HREC-1022-1
LMSC/HREC A784870

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PREDICTION OF ELECTRON DENSITY IN ROCKET EXHAUST PLUMES

October 1967

Contract NAS8-21022

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FOREWORD

This document constitutes an interim report on the investigations and analysis performed by Lockheed Missiles & Space Company, Huntsville Research & Engineering Center, Huntsville, Alabama, under Exhibit A of Contract NAS8-21022 in support of the Aero-Astrodynamics Laboratory at Marshall Space Flight Center, Huntsville, Alabama.

The study, entitled "The Prediction of Electron Density in Saturn IB Rocket Exhausts," was conducted under the technical direction of Dr. R. Farmer and Mr. T. Greenwood, members of the NASA/MSFC technical staff.

Presented herein is a discussion of gasdynamic and thermochemical problems involved with the prediction of electron density prediction for rocket exhaust plumes.

ACKNOWLEDGEMENTS

The authors wish to acknowledge the consultation and guidance of the NASA technical monitors, Dr. R. Farmer and Mr. T. Greenwood, during the course of this study. The contribution of the many Lockheed personnel to the completion of this task is also gratefully acknowledged.

SUMMARY

The prediction of electron density in rocket exhaust plumes is based on a detailed description of several flow regimes and chemistry models. The analytical methods which are discussed in this report represent a set of analysis techniques and computer programs created at Lockheed/Huntsville, General Applied Sciences Laboratories, AeroChem Corp., and NASA/Lewis. To discuss the theoretical approach taken to each of the disciplines involved is beyond the scope of this report. This report, however, is intended to acquaint the reader with the controlling phenomena, basic analytical approach taken to the description of these phenomena, and to present sample calculations that will illustrate important aspects of the problem. For a detailed description of the analytical approach, derivation of equations, assumptions, etc., the reader is referred to the supporting documentation.

The technical discussion is presented in accordance with phenomenological areas and discussion of models for:

- Chemistry and electron prediction
- Chamber region
- Transonic region
- Supersonic region
- Free shear layer region
- Complete integrated analysis

Several important improvements in accepted state-of-the-art methodology are included in the complete model. These include:

- A treatment of combustion inefficiencies
- An improved transonic analysis
- Treatment of fuel striations in the exhaust
- Finite rate shear layer analysis with ignition delays

The analytical model is not without shortcomings. There are certain theoretical flaws which should be overcome. Additional effort should be expended on:

- More complete combustion chamber analysis, considering two- or three-phase flow.
- Exhaust plume Mach disc treatment.
- Exhaust plume intermediate wake description, in which inviscid region and viscous regions are of the same order of magnitude.

Adequate treatment of these problems is beyond the state of the art and will require future attention.

CONTENTS

Section 1
INTRODUCTION

The presence of free electrons in rocket exhausts has an important impact on vehicle/ground communications. The degree of attenuation of communications links, depending on the line of sight, the attitude of the vehicle and the propellant characteristics, may range from slight to total. After the local state properties in the exhaust are known, prediction of electron densities is essentially a chemistry problem. The problem then becomes one of predicting the local flow properties. In the actual case the thermochemistry of the propellant (or propellant/air combination) is an intimate part of the flowfield prediction. Fortunately, the electron-producing reactions are of little importance in determining the global properties of the propellant exhaust. This allows flowfield prediction and electron density prediction to be uncoupled.

The propellant flow passes through several speed regimes, each with its own unique problems which require special attention and special analysis techniques. These regimes are:

- Low speed or combustion region
- Transonic regime
- Supersonic regime
- Mixing region which contains elements of all of the above.

In addition, the question of suitable thermochemistry prevails throughout.

In the subsequent discussion the main features of all disciplines are reviewed and suggested methods of analysis within the framework of available tools will be presented.

Section 2
TECHNICAL DISCUSSION

The prediction of electron densities in a reacting flow is dependent upon thermochemical considerations not only because the presence of electrons in a closed system is solely a thermochemical consideration, but also because the open system local flow conditions depend on the chemistry of the gas. Because this question of chemistry is an overriding one that exists in all flow regimes, it is discussed first.

2.1 THERMOCHEMISTRY MODELS

2.1.1 Inviscid Flows

Flows in a rocket engine from combustion chamber to nozzle exit and through the plume are not always in thermodynamic and chemical equilibrium. These nonequilibrium conditions occur for situations in which the characteristic times for various molecular relaxation processes may be of the same order as the residence time of the fluid. Processes which may be in a state of nonequilibrium include chemical and ionizing reactions and adjustments of molecular vibrational energies. Rotational relaxation is rapid enough to remain close to equilibrium in rocket and plume flows (Reference 1).

For the case of a chemical reaction occurring in an accelerating gas because of decreasing temperature and pressure, the energy released during the reaction is in the form of thermal energy. Within the expanding portion of the nozzle, some of this thermal energy is converted to kinetic energy. The most energy is released from a reaction if it is fast enough to remain close to equilibrium.

Coupling exists between the chemical kinetic and gas dynamic portion of the problem because the rate of energy release affects the flow and the rate of reaction through the temperature. The correct description of the

flows related to rocket engines and the resultant plumes requires that real gas effects and conditions of equilibrium and nonequilibrium be considered.

In an expansion process such as those found in nozzle and plume flows, three regions of flow reaction can be defined. In the first portion of the expansion nozzle, the flow usually stays in a state of equilibrium. Downstream, as the density of the gas falls, the rate of reaction is reduced and significant departures from equilibrium begin to appear. This portion of the flow is designated as a transition region. As the flow continues to expand, the reactions are slowed to such an extent that the fluid appears to be frozen and unchanging in its composition, hence flow in this region is called "frozen flow."

In practice many useful results can be obtained by using the sudden freezing treatment of the gas in the expansion process. The sudden freezing approach reduces the transition region to a point. The entire flow is then calculated using equilibrium conditions upstream of the freezing point and frozen conditions in all calculations downstream of the freezing point.

2.1.2 Viscous Flow

The chemistry model used to describe a shear layer is important since the mixing of gas streams often creates combustible mixtures which drastically change the temperature and electron density profiles. It is difficult to establish a freezing criterion for a mixture in which the atomic composition is constantly changing. Shear flows are, therefore, treated using a chemistry model of either a finite rate reacting gas or an equilibrium gas.

2.2 CHAMBER ANALYSIS

The combustion chamber is one of the most important and least understood aspects of the propulsion system. The reason is that the chemistry, the mixing, the presence of more than one phase and the gas dynamic considerations are all of primary importance and neglect of any of these considerations may introduce severe errors into the analysis. The most common practice for evaluating chamber conditions and, for that matter, flow conditions prior to entrance into the diffuser is by one-dimensional equilibrium techniques.

2.2.1 One-Dimensional Equilibrium Chamber Analysis

The NASA/Lewis Thermochemical Program (Reference 2) is one of the best computer programs available for a one-dimensional analysis. Although it is basically a closed system solution, the assumptions of equilibrium or frozen flow allow an uncoupling of the flowfield equations and the chemistry. By specifying the reservoir conditions and the state properties at a local condition, it is possible to arrive at the kinetic energy of the flow at that point. Under the assumption of one-dimensional flow from a "rest" condition in which the propellants were originally reacted, the properties as a function of area ratio are predicted. The key assumptions in this analysis are

- Perfect mixing,
- Infinite area ratio combustion, and
- Equilibrium (or infinite reaction rate) combustion.

Unfortunately, only the third item is a valid assumption in practical rocket engines. In certain engine/propellant configurations, even that assumption may not be appropriate and a finite rate one-dimensional analysis would be required. If the reactants are uniformly mixed and the equilibrium assumption is valid, steps may be taken to perform a more appropriate one-dimensional analysis of a finite area combustor.

2.2.2 Finite Area Combustor Effects

If the reactants are ignited in an infinite area combustor (or a closed system), then the original combustion pressure is the total pressure of the subsequent expansion. In actuality, however, combustion occurs in a finite area (usually cylindrical) duct which allows the expansion process to begin immediately. The problem then becomes one of combustion in a moving stream. This phenomenon is analogous to heat addition in a moving stream, which in subsonic flow causes a static pressure and total pressure drop along with an increase in velocity. The conservation equations in conjunction with equilibrium thermochemistry and the chamber-to-throat area ratio are sufficient to permit a one-dimensional analysis of the total pressure drop. After the total pressure drop is established, the normal equilibrium thermochemical solution may be re-run at the corrected "combustion pressure."

Usually the momentum of the inlet gases may be neglected. The solution in either case becomes an iterative affair, simultaneously satisfying the conservation equations and the choked-flow condition. An ideal gas analysis of this type was performed by Yedlin (Reference 3) for several Saturn engines.

Figure 1 shows the total pressure drop as a function of area ratio for an isentropic exponent of 1.2. As shown in the figure, the loss approaches 25% as the area ratio approaches unity.

The recommended approach is:

- Perform an equilibrium analysis with the actual injector pressure.
- Choose a representative isentropic exponent and calculate the total pressure drop
- Re-run the thermochemistry solution at the corrected total pressure.

In the recommended procedure the iterative solution was omitted, since in most systems the isentropic exponent changes only slightly, and therefore the first approximation is reasonably accurate. In the event that greater accuracy is needed, effort of the iterative solution is warranted. A more complete treatment of the required calculation is given by Lawrence (Reference 4).

In the above discussions, all of the chemical energy of the reactants was assumed to be made available to the reaction. For a variety of reasons, this may not be true. Droplets and improper mixing are two important mechanisms for withholding some of the energy.

2.2.3 Energy Loss by Withholding Inerts

If a mechanism exists in an engine under consideration which prevents all of the chemical energy from being released in the reaction, this effect will manifest itself in an increased mass flow over that predicted above. This is because the total temperature of the stream is too high. Without regard to the actual mechanism, the amount of fuel and oxidizer which did

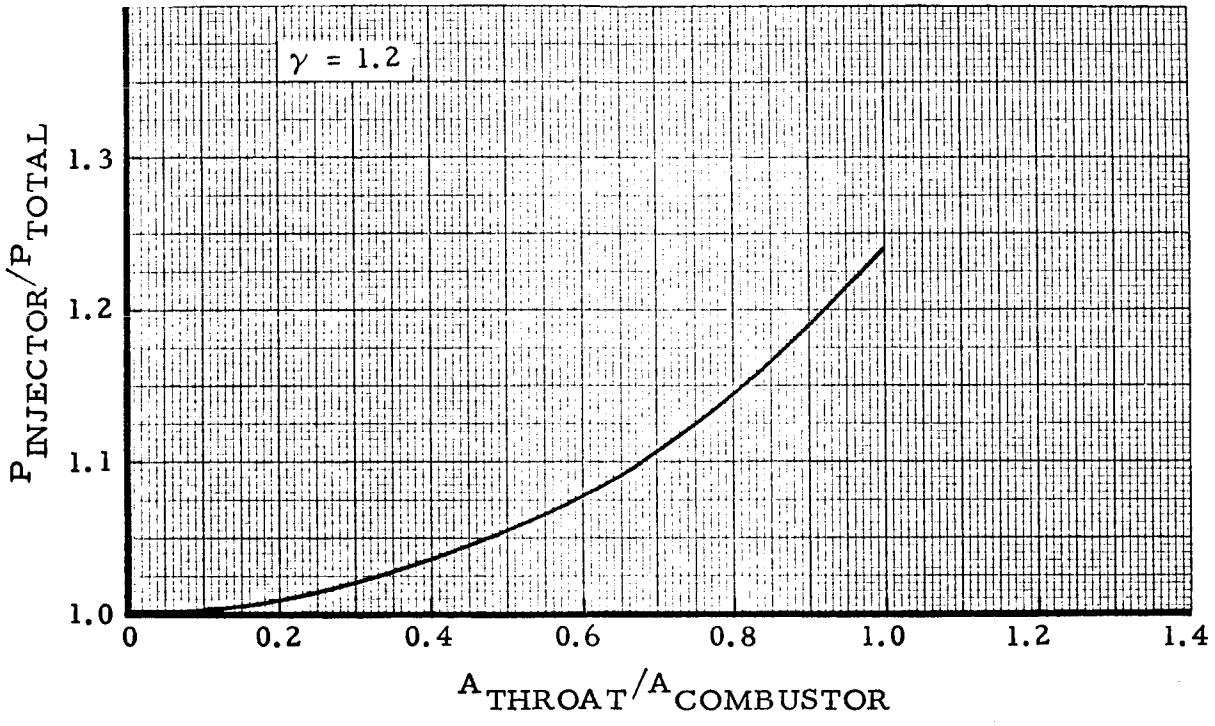


Figure 1 - Total Pressure Loss for Varying Combustor/Throat Area Ratios

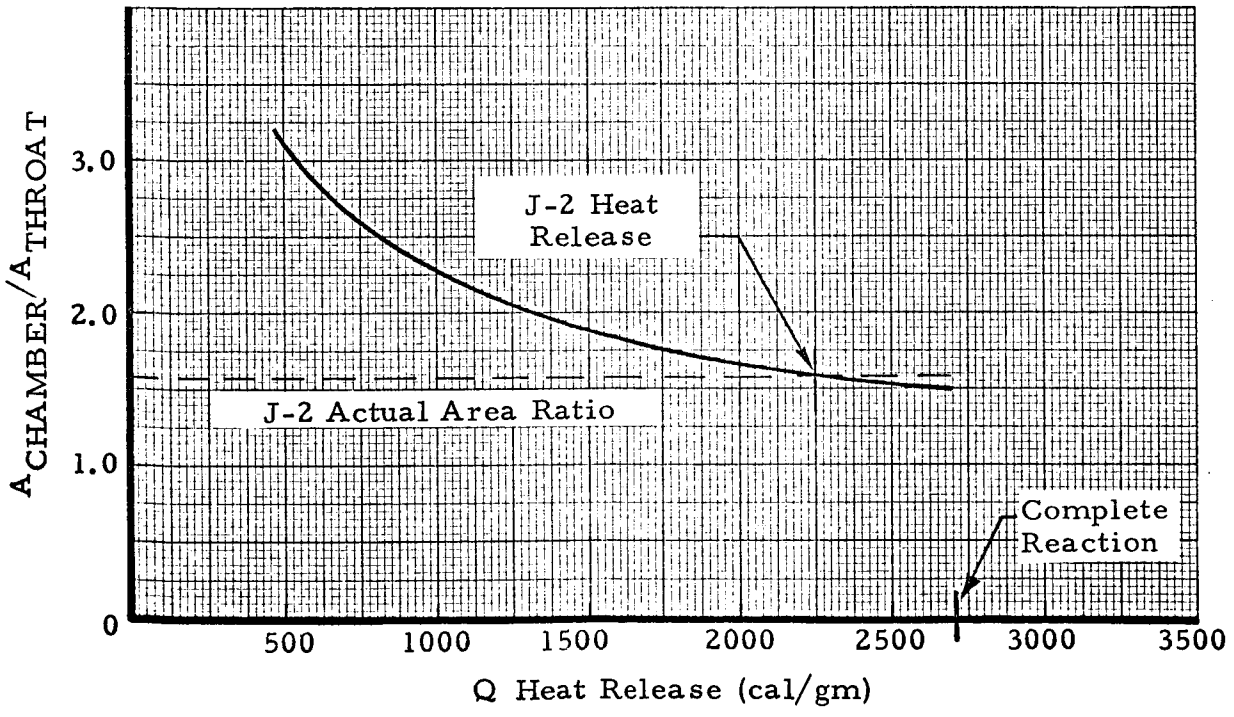


Figure 2 - Area Ratio Required to Choke vs Heat Release

not react can be predicted if the actual flow rate and thrust are known. If portions of the fuel and oxidizer are considered to be inert constituents which withhold their chemical energy from the reaction, the total temperature of the stream is thereby lowered and the mass flow raised.

Since there are two unknowns — amount of fuel and oxidizer which did not react — two measured quantities are necessary to provide a solution. Unfortunately, the thrust is rather insensitive to the amounts of inerts withheld and is therefore a poor choice. However, other parameters are extremely hard to obtain. Temperature measurements in regions where equilibrium assumptions are valid are probably best suited for the extra parameter. A discussion of the phenomena is provided by Lawrence (Reference 4).

Two sample calculations are given for a J-2 engine operating at a mixture ratio of $LOX/H_2 = 5.55$. The first, Figure 2, shows the combustor-to throat area ratio required to choke as a function of the heat released by the combustion. Note that the mass flow rate for this calculation is maintained at the experimental value. The figure indicates that nearly 17% of the available energy is not released in this engine. Figure 3 shows that 24% by weight of the oxidizer or 45% by weight of the fuel must be withheld as inert in order to achieve the correct heat release. The large reduction in hydrogen is due to the fact that the engine is run fuel rich, and a large reduction in the amount of hydrogen available is necessary to significantly affect the combustion process.

The recommended approach is:

- Perform the analysis using measured mass flow and another parameter, such as throat temperature, to ascertain the amount of unreacted propellants.
- If only measured mass flow is available or suitable, withhold inerts in a ratio corresponding to the mixture ratio.

Since the above analysis is dependent on measured quantities, an engine must exist before the calculation can be made. The performance

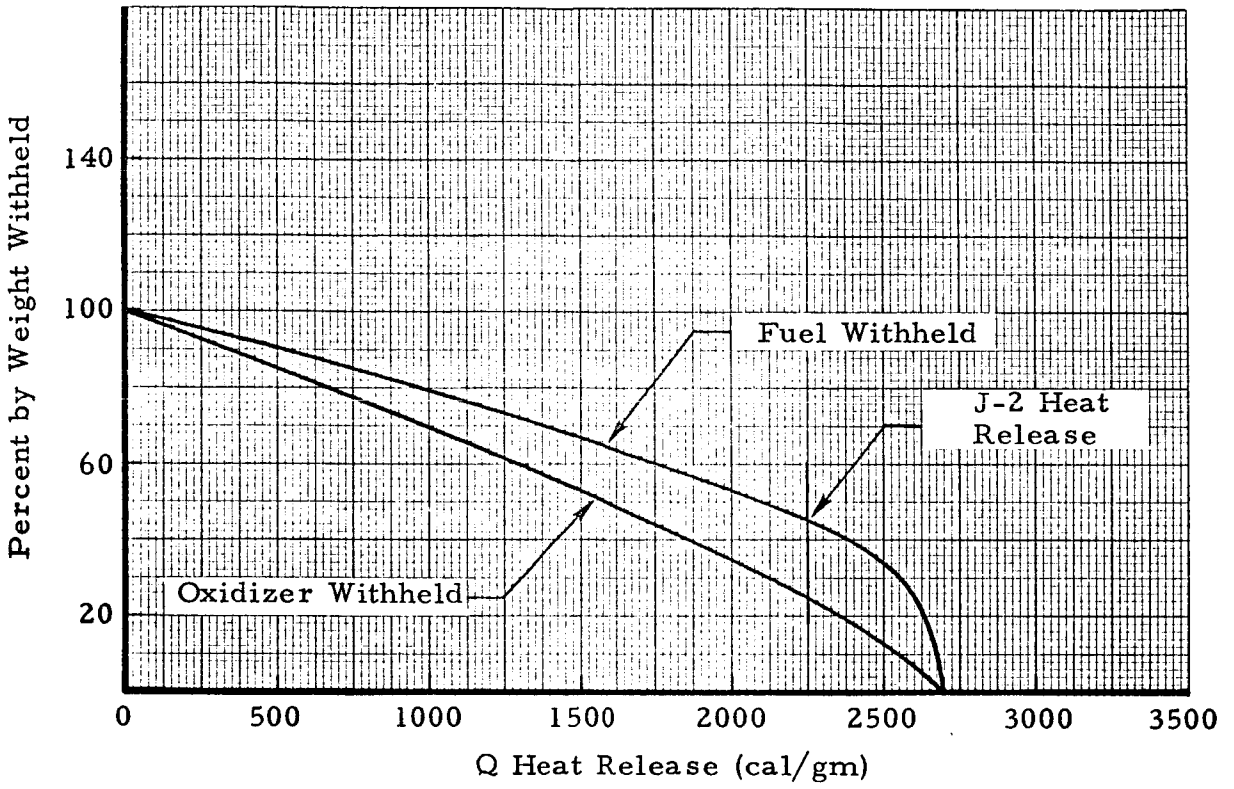


Figure 3 - Percent by Weight Inert vs Heat Release

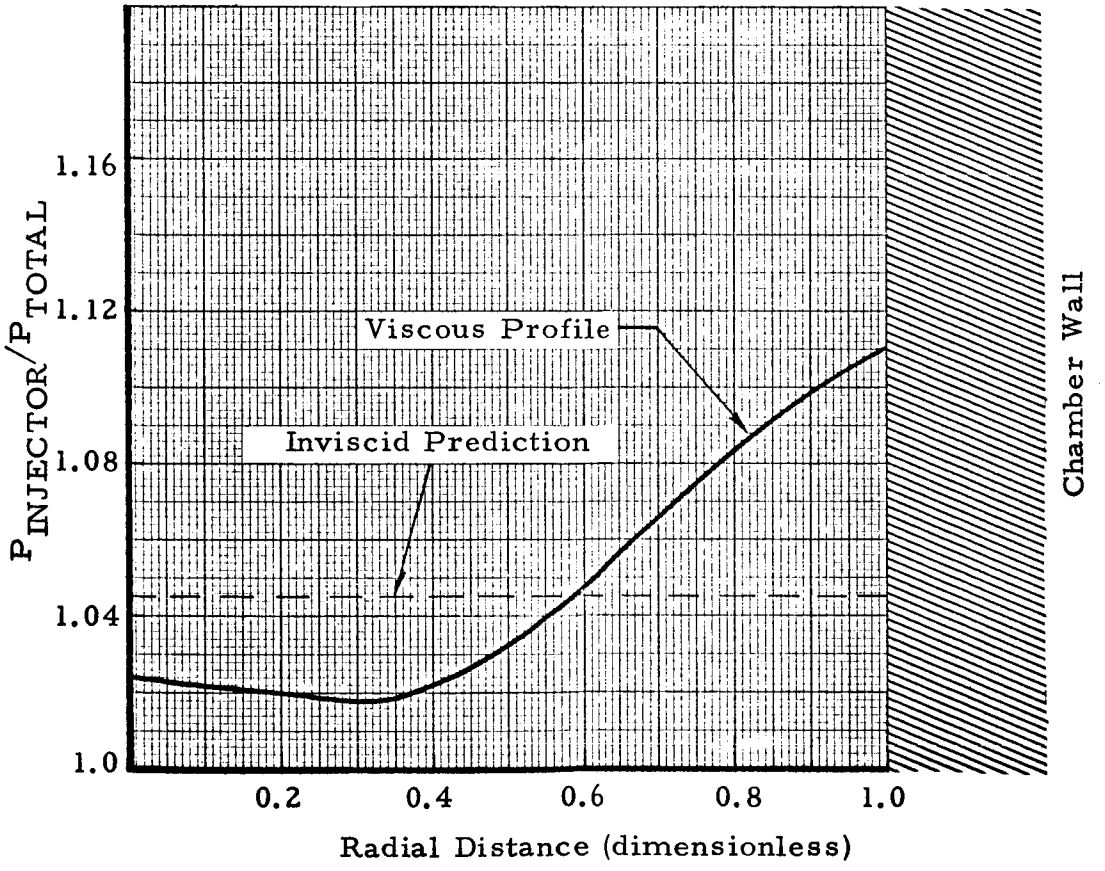


Figure 4 - Radial Variation of Total Pressure at End of Combustor

of engines for which experimental data is not available can only be deduced from data from similar engines. Another drawback of the technique is that it offers no insight into what is actually occurring. If the analyst is convinced that mixing is the controlling mechanism, a more appropriate analysis exists.

2.2.4 Prediction of Energy Loss by Mixing Analysis

Improper mixing of reactants in the combustor arises inadvertently due to combustor design or purposely to cool the combustor walls by injecting a fuel-rich stream along the periphery. In any case, a constant radial pressure in the combustor (which is analogous to a one-dimensional solution with radial variations of mixture) finite rate mixing analysis is provided by the General Applied Sciences Laboratory Finite Rate Mixing Program (Reference 5). An important advantage to this type of analysis is that the elusive extra parameter in the section above is no longer necessary. Specification of the incoming mixture distribution, injector pressure and mass flow rate only are necessary in this analysis. An important consequence of this fact is that these are acceptable preliminary design parameters, allowing proposed propulsion systems to be analyzed. Another important consideration is that information on the mixture ratio distribution at the nozzle entrance is also provided.

In the analysis, the reactants are assumed to be premixed in several strata according to the input information. A hot spot to ignite the propellants is used to begin the finite rate combustion and expansion process. Mixing occurs in a predetermined fashion (by an eddy viscosity model) and the conservation equations are employed to describe the behavior of the flow up to the nozzle inlet.

The appropriateness of this sophisticated analysis depends on whether mixing is, in fact, the dominant mechanism in the combustion inefficiency. If it is not, the analysis is unwarranted since inert phases would have to be

introduced into the analysis as in the previous section. The radial distribution of these inerts would in all probability be unknown, thereby severely compromising confidence in the results.

Figure 4 illustrates the radial total pressure prediction from the GASL analysis. The propellant is C_2H_4-O_2 with an overall mixture ratio of 2.25. Also shown is an ideal gas one-dimensional value computed in the same fashion as Figure 1, but for an isentropic exponent of 1.14. This value was chosen as representative of the entire mixture.

The recommended approach is:

- Choose a reasonable inlet oxidizer/fuel distribution and perform the analysis.
- If the throat choking requirement cannot be satisfied without holding out a significant amount of inerts, revert to the analyses of the previous section.

At this writing, not enough experience has been gained in applying the analysis to actual situations to take a firm stand on the applicability and veracity of the model. Certainly the mixing phenomenon does exist, however, and calculations of this type will aid immeasurably in understanding the combustion process. The only question that remains is whether it is the dominant factor in the energy-release mechanism.

2.2.5 Other Energy Loss Mechanisms

Several other factors which affect the energy release of the combustion system have been investigated. Of these, heat loss by radiation and convection can be rejected. On the other hand, existence of solid and liquid phases, in addition to the gaseous phase, can have a noticeable effect. The complexity of analyses required to treat these phases, coupled with the lack of information concerning their formation, distribution and interaction with the gaseous phase, preclude their inclusion in the analysis at this time.

A large amount of energy at combustion is stored in the vibrational degree of freedom. All indications are, however, that vibrational freezing does not occur until downstream of the throat in typical engines. Further investigation is warranted, however, before a categorical rejection of this source of energy loss is made.

2.2.6 Improvements in the Combustion Chamber Analysis

Improvements in the analysis could certainly be made. Some possible areas of improvement are:

- Include liquid and solid phases in the combustion process.
- Treat the combustion process two dimensionally.
- Couple the transonic solution with the chamber solution (mostly for the benefit of the transonic solution) in order to determine precisely the choking condition.

It would seem, however, that the most significant advancement could be provided by a combined analytical and detailed experimental study. Careful experimentation would provide an insight into the mechanisms that must be considered in the improvement of analysis tools.

2.3 TRANSONIC REGIONS

Many methods have been proposed to analyze the flow conditions in the throat region of a rocket engine. Most attention has been directed toward a polynomial expansion of the potential function as a means of solving for a slightly supersonic line in the vicinity of the throat. This approach is geometry-limited, however, and the quality of the predictions degenerate as the throat radius of curvature becomes smaller and smaller.

For the purpose of this study, a method was needed which could handle practical engine contours, reacting gases, and fuel striations. The techniques of numerically integrating the unsteady equations of motion until the steady-state condition was reached was chosen as the most appropriate method to

achieve the desired results. Actually the technique is, in this case, more properly a relaxation method than a true unsteady solution since the initial guess (the one-dimensional solution) will never occur at any time. The unsteady equations of motion are then used to relax the solution to the steady-state values. The method is applied from the low subsonic through part of the supersonic region, and actual boundary conditions are satisfied throughout. Reacting gases in equilibrium are easily included in the analysis as are fuel striations. An important requirement is that fuel striations, if they exist, should be included in the analysis since they have a severe impact on the plume temperature and concentration predictions.

An effort was therefore initiated to produce a computer program capable of analyzing flows of this type. Such a program was created and several contours were examined. The numerical results obtained are discussed by Stephens (Reference 6). One of the solutions is shown in Figure 5. Back (Reference 7) supplied experimental data for a severely constricted throat, flowing cold air. The comparison of the transonic solution with the experimental values is shown in the figure. The comparison is thought to be excellent, particularly because it is necessary to interpolate between predicted values to produce the plotted contours. There is also some question as to the accuracy of the experimental data in regions in which the flow is inclined with respect to the axis since no attempt was made to align the probe to the local flow direction.

The effect of fuel striations depends on how severe the gradients are and how the propellant gas properties vary with O/F ratio. Stephens compared seventh order boundary layer profiles O/F distribution with a uniform O/F distribution for an F-1 engine. Little effect on the pressure Mach number field was found. The temperature and concentration profiles were significantly altered however. Not enough experience has been gained with the fuel striations calculations to make any categorical statement concerning the pressure/Mach number field for an arbitrary case, although it does appear that these contours are not drastically altered if there is little change in the isentropic exponent with O/F ratio.

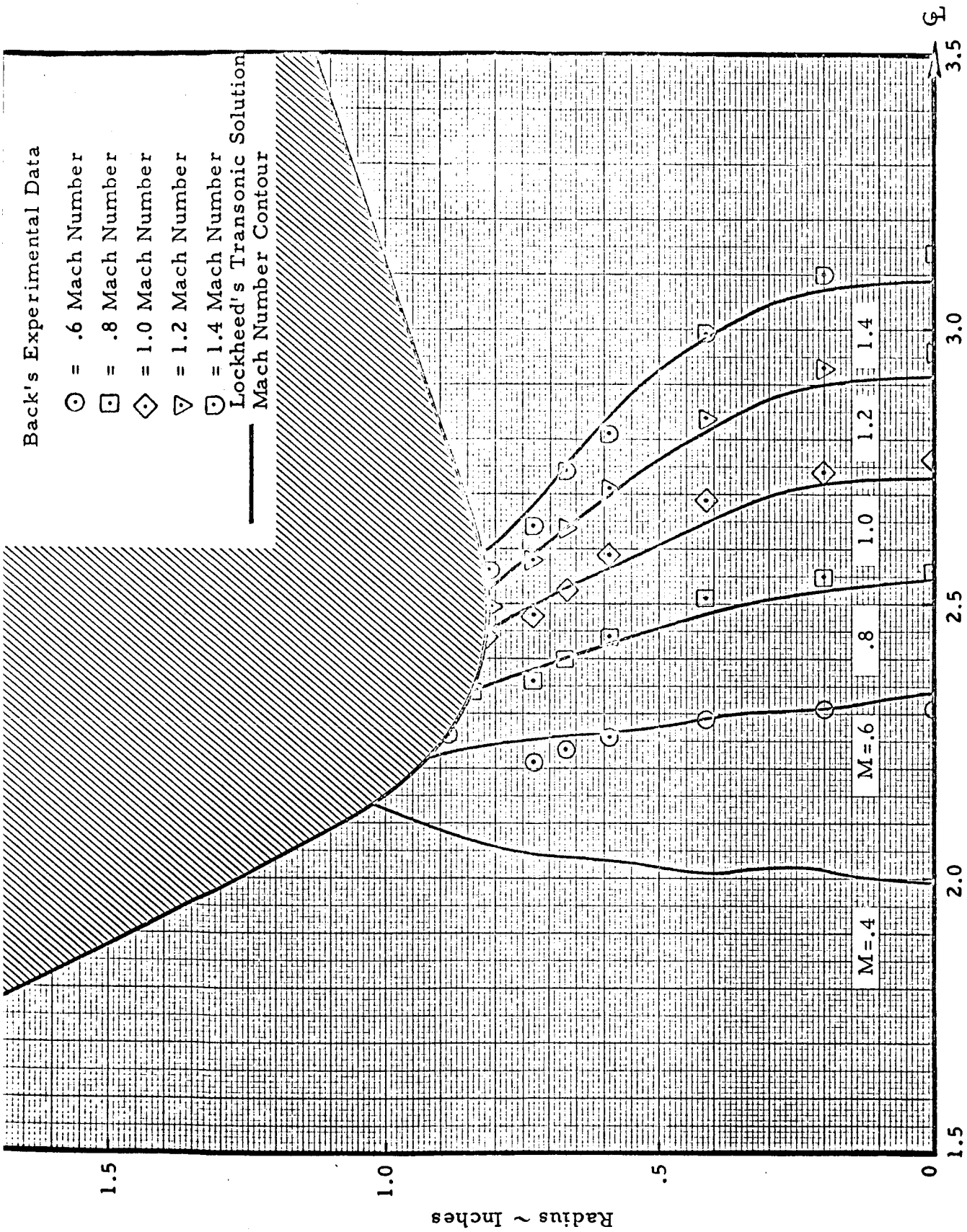


Figure 5 - Comparison of Transonic Mach Number Contours with Back's Data

In an attempt to improve the accuracy and speed of the analysis, a revised method was attempted. This new method has, at present, stability problems which preclude its current use.

When appropriate, a simplification (analogous to one-dimensional flow) can be made which eliminates the need for a transonic solution by assuming that the pressure in the throat is constant. Ratliff (Reference 8) used this assumption for his performance calculation. The Mach number errors created by this approach are known to damp quickly and are therefore of no practical consequence. More important is that there is no method of specifying what the distribution is unless a scaled chamber distribution is used. This method can be used for exploratory calculations, but a detailed analysis should consider the transonic solution.

2.4 SUPERSONIC REGION

Much knowledge and expertize has been accumulated concerning the behavior, and its prediction, of supersonic flow. In particular, the method of characteristics has been shown to be an accurate analysis method for this flow regime. Since the features of rocket nozzle and exhaust fields are, in general, well understood, they are not discussed here. However, several features of the Lockheed/Huntsville method-of-characteristics program which was developed for NASA/MSFC are presented. References 9, 10 and 11 describe the computer program in detail.

The important features of this program, related to electron density prediction, are:

- Both nozzle and plume are treated.
- Equilibrium, equilibrium/frozen, frozen or ideal chemistry may be considered.
- Treatment of shock waves is mathematically and chemically exact.
- Mass entrainment in boundary or shear layer boundary condition may be considered.

- Inviscid analysis of fuel striations can be performed.

Of the above capabilities the fuel-striations option deserves special discussion. The presence of fuel striations, as previously noted, is an important consideration in temperature and concentration profile prediction. Ratliff (Reference 12) has performed several calculations in which the exit temperature was experimentally determined. In particular, Figure 6 illustrates the temperature exit condition for a small liquid rocket motor with a conical diffuser. Shown on the same figure is the temperature distribution predicted for a constant O/F solution at the overall mixture ratio. This prediction shows very little temperature variation across the exit plane, a well-known property of conical nozzles with large area ratios. The experimental data on the other hand indicates large temperature variations.

A parabolic O/F distribution was introduced into the analysis and resulted in the third curve in the figure. An excellent temperature correlation resulted from this distribution. It is interesting to note that in order to achieve this variation, the overall O/F ratio had to be relaxed from the experimental value of 1.6 to a value of 1.4. It is doubtful that the experimental measurement could be that much in error. It is probable that this discrepancy is the same sort of incomplete energy release which has been discussed in the preceding chamber analysis section. Previous experience indicates that the total temperature, hence any stream temperatures, is substantially reduced from the adiabatic flame temperature. Reduction in the O/F ratio, since the mixture is below stochiometric, results in a reduction in temperature. The reduction in available oxidizer also corresponds to the amount of inert oxidizer encountered in the chamber analysis.

2.5 SHEAR LAYER ANALYSIS

The rocket plume interacts with the surrounding atmosphere in a region predominated by viscous shear forces. Three programs are used to calculate the physical properties in such viscous regions. Two of these programs are based upon use of similarity principles and treatment of the

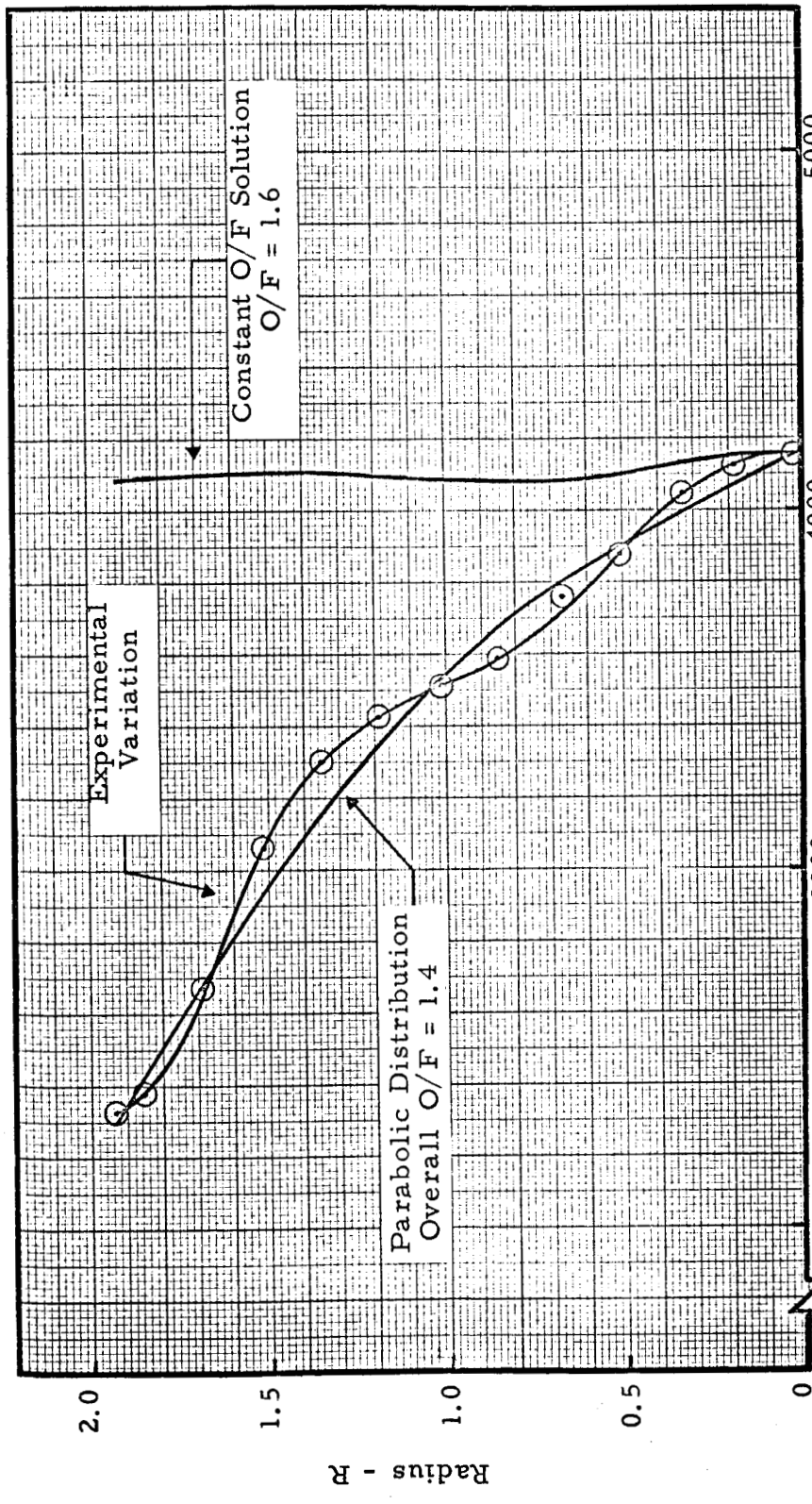


Figure 6 - Variation of Exit Plane Temperature

gases in a chemical equilibrium state. The third program uses the explicit finite difference method of solution for the physical properties of a fluid reacting at a finite rate.

The two programs using equilibrium chemistry and similarity principles while providing relatively rapid results are restricted to cases in which such analyses are applicable. The program which uses finite difference form of solution for a fluid reacting at a finite rate is more generally applicable but, for this generality, speed of calculation is sacrificed. A decision must therefore be made to determine the proper program to apply to a particular problem.

2.5.1 Balanced Jet, Equilibrium Shear Layer

A program to analyze equilibrium shear layers was constructed for rapid calculation of the velocity and temperature distribution within the shear layer. This program is a modification of the GASL finite rate shear layer program (Reference 5). The program treats shear problems which have the following properties:

1. No axial or radial pressure gradient. (This implies a balanced jet situation.)
2. Uniform initial conditions for velocity and fuel percentage.
3. Two-dimensional or axially symmetric mixing of two streams.
4. The chemical system can correctly be described by equilibrium conditions.
5. The fuel is compatible with the NASA/Lewis Thermochemical Program.

The program is based upon an explicit form of iteration of the conservation equations to determine the velocity at preceding positions downstream. The percentage of oxidant and fuel (O/F ratio) mixed through viscous action is keyed to the velocity value thus determined. From this similarity solution approach, the properties of such an equilibrium situation are used to evaluate the temperature within the shear layer. A sample of the results of this program

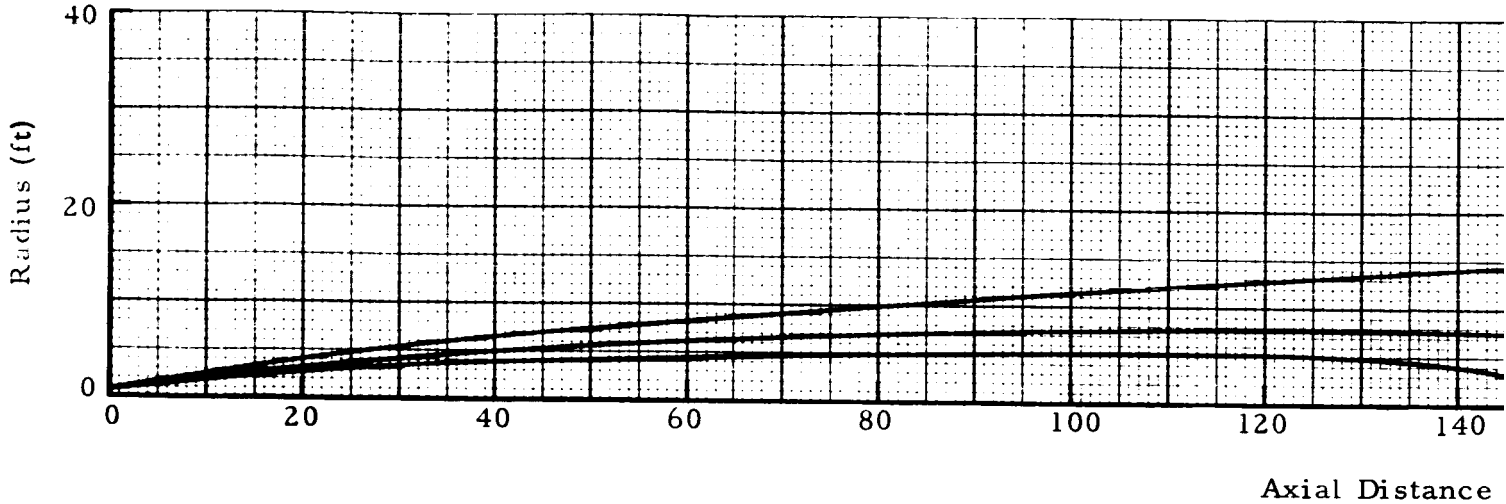
is given by a plot of the static temperature contours for the Apollo Launch Escape engine (Figure 7). This work was reported in Reference 13.

Another program constructed by the Chrysler Corporation provides solution to shear problems which have the properties listed previously. At a sacrifice of speed and simplicity of input in comparison with the above program, the chemical composition is calculated throughout the shear layer and provided as direct output in addition to the velocity and temperature (Reference 14).

2.5.2 Finite Difference, Finite Rate Shear Layer

The two programs described above are for fluids which are in chemical equilibrium. In both cases the application of similarity principles is involved, which makes difficult the treatment of situations other than those in which two uniform streams are mixed. To solve the problem of afterburning and particularly turbine exhaust afterburning, a mixing program was developed by General Applied Sciences Laboratory (Reference 5). This program uses a finite difference solution of the conservation equations rather than a similarity solution to determine the fluid dynamic properties. This procedure allows arbitrary initial profiles to be treated. With this capability, a rocket exhaust, surrounded by turbine exhaust mixing with the atmosphere, can be solved directly. The fluid was treated by a finite rate chemistry model. The inclusion of finite rate reactions makes the program most applicable for treatment of electron density studies. An additional feature of the program is the possibility of solving the shear region in the presence of a prestated axial pressure gradient.

A sample of program results is given for the mixing of hydrogen/oxygen combustion products with preheated air in the form of static temperature contours in Figure 8. These are taken from Reference 15.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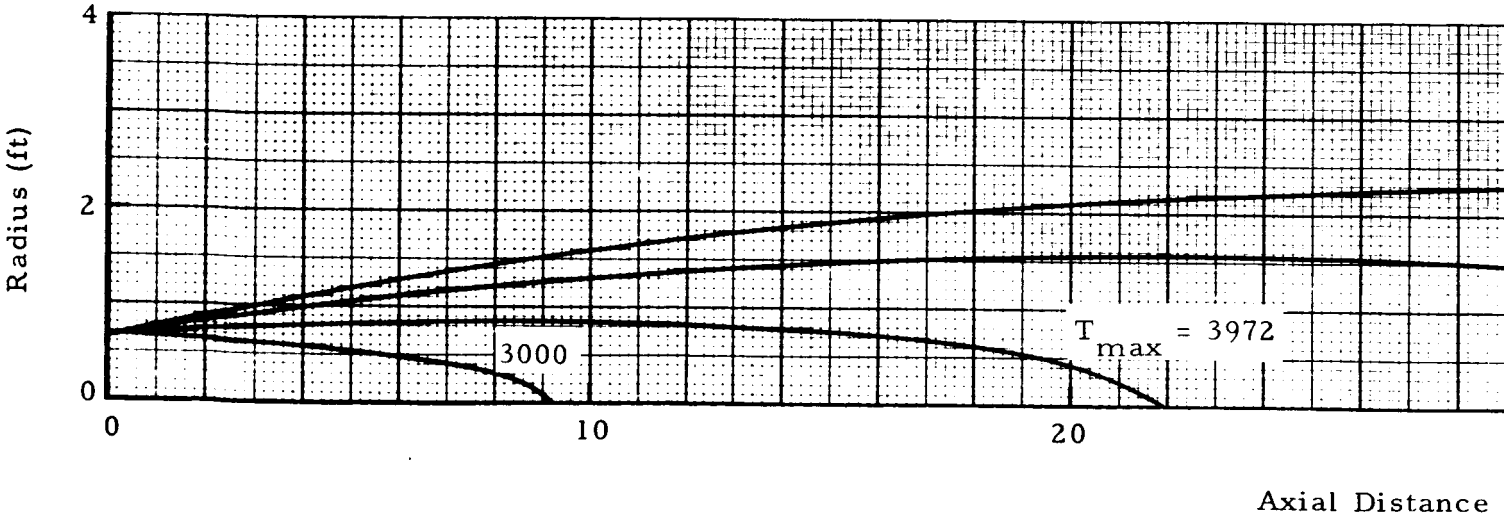
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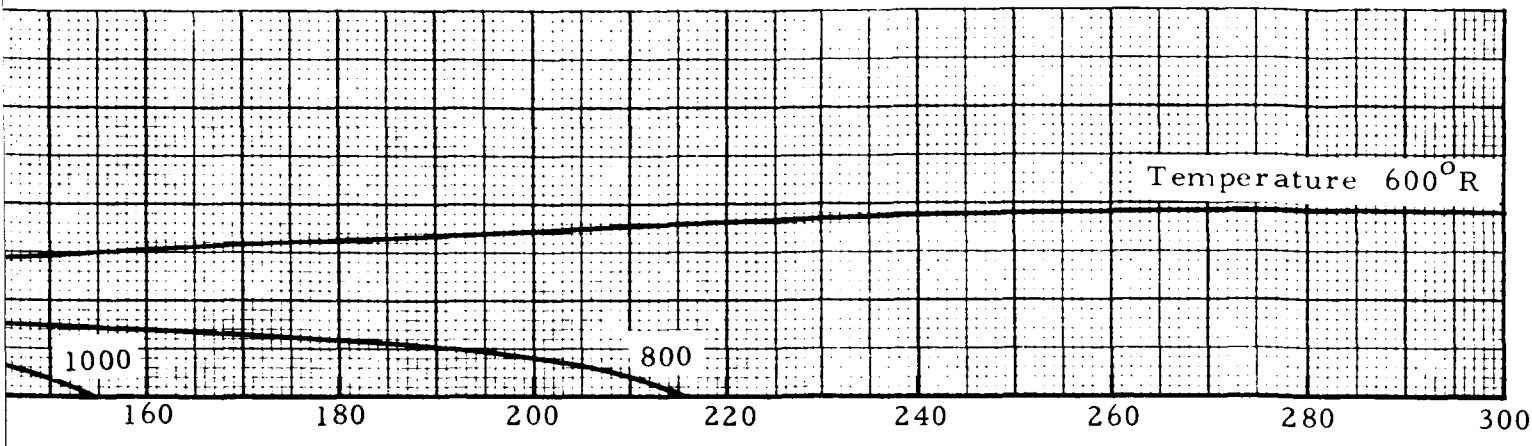
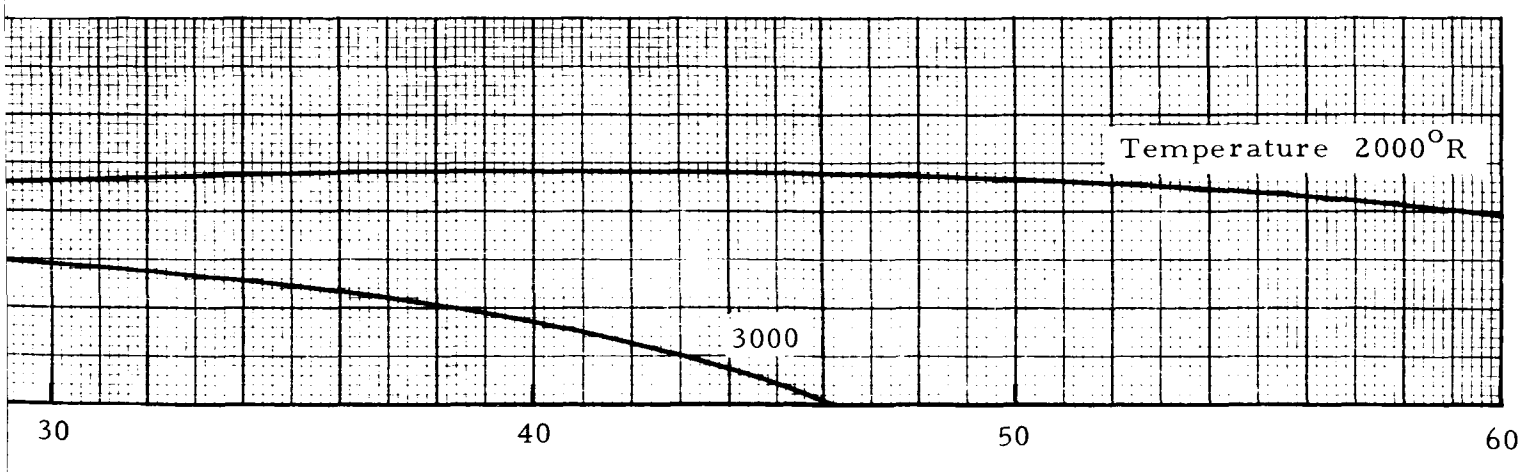
Axial Distance



from Exit Plane (ft)



from Exit Plane (ft)

Figure 7

Apollo Launch Escape, Static Temperature Contours Calculated by the Equilibrium Shear Layer Program

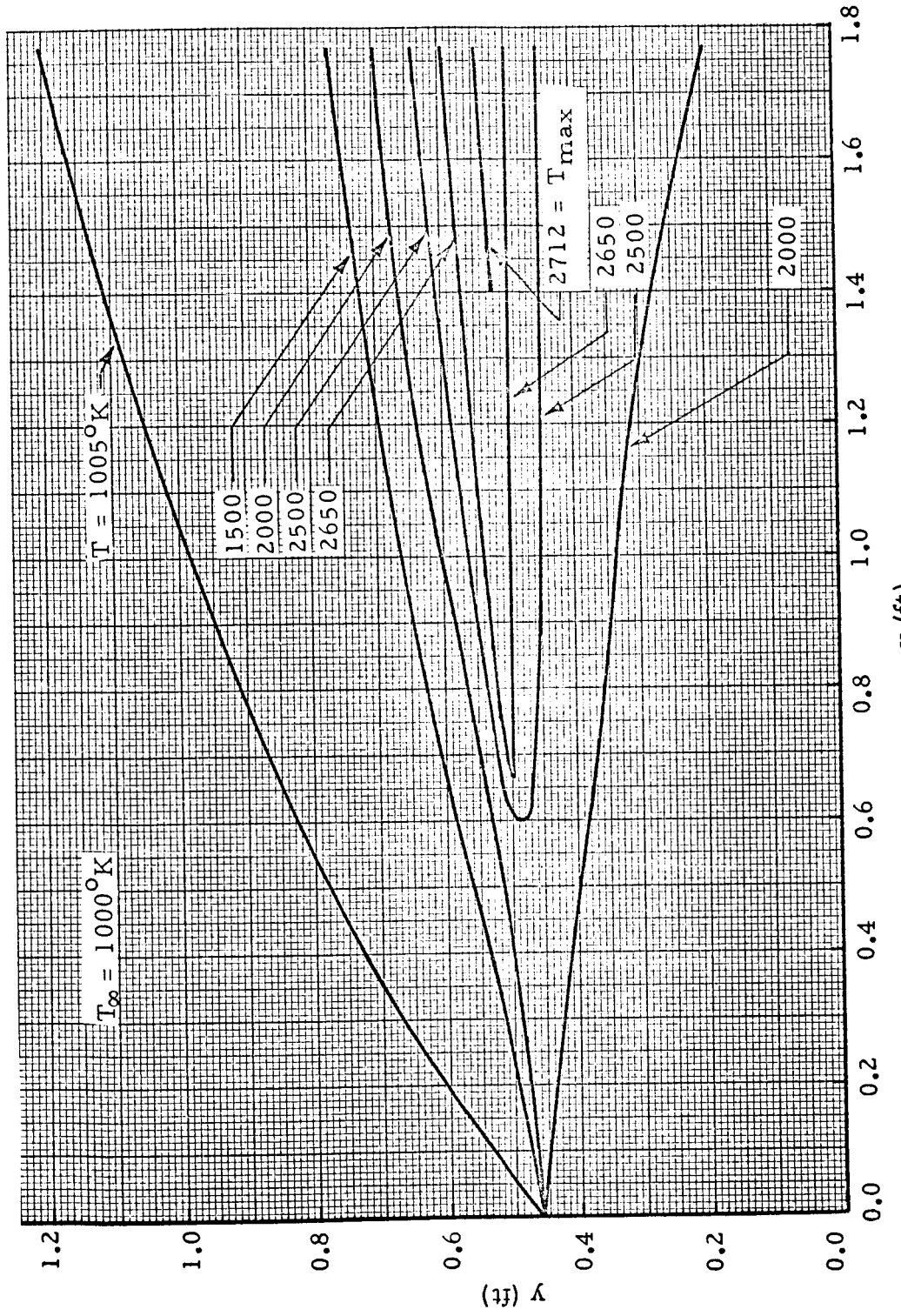


Figure 8 - Temperature Contours Calculated by the Finite Difference Finite Rate Shear Layer Program

2.5.3 Coupling of the Inviscid and Viscous Flows

A complete description of the plume and its interaction with the atmosphere requires a coupled solution of the inviscid region in the plume core and the viscous shear layer on the pheriphery. A direct mathematical coupling of the equations is currently not possible. In place of the direct coupling, a mass capture method has been developed. Through this procedure, the viscous shear layer influences the resulting inviscid solution.

A viscous shear calculation is carried out to determine what portion of the inviscid core would be consumed by the viscous mixing region. The method-of-characteristic solution is then run to evaluate the inviscid core with the exception that the appropriate amount of mass is withdrawn from the boundary region. The calculation continues with the withdrawing of mass until the entire inviscid core has been eliminated and the flow is completely viscous. Properties further downstream are determined by a shear analysis.

The mass capture method is generally applicable to balanced or unbalanced jets. Presently it cannot be applied except to the balanced jet case. This restriction enters because of the inability of the shear programs to calculate cases which contain radial pressure gradients. Rigorous calculations of the shear layer for the underexpanded plumes are unattainable with the programs presently available.

2.5.4 Recommended Approach

While the finite rate mixing program will provide the electron density directly for restricted reactions, it has been found to be impractical to use this program to obtain values far downstream because of excessive computer run times involved. Therefore, the results of the equilibrium shear program developed at Lockheed/Huntsville should be used in conjunction with the output from the NASA/Lewis Thermochemical Program to interpolate upon the temperature to obtain the electron density. This procedure could be simplified with the addition of electron density to the table input of the modified GASL finite rate shear program.

2.6 ELECTRON DENSITY CALCULATION

The electron density in expansive flows as developed for this study is determined by two methods. The simpler of these takes advantage of the freezing point approach described previously. For this approach, an equilibrium solution which includes charged particles such as the NASA/ Lewis Thermochemical Program (Reference 2) is used to evaluate the composition of the fluid, including the electron density up to the freezing point. Composition of the gas remains frozen at the value calculated at the freezing point throughout the remainder of the flow field. This chemical freezing point is determined by using the criteria outlined in Reference 16.

While the above procedure is rapid and relatively simple, electron density in reality is more probably dependent upon the finite rate reactions of fluids in a state of nonequilibrium than in equilibrium. A program which evaluates flow composition of a fluid reacting at a finite rate was constructed by AeroChem Research Laboratories, Inc. (Reference 17). Although this program is based upon the proper procedure, because of the long computer run times required, it has been found to be impractical for complete description of a plume. Usually a limited number of representative streamlines in the plume have been run. The sparsity of data obtained in this fashion makes the use of this sophisticated program restricted in value when an entire plume map of electron density is required.

Shear layers within the flow complicate the solution of electron density because of the constantly changing constituents caused by the mixing action. The AeroChem finite rate program operates only with a fixed atomic composition, which then is acted upon by changing pressures and temperatures.

The recommended procedure for determining electron density in the inviscid portion of the flow is to use the equilibrium-frozen technique using the chemical constituents freezing point criterion. Since there is now no theoretical justification for using this criterion to determine the position of a freezing point in calculation of electron density, at least one streamline

must be concurrently analyzed by the finite rate program to establish a reliable freezing point for the charged species. After the position of the freezing point is determined from the finite rate program, the entire flow field is obtained by using the equilibrium values of electron density up to that point and then maintaining that level of electron density throughout the remainder of the frozen flow. The NASA/Lewis Thermochemical Program is used for this calculation.

2.7 TOTAL INTEGRATED MODEL

The foregoing discussions treated each of the contributing regimes with little or no regard to their interaction in a total integrated model. In this section a systematic approach to the electron density prediction is discussed. Naturally, the purpose and required accuracy of the analysis must be balanced against schedule and cost restrictions.

The first decision which must be made is whether or not to include fuel striations in the analysis. The injector may be designed in such a way that, in addition to thin film wall cooling (which may be ignored in large engines), deeply penetrating mixture gradients will arise. If this is the case, there is little choice but to run the following programs:

- GASL Finite Rate Mixing Program (for combustion chamber and shear layer analysis)
- NASA/Lewis program for various O/F ratios
- Lockheed/Huntsville transonic program with fuel striations
- Lockheed/Huntsville nozzle and plume with fuel striations

If fuel striations are considered to be unimportant, the analysis is considerably simplified. The first and most important question which must be answered is concerned with the chamber analysis. If the combustor-to-throat area ratio is larger, say than four, then the adiabatic flame analysis is appropriate. If the chamber is rather small, it is necessary to employ both the reduced total pressure and inerts calculation. In any event the

the NASA/Lewis program must be exercized to supply thermochemical data to the subsequent flow analysis.

In many cases the transonic solution can be circumvented since in the absence of fuel striations the two-dimensional throat effects are of secondary importance. In most cases the rocket nozzle calculation should be employed. A notable exception to this is large area ratio conical nozzles in which the exit conditions are essentially one dimensional. The two-dimensional effects of the exhaust plume are important, however, so this calculation is always required.

At this stage, no matter what the chamber conditions may be, the inviscid flow field has been established. Whether or not the shear layer analysis should be conducted must now be considered. For high altitude operation this calculation can be sometimes ignored. For low altitude operation it should always be employed. Intermediate altitudes can be simulated by postulating an effective inviscid jet which is balanced at the altitude of interest. If turbine exhausts are dumped into the exhaust plume flows, the shear layer analysis should be included.

Having conducted all the flow analyses, the equilibrium electron density map can be constructed. It is suggested that the center streamline be used in the finite rate AeroChem program to ascertain whether the finite rate effects are important. If a significant electron density variation from the equilibrium analysis is detected, then enough streamlines must be analyzed with the finite rate analysis to establish the overall electron density map. The last calculation should be avoided whenever possible because the finite rate analysis is quite time consuming.

Section 3
CONCLUSIONS AND FUTURE WORK

A series of sophisticated calculations were discussed which, when assembled into a total flow model, represent the state of the art in rocket exhaust plume flow field and electron density analysis. Several important new concepts were introduced which will significantly improve the quality of the prediction over previously accepted procedures.

There are areas in which additional effort is warranted to improve the quality and simplicity of the electron density prediction model. Since the combustion chamber controls the subsequent expansion, any important improvement in the analysis of this region will result in a first-order improvement in the plume analysis. The transonic and supersonic calculations are of excellent quality as long as the simplified chemistry models are acceptable. Continued effort is required on the transonic solution in order to achieve the efficient solution desired. This solution will be far more practical, economically, after numerical problems are resolved. One important shortcoming of the entire analysis is in the prediction of the Mach disc. To achieve an adequate solution to this problem will require much effort.

A method should also be devised to establish a criterion for locating the freezing point in flows with charged species. This can be done by making runs using the NASA/Lewis Thermochemical Program with its equilibrium-frozen procedure and comparing results with the results from finite rate calculations produced by the AeroChem program. Some additional effort should also be expended to facilitate the program-to-program communication, thereby making the entire analysis simple from a user standpoint.

REFERENCES

1. Bray, K. N. C., "Chemical Reactions in Supersonic Nozzle Flows," Ninth Symposium (International) on Combustion, Academic Press, New York, 1963, pp. 770-784.

2. Zeleznik, R. J. and Gordon, S., "A General IBM 704 or 7090 Computer Program for Computation of Chemical Equilibrium Compositions, Rocket Performance, and Chapman-Jouguet Detonations," NASA TN D-1454, October 1962.

3. Yedlin, A. K., "Calculation of the Injector End to Nozzle Stagnation Pressure Ratio," PN5114-2005, Rocketdyne Division, North American Aviation, Canogo Park, Calif., 4 February 1965.

4. Lawrence, T. R., "Investigation of Combustion Chamber Inefficiencies in Rocket Motors," LMSC/HREC A784863, TM 54/20-163, Lockheed Missiles & Space Company, Huntsville, Alabama, November 1967.

5. Edelman, R. and O. Fortune, "Mixing and Combustion in the Exhaust Plumes of Rocket Engines Burning RP1 and Liquid Oxygen," GASL Technical Report No. 631, General Applied Sciences Laboratories, Inc., Westbury, L.I., New York, November 1966.

6. Stephens, J. T., "Analytical Results for Transonic Flow Fields in Rocket Motors," LMSC/HREC A784847, TM 54/20-157, Lockheed Missiles & Space Company, Huntsville, Alabama, October 1967.

7. Back, L., "Transonic Test Dates," Unpublished data obtained from Jet Propulsion Laboratory, Pasadena, Calif., September 1967.

8. Ratliff, A. W., "The Effect of Nozzle Flow Striations on Engine Performance," LMSC/HREC A784646, Lockheed Missiles & Space Company, Huntsville, Alabama, September 1967.

9. Prozan, R. J., "Development of a Method-of-Characteristics Solution for Supersonic Flow of an Ideal, Frozen or Equilibrium Reacting Gas Mixture," LMSC/HREC A782535, Lockheed Missiles & Space Company, Huntsville, Alabama, April 1966.

10. Butler, H. W., "Description of a Digital Computer Program for Nozzle and Plume Analysis by the Method-of-Characteristics," LMSC/HREC A782592, Lockheed Missiles & Space Company, Huntsville, Alabama, December 1966.

11. Ratliff, A. W., "Comparisons of Experimental Supersonic Flow Fields with Results obtained by Using a Method-of-Characteristics Solution," LMSC/HREC A782592, Lockheed Missiles & Space Company, Huntsville, Alabama, April 1966.

12. Ratliff, A. W., "Calculations of Supersonic Nozzle Exhausts with Fuel Striations," Lockheed Missiles & Space Company, Huntsville, Alabama (to be published).

13. Engel, C. D., "Viscous Plumes of Apollo Launch Escape Solid Propellant Engines at Sea Level," TM 54/20-159, LMSC/HREC A784848, Lockheed Missiles & Space Company, Huntsville, Alabama, October 1967.

14. Rozsa, R. B., "Preliminary Results of the Study of Turbulent Jet Mixing Under Chemical Equilibrium Conditions," TB-AE-65-309, Chrysler Corporation Space Division, New Orleans, Louisiana, January 1966.

15. McGimsey, L. R., "Mixing Analysis for Rocketdyne Two-Dimensional Hydrogen/Oxygen Combustion Experiments," LMSC/HREC A784589, TDR 54/20-78, Lockheed Missiles & Space Company, Huntsville, Alabama, 5 August 1967.

16. Sarli, V. J., W. G. Burwell and R. Hofland, Jr., "Evaluation of the Bray Sudden-Freezing Criterion for Predicting Nonequilibrium Performance in Multi-Reaction Rocket Nozzle Expansions," AIAA Propulsion Joint Specialist Conference, Paper No. 65-554, Colorado Springs, Cororado, 14-18 June 1965.

17. Komianos, S. A., G. D. Bleich and H. S. Pergament, "The AeroChem Nonequilibrium Streamline Program with Application to Rocket Nozzles and Exhaust Plumes," AeroChem TN 103, AeroChem Research Laboratories, Inc., Princeton, New Jersey, May 1967.