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## STUDY OF APOLLO WATER IMPACT

FINAL REPORT
VOLUME 10
USER'S MANUAL FOR MODIFICATION CF SHELL OF REVOLUTION ANALYSIS
(Contract NAS9-4552, G.O. 5264)
May 1967


Structures and Materials

Prepared by
A. P. Cappelli
S. C. Furuike (Authors)

Approved by


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## FOREWORD

This report was prepared by North American Aviation, Inc., Space Division, under NASA Contract NAS9-4552, for the National Aeronautics and Space Administration, Manned Space Flight Center, Houston, Texas, with Dr. F.C. Hung, Program Manager and Mr. P. P. Radkowski, Assistant Program Manager. This work was administered under the direction of Structural Mechanics Division, MSC, Houston, Texas with Dr. F. Stebbins as the technical monitor.

This report is presented in eleven volumes for convenience in handling and distribution. All volumes are unclassified.

The objective of the study was to develop methods and Fortran IV computer programs to determine by the techniques described below, the hydro-elastic response of representation of the structure of the Apollo Command Module immediately following impact on the water. The development of theory, methods and computer programs is presented as Task I Hydrodynamic Pressures, Task II Structural Response and Task III Hydroelastic Response Analysis.

Under Task I - Computing program to extend flexible sphere using the Spencer and Shiffman approach has been developed. Analytical formulation by Dr. Li using nonlinear hydrodynamic theory on structural portion is formulated. In order to cover a wide range of impact conditions, future extensions are necessary in the follorsing items:
a. Using linear hydrodynamic theory to include horizontal velocity and rotation.
b. Nonlinear hydrodynamic theory to develop computing program on spherical portion and to develop nonlinear theory on toroidal and conic sections.

Under Task II - Computing program and User's Manual were developed for nonsymmetrical loading on unsymmetrical elastic shells. To fully develop the theory and methods to cover realistic Apollo configuration the following extensions are recommended:
a. Modes of vibration and modal analysis.
b. Extension to nonsymmetric short time impulses.
c. Linear buckling and elasto-plastic analysis

These technical extenpions will not only be useful for Apollo and future Apollo growth configurations, but they will also be of value to other aeronautical and spacecraft programs.

The hydroelastic responsc of the flexible shell is obtained by the numerical solution of the cornbined hydrodynamic and shell equations. The results obtained herein are compared numerically with those derived by neglecting the interaction and applying rigid body pressures to the seme elastic shell. The numerical results show that for an axially symmetric impact of the particular shell studied, the interaction between the shell and the fluid produces appreciable differences in the overall acceleration of the center of gravity of the shell, and in the distribution of the pressures and responses. However the maximum responses are within $15 \%$ of those produced when the interaction between the fluid and the shell is neglected. A brief summary of results is shown in the abstracts of individual volumes.

The volume number and authors are listed on the following page.
The contractor's designation for this report is SID 67-498.

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Unsymmetric Shell of Revolution Analysis

Mode Shapes and Natural Frequencies Analysis

User's Manual for Modification of Shell of Revolution Analysis

User's Manual for Unsymmetric Shell of Revolution Analysis Water Impact Interaction

Comparison With Experiments
User's Manual - No Interaction
User's Manual - Interaction

## Authors

T. Li and T. Sugimura
A. P. Cappelli, and J.P.D. Wilkinson
J. P.D. Wilkinson, A. P. Cappelli, R.N. Salzman
J.P.D. Wilkinson and R.N. Salzman
A. P, Cappelli and S. C. Furuike
A. P. Cappelli, T. Nishimoto, P. P. Radkowski and K. E. Pauley
A. P. Cappelli
A. P. Cappelli and S.C. Furuike
E. Carrion, S. C. Furuike and T. Nishimoto

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#### Abstract

The shell of revolution program deseribed in this report was developed as a basic tool to be used in the elantic, load-deflection analysis of shell structures subjected to arbitrary loads and temperatures. The program is applicable to most aeruspace-type shell elements (e. g. , boosters, reentry vehicles, etc.) as well as ground-based ahells.

The computer program is based on the numerical analysis presented in Reference 1 and is reatricted to linear-elastic thin-shell theory. The analysis utilizes Fourier series expansion technique to separate circumferential variation of froblem variables. A rechuced set of shell field equations for each Fourier harmonic of load resulte from using Fourier approach. The finite difference form of the reduced shell equations are solved by a direct matrix elimination procedure. Solutions for various Fourier harmonics can then be summed to obtain the general solution for arbitrary unsymmetric loads.


In using the program it is necessary to select a mathematical model to represent a physical shell problem. By introducing fictitious subdivisions called shell regions, it is possible to analyse complicated shell configurations as a series of shell regions of simple shapes. The procedures for connecting shell regions require the satisfaction of boundary and junction conditions in the program.

The computer program, which was written in FORTRAN IV and applicable to the IBM 7090/7094 systems, was developed in a general fashion to permit the consideration of variety of thell problems. Wherever possible, time and space-saving techniques have been employed to simplify and reduce the amount of input data to be supplied by the user. Various option techniques have been used to permit more generality and still keep data input at a respectable minimum. The solutions obtained from the program yield deformations, forces, moments, atresses, etc. , at each station of a shell region. This output is presented in tabular form with an option for graphical plotting of results.

The users of this program should be forewarned that the program is only a tool and considerable ingight must be used in relating results to an actual physical shell problem. In turn, the resulte obtained are oniy as good as the mathematical model selected for the problem. The numerical procedure used in the solution of differential ohell equation is an approximate one
(finite differences) and results must be interpreted in terms of round-off errors that are inevitable when ueing approximate numerical techniques.

This report is intended to supply the information necessary for the best utilization of the shell of revolution computer program. Considerable detail has been incorporated in this report to aid not only the engineer but also the programmer in understanding the program. It is hoped that this information will permit the modification and extension of this program to handle various other types of shell response problems (e.g., dynamics, buckling, etc.).

This user's manual has been organized in three basic sections. The first section (I) presents the theory used as á basis of the shell of revolution computer program. For ease of reference. much of the numerical procedure developed in Reference 1 is repeated together with modifications and improvements that have been developed at S\&ID. A general description of the computer program is given in Section II. This section is intended to serve as an aid to the user in establishing a mathematical model for a physical shell problem in terms of the program format. Limitations and general program characteristics are given. Section III gives information for the detailed use of the program. Included are input data shell format, flow diagrams, sample data sheets, example problem, etc. As one becomes familiar with the program, this section will probably be the most used since it gives detailed instructions and characteristics of the program.

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## I. THEJRY

### 1.1 INTRODUCTION

The general numerical procedure developed in Keference 1 for the analysis of unsymmetrical bending of shells of revolution forms the basis of the computer program. Included in the program are extensions and improvements to the basic analysis that were developed at StiD and are reported in References 2 and 3.

The analysis is based on the general firat-order linear shell theory of Sanders (Reference 4), which has been assessed (Reference 5) as the "best" of the many competing thin-shell theories in the literature. All pertinent variables are expanded into Fourier series in the circumferential direction which permit decoup ed sets of ordinary differential equations in terms of the individual Fourier components. Finite difference approximations to these differential equations in the meridional coordinate then are solved using a direct matrix elimination technique (Potter' Method) (Reference 6).

This section will present the general theory which forms the basis of the computer program. Nomenclature and approach similar to that of Reference 1 will be used together with appropriate modifications.

## 1. 2 SCOPE AND LIMITATIONS OF THEORY

The shell theory on which the program is based is restricted to linear, elastic, thin-shell theory. Implied by the above statement and other assumptions introduced in the analysis are the following:
a. The thickness of the shell at any point is small compared to the other dimensions of the shell.
b. Deformations of the shell are small compared to the dimensions of the shell.
c. All portions of the shell deform elastically, obeying Hooke's law.
d. The shell is "complete" as well as axisymmetric, i.e., its only boundaries are at meridian ds and inner and outer surfaces.
e. Each layer of shell material is ast «.: o laze two-dimensional elastic isotropy with respect $2 \quad \therefore \therefore$; sangent to its surface,
but Young's modulus is permitted to be variable (and discontinuous) through the thickness as well ae in the meriodional direction.
f. Poisson's ratio is assumed constant in each shell layer.
g. Arbitrary loads and temperature diatributions are permissible. However, the present analysis is inapplicable when circumferential variation of temperature is sufficiently great to produce appreciabse circumferential changes in Young's modulus. In such cases, average values of Young's modulus can be used to obtain approxmate results.
h. Redundant shell structures can be analysed only indirectly using the program.
i. The effects of transverse shear distortion are neglected in the analysis. A procedure for including these effects is described in Reference 7.
j. Instability is not considered.

## 1. 3 SURFACE GEOMETRY AND COORDINATES



Figure 1-1. Surface Geometry and Coordinates

Material points in the shell can be specified by means of the orthogonal coordinates (s, $\theta$, $\zeta$ ), (see Figure l-1) wheres is the meridional distance measured from a boundary along an axisymmetric reference surface, 0 is the circumferential angle, and $\zeta$ is the normal, outward distance from the reference surface. In homogeneous shelis, the middle surface aiways is used as the reference surface; but when, more generally, the Young's modulus $E$ is variable, the reference surface is best chosen 80 that

$$
\begin{equation*}
\int \zeta E d \zeta=0 \tag{1}
\end{equation*}
$$

where the integration is through the thickness, iThis choice, as will be seen later, simplifies the constitutive rolations of clastic shells.) If the shape of the reference surface is given by $r(s)$, where $r$ is the distance from tne axis, the principal radii of curvature are

$$
\begin{align*}
& R_{\theta}=r\left|1-(\mathrm{dr} / \mathrm{ds})^{2}\right|-1 / 2 \\
& R_{s}=-\left|1-(\mathrm{dr} / \mathrm{ds})^{2}\right| 1 / 2 /\left(\mathrm{d}^{2} \mathrm{r} / \mathrm{ds}^{2}\right) \tag{2}
\end{align*}
$$

Introduce the nondimensional meriodional coordinate $\xi=s / a$, where a is a reference length; then, with $P=r / a$, the nondimensional curvatures $\omega_{\xi}=a / R_{s}$, and $\omega_{\theta}=a / R_{0}$ can be found from the formulas

$$
\begin{align*}
& \omega_{\theta}=\left|1-\left(\rho^{\prime}\right)^{2}\right|^{1 / 2 / \rho}  \tag{3}\\
& \omega_{\xi}=-\left(\gamma^{\prime}+\gamma^{2}\right) / \omega_{\theta} \tag{4}
\end{align*}
$$

where

$$
\begin{equation*}
\gamma=p^{\prime} \rho p \tag{5}
\end{equation*}
$$

In these equations, and henceforth, ( $)^{\prime} \equiv(d / d \xi)()$. Finally, note the Codazzi identity

$$
\begin{equation*}
\omega_{\theta}=Y\left(\omega_{\xi}-\omega_{\theta}\right) \tag{6}
\end{equation*}
$$

and the relation

$$
\begin{equation*}
\rho^{\omega} / \rho=-\omega_{\xi} \omega_{\theta} \tag{7}
\end{equation*}
$$

## 1. 4 EQUILIBRIUM EQUATIONS

The components of membrane force per unit length, transverse force per unit length, moment (about the reference surface) per unit length, and load per unit area (assumed to be applied at the reference surface) are as shown in Figure 1-2.


a)

s)

Figure 1-2. Forces, Moments, and Loads: a) Membrane Forces per Unit Length, b) Transverse Forces per Unit Length, c) Moments per Unit

## Length, d) Loads per Unit Area

In the Sanders theory, the shearing forces $N_{\xi} \theta$ and $N_{\theta \xi}$, as well as the twisting moments $M_{\xi \theta}$ and $M_{\theta \xi}$, ars not handied separately out are combined to provide the modified variables
and

$$
\begin{gather*}
\bar{N}_{\xi \theta}=\frac{j}{2}\left(N_{\xi \theta}+N_{\theta \xi}\right)+\frac{i}{4}\left(\frac{1}{R_{\theta}}-\frac{1}{R_{\xi}}\right)\left(M_{\xi \theta}-M_{\theta \xi}\right)  \tag{8}\\
\bar{M}_{\xi \theta}=\frac{1}{2}\left(M_{\xi \theta}+M_{\theta \xi}\right) \tag{9}
\end{gather*}
$$

With the elimination of the transverse forces $Q_{\xi}$ and $Q_{\theta}$, the equilibrium equations of the Sanders theury (reference 4) can be written, for shells of revulution, as

$$
\begin{gather*}
a \left\lvert\, \frac{\partial}{\partial \xi}\left(\left.\rho N_{\xi}+\frac{\partial}{\partial \theta}\left(\bar{N}_{\xi \theta}\right)-\rho^{\prime} N_{\theta}\left|+\omega_{\xi}\right| \frac{\partial}{\partial \xi}\left(\rho M_{\xi}\right)+\frac{\partial}{\partial \theta}\left(\bar{M}_{\xi \theta}\right)-\rho^{\prime} M_{\theta} \right\rvert\,+\right.\right. \\
\frac{1}{2}\left(\omega_{\xi}-\omega_{\theta}\right) \frac{\partial}{\partial \theta}\left(\bar{M}_{\xi \theta}\right)+a^{2} \rho q \xi=0 \tag{10a}
\end{gather*}
$$

$$
\begin{gather*}
\left.a\left|\frac{\partial}{\partial \theta}\left(\mathbf{N}_{\theta}\right)+\frac{\partial}{\partial \xi}(\rho \overline{\mathbb{N}} \xi \theta)+\hat{\sigma}_{\xi \theta}\right|+\overline{\mathcal{N}}_{\xi \theta} \right\rvert\, \frac{\partial}{\partial \theta}\left(\left.\mathbb{M}_{\theta}+\frac{\partial}{\partial \xi}\left(\rho \bar{M}_{\xi \theta}\right)+\rho^{\prime} \bar{M}_{\xi \theta} \right\rvert\,+\right. \\
\left.\frac{\dot{\rho}}{\bar{L}} \frac{\partial}{\partial \xi} \right\rvert\,\left(\omega_{\theta}-\omega_{\xi} \bar{M}_{\xi \theta} \mid+a^{2} \rho q \theta=0\right. \tag{10b}
\end{gather*}
$$

$$
\frac{\partial}{\partial \xi}\left[\bar{\partial}^{\partial} \bar{\xi}\left(\rho M_{\xi}\right)+\frac{\partial}{\partial \theta}\left(\bar{M}_{\xi \theta}\right)-\rho^{\prime} M_{\theta}\right]+
$$

$$
\frac{1}{\beta} \frac{\partial}{\partial \theta}\left[\frac{\partial}{\partial \theta} \quad\left(M_{\theta}\right)+\frac{\partial}{\partial \xi}\left(\rho \bar{M}_{\xi \theta}\right)+\rho^{\prime} \bar{M}_{\xi \theta}\right]-
$$

$$
\begin{equation*}
a \rho\left(\omega_{\xi} N_{\xi}+\omega_{\theta} N_{\theta}\right)+a^{2} \rho q=0 \tag{10c}
\end{equation*}
$$

## 1. 5 DISPLACEMENTS, ROTATIONS, AND STRAINS

The displacements and rotations of the reference surface (Figure 1-3) are related by the equations

$$
\begin{align*}
& \Phi_{\xi}=\frac{1}{a}\left[-\frac{\partial w}{\partial \xi}+\omega_{\xi} U_{\xi}\right]  \tag{11}\\
& \Phi_{\theta}=\frac{1}{a}\left[-\frac{1}{\rho} \frac{\partial W}{\partial \theta}+\omega_{\theta} U_{\theta}\right]
\end{align*}
$$

The membrane strains of tine reference surface are given by

$$
\begin{align*}
& \epsilon_{\xi}=\frac{1}{a}\left[\frac{\partial U_{\xi}}{\partial \xi}+\omega_{\xi} w\right] \\
& \epsilon_{\theta}=\frac{1}{a}\left[\frac{1}{\rho} \frac{\partial U_{\theta}}{\partial \theta}+\gamma U_{\xi}+\omega_{\theta} W\right] \\
& \epsilon_{\xi \theta}=\frac{1}{2 a}\left[\frac{1}{\rho} \frac{\partial U_{\xi}}{\partial \theta}+\frac{\partial U_{\theta}}{\partial \xi}-Y U_{\theta}\right] \tag{12}
\end{align*}
$$

where ${ }^{f} \xi \theta$ is half the usual engineering shear atrain.

a)


む

Figure 1-3. a) Displacements; b) Rotations

Finally, the measures of bending distortion used in the Sanders theory are

$$
\begin{align*}
& \kappa_{\xi}=\frac{1}{a} \frac{\partial \Phi}{\partial \xi} \xi \\
& \kappa_{\theta}=\frac{1}{a}\left[\frac{1}{\rho} \frac{\partial \Phi_{\theta}}{\partial \theta}+Y \Phi_{\xi}\right] . \\
& \kappa_{\xi \theta}=\frac{1}{2 a}\left[\frac{1}{\rho} \frac{\partial \Phi_{\xi}}{\partial \theta}+\frac{\partial \Phi_{\theta}}{\partial \xi}-Y_{\theta}+\right. \\
& \left.\frac{1}{2 a}\left(\omega_{\xi}-\omega_{\theta}\right)\left(\frac{1}{\rho} \frac{\partial U_{\theta}}{\partial \theta}-\frac{\partial U_{\theta}}{\partial \xi}-Y U_{\theta}\right)\right] \tag{13}
\end{align*}
$$

Then, by the usual Kirchhoff hypothesis (" normals remain normal") and the neglect of terms of order $\zeta / R_{8}$ and $\zeta / R_{\theta}$ relative to unity, the longitudinal, circumferential, and shear strains at a distance $\zeta$ from the reference surface are

$$
\begin{align*}
& { }_{\xi}+\zeta{ }_{\xi}{ }_{\xi} \\
& \varepsilon_{\theta}+\zeta \kappa_{\theta} \\
& { }_{\xi \theta}+\zeta{ }^{\alpha_{\xi}} \tag{14}
\end{align*}
$$

respectively.

### 1.6 CONSTITUTIVE RELATIONS

Neglecting, as usual, the effects of atreses normal to the shell permits the stress-strain-temperature relations to be written as

$$
\begin{align*}
& c_{\xi}+\zeta_{\alpha_{\xi}}=\left|\left(\sigma_{\xi}-v \sigma_{\theta}\right) / E\right|+\alpha T \\
& c_{\theta}+\zeta \kappa_{\theta}=\left|\left(\sigma_{\theta}-v \sigma_{\xi}\right) / E\right|+\alpha T \\
& \epsilon_{\xi \theta}+\zeta \kappa_{\xi \theta}=|(1+v) / E| \sigma_{\xi \theta} \tag{15}
\end{align*}
$$

where the temperature change $T$ may vary with $\zeta$, as well as with $\xi$ and $\theta$. The Young's modulus $E$ and the thermal expansion coefficient $\alpha$ will, however, be permitted to vary only with $\xi$ and $\zeta$. The (modified) forces and moments are approximated closely in the shell by the following integrals through the thickness:

$$
\begin{array}{ll}
N_{\xi}=\int \sigma_{\xi} d \zeta & M_{\xi}=\int \zeta \sigma_{\xi} d \zeta \\
N_{\theta}=\int \sigma_{\theta} d \zeta & M_{\theta}=\int \zeta \sigma_{\theta} d \zeta \\
\bar{N}_{\xi \theta}=\int \sigma_{\xi \theta} d \zeta & \bar{M}_{\xi \theta}=\int \zeta \sigma_{\xi \theta} d \zeta \tag{16}
\end{array}
$$

Then, with the use of the defining relation (Equation 1) for the reference surface, together with the assumption of constant Poisson's ratio, it is found from (Equations 14 through 16) that

$$
\begin{align*}
& \epsilon_{\xi}=\frac{N_{\xi}-\nu N_{\theta}}{\int E d \zeta}+\frac{\int E \alpha T d \zeta}{\int E d \zeta} \\
& \epsilon_{\theta}=\frac{N_{\theta}-v N_{\xi}}{\int E d \zeta}+\frac{\int E \alpha T d \zeta}{\int E d \xi} \\
& \epsilon_{\xi \theta}=\frac{(1+v) \bar{N}_{\xi \theta}}{\int E d \xi} \tag{17}
\end{align*}
$$

and

$$
\begin{align*}
& \alpha_{\xi}=\frac{M_{\xi}-\nu M_{\theta}}{\int \xi^{2} E d \xi}+\frac{\int \zeta E_{a T d} \zeta}{\int \zeta^{2} E d \zeta} \\
& \kappa_{\theta}=\frac{M_{\theta}-\nu M_{\xi}}{\int \xi^{2} E d \xi}+\frac{\int \zeta E \alpha T d \zeta}{\int \zeta^{2} E d \zeta} \\
& { }_{\xi \xi \theta}=\frac{(1+v) \bar{M}_{\xi \theta}}{\int \zeta^{2} E d \xi} \tag{18}
\end{align*}
$$

The complete set of firld equations for the 17 independent variables


## 1. 7 FOURIER EXPANSIONS AND NONDIMENSIONAL EQUATIONS

The independent variables now will be expanded into Fourier series, with appropriate normalization to provide nondimensional Fourier coefficients of roughly comparable magnitudes for the different variables. Letting $\sigma_{0}$ be a reference stress level. $E_{0}$ a reference Young's modulus, and $h_{0}$ a reference thickness, solutions of the field equations will be sought in the following forms:

$$
\begin{align*}
& N_{\xi}=\sigma_{0} h_{0} \sum_{n=0}^{\infty} t_{\xi}^{(n)} \cos n \theta \\
& N_{\theta}=\sigma_{0} h_{0} \sum_{n=0}^{\infty} t_{\theta}^{(n)} \cos n \theta \\
& \bar{N}_{\xi \theta}=\sigma_{0} h_{0} \sum_{n=1}^{\infty} t_{\xi \theta}^{(n)} \sin n \theta  \tag{19}\\
& M_{\xi}=\frac{\sigma_{0} h_{0}}{a} \sum_{n=0}^{\infty} m_{\xi}^{(n)} \cos n \theta \\
& M_{\theta}=\frac{\sigma_{0} h_{0}}{2} \sum_{n=0}^{\infty} m_{\theta}^{(n)} \cos n \theta \\
& \bar{M}_{\xi \theta}=\frac{\sigma_{0} h_{0}}{a} \sum_{n=1}^{\infty} m_{\xi \theta}^{(n)} \sin n \theta  \tag{20}\\
& U{ }_{\xi}=\frac{a_{\sigma 0}}{E_{0}} \sum_{n=0}^{\infty} u_{\xi}^{(n)} \cos n \theta \\
& W=\frac{a \sigma_{0}}{E} \sum_{n=0}^{\infty} w_{0}^{(n)} \cos n \theta \\
& E_{0}  \tag{21}\\
& \sum_{n=1}^{\infty} u_{\theta}^{(n)} \sin n \theta
\end{align*}
$$

$$
\begin{align*}
& \Phi_{\xi}=\frac{\sigma_{0}}{E_{0}} \sum_{n=0}^{\infty} \phi_{\xi}^{(n)} \cos n \theta \\
& \Phi_{\theta}=\frac{\sigma_{0}}{E_{0}} \sum_{n=1}^{\infty} \phi_{e}^{(n)} \sin n \theta  \tag{22}\\
& \epsilon_{\xi}=\frac{\sigma_{0}}{E_{0}} \sum_{n=0}^{\infty} e_{\xi}^{(n)} \cos n \theta \\
& \epsilon_{\theta}=\frac{\sigma_{0}}{E_{0}} \sum_{n=0}^{\infty} e_{\theta}^{(n)} \cos n \theta \\
& \epsilon_{\xi \theta}=\frac{\sigma_{0}}{E_{0}} \sum_{n=1}^{\infty} e_{\xi \theta}{ }^{(n)} \sin n \theta  \tag{23}\\
& k_{\xi}=\frac{\sigma_{0}}{a E_{0}} \sum_{n=0}^{\infty} k_{\xi}^{(n)} \cos n \theta \\
& k_{\theta}=\frac{\sigma_{0}}{a E_{0}} \sum_{n=0}^{\infty} k_{\theta}^{(n)} \cos n \theta \\
& k_{\xi \theta}=\frac{\sigma_{0}}{a E_{0}} \sum_{n=1}^{\infty} k_{\xi \theta}^{(n)} \sin n \theta \tag{24}
\end{align*}
$$

These Fourier expansions are consistent with loadings of the forms

$$
\begin{align*}
& q=\frac{\sigma_{0} h_{0}}{a} \sum_{n=0}^{\infty} p^{(n)}(\xi) \cos n \theta \\
& q_{\xi}=\frac{\sigma_{0} k_{0}}{a} \sum_{n=0}^{\infty} p_{\xi}^{(n)}(\xi) \cos n \theta \\
& q_{\theta}=\frac{\sigma_{0} h_{0}}{a} \sum_{n=1}^{\infty} p_{\theta}^{(n)}(\xi) \sin n \theta \tag{25}
\end{align*}
$$

and a temperature distribution

$$
\begin{equation*}
T=\sum_{n=0}^{\infty} T^{(n)}(\xi, \zeta ; \cos n \theta \tag{26}
\end{equation*}
$$

The various field equations now can be decoupled into separate sets for each Fourier index $n$; for convenience, the superscript ( $n$ ) on Fourier coefficients will be omitted in the equations that follow. The equilibrium equations (Equation 10) lead to

$$
\begin{align*}
& t \xi^{\prime}+\gamma\left(t_{\xi}-t_{\theta}\right)+(n / \rho) t_{\xi}+\lambda 2 \mid \omega_{\xi} m_{\xi}^{\prime}+ \\
& \gamma \omega_{\xi}\left(m_{\xi}-m_{\theta}\right)+(n / 2 \rho)\left(3 \omega_{\xi}-\omega_{\theta}\right) m_{\xi \theta} \mid+p_{\xi}=0 \\
& t \xi \theta^{\prime}+2 \gamma t_{\xi} \theta-(n / \rho) t_{\theta}+\lambda^{2} \mid-\left(n / p j \omega_{\theta} m_{\theta}+\right. \\
& \left.\frac{1}{2}\left(3 \omega_{\theta}-\omega_{\xi}\right) m_{\xi \theta^{\prime}}+\frac{1}{2}\left|\gamma\left(3 \omega_{\theta}+\omega_{\xi}\right)-\omega_{\xi}\right|_{m_{\xi \theta}} \right\rvert\,+p_{\theta}=0 \\
& -\omega_{\xi} t_{\xi}-\omega_{\theta} t_{\theta}+\lambda^{2} \mid m_{\xi} \prime \prime+2 \gamma m_{\xi}^{\prime}-\omega_{\xi} \omega_{\theta} m_{\xi}+ \\
& \left|\omega_{\xi} \omega_{\theta}-\left(n^{2} / \rho 2\right)\right| m_{\theta}-\gamma m_{\theta}^{\prime}+(2 n / \rho) m_{\xi \theta^{\prime}}+ \\
& \left(2 \gamma_{n} / \rho\right) m_{\xi \theta} \mid+p=0 \tag{27}
\end{align*}
$$

where $\lambda=h_{0} / a$, and use has been made of the geometrical identities Equations 6 and 7). The relations (Equations 11 through 13) give

$$
\begin{gather*}
\phi_{\xi}=-w^{\prime}+w_{\xi} u_{\xi}  \tag{28a}\\
\phi \theta=(n / \rho) w+w_{\theta^{u}} u_{\theta}  \tag{28b}\\
e_{\xi}=u_{\xi}+w_{\xi} w \\
e_{\theta}=(n / \rho) u_{\theta}+\gamma u_{\xi}+w_{\theta} w \\
e_{\xi \theta}=\frac{1}{2}\left|u_{\theta}{ }^{\prime}-\gamma u_{\theta}-(n / \rho) u_{\xi}\right| \tag{29}
\end{gather*}
$$

$$
\begin{align*}
k_{\xi}=\phi_{\xi}^{\prime} \quad k_{\theta}= & (n / r) \phi_{\theta}+y \phi_{\xi} \\
k_{\xi \theta}= & \frac{1}{2}\left\{-(n / \rho) \phi_{\xi}+\phi_{\theta}^{\prime}-\gamma \phi_{\theta}+\right. \\
& \left.\left.\left.\frac{1}{2}\left(\omega_{\theta}-\omega_{\xi}\right) \right\rvert\,\left(n u_{\xi} / p\right)+u_{\theta}^{\prime}+\gamma u_{\theta}\right)\right\} \tag{30}
\end{align*}
$$

and finally, the constitutive relations (Equations 17 and 18), inverted to give forces ani moments in terms of strains and bending distortions, lead to

$$
\begin{gather*}
t_{\xi}+b\left(e_{\xi}+v e_{\theta}\right)-t_{T}^{(n)} \quad t_{\theta}=b\left(e_{\theta}+v e_{\xi}\right)-t_{T}(n)  \tag{31}\\
t_{\xi \theta}=b(1-v) e_{\xi \theta}
\end{gather*}
$$

and

$$
\begin{align*}
m_{\xi} & =d\left(k_{\xi}+v k_{\theta}\right)-m_{T}(n)  \tag{32a}\\
m_{\theta} & =d\left(k_{\theta}+v k_{\xi}\right)-m_{T}  \tag{32b}\\
m_{\xi \theta} & =d(1-v) k_{\xi \theta} \tag{32c}
\end{align*}
$$

where

$$
\begin{align*}
& b=\frac{\int E d \zeta}{E_{0} h_{0}\left(1-v^{2}\right)}  \tag{33}\\
& d=\frac{\int \zeta^{2} E d \zeta}{E h_{0}^{3}\left(l-v^{2}\right)}  \tag{34}\\
& t_{T}^{(n)}=\frac{\int E a T^{(n)} d \zeta}{\sigma h_{0}(l-v)}  \tag{35}\\
& m_{T}^{(n)}=\frac{a \int \zeta E \alpha T^{(n)_{d}}}{\sigma_{0} h_{0}^{3}(1-v)} \tag{36}
\end{align*}
$$

(Again, the superscript $(n)$ on $\mathrm{t}_{\mathrm{T}}{ }^{(n)}$ and $\mathrm{m}_{\mathrm{T}}(\mathrm{n})$ will be omitted henceforth.)

For each $n$, the set of fied equations for the 17 Fourier coefficients
 now is given by the 17 equations (Equations 27 through 32):

It may be remarked at this point that the Fourier expansiuns (Equations 25 and 26), which are symmetrical about $\theta=0$ for $q, q \xi$, and $T$ and antisymmetrical for $q$ are not the most general that could exist. For full generality, these expansions hould be augmented by the additional series

$$
\begin{align*}
& \bar{q}=\frac{\sigma_{0} h_{0}}{a} \sum_{n=1}^{\infty} \bar{p}(n)(\xi) \sin n \theta \\
& \bar{q}_{\xi}=\frac{\sigma_{0} h_{0}}{a} \sum_{n=1}^{\infty} \bar{p}_{\xi}^{(n)}(\xi) \sin n \theta \\
& \bar{q}_{\theta}=\frac{\sigma_{0} h_{0}}{a} \sum_{n=0}^{\infty} \bar{p}_{\theta}^{(n)}(\xi) \cos n \theta \\
& \bar{T}=\sum_{n=1}^{\infty} \bar{T}(n)(\xi, \xi) \sin n \theta \tag{25a}
\end{align*}
$$

In this case, the form of the shell field equations can be obtained by setting the Fourier harmonics ( n ) to negative values in Equations 27 through 32. These effects have been neglected in this program but can se included with minor modifications of the program.

### 1.8 REDUCTION TO FOUR SECOND-ORDER DIFFERENTIAL EQUATIONS

The set of field equations obtained constitutes an eighth-order system that can be reduced, in a conventional fashion, to three equations in $u_{\xi}, u_{\theta}$, and $w$. Put a more attractive procedure is to derive four differential equations, each of ser ind order. In the variables $u_{\xi}, u_{\theta}, w$, and $m_{\xi}$. In so doing, it is necessary to eliminate $m_{\theta}$ by meins of the relation

$$
\begin{equation*}
m_{\theta}=v m_{\xi}+d\left(1-v^{2}\right) k_{\theta}-(1-v) m_{T} \tag{37}
\end{equation*}
$$

in order to prevent the ultimate appearance of derivatives of order higher than two. Then, substituting Equatiors 37, 32c, and 31 into Equation 27 and using Equations 28 through 30 to eliminate the membrane strain and
bending distortion gives three of the desired equations; the fourth ceration is given by Equation 32a, again with $k_{g}$ and $k_{0}$ expressed in trams of the displacements. The resultant set then can be written as

$$
\begin{align*}
& a_{1} u_{\xi}{ }^{\prime \prime}+{ }_{2} u_{\xi}^{\prime}+a_{3} u_{\xi}+i_{4} u_{\theta}^{\prime}+a_{5} u_{\theta}+a_{6} w_{6}^{\prime}+ \\
& a 7 w+a g m_{\xi}^{\prime}+a g m_{\xi}=C_{1} \\
& a_{1} 0^{u} \xi^{\prime}+a_{1} 1 u_{\xi}+a_{1} 2^{u}{ }_{\theta}^{\prime \prime}+a_{1} u_{\theta}{ }^{\prime}+a_{14} u_{\theta}+ \\
& a_{15} w^{*}+a_{16} w^{\prime}+a_{17} w+a_{18} m_{\xi}=C_{2} \\
& { }^{1}{ }_{19} u_{\xi}^{\prime}+a_{20} u_{\xi}+a_{21} u_{0}{ }^{\prime \prime}+a_{22} u^{\prime} \theta^{\circ}+a_{23} u_{0}+ \\
& { }^{2} 24^{w^{\prime \prime}}+\mathrm{a}_{25} \mathrm{w}^{\prime}+\mathrm{a}_{26} w+\mathrm{a}_{2} 7^{m} \xi^{\prime \prime}+\mathrm{a}_{28} \mathrm{~m}_{\xi}^{\prime}+ \\
& \mathrm{a}_{29 \mathrm{~m}_{\xi}}=\mathrm{C}_{3} \\
& a_{30} u_{\xi}^{\prime}+a_{31} u_{\xi}+a_{32} u_{\theta}+a_{33} w^{\prime \prime}+a_{34} w^{\prime}+ \\
& a 35 w+a 36 m_{\xi}=C_{4} \tag{38}
\end{align*}
$$

where the a's and $c^{\prime} s$ are given in Appendix A. These equations can be written in the matrix form

$$
\begin{equation*}
E z_{z}^{\prime \prime}+F z^{\prime}+G z=e \tag{i;9}
\end{equation*}
$$

where

$$
z=\left[\begin{array}{l}
u_{\xi}  \tag{40}\\
u_{\theta} \\
w \\
m_{\xi}
\end{array}\right]
$$

and

$$
\begin{align*}
& E=\left[\begin{array}{llll}
a_{1} & 0 & 0 & 0 \\
0 & a_{12} & a_{15} & 0 \\
0 & a_{21} & a_{24} & a_{27} \\
0 & 0 & a_{33} & 0
\end{array}\right] \quad \therefore=\left[\begin{array}{llll}
a_{2} & a_{4} & a_{6} & a_{8} \\
a_{10} & a_{13} & a_{16} & 0 \\
a_{19} & a_{22} & a_{25} & a_{28} \\
a_{30} & 0 & a_{34} & 0
\end{array}\right] \\
& G=\left[\begin{array}{llll}
a_{3} & a_{5} & a_{7} & a_{9} \\
a_{11} & a_{14} & a_{17} & a_{18} \\
a_{20} & a_{23} & a_{26} & a_{29} \\
a_{31} & a_{32} & a_{35} & a_{36}
\end{array}\right] e=\left[\begin{array}{l}
c_{1} \\
c_{2} \\
c_{3} \\
c_{4}
\end{array}\right] \tag{41}
\end{align*}
$$

### 1.9 BOUNDARY CONDITIONS

In the Sanders theory, the expressions for virtual work per unit length at the boundaries $s=0,3$ are

$$
\begin{equation*}
\mp\left(N_{\xi} U_{\xi}+\hat{N}_{\xi \theta} U_{\theta}+\hat{Q}_{\xi} W+M_{\xi} \Phi{ }_{\xi}\right) \tag{42}
\end{equation*}
$$

where

$$
\begin{equation*}
\hat{\mathbf{N}}_{\xi \theta}=\overline{\mathbf{N}}_{\xi \theta}+\left[\left(3 / 2 R_{\theta}\right)-\left(1 / 2 R_{\xi}\right) \mid \bar{M}_{\xi \theta}\right. \tag{43}
\end{equation*}
$$

an'

$$
\begin{equation*}
\hat{Q}_{\xi}=(1 / 2 \rho)\left[(\partial / \partial \xi)\left(\rho M_{\xi}\right)+2\left(\partial \bar{M}_{\xi \theta} / \beta \theta\right)-\rho^{\prime} M_{\theta}\right] \tag{44}
\end{equation*}
$$

are "effective" membrane and transverse shears, respectively, per unit length. (See Figure 1-4.) This form of the virtual work indicates the kinds of boundary conditiong that can be imposed; thus, either $\mathrm{N}_{\boldsymbol{\xi}}$ or $\mathrm{U}_{\xi}$ may be prescribed, either $\hat{N}_{\xi \theta}$ or $U_{\theta}$ may be prescribed, and so on; or, more generally, $N_{\xi}$ and $U_{\xi}$ may be related through an elastic coastraint against meridional displacement; and analogous constraints can link $\hat{\mathrm{N}}_{\xi \theta}$ ard $U_{\theta}, \hat{Q}_{\xi}$ and $W$, and $M_{\xi}$ and $\oplus_{\xi}$. Letting

$$
\begin{aligned}
& \hat{N}_{\xi \theta}=\sigma_{0} h_{0} \sum_{n=1}^{\infty} \hat{t}_{\xi \theta}(n) \sin n \theta \\
& \hat{Q}_{\xi}=\sigma_{0} h_{0} \sum_{n=0}^{\infty} \hat{i}_{\xi}(n) \cos n \theta
\end{aligned}
$$

gives (dropping superscripts)

$$
\begin{align*}
& \hat{t}_{\xi \theta}=t_{\xi \theta}+\left(\lambda^{2} / 2\right)\left(3 \omega_{\theta}-\omega_{\xi}\right) m_{\xi \theta} \\
& \hat{f}_{\xi}=\lambda^{2}\left[m_{\xi}^{\prime}+\gamma\left(m_{\xi}-m_{\theta}\right)+(2 n / \rho) m_{\xi \theta}\right] \tag{46}
\end{align*}
$$

Then the boundary conditions just discussed always can be written (for the nth Fourier components) as

$$
\begin{equation*}
\Omega y+\Lambda z=\ell \tag{47}
\end{equation*}
$$

where



Figure 1-4. Effective Boundary Forces and Moment
and where $\Omega$ and $\Lambda$ are appropriate diagonal matrices, and $\ell$ is a given column matrix. (For example, if $u_{\xi}$ is given, the first diagonal element of $\Omega 2$ is zero, that of $\Lambda$ is unity, and the first element of $\ell$ is the prescribed value of $u_{\xi}$; if there is an elastic constraint on $u_{\xi}$, then the first diagonal element of $\Omega$ is unity, that of $\Lambda$ is the appropriate constraint coefficient, and the first element of $\ell$ vanishes.) But now it is desirable to express the boundary conditions entirely in terms of $z$; from
Equations 28 through 32 and 37, it follows that

$$
\begin{align*}
t_{\xi} & =b_{1} u_{\xi}^{\prime}+b_{2} u_{\xi}+b_{3} u_{\theta}+b_{4} w-t_{T} \\
\hat{t}_{\xi \theta} & =b_{5} u_{\xi}+b_{6} u_{\theta}^{\prime}+b_{7} u_{\theta}+b_{8} w^{\prime}+b_{9} w \\
\hat{\mathbf{f}}_{\xi} & =b_{10} u_{\xi}+b_{11} u_{\theta}^{\prime}+b_{12} u_{\theta}+b_{13} w^{\prime}+b_{14} w+b_{15} m_{\xi} \\
& +b_{16} m_{\xi}+\lambda^{2} \gamma(1-v)_{m_{T}} \tag{49}
\end{align*}
$$

where the b's are given in Appendix A. These equations, together with Equation 28a, give

$$
\begin{equation*}
\mathbf{y}=\mathbf{H z} \mathbf{z}^{\prime}+\mathrm{Jz}+\mathbf{f} \tag{50}
\end{equation*}
$$

where

$$
\left.\begin{array}{l}
H=\left[\begin{array}{llll}
b_{1} & 0 & 0 & 0 \\
0 & b_{6} & b_{8} & 0 \\
0 & b_{11} & b_{13} & b_{15} \\
0 & 0 & -1 & 0
\end{array}\right] f=\left[\begin{array}{c}
{ }^{-t_{T}} \\
0 \\
J
\end{array}\right] \\
\lambda_{1}^{2} \gamma_{1}(1-v) m_{T}  \tag{51}\\
b_{10} \\
b_{2}
\end{array} b_{12} b_{12} \quad b_{14} \quad b_{16}\right]
$$

Hence, the boundary conditions (Equation 47) can be Jritten as

$$
\begin{equation*}
\Omega \mathrm{Hz}^{\prime}+(\Lambda+\Omega J) \mathbf{z}=\ell-\Omega \mathbf{f} \tag{52}
\end{equation*}
$$

### 1.10 SINGULAR POINTS (APEX CONDTION)

If the shell hat a pole (i.e., $r=0$ ), coefficients in the gove.aing differential equations become aingular. An improved procedure for handling such conditions has been outlined in References 9 and 9 and is used in the analysis. The boundary comditions at the apex of a closed shell of revolution are described as follows for each Fourier component ( n )

$$
\begin{array}{ll}
u_{\xi}=u_{\theta}=w^{\prime}=w_{\xi}^{\prime}=0 & \text { for } n=0 \\
u_{\xi}^{\prime}=u_{\xi}+u_{\theta}=w=m_{\xi}=0 & \text { for } n=1 \\
u_{\xi}=u_{\theta}=w=w^{\prime}=0 & \text { for } n=2 \\
u_{\xi}=u_{\theta}=w=m_{\xi}=0 & \text { for } n \geqq 3
\end{array}
$$

These special conditions can be cant in a matrix form identical to Equation 52. For this case, the matrix $\boldsymbol{\Lambda}+\boldsymbol{\Omega} \mathbf{J}$ is not of a diagonal form.

### 1.11 DISCONTINUITY CONDITIONS

The differential equations (39) are not valid at points in the shell in which discontinuities in geometry (and hence in the coefficients) occur; furthermore, $z$ itself is ambiguous at a discontinuity in the inclination of the reference surface, where the directions of $u_{\xi}$ and $w$ change abruptly. Accordingly, special transition equations must be derived which relate $z$ and its derivative on either side of a discontinuity.

In Reference 1, the special case in which reference surfaces coincide across a discontinuity was considered. (See Figure 1-5.) A more general condition, which was treated in Reference 2, occurs when reference surfaces do not coincide at discontinuities. This type of condition is considered for this program and will be referred to as eccentric discontinuities. The effects of external line load end moments applied at the discontinuity are included in the analysis (Figure 1-5). A typical eccentric discontinuity model is shiwn in Figure 1-6. Roman numeral superscripts refer to shell regions; thus, for the example considered, Il denotes values beyond and I values ahead of a discoatinuity. The conconditions of geometrical compatibility are (Figures 1-5 and 1-6)


Figure 1-5. Discontinuity Conditions

Figure 1-6. Eccentric Discontinuity Model

$$
\begin{align*}
u_{\xi}^{I I} & =\left|u_{\xi}^{I}+E_{c c} \phi_{\xi}^{I}\right| \cos \psi-w^{I} \sin \psi \\
u_{\theta}^{I I} & =u_{\theta}^{I}+E_{c c} \phi_{\theta}^{I} \\
w^{I I} & =\left|u_{\xi} I+E_{c c} \phi_{\xi}^{I}\right| \sin \psi+w^{I} \cos \psi  \tag{53}\\
\phi_{\xi}^{I I} & =\phi_{\xi}^{I}
\end{align*}
$$

where $E_{c c}$ is the dimensionless eccentricity of the participating reference surfaces measured along the radius of curvature behind the discontinuity point. It can be noted in Figure 1-6 that a positive value of $\mathrm{E}_{\mathrm{cc}}$ corresponds to an abrupt increase in the radius of a parallel circle as on $\epsilon$ proceeds in the direction of increasing $\xi$. Equilibrium requires that

$$
\begin{align*}
& t_{\xi} I I=t_{\xi} I \cos \psi-\hat{f}_{\xi} I \sin \psi+P_{D} \sin \psi_{0} \\
& \hat{t}_{\xi \theta} I I=\hat{t}_{\xi \theta} I \\
& \hat{\mathbf{I}}_{\xi} I I=t_{\xi} I \sin \dot{\prime}+\hat{f}_{\xi} I \cos \psi-P_{D} \cos \psi_{0}  \tag{54}\\
& m_{\xi} I I=m_{\xi} I-\frac{E_{c c}}{\lambda^{2}} t_{\xi} I-M_{D}
\end{align*}
$$

where $P_{D}$ and $M_{D}$ are Fourier cocfficients of series expansions for externally applied line loads and moments; i.e.,

$$
\begin{aligned}
& \bar{P}_{D}=\frac{o_{0} h_{0}}{a} \sum_{n=0}^{\infty} P_{D} \cos n \theta \\
& \bar{M}_{D}=\frac{o_{0} h_{1} .{ }^{3}}{a} \sum_{n=0}^{\infty} M_{D} \cos n \theta
\end{aligned}
$$

The information in Equations 53 and 54 is reproduced in the equations

$$
\begin{gather*}
y^{I I}=\Psi y^{I}+\Phi_{0} P_{D}  \tag{55}\\
z^{I I}=\Phi z^{I}+y^{I}+\nabla M_{D} \tag{56}
\end{gather*}
$$

where

$$
\begin{align*}
& \Psi=\left[\begin{array}{llll}
\cos \psi & 0 & -\sin \psi & 0 \\
0 & 1 & 0 & 0 \\
\sin \psi & 0 & \cos \psi & 0 \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{57}\\
& \Phi=\mathbf{E}_{\mathbf{c c}}\left[\begin{array}{llll}
0 & 0 & 0 & 0 \\
0 & \left(\omega_{\theta}\right)_{j} I & n / p_{j} I & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array}\right]+\bar{\Psi}  \tag{58}\\
& \#=E_{\mathbf{c c}}\left[\begin{array}{llll}
0 & 0 & 0 & \cos \psi \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sin \psi \\
-1 / \lambda^{2} & 0 & 0 & 0
\end{array}\right] \tag{59}
\end{align*}
$$

$$
\begin{align*}
& \Phi_{0}= {\left[\begin{array}{l}
\sin \psi_{0} \\
0 \\
-\cos \psi_{0} \\
0
\end{array}\right] }  \tag{60}\\
& \underline{n}=\left[\begin{array}{c}
0 \\
0 \\
0 \\
-1
\end{array}\right] \tag{61}
\end{align*}
$$

Combining Equations 55, 56, and 50 then provides the equations relating $\left(z^{\prime}\right)^{I I}$, ( $\left.z^{\prime}\right)^{I}$, and $z^{\prime}$ :

$$
\begin{align*}
& H^{I I}\left(z^{I I}\right)+\left|J^{\prime I I} \Psi-\Psi J^{I}\right| z^{I}-\Psi H^{I}\left(z^{\prime}\right)^{\prime}=\Phi_{0} P_{D}+\Psi f^{I}-f^{I I} \tag{62a}
\end{align*}
$$

where the Roman numerals I and II mean that the matrices $H$, $J$, and $f$ are to be calculated from Equation 51 on the basis of shell properties just behind and ahead of the discontimity, respectively. The equlibrium equations (39), the boundary conditions (52), and the discontinuity conditions (62) will now be cast into a unified set of appropriate finite-difference equations suitable for numerical analysis.

### 1.12 FINITE DIFFERENCE FORMULATION

A finite difference technique will be used in the solution of the shell equations. In treating complicated shell configurations, it will at times be necessary and convenient for analysis purposes to divide the mathematical model of the shell in combinations of smaller region. The dividing line between regions is usually selected at discontinuity regions. (See Section 2.5.) In the finite difference formulation, the path region will be subdivided into ( $N^{P}-1$ ) equai increments of length $\Delta P$. $N^{P}$ corresponds to the number of station or pivotal points considered for the region. The pivotal points are identified along the meridian by the integer index $i$, starting from $i=1$ at $\xi=0$ (station 1) and proceeding to $N$-th station ( $i=N$ ) occurring at the endpoint of the region (see Figure 1-7).


Figure 1-7. Meridional Grid Points

The regions are designated by Roman numeral superscripts I, II, etc., and discontinuity stations by $i=J P$. For the discontinuity junction illustrated in Figure 1-8, the discontinuity $\mathbf{j}^{1}$ would correspond to station $i=N$ of region $I, j{ }^{\text {II }}$ station $i=1$ of region II, etc. The increment $\Delta^{P}$ can be varied from region to region. Thus, it is possible to introduce fictitious discontinuities wherever a change in increment size is considered desirable.

The differential equations (39), boundary conditions (52) (excepting the closed apex condition), and discontinuity conditions (62) are written in finite difference form at all stations on the basis of the usual central difference formulas

$$
\begin{align*}
& z_{i}^{\prime}=\left(z_{i+1}-2 z_{i}+z_{i v 1}\right) / \Delta^{2} \\
& z_{i}^{\prime}=\left(z_{i+1}-z_{i-1}\right) / 2 \Delta \tag{63}
\end{align*}
$$

where the $\Delta$ must, of course, be the one corresponding to the region associated with the station $i$.

Applying the above expressions at the endpoints of a region ( $i=1, N$ ) results in fictitious points occurring outaide the range of the region
(i.e., $i=0, N+1$ ). Figure $1-8$ illustrates the mathematical model used at a discontinuity point with fictitious points $j^{11-1}$ and $j^{1+1}$ reaulting from applicaticn of difference expressions


Fig re 1-8. Finite Difference Stations in Discontinuity Region

The fictitious points can be mathematically eliminated by applying both boundary (or discontinuity) and equilibrium conditions at the endpoints. The details of this type of oparation are described in Reference 2. In the original analysis of Reference 1, a somewhat different approach was utilized in that equilibrium was not satisfied at endpoints. The improved procedure of Reference 2 permits a more accurate representation of shell behavior at endpoints.

In case of a pole condition ( $r=0$ ), the singularity does not permit writing both equilibrium and compatibility; as a result, the procedure must be modified for this case. The approach used for this special case is to express derivatives at endpoints in terms of modified forward (backward) differences. The boundary condition for a pole condition will be written at $i=0$ and $i=N$ with the help of

$$
\begin{align*}
& z_{i}^{\prime}=\left(-3 z_{1}+4 z_{2}-z_{3}\right) / 2 \Delta \\
& z_{N}^{\prime}=\left(3 z_{N}-4 z_{N-1}+z_{N}-2\right) / 2 \Delta \tag{64}
\end{align*}
$$

The order of approximation of these expressions is the same as the usual central difference expreasions and is usually more aceurate than simple forward or backward difference expressions used in Reference 1.

The convention will now be adopted that, at the discontinuity (say $P=1)$, whenever $z_{j}$ is written without a qualifying superscript, it means $z_{j} \frac{f}{j}$; then, whenever $z_{j}^{I I}$ appears, it will be repiaced by utilizing relationship according to Equation 58b. Similar operations would occur for subsequent discontinuities. The results of writing the various difference equations just described can the stated compactly fexcepting for pole conditions) as the following set of algebraic equations for $z_{i}(i=1,2$, $3, .$. N):

$$
\begin{align*}
A_{1} z_{2}+B_{1} z_{1} & =g_{1}(i=1) \\
A_{i} z_{i+1}+B_{i} z_{i}+C_{i} z_{i-1} & =g_{i}(i=2,3, \ldots N-1) \\
B_{N} z_{N}+C_{N} z_{N-1} & =g_{N}(i=N) \tag{65}
\end{align*}
$$

Here, at $i=2,3, \ldots, N-1$ the internal points of the region we have

$$
\begin{align*}
& A_{i}=\left(2 \mathrm{E}_{\mathrm{i}} / \Delta\right)+\mathrm{F}_{\mathrm{i}} \\
& \mathrm{~B}_{\mathrm{i}}=-\left(4 \mathrm{E}_{\mathrm{i}} / \Delta\right)+2 \Delta \mathrm{G}_{\mathrm{i}} \\
& \mathrm{C}_{\mathrm{i}}=\left(2 \mathrm{E}_{\mathrm{i}} / \Delta\right)-\mathrm{F}_{\mathrm{i}} \\
& \mathrm{~g}_{\mathrm{i}}=2 \Delta \mathrm{e}_{\mathrm{i}} \tag{66}
\end{align*}
$$

where the appropriate value for $\Delta$ is used.

At $i=1$, we have been using the procedure outlined above and described in Reference 2 following

$$
\begin{align*}
& A_{1}=\frac{\Omega_{1} H_{1}}{2 \Delta_{1}}+\frac{\Omega_{1} H_{1}}{2 \Delta_{1}} \bar{C}_{1}-1 \bar{A}_{1} \\
& B_{1}=\Lambda_{1}+\Omega_{1} J_{1}+\frac{\Omega_{1} H_{1}}{2 \Delta_{1}} \bar{C}_{1}^{-1} \bar{B}_{1} \\
& g_{1}=\ell_{1}-\Omega_{1} f_{1}+\frac{\Omega_{1} H_{1}}{2 \Delta_{1}} \bar{C}_{1}^{-1} 2 \Delta_{1} e_{1} \tag{67}
\end{align*}
$$

and $i=N$

$$
\begin{align*}
& B_{N}=\Lambda_{N}+s 2_{N} J_{N}-\delta \Omega_{N} H_{N} \bar{A}_{N}^{-1} \bar{\Omega}_{N} \\
& C_{N}=-\frac{s \Omega_{N} H_{N}}{2 \Delta_{N-1}}-\frac{\Omega_{N} H_{N}}{2 \bar{\Delta}_{N-1}} \bar{A}_{N} N^{1} \bar{C}_{N} \\
& \delta_{N}=\ell N-s_{N} N_{N}-\frac{\Omega_{N} H_{N}}{2 \Delta_{N-1}} \bar{A}_{N}^{-1}{ }^{2} \Delta_{N-1} e_{N} \tag{68}
\end{align*}
$$

where the matrices $\bar{A}_{1}, \bar{B}_{1}, \bar{C}_{1}$ and $\bar{A}_{N}, \bar{B}_{N}, \bar{C}_{N}$ are of the form of Equation 66 evaluated at $i-1$ and $N$, respectively.

At discontinuity locations $j^{P}$, considerably more complicated expressions for the matrices are obtainec than ara reported in Reference 1, which arises due to the improved numerical model and the fact that eccentric discontinuities are considered. The details of obtaining these expressions are reported in Reference 2. As a result, the following form of the matrices of discontinuity location is obtained:

$$
\begin{align*}
& A_{j}=\frac{H^{I I}}{2 \Delta^{I I}}+\left(\frac{H^{I I}}{2 \Delta^{I I}}\right) \cdot\left(C^{I I}\right)^{-1} \cdot A^{I I} \\
& B_{j}=\left(\frac{H^{I I}}{2 \Delta^{I I}}\right) \cdot\left(C^{I I}\right)^{-1} B^{I I} \cdot\left[\pi-\left(\frac{6 H^{I}}{2 \Delta^{I}}\right)\left(A^{I}\right)^{-1} \cdot B^{I}+2 J^{I}\right]+|Y| \\
& -|\mathbf{X}| \cdot\left(A^{I}\right)^{-1} \quad B^{I} \\
& \left.C_{j}=-\left(\frac{H^{I I}}{2 \Delta^{I I}}\right) \quad\left(C^{I I}\right)^{-1} \cdot B^{I I}\left(\frac{2 \pi H^{I}}{2 \Delta^{I}} \cdot \frac{2 \Delta H^{I}}{2 \Delta^{I}}\right) \cdot\left(A^{I}\right)^{-1} \cdot C^{I}\right] \\
& -|X| \cdot\left(A^{I}\right)^{-1} C^{I}-|x| \\
& \left.g_{j}=\left(\frac{H^{I I}}{2 \Delta^{I I}}\right) \cdot\left(C^{I I}\right)^{-1} \cdot\left[g^{I I}-B^{I I} \cdot\left(\frac{\left(\underset{A H}{ } H^{I}\right.}{2 \Delta^{I}}\right) \cdot\left(A^{I}\right)^{-1} \cdot g^{I}-B^{I I}\right)\right] \\
& -|x| \cdot\left(A^{I}\right)^{-1} g^{I}-|L| \tag{69}
\end{align*}
$$

where

$$
\begin{align*}
& |x|=(J I I x-\Psi) \cdot\left(\frac{H^{I}}{2 \Delta^{I}}\right) \\
& |Y|=J^{\text {II }}\left(x+2 \mathrm{~J}^{\mathbf{I}}\right)-\Psi \mathrm{I}^{\mathrm{I}} \tag{70}
\end{align*}
$$

The $A_{f} B, C$, and $g$ matrices in Equation 69 are given for either points $j^{I}$ or $j^{\text {II }}$ by Equation 66 with the appropriate superscript attached to $E, F$, $\mathrm{G}_{\mathrm{G}}$ and $\Delta$. At station just past a discontinutty $\left(\mathrm{j}^{I I}+1\right)$ the matrices mus: be modified as follows:

$$
\begin{align*}
& C_{j+1}^{*}=C_{j+1}^{I I}\left\{\left[\left\{+i J^{I}-2 \pi \frac{H^{I}}{2 \Delta^{I}} \cdot\left(A^{I}\right)^{-1} \cdot B^{I}\right]\right.\right. \\
& \left.+\frac{2 H^{I}}{2 \Delta^{I}}\left[I+A^{I^{-1}} C^{I}\right] P_{j-1}^{I}\right\} \\
& g_{j+1}^{*}=g_{j+1}^{I I}-C_{j+1}^{I I} * f^{I}-C_{j+1}^{I I} \cdot\left[\frac{H^{I}}{2 \Delta^{I}} \cdot\left(A^{I}\right)^{-1} g^{I}\right] \\
& +C_{j+1}^{I I}\left\{\frac{7 E H^{I}}{2 \Delta^{I}}\left[I+\left(A^{I}\right)^{-1} \cdot C^{I}\right]\right\} X_{j-1}^{I} \tag{71}
\end{align*}
$$

### 1.13 MATRIX SOLUTION OF DIFFERENCE EQUATIONS

The set of matrix equations (65) will be solved by essenti.nly the same formal procedure that is used in Reference 1 for the analogous equation for the case of axisymmetric lesuire of shells of revolution, this procedure is actually equivalent to solution by the method oi Gaussian elimination used in Reference 1 for the same axisymmetric loading prublem. In its most primitive form, the Gaussian elimination technique would proceed as follows: the first of Equations 65 would be solved for $z_{1}$ in terms of $z_{2}$; this resilt would be substituted into the next equation, and $z_{2}$ would be found in terms of $z_{3}$ and so on; finally, the very last ecuation, together with the result for $\mathrm{z}_{\mathrm{N}}-1$ in terms of zN wolld determine $\mathrm{I}_{\mathrm{N}}$ and then all of the z 's would be calculated in reverse ord:r. A minor modification of this method is desirable, however (aad sometimes essential), in the treatment of Equation 65 for the matrix $B_{0}$ sometimes may be
singular. * Accordingly, the solution is started by the simultaneous solution for $z_{n}$ and $z_{1}$, in terme of 2, ard lisen procecds as just described. Erom

$$
\begin{aligned}
& A_{1}^{Z_{2}}+B_{1}^{z_{1}}=A_{1} \\
& B_{2}^{z_{2}}+C_{2} z_{1}=A_{2}-A_{2} z_{3}
\end{aligned}
$$

it follows that

$$
\begin{equation*}
z_{2}=-\left|B_{1} C_{2}^{-1} B_{2}-A_{1}\right|^{-1}\left|B_{1} C_{2}^{-1} A_{2} z_{3}-B_{1} C_{2}^{-1} g_{2}+g_{1}\right| \tag{72}
\end{equation*}
$$

Now write the general result for $z_{i}$ in terms of $z_{i+1}$ as

$$
\begin{equation*}
2:=-P_{i} z_{i+1}+x_{i} \quad(i=1,2, \ldots N-1) \tag{73}
\end{equation*}
$$

Then, the substitution of $z_{i-1}=-P_{i-1} x_{i}+x_{i-1}$ into the general equation (65) provides the results

$$
\begin{align*}
& P_{i}=\left|B_{i}-C_{i} P_{i-1}\right|^{-1} A_{i} \\
& x_{i}=\left|B_{i}-\tau_{i} P_{i-1}\right|^{-1}\left|B_{i}-C_{i} x_{i-1}\right| \quad(i=2,3, \ldots N-1) \tag{74}
\end{align*}
$$

*This occurs, for example, in the case of a clamped edge, with $u_{\underline{\xi}}=u_{\theta}=w=\phi_{\xi}=0$; then
giving
which is singular.

The recurrence relations (Equation 74), with the initial from (Equation 72),

$$
\begin{align*}
& P_{2}=\left|B_{1} C_{2}^{-1} B_{2}-A_{1}\right|^{-1} B_{1} C_{2}^{-1} A_{2} \\
& x_{2}=\left|B_{1} C_{2}^{-1} B_{2}-A_{1}\right|^{-1} \quad B_{1} C_{2}^{-1} g_{2}-g_{1} \tag{75}
\end{align*}
$$

then provide all the $\mathrm{P}^{\prime} \mathrm{s}$ and $\mathrm{x}^{\prime} \mathrm{s}$ up to $\mathrm{P}_{\mathrm{N}-1}$ and $\mathrm{x}_{\mathrm{N}-1}$. Substitution of ${ }^{2} \mathbf{N}-1=-P_{N-1}{ }^{2} \mathbf{N}+x_{N-1}$ into the last of Equation (66) then gives

$$
\begin{equation*}
z_{N}=\left|\mathbf{B}_{N}-C_{N} P_{N-1}\right|^{-1}\left|g_{N}-C_{N_{N}}{ }_{N-1}\right| \tag{76}
\end{equation*}
$$

and then $z_{N-1},{ }^{z_{N}}$-2, .. $z_{1}$ can be found from Equation 73. Finally, $z_{0}$ is given by

$$
\begin{equation*}
z_{1}=C_{2}^{-1}\left|g_{2}-A_{2} z_{1}-B_{2} z_{3}\right| \tag{77}
\end{equation*}
$$

Thus, the only matrix inversions involved in the solution for all the $z^{\prime}$ s are of $4 \times 4$ matrices, and the process is very well suited for rapid machine computation. The $z_{j}$ obtained at a discontinuity station is, of course, really $\mathbf{z}_{\mathbf{j}}^{\mathrm{I}}$. The value of $\mathbf{z}_{\mathrm{j}}^{\mathrm{II}}$ at this point can be evaluated from Equation 62b. For a singular or pole condition, a slight modification in the elimination procedure is involved to accommodate for firite difference form (Equation 64) applied at an endpoint. The details of this procedire are described in Reference 2.

### 1.14 CALCULATION OF STRESSES

Once the z's have been calculated, the stresses at any point in the shell can be found. The stresses in the present solution are ob:ained from the expansions

$$
\begin{gather*}
\sigma_{\xi}=\sum_{n=0}^{n} \sigma_{\xi}(n) \cos n \theta \\
\sigma_{\theta}=\sum_{n=0}^{\infty} \sigma_{\theta}(n) \cos n \theta \\
\sigma_{\xi \theta}=\sum_{n=1}^{\infty} \sigma_{\xi \theta}(n) \sin n \theta \tag{78}
\end{gather*}
$$

Invertiag the constitutive relations (Equation 15) and using Equations 23, 24, and 26 gives

$$
\begin{align*}
& \sigma_{\xi}(n)=\frac{E_{\sigma_{0}}}{E_{0}\left(1-v^{2}\right)}\left[e_{\xi}^{(n)}+v e_{\theta}^{(n)}+\frac{\zeta}{2}\left(k_{\xi}(n)+v k_{\theta}(n)\right)\right]-\frac{E \alpha T^{(n)}}{1-v} \\
& \sigma_{\theta}(n)=\frac{E \sigma_{0}}{E_{0}\left(1-v^{2}\right)}\left[e_{\theta}^{(n)+v e_{\xi}(n)+\frac{\xi}{a}\left(k_{\theta}^{(n)}+v k_{\theta}(n)\right]-\frac{E \alpha T^{(n)}}{1-v}}\right. \\
& \sigma_{\xi \theta}(n)=\frac{E_{\sigma_{0}}}{E_{0}(1+v)}\left[e_{\xi \theta}(n)+\frac{\xi}{a} k_{\xi \theta}(n)\right] \tag{79}
\end{align*}
$$

Note that $E, u$, and $T^{(n)}$ all may depend on $\zeta$, the distance from the reference surface.

Using Equations 32a, 32b, and 37 (and, again, casually dropping superscripts n) gives

$$
\begin{aligned}
& \mathbf{k}_{\xi}+v \mathbf{k}_{\theta}=\frac{m_{\xi}+m_{T}}{d} \\
& \mathbf{k}_{\theta}+v \mathbf{k}_{\xi}=\frac{m_{\theta}+m_{T}}{d}=\frac{v\left(m_{\xi}+m_{T}\right)}{d}+\left(1-v^{2}\right) \mathbf{k}_{\theta}
\end{aligned}
$$

which, when used in Equation 79, together with the strain-rotationdieplacement equations ( 28 through 30), leads to

$$
\begin{gather*}
{\left[\begin{array}{c}
\sigma_{\xi}(n) \\
\sigma_{\theta}(n) \\
\sigma_{\xi \theta}(n)
\end{array}\right]=K z^{\prime}+L z+\sigma_{T}}  \tag{80}\\
\left.K=\frac{E_{\sigma} \sigma_{0}}{E_{\theta}\left(1-\nu^{2}\right)}\left[\begin{array}{ccc}
1 & 0 & 0 \\
v & 0 & 0 \\
0 & \frac{1-v}{2} \cdot\left[1+\frac{\zeta}{2 a}\left(3 \omega_{\theta}-\omega_{\xi}\right)\right.
\end{array}\right] \begin{array}{c}
\frac{\zeta}{2}(1-v) \frac{n}{\rho}
\end{array}\right] \tag{81}
\end{gather*}
$$



$$
\because=\left[\begin{array}{cc}
\frac{E \sigma_{0} \zeta \mathrm{~m}_{\mathrm{T}}}{E_{0}\left(1-\nu^{2}\right) a d} & -\frac{E \alpha T^{(n)}}{1-v}  \tag{82}\\
\frac{\nu E_{0} \sigma_{0} \zeta \mathrm{~m}_{\mathrm{T}}}{E_{0}\left(1-v^{2}\right) \mathrm{ad}} & -\frac{E \alpha T^{(n)}}{1-v} \\
0
\end{array}\right]
$$

### 1.15 REMARK CONCERNING THE REFERENCE SURFACE

A substantial simplification in setting up the numerical analysis for computation may result from the observation that, in the spirit of thinshell theory, errors of the order of the thickness in the specification of the reference surface can be tolerated in the formulation of the equation of equilibrium. It is recommended accordingly that the key georsetric function $r(s)$ be started with respect to a surface chosen simply according to convenience anywhere in the shell wall. In other words, the condition (Equation 1) need not be imposed insofar as calculations of 1 e various geometrical parameters $\rho, \omega_{\theta}, \omega_{\mathcal{L}}$, and $Y$ are concerned. Of course, if Equation 1 can be satisfied easily in these calcuiations, there is no harm in doing so; but when, for example, the same shell is to be analyzed for several different temperature conditions with different resultant variations of Young's modulus, it is not recommended that new reference surfaces and new variations of $P, \omega_{\theta}$, etc., be calculated for each case. On the other hand, it is essential that the rigorous location of the reference surface enter into Equations 34 and 36 for the nondimensional bending stiffess $d$ and the thermal moment mT. Similarly, the correct value of $\zeta$ as measured from the true reference surface must be used in Equations 80 through 83 for the stresses.

### 1.10 BRANCHING OF SHELL REGIONS

It has been tacitly assumed that the shell under consideration has no more than two boundaries; a multiple-branch shell such as shown in Figure 1-9a may be analyzed, however, by applying appropriate transition conditions at the branch point.

Define separate families of auxiliary matrices $\mathrm{PI}, \mathrm{pII}, \mathrm{pIII}, \mathrm{x}^{\mathrm{I}}$, $x^{I I}$ and $x^{I I I}$ with the properties

$$
\begin{align*}
z_{i}^{I} & =-P_{i}^{I} z_{i+1}^{I}+x_{i}^{I} \\
z_{i}^{I I} & =-P_{i}^{I I} z_{i+1}^{I I}+x_{i}^{\text {II }} \\
z_{i}^{\text {III }} & =-P_{i}^{\text {III }} z_{i+1}^{\text {III }}+x_{i}^{\text {III }} \tag{84}
\end{align*}
$$



Figure 1-9. Branched Shells
where the superscripts refer to the separate branches shown in Figure 1-9a. It is possible to start the calculations of $\mathrm{PI}, \mathrm{xI}$ and PII , $x I I$ at the boundaries of branches $I$ and $I I$ and then leap across the juncture $j$ to the calculation of PIII, xIII. The reverse sweep for the calculation of the $z$ 's then would start at the boundary of branch III and, at the juncture $j$, continue indspendently along the branches I and II back to their respective boundaries. The details of this procedure are herein given. This method can be extended readily to handle a multiplicity of branches as in Figure 1-9b; it will not, kowever, be applicable to closed loops (Figure 1-9c), which muat be treated separately by traditional cut-and-fit methods of indeterminate atructural analyais.

The mathematical model considered for the numerical solution of branched shell problems is shown ir. Figure $1-10$ with the possibility of a concentrated force $P_{D}$ anc. $M_{D}$ applied at the juncture included. The program has been set up to handle 4 shell branches meeting at a common point.


Figure 1-10. Mathematical Model for Branched Shell

By analogy with the previous discussion on discontinuity conditions, we may repeat here for branched shells the compatibility and equilibrium equations in the following manner:

Compatibility:

$$
\left.\begin{array}{l}
u_{\xi}^{I V}=u_{\xi}^{M} \cos \psi M-w^{M} \sin \psi M \\
u_{\theta}^{I V}=u_{\theta}^{M} \\
w^{I V}=u_{\xi}^{M} \sin \psi^{M}+w^{M} \cos \psi^{M} \\
\phi_{\xi}^{I V}=\phi_{\xi}^{M} \tag{85}
\end{array}\right\} \quad(M=I, I I \text { or III })
$$

Fquilibrium: $\quad \mathrm{t}_{\xi}^{\mathrm{IV}}-\sum_{M=1}^{\Psi} \mathbb{M}_{\xi}^{M} \cos \psi M+\sum_{M=1}^{\Psi} \hat{i}_{\xi}^{M} \sin \psi M-\overline{\mathrm{P}} \sin \phi_{0}=0$

$$
\begin{align*}
& \mathrm{t}_{\xi \theta}^{\mathrm{IV}}-\sum_{M=1}^{\mu \mathrm{t}} \mathrm{t}_{\xi \theta}^{M}=0 \\
& \hat{\mathbf{f}}_{\xi}^{\mathrm{IV}}-\sum_{M=1}^{\Psi} \mathrm{t}_{\xi}^{M} \sin \psi^{M}-\sum_{M=1}^{M} \hat{\mathrm{f}}_{\xi}^{M} \cos \psi^{M}+\overline{\mathrm{F}} \cos \phi_{0}=0 \\
& \mathrm{~m}_{\xi}^{\mathrm{IV}}-\sum_{M=1}^{M} \mathrm{~m}_{\xi}^{M}+\bar{M}=0 \tag{86}
\end{align*}
$$

By recalling the definition of the $y$ (Equation 48) and $z$ (Equation 40) matrices and introducing the diagonal matrices

$$
\beta=\left[\begin{array}{llll}
1 & & &  \tag{87}\\
& 1 & & \\
& & 1 & 0
\end{array}\right] \quad \eta=\left[\begin{array}{llll}
0 & & & \\
& 0 & & \\
& & 0 & \\
& & &
\end{array}\right]
$$

Equations 65 and 86 may be recast in the formulas for compatibility

$$
\begin{equation*}
\beta z^{I V}+\eta y^{I V}=\beta \Psi^{I} z^{I}+\eta y^{I}=\beta \Psi^{I I} z^{I I}+\eta y^{I I}=\beta \Psi^{I I I} z^{I I I}+\eta y^{I I I} \tag{88}
\end{equation*}
$$

and for equilibrium

$$
\begin{equation*}
\beta \mathbf{y}^{I V}+\eta \mathbf{z}^{\mathbf{I V}}=\sum_{\mathbf{M}=\mathbf{I}}^{\boldsymbol{I}} \beta \boldsymbol{\Psi}^{\mathbf{M}} \mathbf{y} \mathbf{M}+\eta \mathbf{z}^{\mathbf{M}}+\bar{\Phi} \overline{\mathrm{P}}+\bar{\eta} \bar{M} \tag{89}
\end{equation*}
$$

where

$$
\Phi=\left|\begin{array}{l}
\sin \phi_{0}  \tag{90}\\
0 \\
-\cos \phi_{0} \\
0
\end{array}\right| \quad \bar{\eta}=\left|\begin{array}{l}
0 \\
0 \\
0 \\
1
\end{array}\right|
$$

Introducing Equations 36 into 88 and 89 and noting $\eta f=0 ; \beta f=f$ and $\boldsymbol{\beta} \boldsymbol{\ell}=\boldsymbol{\Psi} \mathbf{f}$, we obtain:
for compatibility:

$$
\begin{align*}
\eta H_{I V}\left(\mathrm{z}^{I V}\right)^{\prime}+\left(\eta J_{I V}+\beta\right) \mathrm{z}^{I V} & =\eta H_{I} \mathrm{z}^{\prime}+\left(\eta J_{I}+\beta \Psi I\right) \mathrm{z}^{I} \\
& =\eta H_{I I} \mathrm{III}^{\prime}+\left(\eta J_{I I}+\beta \Psi I I\right) \mathrm{z}^{I I} \\
& =\eta H_{I I I} z_{I I I}+\left(\eta J_{I I I}+\beta \Psi I I I\right) z^{I I I} \tag{91}
\end{align*}
$$

and for equilibrium

$$
\begin{align*}
\beta H_{I V}\left(z^{I V}\right)^{\prime}+\left(\beta J_{I V}+\eta\right) z^{I V} & =\sum_{M_{I}=I}^{I I}\left[\beta \Psi M_{H_{M}} z^{M^{\prime}}+\left(\beta \Psi M_{M}+\eta\right) z^{M}\right. \\
& \left.+\Psi M_{\mathcal{M}}\right]-f_{I V}+\bar{\Phi} \bar{P}+\bar{\eta} \bar{M} \tag{92}
\end{align*}
$$

A central finite difference acheme is used to obtain the numerical solution of Equations 91 and 92 within the framework of the Gaussian elimination procedure.

To eliminate the fictitious points (they will be used in calcylating for internal forces and atresses at junction) $z_{j+1}, \pi_{j+1}^{\mathrm{I}}, z_{j+1}$ and $z_{j+1}$ that appear, we utilize the equilibrium equations at the enas of the adjoining regions of the juncture in a fashion similar to that used in the discontinuity section. After substituting the expressions for fictitious points in Equations 91 and 92 and recalling the definitions of the A, B, and C matrices (Equations 52), we may write the recursive equation equivalent of Equation 54 for the branched shell. As (for $j^{1 V}$ ):

$$
\begin{equation*}
z_{j}^{I V}=-P_{j}^{I V} z_{j+1}^{I V}+X_{j}^{I V} \tag{93}
\end{equation*}
$$

where

$$
P_{j}^{I V}=L_{M}^{-1}\left\{\frac{\beta H_{I V}}{2 \Delta_{I V}}\left[I+C_{I V}^{-1} A_{I V}\right]-\left[\sum_{M=I}^{I I}\left(K_{M}\right)\right]\left[\frac{\eta H_{I V}}{2 \Delta_{I V}} I+C_{I V}^{-1} A_{I V}\right]\right\}
$$

$$
\begin{align*}
x_{j}^{I V}= & L_{M}^{-1}=\left[\frac{\beta H_{I V}}{2 \Delta_{I V}} C_{I V}^{-1} \mathrm{~S}_{I V}-f_{I V}+\sum_{M=I}^{I I I}\left\{\beta \Psi{ }^{M}-\frac{H_{M}}{2 \Delta_{M}} A_{M}^{-1} \mathrm{SM}_{M}+\Psi^{M} f_{M}\right.\right. \\
& \left.-\frac{\beta \Psi M_{M}}{2 \Delta_{M}}\left[A_{M}^{-1} C_{M}+I\right] x_{j}^{M-1}\right\}-\sum_{M=I}^{I I I}\left\{( K _ { M } ) \left[\frac{\eta H_{I V}}{2 \Delta_{I V}} C_{I V}^{-1} g_{I V}\right.\right. \\
& \left.\left.+\eta \frac{H_{I V}}{2 \Delta_{I V}} A_{I V}^{-1} g_{I V}-\frac{\eta H_{I V}}{2 \Delta_{I V}}\left[A_{I V}^{-1} C_{I V}+I\right] x_{j}^{M-1}\right]\right\}= \tag{94}
\end{align*}
$$

and

$$
\begin{align*}
K_{M}= & \left\{\frac{\beta \Psi^{M} H_{M}}{2 \Delta_{M}}\left[-A_{M}^{-1} B_{M}+A_{M}^{-1} C_{M} P_{j}^{M-1}+P_{j}^{M-1}\right]\right. \\
& \left.+\left(\beta \Psi{ }^{M} J_{M}+\eta\right)\right\} M_{M}^{-1} \\
L_{M}= & \left\{\left[\left(\beta J_{I V}+\eta\right)+\frac{\beta H_{I V}}{2 \Delta I V} C_{I V}^{-1} B_{I V}\right]-\sum_{M=I}^{I I I} K_{M}\left[\frac{\eta H_{I V}}{2 \Delta_{I V}} C_{I V}^{-1} B_{I V}\right.\right.  \tag{95}\\
& \left.\left.+\left(\eta J_{I V}+\beta\right)\right]\right\}
\end{align*}
$$

where

$$
M_{M}=\frac{\eta H_{M}}{2 \Delta_{M}}\left[-A_{M}^{-1} B_{M}+A_{M}^{-1} C_{M} P_{j}^{M-1}+P^{M-1}\right]+\left(\eta J_{M}+\beta \Psi M^{M}\right)
$$

For the remaining branch segments (i.e., $M=I$ II, III), the rollowing recursion formula is used:

$$
\begin{equation*}
\underset{z_{j}^{M}}{M}=Q_{j}^{M} z_{j}^{I V}+P_{j}^{M}{\underset{z}{j+1}}_{I V}^{I V}+X_{j}^{M} \tag{96}
\end{equation*}
$$

where

$$
\begin{aligned}
Q_{j}^{M}= & M_{M}^{-1}\left\{\frac{\eta H_{I V}}{2 \Delta_{I V}} C_{I V}^{-1} B_{I V}+\left(\eta J_{I V}+\beta\right)\right\} \\
P_{j}^{M}= & M_{M}^{-1}\left\{\frac{\eta H_{I V}}{2 \Delta_{I V}}\left[I-C_{I V}^{-1} A_{I V}\right]\right\} \\
X_{j}^{M}= & M_{M}^{-1}\left\{-\eta \frac{H_{I V}}{2 \Delta_{I V}} C_{I V}^{-1} g_{I V}-\eta \frac{H_{M}}{2 \Delta_{M}} A_{M}^{-1} \cdot g_{M}\right. \\
& \left.+\frac{\eta H_{M}}{2 \Delta_{M}}\left[A_{M}^{-1} C_{M}+I\right] X_{j}^{M-1}\right\}
\end{aligned}
$$

and $\left(M_{M}\right)$ is given by Equation 95.
Thus, from a knowledge of $P_{j-1}^{I}, P_{j-1}^{I I}, P_{j-1}^{I I I} \ldots, P_{j-1}^{N-1}$ and $X_{j-1}^{I}$, $X_{j-1}^{I I}, \cdots, X_{j-1}^{N-1}$, the calculation can proceed directly to the determination of the Nth shell region, $\mathbf{P}_{\mathbf{j}}^{\mathbf{N}}, \mathbf{X}_{\mathbf{j}}^{\mathbf{N}}$ and then to the boundary of branch N in the standard fashion.

## APPENDIXIA: FORMULAS FOR COEFFICIENTS

The coefficient $\mathbf{3}_{1}$, 22 . . a 36 in Equation 38 are as follows:

$$
\begin{aligned}
a_{1}= & b \\
a_{2}= & \gamma b+b^{\prime} \\
a_{3}= & v b^{\prime} \gamma-v b \omega_{\xi} \omega_{\theta}-b \gamma^{2}-\frac{(1-v) b n^{2}}{2 \rho^{2}}-\lambda^{2} d(1-v)\left[(1+v) \gamma^{2} \omega_{\xi}{ }^{2}\right. \\
& \left.+\frac{\left(3 \omega_{\xi}-\omega_{\theta}\right)^{2} n^{2}}{8 \rho^{2}}\right]
\end{aligned}
$$

$$
\begin{aligned}
& a_{4}=\frac{(1+v) b_{n}}{20}+\frac{\lambda^{2} \mathrm{~d} \ln (1-1)}{8 f}\left(3 \omega_{\xi}-\omega_{0}\right)\left(3 \omega_{0}-\omega_{\xi}\right) \\
& a_{5}=\frac{\mid n l^{\prime}}{\rho}-\left(\frac{3-v}{2 f}\right)\left(Y l_{n}\right)-\frac{\lambda^{2} d(1-v) \gamma_{n}}{\rho}\left[\frac{\left\{3 w_{\xi}-w_{\theta}\right)\left(3 w_{\theta}-w_{E}\right)}{8}\right] \\
& \left.+(1+\cdots) \omega_{\xi} w_{\theta}\right) \\
& \lambda_{6}=b\left(\omega_{\xi}+v \omega_{\theta}\right)+\lambda^{2} \alpha(1-v)\left[(1+1) \gamma^{2} \omega_{\xi}+\left(n^{2} / 2 \rho^{2}\right)\left(3 \omega_{\xi}-\omega_{\theta}\right)\right] \\
& a 7=b\left|\omega_{\underline{\xi}}^{\prime}+\gamma\left(\omega_{\xi}-\omega_{\theta}\right)\right|+b^{\prime}\left(\omega_{\xi}+v \omega_{\theta}\right) \\
& -\frac{\lambda^{2} d(1-\nu) \gamma_{n} 2}{\rho 2}\left[\frac{3 \omega_{\xi}-\omega_{\theta}}{2}+(1+\nu) \omega_{\xi}\right] \\
& \mathrm{a}_{8}=\lambda^{2} \omega_{\xi} \\
& a_{9}=\lambda^{2}(1-v) \gamma \omega_{\xi} \\
& \mathrm{a}_{10}=-\mathrm{a}_{4} \\
& a_{11}=\frac{b \gamma n}{2} \frac{1}{\rho}(3-v)-\frac{(1-v) n b^{\prime}}{2 \rho}+\frac{\lambda^{2} d(1-v) n}{\rho} \times\left[-(1+v) \gamma \omega_{\xi}{ }^{\omega_{\theta}}\right. \\
& \left.+\frac{Y}{8}\left(6 \omega_{\xi} \omega_{\theta}-7 \omega_{\xi}{ }^{2}-3 \omega_{\theta}{ }^{2}\right)-\frac{\omega_{\xi}^{\prime}}{4}\left(5 \omega_{\theta}-3 \omega_{\xi}\right)\right] \\
& -\frac{\lambda^{2} d^{\prime}(1-v) n}{8 p}\left(3 \omega_{\xi}-\omega_{\theta}\right)\left(3 \omega_{\theta}-\omega_{\xi}\right) \\
& a_{12}=\frac{b(1-v)}{2}+\frac{\lambda^{2} d(1-v)\left(3 \omega_{\theta}-\omega_{g}\right)^{2}}{8} \\
& a_{13}=\left(\frac{1-v}{2}\right)\left(v b+b^{\prime}\right)-\frac{\lambda^{2} d(1-v)}{8}\left(3 \omega_{\theta}-\omega_{\xi}\right) \times\left[2 \omega_{\xi}^{\prime}-\gamma\left(5 \omega_{\xi}-3 \cdot z_{\theta}\right)\right] \\
& +\frac{\lambda^{2} \mathrm{~d}^{\prime}(1-v)}{8}\left(3 \omega_{\theta}-\omega_{\xi}\right)^{2}
\end{aligned}
$$

$$
\begin{aligned}
& a_{14}=-\gamma a_{13}+\left(\frac{1-v}{2}\right) b \omega_{\xi} \omega_{0}-\frac{b n^{2}}{\rho^{2}}-\lambda{ }^{2} d(1-v)\left[\frac{(1+v)_{\omega_{0}}^{2} n^{2}}{\rho^{2}}\right. \\
& \left.-\frac{\omega_{\xi} \omega_{0}}{8}\left(3 \omega_{0}-\omega_{\xi}\right)^{2}\right] \\
& a_{15}=\frac{\lambda^{2} d(1-v)\left(3 \omega_{\theta}-\omega_{\xi}\right) n}{2 \rho} \\
& a_{16}=\frac{\lambda^{2} d(1-v) n}{2 \rho}\left|2(1+v) \gamma \omega_{\theta}-\omega_{\xi}^{\prime}+3 \gamma\left(\omega_{\xi}-\omega_{\theta}\right)\right| \\
& +\frac{\lambda^{2} d^{\prime}(1-\nu)\left(3 \omega_{\theta}-\omega_{\xi}\right) n}{2 p} \\
& a_{17}=-\frac{b n}{} \frac{\left(\omega_{\theta}+v \omega_{\xi}\right)}{\rho}+\frac{\lambda^{2} d n(1-v)}{2 \rho} \times / \gamma \omega_{\xi}{ }^{\prime}-2 \gamma^{2} \omega_{\xi} \\
& \left.-\frac{2(1+v) \omega_{\theta} \mathrm{n}^{2}}{\rho^{2}}+\left(3 \omega_{\theta}-\omega_{\xi}\right)\left(\gamma^{2}+\omega_{\xi} \omega_{\theta}\right)\right) \\
& -\frac{\lambda^{2} d^{\prime} n(1-v) \gamma}{2_{\rho}}\left(3 \omega_{\theta}-\omega_{\xi}\right) \\
& a_{18}=-\left(\nu \lambda^{2} \omega_{\theta} n / \rho\right) \\
& a_{19}=-a_{6} \\
& a_{20}=-b_{\gamma}\left(\omega_{\theta}+v \omega_{\xi}\right)+\lambda^{2} d(1-v) \mid \gamma(1+v)\left(-\gamma \omega_{\xi}{ }^{\prime}\right. \\
& \left.+\gamma^{2} \omega_{\xi}-\left(n^{2} \omega_{\xi} / \rho^{2}\right)+2 \omega_{\xi}{ }^{2} \omega_{\theta}\right)+\left(n^{2} / 2 \rho^{2}\right)\left(\gamma_{\xi}\right. \\
& \left.\left.-\gamma \omega_{\theta}-3 \omega_{\xi}^{\prime}\right)\right\}-\lambda^{2} d^{\prime}(1-v) \mid(1+v) \gamma^{2} \omega_{\xi} \\
& \left.+\left(n^{2} / 2 \rho^{2}\right)\left(3 \omega_{\xi}-\omega_{0}\right)\right] \\
& a_{21}=a_{15}
\end{aligned}
$$

$$
\begin{aligned}
& a_{22}=\frac{\lambda^{-d}(1-\cdot \cdot) n}{2_{\rho}}\left|3 \gamma_{\omega}{ }_{\xi}-\gamma_{\omega_{\theta}} \quad(5+2 v\rangle-\omega_{\xi}\right| \\
& +\frac{i^{2} d^{\prime}(1-\dot{n}}{2 \rho}\left(3 \omega_{0} \omega_{\xi}\right) \\
& a_{23}=-\frac{b_{1}\left(\omega_{\theta}+v_{\omega \xi}\right)}{\rho}+\frac{\lambda^{2} d(1-v) n}{2 p} \times \mid 2(1+v)\left(\omega_{\xi} \omega_{\theta}{ }^{2}-\gamma^{2} \omega_{\xi}\right. \\
& \left.+2 \gamma^{2} \omega_{\theta}-\frac{n^{2} \omega_{\theta}}{\rho 2}\right)+Y \omega_{\xi^{\prime}}+3 \gamma^{2}\left(\omega_{\theta}-\omega_{\xi}\right)+\omega_{\xi} \omega_{\theta}\left(3 \omega_{\theta}-\omega_{\xi}\right) \mid \\
& -\frac{\lambda^{2} d^{\prime}(1-v) n}{2 \rho}\left[2(1+v) Y \omega_{\theta}+Y\left(3 \omega_{\theta}-\omega_{\xi}\right)\right] \\
& a_{24}=\lambda^{2} d(1-v)\left|\left(2 n^{2} / p^{2}\right)+(1+v) y^{2}\right| \\
& a_{25}=-\lambda^{2} d(1-v)\left[(1+\nu)\left(2 \gamma \omega_{\xi} \omega_{\theta}+\gamma^{3}\right)+\left(2 \gamma n^{2} / \rho^{2}\right)\right\} \\
& +\lambda^{2} d^{\prime}(1-v)\left\{(1+v)^{?}+\left(2 \cdot n^{2} / \rho^{2}\right)\right\} \\
& \left.a_{26}=-b i_{\xi}^{2}+2 v \omega_{\xi} \omega_{\theta}+\nu_{\theta}^{2}\right)+\frac{\lambda^{2} d(1-v) n^{2}}{\rho^{2}}\left[(1+v)\left(w_{\xi} \omega_{\theta}-\frac{n^{2}}{\rho^{2}}+2 \gamma^{2}\right)\right. \\
& \left.+2\left(\gamma^{2}+\omega_{\xi} \omega_{\theta}\right)\right] \cdot \frac{\lambda^{2} d^{\prime}(1-v) n^{2}}{\rho^{2}}(3+v) r \\
& a_{27}=\lambda^{2} \\
& a_{28}=\lambda^{2}+(2-v) \\
& 229=-\lambda^{\bar{c}}\left|(1-v) \omega_{\xi} \omega_{\theta}+\left(v n^{2} / \rho^{2}\right)\right| \\
& a_{30}=d \omega_{\xi} \\
& \mathrm{a}_{31}=\alpha\left(\omega_{\xi}+v \gamma \omega_{\xi}\right) \\
& \lambda_{32}=d v n \omega_{\theta} / p
\end{aligned}
$$

$a_{33}=-d$
$a_{34}=-d v Y$
$235=d \nu n^{2} / \rho^{2}$
$a_{36}=-1$
The c's are
$c_{1}=-p_{\xi}+t_{T}{ }^{\prime}-\lambda^{2}(1-v) \gamma \omega_{\xi} m_{T}$
$c_{2}=-p_{\theta}-\left(n / f!t_{T}-\lambda^{2}(1-v)(n / p) \omega_{\theta} m_{T}\right.$
$c_{3}=-p-\left(\omega_{\xi}+\omega \theta_{\theta} \mathbf{t}_{T}-\lambda^{2(1-v) Y M_{T}}+\lambda^{2}(1-v) \mid \omega_{\xi} \omega_{\theta}\right.$
$\left.-\left(n^{2} / \rho^{2}\right)\right] m_{T}$
$c_{4}=\mathrm{m}_{\mathrm{T}}$
Finally, the b's in Equation (49) are
$b_{1}=b$
$b_{2}=v y^{\text {o }}$
$b_{3}=v n b i \rho$
$b_{4}=b\left(\omega \xi+\nu \omega_{\theta}\right)$
$b_{5}=-\frac{b(1-v) n}{2 p}-\frac{d \lambda^{2}(1-v) n}{8 p}\left(3 \omega_{\xi}-\omega_{\theta}\right)\left(3 \omega_{\theta}-\omega_{\xi}\right)$
$b_{6}=\frac{b(1-v)}{2}+\frac{n^{2} d(1-v)}{8}\left(3 \omega_{\theta}-\omega_{g}\right)^{2}$
$b_{7}=-\mathrm{Y}_{6}$
$b_{g}=\frac{\lambda^{2} d(1-\nu) n}{2 \rho}\left(3 \omega_{\theta}-\omega_{\xi}\right)$
$b_{9}=-\gamma_{\mathbf{b}}^{8}$

$$
\begin{aligned}
& b_{10}=-\lambda^{2} d(1-v)\left|(1+v) \gamma^{2} \omega_{\xi}+\left(n^{2} / 2 \rho^{2}\right)\left(3 \omega_{\xi}-\omega_{\theta}\right)\right| \\
& b_{11}=\frac{\lambda^{2} d(1-v) n}{2 \rho}\left(3 \omega_{\theta}-\omega_{\xi}\right) \\
& b_{12}=-\frac{\lambda^{2} d(1-v) Y_{n}}{2 \rho}\left|3 \omega_{\theta}-\omega_{\xi}+2(1+v) \omega_{\theta}\right| \\
& b_{13}=\lambda^{2} d(1-v)\left[\left(2 n^{2} / \rho^{2}\right)+(1+v) \gamma^{2}\right] \\
& b_{14}=-\lambda^{2} d(1-v)(3+v)\left(\gamma n^{2} / \rho^{2}\right) \\
& b_{15}=\lambda^{2} \\
& b_{16}=\lambda^{2}(1-v / \gamma
\end{aligned}
$$

PAGES 40 AND 41 ARE MISSING FROM THE ORIGINAL DOCUMENT.
paragraphs that follow are intended to aid in formulation of the problem for program use and aurment the detail input instructions in Section III. For ease of reference, FORTRAN instruction symbols used in the program and related to the descriptive paragraphs are placed in parentheses following paragraph titles.

### 2.2 PROGRAM CAPABILI - ES AND LIMITATIONS

Before describing s.ae of the general program characteristics, it will perhaps be worthwhi. to list some of the program features that are not generally present in other shell analysis programs. Also included in this list are limitations in the program that have resulted due to theoretical restrictions, computer storage capacity, economic considerations, etc.
a. A shell structure having virtuall any combination of abrupt discontinuities in geometry, loads, temperature, and material properties can be analyzed by breaking the structure into the appropriate regions.
b. The main requircment in each shell region is that geometry, material properties, loads, and temperatures vary smoothly along the generatrix or meridian lize.
c. As many as 50 (estimated) integrally joined shell regions can be analyzed as one shell structure.
d. As many as four regions may be joined at one common junction (branch point).
e. Line loads and line moments can be applied at junctions between regions. The effects of eccentricity of reference surfaces occurring at discontinuity junctions (juncture of two shell regions) is automatically handled by the program. At branch points, these effects can be handled by an approximate procedure or minimized by appropriate selection of junction point.
f. Laminated shell structures consisting of up to three materials broken into as many as six intimately bonded layers can be considered by the computer program.
g. All materials excepting Poisson's ratio can vary from layer to layer through the thickness as well as along the meridional coordinate.
h. As many as 150 integration intervals (station points) can be considered in ruh region.
i. Curve fitting techniques are utilized to reduce amount of input data. A second-degree polynomial fit is uscd.
j. Both unsymmetric surface load and temperatures can be applied to the shell. The variation ui temperature across the cross-section can be continuous and piece-wise linear through each layer.
k. The numerical solution procedure allows for high accuracy without excessive use of computer time. (Approximate machine running time 150 stations per minute for each Fourier harmonic.)

## 2. 3 SIGN CONVENTIONS AND DIMENSIONS

The sign conventions used in the program are illustrated in Figures $1-1$ through $1-10$ in Section $I$. To briefly augment, the stresses $\sigma_{\xi}, \sigma_{\theta}$ and membrane forces $N_{\xi}, N_{\theta}$ are positive when thry tend to produce tension and negative when they are in compression. The moments $M_{\xi}, M_{G}$ are positive in sign when they tend to produce tensile stresses in the inner (bottom) surfaces and compressive stresses in the outer (top) surface. (See Section 2.4.) The extensional displacement $u$ and transverse deflection $w$ are positive when the $\xi$ and $\zeta$ coordinates, respectively, are increased.

In using the program, all data specified must be dimensionally consistent. In the manual, the quantity $P$ will indicate fo rce quantities (e.g., pounds) and L length quantities (e.g., inches). The program output yieids results in force and iength that are consistent with the input quantities.

## 2. 4 RTFERENCE, INNER, AND OUTER SURFACES

The reference surface $\zeta=0$ is chosen such that the requirements of E. uation l be satisfied. The $\operatorname{ross}-\mathrm{sectional}$ properties are then evaluated based upon this reference surface. As discussed in 1.15, a substantial simplification is obtained when specifying key geometric functions (e.g., $r, \omega_{\xi}$ ). if the reference surface is chosen according to convenience anywhere within the shell wall. However, the hell stiffaess parameter should be evaluated systematically along the lines discussed in Section I.

It will be convenient to refer to inner and outer surfaces of the shell. One can keep the inner and outer surface definitions clear by remembering that in directior if :ncreasing value of $\xi$, the outer surface is on the left and the inn. $\quad \rightarrow$ tie right when the geometry is drawn with axial distance in:.. . fr $\quad$ ? to bottom and radial distance frcm left to right
as shown in Figure 1-1. Using the same description, the inner surface will su metimes be referred to as the "bottom" (BOT) surface and outer as the "tod" (TOP) surface.

## 2. 5 SHELL REGIONS (EKK)

In solving a shell problem it is necessary to select a mathematical model to represent the actual shell configuration. It may be necessary or convenient in establishing a suitable mathematical model for complicatej shell configurations to fictitiously divide the shell along its length into a number of "regions." Thus, the first step in the ana!ysio of sheli problems is to delineate the "regions" of the mathematical model. Ideally, this division results in each region being a simple shell element, such as a cylinder, sphere, cone, etc.

The main requirement in delineating a shell region is that shell properties and loads vary smoothly along the generatrix or meridian line in the region. Thus, the logical dividing line between regions would occur at points on the shell where an abrupt discontinuity or a radical change in any of the following exists: (1) geometry; (2) section or material properties; (3) induced or surface loading; (4) temperature distribution; (5) other considerations such as length to radius magnitudes; (6) combinations of 1 through 5. The points at which these fictitious subdivisions occur are called junctions. It will be convenient in the program to differentiate between two types of junction points. The point where one region of the mathematical model is joined to a single other region of the same mathenatical model is termed a discontinuity point or junction. (See Section 1.ll.) Junctions where more than two shell regions meet at a common point are called branch points. (See Section 1.16.) It shou'd be emphesized that it is absolutely essential in treating problems where abrupt discontinuitics in shell properties ( 1 and 2 above) occur to introduce a junction point since a unique solution procedure is required in such cases. For convenience of data input or change in grid increment (Section 1.12), fictitious-type discontinuities may be introduced when desirable.

Theoretically, the limit on the number of shell regions per problem is dictated by the storage capacity on the tapes used. Twenty regions have been used without diffic $\quad y$, and it is estimated that the capability for considering up to 50 regions is possible. With a structural mesh of 150 grid points possibi= 'Section 2.7), it is unlikely that such a large number of regions is necessary in treating even the most complicated of engineering problems.

The program code processes the region data in the orde: in which the regional data is introduced ints the input data deck of punched carts. The
first region is known as region 1 , the second region, regions 2 , etc., even though punched cards clo not carry the number designation of the regions. The complete data information for a particular region must be inputted before the subsequent region data can be considered. The sequence of input of regional data must be consistent with the analytical solution of the problem. The regions should be selected in sequence proceeding in a continuous manner from one boundary to the final boundary. The: procedure for handling branched shell configurations (more than two shells ing) is modified somewhat in that data for each oranch is input up to ne common branch junction point until the next to last branch is completed. The data for the final or closing branch proceeds from junction point to the inal end co.2ditions. (See 2. 10.2.)

The code value EKK represents the number of shell regions selsted for a particular shell problem. The amount of regional data must coincide with the value of r.KK. The examples shown below indicate typical regior delineation for complicated shell configurations.

In Figure 2-1 a five-region shell configuration is shown where four discontinuity junctions have been used to subdivide the methematical model. Junction (1) illustrates a discontinuity point where an abrupt change in the shell section properties occur. A discontinuity of slope detween the reference surface of two joining shells s illustrated by junction (2). At (3) and (4). fictitious subdivisions have been introduced where abrupt changes in load distribution occurred. The arrows on Figure 2-1 indicate the direction of increasing $\xi$ or station number and the sequence of data input. A six-region branched shell is shown in Figure 2-2. The first discontinuity point illustrates the region delineation when two shells of different shapes meet. Branch points (2) and (3) represent common branch points where more than two regions join. This example will be discussed in more detail in Section 2. 10 on junction points.

## 2. 6 FOUKIER COMPONENTS (SUM, ENFO, ENFI, ENFOR, TH「TA)

The computer program permits anaiysis of shells qubijected to unsymmetric loads using a Fourier series technique. This approach described in Section 1.7 permits the analysis of complicated locds by considering the individual contribution of each Fourier component of the Fourier series expansion of the load distribution. The total solution is obtained by summing the Fourier components in the appropriate series expression in the circumferential coordinate.


Figure 2-1


Figure 2-2

When treating more than one Fourier component for a shell problem the code value SUM is set to a nonzero value. A positive value for the SUM indicator indicates the solution will be summed according to the series expressions, Equations 1' through 24, page 7. A negative SUM prints the individual solution tor each discrete Fourier harmonic.
'lhe data value ENFO represents the firgl and lowest Fouried har:nonic (I:NF) considered. Subsequent Fourier components are numbered in increasing order in the ENII(I) data region. Up to $11 \because N F$ values are permissible in this data region. If more than 11 Fourie: barmonices are to be considered, the problem cin be reformulated for the rerinain ing harmonics and the solutions added.

It may be desirable to test the convergence of the Fouricr series solution to obtain intermediate prints of partial Fourier sums. This capat bility is possible using the ENFOR(I. data region where these intermediato prints are permissible. For example, if 10 Fourier compnents are considered. it might be desirable to print the summed solut, ons for the tast three harmonics to compare convergence of results.

In order to detr.rmine the value of solution at circumferential (THETA) locations on the shell, the capability for evaluating the series expressions at discrete THETA values is possible. This data region THETA(I) permits a maximum of 10 circumferential solution printouts.

## 2. 7 STATIONS IN REGIONS (EN)

The machine program achieves a shell solution by integration of finite difference equations along the meridian or arc length distance of the shell. The meridional coordinate $\xi$ on the reference surface has the range $0 \leq \xi \leq \xi_{j}$ for the $j$-th region. The number of integration points (called stations) ]ocated in the region under consideration is assigned the EN code value. The stations are equally spaced with the initial point located on the reference surface at the beginning of the region designated station $1(i=1$ or $\xi=0$ ) and the last or EN-th station at the end of ihe region called station $N(i=N$ or $\dot{\zeta}=\xi_{j}$ ). The numbering of stations proceeds in direction of positive meridional coordinate assigned to the respective region. The naximum number of stations permissible in a region is 150 (minimum 3). The regional input data are specified at stations on the reference surface of each region.

The length of the finite difference "lump" of shell is coraputed internal to the program from the length or wrap distance and the number of stations (IN) in the region. This finite difference increment of integration is defined as DEL in the program and printout. Best results are obtained when the finite difference increments are approximately the same from region to region.

The machine running time increases with the number of integration steps consider $\sim$ d $f=r$ region. The type of shell problem considered should dictate the se of tre $\quad$ eid mesh or number of stations considered. This comes with experience and how the results are to be used. As a general rule, it is recommended that more integration intervals be used where rapid
crange in variables occurs along the length of the shell. Experience with the program indicates that considering 100 stations is probably sufficient for engineering type accuracy of the shell solution in most shell regions. For extremely long shells (e.g., cylinders), it may be necessary to subdivide the shell into more regions in order to obtain a suitable integratior. interval.

### 2.8 GEOMEIRY OF REGIONS (GMI)

Geometric parameters must be defined at each station location. The sign convention for the curvature parameter's, $\omega_{\phi}, \omega_{\theta}$ are illustrated as follows:


In order to assist the analyst in defining the set of geometry parameters with a minimum nuriber of input parameters, several options for specific classes of geometries are made available. The options are described below with their identifying code number (G.MI).

### 2.8.1 Cone-Cylinder Option (GMI $=1.0$ )

This geometry option may be specified for a complete rang of regıunal configurations generated by a straight line, e.g. : circular flates, cones, and cylinders. A miarmum of three input parameters is required. The input purameters required are defined as follows:

1. RAI - Radial distance from axis of revolution to the first station ( $i=1$ ) of the region
2. AXL - meridional length of shell
3. ANX - angle the generstor makes with the axis of revolution

Figure 2-3 illustrates tie geometric parameters used in describing the cone cylinder option. Both RAI and AXL are positive quantities. The parameter ANX - give. in degrees and is positivo clockwise measured from the gene iatc .o the positive $X$ axis as shown in Figure 2-3.


Figure 2-3. Cone Cylinder Geometry

## 2. 8. 2 Sph.ere-Toroid (G.MI $=2.0$ )

This option may be specified for a complete range of regional configuration generated by a circular curve. Four input parar.. ters are necessary for defining a sphere-toroid, as shown in Figure 2-4.


Figura 2-4. Sphere-Toroir Geometry

The input parameters are

1. RC - Radias of curvature of the gel srator
2. ROFF - Offset distance measured from axis of revolution to the center of meridional curvature
3. PHIO - Angular position in degrees of the beginning of a region measured clocicwise positive about the center of curvature from an axis parallel to the axis of revolution
4. PHIN - Angular position of the end of the region

## 2. 8. 3 Discrete Point Option (GMI $= \pm 3.0$ )

This option was developed for use on regions where the generator cannot be described by one of the other options or where a curved generator is given by a set of discrete points. As a consequence of various possible ways the geometry may be supplied to the analyst, several variations of input data format can be accommodated.

On a positive indicator ( $G M I=+3.0$ ), the program will set up the necessary geometric parameter from the input data which describes the generator by discrete radial and axial distances. The input quantities to the program are EM (number of points given), RIPT (radial distance from axis of revolution at input points), XIPT (axial coordinates of the input points). The set of RIPT and XIPT must include the first and last points of the region. XIPT must be given in ascending magnitudes. On a negative indicator ( $G M I=-3.0$ ), the coordinates of the discrete points are given in radial and surface or arc length, the surface length coordinate is input directly in the XIPT locations.

An interpolation routine is used to obtain appropriate geometric parameters at station points from the original input values. The parameters such as curvatures are computed using finite difference forms of the station set. A least squares method is used to minimize the scatter of these computations. To hold the errors in curvatures to less than 10 percent, the number of points described by RIPT and XIPT should be at least as great as the number of stations. For some situations such as locations of major changes ir. the generator curve, it will be necessary to input a denser popu lation of RIPT and XIPT. (See Figure 2-5.) Because of the difficulty involved in the least squares and interpolation routines, extreme care must be exercised in the use of this option in order to obtain an adequate description of shell geometry. A significant improvement in results is obtained if the additional recommendations described below are adhered to.

When the meridional and circumferential radii of curvatures are available, they can be input at discrete points and curve-fit to give a better description of the curvatures. If possible, it is strongly recommended that this capability be used since the errors in curvatures are reduced considerably to better control curvatures and less input points of the generator


Figure 2-5
are required. This data is input in the location RCURV and RCUR? for radius of curvatures $R_{\xi}$ and $R_{\theta}$, respectively (Section 1.3). RCURV and RCURZ values must co-respond with the points described by RIPT and XIPT. This is an optional input to both GMI $=+3.0$ and $G M I=-3.0$. When no values are input at RCURV and RCURZ locations, the curvatures will be computed from the discrete point set of RIPT and XIPT.

### 2.8.4 Conics Options (GMI $=4.0,5.0, \pm 6.0$ )

Several options are made available for the conics class of generator. Three classes of conics are treated: ellipse (GMI = 4.0), hyperbola ( $\mathrm{GMI}=5.0$ ), and the parabolas ( $\mathrm{GMI}= \pm 6.0$ ). The parameters for the conics are taken from the standard form (Figure 2-6).

In Figure 2-6, the coordinates $X^{\prime}$. $Y^{\prime}$ are the standard form coordinates. The input quantities are as follows:

1. RFF is the translation distance of $X^{\prime}$ axis from the axis of revolution.
2. SPNO is the clockwise positive opening angle from positive $X^{\prime}$ to the first station location.
3. SPNN is the positive opening angle from positive $X^{\prime}$ to the last station location.


Figure ?-6
4. A is the semimajor axis parallel to the axis of revolution for the ellipse and hyperbola, $A$ is the distance from the directrix to the focus for the parabolas.
5. B is the semimajor axis perpendicular to the axis of revolution.

### 2.9 END CONDITIONS (BCITP, BCIBM)

Four boundary or end conditions must be supplied at each end of a shell region. From Section 1.9, these conditions are input in matrix form. To simplify the amount of data input a boundary indicator code has becn set up to permit simple call of boundary support conditions. The value BCITP defines the boundary indicator at the lat or top station ( $i=1$ ) of the region and BCIBM the value at the last atation (bottom) ( $\mathrm{i}=\mathrm{N}$ ). The boundary or
end conditions permitted by the code, together with the identifying code number and mathematical description, are as follows:

| BCITP or BCIBM Code No. | Type of End Condition | Mathematical Equivalent |
| :---: | :---: | :---: |
| 1 | Free Support | $\mathbf{t}_{\xi}=\hat{\mathbf{t}}_{\boldsymbol{\xi} \boldsymbol{\theta}}=\hat{\mathbf{f}}_{\xi}=\mathrm{m}_{\xi}=0$ |
| 2 | Roller | $\mathbf{t}_{\boldsymbol{\xi}}=\mathbf{u}_{\boldsymbol{\theta}}=\mathbf{w}=\mathrm{m}_{\boldsymbol{\xi}}=0$ |
| 3 | Clamped (Fixed) | $\mathbf{u}_{\boldsymbol{\xi}}=\mathbf{u}_{\boldsymbol{\theta}}=\mathbf{w}=\phi_{\boldsymbol{\xi}}=0$ |
| 4 | Simple Support ( IVinged) | $\mathbf{u}_{\boldsymbol{\xi}}=\mathbf{u}_{\boldsymbol{\theta}}=\mathbf{w}=\mathrm{m}_{\boldsymbol{\xi}}=0$ |
| 5 | Symmetrical (or Complete)* | $\mathbf{u}_{\xi}=\mathbf{u}_{\theta}=\hat{f}_{\xi}=\phi_{\xi}=0$ |
| 6 | Special | Read Boundary Matrices $\Omega, \Lambda, \ell$ |
| 7, 8 | (to be defined) | Space for additional Boundary Condition |
| 9 | Closed Apex ( $\mathrm{r}=0$ ) | See Section 1.10 for conditions |
| 10 | Branch Point |  |
| 0, >10 | Discontinuity Point |  |

The identifying bcundary matrices for often encountered external support conditions 1 through 5 and 9 are internal to the program and can be called by stipulating the correct code number. Space is available in code numbers 7 and 8 to put in appropriate boundary condition matrices that offer particular interest to the user. Specifying BCITP (or BCIBM) $=6$ permits inputting boundary matrices $\Omega, \Lambda$, and $\ell$ (See Equation 47) directly into the program. This option would be used when considering special bouviaries, spring support conditions, applied load or displacements to boundaries, or any consistent set of end restraint conditions. The details

[^0]of formulating these matrices directly are described .n Section 3.4.5. The indicator value when set equal to 0 ( $0 r>10$ ) indicates a discontinuity condition occurring at the particular boundary location. The program automatically employs the appropriate compatibility relationship as described in Section 1.11. When the endpoint corresponds to a branched junction point (more than two shells coming together), the boundary indicator must be set at 10 and the program will automatically set the solution format to handle branched configurations.

## 2. 10 JUNCTIONS (GPSI, GECX, PD, MD, PSIO)

A junction occurs when one region of the mathematical model is joined to one. two, or three regions of the same mathematical model. It will be convenient to differentiate between two types of junction points. A detailed description of discontinuity and branch type junction points is given in the following paragraphs. Each type requires a different mode of solution in the computer program (Section 1.11 and 1.16). Also discussed below is a description of external line loads and moments that can be applied at junction points.

## 2. 10.1 Discontinuity Junction

By our definition, a discontinuity junction occurs at a point where one region of the mathematical model is joined to another single region of the same mathematical model. Discontinuity junctions are usually selected where abrupt discontinuities in shell properties or loads occur. however, fictitious type discontinuities are sometimes introduced where change in finite difference grid interval is described or for reasons of convenience of inputting data. Types of abrupt discontinuities in shell properties that can be accommodated by the program are illustrated by considering in detail the example shown in Figure 2-7.

Junction(2)(Figure 2-7a) illustrates a discontinsity point occurring between regions II and III due to an abrupt angle change in reference surface caused by two shells of different shape joining at a common point. The angle $\psi$ is coded GPSI in the program and measures the change in slope, i. e., the angle between the normals to the region meridians at the junction point. The discontinuity angle GPSI is referred to the end of the region, e. g., the $\psi$ in Figure 2-7a would be part oi the input data of region II. The discontinuity junction ( 1 ) characterizes a discontinuity point where an abrupt change in the shell cross-sectional (including material) properties occurs. The program will also accommodate eccentric discontinuities, i.e., discontinuities where reference suriaces at a discontinuity junction do not intersect at a common point (see Figure 2-7b). The program automatically compensates for the couple generated by in-plane membrane furces in each region

not being coincident with each other. The eccentricity distance $\mathrm{E}_{\mathrm{cc}}$ (Figure 2-7b) is coded GECX and represents the eccentricity of reference surfaces measured along the radius of curvature ai the end point of a region (e.g., region II of Figure 2-7b). A positive value of $\mathrm{E}_{\mathrm{cc}}$ corresponds to an abrupt increase in the radius of a parallel circle as one proceeds in the direction of increasing $\xi$ and station numbers. This positive direction is shown by directional arrows in Figure 2-7. Foilowing a similar procedure as described above, fictitious discontinuities may be introduced at puints where abrupt variation of load occur or where change in finite difference grid increment is desired.

The existence of a discontinuity junction at the endpoint of a region is specified by the end condition indicator BCITP (or BCIBM). For a discontinuity point, the indication values BCIT'P (or BCIBM) can be set equal to zero or >!). The printout for a discontinuity junction is given by the value $1 \times 10^{10}$.

To illustrate the use of end condition indicators and sequence of data input, the following table has been prepared for sample problem shown in Figure 2-7:

Table 2. 1

| Region | Boundary 2t $\mathbf{i}=1$ <br> (BCITP) | Boundary at $\mathrm{i}=\mathrm{N}$ <br> (BCIBM) | GPSI | GECX |
| :---: | :---: | :---: | :---: | :---: |
| I | 1 | 0 | 0 | $\mathbf{E}_{\text {cc }}$ |
| II | 0 | 0 | $20^{\circ}$ | 0 |
| III | 0 | 0 | 0 | 0 |
| IV | 0 | 0 | 0 | 0 |
| V | 0 | 3 | 0 | 0 |

2.10.2 Branch Junction

A branch point occurs when one region of a mathematical model is joined to two or three regions of the same mathematical model. The program will consider up to four shell regions or branches meeting at a common junction point. In the analys.s of branched inells, a precise order must be followed in the inpuiting of data information. This order can best be exemplified by a typical branched configuration illustrated in Figure 2-2 on page 46. The numbers on each branch identity the regions or branches and indicate the sequence of data input for the regicias comprising the multishell configuration. All required data for a particular region must be input before the nest regional information is considered. The :egions J-III are referred to as starting branches, the last regions are characterized by the fact that the last or N -th station in that region cccurs at the common junction point. A closing branch has its first statios ( $i=1$; at the branch point. The starting and cloding branches must be selected in consistent form with the numerical solution procedure (Figure 2-2). The existence of a branch junction occurring at the endpoints of a region is designated by use of the end condition indicator BCITP (BCIBM) set equal to 10 .

Ths program does not automatically handle eccentricities in reference surfaces occurring at branch point as was done at a discontinuity junction. However, ince line moments can be applied at a junction, it is possible to arcount for the unbalance moment occurring at a branch point due to eccentricities in an approximate manner. This is accomplished by running a multibranch shell case (without eccentricity effects included) and calculating by hand the unbalance moment due to the couple generated by the in-plane membrane forces $N_{\xi}\left(, Q_{\xi}\right.$ contribution) in each region being
dispiaced from each other. Applying the calculated unbalance moment as an externally applied line momint at the junction and rerunning the same case in the program will yield z corrected solution. This trial-dnd-error process can be repeated until tie resulting error is as small as desired. Use of free body diagrams ade helpful in setting up this moci $\because l$.

The procedure for seiting up a branched contigutation in the program can be illustrated by consideration of the example shown in Figure 2-2. In Figure 2-2, the first junction is a discontinuity point with (2) and (3) belang branched points. The arrows on the diagram indicate directiona of increasing $\xi$ or increasing station number for the respective regions. Regions II and III would be starting branches and IV the closing branch associated with junction (2); similarly, IV and V starting and VI closing branches characterizing junction(3). The sequence of input of data with appropriate end and discontinuity conditions can best be illustrated by Table 2-2.

Table 2.2

| Region | BCITP | BCIBM | GPSI | GECX |
| :---: | :---: | :---: | :---: | :---: |
| I | 9 | 0 | 0 | 0 |
| II | 0 | 10 | $315^{\circ}$ | Not possible |
| III | 3 | 10 | 0 | Not possible |
| IV | 10 | 10 | 0 | Not possible |
| V | 1 | 10 | $60^{\circ}$ | Not possible |
| VI | 10 | 9 | 0 | Not possible |

2. 10.3 Discontinuity Loads

The effects of externally applied line loads and moments on a shell response can be determined using the program. The concentrated line load coded PD and moment MD are applied at junction points on the mathematical model. If no geometrical discontinuity exists, a fictitious discontinuity is introduced to incorporate the line load and moment. The program will permit a maximum of 11 Fourier components of PD and MD to be applied to the program. The value of PSIO (in degrees) is the measured angle between the concentrated load direction and the normal to the closing branch at the common branch point. The positive magnitude of $P D$ ( $\bar{P}_{D}$ ) and $M D$ ( $\bar{M}_{D}$ ) is shown in Figure 2-8. For a branched configuration, the load and moment value occurring at a junction can be entered with the regional data of any (one only) of the starting branches; for example, the information could be supplied with data for either of regions I, II, or III. At a discontinuity point, the discontinuity loads would, of course, be supplied with the region preceding the junction point.


Figure 2-8
2.11 PRESSURE LOADS (PILD, PFETB, PTHTB, PNTB)

The values of surface pressure acting on a region are supplied at each station of the region. The sign convention for positive and negative values of pressure is shown in Figure l.2d in its simplified form, internal pressure has a positive value and external presaure a negative value. The normal y and tangential loads $q \xi$ are assumed to be symmetrical about $\theta=0$ and antisymmetrical for circumferential load $\mathbf{q}_{\boldsymbol{\theta}}$, (See Section 1.7.)

To reduce the amount of data load information inpurt into the program and to simpliiy the handling of unaymnetric loads, a pressure load indicator has been introduced. This indicator has the coded value PILD and permits different input format for various types of load information. The dimensional arrays PFETB, PTHTB, PNTB are used for inputting tangential, circumferential, and normal loads, respectively. These arrayn, referred to as load tables, are dimensional for 200 information bits. The detalled prccedure or table setup is given in Section 3.4.7, page 90.

When loads (or more apecifically Fourter coefficients of load) are constant over the region, i.e., do not vary in the meridional coordinate. the PILD indicator is set equal to one. In this case, only one value of prossure load data is required for each Fourier harmonic (ENF) in each loir table. For the case of unsymmetric loade that vary meridionally. tho

Fourior componsents of load can be inputted at seicestod stations atong the moridian. 'The program will automatically compute values at intermodiate
 sot ©øu.l to Iwo whon using this option. Arbitraty unsymmotric loads c:an bo dericribed without prior knowledge of the fourier cocfficients of load distribution by using lll, option three. Discrete values of load are inputted in appropriate loid liblies at specific meridional and circumferential losituns. A lincar intcrpoiation routine yields ncecessary values at intermodi,le lositions. and the program: automatically determines Fourier corlicionts of loads using the lourier-Euler inversion formula. I he inversion integral is comluated numserically with coded valuc ENTH indicating numbor of finite sums taken. It is recommended that the maximum number ") be uscd for this value for general cases.

All pressurc load data are inputted in dimens, enal form (i.e., in units oi $\left.P^{\prime} / I^{\prime}-\right)$ and the program automatic ally performs appropriate operations tos maki cueficients nondimensional (Section 1.7).

### 2.12 IEMPERATURE DISTRIBUTIONS (TBOT, TTOP, I'TP, TIBT)

The temperature of the outer surface, each interface (multilayer shells). and inner surface must be supplied at each station of each region. The temperature data of innei and outer surfaces are inputted in a simi: : manner to pressure loads (see Section 3.4.7). Temperature indicators coded TIBT and TITP for inner and outer surfaces, respectively, are utilized with TBOT and TTOP representing table arrays for inner and outer suriace temperature values.

Temperature distributions across the shell thickness are usually derived from solution of the heat transfer problem. The program handles only shell structural problerns and does not make any heat transfer calculations. However, it does use the given temperature distribution to calculate stresses and deflections due to thermal influences in the shell. Since the temperature must be supplied at each face, there will be one more temperature value at each station than there are layers in the region. The outer and inner surface temperatures are supplied using procedures described above. The internal interface temperatures are supplied using a temperature gradient table for inputting interface temperatures at discrete meridional stations. An irterface defines the surface between two shell layers (Stection 2.13). The gradient value at each interface is prescribed as a prercentage of the total differential between top and bottom surface temperatures. The number of gradient stations considered per region is coded E: : OGR ( 10 maximum), with GSTA being station values at which gradients art: supplied. The gradients are supplied at internal interfaces and GSTA stations counting from first interface beyond the inner surface to the last interface before the outer surface of the shell.

The temperature input data is not curve-fitted directly; instead, the program calculates the thermal load ENT and moment EMT at data input stations and curve-fits using CODIMA to give intermediate station values.

## 2. 13 MULTILAYER SHELLS (ELAY. ENMAT, EMAT)

The computer program permits the analysis of multilayer shell configurations. Laminated shell sections having as many as s.x intimately londed layers can be analyzed. The value assigned to the variable ELAY $i s$ the number of layers in the region. For identification, the layers are numbered consecutively starting from the inner surface. (See Section 2.4.) A repion may consist of various layers of different materials each material haviag different clastic properties. A region of one material may be assumed o be divided into imaginary layers for purposes of determining stress internal to the outside and inside surfaces of the region or handling :on: incar temperature distributions across the thickness. The code value ENMAT indicates the number of different materials considered, for the problem with three being the maximum. The material layer indicator EMAT describes the material for each layer. The material used in a layer are numitref in secuence starting from inner layer and proceeding to outer layer. There are six possible values in the EMAT data locations, i. e., one ior each layer. As an example, let us consider the four-layer shell section shown in Figure 2-9. The layer i fntificaiion is given in Roman numerals. The sequence of data ior EMAT, for example, would be shown 1,2, 1,3, i.e., material 1 in first and third layers, material 2 in second layer, third material in fourth layer.


ELAY $=4$
ENMAT $=3$
EMAT $=1,2,1,3$

Figure 2-9. Layer Shell

$\therefore$ If MATYRIAL, PROPFRTIFS (POIS, FNFI, TMPFI, YMI, FNAI, 'IMI'AI. ALF'F. FNF.S, etc.)

In goneral. the elastic properties for structural material depend on the tomperature of the miterial. In the computer program, the material properties. modulus of elasticity (Young's modulus), and coefficient of thermal expansion are permitted to vary as a function of temperature in the: material. The material properties versus temperature data are read int, the program in the form of tables for cach individual material. The variables FNFI and ENAl describe the number of values of Young's modulus .nd cocificient of thermal expansions. respectively, that will be used in the: t.ibles for the first material. The code value TMPEI represents temperatures of which the YMll (Young's modulus) values are given in the tables fir the first material. TMPAl are temperature values at which the ALFl thermal expansion coefficients are given. In the second material, similar iode instructions are given ly ENE2, TMPE2, YM2, ENA2, TMPA2, ALF2 and so on for the third material.) With the temperature at the layer interiaces and surfaces known, the values of Young's modulus and coefficient oi thermal expansion are determined for each material at each interface by CODIMA curve-fit of the material properiy tables. The material property variation through each layer is obtained by linear interpolation. The value o: Puisson's ratio is assumed constant in each layer and defined in the regional input data as the quantity POIS (six data locations possible, one per layer). The inaccuracies introduced by assuming a constant value of Poisson's ratio for each layer in a region are small and this assumption greatly simplifies the equations of the program. The distribution of material properties must be known before the stiffness properties and thermal loads can be determined at each station on the shell.

### 2.15 STIFFNESS PROPERTIES (EIFH, ENOTH, THS'TA, TH, D, EK)

The stiffness properties of the shell can be evaluated when the material properties and shell thicknesses are known. The stiffness parameters must be supplied at each station in the region. The procedure for input of material property data is given in Section 2.14. For multilayer shells, the program permits the input of shell layer thicknesses in array form and automatically curve-fits data to ascertain thicknesses at intermediate station points. For the case of constant thicknesses, setting the variable EIFH to +1 permits thia use of a simplified data format. For variable thicknesses, EIFH is set to -I to permit reading of layer thickness tables. The quantity ENOTH sets the number of thickness stations given with THSTA being the active station number at which thicknesses are supplied ( 20 maximum). The station number must be the same for all layers. The thicknesses are read in by the quantity TH in order of layers' thicknesses per station, e.g., for a fivelayer shell, the thickness of each layer is read at a specific'station before
precoding to the next station. The order of layer input is consistent with description outlined in Section 2.13. i. e., first laycer at inner surface precoding in ordor to tho last layor on tho outer surface.

The cross-scetional properties are evaluated from Equations 33 and i4. It is assumed that the material properties (and temperature distrihution) varios lincarly across cach layer. Thus, all integrands will be broken up into a sum of linear functions of $\xi$ and the integrals are cvaluated numerically basid on values of material properties at layer and branching suriaces. A similar procedure is used in evaluating the thermal load and moment expression described by Equations 35 and 36.

For the case of constant stiffness properties, the extensional (D) and iloxural ( EK ) stiffncss can be inputted directly into the program by use of the F.N indicator discussed in Section 3.4.6.
$\therefore$. 10 INTERNAL SPRING SUPPORT (GSPRL, GUK, GVK, GWK, GEMK)

The program will allow the consideration of a support spring at any internal station in a region. The station location of the spring is specified by value GSPRL. The values of spring constants for the meridional, circunferential, transverse, and rotational spring supports are given by the symbols GTJK, GVK, GWK, GEMK. This capability would aid in considering shell structures which have internal elastic restraints such as a circumierential ring or other type of elastic support conditions. The program with minor modifications can be extended to handle more internal support points if desired.

## 2. 17 REFERENCE QUANTITIES (SIGO, EO, HO, AU)

SIGO, EO, HO, and AO represent reference stress, Young's modulus, thickness, and length quantities introduced in the analysis to provide nondimensional Fourier coefficients of comparable magnitudes. (See Section l.3.5.) It is usually most convenient to set the value of these quantities equal to one.

## 2. 18 GRAPHICAL PLOTS (PIXI)

This is a program option permitting graphical plottiag of results using the Stromberg Carlson autoinatic plotter. Nonzero values of PIXI will give plots. If no graphs desired set $P I X I=0$.

## 2. 19 SPECIAL INDICATORS (EX, PTHI, PFLAG, STRI)

There are several indicators in the program that yield certain features in the program that cannot be classified completely under the paragraph description presented previously.

The program will permit the stacking of problems so that more than one problem can be run with a job submittal. Theoretically, any number of problems can be stacked. The indicator PTHI is used to eliminate the repetition of data when similar problems are used. A PTHI value equal to zero indicates a normal program path. Positive PTHI values permit skipping of the geometry subroutine with the shell geometry remaining identical to the preceding case. A negative value of PTHI retains all shell properties from preceding case but permits variation of surface loads.

The PFLAG indicator permits the printing of all input data when the value is set to nonzero. In addition, a negative PFLAG will yield print information of a diagnostic type.

The quantity STRI indicates the layer at which a second value of stress across the thickess is desired. The value of stress at the inner surface of the specified layer is printed. Zero value of STRI automatically gives the stress at the outer surface of the shell.

The EX indicator is an option formulated to simplify data when running cases with constant loads and section properties are considered. This option is invaluable in running simple check cases. The details on this use of the EX symbol is given in Section 3.4.6.

## 2. 20 CURVE FITTING

As discussed in previous sections, the shell parameters and loads are curve-fitted using the controlled deviation interpolation method concept (CODIMA). CODIMA basically involves fittit g a second-degree polynomial through three successive points in the data is eld. Thus, the curve passes exactly through the supplied input points. Tise detailed characteristics of CODIMA are outlined in Section 3.7.3 and will not be repeated here. CODIMA was selected because it offers an accurate, efficient, and reliable technique for fitting data. As contrasted to 'least square" techniques, it does not exhibit ill behavior in treating even the most complex of functions. Of most importance, CODIMA fits automatically and does not require additional input construction to be aupplied by the user.

## $\therefore 2 I$ (ir:NFRAL COMMEN'IS

A number of dificulties may arise as a result not of errors in the proseramorits writeup but of certain subtleties connected with shell theory and the construction of the program. It would be foolhardy to attermpt to culiinc all these difficulties and subtleties here. However, it would perhaps be worthwhile to give some simple tips to serve as a reminder in use of the prograti.

1. The user should always check the output data from the program to see if it corresponds to input entered. In using curve-fit techniques (CODIMA) it may be desirable to input more data than absolutely necessary to increase the accuracy of representative results.
$\therefore$ Some difficulty may be encountered in selecting a mathematical model particularly when treating branching configurations; for example, some ambiguity is discovered in the definition of thickness for each shell region in the junction region. Careful study of Section 2.10 .3 with the exercise of good engineering judgment should permit the selection of an adequate engineering model.
2. The user should be reminded that shell theory is two-dimensional and input parameters and results should be interpreted accordingiy. The results, of course, will only be as good as the mathematical model selected.
3. The discrete point option (GMI $=3.0$ ) should be used only when the shell geometry cannot be described by the other geometry options. If this option is used, it is strongly recommended that the capability for input of radius of curvature information be utilized. A dense population of input data must be supplied when using this option in order to guarantee an accurate geometrical representation.
4. For multiregion configurations, extreme care must be exercised to ensure that the geometrical location of a junction point is matched between regions. In addition, best results are obtained if the finite difference intervals are selected to be approximately equal on each region.
5. Some difficulty may be encountered in treating problems having apex-apex or free-free boundary conditions. Rigid body type motion may occur when data are not input precisely. For some problems where some "drift" occurs, it may be possible to supply a nonforce-inducing spring to the shell in order to obtain a zero reference point for displacements.

## III. DETAILED USE: OF THE PROC;RAM

## ․ I INTRODUCTION

The Shell of Revolution Computer Program is written almost entirely in FONRTRAN IV and makes use of the overlay feature of that language. The: cxocption is found in the utility subroutine CRTG, described in Section 3. 7. 5.

The prodram has been checked out in NAASYS, the NAA adaption of the IlSM $7000 ; 7094$ IBSYS/IBJOB system and $2 s$ the NAASYS library routines shown ir the load map, pages 67 to 73 , inclusive, this section.

The NAASYS input tape is 'UNIT05,' the output tape is 'UNIT06, ' and the system CRT file is 'UNIT16.' In addition to these files, the program uses i, $4,7,8,9,10,11,12$, and 13 as scratch tapes or for overlay storage during execution. NAASYS itself, is stored on 'UNIT0l.'

The program is made up of an executive program and eight links, five uf which are called by the executive program, and the other three by the DATLNK subroutine. The name of the main program in each link and a description of its use follows.

| Link No. | Name | Purpose |
| :---: | :---: | :---: |
| 0 | EXECUTIVE | Reads the general data, DA, and controls the <br> flow of execution of the other links. |
| 1 | DATLNK | Acts as a subexecutive program to control <br> GEOM, DATLDS, and DATLYR, the subroutines <br> that set up regional deta. Also reads special <br> data, SDA. Prints Section and Material <br> Properties and Loads. |
| 2 | GEOM | Peads geometry parameters/region, Calcu- <br> lates DEL, R, X, WFE, WTH, GAMA, and <br> RHO. (See program nomenclature, Section <br> 3. l0.) Prints all geometry input and calculated <br> values. |


| link No. | Name | Purpose |
| :---: | :---: | :---: |
| 3 | DATI.DS | Reads pressure loads and temperatures for the inner and outer faces/region, Dl.D. Makes pressure dimensionless and, deper.ding on indicator, sends a constant, curve-fits. or Fourier sums for values at each meridional station. Sets up temperatures at 20 stations for use in DATLYR. Some data prints on indicator. |
| 4 | DATLYR | Reads section and material properties data, DAL/region. Sets up D, EK, ENT, EMT, El, T, ALF at all meridional stations. (The first four mentioned are made dimensionless.) Some data prints on indicator. |
| 5 | PANDX | Forms the $P$ and $X$ matrices of Equations 74 and 75 (Section 1.13) needed in the solution of the difference equations. |
| 0 | INT LD | Uses the $P$ and $X$ matrices from link 5 to form the solution matrix, z. (Equations 76 and 77, Section 1.13) Computes the current Fourier component for the bending moments, transverse shear forces, membrane forces, and stresses. |
| 7 | SUMS | Performs the Fourier summing for unsymmetrical loading conditions. Prints results. Sets up tapes and indicators for next Fourier component. |
| 8 | PIX | Plots shell geometry, displacements and other results from link 6/region. (Results are printed for all THETA values but are plotted for just the first THETA.) |

### 3.2 DECK SET-UP

In Figure 3-1 we have shown the setap of the column binary program deck, with the necessary control cards for each link.

The \$IBJOB, \$ORIGIN, and \$DATA cards are single control cards. The circled numbers found on the first two control cards mentioned indicate the order in which they, plus the associated decks of that link, should be

| IS LIAK | I. PARENT LIAK IS |
| :--- | :--- |
| IS LINK | 2. PARFNT LINK IS |
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| 4 | ／VHI． 1 | 23535 | ／XLO．$/$ | 23536 | ノVL？． 1 | 23537 |
| 0 | ／PR．e 1 | 23541 ＊ | ／mRe． | 23542 | 1MT®O | 23543 ＊ |
| 4 | ／HIGH．O／ | 2？545 | ／2．eptal | 23550 | ／0xxSYY／ | 23551 |
| 2 | ／oxX，YY／ | 23553 | loeeseol | $23554 *$ | ／HOLD．s／ | 23555 |
| 6 | ／YRFGoel | 23557 | ／CTPE．0／ | 23560 | ／CAMV．$/ 1$ | 23561 |
| 2 | 110．CUT／ | 23563 | ICUT． 1 | 23564 | ／LESH．D／ | 235＋5 |
| 3 | ／BUTTeol | 23574 | ／XPND．el | 23575 | 1CHOR 01 | 23576 |
| 7 ＊ |  |  |  |  |  |  |
| 5 | BUMP。O | 23765 |  |  |  |  |
| 0 \％ | BNBCOV | 23770 |  |  |  |  |
| 01 | FAINTY | 24212 |  |  |  |  |
| 21 |  |  |  |  |  |  |
| 7 | ERRLNY | $(242561$ |  |  |  |  |
| 3 | FRRNLY | （244321 |  |  |  |  |
| 71 |  |  |  |  |  |  |
| 71 | HCLOOV | 25552 |  |  |  |  |
| 51 |  |  |  |  |  |  |
| 01 |  |  |  |  |  |  |
| 1 | LINPV | （26）20） |  |  |  |  |
| 31 |  |  |  |  |  |  |
| 61 | Cameav | 27517 ＊ | FRAMEV | 27560 | RESETV | 27562 |
| 2 ＊ | －IDFRM | 27560 |  |  |  |  |
| 71 |  |  |  |  |  |  |
| ． 1 |  |  |  |  |  |  |
| 71 | －EICOV | 30353 |  |  |  |  |
| 71 | SETMOY | 30360 |  |  |  |  |
| 51 | misxyy | 30402 |  |  |  |  |
| 71 VYaxy 30417 |  |  |  |  |  |  |
| 4 | vraxy | 30417 |  |  |  |  |
| 4 | $Y$ MOOV | 36.464 |  |  |  |  |
| 51 | RXPNDV | 30473 |  |  |  |  |
| 7 | VSCALV | 30513 | NXV | 30626 | NYY | 29632 |
| 1 | IYV | 30673 | SCERRV | 30676 ＊ | SERSAV | 30703 ＊ |




FLOTV
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SMXYV
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PAGE 75
04 1 1 (9/96


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67357
51122 THFU
67353 THRU
Map of Core Storage (Cont)
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stacke 1 . For oxample, the socond level subroutines, GFOM, DATI.DS, and DAY' Y R, will be found in the deck before the first-level subroutince, IANDX, becatise tiery are everouted in this orcier.

It is imperative that the utility subroutines be keyt with each link as shown. Tho motrix arithometic subroutines, MAD. MSU, and MMY, are required by $1^{\prime}$ ANIX and INTID, but since only one first-level subroutine m.iy 心c゙upy corcat a giventime, they are entered with the FXECUTIVF. link so that they may be shared by both.

Additional control cards preceding the $\$ 1 B J O B$ card are likely to vary somewhat with the installation. An IBM systems handbook should be consulted.
$\checkmark$ IE4TCS 0 SJUB IRJOB C13504410996 32 1920258B04122FURUIKE . 3305

```
9 1347,35 C $18.309
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- 184302 J \$*
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Additional control cards used at S\&ID

### 3.3 PROGRAM FLOW DESCRIPTION

An overall flow diagram of the paths between the EXECUTIVE program and the first-level subroutines, and between the subroutine DATLNK and its second-level subroutines, is included in Figure 3-2.

A detailed flove diagram of each of these major control type routines, i. e., EXECUTIVE and DATLNK, is also included in Figures 3-3 and 3-4, respectively.

Many comments cards have been included in the listings of the other subroutines to aid in understanding their flow. (See Appendix IIIA, pages 181 through 273.)

### 3.4 INPUT DATA FORMAT

### 3.4.1 Introduction

Two types of data are entered in the program: (1) general data that is read by the EXECUTIVE qrogram and (2) regional data that is controlled by DATLNK. Depending on the values entered for the indicators PTHI (see DA data, Section 3.4.4) and EX (see SDA data, Section 3.4.6), the DATLNK subroutine will call or omit calling GEOM, DECRD(SDA), DATIUS, and


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Figure 3．2．Program Flow


Figure 3.3. Executive Prögram


Figure 3.4. DATLNK - Regional Data Control Program

DATl.YR. These latter four subroutines are cycled per region. A full explanation of the data for each routine, together with sample data sheets, is included in Sections 3. 4. 4 through 3.4.9.

Figure 3-5 shows the possible flow between the various data reading subroutines.

### 3.4.2 DECRD Subroutine

All data, with the exception of the three title cards, is read by means of the DECRD subroutine, available on the NAA library tape.

This routine provides the facility for reading a variable number of pieces of floating point data into specified elements of an aray; these elements may be in either sequential or nonconsecutive locations. Only the information specified is actually read into storage.


The fixed point number (index) in the first field on each card defines the position of the first piece of data on the card. If the index is 1 , the first piece of data will be stored in the first location reserved for the array; if it is 16 , the first word will be placed in the sixteenth position, etc. The rempining fields on each card contain information for the successive locations of the array. If one or more fields are left blank, no information is read into the locations corresponding to these fields; the information already in these locations is unaltered.

The sample data sheets shown in Section 3.6.3 have six fields of 12-card columns each and an identification field of eight columns for sorting purposes.


Th start of region D\& leap. Cycles for EkE regions. See DN data, Prut, path indicator See DH data, sec 3.1.4. Ex, constant data indicator See SDH date, seciona.t.6.
Eat, ne. of temperature stations when zero, ne teaperat...e loads This .adicater s set by the pieguam.

Figure 3. 5. Flow Chart for Data Reading
a. The index must be written to the extreme right of the first field; it may not be zero or blank (ne decimal point).
b. The programmer should keep in mind the way in which FORTRAN stores arrays having dovible or triple subscripts, e.g. $A(1,1)$, $A(2,1), A(3,1), A(1,2), A(2,2)$, etc.
c. The floating point (REAL) data chould be entered with a decimal point (anywhere in the field) and an exponent, when necessary, written to the extreme right of the field and precaded by a 't' or '-'。
d. Reading data is concluded by placing a negative sign in column 1 of the last card to be read.
e. Zcro should always be entered as '0.'. A' -0.1 or '. $0^{\prime}$ ' will be recognized as a blank.

ERROR indication: If the index is zero or blank, the comment "****BAD INDEX ON DECRD CARD ": will be printed, followed by a printout of the columns 1.80 of the defective card. The job will be terminated.

If the data for the array in the CALL statement have been completely read and no negative sign has been encountered in column 1 of last card read, data intended for subsequent CALL's will be read into the incorrect array. When there are no data cards to satisfy the appetite of a CALL DECRD statement, the job will terminate with an end of file tape 5 designation, as shown below.
TRACEEACK - CALLS IN REVERSE ORDER.
CALLING
RCUTINE
FION OR
LINE NO.

If this occurs before all expectud rosulte have beon printed, check the last card of each data bluck for the negative aign in column 1.

### 3.4.3 Data Deck Setup

Data decks should be stacked as follows:

1. Three cards ( 72 columns each) of title data
2. DA, general shell data. read by the EXECUTIVE program
3. GDA, geometry data, read by the GEOM subroutine
4. SDA, special data cases, read in DATLNK subroutine
5. DID, loads data, read in DATLDS subroutine
6. DAL, section properties data, read in DATLYR

With the exception of the three title cards, each group of data listed above should have a minus sign in column 1 of the last card. Groups tiiree th rough six are repeated for additional regions. Remember that some portions may be omitted due to the values of indicators EX or PTHI. (See flow chart, Figure 3-5, Section 3.4.1.)

### 3.4.4 Title Cards and Call DECRD (DA)

Three title cards form the first three carde of any data deck for each case. These cards are usefud in identifying the run at a later date. They may include a bilef problem deacription, the date of the rum, a reference, etc.

Thewe carde may not be omitted, but they may be blank, if desired. If the carde are forgotien, the error indication from DECRD will occur for a multiple case rum, or the job will terminate with an end of file tape 5 designation (at uxplained in Section 3.4.2) provided the DA data for the case was thren cards or legs.

All input data must be dimensionally consistent. It should be noted that all nondimenioneliantion is done internal to the program; thus, all inputs must be supplied with approprist dimensions (e.g., trineverse load $P N$ is input with dimencion $P / L^{2}$ ). In the thetructions that follow, the input quantities in terms of nomenclature of Section I are listed in the description and comments.

| $\begin{aligned} & \text { DFCCRD } \\ & \text { Index } \end{aligned}$ | Name | Descreription nnel Commants |
| :---: | :---: | :---: |
| 1 | EKK | Numbor of regions ( 50 rephons estimated !init) |
| $\pm$ | AO | Reference lenytr (a) |
| 3 | 110 | Ruference thickness (hol) |
| 4 | EO | Reference Young's modulus (E.) |
| 5 | SIGO | Reference stress ( $\omega_{0}$ ) |
| 0 | PIXI | CRT indicator. Plote curves when nonzero. Must be zero when SUM is negative. |
| 7 | PTHI | Path indicator <br> First case = O, "normal" path <br> Following cases: <br> a. Negative - skip GEOM; geometry is the same as preceding case <br> b. Positive - loads change only |
| 8 | SUM | Nonzero for multiple Fourier compowents <br> a. Ponitive - results are summed, with prints given at ENFOR values <br> b. Negative - discrete Fourier componenta. printed each time. so CRT |
| 9 | ENTO | Initial Fourier component ( n ) |
| 10 | ENFI ${ }^{\text { }}$ | Subsequent Fourier components (10 more) |
| 21 | ENF $\boldsymbol{\phi}_{\text {R }}$ | Fourier component print values. Three printe ars permitted. Two intermediate printe of the Fourier numming are possible for checidig convergence. The lant ENFOR given should be the same as the last ENFI |
| 25 | THETA | Circumferential angle (degrees), 10 maximum |

ENFI, ENFOR, and THETA values muat be read for each case. DA(1) through DA(9) are set to sero before reading the first case data but, for multiple case runs, they will retain their values unless changed by the programmer.

## S. H. 5 Gall DP: (RI) (GIDA)

The Gild data arrat is wroed each time before the above staternent is escouled. This me:ans that all cind data must be repeated for multipleregion or multiple-case runs.

| DH:CRD Indin | Name | Description and Comments |
| :---: | :---: | :---: |
| 1 | cimi | Geometry indicator <br> = 1. cone-cyltader <br> = 2. sphere-torold <br> $= \pm 3$. discrete pointe <br> $=i$. ellipae <br> $=5$. hyperbola <br> = *6. parabola |
| $\geq$ | EN | Number of station poilts per region ( 150 maximum) |
| 3 | Priag | Print indicator. Nomere prints all input data. A magative PFLAG printe additional information of a diatmontic type (cee Section 3. 57. |
| 4 | BCITP | Boumdery condition indicetar at first station $\mathrm{i}=1$ (top) |
| § | BCIBM | Bomedary condition finlicator at laet station i at N (bottom) <br> $=1$. free $\left(t_{\xi}, t_{\xi \in}, F_{E}, m_{\xi}=0\right)$ <br> * 2. roller itg, mo w, $\mathrm{m}_{\xi}=01$ <br> $=3$. clamped, fixed $\left(f_{\xi}, u_{\theta}, u_{\theta} w=0\right)$ <br> 4. simply supperted, hinged ( $u_{\xi}, u_{0}, w, m_{\xi}=0$ ) <br> $=5$. complete $\left(f_{\xi}, f_{\xi}, u_{\xi}, v_{\theta}=0\right)$ axiaymmetric load problem only <br> =6. special boundary matrices read in. Must nee 6 whemever nonsero valves are prescribed at boundary values in EM5X or EMNS matrice: <br> =9. closed apex (e.g., ipex of aphere, pole condition). Set one of the apex end conditions to -9 for apex-apex type boundaries. <br> $=10$. brameh point (mowe than 2 regions joining) ** At a branching diecomimuity only one region may have a tap boumelary indicator of 10 <br> $=0,1 . E+10$ discontinuity Henction (two regions joining) |


| 12F:Cにい ludn: | Natme | Description and Comments |
| :---: | :---: | :---: |
| 0 | GPsit | Biscontimuity in slipe at the end of the region (fleg reess) (Ser Figure 2.1 (2) |
| 7 | gricx | Ficentricity of reference surface at a discontinuity point (i : N) (See Section 2.10) |
| : | cisprl | Station location of internal support spring, one per region |
| " | GUk | Spring constant, meridional direction |
| 10 | GVK | Spring constant, circumferential |
| 11 | GWK | Spring constant, normal to shell |
| 12 | GEMK | Spring constant, rotational <br> When GMI = 1. 0; see Section 2.8.1 |
| 15 | RAI | Radial distance from axis of revolution to station 1 (L) |
| 10 | AXL | Meridional length of shell (L) |
| 17 | ANX | Angle the generator makes with the axis of revolution (degrees) <br> When GMI = 2.0; see Section 2.8.2 |
| 15 | RC | Radius of curvature of the generator (L) |
| 16 |  | Offeet distance measured from axis of revolution to center of meridional curvature (L) |
| $1 ?$ | PHIO | Initial opening angle from vertical axis (degrees) |
| 18 | PHIN | Final opening angle from vertical axis (degrees) GMI $=3.0($ or -3.0$)$; see Section 2.8.3 |
| 19 | EM | Number of RIPT's given ( 12 minimum, 150 maximum) |
| 20 | RIPT | Discrete radial distances |


| DECRD <br> Index | Name | Description and Comments |
| :---: | :---: | :---: |
| 170 | XIPT | Discrete axial or vertical distances (or arc lengths) |
| 320 | RCURV | Meridional radii of curvatures |
| 470 | RCURZ | Circumferential radii of curvatures $G M I=4.0,5.0 ; \text { see Section 2.8.4 }$ |
| 796 | RFF | Offset distance from axis of revolution to the parallel coordinate of the standard form |
| 797 | SPNO | Clockwise positive opening angle from the positive vertical standard form coordinate to the first station (degrees) |
| 798 | SPNN | Clockwise positive opening angle from the positive vertical standard form coordinate to the last station (degrees) |
| 799 | A | Semimajor axis parallel to the axis of revolution |
| 800 | B | Semimajor axis perpendicular to the axis of revolution GMI = $\mathbf{4 6 . 0} 0$ see Section 2.8.4 |
| 796 | BfF | Offset distance from axis of revolution to the parallel coordinate of the standard form |
| 797 | SPNO | Clockwise positive opening from the positive vertical standard form coordinate to the first atation (degrees) |
| 798 | SPIN | Clockwise positive opening angle from the positive vertical standard form coordinate to the last station (degrees) |
| 799 | A | Distance from the directrix to the focus, positive in positive direction of the standard form |

Boumdery matrices, when not set by indicator, only the diagonal elements are read. The explanation below is based on the assumption that the user is familiar with Section 1.9, "Boundary Conditions".

| DF:(R1) Inde: | Namb | Description and Comments |
| :---: | :---: | :---: |
| 000 | F:MIX | Diagonal terms of force bounclary matrix ( D ) (i $=1$ or top of ahell) |
| 024 | EM3K | Diagonal terme of dieplacement boundary matrix (A). $i=1$ |
| ais | EM5X | Column boundary matrices ( $)$, top of upen shell ( $i=1$ ); dimensioned for 20 Fourier components of boundary force or diaplacement |
| 30. | EMNI | Like EMIX at i = N (or bottom boundary) |
| 712 | EMN3 | Like EM3X at $\mathrm{i}=\mathrm{N}$ (bottom boundary) |
| 710 | EMN5 | Like EM5X at $\mathrm{i}=\mathrm{N}$ (bottom boundary) |

## 3.4. o Call DECRD (SDA)

The SDA data array is set to zero before the first case and first region data are read. Succeeding regions or cases have just the T, ENT, EMT, PN, PFE, and PTH arrays zeroed. EX is eet to zero on the second pass of an unsymmetrical load case. All oher diata will remain unchanged from the preceding region unless ontered by the programmer. If there are no changes, one data card (with an index mumber) must be read to satisfy the call DECRD (SD.A) statement.

| DECRD <br> Index | Name | Description and Comments |
| :---: | :---: | :---: |
| 1 | EX | Constant ciata indicator* <br> =0. No constints <br> Negative - all conatants. One value is entered for D, EK, E1, T, ALF, DNA, POI and the Forxier component of ENT, EMT, PFE, PTH, EN. These values are modufied by the reference coefficienta where applicable and the entire EN stations are filled with the conitants. |

[^1]| DECRD Index | Name | Descriptios and Comments |
| :---: | :---: | :---: |
| $\begin{array}{r} \text { Th } \\ 18 \text { nonzel } \end{array}$ | followiag except | + 1. - constant section propertios and temperature loads. One value is entered for EI, POI, DNA, ALF, and T. Valves for D, EK, EMT and ENT a re set by Equations 33 through 36 . <br> Values may be read for D and EK by entering the data flag 1. E +10 in SDA (26) and the D and EK values in SDA (27) and SDA (177), respectively. The program multiplies by the appropriate teference coefficients. <br> $=+2$. - constant pressure loads, no temperacure loads. The Fourier component for PN, PFE and PTH are entered as data. The values are multiplied by the reference coefficients and stored for EN stations. <br> ta are read directly into the SDA array only when EX noted. |
| DECRD <br> Index | Name | Description and Comments |
| 25 | POI | Poisson's ratio (v). Not entered for EX $=+2$. |
| 26 | D | Membrane stiffness ( $\mathrm{bE}_{\mathbf{o}} \mathbf{h}_{\mathbf{o}}$ ). Not entered for positive EX, except for data flag use explained in $\mathrm{EX}=+1$. above. |
| 176 | EK | Bending stiffness ( $\mathrm{dE}_{\text {of }} \mathbf{h}_{\mathbf{o}}^{\mathbf{3}}$ ). Same as D . |
| 326 | ENT | Thermal load ( $\mathrm{T}_{\mathbf{T}} \mathrm{m}_{\mathbf{o}} \mathrm{h}_{\mathbf{o}}$ ). Negative EX only. |
| 476 | EMT | Thermal moment ( $\mathrm{m}_{\mathbf{T}} \sigma_{0} \mathrm{~h}^{\mathbf{3}}$ ). Negative mX only. |
| 626 | PFE | Fourier component for urface load applied in meridional direction ( $P_{\xi}{ }^{a / \sigma_{0}} h_{0}$ ). Read for negative $E X$ or $E X=+2$. |
| 776 | PTH | Same as PFE, circumferential direction |
| 926 | PN | Same as PFE, normal direction |


| 11KNR1 Inderx | Name | Dascription and Comments |
| :---: | :---: | :---: |
| 1070 | $E 1$ | Modulus of elasticity (E). Read when BiX is nex:itive: or equal to 11. |
| 1:20 | T | Temperature differential ( $0^{\circ}$ reference temperature). EX neg. or +1. |
| 1370 | A1.F | Coefficient of thermal expansion (a). EX negative or $+1$. |
| 1530 | DNA | Distance from neutral axis. (Value will be negative for inner surface.) EX negative or +1 . |

These data cards (with a "-" in column 1 of the last one) will be succeeded by the following:

When EX is

| -1. | Next region's GDA, geometry data |
| ---: | :--- |
| +1. | This region's DLD, pressure loads data |
| +2. | This region's DAL, Section properties data |
| 0. | This region's DLD, then DAL data |

## 3. 4. 7 Call DECRD (DLD)

The DLD data array is zeroed each time before the above statement is executed. This meates that all DLD data must be repeated for multiple region or multiple efee nvis.


| DPCRI) Index | Name | Description and Comments |
| :---: | :---: | :---: |
| 4 | F.NTII | Number of finite sums taken to evaluate Fourier inversion integral for pressure or temperature cuefficients. For most cases, best results obtained by setting equal to maximum value of 91 . |
| 5 | PFETB | Table for PFE load. The array is dimensioned as 200 and its format is dependent on PILD, as explained below-TAB setup, Section 3.4.7.1 |
| 205 | PTHTB | Table for PTH load. Like PFETB |
| 405 | PNTB | Table for PN load. Like PFETB |
| 005 | TB ${ }^{\text {P }}$ T | Table for temperatures on the inner surfisce. Dimension is 200; format determined by TIST |
| 805 | TT¢P | Table for outer surface temperature. Dimension is 200; format determined by TITP |
| 1005 | PSIO | Angle at which line load is applied at a junction point (see Figure 1-10) |
| 1000 | PD | Magnitude of line loads applied at a junction point. Consecutive locations are used for succeeding Fourier components, 11 maximum |
| 1026 | EMD | Line moment applied at a junction (11 Fourier components possible) |

Don't forget the "-" in column one of the last card.

### 3.4.7.1 Tab Setup

All loade tables-PFETB, PTHTB, PNTB, TBOT, and TTOP-are fimensioned 200. Where Fourier summing is deaired, the values are read for all Fourier components (ENF's) at the same time. The format of the $t=i l e s$ for PFE, PTH, and PN will depend on the value assigned to PILD, $\because$ hile that of TBOT and TTOP are determined by TIBT and TITP, respectively.

TAB(I) Tibles for all loade (temperature) and all FiNF's
Indicator : 1
TAB(I) Constant value for first FiNF
TAB(III) Constant value for second FiNFFitc.

Indicator $=2$
TAB(I) Number of ENF's
TAB(I+1) lst ENF value
TAB(I +2 ) Number of meridional stations where loads are entered
TAB(I+3) Station No. = 1 ** must be 1 .
TAB(I+4) Load at station 1 ,
TAB(I+5) Second station, e.g. 10.
TAB(1+6) Load at atation 10.,
. etc., with station numbers and values interlaced.
-

- **The last station must br. EN, GDA (2)

TAB(2* TAB(1+2)+4) will be like TAB(1+1), i.e., the second ENF value. Repeat the pattern.

Indicator $=3$

| TAB(I) | Nunber of theta rays (circumferential stations) included in the table |
| :---: | :---: |
| TAB(t) | First theta value (degrees) ** must be 0 . |
| TAB(1+2) | Number of stains tc describe the first theta ra ** Must inclin ${ }^{3}$ ais stations listed for all theta rays ( 20 maximum) |
|  | Stations and values interlaced in same manner as for Indicator $=2$. Rules regarding first and last |

$\bullet$
-
-
TAB(2* TAB(1+2)+4)F will be like TAB(I+1) for the second theta and the pattern repeats from there. *中 It is not necesany to include all stations from theta ray one in theta ray two and the succeeding rays but atations 1 and EN must be among those chosen. **The last theta value must be 180. .

The table antorod for an indiciato: 3 will be used to form a matrix,



Th心 matris is formed by double, lincar interpolation of the values. The unterpolation subroutine, DINTRP, will select the lower or upper bound "hou i vilue is aff an end of a theta ray and continue after printing:

## 1.INIIS UF TABLE EXCEEDED BY ARGUMENT $\pm x . x \times x \times E \pm x x$ tn. הxaxtitnx VALUE USED FROM TABLE

This, ui cuurse, wastes time and will not occur if stations along each theta riy stirt with 1 . and end with EN. A more serious error is made when the iirst theta ray is not 0.0 degrees and the last 180.0 degrees. The resulting printont will read:

## ARGUMENT EXCEEDS EXTENT OF TABLE IN DINTRP ARGUMENT $= \pm x$. xoxxaxE $\pm x x$ TABLE VALUES $x$. $\times x \times x \mathrm{E}^{2} \times \mathrm{x}$ ( 6 per_line) <br> and the job is terminated.

When EX is
Next data will be

$$
\begin{array}{cl}
0 . & \text { This region's DAL, section properties data } \\
-1 . & \text { Next region's DGA, geometry data } \\
-1 . \text { or } 2 . & \text { (Should not have had any DLD data.) }
\end{array}
$$

## 3. 4. y Call DECRD (DAL)

The DAL data array is zeroed each time before the above statement is executed. This means that all DAL data must be repeated for multiple region or multiple case runs.

| DECRD <br> Index | Name | Description and Comments |
| :---: | :---: | :---: |
| 1 | ELAY | Number of layers (6 maximum) <br> 2 |
| STRIX | Layer number for second stress print |  |
| EIFH | Thickness indicator <br> $=+1 . ~ c o n s t a n t s ~ a l l ~ s t a t i o n s ~ i n ~ a ~ l a y e r ~$ <br> $=-1 . ~ d i s c r e t e ~ v a l u e s ~ g i v e n ~ a t ~ T H S T A ~ s t a t i o n s ~$ |  |


| DR:C:R Index | Name | Description and Commeats |
| :---: | :---: | :---: |
| 4 | FNSPTI | Number of thickness stations |
| 5 | HHSTA | Station numbers at which thicknesses are given. These are the same foi all layers. ( 20 maximum) First one $=1$., last one $=$ EN. |
| $2 i$ | TH1 | Thicknesses at stations, layers |

$\therefore \%$ The TH array is dimensioned ( $20 \times 6$ ). When EIFH - +1. The sunstunt fur cach layer may be entered in consecutive locations, i. e., the thickness for layer one at DECRD index, 25, thickness for layer two at 26, etc。

When thickness varies along a layer (EIFH =-1) and values are entered at thickness stations (THSTA), they must be entered according to FORTRAN duubly subscripted arrays. Station 1 on the second layer will have a DECRD index 20 locations away from station 1 on layer one (the inner layer). For any given station and layer, the DECRD index $=24+20$ * (layer no. -1 ) + sta. no. (See also the example for entering gradients.)

| DECRD <br> Index | Name | Description and Comments |
| :---: | :--- | :--- |
| 145 | ENMAT | Number of materials considered in problem <br> (3 maximur...) |
| 146 | EMAT | Material indicator/layer (1, 2, or 3) <br> 152 |
| PQRS | Poisson's ratio/layer |  |

The Material Tables data (DECRD indices 158 to 284) for all materials should be entered with the deta for the first region which uses DAL data, whether that region nset all given materials or not.

| DECRD <br> Index | Name | Description and Comments |
| :---: | :---: | :---: |
| 158 | ENEI | Number of Young's moduli for the first material <br> (10 maximum) |
| TMPEI Temperatures at which Young's moduli are given, |  |  |
| frst material (ENEI of them) |  |  |


| DECRD <br> Index | Name | Description and Comments |
| :---: | :--- | :--- |
| 169 | YM1 | Young's modulus for flret material |
| 179 | ENE2 | Same as ENE1, second material |
| 180 | TMPE2 | Same as TMPE1, second material |
| 190 | YM2 | Same as YM1, second material |
| 200 | ENE3 | Same as ENE1, third material |
| 201 | TMPE3 | Same as TMPE1, third material |
| 211 | YM3 | Same 2s YM1, third material |

**When there are no temperature loads, the Young's modulus is considered constant and should be entered at DECRD indices 169, 190, and 211 for materials 1, 2, and 3, respectively.

| DECRD <br> Index | Name | Description and Comments |
| :---: | :--- | :--- |, | ENAI |
| :--- |
| 221 |

":TMPE1, TMPE2, TMPE3, TMPA1, TMPA2, and TMPA3 are used by the curve-fitting ruutine CODIMA, and so the semperatures whould be listod in algebraic ascending order and should bound expected temperatures for all ragions.

| DECRI Indox | Nime | Description and Comments |
| :---: | :---: | :---: |
| 284 | ENQGR | Number of gradient stations (10 maximum) |
| こ心5 | CSTA | Stations at which temperature gradients are given. Same for each interface. First one = 1. Last one $=\mathrm{EN}$ |
| 203 | $\mathbf{G R}$ | Gradients at GSTA stations and "internal" interfaces, counting from the first interface (next to the inner surface) up to and including the last interface (below the outer surface of the shell). Valuds are given os ratio of the total differential between top and bottom surface temperatures. |

**When the gradients are conutant along an interface, ENOGR is entered as 1. and the gradient valuen are entered in the GR array in consecutive locations, each representing the value to be used for one interface. It is not necensary to enter CSTA values.

When the gradiente vary along an interface, and-gradient stations (GSTA) are given, the gradiente themselves must be entered according to the way FORTRAN atores doubly eubecripted arrays. GR (stations, gradient interfaces) $=\mathbf{G R}(10,5)$.

For exmmple, ENOGR $=4$, BLAY : 3., then the DECRD indices for the GR array would be

| Layer | 1. | 10. | 25. | EN |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 295 | (296) | (297) | (298) entered on one card |
| 2 | 303 | (306) | (307) | (308) entered on next card |
| 3 | 315 | (316) | (317) | (318) onternd on third card |

The DECRD index for any layer and atation $=294+10$ (layer no, -1 ) +sta.

The lat DAL data card should have minus ( - ) in column 1. The Acometry data, GDA, for the next region will normally follow except for nubscquent cases where PTHI may not be sero.

When PTII is negative, the geometry dota remains the same and the nost cards will be SDA type. If rme deaires to entr: valures in the EM5X or biliNs boundary matrices withou antering the GEOM subroutine, he may use SDA (2458) and SDA (2494), respectively, when SUM $=0$.; or, when $s$ umming is desired, the values fur the first Pourier component will be entered in SII: ( 2458 ) and SDA (2494) but succeedias Fouxier components for the upper boundary in SDA (780) and lower boundary ia SDA (930).

When PTHI is positive the DAL data will be followed by DLD, luads data for the next region. A positive PTHI does not permit a change in the EM5X and EMN5 boundary matrices.

## 3. 5 OUTPUT FORMAT

Following are sample pages and a description of the output of the prograi... The sample output represents some of the results obtained from the sample problem discussed in Section 3.6. Dive to amount of output information, only a portion of the results will be used to illustrate the output format. Additional results are reported in Section 3.6. The page numbers indicate the start of new pages of the computer output (i.e.. the first print wheel has the carriage control character 1) and do not necessarily correspond to the actual page numbers of the computer output. The latter is a function of the number of meridional stations, the number of regions into which the shell has been divided and the value assigned to the print indicator, PFLAG, entered with the geometry data, GDA. These page numbers and the circled letters that correspond to remarks in the description - re not printed by the computer. The link where the printing occurs and the EFN (external fo mula number) of the FORMAT statements are given for crossreference with the program listings (Section 3. 9).

## Output page 1

Always printed EXEC

A Three title cards. These cards are printed exactly as entered on the data sheets.

B
This space is available for other pertinent comments that wrula not fit the three title cards but that will be useful from a documentation point of view. It is a convenient place to include a sketch of the model assumed in setting up the problem, sucis as identifying the ends and jumetions of various regions and showing the loads and reactions together with their respective points of application.

A This is the value entered as F.NF' comporment.

11 Thercarc 12 vilues printed for Fourier components, but as stated in the input format, only 10 values in addition tos the F.NFO arc provided for. The last one is used as a program indicator and in fact will be "wiped out" by the program, if entered.
(: Space is provided for 10 thetas. The eleventh one is used as an indicator. See B.

1) When $S U M=0$., it is not necessary to enter any ENFOR data.
$E \quad$ When $S U M=0$, only three or fewer ENFOR's should be chosen. This location like $B$ and $C$ should always be -1.0000 E 10 and will be set to this number by the machine.

Output page 3
A Region number will depend on how the data cards were stacked, i. $\epsilon_{\text {. }}$ the first set of data entered is called "l", the second set " 2 ", etc.

B The type of shell is indicated, depending on the $(32,49$ or value read in for the geometry indicator, GMI. 90)

C The value 1.0000E 10 indicates a discontinuity boundary. Any other values were entered as data. See input format for GDA (page )

D This data will vary with the GMI indicator.
E These parameters, together with the finite difference increment, DEL, are computed by the GEOM bubroutine. They are printed at each of the N meridional etations.

| R | Figure 1.1 | Section 1.3 |
| :--- | :--- | :--- |
| X | Figure 1.1 | Section 1.3 |
| WFE | Equation 4 | Section 1.3 |
| WTH | Equation 3 | Section 1.3 |
| GAMA | Equation 5 | Section 1.3 |
| RHO |  | Section 1.3 |

$\theta$

four regions * sphere - cane - cone - cylinde - unstmatitical dress me
loading, temperature loads, layered regions, amundary frrcf
$\infty$
Output Pagu 1
genfanl dara


Output Page 2

There is no sample output page for this, since it would be used very inircquently for diagnostic purposes to check the "summing matrix" that results from linear double interpolation of the load distribution on the shell.

The matrix is dimensioned NFE by NTH, where NFF is the number of stitions entered in the table along the first theta ray (TAB ( $1+2$ )) when the indicitor is set at option 3 ( 20 maximum); and NTH is the fixed point form of DI D(t), ENTH (sec Section 3.4.7), the number of theta increments to sum.

This summing area, called TEMP, is printed station-wise (columnwiscl, aight per line.

Output pige 5
PFLAG $\neq 0$., DATLDS

The output sample for this page represents results printed out for the taibles. It would be used strictly as a check of the data inputted in PFETB, PTHIB, PNTB, TBOT and TTOP. (See Section 3.4.7.) The format would appear as follows:

LOADS TABLES FOR REGION 1
I
PFE PTH
PN
TBOT
TTOP
1
2
-
-
$\cdot$
200
Output page 6
PFLAG $\neq 0$., DATLDS (851)
A Meridional stations, 19 or 20 of them, chosen equally apaced between 1 and EN

B TBOT, temperature on the bottom or inner surface TTOP, temperature on the top or outer suriace. In the example shown the bottom surface temperature data was conctant, and outer surface varied linearly along the meridian, i.e., $\operatorname{TIBT}=1$, TITP $=2$.


**Pages 4, 5 and 6, to rciterate, can be printed only is the DATLDS subroutine has been ertered. (EX will be 0 . or +1 , or, for a succeeding case, PTHI will be greater than zero.)

Output page 7
PFLAG $\neq 0$. , DATLYR

A
First, second, and third materials. Curves are dimensioned for 10 possible values. Zeroes fill the locations wisere no entries have been made.

B The meridional station numbers given here area combination of those set up in DATLDS for the temperature loads (see output page 6) and the thickness stations, THSTA, read as data or set to 1. and EN when the thickness is constant, as it was in the example.

C The temperatures indicate a value at the inner face of the layer

D Printed for all values of PFLAG from here to "Output page 8."

E One value of Poisson's ratio per layer, read lst layer 2nd 3rd 4th 5th 6th
Material indicators have the same format.

F Gradients are entered for interfaces other than the inner and outer faces and at stations com ion to all, thus the printout indicates interfaces
$\begin{array}{lllll}2 & 3 & 4 & 5 & 6\end{array}$
where 1 and 7 would indicate the inner and outer faces, respectively.

DATLNK (102)
G When the DATLYR subroutine has not been entered because EX $= \pm 1$. the Section and Material Properties output will consist of just the printout from this point to "Output page 8."

POL, Poisson's ratio, inner layer POI2, Poisson's ratio for second atress

| l'rimt Symbul | Math Symbol and Fidjuation | D) finition |
| :---: | :---: | :---: |
| 1) | 1; Ficultion 33, Soction 1. 7 | Membranc stiffness |
| F:K | 1 <br> F.quation 34, Section 1.7 | Bunding stiffness |
| F: | F: | Modulus of elasticity (F, 1) |
| Al.F |  | Thermal expansion coefficient |
| IDNA |  | Distance from the neutral ixis to the inner surface |
| 'l' | T |  |
|  | Equation 26, Section 1.7 | Temperature differential |

$\because$ Fur succeding Fourier components none of the Section and Material 'roperties are printed except when PFLAG $\neq 0$.

## Cutput page 8

Always printed DATLNK (104)
A ENF, current Fourier component
B Current Fourier component for a force or moment applied at a junction point, EN.
$C \quad$ Mechanical and thermal loads at each meridional station

| Print Symbol | Math Symbol and Equation | Definition |
| :---: | :---: | :---: |
| $\mathbf{P}$ (PHI) | Equation 25, Section 1.7 | Pressure in the meridional direction |
| $\mathbf{P}$ (THETA) | Equation 25, Section 1.7 | Pressure in the circumferential direction |
| $\mathbf{P}(\mathbf{N})$ | Equation 25, Section 1.7 | Normal pressure |
| ENT | Equation 35, Section 1.7 | Temperature load |
| EMT | Equation 36, Section 1.7 | Temperature moment |


CURVES OF TEMPERATURE VS. THERMAL EXPANSION CTEF.
TABLE OF STATIONS VS. TEMP. AND THICKNFSSES, LAYER 1

THICKNFSS INOICATGR






Thesce matrices are the $: 1$ and $\Lambda$ matrices of Fequation 47. Section 1. $\%$. They are printed for boundary condition indicators equal to 1 . through 6.; thus, a set printed with the last repi:n on the shell would be for a iostom boundary. Branched shells mav have several top boundary prints but any closed apex regions (BCITP $=9$. ) will not be printed.

A Current circumferential angle
13 Current Fourier component. Results are for this component only or represent the Fourier sums to this component, depending on whether $S U M=0$. or $S U M \neq 0$., respectively.


Output page 10

| A | Print Symbol | Math Equation and Symbol | Definition |
| :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & Q(P H I) \\ & Q(T H E T A) \\ & N(P H I) \end{aligned}$ | Figure 2b, Section 1. 3.4 | Transverse forces per unit <br> length <br> Meridional membrane force per unit length |
|  | N(THETA) | Equation 19, <br> Section 1.3. 5 <br> Figure 2a, <br> Section 1.3.4 | Circumferential membrane force per unit length |
|  | $\begin{aligned} & \text { N(PHI, } \\ & \text { THETA) } \end{aligned}$ |  | Membrane force per unit length, shear |



n. กngクE-39




 PIFOKCF:


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 $0.0000000 E-39$
n.0007UJNE-39
$1.0000090 E 00$
$0.0005000 E-39$ 0.nnonodoes-30 (子uoj) 8 ated indzno
bonumpary matrices te*



n.nonoonif-39
LEAVIVG INTLO. LNKG


[^2]

\[

$$
\begin{gathered}
\text { JC jy928.6 } \\
1 \\
\text { IC } 35855^{\circ} 1
\end{gathered}
$$
\]

## $20-10262$

$$
\begin{aligned}
& 119 \\
& 120 \\
& 121
\end{aligned}
$$

$$
\begin{aligned}
& -7.3445 f-72 \\
& -7.36416-02 \\
& -.3832 E-02
\end{aligned}
$$

$$
\begin{array}{r}
0.000 n E-39 \\
0.0000 E-39 \\
-0.000 \cap E-39
\end{array}
$$

[^3]

Output page 11

| A | Print Symbol | Math Equation or Symbol | Definition |
| :---: | :---: | :---: | :---: |
|  | SIG(PHI) | Equation 79, <br> Section 1.4.3 | Meridional stress, inner surface |
|  | SIG(THETA) |  | Circumferential stress, inner surface |
|  | SG(PHI, THETA) |  | Shear stress, inner surface |
|  | SG2(PHI) |  | Nieridional stress, chosen surface |
|  | SG2 <br> (THETA) |  | Circumferential stress, chosen surface |
|  | $\begin{aligned} & \text { SG2(PHI, } \\ & \text { THETA) } \end{aligned}$ |  | Shear stress, chosen surface |

Pages 9, 10, 11 are repeated first for other regions and then for other thetas.

## 3. 6 SAMPLE PROBLEM

To demonstrate the use of the computer program and illustrate the format for data input, the sample problem shown in Figure 3.6 has oeen worked out. This problem is a hypothetical one, selected to illuscrate the use of many options in the program. The problem features an uncymmetrical load distribution, varying temperature loads, branch and eccentric discontinuity junctions, applied boundary forces, discontinuity loads, and others. The details for setting up this problem are Guscribed in the following paragraphs. Sample data sheets are presented in Section 3. 4. 9.

### 3.6.1 Problem Setup

The first step toward setting up this problem is a suitable selection of a inathematical model. For the :hell configuration considered, it will be necessary to divide the shell into at least four regions for computer solution. Using four regions, it will be convenient to draw a line diagram of the geometry denoting the extent of each region, the junction, and appropriate end conditions. This line diagram is shown in Figure 3.7. The arrows indicate direction of increasing meridional coordinate or station numbers. The sequence of input of regional data is given by the numeral designation given the particular regions (i.e., 1-2-3-4). Other sequences for namiering regions are permissible provided the selection is consistent with solution



















 d


Figure 3.7
nrocedure of the program. Referring to the region numbering system show.n in Fisire 3. 7, the problem could be consistently formulated by sequencing the resional input data (with appropriate end condition, of course) in these ioliowing combinations: (2-1-3-4), (4-3-2-1), and (4-3-1-2). The sequences (1-2-4-3) and (2-4-3-1) for example would not offer consistent formulation since a continuous transgression to the next regicr is not possible with this Evrmat. Using the example illustrated in Figure 3.6 let us now proceed to the input of regional data information.

## 3. 0. 2 Regional Data

Let us now consider the individual shell regions that make up the shell coniiguration (Figure $3.8 \mathrm{a}-\mathrm{d}$ ).

Region I


Figuxe 3.8a

RegionIis a sperical shell with an opening angle of 80 degrees. The end condition at ( $i=1$ ) is a closed apex and $\therefore$ quires that BCITP be set equal to 9. Since this region joins to two other shells at $i=N$, its bottom boundary (BCIBM) is set equal to 10 .

The mechanical loading on the shell consists of an unsymmetric external normal pressure load with a distribution given in the form

Figure 3.6

$$
p=-\left(1 \cdot\left(\frac{\xi}{\xi}\right)^{2}\right)(10+; \cos \theta)
$$

where $\xi$ is the are icugth of the shell (dimensionless).
The form of this joad requires that the problem be defined by two Fourier harmonics ( n : 0,1 ) in order to obtain complete solutions. ' C h tomperature applied to this region is a constant temperature differential of 110 degreet $\left(0^{\circ}\right.$ reference) applied to the inner surface and linearly varying tomperature at the outer surface starting from $0^{\circ}$ at the apex to 50 degrees at ytation $i=N$. The number of stations considered in this region is 121. A line load of 10.1 pounds per inch is applied at the branch junction which requires values of $\psi=-10^{\circ}$ and $P_{D}=16.1$ in DLD (1005) a: d DLD (1006), respectively. If desired this load could be read in with data fc. region II. In this case, 4 vouid be set equal to 11 : degrees.


Figure 3.8b

Region II is a conical shell in which the cone angle input (ANX) is 115 degrees. This r . $:$ : il is a inreelayer section with constant temperature of 110 degrees at outer surface and 150 degrees : * inner surface. The middle layer ( 0.1 -inch thick) is constructed of material 2 and layers 1 and 3 ( 0.05 -inch thick) are of material 1 . The temperatures at the interfaces are shown in the accompanying figure and are seffected in the gradient table shown on card 143. A force-free end condition excepting for an applied as isymmetric shear ioad ( 25 pounds $p \in r$ inch) exists at top boundary (station 1). This boundary condition requires tha: BCITP be set equal to 6 , and appropriate diagomal boundary arrays are read in EMIX, EMjX, EM5X array in GDA locations 62)-632. The other endpoint corresponds to a branch junction and BCIBM is set equal to 10.

Region 11


The third region is a aingle-layer conical shell. The end conditions are etipulated at $i=1$ by ectting BCITB $=10$ since this is a brewch point and BCIBM $=0$ at $i=N_{1}$ a diecontinuity point. The temperature of outer surface is assumed to vary limearly from 50 deegrees at etation 1 to 120 degrees at the last etation. The inser surface has a constant temperature of 150 degrees.

Figure 3.8 c
The unsymmetrical pressure load has the distribution

$$
p=-\left(1-\frac{\xi}{\xi}\right)(10+5 \cos \theta)
$$

Material 1 is used and the number of stations have been chosen at 141.

Since an eccentricity in reference surface occurs between Regions III and IV, the eccentricity distance ECX is set equal to 0.031 and discontinuity angle $u=10$ dogrees.

Region IV


Figure 3.8 d

The last region is a two-layer cylindrical shell. The boundary condition at the last station is assumed to be clamped, then $B C I B M=3.0$. $A$ miform internal pressure of 10 psi acts on the section and temperatures of outer and inner surfaces and 120 degrees and 150 degrees, respectively. The outer layer is constructed of ma' rial 2 and the inner layer of material 1. The temperature gradient across the shell thickness is assumed to vary along the meridian of the shell. The values are shown on data sheets.

### 3.6.3 Data Sheets and Results

The regional data for each region are written on standard IBM data sheets. The complete data for the sample problem are shown in the following IBM data form sheete.


$\therefore \boldsymbol{F} \boldsymbol{\sim}$ UR REGI $\mathbf{R}$ (NS
-S
3? CYI INDER *
 - A L PRESSURE,
ILゆADING, TEM PERAT URE I $\boldsymbol{A}$ A 25: D. IAYERED


 S.j: 3 be blank tf desired.
 (B) Identification numbers are $s$ afeguards againat data deck ."scrambling": E.g., a card of GDA data preceding the last (-) card of the DA data, will be read into the DA array: 1--.. The actual values are optional but should be chosen to permit su- additions and sorting of entire deck. - Sample Data Sheet $-\cdots-\cdots$.
FORTRAN FIXED 10 DIGIT LECIMAL DATA


FORTRAN FIXED 10 DIGIT DECIMAL DATA

DESCRIPTION DO NOT KEY PUNCH
 5 TT中P NUMBER IDENTIFICATION DECK NO

$\square \ldots \quad 80.5$

DAL (Section Properties)
Region 1
See Section 3.4.8
1.

Sample Data Sheet (Cont)
FORTRAN FIXED IO DIGIT DECIMAL DATA
programmer

_ $=\cdots=1$
ENE2 (Second Material)


$\therefore-\cdots$ (2) | the tables of temperature vo. |  |
| :--- | :--- |
| Young's Modulus \& the Coef, of |  |
| YM2 (1) | thermal Expansion for the 3 |
|  | materials, are preserved and |
|  | used for all regions and Fourier |
|  | components. Therefore, the |
|  | given temperature range should | | ENAI |
| :--- |
| TMPAI (1) |

Sample Data Sheet (Cont)

FORTRAN FIXED 10 DIGIT DEC:MA, DATA
Date $=\operatorname{sic} 8$ \& 22
30 NOT KEY D :
DID.
(A) Inserts for pg. 4
OAT A






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\text { li } \quad 1
$$

$$
T T
$$

$$
\Rightarrow
$$

FORTRAN FIXED 10 DIGIT UECMAK DGI-

PROGRAMTEA. -_...
-


FORTRAN FIXED 10 DIGIT DECIMAL DATA
programmer ore page 12 of 22 Job no. 2733-01 00 NOT KEY PUNCH
DAL, cont - Region 2 $\square$ -
 — (3rd layer) PQIS (1)
 :
(


$$
\begin{aligned}
& \text { ENMAT } \\
& \text { EMAT (lst layex) } \\
& \hline
\end{aligned}
$$

DECK NO.




Cl

$\because \quad$

 Sample Data Sheet (Cont)
FORTRAN FIXED 10 DIGIT DECIMAL DATA

 630 0
$\frac{(3)}{(4)}$ Sample Data Sheet (Cont)
FORTRAN FIXED 10 DIGIT DECIMAL DATA DESERPPTION 20 NOT KEY PUNCH
Jo8 no. 2733-01


FORTPAN FIXED io cigit lecimal data
programmer
cont. - Region 3 $+$




 $i$ 1

FORTRAN FIXED 10 DIGIT DECIT:AL DATA
$\frac{.22}{\text { NCH }}$
FORTRAN FIXED 10 DIGIT DECIMAL DATA
 ORM H 14-C:77 REV. 7.58 Sample Data Sheet (Cont)
FORTEAN F:X.E IC EHIT DEEMAL DATA EETRIDT:ON 20 NOT KET PIMCH
DLD. coant. - Region 4 _ - - ... $\begin{array}{ll}\cdots & \\ \cdots & -\cdots-\cdots \quad-\cdots\end{array}$ $\because-\pi=\square$



FORTRAN FIXED 10 DIGIT DECIMAL DATA

FORTRAN FIXED 10 DIGIT DECIMAL DATA


FORTRAN FIXED 10 DIGIT DECMMAL DATA
Do not Ker ouncer
Ditz Ceat - Resion 1
$\begin{array}{r}C B(2,13) \\ (2,1) \\ \hline\end{array}$
49:


DESCR:PTION

| $\mathrm{CR}(2,1)$ |
| :---: |
| $(2,1)$ |
| $(3,1)$ |


T
$\rightarrow$ T———n



























 Noco



(IMdJM IIIM



[^4]












Sample Problem Output (Cont)
















Sample CRT Results

- 161 -


Sample CRT Results (Cont)




Sample CRT Results (Cont)


Sample CRT Results (Cont)


Sample CRT Results (Cont)


Sample CRT Results (Cont)


Sample CRT Results (Cont)


Sample CRT Results (Cont)


Sample CRT Results (Cont)


Sample CRT Results (Cont)


Sample CRT Results (Cont)
-


Sample CRT Results (Cont)


Sample CRT Results (Cont)


Sample CRT Results (Cont)

The machine printout fne regicy III immediately follows the data sheets. The complete solution (i.e., $\mathrm{n}_{\mathrm{f}}=0$ and 1 combined) is shown for $0=0$ and 90 degrees. To illustrate the graphical plots, results of particuiar quantities of regions II and IV are shown following printoui. Some results for segions I, II wore discussed in Section 3.4.5 on output form.

### 3.7 UTILITY SUBROUTINES

## 3.7.: MAD, MSU, MMY, INV

These four subroutines perform matrix addition, subtraction, multiplication, and inversion, respectively. They are extremely simple in their approach and must be recompiled to change dimensions for use in other decks. There are no error indications given other thar the usual NAASYS trapping information for underflows, overflows, and divide checiks. When data have been entered correctly, these subroutines will present no problems.

### 3.7.2 DINTRP, ENTERP

These subroutines perform linear double and single interpolation. DINTRP makes use of ENTERP in interpolation for values along a particulax curve.

In DINTRP, when the firet argument is not bounded by the given table (curves), the statement
"ARGUMENT EXCEEDS EXTENT OF TABLE IN DINTRP." is printed, followed by ARGUMENT = 11 PE 12.4) TABLE VALUES (printed 6/line) and the job is terminated.

When the argument in the single interpolation subroutine, ENTERP, exceeds the limits of the table, the roatine selects the value at either end of the table and continues after printing
"LIMITS OF TABLE EXCEEDED BY ARGUMENT = (1 PE 12.4)
(1 PE 12.4) = VALUE USED FROM TABLE"
Values entered in the tables should always be given in increasing algebraic order, both in terms of the numbers used to resignate each curve of the farnily, and the values assigned to the points along the curve.

### 3.7.3 CODIMA

CODIMA is a curve-fitting subroutine has the following properties:
l. he straight portions of any curve defined by three points on a ot:dight line, a stra ht line will be fitted.
2. To the smooth portion of any curve, a smooth curve will be fitted.
3. The method maintains contiruous first derivative except at the ends of a straight segment.
4. The methnat will fit curves with "corners" or "sharp turns" without the large deviation usually found in other methods.

- An interpolatior method is developed in such a way that some of the considerations taken when an engineer fits a curve with a french curve are formulated. This is the CODIM (controlled deviation interpolation method) concept.

The method will interpolate in a more engineering manner in the following respects:

1. The first $\dot{c}$ rivative is continuous except at the ends of straight segment- defired by three points on a straight line.
2. No large deviation will be found when slope changes are large.
3. Ability to change value and slope rapidly.
4. Ability to fit straight lines on straight line port: :s of the curve and fit smooth arcs through the smooth portions of the curve.

The method fits a polynomial through an interval with information given by "previous points" (points to the left) and another polynomial through the interval with information given by "subsequent points" (points to the right). These two polynomials are then compared for compatibility. If they differ, a weighted average of the polynomials is taken in such a way that the polynomial that deviates less from the straight line connecting the points defining the interval is given more weight. For simplicity, parabolas are used over higher-degree polynomials in the CODIMA version.

### 3.7. STCOME

STCOMP is used to cornbine the station numbers at which the thicknesses are enteced in the DAL data region with the station numbers for the
inner and outer layer temperatures (set up in DATLDS) to form a common set of stations to be used in the computations for DNAX, D, EK, ENT, and EMT.

### 3.7.5 CRTG

CRTG is a system of subprograms (some MAP compiled) designed to enable a FORTRAN programmer to use the S-C 4020 CRT plotter for graphing the types most frequently required in engineering and scientific applications.

The ouiput is intended to be imitative of the results obtainable by hand plotting on standard graph paper. Printed and graphical output may be intermixed in any amount.

The system establishes a fairly natural correspondence between the programmer's representation of data and its appearance on the graph. A simple curve may be produced with one CALL statement. For complicated graphs, the full power of FORTRAN may be used to describe the data. Scaling is automatic and includes all curves on a graph.

The drawing of grids and placement of output on the frame are automatic.

Some restrictions of CRTG are as follows:

1. Requires an S-C 4020 to process the output
2. Requires NAASYE and the NAASYS library routines for the S-C 4020
3. Uses the system CRT file, 'UNIT16'
4. Requires the use of nonstandard RETURN statements, a language feature introduced with 7090/7094 FORTRAN IV, Version 13
5. CRTG will fail to express applications that require unusual grids.
6. A special version of NAASYS library routine $D X D Y V$ is required. This is included in the deck.

## 3. 8 ERROR INDICATIONS, PITFALLS, RECOMMENDATIONS

Several of the error indications resulting from improper data input have already been discussed. To reiterate, they were as follows:

1. A bad index on a DECRD card (Section 3.4.2)
2. Omission of the negative sign on the last card of a data a rray (Section 3. 4.2).
3. Omission of some or all of the title cards (Section 3.4.4)
4. Limits of pressure • r temperature tables exceeded by arguments wher using the indicator $=3$ option (Section 3.4.7.1).

One should be very careful to check the ou ${ }^{+}$ut from the program to see that it corresponds to the input that he entered. Better yet, an independent check of input data may prevent a wasted run on the machine. In addition to the four errors indicated above, such things as sign convention, angle measurements, and compatibility of units are common pitfalls.

Shells of Revolution

Shells of Revolution (Cont)


-     -         -             -                 -                     -                         -                             -                                 -                                     -                                         -                                             -                                                 -                                                     -                                                         -                                                             -                                                                 - 

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| ooco 0.060 |




 Matrix Subtract Subroutine


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6210
7000 9001
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SIBFTC CF3P

urve Fitting Subroutine
(Three Points)




Parabolic Curve Fitting Subroutine (Three Points) (Cont)






Geometry Computation Subroutine




 $2(r D A(8)$, GSPRL) $2(T, G A(9), G U K),(G D A(1 \therefore), ~ G V K),(G D A(11), ~ G I N K)$ 4(GDA(16), AXL, ROFF), (GIDA(17), ANX, PHIO), (GDA(IB), PHIN)
5(GDA(29) FM), (SDA(2:), RIPT), (GDA(170), XIPT),
7(GDA(624), EM3x),(GDA(678),EM5X),(GDAi7OR),FMN1),(GDA(71?),FMN3),
8(GDA(7)G), EMNF), (GDA(79K),RFF),(GDA(797), SPN?), (GDA(748), GPNN),
O(Gi)A(790), A) (GDA(8C.i), Fi)

Geometry Computation Subroutine (Cont)

COMMON DA(35), NTPW, NTPR, KTPW, KTPR, SL2, ELAM2, S1, S2,
1 KKE, SQ3, SO4, SO6, ENF, IFR, KLM
COMMON SDA(3098)

$$
\begin{aligned}
& \text { DO } 1 \quad I=1,800 \\
& \text { GDA(I) }=0 . \\
& \text { CALL DECRD }(G D A) \\
& N=E N \\
& N N=N-1 \\
& \text { CALL OPEXO (13) }
\end{aligned}
$$


NOIפヨУ $\forall O S$ O1 $\forall 1 \forall O$ ヨAOW
1
-... -
$-\mathrm{N}$

Geometry Computation Subroutine (Cont)




Geometry Computation Subroutine (Cont)



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00004680
00004690
00004700
0t $1 \rightarrow 0000$
02LT0000
$0 \rightarrow \angle+0000$
$0 \varepsilon \angle+0000$
OSL 20000
00004760
00004770
$06 L 70000$
$08 \angle 70000$
$0 \angle \angle 70000$
$00 \angle 70000$
00004800
00004811 $0 \varepsilon 870000$
02870000
$078+0000$
08870000
00004850
00004860
00004860
00004870
00004870
000004880
$016+0000$
40670000
$068+0000$
000004910
00004920
00004920
00004930
00004940
09670000
$056+0000$
00204970
$00 n 04970$
00004980
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00004990
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m
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$00 n 05020$

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$$
\text { CALL CODIMA }(I J D, \text { STAW, WFW, STAP, WFP, } 4,-100)
$$

WTH(NN) = (HT(MLNH+WT(NND + WINND.1/300











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Linear Interpolation Subroutine $w$ ENTERP**

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-
$$


THSTAI (DAL12S)

$$
4^{.}
$$

藏

(Cont)

Subr. to Set Up D, IK, ENT, EMT, El, T, ALF (Section Properties) (Cont)

---- -



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| :--- |
| 00001034 |
| 00001036 |
| 00001037 |
| 00001039 |
| 000001040 |
| 00001042 |
| 00001043 |
| 00001048 |
| 00001049 |
| 00001050 |
| 00001052 |
| 00001054 |
| 00001058 |
| 00001059 |
| 00001760 |
| 00001062 |
| 0001064 |
| 00001065 |
| 00001068 |
| 00001069 |
| 00001070 |
| 00001072 |
| 00001074 |
| 00001078 |
| 00001079 |
| 00001080 |
| $00 n 01082$ |
| 000001084 |
| 00001086 |

1
0
00
00
0.0
80
80
 $C N$
$C_{0} N$
00
00
00
00
080
808
80
(Const

..._Subr. to Set Up D, IK, ENT, EMT, E1, T, ALF (Section Properties) (Cont)

Subr. to Set Up D, IK, ENT, EMT, E1, T, ALF (Section Properties) (C.nt)


Sub.-outine to Combine the Two Stion are Given at Different Stations (Cont)
$\underset{\text { 를 }}{0}$


IFt PFLAK, $1 \cdot 3,3$
1 WRITFI6,
2 FOFMATI $1 / 10 \times .19$
2000.9400
0 Onnob401



[^5]


$$
\text { EM4 } 1.1 \text { ) }=10 \text { BOTTOM OR TOP BOUNDARY, CLUSED }
$$
$C^{260 \text { TFIT NE. } 1160 \text { TO } 800}$

## 90, 210, 230



Set Ūp $\mathbf{P}$ and $\mathbf{X}$ Matrices (Cont)
$\qquad$


 NONNNNNNNNNNNNNNNNNWNNNNNNNNNNNNNNNN










$\begin{array}{ll}N \\ 0 & 0 \\ 0 & \\ 0 & \\ 0 & \\ 3 & \\ 2\end{array}$


00005101
0006511,2
000.05163
00005114

0
$N$
$n$
0
0
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88
08

00005123.

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ลัลี่


8
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8
8
00045160
00005170
00005279
00005270
00005272
00005280

 COMPLETE (CLUSED)
-....


FIXED OR CLAMPED BUUNUARY

- $-\cdots$

$\therefore$

Set Up P and X Matrices (Cont)

Set Up $P$ and $X$ Matrices (Cont)




[^6]

$F$









600706
300700 104009
0004050
 $00 \% 14018$
0004013

$30500 \quad 310-1=10 \mathrm{~N}$

C SAVF MATRICFS FOR $2(N)$ IN BRANCHING


|  | -. |  | FORM 7. AT J+ Discontinulty |
| :---: | :---: | :---: | :---: |
|  | CALL MAO | . (4,1: DIS3.DIS4,EM6) |  |
|  | CALL MMY | (4,491, D152,20.0153) |  |
|  | CALL MSU | (4.1, EM6.DIS3,EM6) |  |
|  | Call mmy | (4.4,1, DISI,Z.DIS3) |  |
|  | CALL MAD | (4,1, EM6,DIS3,EM6) |  |
|  |  |  | MOVF J- ANE J+ 2 VALIIFS |

## 

## C


$\mathrm{N}+1$
Deflections and. Internal Loads (Cont)




GENERAL DFRIVBTiVES,

> -
$u^{9}$
x2770) (18-146)2

general eouations

GENERAL DFRIVBTIVE:
$=-W P+W F E(I) * Z(1,1)$

545FEFE

## SII*Z(2,I) + GAMA(I)*Z(1,I) + WTHD $(I) * Z(3, I) ~$



ENFT(I) $=-$ D(I)*S4*(VP-GAMA(I)*2(2,1)-S11*2

$\begin{aligned} & 1 \text { NWHDII) } \\ & 548-515=5 \\ & S 7=S I \\ & S Q=E 1 \\ & 5 R=S I \\ & S 10=5\end{aligned}$
SヨSS- 15 INZ yO_ 135

$\omega$
DNAR(I)/AO -
DNA2(I)
$15 * S 3$

Deflections and Internal Loads (Cont)




Variable Theta, Summing

## BACKSFAtE 8 NOT

C-T30 WRITE 16,733 : KR, THETX, ENF, II USUM(II) VRINT RESULTS 130 EMFEIIIS EMTHIIID EMFTIII I $=1$ IN I EMFEIIIs EMTHIII, GAFTTII, $I=1$ NN: DEFLECTIONS AND INTERNAL
 2 GHM(THETA), $3 X \in$ 22HM(PHI,THETA) $/ \prime(14,6 E 13.4)$ ) WRITE (6, 735) \{I, OFE(I), QTH(II, ENFEIII, ENTH(I), ENFTII),


 (PH1), $4 X, 10 H 5 G 2(T H E T A), 2 X$,
$(14,106 E 13.4)$,
149 1-0.13041,
Variable Theta,


## wronoogoooo mutogonoo   88 60 8

Surnming (Cont)

IBFTC CRT
 CRT Package


## 

## APPENDIX IIIB

## PROGRAM NOMENCLATURE

In this section are listed all variables that are used in the FORTRAN IV program and their related definitions. The appropriate mathematical equivalents from Section 1.0. are included where applicable.

## A

A 4-x-4 matrix, defined in Equation 66, Section 1. 12; constant in conics computation

AO a Reference length (L)
A2 4-x-4 matrix, used to preserve A matrix for meridional
A2cds
A2SIN $\operatorname{Sin}^{2} \theta_{1}$, variable in conics computations.
AA2 Constant in conics computation
A cads.
AC $\mathrm{C} S \mathrm{SP}$
AI

AJI Used in computing $X$ distance for general discrete points, GEOM

AK Used in computing $X$ distance for general discrete points, GEOM

ALF $\quad \alpha \quad$ Thermal expansion coefficient for $N$ summations
ALFI Thermal expansion coefficient data for material 1

| ALF2 | Thermal expansion coefficient data for material 2 |
| :---: | :---: |
| ALF3 | Thermal expansion coefficient data for material 3 |
| ALFA | DEL/DPREV |
| A.LFA2 | ALFA **2 |
| ALFI | Inner coefficient of thermal expansion/layer |
| ALF $\phi$ | Outer coefficient of thermal expansion/layer |
| AM. | Value of signed variable |
| AMB2 | $A^{2}-B^{2}$ constant in hyperbola co- sutation |
| AMU | Sign control variable in conics |
| ANGSP | Angle span for sphere-toroid, GEOM |
| ANX | Angle between the gene rator and axis of revolution, cone-cylinder, GEOM |
| APB2 | $A^{2}+B^{2}$ constant in ellipse computation |
| APHI | GEOM parameter for computing $X$ distance in spheretoroid shape |
| ASI | 4-x-4 matrix, used to preserve A matrix at top, for open shell or discontinuity |
| ASINP | SIN(APHI) see APHI |
| ASN | 4-x-4 matrix, used to preserve A matrix at bottom, for open shell or discontinuity |
| ASSIN | $\sin ^{2} \theta_{1}$, variable in parabola computation |
| AXL | Axial surface length |
| B |  |
| B | 4-x-4 matrix, defined in Equation 66, Section 1. 12; constant in conics |


| B2 | 4-x-4 matrix, usud to preserve A matrix for station 2 ; also used in stresses |
| :---: | :---: |
| B2MSMS | Variable in hyperibola computation |
| BB2 | $B^{2}$ |
| BCD | Three title cards read in executive program |
| BCIB | Boundary condition indicator, bottom, ( $i=N$ ) SDA data |
| BCIBM | Boundary condition indicator, bottom, (i=N) GDA data |
| BCIT | Boundary condition indicator, top (i=1)SDA |
| BCITP | Boundary condition indicator, top (i=1) GDA |
| BC\&SP | COS(PHIO), sphere-toroid, GEOM, see PHIO |
| BETA | Cylindrical coordinate variable describing conics |
| BETADP | Second derivative of BETA |
| BETAP | Derivative of BETA with respect to angular variable of BETA description |
| BI | 4-x-4 matrix, used to preserve B matrix for bottom discontinuity |
| BJR | Index in conics when subdividing cylindrical coordinate range |
| BM | Value of signed variable |
| BMU | Sign control variable in conics |
| BфT4 | 4-x-1 matrix, used to preserve solutiors for bottom discontinuity point |
| BP $\mathbf{b}^{\mathbf{l}}$ | First derivative of the membrane stiffness |
| BPHI | GEOM parameter used in computing $X$ distances for sphere-toroid |
| BQ | Same as B2, see COMMON region, INTLD |

BS1

BSINP
BSN

BTA2
BTAP2
C
C
C2
CHI
CHII
CHI2
CI

CS1

CSN
4-x-4 matrix, used to preserve C matrix at bottom, for open shell or discontinuity

D
D d Membrasè stiffness (dimensionless in program),
D d $\quad \begin{aligned} & \text { Membranee } \\ & \text { Equation } 33\end{aligned}$
DA
4-x-4 matrix, used to preserve B matrix at top, for open shell or discontinuity

SIN(PHIO), see PHIO
4-x-4 matrix, used to preserve B matrix at bottom, open shell or discontinuity

Variable in the conics option
Variable in the conics option

4-x-4 matrix, defined in Equation 66, Section 1.12
4-x-4 matrix, used to preserve $C$ matrix for station 2
4-x-4 discontinuity matrix, Equation 57
4-x-4 discontinuity matrix, Equation 58
4-x-4 discontinuity matrix, Equation 59
4-x-4 matrix, used to preserve C matrix for bottom discontinuity

Parabolic curve fitting subroutine (see page 178, Section 37. 3)
COS(ANX) used to compute W TH for cone-cylinder, GEOM $\operatorname{COS}\left(\eta_{\theta}\right)$ used in Fourier summing

4-x-4 matrix, used to preserve C matrix at top, for open shell or discontinuity

D

General data area, read in executive program

| DAL | Section properties data/region, read in DATLYR subroutine |
| :---: | :---: |
| DATLDS | Data loads subroutine sets PN, PFE, PTH, TBT, TTP (pressure and temperature loads) |
| DATLNK | Regional data read wibroutine, sub-executive program for GEOM, DATI.DS, DATIYR |
| DATLYR | Section properties subroutine |
| DDL | Used in GEOM (2040) |
| DECRD | Data read subroutine (see explanation page 79) |
| DEL $\boldsymbol{\Delta}$ | Interval size between meridional stations |
| DEL2X | 2. * DEL |
| DELSQ | DEL ** 2 |
| DELTH | Circumferential increment for Fourier summing for loads |
| DENM | Denominator for computing GAMA in GEOM subroutine |
| DENMP | Denominator quantity for finite difference first derivatives in discrete points option |
| DEN¢M | Denominator for computing WFE in GEOM subroutine |
| DEN\&MP | Denominator for finite difference and derivatives in the discrete points option |
| DINTRP | Linear double interpolation subroutine |
| DIS 1, 2, 3, 4 | Discontinuity matrices formed at top discontinuity point in PANDX |
| DLD | Data area in DATLDS subroutine/region |
| DLR | Used in GEOM (2060); intermediate radial increment in d'screte point option |
| DLS | Used in GEOM (2062); intermediate arc length increment in discrete point option |



| ELD |  | 4-x-1 matrix used at top discontinuity point |
| :---: | :---: | :---: |
| EM |  | Number of radii entered for discrete point geometry case |
| EM1 | $\boldsymbol{\Omega}$ | 4-x-4 diagonal boundary force matrix ( $i=1$ ) (Equation 47) |
| EMIN | $\boldsymbol{\Omega}$ | 4-x-4 diagonal boundary force matrix (i $=\mathbb{N}$ ) (Equation 47) |
| EM1X | $\Omega$ | EM1 when read as data in GDA. area |
| EM2 | H | 4-x-4 matrix (Equation 50) |
| EM3 | $\Lambda$ | 4-x-4 diagonal boundary displacement matrix ( $i=1$ ) (Equation 47) |
| EM3N | $\Lambda$ | 4-x-4 diagonal boundary displacement matrix ( $\mathrm{i}=\mathrm{N}$ ) (Equatinn 47) |
| EM3X |  | EM3 in GDA data area |
| EM4 | J | 4-x-4 buundary matrix (Equation 51) |
| EM5 | 1 | 4-x-1 boundary matrix ( $\mathrm{i}=1$ ) (Equation 47) |
| EM5N | 1. | 4-x-1 boundary matrix ( $\mathrm{i}=\mathrm{N}$ ) (Equation 47) |
| EM5X |  | EM5 in GDA data area |
| EM6 | f | 4-x-1 boundary matrix (Equation 51) |
| EMAT |  | Material indicator/layer (1, 2, or 3) |
| EMD | $\mathbf{M}_{\mathbf{D}}$ | Moment at bottom discontinuity point |
| EMFE | $m_{\xi}$ | Bending moment per unit length, meridional direction |
| EMFEP |  | First derivative of EMFE |
| EMF'T | $m_{\xi \theta}$ | Bending moment, shear |
| EMFTP |  | Fixst derivative of EMFT |
| EMEK |  | Spring value-moment at location SPRL, |
| EMN1 |  | EMIN, when read as data in GDA area |


| EMN3 |  | EM3N, when read as data in GDA area |
| :---: | :---: | :---: |
| EMN5 |  | EMSN, when read as data in GDA area |
| EMT | $\mathbf{m}_{\mathbf{T}}$ | Temperature moment, Equation 36, Section 1.7 |
| EMTHH | $\mathrm{m}_{\boldsymbol{\theta}}$ | Bending moment per unit length, circumferential direction |
| EMTHP |  | First derivative of EMTH |
| EMTP |  | First derivative of EMT |
| EN | N | Number of meridional points/region (150 maximum) |
| ENA1 |  | Number of thermal expansion coefficients given for first material ( 10 maximum) |
| ENA 2 |  | Number of thermal expansion coefficients given for second material ( 10 maximum) |
| ENA3 |  | Number of thermal expansion coefficients given for third material ( 10 maximum) |
| ENE1 |  | Number of Young's moduli given for first material ( 10 maximum) |
| ENE2 |  | Number of Young's moduli given for secend material ( 10 maximum) |
| ENE3 |  | Number of Young's moduli given for tiiird material ( 10 maximum) |
| ENF | $n$ | Current Fourier romponent |
| ENFO |  | Initial Fourier component |
| ENFE | $t_{\text {t }}$ | Fourier component for membrane force, meridional direction |
| ENEI |  | Subsequent Fourier components ( 10 maximum) |
| ENF¢R |  | Fourier component pxint vaiues (3 possible) |
| ENFT | $t_{\xi \theta}$ | Fourier coefficient for membrane shear force |


| ENMAT | Number of materials (3 maximum) |
| :---: | :---: |
| ENQGR | Number of gradient stations (10 maximum) |
| EN¢T | Number of st:'ions for temperatures given in TBOT and TTOP |
| EN¢TH | Number of stations for thicknesses given in TH (20 maximum) |
| ENT $t_{\text {t }}$ | Temperature load (nondimensional) Equation 35, Section 1.7 |
| ENTERP | Single, linear interpolation subroutine |
| ENTH | Number of theta values to use in Fourier summing for loads, also Fourier coefficient for membrane force, circumferential direction |
| E¢ | Outer modulus of elasticity/station/layer |
| EX | Constant data indicator. Use only when $S U M=0$. <br> 0 . = no constants, $-=$ all constants, $+1=$ section properties and teraperature loads constant, $+2=$ symmetrical pressure loads, no temperature loads. SDA(1) |
| $\underline{F}$ |  |
| F | 4-x-4 matrix, see (Equation 41) B \& R |
| FEFE $\phi_{\xi}$ | Fourier coefficient for rotation, meridional |
| FETH $\boldsymbol{\phi}_{\boldsymbol{\theta}}$ | Fourier coefficient for rotation, circumferential. |
| FETHP | First derivative of FETH |
| FI | 4-x-1 matrix, used to preserve $f$ matrix (Equation 51, Section 1.12) for bottom discontinuity point |
| G |  |
| G | 4-x-1 matrix g in Equation 66, Section 1.12 |
| G2 | 4-x-1 matrix, used to preserve $g$ matrix for station 2 , also used in stresses |


| GA |  | 4-x-4 matrix, Gin Equation 41, Section 1.8 |
| :---: | :---: | :---: |
| GAM |  | Current GAMA value |
| GAM2 |  | GAM ** 2 |
| GAMA | $\gamma$ | Geometry parameter at stations |
| GAMRX |  | Intermediate variable for sign check |
| GDA |  | Data area in GEOM subroutine/region |
| GECX |  | Eccentricity of reference surface at bottom discontinuity point |
| GEMK |  | Spring value - moment |
| GE $\mathbf{Q M}^{\prime}$ | : | Geometry subroutine |
| GE M $^{\text {I }}$ |  | GMI in SDA region |
| GI |  | 4-x-1 matrix, used to preserve g matrix (Equation 66, Section 1.12 for bottom discontinuity point |
| GMI |  | Geometry indicator. 1. = cone-cylinder, 2. = spheretoroid, 3. = general discrete points |
| GPSI | $\psi$ | Angle of inclination at discontinuity (degrees) |
| GQ |  | 4-x-1 matrix, used to preserve g matrix for station 2, see G2 |
| GR |  | Values of gradients at GSTA stations and internal interfaces |
| GS1 |  | 4-x-1 matrix, used to preserve $g$ matrix when $I=1$ at open boundary or discontinuity |
| GSN |  | 4-x-1 matyix, used to preserve g matrix when $I=N$ at open boundary or discontinuity |
| GSPRL |  | Location of spring (one per region) |
| GSTA |  | Temperature gradient stations (same for each interface) |



| IL | GEOM - DO inder for EFN 80 <br> DATLDS - TLOC subscript to pick up NDSTA for next ENF/load increment $=1$ |
| :---: | :---: |
| INA1, 2, 3 | Fixed point form of EriAl, 2, 3 |
| INC | Increment between temperature stations |
| INDC | Fixed point form of loads indicators |
| INE1, 2, 3 | Fixed point form of ENE1, 2, 3 |
| INTLD | Subroutine which computes deflections, internal loads and stres-es |
| INV | iNatrix inversion subroutine |
| IPRS | Fised point form of PILD, the pressure indicator |
| IRGN | Region number in argument list of GEOM subroutine |
| ISDA | SDA location for storing pressures, incremented by PN dimension |
| ISEC | Region DO loop index in PANDX subroutine |
| ITB | TAB subscript for pressure tables, incremented by PNTB dimension |
| ITBT | Fixed point form of TIBT, bottom surface temperature indicator |
| ITPF | Fixed point form of TITP, top surface temperature indicator |
| IX | DATLDS - subscript used to store loads in SDA data area PIX - DO luop index for region counter |
| IXL | Path indicator in INTLD for second interface stress calculations |
| IXX | Fixed point form of material indicator/layer |
| 12 | Subscript for Z solution; used to step backwards through a region |

J

JI
JKL
JR
JTD
JTDi
K
K
K0

K1
K2

KK

KKE

KLM
KP1
KS
KTPR Tape number 12 or 13 where SDA data is stored. On subsequent passes during Fourier summing the number are interchanged

Subscript of TAB used to pick up the number of stations in DATLDS


| NFE | Number of meridional stations for Fourier summing of the loads |
| :---: | :---: |
| NLAY | Fixed point form of ELAY, the number of layers/region |
| NLAY1 | Number of interfaces, ELAY + 1. |
| NMAT | Fixed point form of ENMAT, the number of materials |
| NN | DO loop upper level for RHOX calculation in GEOM |
| NQGR | Fixed point form of ENOGR, number of gradient stations |
| N¢STA | Number of stations where loads are given |
| N $\mathbf{T P}^{\text {P }}$ | Tape number for Fourier components. 3 = several ENF's, 8 = one ENF value |
| NS | DO loop index, region number counter, DATLNK at EFN 1 |
| NSM | Index; discrete point option |
| NTH | Fixed point forms of ENTH, number of thetas to sum |
| NTPR | Tape number 9 or 10 used during Fourier summing to |
| NTPW | store sums/region/theta |
| NX | Temporary save location when interchanging NTPR and NTPW |
| $\Phi$ |  |
| фPEXQ | Optional error exit subroutine. Used to take square root of the absolute value of a negative argument in GEOM |
| P |  |
| P | Three-dimensional array (Equations 4, 4, 150) used to store the $P$ matrices (Equation 74) at each meridional station/ region |
| P0 | 4-x-4 matrix for $P$ at the ( $\mathrm{N}-1$ )st station of previous region |
| PANDX | Subroutine for generating the $\mathbf{P}$ and X matrices of Equation 74 |

## PD $\quad \mathbf{P}_{\mathbf{D}}$ <br> PFE $\quad \mathbf{P}_{\boldsymbol{\xi}}$

PFETB
PFEX
PFLAG
PHIO
PHIN
PI
PILD

PIX
PIXI
PM

PN $\quad \mathbf{P}$
PNTB Data table of PN values, DLD data area
PNX Table location for PN values for next Fourier component
P $\phi \quad \nu \quad$ Poisson's ratio for the inner layer
PфI2
PфIS
PфIX
PR $\boldsymbol{\phi}$

PSI $\psi \quad$ Discontinuity angle at the bottom of a region (degrees)
PSIO $\psi_{0}$
Pressure at a point of discontinuity
Fourier component for load in the meridional direction
Data table of PFE values, DLD data area
Table location for PFE values for next Fourier component Print indicator

Initial opening angle from vertical axis for sphere or toroid Final opening angle from vertical axis for sphere or toroid

Determinant value, in argument list of INV subroutine
Pressure indicator for type of data in tables, see explanation of DLD data

CRT subroutine
CRT indicator; plots curves when nonzero
COMMON location for preserving material properties data Fourier component for load in the normal direction

Poisson's ratio for the second stress layer
Poisson's ratio/layer in DAL data area
Temporary storage location for POI in INTLD at EFN 1002
Intermediate variable for a sign check in GAMA computation of discrete point option

Angle at which load is applied at discontinuity point (degrees)

| PSIM |  | 4-x-1 matrix, moment at discontinuity point |
| :---: | :---: | :---: |
| FsIP |  | 4-x-1 matrix, pressures at discontinuity point |
| PTH | $P_{\theta}$ | Fourier component for load in the circumferential direction |
| .PTHI |  | Path indicator for multiple case jobs |
| P THTB |  | Data table of PTH values, DLD data area |
| P THX |  | Table location for PTH values for next Fourier component |
| $\underline{Q}$ |  |  |
| QFE | $\hat{f}_{\xi}$ | Transverse force per unit length in meridional direction |
| QTH | $\hat{\mathbf{f}}_{\boldsymbol{\theta}}$ | Transverse force per unit length in circumferential direction |
| $\underline{R}$ |  |  |
| R | $\mathbf{r}$ | Normal distance from shell to axis |
| RA1 |  | Radius of cone or cylinder at station 1 |
| RC |  | Radius of curvature of sphere or toroid |
| RCRV |  | Interpolated station values of meridional radius of curvature |
| RCRZ |  | Interpolated station values of circumferential radius of curvature |
| RCURV |  | Input values of meridional radius of curvature |
| RCURZ |  | Input values of circumferential radius of curvature |
| RFF |  | Standard form coordinate of conics offset from axis of revolution |
| RH $\boldsymbol{\phi}$ |  | Current RHOX value set for each station in PANDX |
| RH中P |  | First derivative of RHO |
| RHOX | $\rho$ | R/A0 |

RIPT Discrete radii for general shell shape, GDA data area
RJ

RR Intermediate radius designation at stations for smoothing in discrete points

RRJ Intermediate radius designation for smoothing in discrete points

S
S

S1
S2
S3

S4
S5

DATLYR: 310, 348 PANDX: 100, 410 INTLD: 545, 575

| S12 | DATLYR: 305, 347 PANDX: 430 |
| :---: | :---: |
| S13 | DATLYR: 305, 348 PANDX: 430 INTLD: 552, 559 |
| S15 | PANDX: 100, 140 INTLD: 548, 549 |
| S16 | DATLYR: 348 |
| S17 | DATLYR: 347 |
| S18 | DATLYR: 348 |
| S20 | INTLD: 490 |
| S21 | INTLD: 553 |
| S22 | INTLD: 553 |
| S91 | DATLYR: 351 |
| 592 | DATLYR: 351 |
| S93 | DATLYR: 351 |
| S94 | DATLYR: 351 |
| S101 | DATLYR: 360 |
| S102 | DATLYR: 360 |
| S103 | DATLYR: 360 |
| S104 | DATLYR: 360 |
| S105 | DATLYR: 360 |
| S106 | DATLYR: 360 |
| S107 | DATLYR: 360 |
| S108 | DATLYR: 360 |
| SARB | Discrete point option in GEOM |
| SDA | Regional data axea, all parameters used in PANDX, |


| SGFE2 | $\sigma_{\xi 2}$ | Meridional stresses for chosen second layer |
| :---: | :---: | :---: |
| SGFT2 | $\sigma_{\xi 6}$ : | Shear stresses for chosen second layer |
| SHTH2 | ${ }_{02}$ | Circumfereitial stresses for chosen surface |
| SLGO | $\sigma_{0}$ | Reference stress (psi) |
| SIGFE | $\sigma_{\xi}$ | Meridional stresses for inner surface |
| SIGFT | $\sigma_{\xi \theta}$ | Shear stresses for inner surface |
| SIGTH | $\sigma_{\theta}$ | Circumferential stresses for inner surface |
| SINFI |  | SIN (ANX), used to computed R in cone-cylinder, GEOM |
| SINNT |  | SIN ( $\mathrm{n}_{\boldsymbol{\theta}}$ ), used in Fourier summing |
| SKIP |  | Path indicator, = 1. for fictitious discontinuity, PANDX |
| SL1 |  | Paih indicator, $=0$, for printing when SUM=0. |
| SL2 |  | Path indicator, =0, after the first pass when sumining |
| SPN0 |  | Initial opening angle of conics (station 1) |
| SPNN |  | Terminal opening angle of conice (station $n$ ) |
| SPRL |  | Location of spring, SDA. (3) |
| SQ3 |  | Summing coefficient A0 * SIGO/E0 |
| SQ4 |  | Summing coeifficient SIGO $\%$ H0 ** $3 / 1.0$ |
| SQ6 |  | Summing coefficient SIGO* HO |
| STA |  | DATLDS, stations at which loads are given in loads tables; DATLYR, combined TSTA and THSTA |
| STAP |  | Temporary array stations of apex interpolation in discrete point option |
| STAW |  | Temporary array stations of apex interpolation in discrete point option |


| STAX | Array of meridional stations, PIX |
| :---: | :---: |
| STC ${ }^{\text {M }}$ | Subroutine to connbine thickness and temperat re stations |
| STN | Temperature loads station numbers |
| STRI | Layer number for second stress print (other = inner surface) |
| STRIX | STRI in DAL data area |
| SUM | Indicator, nonzero for multiple Fourier components <br> $+=$ summing, with prints at ENFOR values <br> - = discrete Fourier values, printed each time, no CRT |
| SUMN | Auxil'ary array for summing current Fourier components with previous sums |
| SUMS | Subroutine which sums Fourier components and prints results |
| SURB | Arc length in intermediate arc length computation in discrete point option |
| SURF | Arc length to station location |
| SURN | Arc length in intermediate arc length computation in conics option |
| T |  |
| T | Temperature change for N summations, SDA (1226) |
| T2 | T for second surface where stresses are computed, SDA (2799) |
| TAB | Data tables for pressure and temperature loads |
| TBфT | Loads table for the temperatures on the inner surface |
| $\text { TB } \phi \mathbf{T X}$ | Table location for TBOT values for next Fourier component |
| TBT | Temperatures for inner surface at STN stations, DATLDS |
| TEMP | NFE-x-NTYi loads array resulting from double interpolation of data |


| TH |  | Thicknesses at stations, layers |
| :---: | :---: | :---: |
| THETA | $\theta$ | Circumferential angles, ten maximum (degrees) |
| THETX |  | Circumferential angles for summing loads |
| THEX |  | Theta value read from tape, PIX |
| THK |  | Thicknesses at combined stations (STA)/layer |
| THSTA |  | Statior numbers at which thicknesses are given |
| TIBT |  | Temperature indicator, inner face (see explanation for DLD data) |
| TITP |  | Temperature indicator, outer race (see explanation for DLD data) |
| TLゆC |  | Table locations, PFEX, PTHX, PNX, TBOTX, TTOPX |
| TMP |  | Temperatures at combined stations (STA)/layer |
| TMPA1 |  | Temperatures $a^{+}$which thermal expansion coefficients are given, material 1 |
| TMPA2 |  | Temperatures at which thermal expansion coefficients are given, material 2 |
| TMPA3 |  | Temperatures ait which thermal expansion coefficients are given, material 3 |
| TMPE1 |  | Temperatures at which Young's moduli are given, material l |
| TMPE2 |  | Temperatures at which Young's moduli are given, material 2 |
| TMPE3 |  | Temperatures at which Young's moduli are given, material 3 |
| TSTA |  | Station numbers at which temperatures are given |
| TT¢P |  | Loads table for the temperatures on the outer surface |
| TTфPX |  | Table location for TTOP values for next Fourier component |
| TTP |  | Temperatures for outer suriace at STN stations; DATLDS |

## $\underline{\mathbf{U}}$

UK

VAL
VK
VP
VSUM
W
WF

WFE $\quad \omega_{\xi}$
WFEN WFE at last point, previous region
WFEP First derivative of WFE
WFP Intermediate designation of meridional curvatures in the discrete point option

WFW Intermediate designation of meridional curvatures in the discrete point option

WK
WP
WSUM Array for summing $W$ deflections
WT Intermediate designation circumferential curvatures in discrete points for amoothing

| W TH | Current W THD value |
| :---: | :---: |
| WTHD | Nondimensional curvature in the circumferential direction |
| $\underline{x}$ |  |
| X | Two-dimensional array (Equations 4, 150) used to store the $X$ matrices (Equation 74) at each meridional station/region |
| X0 | 4-x-1 matrix for $X$ at the ( $\mathrm{N}-1$ ) st station of the previous region |
| XD | 4-x-4 matrix used at top discontinuity point |
| XIPT | Discrete X distances, GEOM or arc lengths |
| XJ | Intermediate $\mathbf{X}$ distances for better curve fitting of discrete points |
| XSI | X distance array used with R's to plot shell shape/region |
| $\underline{Y}$ |  |
| YD | 4-x-4 matrix used at top discontinuity point |
| YM1 | Young's moduli data, entered for TMPEl temperatures, first material |
| YM2 | Young's moduli data, entered for TMPE2 temperatures, second material |
| YM3 | Young's moduli data, entered for TMPE3 temperatures, third material |
| YMX | Constant Young's modulus when there are no temperature loads |
| $\underline{2}$ |  |
| Z | Two dimensional array (Equations 4, 150) of solutions (Equation 73) |
| z0 | 4-x-1 matrix for $Z$ at the ( $\mathrm{N}-1$ )st station of the previous region |

Z1M1 Solution matrix for fictitious point before station 1
ZETA Intermediate cylindrical coordinates in conics option
ZNP 1
ZTA Solution matrix for fictitious point after station $\mathbf{N}$ Station interpolated cylindrical coordinates in conics option

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[^0]:    *Special condition when shell has a plane of symmetry about the normal to the axis of revolution. Use only for axisymmetric loads ie.g., complete sphere can be treated as hemisphere).

[^1]:    *The constant data indicator for Ex 0 can be ueed oaly when SIM $=0$. When SUM $\ddagger 0$, EX must be set equal to sero and mommal input format following.

[^2]:    "品 0
    0
    $i$
    $i$
    0
    0
    0
    0
    0
    8
    $c$
    0
    0
    0
    0

[^3]:    Output 1'in!

[^4]:    Sample Problem Output (Cont)

[^5]:    Set Up P and X Matrices (Cont)

[^6]:    DO Loop Subroutine (Cont)

