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Har. copy (HC) 3.00

Microfiche (MF) .65

ff 653 July 65

N 68-11964	
(ACCESSION NUMBER)	(THRU)
21	1
(PAGES)	(CODE)
CR-91118	24
(NASA CR OR TMX OR AD NUMBER)	(CATEGORY)

FF No. 602(C)

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Alpha Particle Elastic Scattering*

by

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* This research was supported in part by the Atomic Energy Commission under contract ATC(11-1)-1190 and by the National Aeronautics and Space Administration under contract NsG-233-62.

ABSTRACT

The elastic scattering of alpha particles by alpha particles is calculated from threshold to 100 MeV laboratory energy using both the Schroedinger equation and the N/D equations. The N/D equations do not yield very accurate results. The Schroedinger equation solution extends an earlier calculation of Preist to higher energies, higher partial waves and uses more recent values of meson nucleon coupling constants. With slight changes in their values we obtain good agreement with experimental phase shifts to 100 MeV for $\ell = 0, 2, 4$.

I. INTRODUCTION

The elastic scattering of alpha particles by alpha particles has been measured experimentally up to one hundred and twenty MeV lab energy.[1] Several theoretical calculations have been made.[2,3] Okai and Park, using a Guassian type potential for the two nucleon interaction, with five of six parameters determined by low energy data, solve the integro-differential equation which results when exchange forces are allowed for. Preist neglects exchange and uses the one boson exchange potential for the σ and the ω mesons, the coupling constants being taken from early phase shift fits of nucleon nucleon data.[8] A more recent fit[4] in which the I=0 mesons are the σ, the ω and the φ mesons, gives very different values for the coupling constants, in particular the extreme repulsion of the ω particle is considerably reduced. Using the more recent values, we calculate phase shifts for ℓ = 0, 2, 4 partial waves for lab energies between 0 and 100 MeV.

Writing the potential as a Fourier transform,

$$V(r) = \frac{-i}{\pi} \frac{1}{r} \int_{-\infty}^{+\infty} k\,dk\, e^{ikr}\, V(k) \tag{1}$$

the central part of the two nucleon potential due to scaler or vector meson exchange is[9]

$$V_s(k) = -\left[g_s^2 \left(1 - \frac{1}{q}\frac{m_s^2}{m^2}\right)\right] \frac{1}{k^2 + m_s^2} = -\frac{c_s^2}{k^2 + m_s^2}$$

$$V_v(k) = \left[g_v^2 \left(1 + \frac{1}{4}\frac{m_v^2}{m^2}\right)^2\right] \frac{1}{k^2 + m_v^2} = \frac{c_v^2}{k^2 + m_v^2} \qquad (2)$$

This one boson exchange potential is a non-relativistic reduction of the field theoretic potential. It is stated to be accurate to order (meson mass/nucleon mass) squared. In the case of the ω and the φ mesons this is certainly meaningless and we report only values of c_s^2, c_v^2. To obtain the potential between two alpha particles we multiply (2) by the square of the form factor f^2 of the alpha particle. We use the form factor of Preist[3]

$$f(k^2) = 4\left(\frac{W^2}{W^2 + k^2}\right)^4 \qquad (3)$$

for the mesons and

$$f(k^2) = 2\left(\frac{W^2}{W^2 + k^2}\right)^4 \tag{4}$$

for the photon. The factors come from 4 nucleons and 2 protons in each alpha particle. In units in which the pion mass is one, w^2 has the value 77.07.

The ambiguity in the non-relativistic reduction of the potential might be avoided by solving the N/D equations and using the full relativistic form. We will conclude, however, that the N/D equations are not sufficiently accurate.

Both the Schroedinger equation

$$\frac{d^2u(r)}{dr^2} + [q^2 - MV(r) - \frac{\ell(\ell+1)}{r^2}]\ u(r) = 0 \tag{5}$$

and the N/D equations[4]

$$N_\ell(s) = B_\ell(s) - \frac{1}{\pi}\int_0^\infty \frac{\rho(s')ds'}{s' - s}\left[\frac{s+s_o}{s'+s_o}\ B_\ell(s) - B_\ell(s')\right] N_\ell(s')ds' \tag{6}$$

$$D_\ell(s) = 1 - \frac{(s+s_o)}{\pi}\int_0^\infty \frac{\rho(s')\ N_\ell(s')ds'}{(s'-s)\ (s'+s_o)} \tag{7}$$

have been solved numerically. Here with M the mass of the alpha particle η is the electrostatic constant ($\eta = \frac{Me^2}{2q} = .4032/q$); $s = q^2$, the momentum squared; $\rho(s)$ the phase space factor modified by coulomb effects

$$\rho(s) = \sqrt{\frac{s}{s+4.M^2}}\; q^{2\ell} \prod_{j=1}^{\ell} \left(1+\frac{\eta^2}{j^2}\right) \frac{2\pi\eta}{e^{2\pi\eta}-1} \tag{8}$$

and $B_\ell(s)$ the Born term[6]

$$B_\ell(s) = -\frac{2^{2\ell} M (\ell!)^2}{((2\ell+1)!)^2} \int_0^\infty e^{2iqr} \left[{}_1F_1(\ell+1+i\eta,\ 2\ell+2, -2iqr)\right]^2 \times V(r) r^{2\ell+2}\, dr \tag{9}$$

where ${}_1F_1$ is the confluent hypergeometric function[5]

$${}_1F_1(\ell+1+i\eta,\ 2\ell+2,\ -2iqr) = \frac{\Gamma(2\ell+2)}{\Gamma(\ell+1+i\eta)\,\Gamma(\ell+1-i\eta)} \int_0^1 e^{-2iqrt}\, t^{\ell+i\eta}\, (1-t)^{\ell-i\eta}\, dt \tag{10}$$

To evaluate the Born term 4 integrals must be performed (over r, k, t, t′). The r integration and one of the t integrations can be done analytically. Making a change of variables yields

$$B_\ell(s) = C \int_{-\infty}^{+\infty} k\, dk\, \frac{1}{k^2+\mu^2} \left(\frac{W^2}{W^2+k^2}\right)^8 J \tag{11}$$

$$J = -\left(\frac{k}{2q} - 1\right)^{-2i\eta} \frac{1}{2^\ell} \int_{-1}^{+1} \left[\frac{(t-1)}{(t+1)}\, \frac{1}{2}\left|t+1-\frac{k^2}{2q^2}\right|\right]^{i\eta} \frac{(t^2-1)^\ell\, dt}{\left(t+1-\frac{k^2}{2q^2}\right)^{\ell+1}} \tag{12}$$

$$C = \frac{i}{\pi} Mg^2\, 2^{2\ell}\, (-1)^{\ell+1} \frac{(\ell!)^2}{|\Gamma(\ell+1+i\eta)|^2} \frac{1}{(2q)^{2\ell+2}} \tag{13}$$

We notice that for $\eta=0$ (no coulomb effects) and $w = \infty$

$$J = 2(-1)^{\ell+1} Q_\ell\left(1 - \frac{k^2}{2q^2}\right) \tag{14}$$

$$B_\ell(s) = -\frac{Mg^2}{2q^{2\ell+2}} Q_\ell\left(1 + \frac{\mu^2}{2q^2}\right) \tag{15}$$

II. NUMERICAL PROCEDURE

1. Schroedinger Equation

The potential, equation 1, has been evaluated numerically. Because of the rapid decrease of the integrand due to the form factor, it is sufficient to do the integral from $k = 0$ to $k = w$. A twelve point per cycle of the trignometric function in the integrand Simpson's rule is used. The wave function given by the Schroedinger equation is numerically integrated out to $r = 7\mu$-1 (μ, pion mass) and matched to

$$u(r) = N(\cos\delta_\ell \, F_\ell(qr) + \sin\,\delta_\ell \, G_\ell(qr)) \tag{16}$$

the Coulomb wave functions being generated by the procedure described in reference 7. A three point formula based directly on a second order differential equation is used. A step size .02 from r=0 to 1 and a step size .06 from r=1.0 to 7.0 is used. About one minute is required on a IBM 7044 computer, most of the time being consumed in evaluating the potential.

2. Born term $B_\ell(s)$

The k integration, equation 11, is trivial but in the t integrand, equation 12, there are singularities at the end-points $t = \pm 1$. The following procedure is used to evaluate the t integral numerically.

For $|k^2/2q^2| < 5$ we make a polynomial fit

$$\left[\frac{1}{2}\left(t+1-\frac{k^2}{2q^2}\right)\right]^{i\eta} = a_1 + a_2 t + \ldots + a_n t^{n-1} \tag{17}$$

and do the residual integral I_k analytically

$$I_k = \frac{1}{2^\ell}\int_{-1}^{+1}\left[\frac{1-t}{t+1}\right]^{i\eta} \frac{t^{k-1}(t^2-1)^\ell\, dt}{\left(t+1-\frac{k^2}{2q^2}\right)^{\ell+1}} \tag{18}$$

The I_k can be expressed as a finite series of terms which can be generated by recursion relations. These recursion relations become unstable when $|k^2/2q^2|$ is greater than 5. Legendre polynomials which are orthogonal on the integual $t = -1$ to $+1$ are used to make the fit, equation 17, and the a_k can be generated by recursion relations. Then

$$J = -\left(\frac{k}{2q} - 1\right)^{-2i\eta} e^{-\pi\eta} \Sigma a_k I_k \tag{19}$$

For $|k^2/2q^2| > 5$ we make a polynomial fit

$$\left[t+1-\frac{k^2}{2q^2}\right]^{i\eta-\ell-1} = a_1 + a_2 t + \ldots + a_n t^{n-1} \tag{20}$$

and do the residual integrals I_k analytically

$$I_k = \frac{1}{2^\ell} \int_{-1}^{+1} \left(\frac{1-t}{t+1}\right)^{i\eta} t^{k-1}(t^2-1)^2 \, dt \tag{21}$$

The I_k can be expressed as a finite series of terms which can be generated by recursion relations. To make the fit, equation 20, we use a Taylor series about $t = 0$. Then

$$J = -\left(\frac{k}{2q} - 1\right)^{-2i\eta} \frac{e^{-\pi\eta}}{2^{i\eta}} \Sigma a_k I_k \tag{22}$$

Finally, for $s = q^2 = 0$ a value can be obtained by using the limit as $q \to 0$ of the confluent hypergeometric function

$${}_1F_1(\ell+1+i\eta, 2\ell+2, -2_i kr) \xrightarrow[q\to 0]{} (2\ell+1)! \sum_{k=0}^{\infty} \frac{(2q\eta r)^k}{(2\ell+k+1)!\, k!} \tag{23}$$

to obtain

$$B_\ell(0) = C \int_{-\infty}^{+\infty} kdk \frac{1}{k^2+\mu^2} \left(\frac{W^2}{W^2+k^2}\right)^8 J \tag{24}$$

$$J = \frac{1}{(-ik)^{2\ell+2}} \sum_{k=0}^{\infty} \left(\frac{2\eta q}{-ik}\right)^k \frac{1}{(2\ell+1+k)!} \begin{pmatrix} 4\ell+2+2K \\ K \end{pmatrix} \tag{25}$$

$$C = -Mg^2 \, (\ell!)^2 \, 2^{2\ell} \left(\frac{-i}{\pi}\right) \tag{26}$$

The inner integral, equation 12, need be done only once. It has the same value for each of the three mesons and the photon. It does not need to be evaluated at each point desired in solving equation 7 as it varies smoothly and can be fit, to better than one percent, to a set of pole terms of first and second orders

$$B_\ell(s) = \sum_{i=1}^{5} \frac{a_i}{s+b_i} + \sum_{i=1}^{5} \frac{a'_i}{(s+b_i)^2} \tag{27}$$

In the calculations below, B is evaluated at 11 points between 0 and 100 MeV laboratory energy and equation 27 is used to interpolate 40 points.

The accuracy of the above procedure for evaluating the Born term can be checked by using the Schroedinger equation. If the coupling constants are multiplied by a small constant, say .001, both the Schroedinger equation and the N/D equations should yield essentially the same result, the first Born approximation. It is found when this is done, that the resulting phase shifts agree to better than one per cent.

As an aid to the numerical accuracy of the above procedure, the k integration equation 9 is done off the real axis. This places a lower limit on the magnitude of the quantity in parenthesis in equations 17 and 20 facilitating their fit to a polynomial. Further, it simplifies the part of the coulombic potential due to the non-point nature of the alpha particle. With the k integration due off the axis (that is, a pole at k=0 removed), this is just given by equation 11 with μ=0. This coulombic residue is unimportant and could be ignored altogether.

3. The N/D Equations

The integral equation, equation 6, for the N function is non-singular. The integral is treated by Simpson's rule: 40 points are chosen between 0 and 100 MeV laboratory energy, 10 between 100 and 200 MeV, and 10 above 200 MeV; 60 points in all. The resulting 60 by 60 matrix equations are solved by Guass Jordon reduction. The D function is then obtained by a quadrature, the singular point of the principle value integral being passed over by making an analytic approximation at that point. The phase shift is determined by [4]

$$\frac{1}{Cq^{2\ell+1}} \; e^{i\delta_\ell} \sin\delta_\ell = \frac{N_\ell(s)}{D_\ell(s)}$$

$$\text{where} \quad C = \prod_{j=1}^{\ell} \left(1 + \frac{\eta^2}{j^2}\right) \frac{2\pi\eta}{e^{2\pi\eta}-1} \quad .$$

The calculation takes approximately one minute on a IBM 7044 computer, most of the time being consumed is generating the Born term.

As mentioned, the accuracy of the two programs, the Schroedinger solution and the N/D solution can be checked against one another by multiplying the coupling constants by a small number. When this is done the phase shifts agree to within a per cent.

Comparing the two methods, the Schroedinger equation is slightly quicker and presumably more accurate; the N/D equation yields phase shifts at a much larger number of points.

III. RESULTS

Preist used the early values[8] $g_\sigma^2 = 5.61$, $g_\omega^2 = 16.7$. To obtain a fit he had to reduce the attractive g_σ^2 to 2.95. We show his results in Figure 1 for the $\ell = 0,2$ waves. The $\ell = 4$ phase shift only rises to 60° at higher energies for these values while the experimental value is 140°. Using a simple gradient search program, we tried to improve the fit below 40 MeV for $\ell = 0,2$. The attractive g_σ^2 climbed back to 5.45 and the repulsive g_ω^2 nearly tripled to 45.4. The large changes in the coupling constants needed to fit α-α data indicates they are not good values.

In Figure 2 we show the results using the more recent coupling constants of Scotti and Wong. We take as the masses of the mesons $m_\omega = 780$ MeV, $m_\sigma = 437$ MeV, $m_\omega = 780$ MeV. Scotti and Wong give the values

$$g_\sigma^2 = 3.05 \qquad g_\omega^2 = 2.77 \qquad g_\varphi^2 = 2.26$$

for their fit to nucleon nucleon data. To obtain the fit shown in Figure 2, we use the values

$$c_\sigma^2 = 2.66 \qquad c_\omega^2 = 3.07 \qquad c_\varphi^2 = 2.95$$

The small change in the coupling constants particularly when the ambiguity in the non-relativistic reduction is recalled, is further confirmation of their values. Also shown in Figure 2 is the N/D result. The potential, it would seem, is too strong for the approximation of replacing the left hand cut by its nearest singularity to be very accurate.

We have neglected exchange effects in this calculation. Expecting exchange effects to be more important at lower energies than at higher, we mention that we have difficulty in fitting the d wave below 10 MeV (we obtain $\delta_2 = 11°$ at 5 MeV against an experimental $\delta_2 = 37.5°$) and the $\ell=4$ wave below 30 MeV (we have $\delta_4 = 5°$ at 20 MeV with an experimental $\delta_4 = 27.7°$). In Figure 3 we show an attempt to improve the fit for $\ell=0$ and 2 at lower energies. There is no improvement.

ACKNOWLEDGMENTS

The author would like to thank Dr. J. A. Van Allen for his kindess in helping him obtain funds for the computer work.

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FIGURE CAPTIONS

Figure 1 s and d phase shifts below 40 MeV. The solid lines are Preist results, the dashed lines the result of a gradient search. The crosses indicating the trend of experimental data are the values fitted.

Figure 2 $\ell = 0, 2, 4$ phase shifts below 100 MeV. The solid lines are the Schroedinger equation result, the dashed, the N/D equations results. The crosses indicate the trend of experimental data and are the values fitted.

Figure 3 s and d waves below 40 MeV.

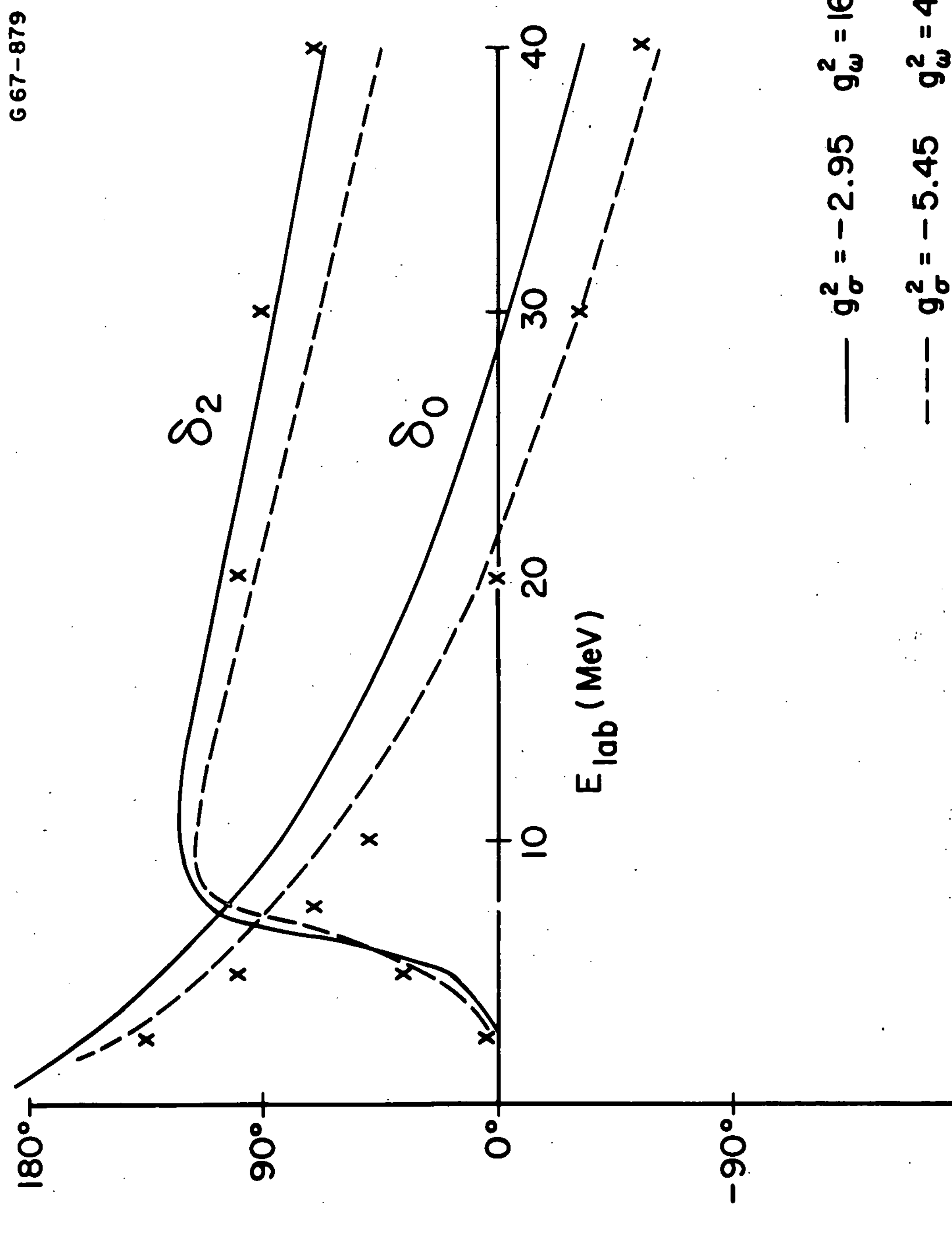
G 67-879
180°
90°
0°
-90°
-180°
δ_2
δ_0
10
20
30
40
E_{lab} (MeV)
$g_\sigma^2 = -2.95 \quad g_\omega^2 = 16.7$
$g_\sigma^2 = -5.45 \quad g_\omega^2 = 45.4$

FIGURE 1

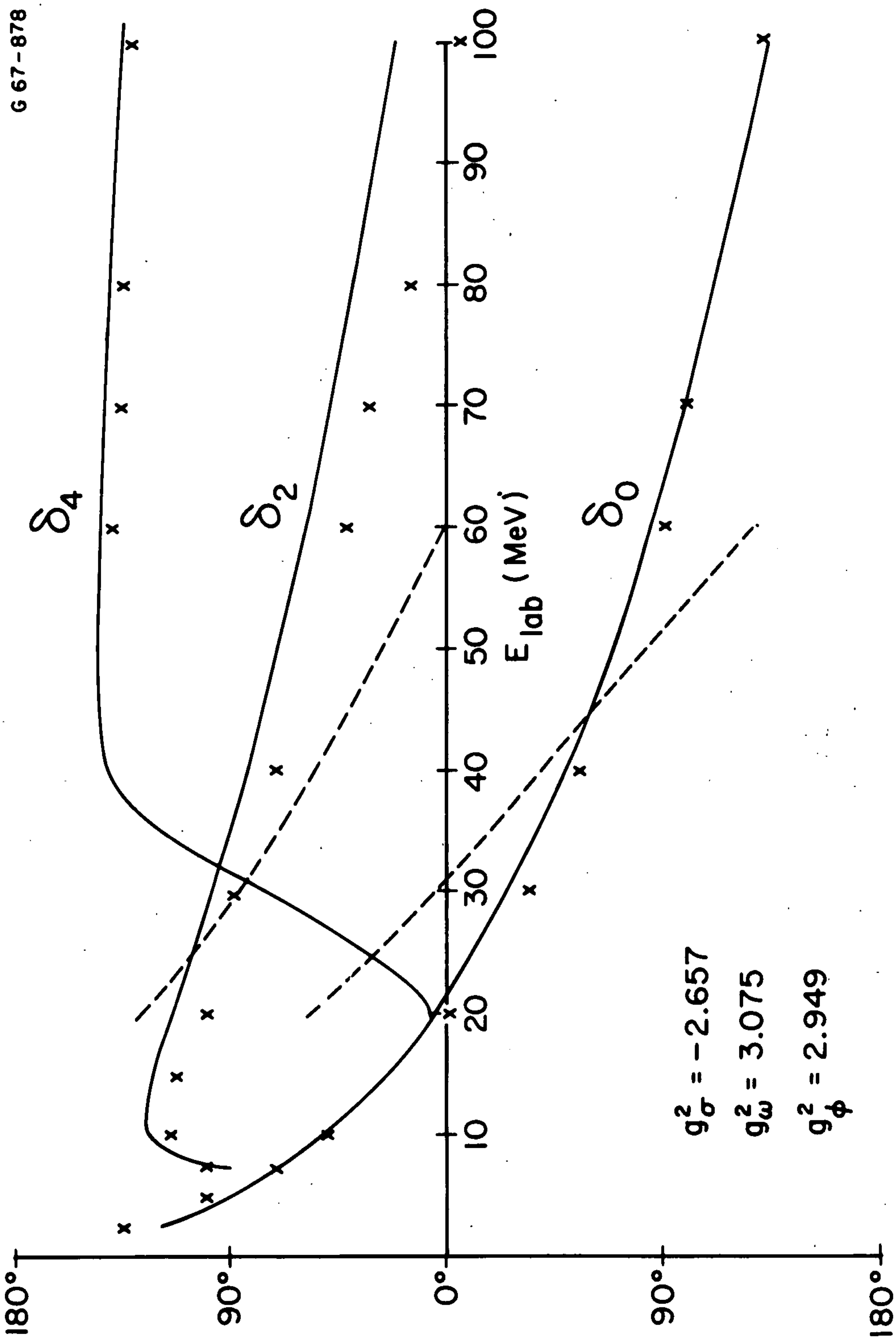
G 67-878
δ_4
δ_2
δ_0
E_{lab} (MeV)
10
20
30
40
50
60
70
80
90
100
180°
90°
0°
90°
180°
$g_\sigma^2 = -2.657$
$g_\omega^2 = 3.075$
$g_\phi^2 = 2.949$

FIGURE 2

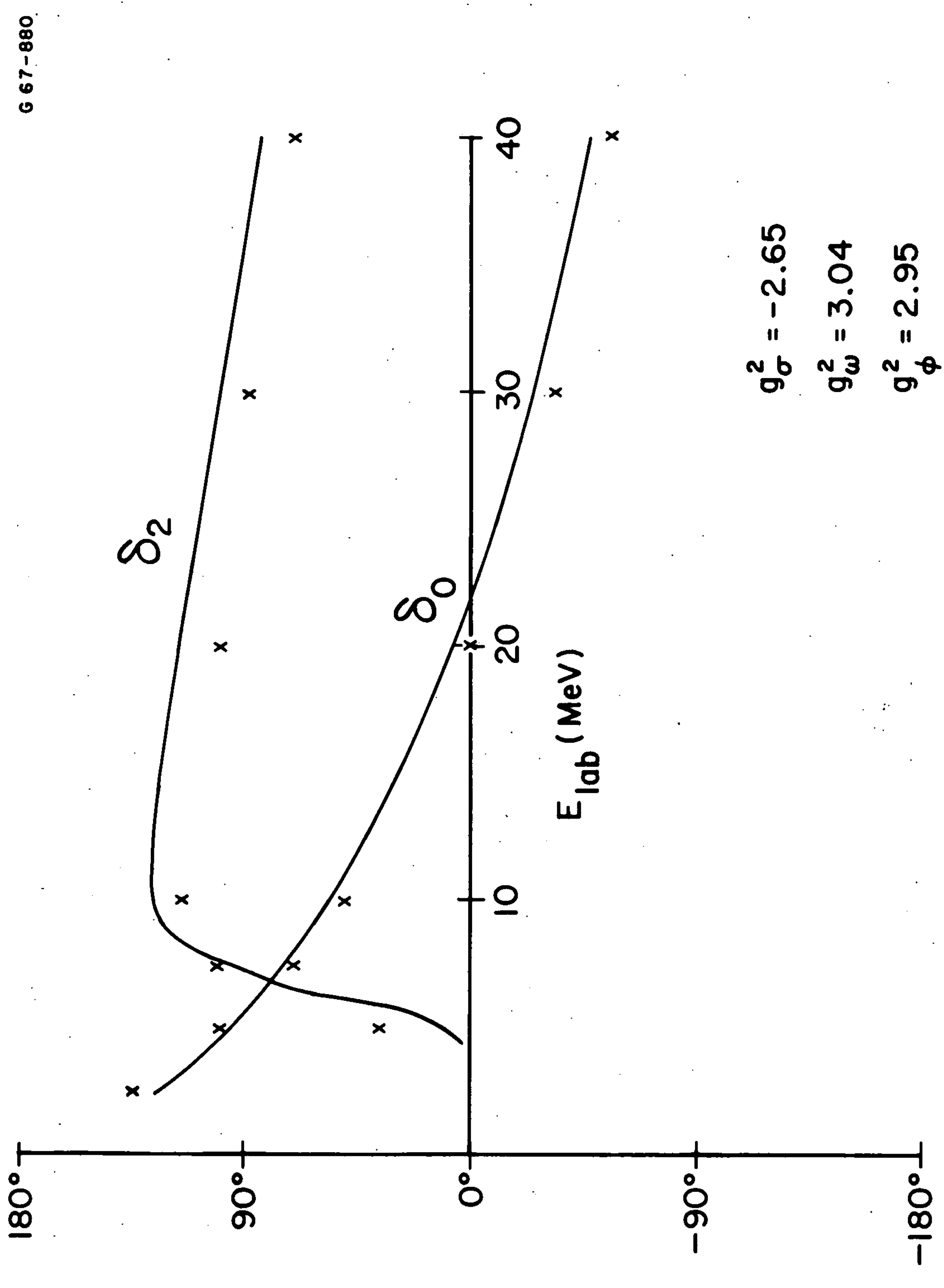
G 67-880
180°
90°
0°
-90°
-180°
10
20
30
40
E_lab (MeV)
δ2
δ0
g²σ = −2.65
g²ω = 3.04
g²φ = 2.95

FIGURE 3