DYNAMICS OF SHELLS

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SUMMARY

This report presents a review of work accomplished under the present contract, covering the period 16 March 1964 through 14 July 1967, and is based on the summaries of previous technical reports and published papers. Some recommendations for future research are given.

INTRODUCTION

The present program was initiated with the objective of gaining a better understanding of the vibrational characteristics, both normal modes and natural frequencies, of thin shells. Particularly, we were interested in shell geometrical configurations for which satisfactory analytical and experimental results were not already available. To a greater or lesser degree, this included all shell forms other than the right circular cylinder as was pointed out by a literature survey (Technical Report No. 1)* made early in the program. The primary emphasis then was on the effects of the geometry of the midsurface of the shell. Otherwise, the shells were assumed to be homogeneous, isotropic and of uniform wall thickness. Later in the program, stiffened shells were also included.

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*A list of all technical reports issued on this project is given later in this report.

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Specific shell geometries studied included the truncated cone (Technical Reports Nos. 2, 3, 4), a joined cone-cylinder (Technical Report No. 6) and the complete sphere (Technical Report No. 8). The analytical procedure developed to solve the eigenvalue problem governing the linear vibration of simple shells of revolution may deserve some comment here. The basic procedure is not new, but it does not appear to have been used to any extent by previous workers in shell dynamics. The governing linear differential equations are formulated in matrix form. The unknown modal functions are then expressed in Fourier expansions with undetermined parameters which satisfy all the boundary conditions exactly (the number of boundary conditions is ten for refined shell theory, eight for bending theory, and six for membrane theory). These Fourier expansions are then substituted into the governing matrix equations, and a generalized Galerkin procedure is used to generate a coefficient matrix, the solution of which yields the eigenvalues and eigenvectors of the associated vibration problem. An advantage of this procedure is that, through the use of matrix operations in satisfying the governing differential equations, the procedure of calculating the final matrix equation is systematic and suitable for computer automation. For example, in the solution of the supported conical shell, the matrix was generated by a set of twenty-five algebraic formulas which remain unchanged if the number of terms retained in the series representation of mode functions is increased for higher accuracy. The solution in this case was also applicable for the entire range in cone angle from the flat annular plate to the right circular cylinder.

While the method is generally applicable for any set of homogeneous boundary condtions, difficulty was encountered in the case of flexural vibrations of cones with free edges. For this boundary condition, the vibration modes are nearly inextensional and the frequencies are very low, making the governing matrix nearly singular. In other words, the low frequencies of the flexural modes are poorly defined, at least numerically, by the resulting matrix which contains many large eigenvalues corresponding to other modes. As a result, the numerical difficulties in dealing with a large, nearly singular matrix offset the advantages of the procedure in this case. The procedure cannot be applied to a shell with discontinuities such as the composite shell discussed in Technical Report No. 6 since the mode functions with discontinuities cannot be expanded into sufficiently differentiable series.

In summary, the procedure may be considered as complementary to the well-known Rayleigh-Ritz method or the other numerical schemes such as finite difference, finite element, or direct numerical integration schemes.

Consideration of shells composed of segments of different geometry, such as the cone-cylinder, led to questions concerning the compatibility of the different sets of governing differential equations relating to the different segments; particularly, if there was consistency in the retention or exclusion of terms of higher order in the particular shell equations employed. This resulted in a close look at the derivation of second-approximation theory for elastic shells as presented in Technical Report No. 5. The new approach was based on the concept that the stress and displacement components in three-dimensional elasticity can be expanded into infinite series of Legendre polynomials of a dimensionless thickness variable; these series then converge uniformly over the thickness interval. This is contrasted to the usual power series expansion about the shell midsurface. The results obtained were then compared with other higher order shell theories.

During the final year of the program, attention was turned toward stiffened shells. Specifically, the question of the limitations in using equivalent orthotropic shell theory, or other theories which neglect the effect of the discreteness of the stiffeners, in modifying the detailed mode shapes for the vibrating shell, was considered. In Technical Report No. 7, an "exact" method was developed for calculating the frequencies and mode shapes of ring-stiffened, circular cylindrical shells. The method is "exact" in the sense that the motion of discrete ring and shell segments are accounted for completely within the scope of the linear ring and shell equations employed. The solution in terms of the eigenvalues and eigenvectors is obtained from an appropriate coefficient matrix; thus, there is no a priori assumption as to the mode shapes assumed by the inter-ring shell segments. In Technical Report No. 9, the solution was extended to include eccentricity of the rings, and experimental evidence was presented to confirm the conclusions of the analytical procedures, both for widely spaced stiffeners and for closely spaced stiffeners. In both cases, inter-ring panel deformations were found to be significant in certain ranges of the parameters.

Before concluding this introduction, the writer would like to emphasize his opinion that optimum progress can be obtained on problems such as those considered here or, in fact, in the understanding of any physical phenomena, through the continuous interplay between the analysis and the experiment. The closer the proximity, in both time and space, between these two otherwise separate activities, the more rapid will be the progress toward solution, the greater the chances for success, and the greater the opportunity for discovering and defining new phenomena. Accordingly, the total effort expended in coming to a true understanding of the problem and its solution will be minimized. Those working on the problems summarized herein certainly found this to be the case.

SUMMARY OF TECHNICAL REPORTS AND PUBLISHED PAPERS

Technical Reports

The following is a list of all technical reports issued during the course of this program including the abstract:

Technical Report No. 1

> "A Survey of the Literature on the Vibrations of Thin Shells" by William C. L. Hu, June 30, 1964.
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> > A survey of the literature pertaining to free vibrations of thin elastic shells is presented with particular attention to shells of different geometries. The abundant literature on the vibration of circular cylindrical shells is reviewed only to the extent that it sheds light on the general shell vibration problem. Only limited information concerning other shell configurations exists in

the published literature. Of these, the spherical shell and the truncated conical shell have received by far the most attention; however, even for these shells, complete correlations between the analytic results and experimental data have yet to be made. For these and other shell geometries, there is a dire need to develop new approximate techniques which enable one to solve practical problems with acceptable accuracy.

Technical Report No. 2

"Free Vibrations of Conical Shells" by William C. L. Hu, October 15, 1964, (also, NASA TN D-2666).

A method is presented for calculating the natural frequencies and associated modes of axisymmetric and nonsymmetric vibrations of truncated conical shells. The effects of transverse shear deformation and rotatory inertia are included in the formulation. The determination of the natural frequencies and mode functions is reduced to the calculation of eigenvalues and associated eigenvectors of a coefficient matrix, whose size depends on the number of terms retained in the Fourier expansions of the mode functions. Numerical examples are given to illustrate the calculation procedure. Axisymmetric vibrations of free-free conical shells are investigated based on a five-term truncation of the Fourier series of the mode functions, with special emphasis on the variation of the frequency spectrum with respect to the semivertex angle and the completeness parameter of the conical shell.

Technical Report No. 3

"Nonsymmetric Transverse Vibrations of Truncated Conical Shells" by U. S. Lindholm and W. C. L. Hu, March 31, 1965.

A combined theoretical and experimental study is presented of resonant frequencies and associated mode shapes of truncated conical shells over a wide range of geometrical and modal parameters. The theoretical analysis incorporates the effects of bending and membrane rigidity, and of inertia terms due to transverse motion, as well as meridional and circumferential in-plane motion. The experimental results were obtained for four conical shell models, with semivertex angles 14°, 30°, 45°, and 60°. The correlation of both resonant frequencies and mode shapes is very good.

Technical Report No. 4

"Flexural Vibrations of Conical Shells with Free Edges" by W. C. L. Hu, J. F. Gormley, and U. S. Lindholm, July 15, 1965, (also, NASA CR-384).

> Experimental data are presented for the resonant frequencies and associated mode shapes of truncated conical shells with free edges in transverse vibration. A wide range of the geometrical and modal parameters is covered. A semiempirical frequency equation is developed which can be used to predict the first axial mode resonances with satisfactory accuracy.

Technical Report No. 5

"A Rigorous Derivation of Second-Approximation Theory of Elastic Shells" by William C. L. Hu, November 10, 1965.

> A rigorous derivation and new analytical viewpoint of linear shell theory are presented which aim at resolving some fundamental difficulties in elastic shell theory. The approach is based on the concept that the stress and displacement components in three-dimensional elasticity can be expanded into infinite series of the Legendre polynomials of a dimensionless thickness variable, which converge uniformly and rapidly in the thickness interval. The shell equations are derived through integration of the linear elasticity equations. The orthogonality property of the Legendre polynomials uncouples most higher order terms during the integration process. A minimum number of assumptions are then introduced after the integration and only when necessary. The a priori Kirchoff-Love hypothesis is replaced by a more rigorous accuracy criteria.

Technical Report No. 6

"A Study of Joint Discontinuity in Vibrations of Composite Shells" by William C. L. Hu, May 12, 1966.

> The vibrational characteristics of composite (cone-cylinder) shells are investigated analytically and experimentally. The behavior of the circular joint connecting the conical and cylindrical shell components is discussed in detail. Both the analysis and the experimental results reveal that the derivatives of the mode functions are discontinuous at the joint, where a V-shaped minimum of normal displacement was observed in all modes

being excited. It is evident that dynamic stress concentration is involved even in free vibrations of such shell structures. The jump conditions are formulated for the cone-cylinder joint, and various approaches of solving this problem are discussed.

Technical Report No. 7

"Vibrations of Ring-Stiffened Cylindrical Shells - An 'Exact' Method" by William C. L. Hu and Thein Wah, October 1966.

An analytical method is presented for the determination of the vibrational characteristics of ring-stiffened cylindrical shells. The stiffening rings, which are assumed to be uniform and evenly spaced, are treated as discrete members, and the inter-ring shell motions are fully and exactly accounted for within the linear shell theory used in the analysis. The flexural, extensional, and torsional rigidities of the rings are retained in the analysis as well as all in-plane inertia terms. Numerical comparison is made with the results from existing energy methods and orthotropic model approach. It is found that for high circumferential wave numbers, the inter-ring displacements may become very pronounced.

Technical Report No. 8

"Nonsymmetric Modes and Frequencies of Complete Spherical Shells" by John F. Gormley and William C. L. Hu, April 1967.

A matrix method is applied to calculate the mode shapes and frequencies of nonsymmetric vibrations of complete spherical shells. Based on a thin shell theory incorporating transverse shear and rotatory inertia, the solution is obtained through Fourier expansions of the meridional mode functions, which are also expressible in terms of associated Legendre functions. As has previously been shown within the framework of membrane theory, the frequency spectra of nonsymmetric modes ($n > 1$) are found to degenerate to that of axisymmetric vibration for the refined shell theory. Numerical results of nonsymmetric mode shapes are presented.

Technical Report No. 9

"An Analytical and Experimental Study of Vibrations of Ring-Stiffened Cylindrical Shells" by William C. L. Hu, John F. Gormley, and Ulric S. Lindholm, June 1967.

A combined analytical and experimental study is presented of the nonsymmetric vibrations of ring-stiffened cylindrical shells. In the analysis, the uniform and evenly spaced stiffeners are treated as discrete members. Through the application of a difference-equation method developed in a previous report, the resonant frequencies and axial mode shapes are calculated for specific examples. The previous analysis is extended herein to include eccentricity of the ring stiffeners with respect to the midsurface of the shell. The results of this analysis show that the usual plot of frequency versus circumferential wave number, n, for ring-stiffened cylindrical shells may be divided into three regions: For very low values of n, the frequencies are controlled dominantly by the membrane stiffness of the shell with the rings acting as added masses; for intermediate values of n, the in-plane bending stiffness of the rings is dominant and controls the frequencies; for very high values of n, the rings become effectively rigid, having very little motion, and the frequency is essentially controlled by the panel vibration of a single bay. This third region of panel vibration will not be predicted by the usual orthotropic or "averaged" methods for stiffened shell analyses. Eccentricity of the stiffeners with respect to the midsurface of the shell was found to have only small effect on the resonant frequencies. Experimental evidence is presented to help corroborate the analytical conclusions reached. The experiments are generally in good agreement with the theory presented.

Published Papers

The following published papers or notes have resulted from work completed on this project:

1. "Comments on Vibrational Characteristics of Thin-Wall Conical Frustrum Shells" by W. C. L. Hu, AIAA Journal, 3, 6, p. 1213, June 1965.

2. "Comments on 'Axisymmetric Vibrations of Thin Elastic Shells' by DeSilva and Tersteeg" by W. C. L. Hu, Journal of the Acoustical Society of America, 38, 2, pp. 365-366, August 1965.

3. "A Rigorous Derivation of Second-Approximation Theory of Elastic Shells" by W. C. L. Hu, Proceedings of the Fifth U. S. National Congress of Applied Mechanics, p. 282, June 1966.

4. "Nonsymmetric Transverse Vibrations of Truncated Conical Shells" by U. S. Lindholm and W. C. L. Hu, International Journal of Mechanical Sciences, 8, pp. 561-580, September 1966.

5. "An Experimental Study and Inextensional Analysis of Vibrations of Free-Free Conical Shells" by W. C. L. Hu, J. F. Gormley, and U. S. Lindholm, International Journal of Mechanical Sciences, 9, 3, pp. 123-136, March 1967.

6. "Experimental and Analytical Study of Vibrations of Joined Shells" by W. C. L. Hu and J. P. Raney, AIAA Journal, 5, pp. 976-980, May 1967.

7. "Vibration Analysis of a Stiffened Cylinder Including Inter-Ring Motion" by T. Wah and W. C. L. Hu, Journal of the Acoustical Society of America, January 1968.

DISCUSSION OF CONSTRUCTION OF THIN SHELL MODELS

During the course of our experimental investigation of the vibration of thin shell models, we have investigated and used various methods of fabrication. A brief discussion here of these methods may be useful. Additional attention will be given to a chemical plating method because it has not been covered in our formal technical reports, and some effort was expended in its development.

In some of our early work, we attempted machining circular cylindrical shells from tubular stock material. This method had the advantage of avoiding any joint or seam discontinuity in the shell. The disadvantages are that it is restricted to relatively small diameters, is quite expensive, and it is very difficult to hold an acceptable tolerance on the wall thickness dimension for thin shells.

Subsequently, we have had very good success in forming circular cylindrical and conical shells from flat rolled sheet stock. The initial sheet stock can be obtained with very good tolerance on the thickness dimension. This sheet is then rolled into the desired shape and joined along one seam which is usually a generatrix of the shell. To minimize the discontinuity in dimension or effective stiffness along this seam, a method of forming an arc-welded, butt joint was developed which has proved very successful. Butt welds on material (stainless steel) as thin as 0.005 inch have been made. Also, the size or diameter of the shell is not restricted as in the machined shell. While some special fixturing including heat sinks is required for the welding, the complete shell models are relatively inexpensive. There are some discontinuities at the seam and changes in radius of curvature resulting from this method of fabrication which can be detected in careful mode shape measurements during vibrations, but these are considered to be minimal. With this method, the shell geometries are restricted to those which are elastically formable from flat sheet.

Very high quality cylindrical shells are being manufactured in some laboratories by the process of electroforming the shell on a mandrel which is later removed. This technique was developed to produce shells of high accuracy for buckling stability studies. One limitation is that the amount of material deposited by electroforming is largely determined by the current density on the plated part. Therefore, in order to obtain a shell of uniform thickness, the anode must conform closely in shape to the cathode to insure a uniform electrical potential over the entire surface. A conforming anode can be made easily for simple shapes such as cylinders, but there can be considerable difficulty in manufacturing more complicated shapes. This method requires specialized equipment to the extent that the initial setup is quite expensive.

The above methods are not readily adaptable to shells of compound or nonzero Gaussian curvature, or completely closed shells such as spherical, ellipsoidal or toroidal shells. Some experimental shell models of these shapes can be fabricated from flat sheet stock by spinning, or otherwise forming the material against a die of the desired shape. High dimensional accuracy of the radii can be obtained; however, the wall thickness of the shell cannot be accurately controlled by these forming processes, as the variation in the shell wall thickness will be determined by the local plastic stretching required to obtain the desired shape.

It was proposed during the present program to fabricate and test a complete toroidal shell model. We desired a continuous model of uniform wall thickness without seams. Of the above-mentioned methods of fabrication, only the electroforming method appeared even feasible. The problems with this method involved: (1) removing the mandrel, (2) assuring reasonable tolerances on the wall thickness because of the geometry and required electrode configurations, and (3) cost (an estimate from one laboratory already set up for this process for one toroidal model was $10,000, with no guarantee of the degree of success). For these reasons, we spent some effort to develop an alternative electroless chemical plating method which appears to have certain advantages. This technique deposits a nickel plating on a mandrel by an autocalytic reduction process. It is a standard commercial process for plating but has not as yet been used in shells research. The electroless plating has the following advantages over electrodeposited metal:

(1) The plating deposits with uniform thickness on all exposed parts of the mandrel. No buildups of plating occur on points or edges, i.e., regions of sharp curvature.

(2) No electrical equipment is required.

(3) Conforming electrodes are not required, thus allowing more complex shapes to be formed.

(4) Total equipment required is less complex and less expensive.

Briefly, the plating process involves immersing the part to be plated in a chemical bath of the nickel plating solution*. The bath is maintained at a controlled elevated temperature (approximately 200° F). With many metals, e.g., iron, steel, aluminum or magnesium, the plating process will start as soon as the part is introduced into the bath. For other metals such as copper, plating must be initiated with a small electric current or merely contact with an aluminum wire which is itself being actively plated. Metals such as bismuth, tin and lead tend to poison the plating solution and must be flashed with copper before they can be plated with the electroless nickel. The nickel is in an amorphous state as plated but can be made crystalline and magnetic by heat treatment at 400°C for 1 hour in a neutral atmosphere.

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*We obtained the plating solution initially from Krel Laboratories, Inc., Chicago, Illinois, and subsequently from R. O. Hull and Company, Cleveland, Ohio.

For a completely closed shell, such as the toroid, the mandrel to be plated must, of course, be removed after the shell is completely formed. In this regard, we took the following approach. A solid toroidal mandrel was cast from a low melting point alloy, Cerrotru, a lead-bismuth alloy. Since this mandrel material could not be plated directly, it was flashed with copper to about 0.0005-inch thickness. The nickel plating was then applied to the copper. The mandrel was removed from the nickel shell by melting and extraction through a small hole. This was accomplished by first chilling the entire system to 0° F, and then rapidly immersing it in a bath of mineral oil at 400° F. As the outer regions of the mandrel start to melt, it is rotated rapidly so that the melt flows centrifugally through a small hole in the outer diameter of the toroid. Rapid heating was necessary to avoid the equilibrium thermal expansion of the Cerrotru which could possibly rupture the nickel shell.

The above procedure, both the plating and mandrel extraction, worked satisfactorily on a small toroidal model of about 3 inches outside diameter. When the process was scaled up to plate a larger model of about 9 inches, we were unsuccessful in the plating operation. All of the reasons for the lack of success in plating the larger model are uncertain. One of the primary reasons appears to be the difficulty in obtaining a uniform and complete copper flashing of the larger surface area. Since the Cerrotru is incompatible with the plating solution, this becomes very critical. Other factors, such as temperature control and local overheating, improper pH control of the solution, adequate filtering, etc., also become greater problems with the larger model.

Because of other project commitments and overall objectives, it was decided not to pursue this task beyond this point. It is felt, however, that the electroless plating process is feasible and advantageous for forming shell models of compound curvature or closed shells. Based on our limited experience, we would discourage the use of lead base alloys for the mandrel material and recommend that the nickel plating be done directly on an aluminum mandrel. The aluminum mandrel can be removed by differential chemical etching. Sodium hydroxide, for instance, attacks aluminum vigorously but does not affect the nickel plate.

RECOMMENDATIONS FOR FUTURE WORK IN SHELL DYNAMICS

Based upon our overall work in shell dynamics, the following problems are suggested as being worthwhile for future research. The list is by no means exhaustive but represents what we believe to be some practical problems which should be amenable to existing methods of analysis incorporated with a little ingenuity and some experimental data.

Free Vibrations of Nonrotationally Symmetric Shells

Shell geometries treated in the literature are almost exclusively shells of revolution; i.e., they have one axis of symmetry. The mode function in the plane normal to the axis of symmetry is then separable, having harmonic periodicity. In the absence of this axial symmetry, the mode functions in all the space variables become unknown and may be coupled. One useful series of studies could start with the cylindrical shell of noncircular cross section; an elliptical section is an obvious choice because it degenerates into the circular cylinder as a limiting case. The next step, to a shell of nonconstant section, would be the elliptical cone. Eventually, these studies would lead to shapes of even more complex geometries such as are used or contemplated for lifting body reentry vehicles.

Free Vibrations of Nonisotropic, Laminated, or Sandwich Shells

Shells of these constructions are becoming more evident in practice. Analytical techniques for their analysis with experimental verification are not yet available to the extent warranted by expected extended usage of composite shells.

Transient Response of Shells

The release of tie-downs at liftoff, stage separation, docking, and landing impact are examples of operations that produce transient or impulsive-type loads on space vehicle, shell-type structures. It is of interest to determine the applicability of a normal mode type analysis to determine the transient response of shells. Because of the multiplicity of modes within any frequency band for shells, the practical applicability of the normal mode approach may be in question and should be determined.

Dynamic Response of Prestressed Shells

Shell structures in service are often subject to static or quasi-static loading in addition to transient, oscillatory forces. For instance, a single, cylindrical tank section of a launch vehicle is subject to axial compressive stresses arising from the axial thrust and inertia of the vehicle, as well as circumferential hoop tension from the internal ullage pressure in the tank. When oscillatory forces are superimposed on these constant prestresses, the possibility is always present for dynamic instabilities to occur. These instabilities can lead to parametric resonances and subsequent nonlinear behavior in the unstable regions. The response of shells under multidimensional prestress in combination with oscillatory loading is therefore of interest.

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