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AN ANALYSIS OF THE SIDE-LEAKAGE EFFECT IN HIGH-SPEED GAS LUBRICATED SLIDER AND PARTIAL ARC BEARINGS

H. G. Elrod, Jr., et al

The Franklin Institute Research Laboratories
Philadelphia, Pennsylvania

August 1967

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Technical Report

Interim Report
I-A2049-29

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H. G. Elrod, Jr.
J. T. McCabe

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BENJAMIN FRANKLIN PARKWAY • PHILADELPHIA, PENNA. 19103

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ABSTRACT

The side leakage effect in a high speed gas lubricated bearing is treated by two approximate methods. The first is a linearized "ph" solution of the asymptotic differential equation. The second is a Karman-Cohlhausen type boundary layer analysis. Both methods are applied to the flat slides and the results are shown to favorably agree with those of an accurate finite difference solution.

The asymptotic approach is recommended for high speed (high Λ) cases rather then the finite difference method because as Λ increases the accuracy of the asymptotic solution increases while the accuracy of the finite difference solution degenerates.

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NOMENCLATURE

G	=	film-shape function, defined in Eq. 29.
h	=	local film thickness; h_1, at leading edge; h_2, at trailing edge
H	=	h/h_1
L	=	length of bearing in direction of motion
L_D	=	load deficiency due to leakage on one side
L_∞	=	load for $\Lambda \to \infty$
$\dot{m}$	=	mass flow per unit film width
p	=	fluid pressure; p_a, ambient; p_∞, corresponding to $\Lambda \to \infty$
s	=	Laplace transform parameter
T_D	=	Deficiency moment, defined in Eq. 27
V	=	surface velocity
W	=	width of bearing, transverse to motion
x	=	distance from leading edge
z	=	distance from one side of bearing
α	=	$(L/h_1)(dh/dx)$ for flat slider
β	=	$\zeta\sqrt{\alpha}$
δ^*	=	penetration thickness, defined in Eq. 17
ε	=	amplitude parameter, defined in Eq. 29
ζ	=	$\sqrt{\Lambda}\ \frac{z}{L}$
Λ	=	Compressibility No. = $6\mu VL/p_a h_1^2$
μ	=	absolute fluid viscosity

NOMENCLATURE (CONT.)

ξ	=	x/L
ρ	=	fluid density
τ	=	dummy variable corresponding to ξ
ψ	=	$\frac{ph}{p_a h_1}$; ψ', disturbance value of ψ

1. INTRODUCTION

When operated at high speed, gas-lubricated bearings manifest boundary-layer effects at their exposed film edges similar to such effects observed in fluid mechanics and heat transfer. Two previous reports from The Franklin Institute Laboratories have dealt with various aspects of this topic. The first of these (ref. 1) in 1962 considered the side-leakage effect in high-speed cylindrical journal bearings. In this report it was shown that the extent of axial side-leakage penetration into the bearing interior varies inversely as the square root of the rotational speed. The interior of the bearing operates as an "infinite bearing" with the pressure-film-thickness product, ph, determined by the mass content rule. In the second report (ref. 2) of 1964 the trailing-edge effect in high-speed slider and partial-arc bearings was analyzed. It was there proved that the "ph" product is equal to the leading edge value except near the trailing edge in a transition region having a thickness varying inversely as the speed. The preliminary analysis of this region by Gross and Zachmanoglou (ref. 3) was confirmed and made rigorous. Side leakage effects were not treated. In a recent report DiPrima (Ref. 4) examined the coupled effects of high speed and high squeeze-frequency. Where his results overlap those in Refs. 1 and 2, they agree.

The purpose of the present work is to provide a complementary analysis for the side edge effects in slider and partial arc bearings. Actually, because the side effects vary in magnitude accordingly to the reciprocal square root of the speed, these effects usually dominate over those occurring at the trailing edge. Specific numerical results are given here for the flat slider bearing. The value of the analysis is three fold. First, it provides insight concerning the physical phenomena in the bearing. Second, it provides specific results applicable to bearing design.

Third, it provides an independent means of arriving at results heretofore obtained only by finite-difference digital computer calculations.

The side leakage effect is analysed by two different approximate methods. In the first of these the asymptotic differential equation for "ph" is linearized. In the second method, a Karman-Pohlhausen type of boundary layer analysis is employed. General results are obtained by both methods, and then applied to the flat slider. The theoretical flat-slider results are inter-compared, and then compared with several accurate finite-difference calculations. All comparisons show good agreement.

2. ASYMPTOTIC EDGE ANALYSIS

Figure 1 is a plan view of a slider or partial-arc bearing. It defines the coordinates to be used, and shows that along each side of the bearing a layer develops in which the effects of side leakage and reduction in load-carrying capacity are present. The interior of the bearing film is unaffected. There is a striking parallel between this situation and the development of a boundary-layer over a flat plate (Blasius problem).

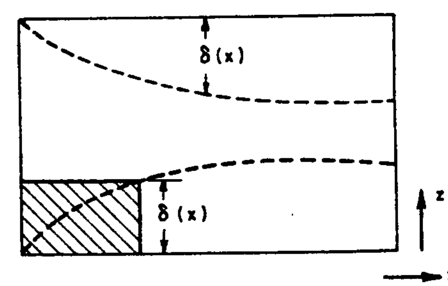

Figure 1 - Plan View of Slider Bearing Showing Areas Affected by Side Leakage

Let us begin our analysis with the following well-known equation for the mass flow per unit film width in a bearing:

$$\dot{m}_i = \frac{\rho V_i h}{2} - \frac{h^3}{12\mu} \rho \frac{\partial p}{\partial x_i} \quad . \qquad (1)$$

Here V_i is the component of the surface speed in direction x_i. The remaining notation is standard and defined in the table of Nomenclature.

Consider for a moment how the terms in this equation may inter-relate as the speed V_1 is allowed to approach infinity. Presume temporarily that the pressure gradient and the fluid density remain bounded under such circumstances. Equation (1) then reduces to Eq. (2). Thus:

$$\dot{m}_1 = \frac{\rho V_1 h}{2} \quad , \tag{2}$$

Or:

$$\rho h = \frac{2\dot{m}_1}{V_1} \quad . \tag{3}$$

Since the mass flux $\dot{m}_1$, is continuous and the film thickness, h, is a well-behaved function of position, the density, ρ, is also well-behaved. In the case of a perfect gas, the pressure in an isothermal film must then also be well-behaved. Now these findings are entirely consistent with the original temporary hypothesis, so it can be anticipated that there is a tendency with a gas for the Couette portion of the flow to dominate the pressure induced portion as $V_1 \to \infty$.

Let us now make a mass balance on a small rectangular section of bearing film with $V = V_x$ large and $V_z = 0$ (see Figure 2).

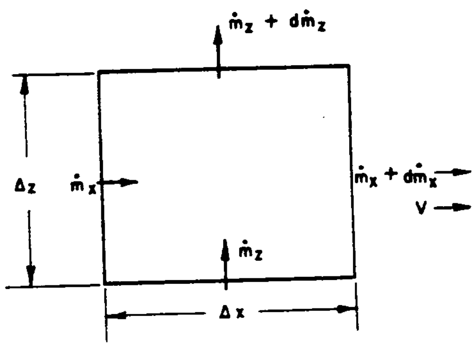

Figure 2 - Elementary Mass Balance for Deriving Edge-Effect Differential Equation

$$\frac{\partial m_z}{\partial z} + \frac{\partial m_x}{\partial x} = 0 \ . \tag{4}$$

Therefore:

$$\frac{\partial}{\partial z}\left(-\frac{h^3}{12\mu}\rho\frac{\partial p}{\partial z}\right) + \frac{\partial}{\partial x}\left(\rho\frac{V h}{z}\right) = 0 \ , \tag{5}$$

Or:

$$\frac{\partial}{\partial z}\left(ph^3\frac{\partial p}{\partial z}\right) = 6\mu V\frac{\partial}{\partial x}(ph) \quad , \tag{6}$$

for isothermal gas flow. This is the differential equation used in ref. 1, but derived there from mathematical, rather than physical, considerations.

$$\left.\begin{aligned} &\text{With } H \equiv h/h_1,\ \xi \equiv \frac{x}{L},\ \Lambda \equiv \frac{6\,VL}{p_a h_1^2}\ , \\ &\psi \equiv \frac{ph}{p_a h_1}\ ,\quad \zeta \equiv \sqrt{\Lambda}\ \frac{z}{L}, \end{aligned}\right\} \tag{7}$$

Equation (6) becomes:

$$\frac{\partial\psi}{\partial\xi} = \frac{\partial}{\partial\zeta}\left(H\,\psi\,\frac{\partial\psi}{\partial\zeta}\right) \ . \tag{8}$$

$$\left.\begin{aligned} &\text{Boundary conditions at the film edges for } \psi\,(\xi,\zeta) \text{ are:} \\ &\qquad \psi\,(0,\zeta) = 1\ , \\ &\qquad \psi\,(\xi,0) = H\ . \end{aligned}\right\} \tag{9}$$

In the interior of the film (i.e., remote from the sides) the mass flux $\dot{m}_x$ must be constant at its Couette value. Therefore:

$$\left.\begin{aligned} &\rho\,\frac{hV}{2} = \text{constant}, \\ \text{or:}\quad &p_\infty h = p_a h_1\ , \\ \text{That is:}\quad &\psi \to 1 \text{ as } \zeta \to \infty \end{aligned}\right\} \tag{10}$$

In the case of flat slider bearings, a universal load deficiency curve can be constructed. For such bearings,

$$\frac{h}{h_1} = H = 1 - \alpha \frac{x}{L} = 1 - \alpha\xi \ , \tag{11}$$

Therefore:

$$dH = -\alpha d\xi \ . \tag{12}$$

Equation (8) becomes:

$$-\alpha \frac{\partial\psi}{\partial H} = H \frac{\partial}{\partial\zeta}\left(\psi \frac{\partial\psi}{\partial\zeta}\right) . \tag{13}$$

Now let $\beta \equiv \xi\sqrt{\alpha} = \frac{z}{L}\sqrt{\alpha\Lambda}$, (14)

so that:

$$-\frac{1}{H} \frac{\partial\psi}{\partial H} = \frac{\partial}{\partial\beta}\left(\psi \frac{\partial\psi}{\partial\beta}\right) . \tag{15}$$

The boundary conditions for $\psi(H,\beta)$ are:

$$\left.\begin{aligned} &\psi(1,\beta) = 1 \ , \\ &\psi(H,0) = H \ , \\ &\psi(H,\beta) \to 1 \text{ as } \beta \to \infty \ . \end{aligned}\right\} \tag{16}$$

No free parameters appear in the problem.

The support pressure at points unaffected by side leakage is $p_\infty - P_a$. The loss in support at affected points is $p_\infty - p$. A measure of the leakage penetration is, therefore:

$$\delta^* \equiv \int_0^{w/2} \frac{(p_\infty - p)\, dz}{p_\infty - p_a} \ , \tag{17}$$

$$= \frac{L}{\sqrt{\alpha\Lambda}} \int_0^\infty \frac{(p_\infty h - ph)\, d\beta}{p_\infty h - p_a h} \ , \tag{18}$$

$$= \frac{L}{\sqrt{\alpha\Lambda}} \int_0^\infty \frac{(1 - \psi)\, d\beta}{(1 - H)} \ , \tag{19}$$

therefore:

$$\frac{\delta^*}{L} \sqrt{\alpha\Lambda} = \int_0^\infty \frac{(1 - \psi)\, d\beta}{1 - H} \tag{20}$$

Integration removes any dependence on "β", so that $\sqrt{\alpha\Lambda}\,(\delta^*/L)$ is a universal function of "H" for all high-speed flat slider bearings, as shown in Figure 3.

The total loss in load supports is given by:

$$L_D = \int_0^L \int_{Z=0}^{w/2} (p_\infty - p)\, dx dz \ , \tag{21}$$

Or:

$$L_D = \int_0^L (p_\infty - p_a)\, \delta^* dx \tag{22}$$

$$= \int_0^1 \frac{(p_\infty h - p_a h)}{h}\, \delta^* d\xi \ , \tag{23}$$

$$= -L\, p_a h_1 \int_1^{H_2} \frac{(1 - H)}{h}\, \delta^* \frac{dH}{\alpha} \ , \tag{24}$$

$$L_D = \frac{1}{\alpha}\left(\frac{L^2 p_a}{\sqrt{\alpha\Lambda}}\right) \int_{H_2}^1 \frac{(1 - H)}{H}\, \frac{\delta^*}{L} \sqrt{\alpha\Lambda}\, dH \tag{25}$$

Or:

$$\frac{L_D \sqrt{\Lambda}}{p_a L^2} = f_1(H_2) \ ; \ H_2 = h_2/h_1 \ . \tag{26}$$

This universal curve is presented in Figure 4.

In a like way, the deficiency of moment,

$$T_D = \int_0^L \int_0^{w/2} (p_\infty - p)\, x\, dz dx \ , \tag{27}$$

can be shown to be given by:

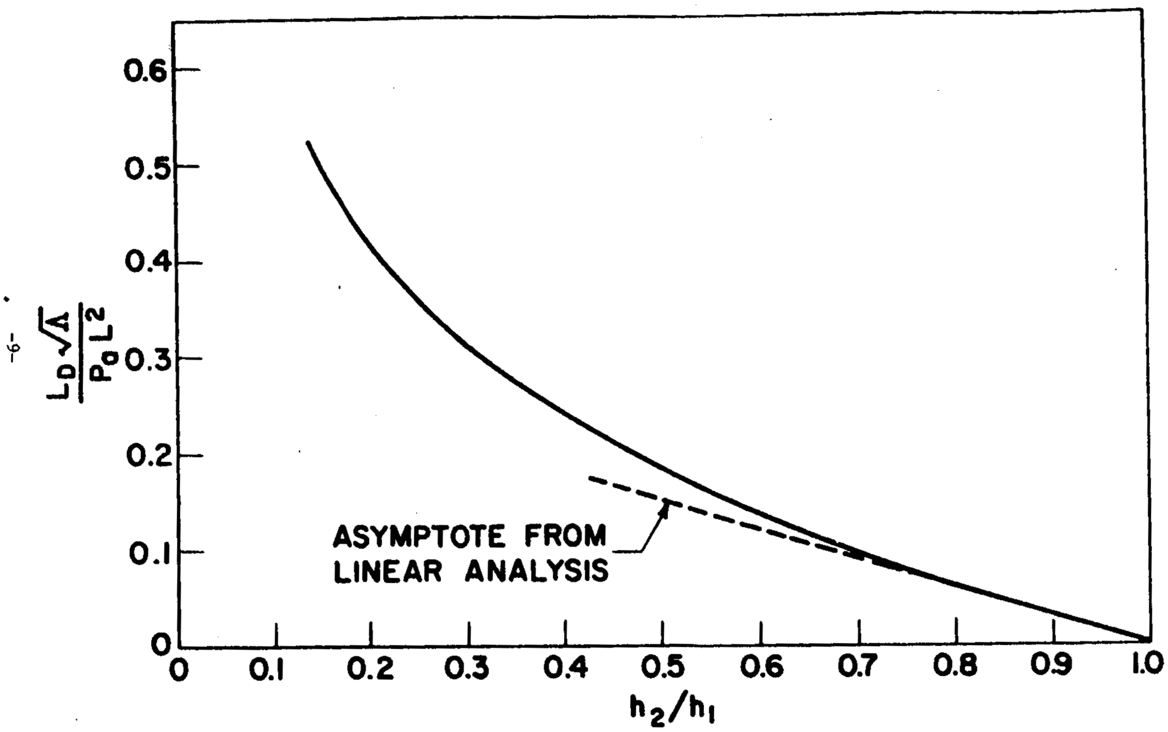

Fig. 4 - Load Deficiency Due to Slide Leakage - Flat Slider
(one side only)

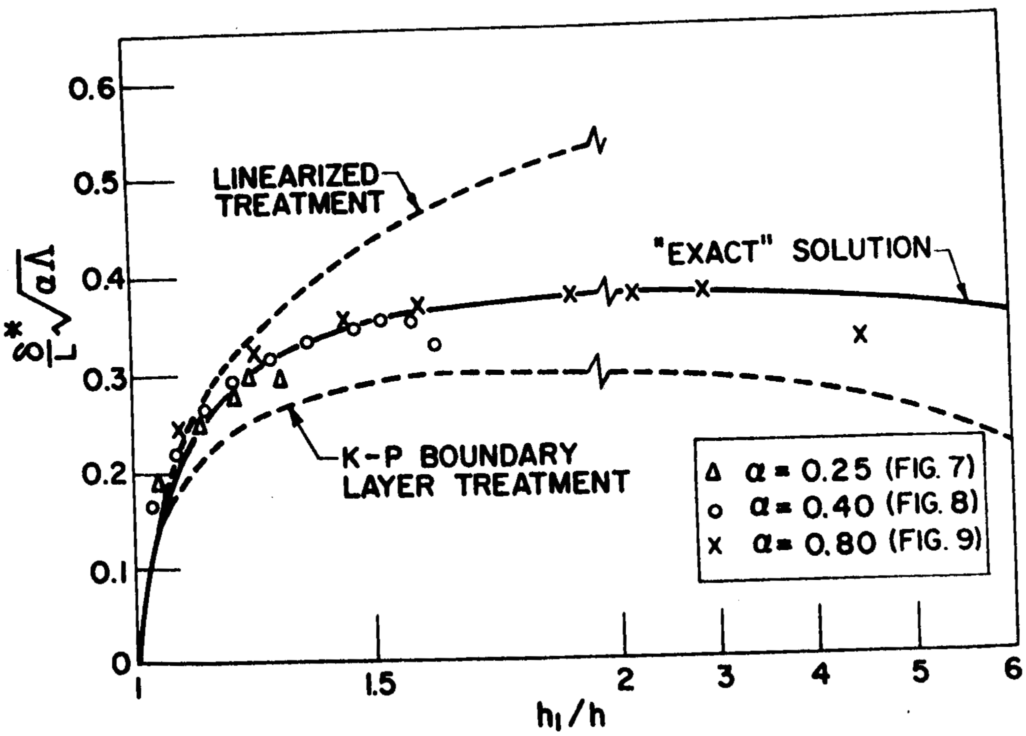

Fig. 3 - Leakage Penetration for Flat Slider

$$\frac{\alpha^2 \sqrt{\alpha\Lambda}\; T_D}{p_a L^3} = f_2(H_2) \qquad (28)$$

The function $\frac{T_D \sqrt{\Lambda}}{p_a L^3}$ is shown in Figure 5.

The functions H, 1/H, $\sqrt{\alpha\Lambda}$ δ^*/L, $\frac{\sqrt{\Lambda}\, I_D}{p_a L^2}$, and $\frac{\sqrt{\Lambda}\, T_D}{p_a L^3}$ are also tabulated in Appendix I.

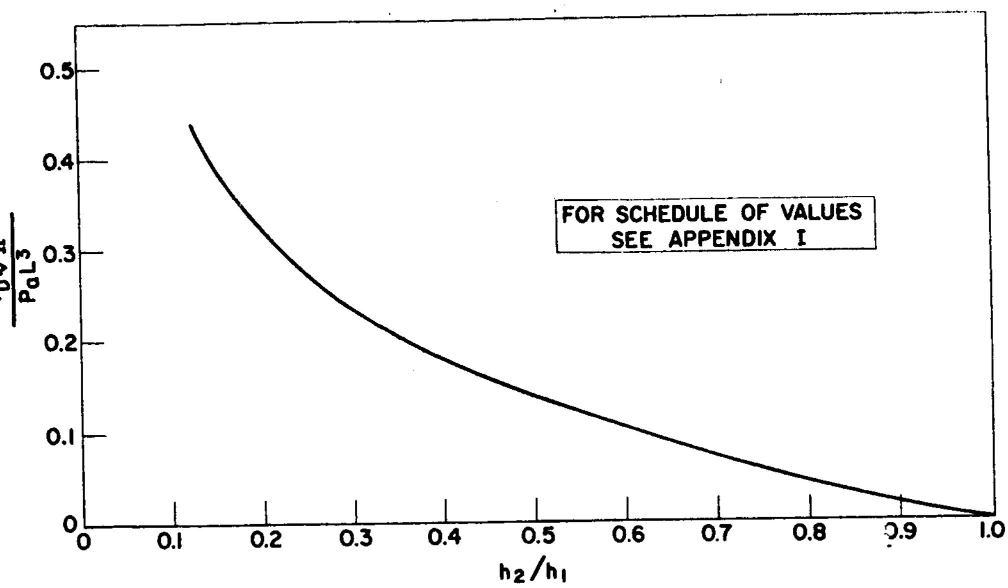

Fig. 5 - Moment Deficiency Due to Side Leakage - Flat Slider (one side only)

3. LINEARIZED ANALYSIS

An analytical solution can be found for slider bearings with film shape:

$$H = 1 - \varepsilon\, G(\xi);\ \varepsilon \ll 1 \tag{29}$$

where $G(\xi)$, the shape function is "smooth" with $G(0) = 0$. The solution is found from the perturbation expansion:

$$\psi = 1 - \varepsilon\, \psi_1(\xi,\zeta) - \varepsilon^2 \psi_2(\xi,\zeta) \ldots\ldots \tag{30}$$

To first order in "ε", Eq.(8) becomes:

$$\frac{\partial \psi_1}{\partial \xi} = \frac{\partial^2 \psi_1}{\partial \zeta^2}\,, \tag{31}$$

and the boundary conditions are:

$$\left.\begin{aligned} &\psi_1\,(0,\zeta) = 0\,, \\ &\psi_1\,(\xi,0) = G(\xi)\,, \\ &\psi_1\,(\xi,\zeta) \to 0 \text{ as } \zeta \to \infty\,. \end{aligned}\right\} \tag{32}$$

The above problem is readily solved by the Laplace transform method. Thus, the transform of Eq. 31, with "s" as parameter, is:

$$s\,\bar{\psi}_1 = \frac{d^2\bar{\psi}_1}{d\zeta^2}\,. \tag{33}$$

A solution of Eq. (33) vanishing at infinity is:

$$\bar{\psi}_1 = f(s)\ \varepsilon^{-\zeta\sqrt{s}} \tag{34}$$

Along $\zeta = 0$, $\bar{\psi}_1$ must become the transform of $G(\xi)$.

Therefore: $$\bar{\psi}_1 = \overline{G(s)}\ \varepsilon^{-\zeta\sqrt{s}}\,. \tag{35}$$

Now the inverse of $\varepsilon^{-\zeta\sqrt{s}}$ is:

$$\mathcal{L}^{-1}(\varepsilon^{-\zeta\sqrt{s}}) = \frac{1}{2\sqrt{\pi}} \frac{\zeta}{\xi^{3/2}} \varepsilon^{-\frac{\zeta^2}{4\xi}} \quad \text{for all } \zeta > 0. \tag{36}$$

Consequently, ψ_1 can be written as the following convolution integral:

$$\psi_1(\xi,\zeta) = \int_{\tau=0}^{\xi} G(\xi - \tau) \frac{1}{2\sqrt{\pi}} \frac{\zeta}{\tau^{3/2}} \varepsilon^{-\frac{\zeta^2}{4\tau}} \, d\tau. \tag{37}$$

Integration by parts yields:

$$\psi_1 = \int_{\tau=0}^{\xi} \operatorname{erfc}\left(\frac{\zeta}{2\sqrt{\tau}}\right) G'(\xi - \tau) \, d\tau \tag{38}$$

In this form, the expression for ψ_1 need not be restricted to $\zeta > 0$.

The leakage penetration is given by:

$$\delta^* = \frac{L}{\sqrt{\Lambda}} \int_0^\infty \frac{(1 - \psi)}{1 - H} \, d\zeta \quad , \tag{39}$$

where, since $1 - \psi = \varepsilon \, \psi_1$ to $0(\varepsilon)$,
and $1 - H = \varepsilon \, G(\xi)$,

it is found from Eq. (38) that:

$$\sqrt{\Lambda} \, \frac{\delta^*}{L} = \frac{2}{\sqrt{\pi}} \frac{1}{G(\xi)} \int_{\tau=0}^{\xi} \sqrt{\tau} \, G'(\xi - \tau) \, d\tau \ . \tag{40}$$

A second integration gives:

$$\frac{\sqrt{\Lambda} \, L_D}{\varepsilon \, p_a L^2} = \frac{2}{\sqrt{\pi}} \int_{\tau=0}^{1} G(\xi - \tau) \sqrt{\tau} \, d\tau \ . \tag{41}$$

For the specific case of a flat slider bearing, the foregoing expressions are easily evaluated. Thus:

$$\sqrt{\Lambda} \, \frac{\delta^*}{L} = \frac{4}{3\sqrt{\pi}} \sqrt{\xi} \ , \tag{42}$$

Or:

$$\sqrt{\varepsilon\Lambda} \, \frac{\delta^*}{L} = \frac{4}{3\sqrt{\pi}} \sqrt{1 - H} \ , \tag{43}$$

And:

$$\frac{\sqrt{\Lambda} \, L_D}{\varepsilon p_a L^2} = \frac{8}{15\sqrt{\pi}} \ , \tag{44}$$

Or:

$$\frac{\sqrt{\Lambda} \, L_D}{p_a L^2} = \frac{8}{15\sqrt{\pi}} (1 - H) \ . \tag{45}$$

The initial portion of the curve in Figure 4 is confirmed by Eq. (45).

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4. "KARMAN-POHLHAUSEN" ANALYSIS

It is instructive to employ a Karman-Pohlhausen type of approximate analysis to obtain expressions for the leakage penetration and load loss. Consider the "boundary-layer" $\delta(x)$ shown in Figure 6. The transverse pressure distribution at some station, x, must:

a) give $p\ (x, o) = p_a = \text{const.}$,
b) give $p\ (x, \delta) = p_\infty(x) = p_a h_1/h$,
c) provide "smoothness" at the junction $z = \delta$, here taken to mean $\frac{\partial p}{\partial z} = 0$ at $z = \delta$.

Further conditions might be imposed, producing theories of more accuracy, but the foregoing three conditions are taken to suffice for present purposes.

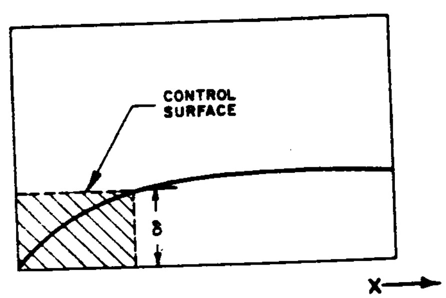

Figure 6 - Sketch Showing Control Volume Used to Develop K - P Equation

Therefore:

$$p = p_a + 2\,(p_\infty - p_a)\eta \;-\; (p_\infty - p_a)\,\eta^2 \tag{46}$$

where: $\eta = z/\delta$

Next let us write a mass balance for the cross-hatched region in Figure 6. Thus:

$$\frac{\rho_a h_1 \delta V}{2} = \int_0^{\delta} \frac{\rho h V}{2}\,dy + \int_0^{x} \frac{h^3}{12\mu}\,\rho_a\,\frac{\partial p}{\partial y}\,dx, \tag{47}$$

Or:

$$\frac{p_a h_1 V}{2}\,\delta = \int_0^{\delta} \frac{phV}{2}\,dy + \int_0^{x} \frac{h^3}{12\mu}\,p_a \left(\frac{\partial p}{\partial y}\right)_0 dy\ , \tag{48}$$

$$= \frac{Vh\delta}{2}\int_0^1 p\,d\xi + \frac{p_a}{12\mu}\int_0^x \frac{h^3}{\delta}\left(\frac{\partial p}{\partial \eta}\right)_0 dx\ , \tag{49}$$

Substitution of Eq. (46) into this equation gives:

$$\frac{p_a h_1 V\delta}{2} = \frac{Vh\delta}{2}\left[p_a + (p_\infty - p_a) - \frac{1}{3}(p_\infty - p_a)\right]$$

$$+ \left\{\int_0^x \frac{h^3}{\delta}\,2\,(p_\infty - p_a)\,dx\right\}\frac{p_a}{12\mu}\ . \tag{50}$$

Regrouping, and making use of Eq. (50), one gets:

$$\frac{hV\delta}{2}\left[\frac{1}{3}(p_\infty - p_a)\right] = \frac{2p_a}{12\mu}\int_0^x \frac{h^3}{\delta}(p_\infty - p_a)\,dx\ . \tag{51}$$

Now let us write: $h(p_\infty - p_a) = h_1 p_a - h p_a$

$$= p_a(h_1 - h) \tag{52}$$

and differentiate Eq. (51). Thus:

$$\frac{d}{dx}\left\{\delta V(h_1 - h)\right\} = \frac{p_a}{\mu}\,\frac{h^2}{\delta}\,(h_1 - h)\ , \tag{53}$$

or:

$$V(h_1 - h)\frac{d\delta}{dx} + \delta\,\frac{d}{dx}V(h_1 - h) = \frac{p_a}{\mu}\,\frac{h^2}{\delta}\,(h_1 - h)\ , \tag{54}$$

or:

$$\frac{1}{2}\frac{d\delta^2}{dx} + \delta^2\,\frac{d\left[\ln V(h_1 - h)\right]}{dx} = \frac{p_a h^2}{\mu V} \tag{55}$$

A solution for the homogeneous linear equation in "δ^2" is:

$$\omega = \frac{1}{V^2(h_1 - h)^2}\ . \tag{56}$$

With $\delta^2 = \omega v$ we find:

$$\frac{\omega}{2}\frac{dv}{dx} = \frac{p_a h^2}{\mu V}\ . \tag{57}$$

Since $\delta^2 = 0$ at $x = 0$, v must also vanish. Therefore:

$$v = \frac{2p_a}{\mu V}\int_0^x h^2V^2(h_1 - h)^2\,dx \tag{58}$$

and:

$$\delta^2 = \frac{1}{V^2(h_1 - h)^2}\left(\frac{2p_a}{\mu V}\right)\int_0^x h^2V^2(h_1 - h)^2\,dx\ . \tag{59}$$

Finally:

$$\left(\frac{\delta}{h_1}\right)^2 = \left(\frac{2p_a h_1}{\mu V}\right)\frac{\displaystyle\int_0^{x/h_1}\left(\frac{h}{h_1}\right)^2\left(1 - \frac{h}{h_1}\right)^2 d\left(\frac{x}{h_1}\right)}{\left(1 - \frac{h}{h_1}\right)^2} \tag{60}$$

The leakage penetration, δ^*, is:

$$\delta^* = \int_0^{W/2} \frac{(p_\infty - p)dz}{p_\infty - p_a} = \frac{1}{3}\delta \quad . \tag{61}$$

The total load deficiency, from entrance to exit is:

$$L_D = \int_0^L (p_\infty - p)\, \delta^*\, dx \quad , \tag{62}$$

Or:

$$L_D = \frac{p_a h_1^2}{3} \int_0^{L/h_1} \left(\frac{h_1}{h} - 1\right)\left(\frac{\delta}{h_1}\right) d\left(\frac{x}{h_1}\right) \tag{63}$$

For the flat slider bearing it is easy to apply Eqs. (60) and (63). The expression for δ^* becomes:

$$\sqrt{\alpha\Lambda}\,\frac{\delta^*}{L} = \frac{2}{\sqrt{3}}\sqrt{\theta\left(\frac{1}{3} - \frac{\theta}{2} + \frac{\theta^2}{5}\right)} \quad , \tag{64}$$

with $\theta \equiv 1 - h/h_1$,

and:

$$\frac{\sqrt{\Lambda}\, L_D}{p_a L^2} = \frac{\frac{2}{\sqrt{3}} \int^{\theta} \frac{\sqrt{\theta(\frac{1}{3} - \frac{\theta}{2} + \frac{\theta^2}{5})}\ d\theta}{1 - \theta}}{(1 - H)^{3/2}} \tag{65}$$

Both the penetration and load deficiency are in reasonable agreement with the earlier more accurate analyses. It is particularly worthy of note that the K-P method faithfully reproduces the non-linear result that the penetration attains a maximum.

5. COMPARISON WITH FINITE-DIFFERENCE RESULTS AND TYPICAL BEARING CALCULATION

To verify the foregoing analysis and to show its application, several cases of flat high-speed slider bearings were calculated by direct finite-difference solution of the two-dimensional Reynolds Equation. The pressure contours are presented in Figures 7, 8, and 9 as a demonstration of the effect being studied and to provide a permanent record of the results. Unusual care was taken to obtain good accuracy at high Λ by using a fine calculation grid, 54 x 34 for a bearing.

The correlations of penetration depth and load deficiency (Figs. 3 and 4) were checked using the above finite-difference data, with the good results shown. Accordingly, these correlations can be used with confidence for high-lambda flat sliders. A criterion for applicability of the correlations should be that the boundary-layer δ does not reach the line of bearing symmetry. Since $\delta^* \cong \frac{1}{3}\delta$, this condition becomes:

$$\sqrt{\alpha\Lambda}\ \frac{\delta^*}{L} < \frac{1}{6}\left(\frac{W}{L}\right)\sqrt{\left(1 - \frac{h_2}{h_1}\right)\Lambda}$$

For further comparison, there are the results of Gross and Michael (Ref. 4). Figure 10 presents a comparison between load coefficients obtained by their finite-difference technique and the present asymptotic results. Except at high Λ, the agreement is excellent. The discrepancies at high Λ are perhaps due to numerical difficulties in handling the excessively high pressure gradients in this range.

To illustrate use of the curves in Figures 3, 4 and 5, let us calculate the load coefficient and location of the center of pressure for a flat slider operating with:

$$\Lambda = 25$$
$$\frac{W}{L} = 1$$
$$h_2/h_1 = \frac{1}{2}$$

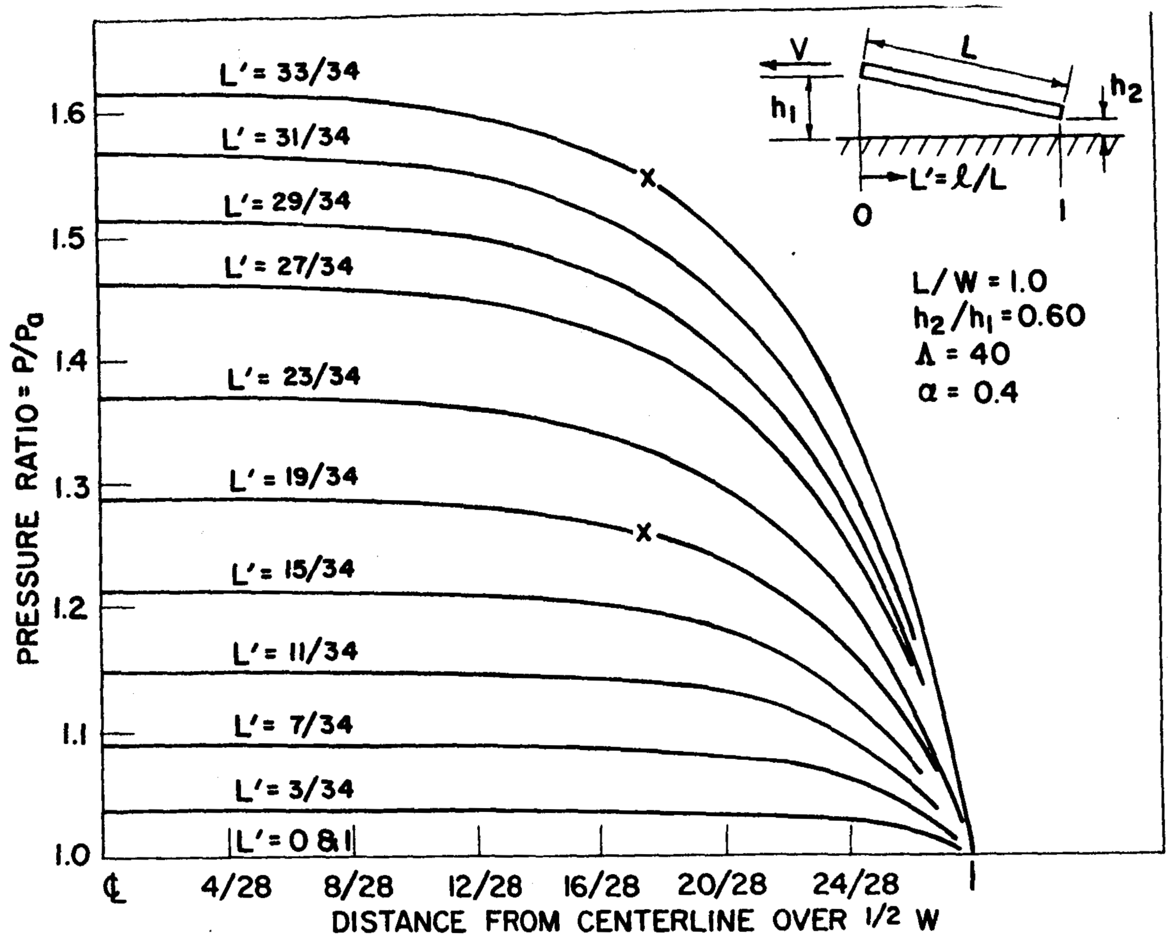

Fig. 8 - Pressure Profiles for Flat Slider Bearing, Finite Difference Solution for α = 0.4

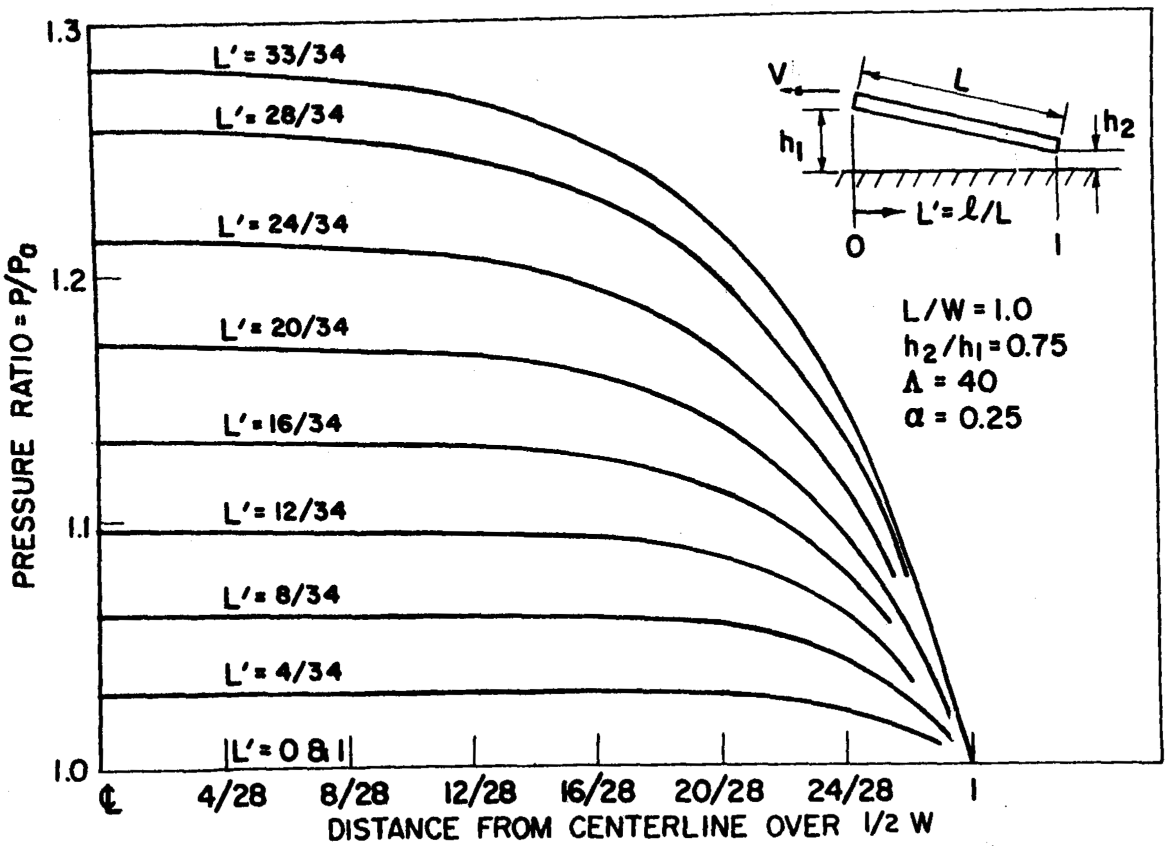

Fig. 7 - Pressure Profiles for Flat Slider Bearing, Finite Difference Solution for α = 0.25

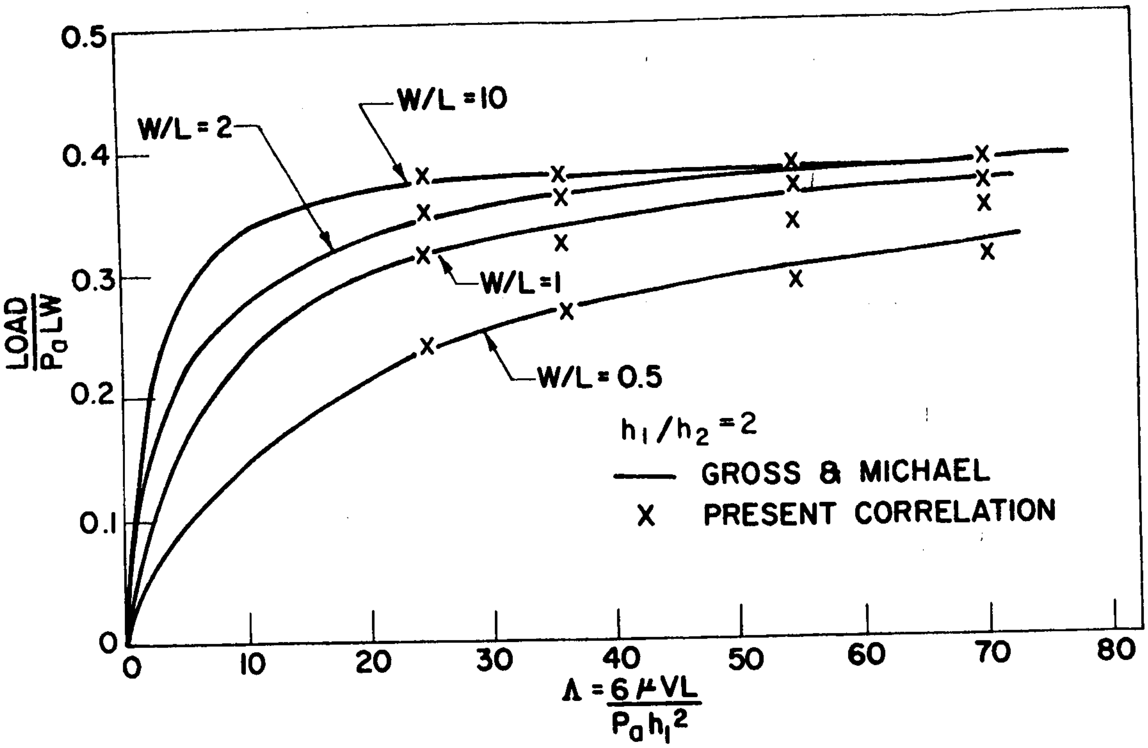

Fig. 10 - Comparison with Finite-Width Flat-Slider Results of Gross and Michael

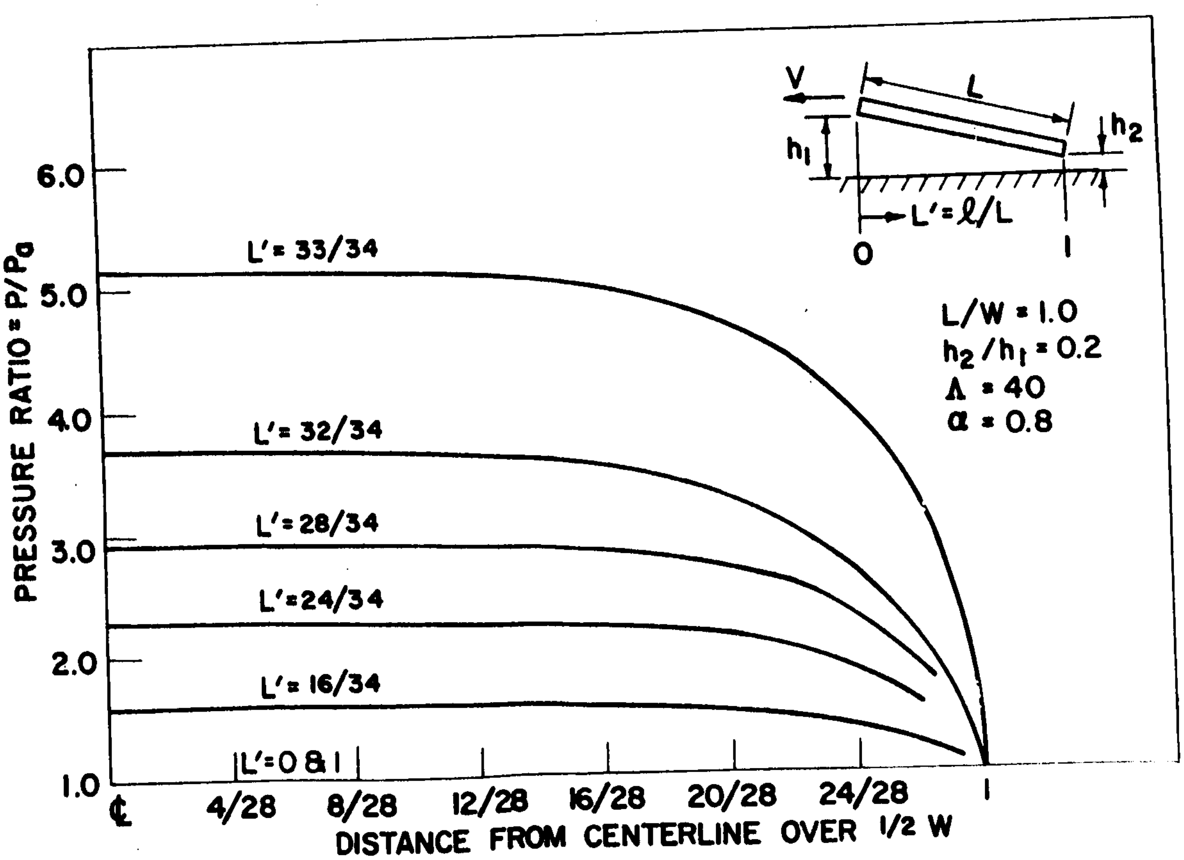

Fig. 9 - Pressure Profiles for Flat Slider Bearing, Finite Difference Solution for $\alpha = 0.8$

Let us first check on the applicability of the asymptotic formula. From Figure 3, with $h_1/h_2 = 2$ one finds:

$$\sqrt{\alpha\Lambda}\ \frac{\delta^*}{L} = 0.375 \ .$$

On the other hand,

$$\frac{1}{6}\ \frac{W}{L}\sqrt{\left(1 - \frac{h_2}{h_1}\right)\Lambda} = \frac{1}{6} * 1 * 5 * \sqrt{1 - 0.5.}$$

$$= 0.590$$

Since $0.375 < 0.590$, it is permissible to proceed.

According to Figure 4,

$$\frac{\sqrt{\Lambda}\ L_D}{p_a L^2} = 0.186$$

Now

$$\frac{L_\infty}{p_a WL} = \left(\frac{\ell n \frac{1}{H_2}}{1 - H_2} - 1\right)$$

Therefore:

$$\frac{\text{Load}}{p_a WL} = \left(\frac{\ell n \frac{1}{H_2}}{1 - H_2} - 1\right) - 2\left(\frac{\sqrt{\Lambda}\ L_D}{p_a L^2}\right)\ \frac{L}{W}\ \frac{1}{\sqrt{\Lambda}}$$

$$= \frac{\ell n\ 2}{1 - .5} - 1 - \frac{2 * .186 * 1}{5}$$

$$= 0.312$$

Next, according to Figure 5,

$$\frac{\sqrt{\Lambda}\ \ T_D}{p_a L^3} = 0.132$$

Now

$$\frac{T_\infty}{p_a WL^2} = \frac{L_\infty}{p_a WL(1 - H_2)} - \frac{1}{2}$$

$$\therefore\ \frac{T_D}{p_a WL^2} = \frac{\left\{\frac{\ell n \frac{1}{H_2}}{1 - H_2} - 1\right\}}{(1 - H_2)} - \frac{1}{2} - \frac{2}{\sqrt{\Lambda}}\ \frac{L}{W}\left(\frac{\sqrt{\Lambda}\ \ T_D}{p_a L^3}\right)$$

$$= 0.220$$

The location of the center of pressure, x_p, is given by:

$$\frac{x_p}{L} = \frac{0.220}{0.312} = 0.705$$

virtually unchanged grom the value for $\Lambda \rightarrow \infty$.

6. SUMMARY

The side-leakage effect in flat slider and partial-arc bearings has been analyzed for high Λ, with specific numerical results being given for the flat slider bearing. These results are presented in Figures 3 and 4. They have been confirmed by independent finite-difference calculations, and can be used to make rapid performance calculations for high-speed flat slider bearings of finite width.

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APPENDIX

COMPUTER PROGRAM AND NUMERICAL RESULTS

H	$\frac{1}{H}$	$\sqrt{a\Lambda}\ \frac{\delta^*}{L}$	$\frac{L_D\sqrt{\Lambda}}{P_a L^2}$	$\frac{T_D\sqrt{\Lambda}}{P_a L^3}$
.98994949-00	.10101525+01	.70710676-01	.35986079-02	.35986076-02
.97979590-00	.10206207+01	.10579728+00	.64017724-02	.51327547-02
.96953598-00	.10314212+01	.12887201-00	.94788295-02	.71546114-02
.95916630-00	.10425721+01	.14886425-00	.12642646-01	.93314255-02
.94868330-00	.10540925+01	.16579264-00	.15881652-01	.11597736-01
.93808316-00	.10660036+01	.18113207-00	.19186997-01	.13931312-01
.92736186-00	.10783277+01	.19490509-00	.22559300-01	.16324498-01
.91651514-00	.10910894+01	.20765226-00	.25998986-01	.18774217-01
.90553852-00	.11043153+01	.21939524-00	.29508557-01	.21280133-01
.89442720-00	.11180340+01	.23039462-00	.33090480-01	.23842964-01
.88317610-00	.11322770+01	.24066993-00	.36748060-01	.26464441-01
.87177979-00	.11470787+01	.25036605-00	.40484748-01	.29146662-01
.86023253-00	.11624764+01	.25950031-00	.44304561-01	.31892342-01
.84852815-00	.11785113+01	.26816032-00	.48211753-01	.34704503-01
.83666003-00	.11952286+01	.27636106-00	.52211090-01	.37586609-01
.82462113-00	.12126781+01	.28415866-00	.56307715-01	.40542500-01
.81240385-00	.12309149+01	.29156587-00	.60507306-01	.43576442-01
.80000001-00	.12500000+01	.29861999-00	.64816043-01	.46693139-01
.78740079-00	.12700013+01	.30533142-00	.69240723-01	.49897783-01
.77459668-00	.12909944+01	.31172564-00	.73788841-01	.53196157-01
.76157732-00	.13130643+01	.31781099-00	.78468648-01	.56594626-01
.74833149-00	.13363062+01	.32360475-00	.83289274-01	.60100274-01
.73484693-00	.13608276+01	.32911330-00	.88260865-01	.63720999-01
.72111027-00	.13867505+01	.33434831-00	.93394750-01	.67465650-01
.70710680-00	.14142135+01	.33931410-00	.98703568-01	.71344115-01
.69282034-00	.14433756+01	.34401792-00	.10420154+00	.75367564-01
.67823301-00	.14744195+01	.34846214-00	.10990474+00	.79548635-01
.66332497-00	.15075567+01	.35265053-00	.11583141+00	.83901753-01
.64807408-00	.15430335+01	.35658322-00	.12200235+00	.88443365-01
.63245555-00	.15811388+01	.36026067-00	.12844148-00	.93192467-01
.61644141-00	.16222142+01	.36368070-00	.13517644-00	.98171052-01
.60000002-00	.16666666+01	.36684044-00	.14223944-00	.10340484+00
.58309521-00	.17149858+01	.36973467-00	.14966829-00	.10892414+00
.56568544-00	.17677669+01	.37235685-00	.15750782-00	.11476496+00
.54772257-00	.18257418+01	.37469777-00	.16581172-00	.12097062+00
.52915028-00	.18898223+01	.37674616-00	.17464501-00	.12759376-00
.50990198-00	.19611612+01	.37848727-00	.18408753-00	.13469928-00
.48989797-00	.20412414+01	.37990293-00	.19423874-00	.14236836-00
.46904159-00	.21320070+01	.38096989-00	.20522477-00	.15070447-00
.44721362-00	.22360678+01	.38165896-00	.21720888-00	.15984215-00
.42426410-00	.23570224+01	.38193248-00	.23040760-00	.16996089-00
.40000002-00	.24999998+01	.38174141-00	.24511668-00	.18130732-00
.37416577-00	.26726121+01	.38102020-00	.26175525-00	.19423339-00
.34641020-00	.28867509+01	.37967841-00	.28094538-00	.20926560-00
.31622782-00	.31622771+01	.37758556-00	.30366862-00	.22724251-00
.28284275-00	.35355334+01	.37454179-00	.33161069-00	.24962061-00
.24494903-00	.40824819+01	.37021127-00	.36805464-00	.27927800-00
.20000008-00	.49999980+01	.36394167-00	.42089189-00	.32327422-00
.14142142-00	.70710642+01	.35405552-00	.51983330-00	.40907448-00

COMPUTER PROGRAM USED TO OBTAIN SIDE-LEAKAGE EFFECTS

```
      DIMENSION PSI(101,51),P(101,51),H(51),EL(51),DELSTR(51),DEN(51),AL
     1OAD(51),AMOM(51)
    1 DELTAT=.01
    2 DELTAZ=SQRT(.02)
    3 DO 7 K=1,51
    4 TEMP=1.-2.*FLOAT(K-1)*DELTAT+.00000001
    5 H(K)=SQRT(TEMP)
    6 PSI(1,K)=H(K)
    7 PSI(101,K)=1.
    8 DO 9 K=1,101
    9 PSI(K,1)=1.
   10 DO 16 N=1,49
   11 DO 12 M=2,100
   12 PSI(M,N+1)=.5*PSI(M,N)*(PSI(M+1,N)+PSI(M-1,N))+.125*
     1(PSI(M+1,N)-PSI(M-1,N))**2 + PSI(M,N)*(1.-PSI(M,N))
   13 SUM=0.
   14 DO 15 M=1,100
   15 SUM=SUM+.5*(PSI(M,N+1)+PSI(M+1,N+1)-2.)*DELTAZ
   16 DELSTR(N+1)=SUM/(H(N+1)-1.)


  101 FORMAT(5E18.8)
      DO 30 K=1,51
   30 EL(K)=1./H(K)
      DO 60 K=1,51
   60 DEN(K)=(DELSTR(K)*(EL(K)-1.))/ H(K)
      DEF=0.
      DO 70 K=1,50
      DEF=DEF+.5*DELTAT*(DEN(K)+DEN(K+1))
   70 ALOAD(K+1)=DEF
      DO 80 K=1,51
   80 DEN(K)=(1.-H(K))*DEN(K)
      DEF=0.
      DO 90 K=1,50
      DEF=DEF+.5*DELTAT*(DEN(K)+DEN(K+1))
   90 AMOM(K+1)=DEF
      DO 200 K = 2,50
      PP = 1.-H(K)
      ALOAD(K) = ALOAD(K)/PP**1.5
  200 AMOM(K) = AMOM(K)/PP**2.5
      DO 100 K=2,50
  100 WRITE(6,101)H(K),EL(K),DELSTR(K),ALOAD(K),AMOM(K)
      STOP
      END
```


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The side leakage effect in a high speed gas lubricated bearing is treated by two approximate methods. The first is a linearized "ph" solution of the asymptotic differential equation. The second is a Karman-Cohlhausen type boundary layer analysis. Both methods are applied to the flat slides and the results are shown to favorably agree with those of an accurate finite difference solution.

The asymptotic approach is recommended for high speed (high Λ) cases rather than the finite difference method because as Λ increases the accuracy of the asymptotic solution increases while the accuracy of the finite difference solution degenerates.

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