\right)$.
e. User compiled call functions are used in place of tables to input data of a 2 and 3 variable nature.
f. As many as 100 spacial integration ints 1 s can be considered.
g. The Fourier expansions can be taken to 10 terms.

### 2.3 SIGN CONVENTIONS AND DIMENSIONS

The sign conventions used in the programs are illustrated in Figures (1.2, 1.3) in Section 1.0. To augment briefly the stresses $\sigma_{\xi}, \sigma_{\theta}$ and membrane forces $\mathrm{N}_{\xi}, \mathrm{N}_{\theta}$ are positive when they tend to produce tension and negative when they are in compression. The moments. $\mathrm{M}_{\xi}, \mathrm{M}_{\theta}$ are positive in sign when they tend to produce tensile stresses in the inner surfaces and compressive stresses in the outer surface (see Section 2.4). The extensional displacement $u$, transverse deflection $w$, and meridional rotation $\Phi \xi$, are positive when the $\xi$ and $\zeta$ coordinates are increased respectively.

In using the program, all data specified must be dimensionally consistent.

### 2.4 REFERENCE, INNER, AND OUTER SURFACES

The reference surface $\zeta=0$ is chosen such that the requirements of limitation (c) above be satisfied. The cross sectional properties are then evaluated based upon this reference surface. As discussed in 1.15, a substantial simplification is obtained when specifying key geometric functions (. . g., $\rho, \gamma$ ), if the reference surface is chosen according to convenience anywhere within the shell wall. However, the shell stiffness parameter should be evaluated systematically along the lines discussed in Section 1.6 Eq. 1. 17.

It will be convenient to refer to inner and outer surfaces of the shell. One can keep the inner and outer surface definitions clear b; remembiring that in direction of increasing value of $\xi$ the outer surace is on the left and inner surface is on the right when the geometry is drawn with axial distance increasing from top to bottom and radial distance from left to righi as shown in Figure 1. 1.

## 2. 5 GEOMETRY (GIN)

Geometric parameters must be defined at each station location. The sign convention for the curvature parameters, $\omega_{\xi}, \omega \theta$ are defined in the figures below.

$\omega_{\xi}>0 \quad \omega_{b}>0$

$\omega_{\xi}=0 \quad \omega_{\theta}>0$

$\omega_{\xi}>0 \quad \omega_{\theta}<0$

$\omega_{\xi}<0 \quad \omega_{\theta}<0$

In order to assist the analyst in defining the set of geometry parameters with a minimum number of input parameters, several options for specific classes of geometries are made available. The options are described below with their identifying code number (GIN).

### 2.5.1 Cone-Cylinder Option (GIN) $=1.0$ )

This geometry option may be specified for a complete range of regional configurations generated by a straight line, e.g., circular plates, divergent cones, cylinders and convergent cones. A minimum of 3 input parameters are required. The input parameters required are defined as follows:

1. RA1-Radial distance from axis of revolution to the first station ( $i=1$ ) of the region.
2. AXL - meridional length of shell.
3. ANX - angle the generator makes with the axis of revolution.

Figure 2.3 illustrates the geomet:ic parameters used in describing. the cone cylinder option.


Figure 2.1 Cone Cylinder Geometry
Both•RAI and AXL are positive quantities. The parameter ANX is given in degrees and is positive clockwise measured from the generator to the positive $X$ axis as shown in Figure 2. 1.

### 2.5.2 Sphere-Toroid $(G I N=2.0)$

This option may be specified for a complete range of regional configuration generated by a circular curve. Four input parameters are necessary for defining a sphere-toroid as shown in Figure 2, 4.


Figure 2.2 Sphere Foroid (icomolm

The input parameters are:

1. RC - Radius of curvature of the generator
2. ROFF - Offset distance measured from axis of revolution to the center of meridional curvature.
3. PHIO - Angular position in degrees of the beginning of a region mieasured clockwise positive about the center of curvature irom an axis parallel to the axis of revolution.
4. PHIN - Angular position of the end of the region.

## 2. 5. 3 Discrete Point Option (GIN $= \pm 4.0$ )

This option was developed for use on regions where the generator cannot be described by one of the other options or where a curved generator is given by a set of discrete points. As a consequence of various possible ways the geometry may be supplied to the analyst, several variations of input data format can be accommodated.

On a posicive indicator ( $G I N=+4.0$ ), the program will set up the necessary geometric parameter from the input data which describes the generator by discrete radial and axial distances. The input quantities to the program are EM (number of points given), RIPT (radial distance from axis of revolution at input points), XIPT (axial coordinates of the input points). The set of RIPT and XIPT must include the first and last points of the region. XIPT must be given in ascending magnitudes. On a negative indicator (GIN $=-4.0$ ), the coordinates of the discrete points are given in radial and surface or arc length, the surface length cocrdinate is input directly in the XIPT locations.

An interpolation routine is used to obtain appropriate, eometric parameters at station points from the original input values. The parameters such as curvatures arc computed using finite difference forms of the station set. A least squares method is used to minimize the scatter of these computations. Tc hold the errors in curvatures to less than 10 percent, the number of points described by RIPT and XIPT should be at least as great as the number of stations. For some situations such as locations of major changes in the generator curve, it will be necessary to input a denser populatior, of RIPT and XIPT. (See Figure 2.3.) Because of the difficulty involved in the least squares and interpolation routines, extreme care must be exercised in the use of this option in order to obtain an adequate description of shell geometry. A significant improvement in results is obtained if the additional recommendations described are adhered to.


Figure 2.3
When the mexidional and circumferential radii of curvatures are available, they can be input at discrete points and curve-fit to give a bette: description of the curvatures. If possible, it is strongly recommended that this capability be used since the errors in curvatures are reduced considerably to better control curvatures and less input point: of :he generator are required. This dara is infut in the location RCURV and RCOFZ for radius of curvatures $\mathrm{R}_{\xi}$ and $\mathrm{R}_{\theta}$, respectively (Section !. 3). RCURV and RCURZ values must correspond with the pcints described lav RIPT and XIPT. This is an optional input to both GIN $=+4.0$ and GIN $=-4.0$. When no values are input at RCURV and RCURZ locations, the curvatures will be computed from the discrete point set of RIPT and XIPT.

## 2. 6 CALL FUNCTION PARAMETERS

Input data of a multivariable nature are defined to the computer programs by the use of call cunction subroutines. These parameters are described functional in terms of the meridional and circumferential variables (PEL, ZTA). The time variable (TU) is available for desc tibing the lnading history. Call function subroutines are to be coded and compiled by the user. Specific instructions for the coding of these sabroutines are given in Section 3. Data that will be input in this manner are described in the following sections.

### 2.6.1 Stiffness Properties

The stiffness parameters axe described in Section 1.6, Equations 1.16. It is sufficient to describe the inplana stiffness $B_{1}$, and the bending stiffness $\Sigma_{1}$ and Poisson's ratio in order to define all: - necessary parameters. The

## 'REPRODUCIBILITY OF THE ORIGINAL PAGE IS POOR

function (BFCN, Static, DBBDD, Dynamics) will be written to define these parameters over the reference surface of the shell.

$$
\begin{aligned}
& B_{1}=B_{1}(\xi, \theta) \\
& D_{1}=D_{1}(\xi, \theta)
\end{aligned}
$$

### 2.6.2 Thermal Loads

The thermal load terms are described in Section 1. 6, Equation 1.15 The call function (TFCN, Static, DTMP. Dynamics) will be written to define these parameters over the reference surface of the shell.

$$
\begin{aligned}
\mathbf{N}_{\xi}^{T} & =\mathbf{N}_{\xi}^{T}(\xi, \theta) \\
\mathbf{M}_{\xi}^{\mathbf{T}} & =\mathbf{M}_{\xi}^{\mathrm{T}}(\xi, \theta)
\end{aligned}
$$

### 2.6.3 Elastic Foundations

The elastic foundation parameters are defined in Section 1.6, Equation 1.6. For the static program these parameters will be defined in function subroutine KFCN. For the dynamic program these parameters will be defined in DKMP.

### 2.6.4 External Damping

The external damping parameters are defined in Section 1.6, Equation 1.6. In the Dynamics program this function subroutine, DKMP will define both the elastic foundation terms and the external damping parameters.

### 2.6.5 Mass Properties

The mass properties for the Dynamic program defined in Section 1.6, Equation 1.6 are defined to the computer program in the subroutine DMASS.

### 2.6.6 Pressure Loads

The pressure loading for the static case is defined to the program in the subroutine PFCN. For the dynamic case the time dependent loading is defined to the computer program in the subroutine PRSS. Positive pressure yiclds positive displacement.

## "REPRODUCIBILITY OF THE ORIGINAL PACE IS POOR:"

### 2.6.7 Parameters for Stresses

For the static program, the parameters necessary for the computation of stresses are defined to the computer program by the following subroutines, STRIFN, STR2FN, TTF1, TTF2. These subroutines define the following parameters:

STRIFN, for the first stress print layer, EIl, Young's Modulus, DN1, distance from the reference surface to the stress print locations and POISl, Poisson's ratio. .

STR2FN, these are parameters necessary for stresses at a second layer. The definitions of EI2, DN2, POIS2 currespond to the first stress print parameters.

TTF1, this subroutine defines the temperature TMP1 and thermal expansion coefficient ALFl at the first stress print location.

TTF2, these parameters are for the second stress print and the TMP2, ALF2 have definitions corresponding to TTF1.

### 2.7 STATIONS IN REGIONS: (EN)

The machine program achieves a shell solution by integration of finite difference equations along the meridian or arc length distance oi the shell. The number of integration points (called stations) located in the region under consideration is assigned the EN code value. The stations are equally spaced with the initial point located on the reference surface at the beginning of the region designated station $1(i=1$ or $s=0)$ and the last or EN - th station at the end of the region called station $N(i=N$ or $s=\bar{s})$. The numbering of stations proceeds in direction of positive meridional coordinate assigned to the respective region. The maximum number of stations permissible in a region is 100 (minimum 5). The regional input data are specified at stations on the reference surface of each region.

The length of the finite difference "lump" of shell is computed internal to the program from the length or wrap distance and the number of stations (EN) in the region. This finite difference incroment of integration is defined as DEL in the program and printout.

The machine running time increases with the number at intergrtion steps considered per region. The type of shell problemi considerad should dictate the size of the grid mesh or number ol stations considered. lhas comes with experience and how.the results atre tobe usid ds a gumeral rule, it is recommended that more integration inturnts lur usid whore raphat change in variables occurs along the length of the sholl

### 3.0 DETAILED USE OF PROGRAMS

### 3.1 IN~FODUCTION

The Unsymmetric Shell Computer Program is written in the FORTRAN IV language and makes use of the overlay feature and the ALTIO option of that language. ALTIO sacrifices speed of execution in favor of providing an additional 1900 core locations. (It does not make use of buffered inputoutput.) If the programs are to be compiled for use without the ALTIO option it will be necessary to add an additional BACKSPACE command at each location where the backspace moved the tape over an END OF FILE. These have. been noted in the listings.

The program has been checked out in NAASYS, the NAA adaption of the IBM 7090/7094 IBSYS/IBJOB system and used the NAASYS library routines shown in the load map, Section 3.12 m .

The NAASYS input tape is 'UNiTG5'; the output tape is 'UNIT06'. In addition to these files, the program $u$ e. units 3, 7, 8, 9, 10, 11, 12 and 13 as scratch tapes or for overlay storage during execution. NAASIS itself, is stored on 'UNIT01'.

The program is made up of an executive program and eleven links, four of which are called by the executive program, three by the DATLNK subroutine and four by the PANDX subroutine. The name of the main program in each link and a description of its use, follows.

| Link No. | Name | Purpose |
| :---: | :---: | :---: |
| 0 | Executive | Reads the title cards and general data; sets up tape numbers and rewinds all tapes; controls the flow of execution of other links. |
| 1 | DATLNK | Acts as a sub-executive program to control GEOM, DATLYR and DATLD so that these three subroutines may share common auxiliary routines CODIMA, FNTERP and DINTRP. |
| $\angle$ | GEOM | Reads geometry parameters, GMDA. Sets up top and bottom boundary milriars. Gilculates DFL, R; XSI, WTIIX, WFFX, G^M^, RIIOX, WFFiPX and stores the latter 5 arrays on lipe. Prints genceral data, boundary matrices and perifinent geomeliy diata on indicator. |


| Link No. | Name | Purpose |
| :---: | :---: | :---: |
| 3 | DATLYR | Sets up stiffness and temperithre coefficients by function subprograms. Thesc are stored on tape. |
| 4 | DATLD | This is the Fourier Load coefficients generator. Pressures, PFE, PTH and PN are supplied by function subprograms. These coefficients are preserved on tape. |
| 5 | PANDX | A sub-executive program which, together with the subroutines it controls--RSLT, FGHPE, DYLMN, GMTX--generates the recursion terms and the ' $P^{\prime}$ and ' X ' matrices of equations 1.60 Section 1.10 . |
| 6 | RSLT | Sets up the boundary matrices needed in computing ' $P$ ' and ' $X$ ' at the top and bottom of the shell. (See Section 1.10.) |
| 7 | FGHPE | Sets up the $F, G$, and $H$ matrices in equilibrium equations 1.34 of Section 1.7. |
| 8 | DYLMN** | Sets up the L, M, and $N$ matrices for the dyndmic response equations of Section 1.9. These are written on tape 13. |
| 9 | GMTX* | Forms the PE matrix of equation 1.34 of Section 1.7. Computes ' $\mathrm{G} * *$ ', ' $\mathrm{B} * *{ }^{\prime}$, ' $\mathrm{P}^{\prime}$ and ' X ' matrices. |
| 10 | ZMTRX | Computes ' $Z$ ', the solution matrix. |
| 11 | SUMS | Sums the Fourier components and computes internal loads, i.e., bending moments, transverse shear forces, and membrane forces. (In the static version, stresses may also be computed in this link with the parameters defined by function subprograms.) |
| *Dynamics version only |  |  |

In Figure 3A of this section we have shown the set-up of the columnbinary program deck with the necessary control cards linr cold link. . I list of the deck names corresponding to the subprogr.um momes is givon in the tabie at the end of the section.


TABLE OF SUBROUTINE NAMES AND DECK NAMES

| Subroutine Name | Static Deck | Dynamics Deck |
| :---: | :---: | :---: |
| DATLNK | DLNK | DLKDY |
| GEOM | GMTRY | GMYD |
| DATLYR | STIFF | STFDY |
| DATLD | LOAD | DLDY |
| PANDX | PEANDX | PXDYN |
| RSLT | BNDRY | BNDD |
| FGHPE | FGMTX | FGHDY |
| DYLMN |  | LMNDY |
| GMTX |  | GDYN |
| ZMTRX | SOLTN | SLND |
| SUMS | SUSM | SUMD |

The \$IBJOB, \$ORIGIN and \$DAT. cards are single control cards. The circled number found on the first two mentioned control cards, indicate the c-der in which they, plus the associated decks of that link, should be stacked. Fur example, those second level subroutines preceded by a \$ORIGIN DELTA card - GEOM, DATLYR and DATLD will be found in the deck before the first level subroutine, PANDX, because they are executed in this order.

It is imperative that the utility subroutines be kept with each deck as shown. Si. ce only one link of the same level may occupy core at a given time, the utility subprograms CODIMA, ENTERP and DINTRP are stored with the DATLNK link so that they may be shared by GEOM, DATLYR and DATLD.

Additional control cards preceding the \$IBJOB card are likely to vary somewhat with the installation. An IBM systems handbook should be consulted. The $\dot{c} a r d s$ used at Space and Information Systems Division are shown in Figure 3B, below.


Figure 3B

### 3.3 DATA DECK SET-UP

Data decks should be sacked as follows:

1. Three cards ( 72 columns each) of title data.
2. GDA, general shell data, read by the EXECUTIVE program.
3. GMDA, geometry data, read by the GE $\varnothing \mathrm{M}$ subroutine.

All other parameters such as stiffnesses, springs, damping, mass, temperature and pressure loadings, and section properties used in stress computations are defined by function subprograms. These subprograms will be written by the user and compiled for each particular run. A discussion of how to write the various functior, subprograms is found in Section 3.4.

With the exception of the thres title cards, each group of data listed above should have a minus sign in column 1 of the last card, since this data is read by the DECRD subroutine. See Section 3.5.1. In the instructions below, the DECRD index of eac.2 innut quantity is given.

### 3.3.1 Title Cards

Three title cards form the first three cards of any data deck for each case. These cards are useful in identifying the run at a later date. They may include a brief problem description, the date of the run, a reference, etc.

These cards may not be omitted, but th.y may be blank, if desired If the cards are forgotten, the error indication from DECRD may occur for a multiple case run in which title cards are present for the second case, or the job may terminate with an EXECUTIDN ENDED designation (as explained in Section 3.5.1). A more serioas situation occurs when the data from subsequent subroutines are read out of turn and the program is placed in an endless loop.

### 3.3.2 GDA, General Shell Data

All input data must be dimensionally consistent. It should be noted that all nondimensionalization is done internal to the program, thus all inputs must be supplied with appropriate dimensions (e.g., transverse load PN is input with dimensions $P / L^{2}$ ).

| DECRD <br> Index | Name | Description and Comments |
| :---: | :--- | :--- |
| 1 | AO | Reference length (a) |
| 2 | H0 | Reference thickness ( $h_{0}$ ) |
| 3 | E0 | Reference Young's modulus (E $E_{0}$ ) |
| 4 | SIG0 | Reference stress ( $\sigma_{0}$ ) |
| 5 | PQI | Poisson's ratio ( 1 ) |
| 6 | ENF | The number of Fourier components (10 maximum) |


| DECRD <br> Index | Name | - Description and Comments |
| :---: | :---: | :---: |
| 7 | BCIT | Boundary condition indicator at first station. |
|  |  | $=1$. closed apex |
|  |  | $=2$. pinned |
|  |  | $=3$. clamped |
|  |  | $=4$. free |
|  |  | $=5$, roller w |
|  |  | $=6$. roller u |
|  |  | $=10$. special boundary matrices read in with geometry data. Must use lowhenever non-zero values are prescribed in boundary matrices $\ell_{0}$ or $\ell_{n}$. |
| 8 | BCIB | Boundary condition indicator at last station. (As BCIT) |
| 9 | PFLAG | Print indicator for input data. |
|  |  | $\begin{aligned} & =0 . \text { prints general data and boundary } \\ & \text { matrices } \end{aligned}$ |
|  |  | $=$ 1. prints above information and input data for the particular geometry configuration selected by GIN and the computed values for $r, x, \omega_{\theta}, \omega_{\phi}, \rho$ and $\gamma$. |
|  |  | $=-1$. prints all of above information and stiff. ness coefficients, elastic coefficients, thermal load and moment and pressure loads. |
| 10 | CEXT | Number of time cycles |
| 11 | DELT | Time increment in seconds |


| DECRD <br> Index | Name | Description and Comments |
| :---: | :---: | :---: |
| 12 | THT | Circumferential angle $\theta$ (degrees). Five values of THETA may be chosen. The deflections, rotations, internal loads and stresses will be printed at these values. |
| 18 | FPRNT | Fourier component print values. Three prints are permitted. Two intermediate prints of the Fourier summing are possible for checking convergence. The last FPRNT value given should be the same as the value given for ENF in the general data, $i$.e., GDA (6). |

Note: If THT and FPRNT are not entered, the program will set $\operatorname{THT}(1)=0.0$ and $\operatorname{FPRNT}(1)=$ ENF.

The last card of GDA data should have a minus (-) in cc’umn 1.

### 3.3.3 GMDA, Geometry Data

The GMDA data array is zeroed before the data is read. This means that any data with a value $=0$. need not be entered.

| DECRD <br> Inder | Name | Description and Comments |
| :---: | :---: | :---: |
| 1 | GIN | Geometry indicator |
|  |  | $=1$. cone-cylinder |
|  |  | $=2$. sphere-toroid |
|  |  | $= \pm 4$. discrete points |
| 2 | EN | Number of meridional stations (100 maximum) |
| 3 | ENLAY | Number of layers |

When GIN $=1.0$; see Section 2.5.1
$4 \mid$ RAl $\mid$

Radial distance from axis of revolution to station 1. (L) ${ }^{*}$
*L - unit of length

| DECRD <br> Index | Name | Description and Comments |
| :---: | :---: | :---: |
| 5 | AXL | Meridional length of shell (L) <br> 6 |
| ANY | Angle the generator makes with the axis of revolution <br> (degrees) |  |

When $\cdot$ GIN $=2.0$; see Section 2. 5.2

| 4 | RC | Radius of curvature of the generator (L) <br> 5 |
| :--- | :--- | :--- |
| 6 | ROFF | Offset distance measured from axis of revolution to <br> center of meridional curvature (L) |
| PHIO | Initial opening angle from the vertical axis (degrees) |  |
| PXIN | Final opening angle from the vertical axis (degrees) |  |

When GIN $=$ 4. 0 ; see Section 2.5.3

| 8 | EM | Number of RIPT's given (12 minimum, <br> 100 maximum) |
| ---: | :--- | :--- |
| 9 | RIPT | Discrete radial distances <br> 209 |
| 309 | XIPT | Discrete axial or vertical distances (or arc lengths) |
| RCURV | Meridional radii of curvature <br> RCURZ | Circumferential radii of curvature |

Boundary matrices, when not set by indicator: only the elements which might possibly be non-zero are included in the arrays. The explanation below assumes the user is familiar with Section 2. 8, Boundary Conuitions.

| 409 | BNDTX $\begin{aligned} & (1)= \\ & \Omega(1,1) \end{aligned}$ | Elements of force boundary matrix $\left(\Omega_{0}\right)$ at station |
| :---: | :---: | :---: |
| 410 | $\Omega(1,2)$ |  |
| 411 | $53(2,2)$ |  |
| 412 | $\Omega(3,3)$ |  |



The last card of GMDA data should have a minus ( - ) in column 1.

### 3.4 FUNCTIONAL SUBPROGRAMS

The FUNCTION subprogram is an independently written and compiled program that is executed wherever its name appears in an arithmetic statement in another routine. The object (binary) or source (FORTRAN) deck must be included in the job deck even though the FUNCTION value equals zero; otherwise, the program that references the FUNCTION will not be executed.

## "REPRODUCIBILITY OF THE ORIGINAL PAGE IS POOR;

Since a FUNCTION is a separately compiled subprogram, the variables and statement numbers within it do not relate to any other component of the program. Communication between the FUNCTION and its calling program takes place at execution time. The value of the FUNCTION is always returned to the calling program; values may also be returned via the FUivCTION arguments and/or variables assigned to a common region.

General Form:


END
Name is a subprogram name ( $1-6$ alphanumeric characters, the first of which is alphabetic). FUNCTION names used in this program are listed at the end of this section.
a, $b,--\underline{n}$ are nonsubscripted variable or array names used to transfer values between the FUNCTION subprogram and the program that references it. There must be at least one argument.

Basic Description:
The FUNCTION statement must be the first statement in the subprogram definition. (Comments cards excluded.) The name must not be the same as a library or built.-in function unless the name is included in an EXTERNAL statement which precedes the first use of the FUNCTION by the calling program.

The FUNCTION subprogram may contain any FORTRAN statement except another FUNCTION statement, a.SURROUTINE statement, or a BLOCK DATA subprogram.

A variable in a COMMON block may be referenced if the rules for use of COMMON are followed.

The name of the FUNCTION must be assigned a value at least once within the subprogram. This value may be assigned by the appearance of the function name on the left side of an arithmetic statement (e.g. SOMEF $=\mathrm{X} / \mathrm{Y}$ ), or by its appearance in the list of a READ statement within the subprogram.

Arguments:
The relationship between variable names used as arguments in the calling program and the variables used as arguments in the FUNCTION subprogram is illustrated in the following example.

```
Calling Program
            •
            •
    A = SOMEF (B,C)
        -
        -
        •
    Subprogram
        FUSVTION SOMEF (X, Y)
    50 SOMEF = X/Y
    55 RETURN
        END
```

The value of the variable $B$ of the calling program will be used as the value of the subprogram variable $X$; and $C$ for $Y$. Thus, if $B=10.0$ and $C=5 . G$, then $A=2.0$.

The arguments used in the unsymmetrical shell of revolution program are PEL, ZTA and TU, where PEL is the meridional distance to a station, ZTA is the circumferential distance and TU is the elapsed time. If additional values are necessary to define a. FUNCTION it will be necessary to include a READ statement in the scioprogram and to supply the required data cards to satisfy the REAL at execution time.

## RETURN and END Statements:

A FUNCTION subprogram must contain an END statement and at least one RETURN statement. The END statement specifies the physical end of the subprogram for the compiler. The FETURN statement signifies a logical conclusion of the computation and returns any computed value and control to the calling program.

## Multiple Entries:

The normal entry into a FUNCTION subprogram is marie by a function reference in an arithmetic expression. Entry is made at the first executable statement following the FUNCTION statement. It is also possible to enter a FUNCTION by means of a reference to an ENTRY statement in the FUNCTION.

## General Form

ENTRY name ( $a, b, \ldots n$ )
name is the name of an entry point. It must follow all the rules given for the FUNCTION name.
$\underline{a}, \underline{b}, \ldots \underline{n}$ are the dummy arguments corresponding to actual arguments supplied by a function reference.

The ENTRY statement is not executable. When inserted in a series of statements, the ENTRY statement has no effect on the logical flow of the subprogram. Entry to the subprogram is made at the first executable statement following the ENTRY statement, so it is easy to start using a subprogram at any desired point.

Within a FUNCTION subprogram, only the FUNCTION name may be used as the variable to carry a result back to the calling program. The ENTRY name may not be used for this purpose. The following example illustrates this rule.

```
Calling Program
            •
    A = ONEF (B,C)
    •
    •
    F = TWOF (D,E E)
        .
        -
Subprogram
    FUNCTION ONEF (X, Y)
    5ONEF=X * Y
    10RETURN
```

```
        ENTRY TWOF (X, Y)
    ONEF = X + Y
    20 RETURN
        END
```

Note the use of the primary function name to return the function value to the calling program, even when the reference was to an ENTRY name.

Additional examples showing the use of FUNCTION subprograms with multiple ENTRY statements may be found in Sections 3.6, 3. 7, 3.8 and 3.9.

Following is a table which gives the FUNCTION names, ENTRY names, calling program names and the assigned deck names (\$ IBFTC manes) for the static and dynamic versions of the Unsymmetrical Shell of Revolution Program. For their definitions see Program Nomenclature, Section 3. 13.

STATIC DYNAMIC

Calling Program DATLYR
FUNCTION BBB (PEL, ZTA) BFCN DBBDD ENTRY DDD (ZTA)

FUNCTION ENTT (PEL, ZTA) TFCN DTMP
ENTRY EMTT (PEL, ZTA)

| FUNCTION | *DKKl (PEL, ZTA) | KFCN | DKDMP |
| :---: | :---: | :---: | :---: |
| ENTRY | *DKK2 (PEL, ZTA) |  |  |
| ENTRY | DKK3 (PEL, ZTA) |  |  |
| ENTRY | DMPl (PEL, ZTA ) |  |  |
| ENTRY | DMP2 (PEL, ZTA) |  |  |
| ENTRY | DMP3 (PEL, ZTA) |  |  |

FUNCTION DMMl (PEL, ZTA)
ENTRY DMM4 (PEL, ZTA)
DMASS

Calling Program DATLD
FUNCTION PPPN (PEL, ZTA)
ENTRY PPPH (PEL, Z'TA) PFCN
ENTRY PPPF (PEL, ZTA)
*Deleted in the static deck because of core storage problems, therefore DKK 3 becomes the FUNCTION name.

## STATIC DYNAMIC

FUNCTION PPPN (PEI, ETA, TU)
ENTRY MPH (PEI, ETA, TU)
ENTRY PPPF (PEI, ETA, TU)

Calling Program SUMS

| FUNCTION | ELl (PEL, ZTA) |  |
| ---: | :--- | ---: |
| ENTRY | DN1 (PAL, ZTA) | STRIFN |
| ENTRY | POIS1 (PEI, ZTA) |  |
| FUNCTION | EI2 (PEI, ZTA) |  |
| ENTRY | DN2 (PEI, ZTA) | STR2FN |
| ENTRY | POIS2 (PEI, ZTA) |  |
| FUNCTION | TMP1 (PEI, ETA) |  |
| ENTRY | ALF1 (PEI, ZTA) | TTF1 |
| FUNCTION | TMP2 (PEI, ZTA) |  |
| ENTRY | ALF2 (PEI, ZTA) | TTF2 |

### 3.5 UTILITY SUBROUTINES

### 3.5.1 DECRD Subroutine

All data, with the exception of the three title cards, is read by :means of the DECRD subroutine, included with the symbolic decks.

This routine provides the facility for reading a variable number of pieces of floating point data into specified elements of an array; these edements may be either in sequential or in nonconsecutive locations. Only the information specified is actually read into storage.

$$
\begin{aligned}
& 16 \\
& 4 \cdot 6 \\
& 0 \cdot \\
& 1 \cdot \\
& 3 \cdot 3
\end{aligned}+178
$$

The fixed point number (index) in the first field on each card defines the position of the first piece of data on the card. If the index is 1 , the first piece of data will be stored in the first location reserved for the array; ir it is 16 , the first word will be placed in the 16 th position, etc. The remaining fields on each card contain information for the successive locations of the array. If one or more fields are left blank, no information is read into the locations corresponding to these fields; the information already in these locations is unaltered.

The sample data sheets shown in Section 3.6.2 have 6 fields of 12 card columns each, and an identification field of 8 columns for sorting purposes.
a. The index must be written to the extreme right of the first field; it may. not be zero or blank. (No decimal point)
b. The programmer should keep in mind the way in which FORTRAN stores arrays having double or triple subscripts, e.g., A(l, l), $A(2,1), A(3,1), A(1,2), A(2,2)$, etc.
c. The floating point (REAL) data should be entered with a decimal point (anywhere in the field) and an exponent, when necessary, written to the extreme right of the field and preceded by a ' + ' or '-'.
d. Reading data is concluded by placing a negative sign in column 1 of the last card to be read.
e. Zero should always be entered as '0.'. A' 0 . ' or '. $0^{\prime}$ will be recognized as a blank.

ERROR indication: If the index is zero or blank, the comment $" \% \% * \% B A D$ INDEX ON DECRD CARD=" will be printed, followed by a printout of the columns 1-80 of the defective card. The job will be terminated.

If the data for the array in the CALI statement has been completely read, and no negative sign has been encountered in column 1 of the last card sent, data intended for subsequent CALL's will be read into the incorrect array. When there are no data cards to satisfy the appetite of a CALL DECRD statement, the job will terminate with an EXECUTION ENDED designation.

If this occurs before all expected results have been printed, check the last card of each data block for the negative sign in column 1.

In order to use the DECRD routine in conjunction with the ALTIO option, it is necessary to physically load a copy of the routine with the program deck. ALTIO will not cause the proper adjustments to DECRD from the library tape at load time, so drop-in decks must be used.



### 3.5.2 MAD, MSU, MMY, INVMS

These four subroutines perform matrix addition, subtraction, multiplication and irversion, respectively. They are extremely simple in their approach and must be recompiled to change dimensions for use in other decks. There are no error indications given in the lst three routines other than the usual NAASYS trapping information for underflows, overflows and divide checks. When data has been entered correctly, these subroutines will present no problems.

The inversion routine has an error indicator, IX, which is set at 0 or -1 for a singular matrix. This is returned to the calling program through the argument list and may be tested after return. The Unsymmetrical Shell of Revolution Program makes such a test, prints the comment "SINGULAIN MATRIX I $=X X^{\prime \prime}$ and terminates the job. When thiserror occurs, check the data-especially any special boundary matrices e ered, and the FUNCTION subprograms.

The Dynamics version of the program uses a MAP (machine language coded) copy of the INVMS subroutine. The FORTRAN language routine causes the GDYN link to exceed core storage. Listings for both versions are included.


## REPRODUCIBILITY OF'THE ORIGINAL PAGE IS PÖOR



$$
\text { IDFE NO. } 8 K-Q \cap 1
$$

SIBFTC JNVNSP


$$
\begin{aligned}
& \quad J=1 ; M \\
& =A(1, J)
\end{aligned}
$$

invsoneno. $C$ OF MATRIX USING SINGLE PRFCISION OPERATIONS


INVSOOng
INVOnMa

INVSOnIO | INVSONI |
| :--- |
| INVSOOII |
| INVSOOI2 |

 Mi NuMBER OF ROWS
IX ERROR INDIGATOR
D WORKING ARRAY.MI

$$
\begin{aligned}
& \text { IRROR INDICATOR } \\
& \text { NORKING ARRAY. MUST BF SINGLE PRECISION AND DIMENSIONED MI X YI } \\
& \text { SUBROUTINE INVMS (A,N,MI,IX,D,AI) }
\end{aligned}
$$

$$
\begin{aligned}
& \text { EQUIVALENCF } 111,14,17 \text { ), (12,15,j,K): (13,L), } \\
& \text { (PRODOM(ILT,AMX) }
\end{aligned}
$$ C $\quad M=M L$

$$
\begin{aligned}
& I=1, M \\
& J=1, M
\end{aligned}
$$





$$
W^{6} I=1 \quad 000 \tau 00
$$

$$
\begin{aligned}
& \varepsilon \\
& \varepsilon \\
& 1 W=O \\
& W
\end{aligned}
$$

$$
\text { GO TO } 17
$$





1
$i$




$$
\bar{i}
$$





## $\rightarrow$




### 3.5.3 DINTRP, ENTERP

These subroutines perform linear double and single interpolation. DINTRP makes use of ENTERP in interpolicion for values along a particular curve. They are included for use by FUNCTION subprograms called by DATLY: or DATLD.

In DINTRP when the first argument is not bounded by the given table (curves) the statement
"ARGUMENT EXCEEDS EX'TENT OF TABLE IN DINTRP." is printed, followed by

ARGUMENT $=$ (1 PE 12.4)
TABLE VALUES (printed 6/line)
and the job is terminated.
When the argument in the single interpolation subroutine, ENTERP, exceeds the limits of the table, the routine selects the value at either end of the table and continues after printing.
"LIMITS OF TABLE EXCEEDED BY ARGUMENT = (1 PE 12.4) (1 PE 12.4) = VALUE USED FROM TABLE"

Values entered in the tables should always be given in increasing algebraic order, both in terms of the numbers used to designate each curve of the family, and the values assigned to the points along the curve.



### 3.5.4 CODIMA

CODIMA is a curve fitting subroutine which has the following properties:

1. To the straight portions of any curve defined by three points on a straight lire, a straight line will be fitted.
2. To the smooth portion of any curve, a smooth curve will be fitted.
3. The method maintains continuous first derivative except at the ends of a straight segment.
4. The method will fit curves with "corners" or "sharp turns" with. out the large deviation usually found in other methods.

An interpolation method is reveloped such that some of the considerations taken when an engineer fits a curve with a french curve are formulated. This is the CODIM (controlled deviation interpolation method) concept.

The method will interpolate in a mose engineering manner in the sense that:

1. The first derivative is continuous except at the ends of straight segments defined by three points on a straight line.
2. No large deviation will be found when slope changes are large.
3. Ability to change value and slope rapidly.
4. Ability to fit straight lines on straight line portic... $\therefore$. *he curve and fit smooth arcs through the smooth portions of the curve.

The method fits a polynomial through an interval with information given by "previous points" (points to the left) and another polynomial through the interval with information given by "subsequent points" (points to the right). These two polynomials are then compared for compatibility. If they differ, a weighted average ni the polynomials is taken in a way such that the polynomial that deviates less from the straight line connecting the points defining the interval is given more weight. For simplicity, parabolae are used over higher degree polynomials, in the CODIMA version.
NO. OF POINTS TO ITERPOLATE
LOCATION OF POINTS TO BE INTERPOLATED.
LOCATION OF POINTS TO BE INTERPOLATED
ANSWERS
INDEPENDENT ARGUMENT



| CODIMO40 |
| :--- |
| CODIMO49 |
| CODIMO50 |
| CODIMO59 |



CODIM110
CODIM120


CODIM150
CODIM160
08IWIOOS






### 3.5.5 FILE

FILE is a function subprogram by which any number of files for end of file marks) on a tape may be skipped, as specified by the programmer. Alternatively, the tape may be backspaced any number of files. The tape is positioned at the beginning of the desired file ready to read or write the first record. (Extent: 45 locations.)

Availability: On the FORTRAN library tape.
Use: The specified tape can be positioned as indicated by means of the statemert:
$A=\operatorname{FILE}(I, J)$.
A Used to make the stat.ment format consistent with FORTRAN rules.

I FORTRAN tape number of one of the available tapes; must be a positive fixed point constant or variable.
$J \quad$ The number of files to be skipped or backspaced, including the file in which the tape is positioned. It must be a fixed point FORTRAN expression.
$J>0$, skip $\mathrm{J}<\mathrm{O}$, backspace
I should never be zero. If a zero argument is used, one file will be skipped either forward or backward depending on the sign of zero.

## Examples:

a. Tape 4 is positioned within or at the end of file 2 and we wish to get to file 5. Then $J=3$.
$A=\operatorname{FILE}(4,3)$
b. Tape 4 is positioned at the beginning of file 12 and file 16 is desired. Then $J=4$.

$$
A=\operatorname{FILE}(4,4)
$$

c. Tape 4 is positioned anywhere within file 7 and file 4 is desired. Then $J=-4$.
$A=\operatorname{FILE}(4,-4)$
d. Tape 4 is positioned anywhere within file 7 and the beginning of file 7 is desired. Then $J=-1$.

$$
A=F I L E(4 ;-1)
$$

Even if the tape is at the beginning of a file; you must count that file in computing the value of J. In the last example, if tape 4 is at the beginning of file 7 and a $J$ of -1 is given, the next file to be read or written would still be file 7 .



### 3.6 SAMPLE PROBLEM 1

### 3.6.1 Problem Description and Set-Up

A free cylinder under thermal loading is considered. The geometry and loading is shown in the following figure

$\mathrm{BBB}=2.4725 \times 10^{7}$ (in plane)
$\mathrm{DDD}=1.8887 \times 10^{6}$ (bending)
Thermal Load
$\mathrm{ENTT}=9.375 \times 10^{4} \quad$ (in plane)
$\mathrm{EMTT}=1.0045 \times 10^{4}$ (bending)
Stress Output Parameters

$$
\begin{array}{rlrl}
\text { EI1 } & =3 \times 10^{7} & \mathrm{EI} 2 & =1.5 \times 10^{7} \\
\text { DN1 } & =-0.4167 & \mathrm{DN} 2 & =0.5833 \\
\text { POIS1 } & =0.3 & . \text { POIS2 } & =0.3
\end{array}
$$

### 3.6.2 Data Sheets



### 3.6.3 Functional Subprogram Used,







### 3.6.4 Output



BOTTOM BOUNDARY **

geometrr data fir cone - cylinder.


THE COEFFICIENTS ARE
CHECKPRINT OF LOAD COEF. STORED IN GROUPS OF $\triangle L L ~ E N F ~ P E R ~ M E R I D I O N A L ~ S T A T I O N: ~$
FIR STATION I THROUGH STATION 21




3


| STATION 2 |  |  | NIXT, THETAI D. OOMNONAF-39 | $\begin{gathered} 01 \times 11 \\ -9.2257>02 \mathrm{E} \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| --.. | $\begin{gathered} \text { N(X1) } \\ 1.0485137503 \end{gathered}$ | $\begin{aligned} & \text { NITHETA) } \\ & \text {-2.0540514E } 04 \end{aligned}$ |  |  |  |
| $\begin{aligned} & \text { M1xII } \\ & -6.9193945 F \approx ? \end{aligned}$ |  | $\begin{aligned} & \text { MTTRETAT } \quad 1 \\ & -7.2300801 F \quad 73 \end{aligned}$ | $\begin{aligned} & \text { MTXI THETA: } \\ & 0.00 \text { OCONOF-39 } \end{aligned}$ | $\begin{aligned} & \text { OTHFFTAI } \\ & 0.0000000 \mathrm{E}-39 \end{aligned}$ | SIG(THETA,ETA) <br> 0.0000000E-39 |
| Stresses for layer 1 |  |  |  | $\begin{gathered} \text { SIG(XI, ETA) } \\ -1.2300967 E 03 \end{gathered}$ |  |
| $\begin{array}{cc} \text { SIG(XI) } \\ 5.8375792 F & 04 \end{array}$ |  | $\begin{aligned} & \text { SIG(THFIA) } \\ & 6.5179318 E \text { O4 } \end{aligned}$ | $\begin{array}{r} \text { SIG\{XI, THFTA\} } \\ - \text {-n. } O D \cap O D O F-39 \end{array}$ |  |  |
| STRESSES FOR CAYER ${ }^{2}$ |  |  |  |  |  |
| $\begin{array}{cc} \text { SIG1XII } \\ 3.6717552 E & \text { S1G(THETA) } \\ -5.0065183 E & \end{array}$ |  |  | $\begin{aligned} & \text { SICIXI, THETA) } \\ & 0.00000 \text { I } \end{aligned}$ | $\begin{gathered} S[G(x I, E T A 1 \\ -6.1504833 F \text { oz } \end{gathered}$ | $\begin{aligned} & \text { SIGITHETA,ETA1 } \\ & 0.0000000 \mathrm{E}-39 \end{aligned}$ |
| Station 1 |  |  |  |  |  |
| $\begin{aligned} & \text { N(x1) } \\ & -7.6171875 E-02 \end{aligned}$ |  | $\begin{aligned} & \text { NITHETAI } \\ & -5.2677420 E \text { O4 } \end{aligned}$ | N(XI, THETA) <br> n. OnO0000F-39 | $\begin{gathered} 01 \times 11 \\ 3.6267831 \mathrm{E}-02 \end{gathered}$ |  |
| $\begin{aligned} & \text { M×11 } \\ & -1.1962891 E-02 \end{aligned}$ |  | $\begin{aligned} & \text { MITMETA) } \\ & -T .0315019 F 03 \end{aligned}$ | M(XI, THETA) D. NDODONDE-39 | Q(THETA) <br> -0.0000n00f-39 |  |
| STRESSES FTR LAVER |  |  |  |  |  |
|  | $\begin{gathered} \text { SIG1xil } \\ \text { 5.: } 945470 \mathrm{E} \end{gathered}$ | $\begin{gathered} \text { SIG(THETA) } \\ 3.7847100 E \text { O4 } \end{gathered}$ | SIG(XI, THETA) $-0.000 \cap 000 \mathrm{E}-39$ | $\begin{gathered} \text { S1G8XI,ETAI } \\ 4.8357133 \mathrm{~F}-02 \end{gathered}$ | $\begin{aligned} & \text { SIG(THETA,ETA) } \\ & -0.0000000 E-39 \end{aligned}$ |
| STRFSSES FOR LAYER ? |  |  |  |  |  |
|  | $\begin{gathered} \text { SIG I XI) } \\ 6.995361 F \end{gathered}$ | $\begin{gathered} \text { STG(TiserA) } \\ -6.4419699 E \text { O4 } \end{gathered}$ | SIGIXI,THETA) O. AOMODOOE-39 | $\begin{gathered} \text { SIG(XI FTA) } \\ 2.4178567 E-02 \end{gathered}$ | $\begin{gathered} \text { SIGITHETA, ETA) } \\ -0.0000000 \mathrm{E}-39 \end{gathered}$ |

### 3.7 SAMPLE PROBLEM 2

### 3.7.1 Problem Description

An Apollo-like shill pinned at the edge is considered.


Normalizing constants

$$
\begin{aligned}
a_{0} & =1 \\
\mathrm{~h}_{0} & =1 \\
\mathrm{E}_{0} & =1 \\
\sigma_{0} & =1
\end{aligned}
$$

No Thermal Loads

## Stiffness Parameter

$$
\begin{array}{ll}
\mathrm{A}=\xi^{2}: 5625-2(75) \xi, \operatorname{Cos} \theta \\
\text { if } \mathrm{A}<4425, & \mathrm{BBB}=3.296 \times 10^{6} \\
- & \mathrm{DDD}=3.296 \times 10^{6} \\
\text { if } 4425<\mathrm{A}<7225, & \mathrm{BBB}=1.9779 \times 10^{6} \\
& \mathrm{DDD}=1.9779 \times 10^{6} \\
\text { if } 7225<\mathrm{A}<11025, & \mathrm{BBB}=1.386 \times 10^{6} \\
& \mathrm{DDD}=1.386 \times 10^{6} \\
\text { if } \mathrm{A}>11025, & \mathrm{BBB}=0.79116 \times 10^{6} \\
& \text { BD }=0.79116 \times 10^{6}
\end{array}
$$

P essure parameters

Normal pressure definition

$$
\begin{aligned}
& B=\xi^{2}+(\equiv 0.492)^{2}-2(30.492) \xi \operatorname{Cos} \theta \\
& \text { if } B<400 \quad P P P N=100.0 \\
& \text { if } B \geq 400 \quad P P P N=0.0 \\
& \text { also circumferential and meridional pressures definition } \\
& \quad P P P H=P P P F=0.0
\end{aligned}
$$

Stress output parameters

$$
E I I=E I 2=29.5 \overleftarrow{\times} 10^{6} \text { Young's Modulus of stress output location }
$$

Distance to neutral surface

```
A = 自2+5625.-2(75)\xi\operatorname{Cos}0
    if A<4425 DN1 = -1.025, DN2 = +1.0̇25
    if 4425<A<7225, DN1 = -1.005, DN2 = +1.005
    if 7225<A<l1025, DN1 = -0.995, DN2 = +0.995
    if A>11025, DNI = -0.987, DN2 = +0.987
```


### 3.7.2 Data Sheets


PROGRAMMER

programmer


### 3.7.3 Function Subprograms Used


SIBFTC TFCN




IRFTC STRIFN










### 3.8 SAMPLE PROBLEM 3

### 3.8.1 Problem Description

A uniform cylinder under semisinusoidal time dependent loading is considered.


$$
\begin{aligned}
v & =1 / 6 \\
a_{0} & =1 \\
\mathbf{h}_{0} & =1 \\
E_{0} & =1 \\
\sigma_{0} & =1
\end{aligned}
$$

Stiffness Parameters

$$
\begin{aligned}
& \mathrm{BBB}=12.0925 \times 10^{6} \\
& \mathrm{DDD}=16.56 \times 10^{4}
\end{aligned}
$$

Mass properties $\mathrm{DMMl}=3.49 \times 10^{-3}$
No thermal loading

No elastic foundation

No external damping

$$
P P P N=1000 \operatorname{Sin}(1003 \mathrm{TU})
$$


if $\mathrm{TU} \geq .003113$

```
PPPN = 0.0
PPPH = PPPF = 0.0
```

DEEK NO. - PROGRAMMER



[^1]3.8.3. Function Subprograms Used

SIBFTC DKUMP

0(12, 1900i
0! ! : ... !


3.8.4 Output


ROTTIM RISUNDARY **


 FOR STATION - I THROUGH STATION 11






 :
 2.2277777E

 $\begin{array}{r}1.2092497 \mathrm{~F} \\ 1.2092497 \mathrm{~F} \\ 1.27 \\ 1.2092457 \mathrm{~F} \\ \hline 1.2792497 \mathrm{~F} \\ \hline\end{array}$ - 0.0 nncooiex-39






 - O. nnocoone- $\quad$ C. -0.0nOC OODE-39












-n.n0000nnc- 39
$-3.2122134 E-06$
$-8.4332169 E-07$
$-1.8189894 E-12$
6.4337896F-07
11
0.000 COODF-39 0.0000000E-39
-2. MATRIX FBR STATION
$-5.4073139 \mathrm{~F}-05-0.0000$


\footnotetext{


> MATRIX FTR STATIGN $\quad 6 \quad . . . . .$.
> $-9.094947 \mathrm{DE}-13 \quad-0.00000 \cap O E-39$

$$
\begin{aligned}
& \text { 2.: MATRIX FGR STATION } \\
& 3.8498592 E-07 \\
&-0.000000 \cap F-39
\end{aligned}
$$

$$
\text { Q2' MATRIX FGR STATION } 8
$$

$$
\begin{aligned}
& -2.9512694 E-06 \quad-0.0000000 E-39 \\
& \cdots \quad-3.8498865 E-07 \quad-0.0000000 E-39 \\
& \cdots \\
& \cdots
\end{aligned}
$$


$\stackrel{\rightharpoonup}{*}$
नZ̈MATR̈IX FGR STATIGN

$$
5.2341662 F-n 3
$$

5.32328 SOE-03
$4.7842582 F-03$ 68-3000000000-

2
68-300000UN*0- 20-32592156*2


## - CZ MAT̄TXX for statión <br> $$
\text { - } Z^{\circ} \text { MATVTX F } \overline{O R} \text { STATIONN }
$$

$$
1.73 ? 6853 E-05
$$

$$
-3
$$



90-316く2596*9-

### 1.7655379E-03

 ... $-\cdots .{ }^{-} \quad .$.



$$
-9.0949470 F-13
$$

-     - ODoñooe-39

$$
5.2030697 \mathrm{E}-\mathrm{n} 3
$$

$$
\begin{array}{r}
\text { PHilxin' } \\
-1.818989 \mathrm{E}-12
\end{array}
$$

$$
\begin{aligned}
& \text { PHIITHETAI } \\
& -0.0 \text { OOOOOE- } 39
\end{aligned}
$$


1.737E853E-n5 -n. OnODONOE-39


STATIGN
THETA $=$
ENF $=$
5.40
NDIIVIS
THETA $=$
ENF $=$
-8.5
 -

$$
4.784{\underset{V}{2}}_{582 E}^{5}=03
$$

$$
-6.965 \grave{2} 29 \bar{A} \bar{E}-06
$$

STATION 11

$\left\lvert\, \begin{array}{ccc:c} & 1 & \\ & \vdots & \\ \vdots & & \\ \vdots & & \\ \vdots & \vdots & & \\ \vdots & \vdots & 1 & 1\end{array}\right.$



01X11
R. $669504 ट E-03$
o(thetai:
N(XI,THETAI
O.OODOOONE-39
M(XI, THETA)
O.OODOOONF-39 N(XI, THETA)
0.00000 NOE -39
M(XI, THETA)
0. ONONONOE 39


M(THETA)
$1.3186140 \mathrm{C}-01$
N(XI, THETA)
0.0 OOOOODE-39
M(X1 THETA)
0.000 OnOOE-39
NIXI,THETAI
$0.900 \mathrm{OOOOF}-39$
M(XI, ThFTA)







n. OnOnnONE-39 n. Oncononnt- 39
n. nnonnone- 39
8.430R790E O?
-. -...........
$.000 n O D N E-39$
$.00000 C O E-39$
$.000000 N E-39$
.700000 E- 39
FGR STATIGN 1 THRGUGH STATIGN 11

0.77000 nine- 39
C. nOOOCOOE- 39
$\frac{0.00 n n 000 \mathrm{E}-39}{0.0 n 000 n 0 \mathrm{E}-39}$
$-\frac{8.4308750 E}{8.4308790 E} 02$
-- - ....-......--



$$
-\cdots-\cdots-\cdots
$$



$$
\cdots \quad-\cdots-\cdots
$$






－－－．．．－
ก．DOOOOMOF－39
0．nOnnの明E－39
－n．0n000000F－39
$-0.0 n 00000 \mathrm{~F}-39$
$\cdots$ $7.2956723 \mathrm{E}-05$
$-1.5654974 \mathrm{E}-04$
$-1.5654974 E-04$
$-8.6705197 E-04$
$-8.6705197 E-04$
$\ldots$
$-1.1038859 E-03$
3．1565081E－n2

20－30てカモ685・く
1．12686639F－09



$$
0.0000000 \mathrm{v}-39
$$



Phittheta) PH 1 (1)
B.671 $2.5893420 \mathrm{E}-0$ ?
PHI(XI)
0.0000000 E-39
0.0 OUOOODE-39
-..
-


$$
\begin{aligned}
\text { StAtion } & 11 \\
\cdots & =0.00 \\
\cdots & 0.0
\end{aligned}
$$

0.000 UnNOF-39



CNF $=$ CO:

. V000000F-39
1
$\vdots$



$$
3.1131264 \mathrm{E}-02
$$

PHITTHETAS
n. OnOONOF- 39

PHIXII
$2.1827873 E-11$
$68-300000000$
$68-30000000 \cdot 0-$

.- . . .-.-...-. .-.........
 2.2956723世-0. $20-31805991^{\circ} \varepsilon$
$0.0000000 \mathrm{E}-39$
$3.9743143 \mathrm{E}-05$
STATION 5 .


| $\begin{aligned} & \text { PH11 } \times 11 \\ & 2.2956723 E-05 \end{aligned}$ | PHIITHEXVA) $0.0000000 E-39$ |
| :---: | :---: |

- 





－O．iOnOOORJE－39
01×15
6．1373189E－03
O（THETA）
－0．OOOONODF－39
$0(\times 1)$
$0.6360177 E-05$
O（THETA）
$-0.0000000 \mathrm{E}-39$
$0(X 1)$
$-6.0 \_3427 E-03$
Q（THETA）

- n．nOnOOOOE－39
$01 \times 11$
$-8.9919292 \mathrm{E}-02$
0.00000 ONE -39
N（XI．THETAA）
O．OOONONOF－39
M（XI，THETA）
O．OnOOONOF－${ }^{2} 9$
NCXI，YHETA
MIXi THETAI：
$n$ OOONONOF－ 39
－0nonnonf－39
N（XI，THETA）
O．OONONCOE－39
M（XI，THETA）
O．CONONOOE－39
N（XI，THETA）
O．ONOnNONE－39
i． $3047171 \mathrm{~F}-01$

$$
\text { STATISN } 7
$$

$.6285702 F \mathrm{O}_{3}$



|  | M（XI）MITHETA） |  |
| :---: | :---: | :---: | :---: |
| $-1.5945630 E-01$ | $-2.6581365 F-02$ |  |


M（THETA）
$-1.7183610 E-C 2$

ルーヨてくと」．9 $2 \theta^{\circ}-2$
9 NOILVIS
STATION 5

| N（XI） |
| :--- |
| $\cdots$ |

$\underset{-1.030 \operatorname{cin} 4 E-n 1}{ }$

DRTHETAS
-0. COOOOONE-39
$10-31 \angle 1 \angle \ni 0 E \cdot I$
$1 \nabla 1 \exists H 1 / W$


0(xis)
2. $3058344 \mathrm{E}-01$

MIXI, THETA)
O.ONOONOOE-39
N(XI, THETAY
O.OOOONOOE-39
MIXI.THETA!
O.OOONOOOE-39


NiXI. THETA)
0.0000 OOOF -39

co $318 \angle 00 \angle \varepsilon{ }^{\circ} 8$


$$
\begin{gathered}
-- \\
\operatorname{STAVION} \\
\hline
\end{gathered}
$$

[^2]THE COEFFTITTANTS ARE
statiom.

ハ!
$10002000^{\circ}$



n. nongnnot-3n




02
02
$$
02
$$
\[

$$
\begin{aligned}
& i 2826 \\
& 38820
\end{aligned}
$$
\]

FGR STATIGN I THRGUGH SIATIGN 11
 $\qquad$ --




| n. OOOOONOF-39 |
| :---: |
|  |  |
|  |
| ．．．－ |
| －n．0000nOne－39 |
| ．－－．．．．．．．．－－．－－．． |
| －0．0nOOONOE－39 |

4．1839696E－05
$-5.5042517 E-04$
$-2.1914624 E-n 3$
$-2.7384744 E-03$
$8.1248075 \mathrm{~F}-\mathrm{n2}$
$8.0825035 \mathrm{~F}-07$
$6.0952132 \mathrm{~F}-\mathrm{n?}$
$4.325784 .2 \mathrm{~F}-09$

| 68－300000000 | 11－3Eヶら9E66・カー |
| :---: | :---: |
| 1 | NOLIVIS |
| 6と－30600000•0－ | －E0－3くLOJUもt＊て |
|  | Nillous \％ |
| ot－3000000060 |  |
| $\varepsilon$ | NOHVIS． |
| 68－30000600．0 | カレ－3y＜Lとてくを・2 |
| 力 |  |



PHIIXII
$2.7384748 \mathrm{E}-03$
n. ORONOONE. 39



- STATION $10 . \cdots$



$\stackrel{V}{n} 0.0009000 \mathrm{E}-39$

PHIIXII
$2.1914625 \mathrm{E}-03$

$\vdots$
(IX)IHd

3
$>$
3 ก. $\begin{array}{lll}\text { STATION } & 9 \\ \text { FNF } & =0.0 \\ & 0.0 \\ & & 0.0\end{array}$








[^3]
### 3.9 SAMPLE PROBLEM 4

### 3.9.1 Problem Description

The Apollo-like shell of sample problem 1 , Section 3.7 is now subjected to hydrodynamic pressures compacted from a rigid shell solution.


All the input parameters are the same as sample problem 1 except for the subroutine describing pressures.

$$
\begin{array}{ll}
\mathrm{V}_{\mathrm{o}}=360 \mathrm{in.} / \mathrm{s} & \text { Initial impact velocity } \\
\mathrm{R}_{\mathrm{c}}=175.6 \mathrm{in.} & \text { Radius of curvature } \\
\rho_{\mathrm{w}}=62.5 / 17280 \# / \mathrm{in}^{3} & \text { Fluid densi.iy } \\
\mathrm{W}_{\mathrm{T}}=10,000 & \text { Weight of body }
\end{array}
$$

inside wetted region

$$
\operatorname{PPPN}=\frac{\sqrt{2} \rho_{W} n^{3 / 2} R_{r} 1 / 2\left[1-\gamma\left(2-3 r^{2} / a^{2}\right)\right]}{\pi t^{1 / 2}\left(1-r^{2} / a^{2}\right)^{1 / 2}(1+\gamma)^{2}}
$$

where

$$
t=\frac{1}{R_{c} V_{o}}\left(\frac{3 W_{T}}{4 \sqrt{2} \rho_{W}}\right)^{2 / 3}, \quad Y=\frac{8 \sqrt{2} \rho_{w}\left(R_{c} V_{o} t\right)^{3 / 2}}{3 W_{T}}
$$

outside wetted region

$$
P P P N=0.0
$$

also

```
PPPH = PPPF = 0.0
```

The mass properties of the Apollc-like shell.

$$
D M M 1=1.4648 \times 10^{-3}\left(t_{f}\right)+4.7443 \times 10^{-5}
$$

where

$$
A=\xi^{2}+5625-2(75) \xi \operatorname{Cos} \theta
$$

$A<422 i ; \quad t_{f}=0.05$
$4225<\mathrm{A}<7225 \quad \mathrm{t}_{\mathrm{f}}=0.03$
$7225<\mathrm{A}<11025 \mathrm{t}_{\mathrm{f}}=0.02$
$A>11025 \quad t_{f}=0.012$

### 3.9.2 Data Sheets



### 3.9.3 Function Subprogram Used





### 3.10 ERROR INDICATIONS, PITFALLS, RECOMMENDATIONS

Several of the error indications resulting from improper data input have already been discussed. To reiterate, they were:

1. A bad index on a DECRD card (Section 3.5.1)
2. Omission of the negative sign on the last card of a data array (Section 3.5.1)
3. Omission of some or all of the title cards (Section 3.3.1).

One should be very careful to check the output from the program to see that it corresponds to the input that he entered. Better yet, an independent check of input data may prevent a wasted run on the digital computer.

The amount of data entered by the DECRD routine is relatively small but such things as sign convention, angle measurements, and compatibility of units are common pitfalls.

Many of the needed parameters are suppied to the program by use of FUNCTION subprograms. Before wríting such subprograms, Section 3.4, 3.6.3, 3.7.3, 3.8.3 and 3.9 .3 should be studied. A check list is given below.

1. Are the FUNCTION and ENTRY names spelled the same as those given in the table of Section 3. 4 ?
2. Does the number of arguments agree with the number in the calling program? Note that ENTRY DDD has just one argument, (ZTA), and the pressure functions for the Dynamics version require the added argument TU.
3. Has the FUNCTION name been used to return the function value to the calling program even when reference was made to an ENTRY name?
4. Does the FUNCTION subprogram contain at least one RETURN statement and an FND statement?
5. If aflitional data were needed to define the function have the data carls been included with the data deck?

### 3.11 PROGRAM LISTINGS FOR STATIC VERSION

### 3.11a Main Program




## 3. ile GDCM Subroudive (GMTRY)









### 3.11d DATLYR Subroutise (sTIFE)


NMF $=$ ENF
NNF:
IN $=1$
*
LESS THAN: OML HLOCK. MOSRE



## 3. 11e DATLD Subroutine (LOAD)




SIBFTC DEANDX

*     * 






*
RECURSION FORMS









 いいひひ

[^4]

> IRTF -NF 1 - FIRST STATION IN A SFCTION INOT REGIONI
IRTE -ED. 1 - LAST STATION IN PREVIOUS SECTION.

IF IRTE. NF. 11 GO TO 110
CIFIK LAE. LAST STATION-PREVIOUS BLOCK


$$
\begin{aligned}
& \text { TK } \triangle \text { NE } 1 \\
& =M M-1
\end{aligned}
$$

$$
\begin{aligned}
& \text { FIRST STATION - NOT FIRST RLOCK } \\
& 110 \text { IF IK ANF. } 1 \text { •OR• J AF. } 11 \text { GO TO } 111 \\
& \text { MM }=M M+1
\end{aligned}
$$





$\boldsymbol{v}$
 G13KMJ $=$ (SCG13(K4)-P2SG13(KMJA) 1 IDEL? GP1KMJ $=$ SSCG1(K4) - P2SCGI(KMJA) 1 DEL2 120 BBIP $=0.5 *(B P I K P J+F C T R *$ BPIKMJI $=0.5$ * (DP1KPJ + FCTR * DPIKMJ) $=0.5$ * IFCTR * G13KMJ + FCTR2 " G13KPJI

$$
\square
$$ GG1K, = 0.5 * (FCTK * :2SCGI KMMA ) + FCTRZ * P2SCG1 (KPJ)) OCK $3 \mathrm{~K}=\mathrm{L}$

GO in 150

$$
10139
$$

135 BRIK) $=0.5 *(S C B 1(K 2)+$ FCTR * SCBI:K5) $)$


| $c$ |
| :--- |
| 5 |




### 3.11g RSLT Subroutine (BNDRY)





'REPRODUCIBILIIY OF IHF ORIGINAI PAGE IS POOF



## 3. 11h FGHPE Subroutine (FGMTX)







REPRODUCIBILITY OF IHF ORIGINAI PAGG IS POOR


## 3. 11i ZMTRX Subroutine (SOLTN)











### 3.12 PROGRAM LISTANGS FOR DYNAMECS VEREICN

3. 12a Executive Proyram

-86806000
02600000











### 3.12d DATLYR Subroutine (STFDY)



00000520

00000569
00700570
00000580





## 3. 12e DATLD Subroutine (DLDY)







| 00000789 |
| :--- |
| 00000790 |
| 00000800 |
| 00000809 |
| 00000810 |
| 00000811 |
| 00000812 |
| 00000813 |
| 00000814 |
| 00000815 |
| 00000816 |
| 00000817 |
| 00000900 |
| 00000910 |
| 00000920 |
| 00000930 |
| 00000939 |
| 00000940 |
| 00000950 |
| 00000960 |
| 00000970 |
| 00000980 |
| 00000990 |
| 00000999 |
| 00001000 |
| 00001469 |
| 00001019 |
| 00001420 |
| 00001030 |
| 00001040 |
| 00001048 |
| 000001049 |
| 00001050 |
| 00001056 |
| 00001057 |
| 00001058 |
| 00001059 |
| 000001980 |
| 00001069 |
| 00001070 |
| 00001080 |
| 00001090 |
| 00001100 |
| 0 |




IN $=1$
MAN $=0$
IFLG $=0$
IF ITCTR ©EOQ 1.01 GO TO A
READ 191 PFE PTH, PN
10s 0I) $53 y n s s 3 y d$ ov3y
READ PRESSURES 110,50 ) TIME CYCLE_1
READ STIFFNESS COEFF IIOOI
IN - INITIAL STATION IN BLOCK
LN - LAST STATION IN BLOCK.
Nd HAd 3 ad IE 1 OV3甘
10 READ 191 LN IN. COEFC

$\omega \omega \omega$











### 3.12g RSLT Subroutine (BNDD)






## 3. 12h FGHPE Subroutine (FGHDY)




## 3. 12i DYLMN Subroutine (LMANDY)




## 3. 12j GMTX Subroutine (GDYN)

 ab00nnnc octonnono 00061110
00000120 cetnmono $=$
3
$?$
0 $1610 n o n o$
$0 G I n n c o o$ N C $C$
0
0
0
0
0 20
5
0
0
0
08
30


 | N |
| :--- |
| 0 |
| 0 |
| 0 |
| 0 | 20

0
0
0
0
0
0
0
0 $\infty$
N
N
0
0.
0.
0.
0.
 0
$N_{0}$
m
0
0
0
0
0
0
0
 $0=$
0
0
3
3
0
0
0 0
0
3
3
0
0
0
SIEFTC GUYN

COMMON GDA(25), DEL, N, NNF, NTH,
C RMTX(160), TCTR, KZTW, KZTR, KPTW, KPTR
COMMON /PXCMN/ MM, LLN, I, IFLG, LN, IRSFG, IFGFG,

 4 WTH, WFE, GAM, RHO, WFEP, LAM, LAM2, LAMO2, LAM2O2, LAM2O4 6 UCMIKJ, UCM2KJ, DCM3KJ, UCM4KJ, UCMSKJ, UCCIKJ, DCC2KJ, UCC3KJ, 7 DCKIKJ, DCK2KJ, DCK3KJ

## COMMON /BLOCK1) EMM2 150.50 ) COMMON /BLOCK21 EMM3

RERL LAM, LAMO2, LAM2O2, LAM2O4, LAM2OB, LAM2, KORO, KURO2,
1 MFETK, MTHIK, MFETP, LT, LLI

$\qquad$


READ L MATRIX
READ M MATRIX
READ N MATRIX



P ANO $x$ ARE REAUY TO GE CREATEU
P
P ANO $x$ ARE REGUV TO be create

INVERSE REPLACES B ON TAPE

$$
\begin{aligned}
& \text { READ GO }
\end{aligned}
$$

$$
\begin{aligned}
& \begin{array}{l}
\text { CALL MMY (KK5,KK5,KK5, AL, C, EMM2) } \\
\text { CALL MMY (KK5, KK5, KK5, EMM2, AP AL) }
\end{array} \\
& \text { READ } 131 \text { P } \\
& \text { READ SPACE } \\
& \text { READ } 131 \text { P } \\
& \begin{array}{l}
\text { BACKSPACE } 3 \\
\text { BACKSPACE } 3
\end{array} \\
& \begin{array}{l}
\text { CALL MSU (KK5, KK5, P, AL, AL, } \\
\text { CALL MMY (KK5, KK5,KK5, EMM2, B, F }
\end{array} \\
& \text { CALL MMY IKKS. KKSoKKS.EMM2. Be FI } \\
& \text { PE, EMM3) } \\
& \begin{array}{l}
\text { CALL MMY (KK5, KK5, 1, EMM2, PE, EMM 3) } \\
\text { CALL MSU (KKS, 1, EMMA, EMM3, EMM4) } \\
\text { CALL INVNMM (B, 50, KK5, IERR) } \\
\text { IF (IERR ©LE, O) GO TO } 320
\end{array}
\end{aligned}
$$










$$
\begin{aligned}
& \text { BACKSPACE } 3 \\
& \text { WRITE } 1 \text { 3: A, B, C, PE }
\end{aligned}
$$

$$
510 \text { IF ITCTR OGT. 2.0) GO TO } 514
$$








## 3. 12k 2MTRX Subroutine (SLND)



## 








808
0
3
3
80
00
 (

 00002200 0122000 00002220
00002230 00002240 2
2
2
8
8
8 0
$N$
$N$
$N$
0
0
8
8 0
$\mathbf{N}$

0
0
0
0
 10
N
N
80
88
80
08 $00 \varepsilon 20000$ 0

N
N
0
0
8
8 0
N
N
N
N
08
0.8 0
0
0
N
N
0
88
88
80 08
${ }^{\circ}$
8
88
88
8 80
0
0
08
08
08
08 c
N
N
0.
0.8

0.8 | 0 |
| :--- |
| $N$ |
| 0 |
| 0 |
| 0 |
| 0 |

 08
10
0
08
88
88
08 $00220000{ }^{0}$ 0.8
y
8.
88
80 0888
689
8888
8888 8
88
88

## COMPUTATION OF STRAINS ** (STR 139. PG.IT1 **


COMPUTATION OF INTERNAL FORCES ** (STR 139. PG.12) **
AIIL,ITI ESI + B3ILOITI EETH - ENTSIILOITI
111671H11N3-H13*1116712日 + 1S3.111671E日
DIIL,ITI*EKSI + D3IL.ITI EKTH - EMTSIILOITI

$\begin{aligned} & \text { GI3(L,ITH EKSTH } \\ &= G 3(L, I T) * G T H\end{aligned}$


WRITE (6.270) (EMFE(JI, EMTHIJ) EMFIIJI QIHIJI, JEIGIITI

 ( L L LI34d "xzII (VIJMIIOHP 275 COMTINUE


- EAN

$N N$
NN
$0 N$

-1 IMX
-1 ITCN




### 3.13 PROGRAM NOMENCLATURE (4)



AO
A0 (50, 50)

Al (50, 50)
ALF1

ALF2
ALMO (5, 5)

ALM1 (5, 5)
ALMN (5, 5)

ANX

AXL

## (B)

B**

BO (50, 50)

B1 (50, 50)
BBB

Reference length
Ao Boundary matrix, see Equation (1.46), Section 1.8

Another designation for A0 (50, 50)
ENTRY to function subprogram TMP1, used to clefine the coefficient of thermal expansion for the stress calculations of layer one

As ALFl, for layer two
Boundary displacement matrix at the first meridional station, Equation (1.36), Oth harmonic

As ALMO (5, 5) for the lst harmonic
As ALMO $(5,5)$ for the terminal boundary

Angle between the generator and axis of revolution; cone-cylinder option of GEOM.

Axial surface length

Modified B matris for dynamic response, Equation (1.56), Section 1.9

Bo Boundary matrix; see Fquation (1. 4c), Section 1.8

Same as BO (50, 50)
FUNCTION subprogram to define membrane stiffness, Equation (i. 28).

BCD (36)

BCIB

BCIT
BFCN

BMTX (160)

BN (50, 50)

BNDTB (17)

BNDTX (17)
(C)

C0, Cl $(50,50)$

CEXT

CN (50, 50)

CODIMA

COEFC
(D)

DATLD

Three title cards read in executive program

Boundary condition indicator for the terminal boundary; entered with the general data, GDA. See Section 2.8.

As BCIB, for the initial boundary
Deck name for the static version of subprogram, BBB

C $\varnothing$ MM $\varnothing \mathrm{N}$ array which includes $\Omega, \Lambda$ and $\ell$ boundary matrices for the $O t h$ and 1 st harmonics
$\mathrm{B}_{\mathrm{N}} \quad$ Terminal boundary matrix; Equation (1.46), Section 1.8

Special boundary array for station $N$ read with the geometry data, GMDA. See Sections 1.8, 2.8.

As BNDTB, for station one

Co Boundary matrices; see Equation (1.46). Section 1. 8.

Number of time cycles desired. Read with GDA data, Section 3.3.2.
$\mathrm{C}_{\mathrm{N}} \quad$ Terminal boundary matrix, Equation (1.46), Section 1.8.

Parabolic curve fitting subroutine, see Section 3.5.4.

Stiffness coefficient array set up in the DATLYR subroutine.

$$
\begin{aligned}
& \text { Subroutine used to set up prencure } \\
& \text { loads }
\end{aligned}
$$

| DATLNK |  | Sub-executive subroutine which monitors GEOM, DATLYR and DATLD |
| :---: | :---: | :---: |
| DATLYR |  | Subroutine which sets up the stiffnes: coefficient array, COEFC |
| DBBDD |  | Derk name for the Dynamics version o: sibprogram. Bis |
| DCC1, 2, 3 | $\mathrm{D}_{\mathrm{m}}$ | External damping coeffir: :-rnts found in the COEFC array |
| DCK1, 2, 3 | $\mathrm{K}_{\mathrm{m}}$ | Spring coefficients, in COEFC |
| DCM1, 4 | $\mathrm{Mm}_{\mathrm{m}}$ | Mass coefficients of translation and rotation, in COEFC |
| DDD |  | ENTRY to function subprogram BBB, used to define the bending stiffness coefficients of the COEFC array |
| DECRD |  | Data read subroutine; see Section 3.5.1 |
| DEL | $\Delta$ | Interval size between meridional stations |
| DELT | ¿ | Time increment (seconds), read with GDA data. See Section 3.3.2. |
| DELTH |  | Interval size between circumferential stations. Constant $=\mathbf{2}^{\circ}$. |
| DKDMP |  | Deck name for the Dynamics deck which sets up spring and damping coefficients of the array COEFC |
| DKK1, 2, 3 |  | FUNCTICN and ENTRY points to the subprogram which sets up spring coefficient |
| DMASS |  | Deck name for Dynamics deck which forms the mass coefficients |
| DMM1, 4 |  | FUNCTION and ENTRY points of deck DMASS |

DMP1, 2, 3

DN1

DN2

DPRSS

DTMP

DYLMN
(1)

EO
EII, 2

EM

EMTT

FR
FNF
F.NTH

ENTRY points to deck DKDMP to set up the translational damping coefficients.

ENTRY point to function subp:ogram, EII. Sets up distance from neutral axis of the first layer used in stress calculations.

As DN1 for the second stresses

Deck name for the Dynamics version of PPPN which supplies the pressure loadings

Deck name for the Dynamics version of ENTT which gives the temperature load and moment

Subroutine subprogram in the Dynamics deck which sets up the $L, M$ and $N$ matrices of Equation (1.51). Section 1.9

Reference Young's Modulus
FUNCTICN subprograms used to define the moduli of elasticity for the two layers at which stresses are desired

Number of radii entered for the discrete point geometry option

ENTRY point to subprogram ENTT; used to set up temperature moments

Number of meridional stations
Number of Fourier harmonics where the first one is the oth one

Number of circumferential stations, internally set at ninety

| ENTT |  | FUNCTION subprogram which sets the value of the temperature load, ${ }^{t} T^{\prime}$, Equation (1.15), Section 1.6 |
| :---: | :---: | :---: |
| (3) |  |  |
| F(50, 50) | F | Matrix of equilibrium Equation (1.34), Section 1.7 |
| FGHPE |  | Sutroutine which forms the F,G and $H$ matrices of Equation (1.34). In the Statics version the force matrix PE is also set up here. |
| FPRNT |  | Fourier component print values, read with GDA data. See Section 3.3.2. |
| © |  |  |
| G (50,50) | G | Matrix of equilibrium Equation (1.34). Section 1.7 |
| G** | $g_{i, j}^{*}$ | Modified g matrix for dynamic response, Equation (1.56), Section 1.9 |
| GAMA | $\gamma$ | P/P |
| GDA |  | General data array, read by the executive program |
| GEOM |  | Geometry subroutine |
| GIN |  | Geometry indicator read with GMDA data; see Sertion 2.5 |
| GMDA |  | The geometry data array; see Section 3.3.3 |
| GMI |  | \|GIN ${ }^{\text {c }}$ |
| GMTX |  | Dynamics subroutine subprogran which modifies the $B$ and $g$ matrices and forms the $P$ and $X$ matrices of Equation (1.60), Section 1. 10 |


| Hi50, 50) | H | Equilibrium matrix of Equation (1.34), Section 1.7 |
| :---: | :---: | :---: |
| H0 | $\mathrm{h}_{0}$ | Reference thickness |
| (1) |  |  |
| 1 |  | Station number within each block of stiffness coefficients, COEFC, in PANDX subroutine |
| IFDYN |  | PANDX flag which determines when $\mathrm{L}, \mathrm{M}$ and N matrices of the Dynamics routine have been completed |
| IFGFG |  | PANDX flag to determine entry into the FGHPE subroutine |
| IFLG |  | Stiffness coefficient block indicator; used in PANDX |
| IFMX |  | PANDX flag which shows whether the PE (force) matrix is complete |
| IRSFG |  | PANDX flag for boundary computations |
| IRTE |  | PANDX indicator for iirst or last station within a cr. ficient block |
| (3) |  |  |
| JPATH |  | Path indicator through the Fourier harmonic loop of PANDX; used to skip setting up the boundary conditions and the FGHPE subroutine |
| (1) |  |  |
| KFCN |  | Deck name for the Statics version of subprogram DKKI |
| KPATH |  | Path indicator through the Fourier harmonic loop of PANDX; used to skip boundary conditions and the DYLMN subroutine |


| KPTR, KPTW |  | Variable tape numbers 10 or 12 used in the Dynamics version for storing $P$ and $X$ matrices by subroutine GMTX |
| :---: | :---: | :---: |
| KZTR, KZTW |  | Variable tape number 8 or 11 used in GMTX for preserving the three previous solution matrices ( $Z$ ) at each tiree interval |
| (1) |  |  |
| $L$ (50,50) | $\mathrm{L}_{\mathrm{i}}$ | Matrix used in the Dynamics version to modify the g matrix, Equation (1.51). Section 1.9 |
| LLO (5) | $\ell 0$ | Initial boundary, \& matrix; Equation (1.38), Section 1.7 |
| LLLN (5) | $\ell_{N}$ | Terminal boundary, f matrix |
| (N) |  |  |
| M (50, 50) | $\mathbf{M}_{\mathbf{i}}$ | As L (50,50) |
| M (THETA) | $M_{6}$ | Bending moment per unit length in the circumferential direction |
| M (XI) | ${ }^{M} \underline{\underline{E}}$ | Bending moment per unit length in the meridional direction |
| M (XI, THETA) | $\bar{M}_{\xi \theta}$ | Bending moment; shear |
| MFE |  | Current temperature moment at a given meridional station and for $k$ harmonics |
| $\mathbf{M M}$ |  | Absolute station number in PANDX subroutine |
| MMN |  | Station number within block of loads in PANDX |
| MTP |  | First derivative of the temperature moments |

$N(50,50)$
N(THETA)
$N(X I)$
N(XI, THETA)
NNF

NTH
(d)
$\varnothing$ MGO $(5,5)$
$\phi$ MG1 $(5,5)$
$s_{2}$
$\varnothing$ MGN $(5,5)$
(P)

P(50, 50)
PANDX

PF. (50)
n

Fixed point form of EN, number of meridional stations
$\mathrm{N}_{\mathrm{i}} \quad$ As $\mathrm{L}(50,50)$
$\mathrm{N}_{\theta} \quad$ Membrane force, circumferential
$\mathbf{N}_{\xi} \quad$ Membrane force, meridional
$\bar{N}_{\xi \theta} \quad$ Membrane shear force
Fixed point form -f ENF, number of Fourier harmunics

Fixed point form of ENTH, number of circumferential stations $=90$

Boundary force matrix for the 0 th harmonic; Equation (1.36), Section 1.7

As $\varnothing \mathrm{MGO}(5,5)$ for the 1 st harmonic
$\Omega_{3} \quad$ As $\varnothing \mathrm{MGO}(5,5)$ for the terminal boundary

Matrix of Equation (1.60), Section 1.10
Sub-executive subroutine which directs the formation of the various matrices needed in computing the $P$ and $X$ matrices of Equation (1.60), Section 1.10. This subroutine calls the RSLT, FGHPE, DYLMN and GMTX subroutines, the latter two in the Dynamics version only.

Force matrix of Equation (1.34). Section 1.7

| PEL |  | Meridional distance to a station from the initial atation |
| :---: | :---: | :---: |
| PFCN |  | Deck name for the Statics version of subroutine PPPN |
| PFE | $\mathrm{P}_{\xi}$ | Fourier component for load in the meridional direction |
| PFLAG |  | Print flag, read with GDA, general data. See Section 3. 3.2. |
| PHIO | $\phi_{0}$ | Initial opening angle from verical axis for sphere or toroid |
| PHIN | $\phi_{\mathrm{N}}$ | Final opening angle from vertical axis for sphere or toroid |
| PHI (THETA) | $\Phi_{\theta}$ | Rotation in the circumferential direction |
| PHI (XI) | $\Phi_{\xi}$ | Rotation in the meridional direction |
| PHS |  | Program name for PHI (XI) |
| PHT |  | Program name for PHI (TIFFTA) |
| PN | $p_{\vdots}$ | Fourier component for load in the normal direction |
| PCI |  | Poisson's ratio |
| POIS 1 |  | ENTRY point to function subprogram. EIl. Sets up Poisson's ratio for use in the stress calculations for the first layer. |
| POIS2 |  | As POIS for the second stresses |
| PPPF |  | ENTRY point to function subprogram, PPPN. Defines the meridional pressure. |
| PPPH |  | As PPPF for circumferential presure |


| PPPN |  | FUNCTI $\varnothing$ N subprogram for pressure loadings. See Section 1.5 for sign convention. |
| :---: | :---: | :---: |
| PTH | $\mathrm{P}_{\theta}$ | Fourier component for load in the circumferential direction |
| Q |  |  |
| Q (THETA) | $Q_{6}$ | Transverse force per unit length in the circumferential direction |
| O (XI) | $Q_{\xi}$ | Transverse force per unit length in the meridional direction |
| (A) |  |  |
| R | $r$ | Normal distance from axis to shell |
| R (50, 50) | R | Boundary matrix of Equation (1.46), <br> Section 1.8 |
| RAl |  | Radius of cone or cylinder at station 1 |
| RC |  | Radius of curvature of sphere or toroid |
| RCURV |  | Input values of rerisional :-adius of curvature |
| RCURZ |  | Input values of circumferential radius of curvature |
| RHOX | $\rho$ | R/AO |
| RIPT |  | Discrete radii for general shell shaje |
| R $\quad$ FF |  | Offset distance of center of curvature from axis of revolution, for toroids |
| RSLT |  | Subroutine which computes the bounda:y matrices for the $P$ and $X$ matrices |


| (S) |  | Oi |
| :---: | :---: | :---: |
| $S(50,50)$ | S | Roundary matrix of Equation (1, 46), Section 1.8 |
| scmi, 2, 3 | $\mathrm{B}_{\mathrm{m}}$ | Membrane stifiness coefficients |
| SCDI, 2, | $\mathrm{D}_{\mathrm{m}}$ | Bending stiffness coefficients |
| SC.G1, 2,3 | $G_{m}$ | Shear stiffness coefficients |
| sc.ali | $\mathrm{G}_{13}$ | Sheir. twist stiffness coefficient |
| SCMITSI | $M_{\xi(n)}^{T}$ | Meridional thermal moment coefficients |
| SCMITH | $M_{\theta(n)}^{T}$ | Circumferential thermal moment coefficients |
| SCNTSI | $\mathfrak{t}_{\xi(n)}^{T}$ | Meridional thermal load coefficients |
| SCNTTH | ${ }^{t}{ }_{\theta(n)}^{T}$ | Circumferential thermal load coefficients |
| SIG 0 | $\sigma_{0}$ | Referencestress |
| SIG (THETA) | $\sigma_{\theta}$ | Circumferential stress |
| SIG (THETA, ETA) | ${ }^{\top} \theta \zeta$ | Circumferential transverse shear stress. |
| SIG (XI) | $\sigma_{\xi}$ | Merizional stress |
| SIG (XI. ETA) | ${ }^{\top}{ }_{\xi}{ }_{\zeta}$ | Meridional transverse shear stress |
| SIG (XI, THETA) | $\sigma_{\boldsymbol{\xi} \theta}$ | In-plane shear stress |
| SUMS |  | Subroutine which does the Fourier summing for the deflections and rotations, computes the internal loade and, in the Static version, stresses |
| ( ${ }^{\text {a }}$ |  |  |
| TCTR |  | Time cycle number |

TFCN

TFE

THT

TMP1, 2

TTF1, 2

TTP

TU
(U)

U
(V)

V
(4)

W

W (THFTA)

W (XI)

WDOT

WFFPX

Deck name for the Statics version of funcion subprogram, ENTT

Current temperature load at a given meridional station and for $k$ harmonics

Circumferential angle THETA
(degrees) at which print-outs of results are desii ed. (5 permitted)

FUNCTION subprograms used to define the temperature for the two layers at which stresses are desired

Deck names for function subprograms, TMP1 and TMP2, respectively

First derivative of the temperature loads
t. Current time
$u_{\xi} \quad \begin{aligned} & \text { Meridional displacement; see } \\ & \text { Figure } 1.2\end{aligned}$ Figure 1.2

Circumferential displacement; see
$\begin{array}{ll}u_{\theta} & \text { Circumfere } \\ & \text { Figure } 1.2\end{array}$
w
${ }^{\omega} \xi$
$\frac{\partial W}{\partial t} \quad$ Velocities
$\frac{\partial \omega_{\xi}}{\partial \xi}$
$\omega_{\theta} \quad$ Circumferential curvature, print heading
Normal displacement; see Figure 1.2 ,

Meridional curvature, print heading

First derivative of meridional
curvatures

| WFEX | $\omega_{\xi}$ | Program name for W (XI) |
| :---: | :---: | :---: |
| WTHX | ${ }^{\omega} \theta$ | Program name for W (THETA) |
| (3) |  |  |
| X (50) | x | Matrix of Equation (1.62), Section 1.10 |
| XI | $\boldsymbol{\xi}$ | Meridional distances to stations computed in GEOM |
| XIPT |  | Discrete XI distances or arc lengths array intered with GMDA |
| (2) |  |  |
| Z (50) | 2 | Solution matrix, Equation (1. 38), Section 1.7 |
| ZMTRX |  | Subroutine which solves for the $Z$ matrix using the $P$ and $X$ matrices |
| ZP |  | Solutions for previcus time cycle |
| ZPP, ZPPP |  | Solution matrices for the secord and third previous cycles |
| ZTA | $\zeta$ | Circumferential distance to a station |
| 2X (50) |  | Another name for Z (50) |


[^0]:    *See Appendix

[^1]:    ORM 144-E-17 REV. 75

[^2]:    

[^3]:    ENO EXECUTGN

[^4]:    IS COEFF BLOCK COMPLETED
    no
    
    

